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THE ELASTIC STABILITY OF TEE STIFFENERS

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Introduction

Most modern structural design places a premium on weight economy. This is true in the construction of modern surface ships and submarines, and is particularly so in the construction of airplanes and lighter-than-air ships. Weight economy is usually best achieved by the use of reinforced monocoque construction in which the shell plating itself is required to carry its full share of the load. With the proper choice and spacing of stiffeners, the entire structure can be designed to carry average compressive stresses approaching the compressive yield point of the material.

The most fundamental problem in the design of reinforced monocoque construction is the determination of the proper thickness of plating and spacing of stiffeners. This subject has been exhaustively treated [1]* and will not be considered here. The next problem is the proper choice of stiffeners to support the plating adequately. One element of this problem, the required moment of inertia of the stiffeners, has been investigated by Timoshenko [2] and Barbré [3]. However, another equally important element has not yet been considered. Adequate moment of inertia, though necessary, is not sufficient to assure stability of the stiffener; in addition, the stiffener must be properly proportioned. It is easily possible for a stiffener to have adequate moment of inertia and yet be so poorly proportioned that it will be distorted badly or crippled locally before the plating which it supports can develop its full strength. It is the purpose of this paper to investigate the relative proportions of a Tee stiffener, required to assure its stability. The results of this investigation are combined with previously established moment of inertia requirements into a single chart from which a stiffener can be selected that is suitable in all respects.

To fix our ideas, we shall consider the simple Tee stiffener shown in Fig. 1. In general, a Tee stiffener can be considered as a flat plate (the web) with one longitudinal edge supported by the plating to which it is attached and from which it derives some resistance to rotation, and the other edge elastically supported by the flange, the resistance to rotation depending upon the torsional rigidity of the flange. A stiffener attached to plating of about the same buckling strength can be considered as simply supported at the toe, since the plating at its buckling stress can offer little or no resistance to rotation.

The maximum attainable strength of a stiffener is determined by the buckling strength of the web, adequately supported. If the stiffener is properly proportioned, it will develop, under compressive loading, the full buckling strength of the web; if not, it may fail prematurely in any one, or in any combination, of five different ways:

* Numbers in brackets refer to list of references at end.

Primary failure [4]

1. Column buckling of the stiffener in the plane of the web, carrying the attached plating with it.
2. Column buckling of the stiffener normal to the web.
3. Twisting of the stiffener about the line of attachment to the plating.

Secondary failure

4. Premature plate buckling of the web normal to its plane.
5. Local buckling of the flange.

If the stiffener does not fail prematurely in any of these five different ways, the maximum buckling strength of the web will be attained. We shall now investigate the flange proportions required for a stiffener to develop the full buckling strength of the web.

The Stability of the Web of a Tee Stiffener.

Let us ignore for the present the possibility of primary failure of the stiffener either by twisting, or by column buckling in the plane of the web, and investigate the effect of the flange upon the buckling strength of the web. We shall also ignore temporarily the possibility of local buckling of the flange. We will consider the case of a Tee stiffener loaded by uniform compressive forces acting in the middle plane of the web parallel to the longitudinal axis as indicated in Fig. 1.

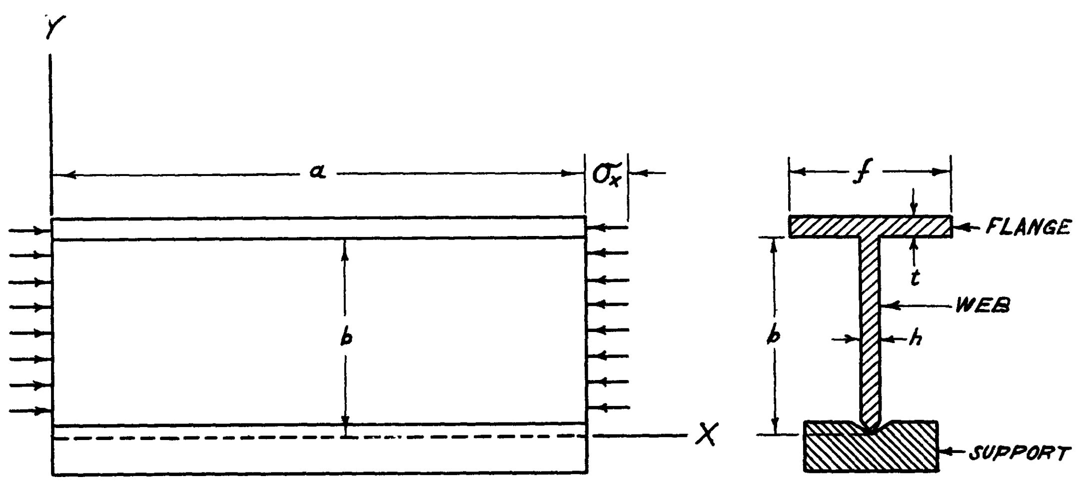


Fig. 1.

The differential equation of the deflection surface of the web plate, the compressive force being considered positive for compression, becomes [1, p 337] [5]

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = -\frac{\sigma_x h}{D}\frac{\partial^2 w}{\partial x^2} \tag{1}$$

where, following the notation in [1], w denotes the deflection of the web plate normal to the XY plane, σ_x the uniform compressive stress acting on both web and flange, and $D = E h^3/12(1-\nu^2)$, the flexural rigidity.

The ends of the plate $x = 0$, $x = a$, and the edge $y = 0$, are assumed to be simply supported, i.e., free from bending moment and normal displacement. Expressed analytically [1, p 298]

$$\left.\begin{aligned} &w=0\,;\ \frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2} = 0 \qquad \text{for } x=0,\ x=a \\ &w=0\,;\ \frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2} = 0 \qquad \text{for } y=0 \end{aligned}\right\} \tag{2}$$

The solution of Eq. (1) satisfying the boundary conditions Eq. (2) can easily be shown to be [1, p 337, 338]

$$w = (A \sinh \alpha y + B \sin \beta y) \sin kx \tag{3}$$

where

$$\alpha = k\sqrt{\mu+1}\ ;\ \beta = k\sqrt{\mu-1}\ ;\ k = m\pi/a\ ;\ \mu = \sqrt{\frac{\sigma_x h}{Dk^2}}\ ;$$

m is the number of sinusoidal half-waves into which the plate buckles; A and B are constants of integration.

The edge of the web plate, $y = b$, is assumed to be elastically supported by the flange, i.e., the normal displacement and bending moment are proportional to the load applied to the flange by the web. These boundary conditions lead to the equations

$$D\left[\frac{\partial^3 w}{\partial y^3} + (2-\nu)\frac{\partial^3 w}{\partial x^2 \partial y}\right] - EI\frac{\partial^4 w}{\partial x^4} - A\sigma_x\frac{\partial^2 w}{\partial x^2} = 0 \tag{4}$$

$$-D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) = C\frac{\partial^3 w}{\partial x^2 \partial y} \tag{5}$$

where EI, C, and A are respectively the flexural rigidity, torsional rigidity, and area of the flange [1, p 346; p 343] [6] [7].

The introduction of the boundary conditions Eqs. (4) and (5) into Eq. (3) yields the expressions

$$(\alpha t \cosh \alpha b - k^4 \varepsilon \sinh \alpha b)A - (\beta s \cos \beta b + k^4 \varepsilon \sin \beta b)B = 0 \qquad (6)$$

$$(r\alpha \cosh \alpha b + s \sinh \alpha b)A + (r\beta \cos \beta b - t \sin \beta b)B = 0 \qquad (7)$$

wherein

$$s = \alpha^2 - \nu k^2; \quad t = \beta^2 + \nu k^2; \quad \varepsilon = \frac{EI}{D} - \frac{A\sigma_x}{Dk^2}; \quad r = \frac{Ck^2}{D}$$

Eqs. (6) and (7) are simultaneous, homogeneous, linear equations in A and B from which the values of the constants A and B to be used in Eq. (3) can be determined. The buckled form of equilibrium of the plate is possible only when A and B have values different from zero, i.e., when the determinant of the two equations vanishes. Then

$$\begin{vmatrix} \alpha t \cosh \alpha b - k^4 \varepsilon \sinh \alpha b, & -(\beta s \cos \beta b + k^4 \varepsilon \sin \beta b) \\ r\alpha \cosh \alpha b + s \sinh \alpha b, & rb \cos \beta b - t \sin \beta b \end{vmatrix} = 0 \qquad (8)$$

The determinant when expanded becomes

$$\left[\sinh \alpha b \sin \beta b\right]\left[\beta(1+\mu-\nu)^2 \cot \beta b - \alpha(1-\mu-\nu)^2 \coth \alpha b + 2\mu k^2 \varepsilon \right.$$
$$\left. + 2r\mu\sqrt{\mu^2-1} \coth \alpha b \cot \beta b + \varepsilon r(\alpha \coth \alpha b - \beta \cot \beta b)\right] = 0 \qquad (9)$$

Eq. (9) gives the relations between the variables involved when the web plate of a Tee stiffener, simply supported at the ends and one longitudinal edge, and elastically supported at the other edge by a flange, is in a condition of unstable equilibrium. Since α, β, and μ, contain σ_x, Eq. (9) can be used to calculate the critical value of σ_x for a Tee stiffener with any given dimensions and elastic properties. It should be noted that the quantity m involved in k must be determined in any given case by the usual instability condition that it be the integral value for which σ_x is a minimum. This equation is applicable to a Tee stiffener, bulb plate, and other sections with a symmetrical flange, when welded to plating of about the same buckling strength as the stiffener, since the plating at its buckling stress can offer no more than simple support to the stiffener.

The appearance of σ_x in three variables makes calculation very tedious and it is advantageous to use the following notation introduced by Miles [8].

$$\phi = kb = m\pi\frac{b}{a}; \quad \psi = kb\mu = b\sqrt{\frac{\sigma_x h}{D}}; \quad \theta = \frac{\varepsilon}{b}$$

whence

$$\theta = \frac{EI}{bD} - \frac{A\psi^2}{bh\phi^2}; \quad r = \frac{C\phi^2}{Db^2}; \quad \alpha = \frac{\sqrt{\phi}}{b}\sqrt{\psi+\phi}; \quad \beta = \frac{\sqrt{\phi}}{b}\sqrt{\psi-\phi}$$

Eq. (9) is satisfied by setting either factor equal to zero. The equation obtained by setting the first factor equal to zero will be discussed later. The equation obtained by setting the second factor equal to zero, using the preceding notation, becomes

$$\begin{aligned}&\sqrt{\psi-\phi}\,[\psi+(1-\nu)\phi]^2 \cot\sqrt{\phi\psi-\phi^2}\\ &-\sqrt{\psi+\phi}\,[\psi-(1-\nu)\phi]^2 \coth\sqrt{\phi\psi+\phi^2} + 2\phi^{5/2}\psi\theta\\ &+2\phi^{3/2}\psi\frac{C}{Db}\sqrt{\psi^2-\phi^2}\coth\sqrt{\phi\psi+\phi^2}\cot\sqrt{\phi\psi-\phi^2}\\ &+\phi^4\theta\frac{C}{Db}\left(\sqrt{\psi+\phi}\coth\sqrt{\phi\psi+\phi^2} - \sqrt{\psi-\phi}\cot\sqrt{\phi\psi-\phi^2}\right) = 0 \qquad (10)\end{aligned}$$

Eq. (10) is expressed in terms of only two variables, a "stress factor" ψ and an "aspect factor" ϕ. The minimum value of ψ determined from Eq. (10) with ϕ restricted to integral multiples of π b/a, i.e., m restricted to integral values, yields the critical or buckling stress of the web of a Tee stiffener for any given dimensions and physical properties.

Eq. (9) can be obtained from Chwalla's general solution of the problem of the compressed rectangular plate, stiffened at the longitudinal edges [9]. However, the general solution offers an extremely cumbersome and indirect method of obtaining this equation.

Special Cases.

It is of interest to pause and examine Eqs. (9) and (10) for certain limiting values of flexural rigidity, torsional rigidity, and area, EI, C, and A, of the flange of the stiffener.

I. The Bryan Case — Simple Support. The flange has zero torsional rigidity and infinite flexural rigidity, C = r = 0, EI = A = ε = θ = ∞, i.e., the flange offers simple support to the plate. This condition is satisfied when the first factor of Eq. (9) is equated to zero, i.e., when

$$\beta b = n\pi \quad \text{or} \quad \psi = \frac{n^2\pi^2}{\phi} + \phi \qquad (11)$$

The minimum value of σ_x for integral values of m and n is the critical buckling pressure. This minimum value can be obtained only when n = 1, i.e., the web may buckle in several (m) half-waves in the direction of loading but only in one (n = 1) half-wave in the direction perpendicular to it.

The critical value of the buckling stress, therefore, becomes

$$\sigma_{cr} = \frac{\pi^2 D}{b^2 h}\left(m\frac{b}{a} + \frac{1}{m}\frac{a}{b}\right)^2 = k\frac{\pi^2 D}{b^2 h} = k\frac{\pi^2 E}{12(1-\nu^2)\left(\frac{b}{h}\right)^2} \qquad (12)$$

which is Bryan's formula [10]. K = 4 for long plates or for plates with integral a/b ratios. For steel with E = 29 x 10^6 lb per sq in and ν = 0.3,

$$\sigma_{cr} = \frac{105 \times 10^6}{(b/h)^2} \qquad (13)$$

II. The Timoshenko Case — Free Edge. The flange is removed, C = r = EI = A = ε = θ = 0. Eq. (9)(second factor) then reduces to

$$\beta(1+\mu-\nu)^2 \cot\beta b - \alpha(1-\mu-\nu)^2 \coth\alpha b = 0 \qquad (14)$$

which corresponds to Timoshenko's equation for the buckling strength of a flat plate simply supported at one longitudinal edge and free at the other [11].

III. The Miles Case. The flange has zero torsional rigidity and finite flexural rigidity, C = r = 0, 0 < (EI, A, ε , θ) < ∞ . Eq. (10) then reduces to

$$\sqrt{\psi-\phi}\left[\psi+(1-\nu)\phi\right]^2 \cot\sqrt{\phi\psi+\phi^2}$$
$$-\sqrt{\psi+\phi}\left[\psi-(1-\nu)\phi\right]^2 \coth\sqrt{\phi\psi+\phi^2} + 2\phi^{5/2}\psi\theta = 0 \qquad (15)$$

Eq. (15) corresponds to the expression developed by Miles for the polar symmetrical buckling of a plate elastically supported at the longitudinal edges with negligible resistance to torsion [8]. This equation will be discussed later.

IV. The flange has infinite flexural rigidity and finite torsional rigidity, EI = A = ε = θ = ∞ , 0 < (C, r) < ∞ . Eq. (10) then reduces to

$$2\psi + \phi^{3/2}\frac{C}{Db}\left(\sqrt{\psi+\phi}\ \coth\sqrt{\phi\psi+\phi^2} - \sqrt{\psi-\phi}\ \cot\sqrt{\phi\psi-\phi^2}\right) = 0 \qquad (16)$$

V. The flange has infinite flexural and torsional rigidity, $EI = A = \varepsilon = \theta = C = r = \infty$, i.e., the web plate is built in or clamped at the flange edge. Eqs. (10) and (16) then reduce to

$$\cot \sqrt{\phi\psi - \phi^2} = \frac{\sqrt{\psi + \phi}}{\sqrt{\psi - \phi}} \coth \sqrt{\phi\psi + \phi^2} \qquad (17)$$

Effect of Flexural Rigidity of Flange upon the Buckling Strength of the Web.

We shall now investigate the influences of the flexural rigidity of the flange upon the buckling strength of the web, holding the torsional rigidity, C, at some constant value. The constant torsional rigidity factors chosen are C/Db = 0 and C/Db = 2 since they represent the upper and lower limits usually encountered in practice. In Fig. 2 are shown two sets of curves plotted from Eq. (10) using θ as a parameter, one set for C/Db = 0, the other for C/Db = 2. The similarity of these two sets of curves is at once apparent and general conclusions can be drawn that will be valid for both and also for intermediate values of C/Db.

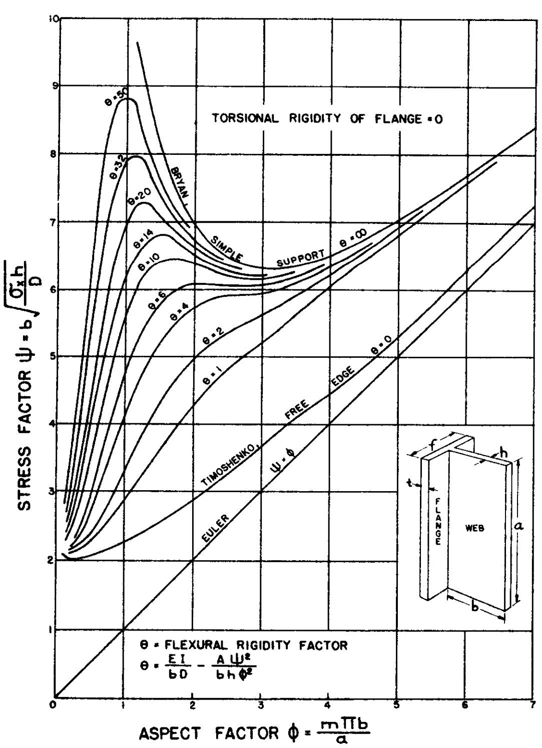


Fig. 2a

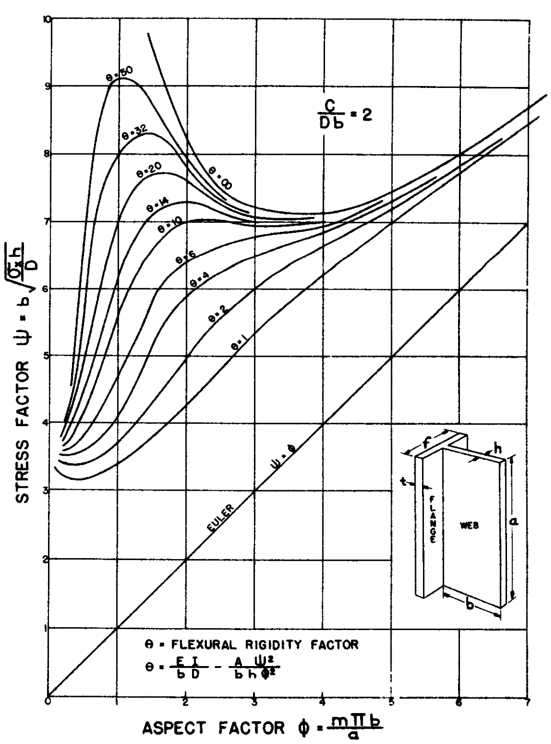


Fig. 2b

Charts Showing Effect of Flange Dimensions on Buckling Strength of Web of Tee Stiffener.

The case for $C/Db = 0$, represented by Eq. (15), was treated by Miles [8] and the curves shown in Fig. 2a are identical with those developed by him except for a change of scale resulting from a slightly changed notation. In this figure the limiting curve $\theta = \infty$ gives the buckling strength of the web when the flange has infinite flexural rigidity, i.e., the web plate is simply supported at all four edges. This curve is plotted from Eq. (11) and represents the Bryan critical strength. The curve $\theta = 0$ represents Timoshenko's equation for a plate with one edge free, Eq. (14). The curves for intermediate values of θ indicate the increase in web buckling strength to be expected as the flexural rigidity of the flange is increased. The curve $\psi = \phi$ was not obtained from the foregoing equations but represents Euler's column curve for a plate. It indicates the strength of the stiffener when both the flange and attached plating are removed.

The curves of Fig. 2 show that the value of ϕ is quite restricted for moderately large values of θ, since for a given a/b, the integer m, and hence ϕ, must have such values as to make ψ a minimum. It will be observed in Fig. 2a that all the curves for θ greater than about 10 have a distinct minimum at $\phi = \pi$. In Fig. 2b the minimizing ϕ is slightly greater. In both cases the ratio of the critical (minimum) value of ψ to the minimizing value of ϕ can be taken as 2. It may be noted that the restriction $\phi = \pi$ implies $m = a/b$, i.e., the plate buckles in square bulges.

Flange Proportions Required to Prevent Web Buckling.

The fact that the critical value of ψ occurs in the region $\phi = \pi$, and that the ratio of the critical value of ψ to the minimizing ϕ can be taken as 2 for all Tee stiffeners used in practice, makes it possible to obtain a simple analytical expression for the flange dimensions required to develop the full buckling strength of the web. Although theoretically this full buckling strength is obtained only for flanges of infinite flexural rigidity, $\theta = \infty$, practically, as can be seen from Fig. 2, the same buckling strength can be obtained for finite values of θ. In fact, it will be observed that in the region $\phi = \pi$ the value of ψ is practically constant for all values of $\theta \geqq 20$. Although this statement could be made for a smaller value of θ, since the final result will not depend greatly upon which value is used, the safer value $\theta = 20$ was selected arbitrarily as the minimum value that would develop the full buckling strength of the web. Hence, from the definition of θ we can write

$$\frac{2.73\, I}{b\ h^3} - \frac{A}{bh} \geqq 5 \qquad (18)$$

as the required condition that the flange will adequately support the web plate until its maximum buckling stress is developed. In addition it follows from the derivat that Eq. (18) represents also the condition that the flange, and hence the stiffener as a whole, will not buckle in a plane normal to the web. Eq. (18) is valid for all types of symmetrical flange stiffeners attached to plating at the toe, i.e., for Tee stiffeners, bulb-plates, etc. since it depends only upon the flexural rigidity and area of the flange and not upon its shape. Moreover, it gives results which are on the side of safety for I-sections, H-sections, and other types of symmetrical sections. Consequently, a symmetrical flange stiffener of any type will develop the full buckling strength of the web provided Eq. (18) is satisfied, assuming for the time being that the torsional stiffness and radius of gyration are sufficient to prevent twisting failure of the stiffener as a whole, or column failure in the plane of the web.

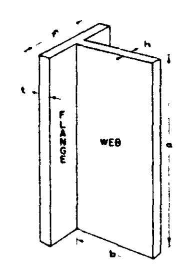
For the particular case of a Tee stiffener with the notation shown in Fig. 1, Eq. (18) reduces to, (expressing I and A in terms of the dimensions)

$$\frac{f^3}{h^3}\frac{t}{h} - 4.4\frac{f}{h}\frac{t}{h} - 22\frac{b}{h} = 0 \qquad (19)$$

Eq. (19) can be readily solved for f/h for any given value of b/h when t/h is given a fixed ratio. Moreover, the value of f/h is not especially sensitive to change of t/h within the limits commonly encountered in practice. Several values of flange width required to develop the full Bryan critical strength of the web of a Tee stiffener are given in Table I.

TABLE I

Flange Width Required to Develop Maximum Buckling Stress in Web of Tee Stiffener Constructed of Steel.

Web, b/h	Required Flange Eq. (19) f/h		Web Buckling Stress		Stiffener Notation
	t=h	t=1.2h	Bryan Eq. (13)	Increased as per Fig. 3	
10	6.3	5.9	yield	yield	
20	7.8	7.4	"	"	
30	8.9	8.4	"	"	
40	9.7	9.2	65,600	"	
50	10.5	9.9	42,000	47,000	
60	11.1	10.5	29,200	33,000	
80	12.2	11.5	16,400	18,000	
100	13.1	12.4	10,500	11,500	

A Tee stiffener with the flange proportions indicated in Table I will be insured against collapse by either of two of the types of failure listed in the introduction: type 4, premature plate buckling of the web, and type 2, column buckling normal to the web. Additional flange requirements, which in some cases increase the values of f/h above those given in Table I, must be imposed to insure against the other types of failure. Before considering these requirements, it is advantageous to make a slight digression.

<u>Increase in Buckling Strength of the Web due to Torsional Rigidity of Flange.</u>

From a comparison of the two charts, Fig. 2a and 2b, it is seen that the buckling strength of the web is considerably greater for the case C/Db = 2 than for C/Db = 0, i.e., the greater torsional rigidity of the flange increases the maximum buckling strength of the web. It is desirable now to investigate this relation in more detail. To do this, it is not necessary to construct charts similar to Fig. 2 for other values of C/Db, since we are now interested primarily in stiffeners for which $\theta \geqq 20$. As the value of ψ is not appreciably increased by further increase of θ, we may assume θ as infinite in Eq. (10), and use Eq. (16) for calculating the buckling strength of the web.

A series of values of C/Db have been selected and the corresponding values of ψ calculated from Eq. (16). The percentage increase of web buckling stress over the Bryan critical stress has been calculated also. Since the web buckling stress is proportional to ψ^2, this increase over the Bryan critical stress is in the ratio $\psi^2/4\pi^2$. These values are listed in Table II and shown graphically in Fig. 3.

TABLE II

Buckling Stress Factors for Web of Tee Stiffener for Several Values of Torsional Rigidity.

C/Db	0.1	0.5	1	2	5	∞
ψ	6.42	6.75	6.94	7.11	7.22	7.30
$\psi^2/4\pi^2$	1.04	1.15	1.22	1.28	1.32	1.35

It is seen that due to the torsional rigidity of the flange the web can develop critical stresses considerably in excess of the Bryan critical stress even for flanges of moderate dimensions. This increase varies from 10 to 30 per cent for flanges usually encountered in practice. Consequently, the flange proportions listed in Table I will insure against column failure of the flange in a plane normal to the web until the web stress has exceeded the Bryan critical stress by the amounts shown in Fig. 3.

The Twisting Stability of Tee Stiffeners.

In the preceding investigation of the buckling strength of the web of a Tee stiffener, the possibility of twisting failure of the stiffener as a unit was ignored. The twisting stability of a Tee stiffener will now be briefly discussed. Wagner has investigated the critical twisting strength of open-section columns which are free to rotate about the shear center or center of twist [12]. This work has been extended by Wagner and Pretschner [13], Lundquist and Fligg [4] and Kappus [14] to include the case of open-section columns which are free to twist about an arbitrary axis only, such as the line of attachment of plating, and which have different degrees of end fixation.

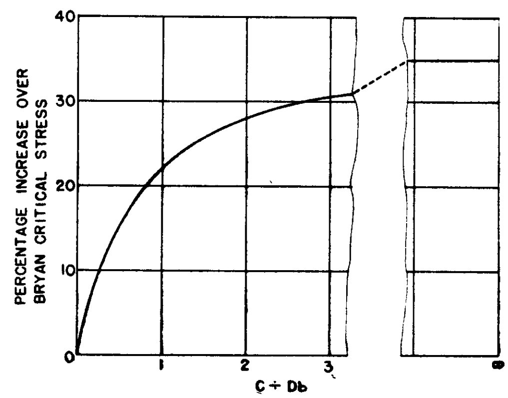


Fig. 3. Percentage increase in buckling stress of web of Tee stiffener due to torsional rigidity of flange.

The modified Wagner formula for the critical twisting stress is

$$\sigma_T = \frac{1}{I_p}\left(C_s + \frac{\pi^2 E}{a^2} C_{BT}\right) \tag{20}$$

where σ_T is the average compressive stress causing twisting instability, C_s the torsional rigidity of the section, $\underline{a}$ the effective length, I_p the polar moment of inertia about the arbitrary axis of twist, and C_{BT}, called the torsion bending constant, is defined as

$$C_{BT} = C_B + C_T = \int w^2 t \, ds + \frac{t^3}{12}\int s^2 \, ds \tag{21}$$

where w is the normal displacement of the end cross-section per unit twist and s is the distance taken along the cross-section.

For a Tee section which can rotate only about its toe due to attachment to plating, Eq. (21) becomes, since w = b s, [13] (see Fig. 1)

$$\begin{aligned} C_{BT} &= \int_{-f/2}^{f/2} b^2 s^2 t \, ds + \frac{h^3}{12}\int_0^b s^2 \, ds + \frac{t^3}{12}\int_{-f/2}^{f/2} s^2 \, ds \\ &= \frac{f^3 b^2 t}{12} + \frac{b^3 h^3}{36} + \frac{f^3 t^3}{144} \end{aligned} \tag{22}$$

The third term on the right side can be neglected and the second term is negligible except for large values of b/h.

For the Tee section shown in Fig. 1 Eq. (20) then becomes

$$\sigma_T = \frac{3E\left[\frac{1}{7.8}\left(\frac{b}{h}+\frac{f}{h}\frac{t^3}{h^3}\right)+\frac{\pi^2 b^2}{12\,a^2}\left(\frac{f^3 t}{h^3 h}+\frac{1}{3}\frac{b}{h}\right)\right]}{\frac{b^3}{h^3}+3\frac{b^2}{h^2}\frac{f}{h}\frac{t}{h}+\frac{f^3}{h^3}\frac{t}{h}\frac{1}{4}}$$

$$= \frac{3E}{k}\left(B+C\right) \qquad (23)$$

Thus, the critical twisting stress σ_T for a Tee stiffener simply supported at the toe is expressed in terms of the non-dimensional ratios b/h, f/h, t/h, and a/b.

If the stiffener is not pin-ended, as implied in the foregoing equations, the effective column length is no longer the total length and Eq. (23) becomes

$$\sigma_T = \frac{3E}{k}\left(B+\gamma C\right) \qquad (24)$$

where the value of the coefficient γ depends upon the degree of end fixation. For pin-ends, γ = 1; for built-in ends, γ = 4; for intermediate degrees of end fixation, the value of γ lies between 1 and 4. For design purposes the value γ = 2 has been chosen for the following reasons:

The use of unity as a coefficient of C is on the side of safety but far too conservative. The shell plating to which the stiffener is attached, even though buckled, offers considerable resistance to twisting of the stiffener, since the twist length of the stiffener may be many times the bulge length of the plate. Moreover, the end cross sections of the stiffener are invariably restrained somewhat from warping. Consequently, we are justified in assuming a greater value than unity for the coefficient of C. Very few experimental data are available in the literature, but these data [13] and general experience with structural assemblies under compressive loading indicate that a coefficient of 2 applied to the torsion-bending constant represents a conservative figure for design purposes.

The necessary and sufficient condition that the flange of a Tee stiffener have dimensions sufficient to insure that the stiffener will develop its full web strength before it fails by twisting instability is

$$\sigma_T \geqq \sigma_{cr} \qquad (25)$$

The value of σ_T is obtained from Eq. (24) and the value of σ_{cr} from Eq. (12) increased by the amount shown in Fig. 3.

Determination of Required Flange Dimensions.

We are now in a position to design a suitable flange for a Tee stiffener of given web dimensions. The flange must be so proportioned that it will insure against all of the types of failure listed in the introduction, until the web has reached its full buckling strength. These required conditions will here be considered one by one.

To insure against premature buckling of the web, (type 4 failure), Eq. (19) must be satisfied. Eq. (19) also insures against column buckling in a plane normal to the web (type 2 failure).

To insure against failure by twisting instability (type 3 failure), Eq. (25) must be satisfied.

To insure against local buckling of the flange (type 5 failure), it is necessary only, as can be shown by Eq. (14), that the unsupported width of flange shall not exceed 15 thicknesses, i.e., that f/h shall not exceed 30.

The restrictions imposed by all these conditions automatically insure against buckling of the stiffener in the plane of the web (type 1 failure), as can be readily verified by calculations.

It is possible to construct a single chart which will combine all of the foregoing requirements, so that suitable flange proportions can be read directly from curves. Such a chart is represented in Fig. 4. Since, as seen in Eqs. (23) and (25), we have to deal with four non-dimensional variables as well as with the physical properties of the material, certain of these variables must be held constant and the resulting curves are applicable only to those restricted conditions. In Fig. 4 the ratio t/h, the elastic modulus and the yield point are held constant. The resulting curves are applicable to Tee stiffeners of uniform thickness throughout, t = h, and constructed of medium steel with an elastic modulus of 29×10^6 lb per sq in. and a yield point of 40,000 lb per sq in. or less.

To construct the chart, we first plot a series of constant f/h curves representing Eq. (25). Such curves will prescribe stiffener dimensions which insure against twisting instability.

This basic series of curves is then cut off at the bottom at such points as to satisfy Eq. (19), thus insuring against premature buckling of the web, and against column buckling in a plane normal to the web.

The maximum value of f/h included in this chart is 30, thus insuring against local buckling of the flange.

Calculations show that the slenderness ratios of all stiffeners permitted by the chart are small enough to insure against column buckling of the stiffeners in the plane of the web.

The resulting flange proportions indicated by the chart, Fig. 4, are thus adequate to assure the stability of the stiffeners in all respects until the web has reached its full buckling strength.

Applications to Design of Stiffeners.

An examination of the chart, Fig. 4, reveals several important facts. First,

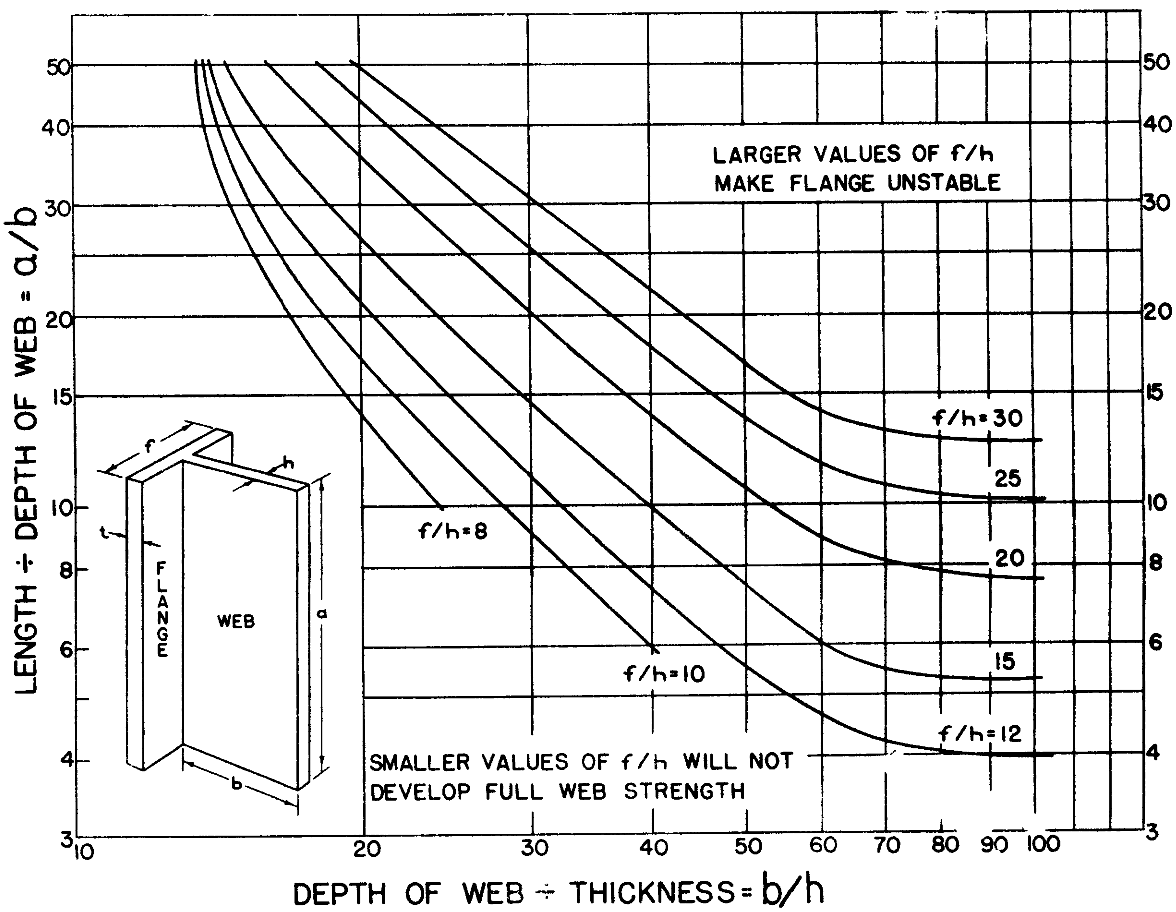


Fig. 4. Flange proportions required to develop maximum buckling strength of web of a Tee-stiffener when constructed of steel with yield point of 40,000 lbs per sq in. or less.

the range of f/h that will develop the full buckling strength of the web is rather limited, varying from 8 to 30. Values of f/h below 8 or 10 should not be used since they do not meet the requirements of Eq. (19). Values of f/h above 30 should not be used because the flange itself will buckle.

Second, stiffeners with b/h greater than 50 or 60 cannot be used to advantage. If deeper webs are required, they should be reinforced by an intermediate longitudinal stiffener.

Although f/h values less than 8 will not develop the full buckling strength of the web of a stiffener, it is to be observed from Fig. 2a that even very small edge reinforcement (say θ = 1) is vastly superior to a free edge, θ = 0. This explains why the use of such simple stiffening as a bead of weld laid down along the free edge of a flat bar stiffener will often greatly increase the stability of the member. Bulb plates can often be used to advantage.

Previous symmetry requirements exclude stiffeners such as Zees and angles for which one of the principal axes of inertia is inclined to the plane of the plate. These sections require special treatment.

Extension of Results to the Plastic Range.

Although the foregoing derivation is based upon purely elastic behavior of the material, and the design chart Fig. 4 is for steel with a fairly sharp yield, the basic equations can be applied with safety to materials having a non-linear stress-strain curve, provided the elastic modulus E is replaced by the correct reduced modulus $\bar{E}$ [1, p 386]. Consequently design curves similar to Fig. 4 can be derived for aluminum and other non-ferrous materials provided compressive stress-strain curves of the material are available. Such curves, although not strictly correct, will err on the side of safety.

Conclusions.

The flange proportions required to insure that a medium steel Tee stiffener will be stable until the web has developed its full buckling strength can be selected directly from the chart, Fig. 4. The chart prescribes stiffener proportions which insure against premature failure in any of the ways listed in the introduction of this paper.

The maximum buckling stress of the web of ordinary Tee stiffeners is from 10 to 30 per cent above the Bryan critical stress, due to the effect of the torsional rigidity of the flange.

The flange proportions indicated by Fig. 4 are on the side of safety for all types of medium steel stiffeners such as I-sections, H-sections, etc. for which a principal axis of inertia is perpendicular to the plane of the plate.

The equations developed can be applied to materials with non-linear stress-strain characteristics provided proper values of reduced modulus are used.

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