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A COMPARISON OF THREE METHODS OF ROLLING SHIP MODELS

BY J. G. THEWS AND L. LANDWEBER

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SYMBOLS

D	Depth of water in feet
g	Acceleration of gravity
GM	Metacentric height
h	Half draft of ship
H	Wave height, crest to trough
L	Wave length, crest to crest
m	Mass of each pair of weights of rolling gear
N	Number of rolling cycles
s	Eccentricity of the c.g. of each pair of weights of the rolling gear
T	Period of rolling cycle
W	Displacement of ship or model
x	Horizontal distance in the direction of motion of a wave
y	Vertical height of a point on a wave profile
z	Distance between weights along the shafts of the mechanical rolling gear.
γ	Maximum wave slope
θ	Amplitude of roll (angle of heel in rolling) in degrees
μ	$d(\log_e \theta)/dN$
ϕ	arctan γ
ω	Angular speed of shafts of rolling gear.

A COMPARISON OF THREE METHODS OF ROLLING SHIP MODELS

Old and important as it is, the practical side of the subject of the rolling of ships is essentially still on a hit and miss basis. On the other hand, the theory of rolling as developed by Wm. Froude in 1860 is a good approximation and one perhaps as simple and sound as can be developed for a subject of this nature.

To apply the theory to a practical case, data on the roll-damping qualities of the ship in question are required. These data can be obtained experimentally, the most practicable way being to test suitably sized models of the ship in a model basin. Assuming this to be done, it is then possible to predict the rolling behavior of the ship in uniform, synchronous waves.

By themselves, however, these predictions are fairly meaningless. They will not predict what the form will do under sea conditions. Indeed, each class or type of ship has its own requirements in regard to rolling behavior, and many factors go to determine what the behavior in any given case will be and what it ought to be.

At present there is no criterion or standard of performance as to what the rolling qualities of any classification of ships should be. Phrases such as the following are commonly heard: A good sea boat; will ride out any storm; a notoriously bad roller; a comfortable ship; a good gun platform; too stiff. The beginning of a standard of comparison could be made by obtaining information like this, together with actual quantitative data on the roll - damping qualities of the subject ships, and classifying this according to the type and size of ship and the sea routes on which each is normally used. Models of other ships may then be tested in rolling, and by comparing the results with those of similar ships in the standard and noting the seas in which they are to operate, it should be possible to predict whether or not the tested forms will meet practical conditions satisfactorily, or whether or not some changed feature is an improvement from the point of view of rolling behavior. These predictions would then have considerable practical significance.

A small collection of roll damping information from some actual ships has been made. This has been presented in a table in U.S. E.M.B. Report No. 430. These data are not equally reliable in all cases. Also, very little information is extant that indicates the degree to which each case is considered satisfactory or desirable from the aspect of rolling behavior. Perhaps rather than being the start towards some standard, this small collection serves better to emphasize the scarcity of actual ship rolling information that exists or is available.

This investigation concerns itself with the first part of the problem of predicting the rolling of ships for given standard conditions. Various methods are available for testing a model in rolling, and each method has its virtues. The purpose here is to reduce three methods to a common base of comparison and to note how consistent they are experimentally.

The methods of testing in rolling are:

(a) By means of uniform, synchronous waves.

(b) From the model's declining angle curve.

(c) By means of a mechanical rolling gear mounted in the model.

Apparatus and Procedure

Experiments using the various methods of studying rolling are performed in a small model basin, 80 ft. long by 7 ft. wide by 4.5 ft. deep. A convenient length of a model to be tested is six feet. It can be placed transversely to the basin's length with the bow and stern each about six inches from the basin's wall. In this position, the model is placed in a light, elastic rigging, the purpose of which is to restrain drift and maintain the proper orientation of the model in the basin, and to support a quadrant carrying an angular scale divided in degrees. The angle of heel of the model is obtained directly by reading the travel over the angular scale of a pointer secured to the model.

The metacentric height (GM) is computed from the angle of heel due to a known change in position of a small weight in the model. The period of a complete roll (T) is determined for each condition by measuring with a stop watch the time for ten complete rolls.

The analysis of the data for each method has for its aim the derivation of a curve of predicted rolling in synchronous waves; i.e., a curve relating θ with γ, where θ is the amplitude of roll of a ship in uniform synchronous waves of maximum wave slope γ. For various reasons the relation between θ and γ is presented as a plot of θ against the abscissa $T\sqrt{GM}\cdot\sqrt{\gamma}$, the square root being introduced to straighten out the curves of predicted rolling which, in general, seem to approximate a parabolic law; and the coefficient $T\sqrt{GM}$ being introduced to facilitate observation of the effect of T and GM on the predicted rolling. In computing this abscissa, the values of T and GM for small amplitudes are used.

In loading the model, care is taken that the general distribution of weights around the longitudinal axis is preserved through each of the three methods. The need for this precaution is indicated in Fig. 7 where the effect on rolling of varying the longitudinal distribution of weights is shown.

Further details concerning the apparatus and procedure are described in the separate discussions of the different methods of studying rolling.

A. The Wave-Maker

Waves are generated by the vertical harmonic motion of a plunger extending across the width of the basin. The plunger is driven by a 1 HP motor through an eccentric, imparting a simple-harmonic motion to the plunger. The period of the waves generated is controlled through a variable resistance in the field circuit of the motor while the wave amplitude is varied by changing the eccentricity.

To prevent the reflection of waves from the wall at the other end of the basin it was necessary to provide a beach extending from the water's surface to the bottom of the tank, to lay a dozen layers of iron mesh across this beach, and to move the beach forward to leave three feet of dead water behind it.

Further difficulty was experienced in that the primary wave seemed to have irregular secondary waves superimposed upon it. Attributing this to a self-reflection of the generated wave at the face of the wave-maker, some success in eliminating these secondary waves was attained by laying a series of hinged boards across the basin near the plunger.

Considerable interaction between the model and the wave-maker was observed. For, while the waves were of uniform height when the model was out of the tank, when the model was in position in the basin and the waves caused it to roll, the wave amplitudes kept changing slowly as did the noise of the wave-maker, just as energy surges to and fro between the elements of a double pendulum.

In computing the maximum wave slope γ, it is a close approximation for waves of small height compared to their length, to assume the wave profile to be a sine curve.

Take the instantaneous equation of a wave profile to be

$$y = \frac{H}{2} \sin 2\pi x/L \quad \ldots \ldots \ldots \ldots \ldots \quad (1)$$

where y is the vertical height of a point on the wave profile

x measures horizontal distance in the direction of motion of the wave

H is the wave height, crest to trough

L is the wave length, crest to crest

Then $$dy/dx = \frac{\pi H}{L} \cos 2\pi x/L,$$

and hence the maximum wave slope γ is

$$\gamma = \pi H/L \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (2)$$

Thus it is necessary to measure H and L to compute γ.

H is determined by measuring the vertical motion of a light float set parallel to the wave crests near the model.

Wave-lengths were calibrated against wave-period (measured with a stop watch) by measuring simultaneously the positions of two wave crests. A plot of the data for measured wave length against period is shown in Fig. 1, where the computed curves of wave length against period for a trochoidal wave in deep water and in shallow water four feet deep are also shown. The values from the latter curve, as tabulated in Table 1, were used in determining wave slopes. This is justified by the agreement between that curve and the experimental spots.

The following procedure is used to obtain rolling data with the wave-maker for any condition of the model. With a given eccentric setting on the wave-maker,

TABLE I

WAVE LENGTH AGAINST PERIOD FOR A TROCHOIDAL WAVE IN WATER 4 FT. DEEP

$$T^2 = (2\pi L/g)\coth(2\pi D/L)$$

T(sec.)	L(ft.)	T	L	T	L
1.30	8.6	1.65	13.3	2.00	17.9
1.31	8.7	1.66	13.5	2.01	18.0
1.32	8.8	1.67	13.6	2.02	18.2
1.33	9.0	1.68	13.8	2.03	18.3
1.34	9.1	1.69	13.9	2.04	18.5
1.35	9.2	1.70	14.0	2.05	18.6
1.36	9.4	1.71	14.2	2.06	18.7
1.37	9.5	1.72	14.3	2.07	18.9
1.38	9.6	1.73	14.4	2.08	19.0
1.39	9.8	1.74	14.6	2.09	19.1
1.40	9.9	1.75	14.7	2.10	19.2
1.41	10.0	1.76	14.9	2.11	19.4
1.42	10.2	1.77	15.0	2.12	19.5
1.43	10.3	1.78	15.1	2.13	19.7
1.44	10.4	1.79	15.3	2.14	19.8
1.45	10.6	1.80	15.4	2.15	19.9
1.46	10.7	1.81	15.5	2.16	20.0
1.47	10.9	1.82	15.7	2.17	20.2
1.48	11.0	1.83	15.8	2.18	20.3
1.49	11.1	1.84	15.9	2.19	20.4
1.50	11.3	1.85	16.1	2.20	20.5
1.51	11.4	1.86	16.2	2.21	20.7
1.52	11.5	1.87	16.3	2.22	20.8
1.53	11.7	1.88	16.5	2.23	20.9
1.54	11.8	1.89	16.6	2.24	21.1
1.55	12.0	1.90	16.7	2.25	21.2
1.56	12.1	1.91	16.8	2.26	21.4
1.57	12.2	1.92	16.9	2.27	21.5
1.58	12.4	1.93	17.0	2.28	21.6
1.59	12.5	1.94	17.1	2.29	21.8
1.60	12.6	1.95	17.3	2.30	21.9
1.61	12.8	1.96	17.4	2.35	22.7
1.62	12.9	1.97	17.5	2.40	23.3
1.63	13.1	1.98	17.7	2.45	24.0
1.64	13.2	1.99	17.8	2.50	24.6
1.65	13.3	2.00	17.9	2.55	25.2

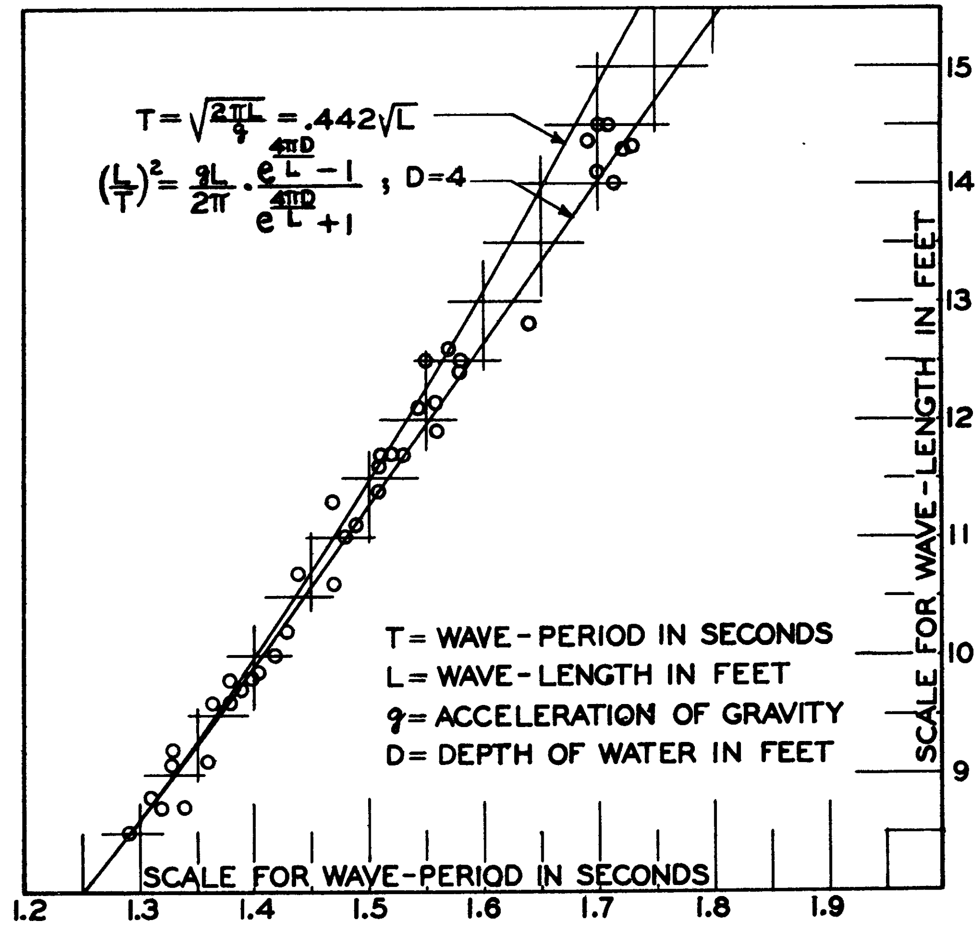

FIG. 1 CALIBRATION OF WAVE MAKER

waves are generated and the model rolls in a train of waves. When the amplitude of roll is fairly steady, readings are taken for θ, H and T. A series of such readings is made, varying T in the neighborhood of synchronism between the waves and the model. This process is repeated for each of the eight eccentric settings of the wave-maker.

Of the data obtained in this way, only those at synchronism are desired. Since it is difficult to ascertain from direct observation which of the readings correspond to synchronism, the following method is used. All the data for a given model condition are plotted as θ against $T\sqrt{GM}\cdot\sqrt{\gamma}$, the latter being computed from equation (6). Since, for a given γ, θ is a maximum at synchronism, it is clear that the upper envelope of all the spots so plotted will give the curve relating θ and γ at synchronism. The detailed analysis of a set of data is shown below:

Analysis of Data from Wave Maker

Model 3399, equipped with combination bilge and docking keels, loaded to a displacement of 52. lb. and with its GM and period adjusted to 0.812 in. and 1.68 sec., respectively, was rolled by waves generated by the wave-maker. The data and analysis for two of the eccentric positions of the wave-maker are shown in Table II.

TABLE II

θ (deg.)	H(in.)	T(sec.)	L(ft.)	γ (deg.)	$T\sqrt{GM}\cdot\sqrt{\gamma}$
			FROM ECC. 3		
6.8°	.73"	1.70	14.0'	.78°	1.33
7.1°	.70"	1.68	13.8'	.76°	1.32
6.6°	.65"	1.66	13.5'	.72°	1.39
			FROM ECC. 8		
13.7°	2.61"	1.73	14.4'	2.72°	2.50
12.0°	2.53"	1.78	15.1'	2.51°	2.39
12.3°	2.64"	1.78	15.1'	2.62°	2.45
13.3°	2.60"	1.725	14.3'	2.73°	2.50
13.1°	2.44"	1.66	13.5'	2.71°	2.47
12.5°	2.52"	1.71	14.2'	2.66°	2.47
12.9°	2.55"	1.70	14.0'	2.73°	2.50
13.9°	2.66"	1.63	15.1'	2.64°	2.47

The original experimental data consist of the values of θ, H and T entered in the first three columns. Values of L in column 4, corresponding to T, were read from Table I. γ and $T\sqrt{GM}\cdot\sqrt{\gamma}$ were then computed from eq. (2). Results from all the eccentrics are shown plotted in Fig. 5B. The curve for rolling in synchronous waves is the upper envelope of these spots. The envelope curve drawn in Fig. 5B was first cross-faired with the curves for other model conditions.

The curve of predicted rolling obtained in the manner described above made use of the slope of the wave at the surface. According to wave theory, however, the effective wave slope producing rolling is that at the half draft of the ship. Denoting this wave slope by γ, we have

$$\gamma = \gamma_0 e^{-\frac{2\pi h}{L}} \doteqdot \gamma_0 \left(1 - \frac{2\pi h}{L}\right)$$

where γ_0 is the maximum wave slope at the surface
and h is the half-draft of the ship.

In the above example, h = .128 ft. and L = 13.8 ft. (from Table I). Hence, $\gamma = \gamma_0 \left(1 - \frac{2\pi \times .128}{13.8}\right) = .941\,\gamma_0$. The dashed-curve in Fig. 5B represents the corrected predicted rolling curve.

DECLINING ANGLE CURVES

Procedure

The declining angle curve of a model is obtained by heeling the model over to a large angle, releasing it, and reading the successive amplitudes of roll to a side. The declining angle curve is then given as a curve faired through a plot of these angles, θ, against the number of complete rolls, N.

To arrive at the predicted rolling in synchronous waves from these data, the procedure is to compute γ from the relation

$$\pi\gamma = \frac{d\theta}{dN} \tag{3}$$

where $\frac{d\theta}{dN}$ is the slope of the declining angle curve corresponding to an angle θ. Thus θ is expressed in terms of γ.

ANALYSIS OF DECLINING ANGLE DATA

Model 3399, equipped with combination bilge and docking keels, loaded to a displacement of 52.0 lb. and with its GM and period adjusted to 0.812 in. and 1.68 sec., respectively, was heeled over to an angle of 25° in quiet water and then released. The successive maximum angles of roll, θ, were read on the side opposite to the initial heel.

The amplitudes of roll, θ, are recorded against the number of the roll, N, in columns 1 and 2 of Table III. θ is plotted against N on semi-log coordinate paper in Fig. 2. The curve that is faired through the spots is the declining angle curve.

To compute γ , it is necessary to find dθ/dN from the declining angle curve, as is seen from equation (3). The direct graphical determination of dθ/dN from the declining angle curve on uniform coordinate paper is not feasible because the curve so plotted is too steep to permit accurate graphical measurements of slopes. Rather, the curve is flattened as shown drawn in Fig. 2 and dθ/dN computed from the identity

$$d(\log_e\theta)/dN = \frac{1}{\theta}\ \frac{d\theta}{dN} \tag{4}$$

where $d(\log_e\theta)/dN$ ($= \mu$, say) is determined graphically from the semi-log plot.

The procedure for obtaining μ for $\theta = 5^\circ$ is indicated in Fig. 2. The value of N_o is found where a line through the point $\theta_o = e$ (=2.718...) on the θ axis, parallel to the slope of the declining angle curve at $\theta = 5^\circ$, intersects the N axis. In the case shown, the value $N_o = 5.38$ was found. This is equal to the

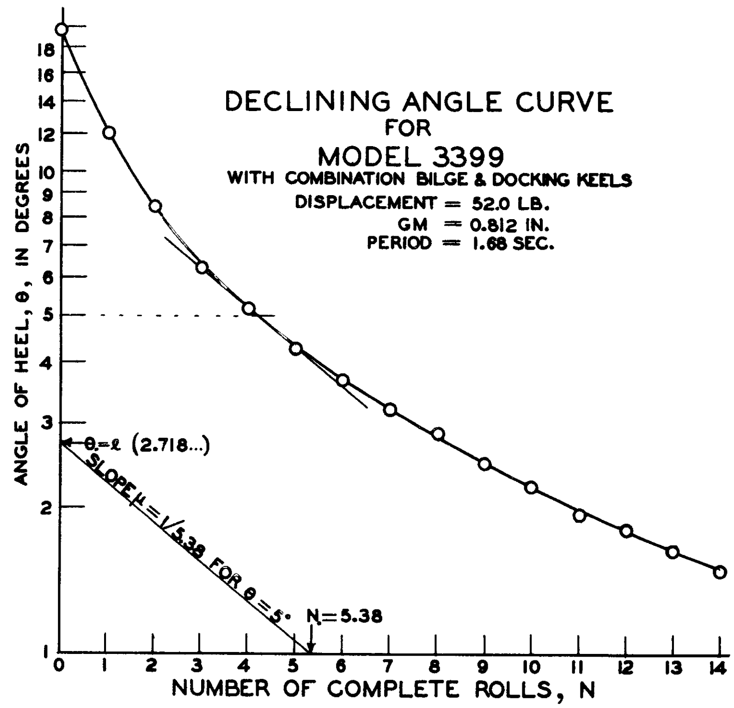

FIG. 2. DECLINING ANGLE CURVE

TABLE III

ILLUSTRATION OF ANALYSIS OF DECLINING ANGLE CURVE DATA
FOR MODEL 3399 WITH COMBINATION BILGE AND DOCKING KEELS, WITH
T = 1.68 SEC., GM = 0.812"

ORIGINAL DATA		FROM FAIRED CURVE			COMPUTED RESULTS	
N	θ	θ	$1/\mu$	$\mu\theta$	$\gamma = \mu\theta/\pi$	$T\sqrt{GM}\cdot\sqrt{\gamma}$
1	19.7	2	9.80	.204	.0650	.386
2	12.1	3	7.63	.393	.1251	.536
3	8.3	4	6.19	.646	.206	.687
4	6.3	5	5.38	.930	.296	.824
5	5.15	6	4.62	1.300	.414	.974
6	4.3	7	4.00	1.750	.557	1.130
7	3.7	8	3.39	2.36	.751	1.313
8	3.2	9	3.00	3.00	.955	1.480
9	2.85	10	2.80	3.57	1.137	1.614
10	2.5	12	2.41	4.98	1.586	1.906
11	2.23	14	2.15	6.51	2.07	2.180
12	1.95	16	1.96	8.16	2.60	2.440
13	1.80	18	1.88	9.58	3.05	2.644

inverse of the slope, $1/\mu$, since here

$$d(\log_e \theta)/dN = \frac{\log_e \theta_o}{N_o}$$

(where θ_o and N_o are the values where the tangent to the curve, or a line parallel to the tangent, intersects the θ and N axes) and $\log_e \theta_o = 1$ when $\theta_o = e$. $d\theta/dN$ is then readily computed from equation (4) as $\mu\theta$, and γ is then given by equation (3). The calculations for values of θ ranging from 2^o to 18^o are shown in columns 3 to 6 of Table III. The results are shown as a plot of θ against $T\sqrt{GM}\cdot\sqrt{\gamma}$ in Fig. 5B.

C. The Mechanical Rolling Gear

The rolling gear consists of two pairs of rotating weights mounted on two shafts which are normal to the deck of the model. The weights are adjustable eccentrically. A pair of the weights are on each shaft, their positions and orientations being such that the shafts are balanced statically but not dynamically. The shafts are driven by a shunt motor (.005 H.P.) of adjustable speed, and are geared together so as to rotate with the same speed but in opposite directions, (this latter to eliminate longitudinal torque components). See Fig. 3.

The transverse oscillating couple as produced by the rotation of the dynamically unbalanced shafts causes the model to roll. The amplitude of the

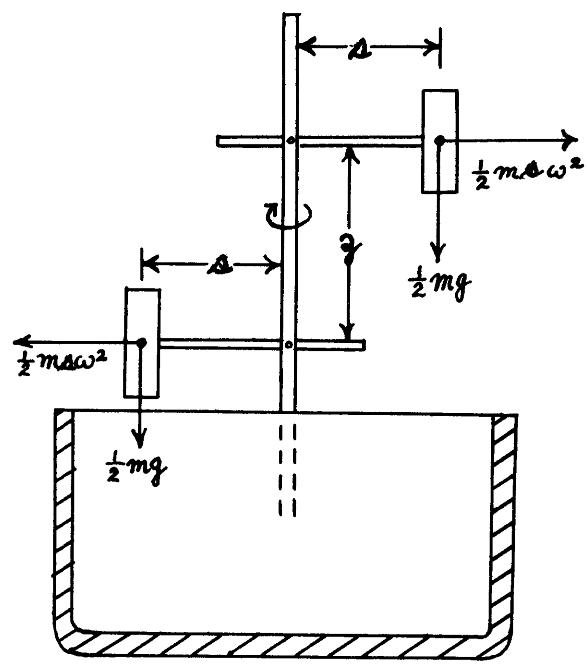

FIG. 3. SECTION OF MODEL WITH MECHANICAL ROLLING GEAR

periodic torque is given by the expression:

$$L = ms\omega^2 z \tag{5}$$

where m is the mass of each pair of weights, 1.216 lb.

s is the eccentricity of the c.g. of each pair

ω is the angular speed of the shafts, and

z is the distance between weights along the shafts.

The procedure in using the rolling gear consists in setting the weights at successive eccentricities (all four weights having the same eccentricity at any one setting) and noting the period and the model's amplitude of roll to a side when resonance between the rolling model and the rotating weights is established.

To predict rolling in synchronous waves from the rolling due to the mechanical gear, the procedure is to equate the torque exerted by the rolling gear to the expression, from Froude's theory, for the torque exerted by a wave in synchronism with the ship,

$$L = W\ GM \sin \phi \tag{6}$$

where W is the ship's displacement, and

$\phi = \arctan \gamma$.

Since ϕ is small, $\sin \phi$ may be replaced by γ in (6), giving for the maximum slope of the equivalent wave

$$\gamma = \frac{ms\omega^2 z}{W \cdot GM} \tag{7}$$

Entering the original data in equation (7), a relation is obtained between θ and γ , giving the predicted rolling in synchronous waves.

Analysis of Mechanical Rolling Gear Data

Model 3399, equipped with combination bilge and docking keels, loaded to a displacement of 52.0 lb. and with its GM and period adjusted to 0.812 in. and 1.68 sec., respectively, was rolled by the mechanical rolling gear.

The data and analysis are presented in Table IV. The eccentric weights were set successively at positions s corresponding to the ms values in col. 1, and for each ms value the period was varied until the maximum amplitude of roll (resonance) was obtained. The period and amplitude at resonance are entered in columns 2 and 3. The values of $T\sqrt{GM} \cdot \sqrt{\gamma}$ can be calculated directly from the equation,

$$T\sqrt{GM} \cdot \sqrt{\gamma} = 1.244\sqrt{ms} \tag{8}$$

which is derived as follows:

By (5), $\gamma = \frac{ms\omega^2 z}{W \cdot GM}$ radians. Hence, changing to degrees and putting $\omega = \frac{2\pi}{T}$,

we have

$$\gamma = \frac{180}{\pi} \times \frac{msz}{W \cdot GM} \times \frac{4\pi^2}{T^2} \tag{9}$$

$$\therefore \quad T\sqrt{GM} \cdot \sqrt{\gamma} = \sqrt{\frac{720\pi z}{W}} \cdot \sqrt{ms} \tag{10}$$

Putting z = 13.72" and w = 52.0 x 386 in.lb.sec. units in (8), equation (6) is obtained. $T\sqrt{GM} \cdot \sqrt{\gamma}$ was computed in this way (col. 4). The results are plotted in Fig. 5B.

TABLE IV

ms	θ	T	$T\sqrt{GM} \cdot \sqrt{\gamma}$
.30 in.lb.	5.0°	1.68 sec.	.681
.61	7.0	1.68	.972
.91	8.5	1.68	1.19
1.22	9.7	1.68	1.37
1.52	10.9	1.68	1.53
1.82	11.9	1.68	1.68
2.13	12.9	1.68	1.815
2.43	13.9	1.68	1.94
2.74	14.7	1.67	2.06
3.35	16.2	1.65	2.28
3.50	16.7	1.64	2.33

Experimental Results

A. Tests on Battleship Model 3399.

A series of tests was made on a six-foot battleship model (No. 3399) for the following conditions:

Displacement = 52.0 lb.
Mean draft = 3.1 in.
Wetted surface area = 6.0 sq. ft.

TABLE V

BARE HULL		WITH COMBINATION BILGE AND DOCKING KEELS	
T(sec.)	GM(in.)	T(sec.)	GM(in.)
1.65	0.758	1.68	0.758
—	—	1.68	0.812
1.68	0.85	1.68	0.85
1.66	1.09	1.67	1.095
1.50	0.92	1.53	0.92
1.94	0.92	1.98	0.92

It will be observed that the period was kept approximately constant in the tests listed in the first four rows above, while GM was held fixed in those in the last two rows.

For each of these conditions the model was tested for rolling characteristics by the three methods of studying rolling. The results are shown in Figs. 4 and 5 as a plot of θ against $T\sqrt{GM}\cdot\sqrt{\gamma}$. All the curves drawn are cross-faired.

B. These three methods of studying rolling were also applied in the case of another model. The lines of this model are those of the U.S.S. LEXINGTON. This model was tested in rolling when loaded as is the actual ship, U.S.S. LEXINGTON, and also when loaded as the Conte di Savoia, a ship whose lines are somewhat similar.

Results for these two cases, together with one set from a series of runs on 3399 are shown in Figures (6), (8) and (9). The set chosen for 3399 is the one which most nearly fits actual battleship loading. Thus, these three sets may be taken as showing the nature of rolling results and their agreement for actual ships as determined by the three methods of rolling.

Discussion

The three methods of studying rolling have few similarities. Each requires its own apparatus, a different set of data, and an independent set of assumptions for analysis. For the sake of comparison, however, we can inquire as to the reliability of the initial data. Also the question as to the plausibility of the assumptions is of interest, although more academic. Finally, the predicted rolling results can be compared, taking the results from the wave-maker as the standard.

Of the three methods, the rolling of a model by generated waves is the most direct. The difficulty here lies in generating a pure train of synchronous, uniform waves in a closed tank. In addition it has been mentioned that considerable interaction between the model and the wave-maker was observed during rolling. These experimental difficulties were overcome to a great extent by preventing wave reflections (as has been described) and by taking simultaneous readings for the original data during an interval of steady rolling.

The method of analysis adopted, it is believed, combines and fairs the experimental results in such a way as to bring the final predictions near the real answer. However, because of the difficulty of determining resonance, complete consistency cannot be expected from individual tests. The fact that the above results from the wave-maker are fairly consistent may be partly attributed to the fact that an extensive series of tests was made and cross-faired. The principal value of the method is to serve as a standard of comparison for the other two methods and thus to show how nearly their results agree with the direct results from rolling in uniform, synchronous waves.

The declining angle curve method of predicting rolling is a simple and quick way of getting an answer. Data are obtained easily. Difficulty may be encountered in their analysis, however, since the declining angle curve is not easily faired correctly and predicted rolling results are especially sensitive to fairing.

In its nature, the method is an indirect mathematical one. The model is not made to roll by waves or any substitute therefor. The method is based on the equating of the loss of angle per roll, which is assumed to be given experimentally by the slope of the declining angle curve, with an expression for the increment in angle per roll due to a wave, as given by a simplified and approximate theory of rolling.

To test Froude's assumption that the angular increment per roll due to a wave is $\pi\gamma$, the following experiment was performed on Battleship Model 3399 in its bare hull condition. The model was held stationary in a train of waves, released when the wave slope at the model was a maximum, and the successive amplitudes of roll read. These successive amplitudes were plotted against the number of rolls N to give an "increasing angle curve." The angular increment per roll due to the wave is then given by the slope $d\theta/dN$ of this θ, N curve at the zero angle, since only there is the slope unaffected by the damping of the fluid. The results obtained from various tests are shown in Table VI.

TABLE VI

GM(in.)	T (sec.)	$\pi\gamma$ *(deg.)	$d\theta/dN$	$d\theta/dN/\pi\gamma$
1.09	1.66	4.6	3.3	.72
1.09	1.66	5.7	4.4	.77
1.09	1.66	7.85	5.9	.75
.85	1.64	2.35	1.85	.79
.85	1.62	4.53	3.15	.70
.85	1.61	5.75	4.1	.71
.85	1.62	3.72	2.45	.66
.85	1.62	3.09	2.10	.68

* corrected to wave slope at half-draft of model.

These results indicate that for the above model conditions, the value of γ as given by equation (3) is about 28% too small. It is seen from Figs. 4 and 5 that that correction would suffice to eliminate the discrepancy between the predicted results from the wave-maker and the declining angle curve.

Considering the approximations involved, the results obtained from this method, as shown in Figs. 4 – 9, are good. On the average, the angles of roll of Model 3399 predicted on the declining angle curve method are 10% greater than those obtained from rolling in waves (as corrected for the wave slope at the half-draft). An examination of Figs. 4 – 6 shows that this error is less for the smaller angles and greater for the larger angles. This is to be expected from the fact that since the model is released from a large angle in quiet water, damping is less, and hence the results at the large angles must be considered uncertain.

Thus the objections to the declining angle method are threefold:

(1) The method is uncertain at the larger angles.

(2) Difficulties in the analysis of the data may lead to erratic results in individual tests, as in the bare hull condition of Model 3454 in Fig. 9.

(3) There is no measure of the error introduced by the basic equation (3) for a given model and condition of loading.

The mechanical rolling gear method is noteworthy for its directness and simplicity. The model is actually made to roll by a harmonic torque whose magnitude is known and under complete control. Angles are measured directly at resonance, and the analysis of the data offers no difficulties.

To all appearances and considerations, the mechanical rolling gear gives consistent and comparable results at all times. This assumes that the gear is in good operating condition and that it is handled with a reasonable degree of skill, especially in the matter of adjustment of the positions of the weights, and that the apparatus is statically balanced at all times and dynamically balanced for the zero position of the weights. It is suggested that the apparatus might be simplified and improved, especially in obtaining small values of the torque, by redesigning the gear so that the torque is varied by changing the weights at a fixed eccentric instead of changing the eccentricities of constant weights. This can be accomplished without change in displacement or GM.

The predicted rolling results on this method in Figs. 4 – 9, while consistent among themselves, gave angles of roll about 20% greater than those from rolling in waves. An examination of the procedure followed in analyzing the original data indicates, as the most probable source of this discrepancy, the value for the torque of a wave assumed in equation (6).

The expression for the torque in equation (6) can be derived from Froude's theory by considering a ship on a wave at the instant of maximum wave slope, when, in synchronism, the ship is vertical and the heeling torque of the wave is greatest. Froude's assumptions lead to the conclusion that the resultant force on the ship is horizontal, passing through the metacentre, of magnitude $W \sin \phi$, and hence exerts the torque $W \sin \phi \cdot GM$ about the centre of gravity. Concerning the point of action of this resultant force, D. W. Taylor said,*

"If this resultant force should pass below the centre of gravity, instead of above, the ship would actually tend to roll toward the wave instead of away from it. In practice it would seem that for most ships the resultant is not far from

*"Calculations of Ship's Forms and the Light Thrown by Model Experiments upon Resistance, Propulsion and Rolling of Ships." 1915.

the metacentre. Froude's theory seems virtually to assume that the resultant passes always through the metacentre. It is quite possible, however, that particularly when the ship is shaped with this in view the resultant will really pass below the metacentre. This would account for the fact that reduction in metacentric height appears often to reduce rolling more than can be accounted for by the change on metacentric height alone."

These remarks indicate that the ratio between predictions from the mechanical gear and the wave-maker is proportional to the distance, h, that the resultant force passes above the centre of gravity. It would be expected then that this percentage deviation would differ for different models and different conditions of loading. The variations for the models and conditions tested are shown in Table VII.

TABLE VII

RATIO OF h TO GM AS DETERMINED FROM RATIO OF γ's FROM MECHANICAL GEAR AND WAVE MAKER, FOR $\theta = 12°$.

Model	h/GM
Model 3399, bare hull	.64
Model 3399, combination bilge and docking keels	.67
Model 3454 as Conte di Savoia, bare hull	1.08
Model 3454 of Lexington, bare hull	.94
Model 3454 of Lexington 18" bilge keels	.88
Model 3454 of Lexington 42" bilge keels	.77

Thus the mechanical rolling gear method has overcome the experimental difficulties involved in working with a wave maker, and avoided the devious procedure characterizing the declining angle curve method. However, until some way is devised, theoretical or experimental, of correcting Froude's theory, the method cannot yield absolute results. Perhaps its greatest value is in its capacity of giving comparative data, which the method does with great accuracy and consistency.

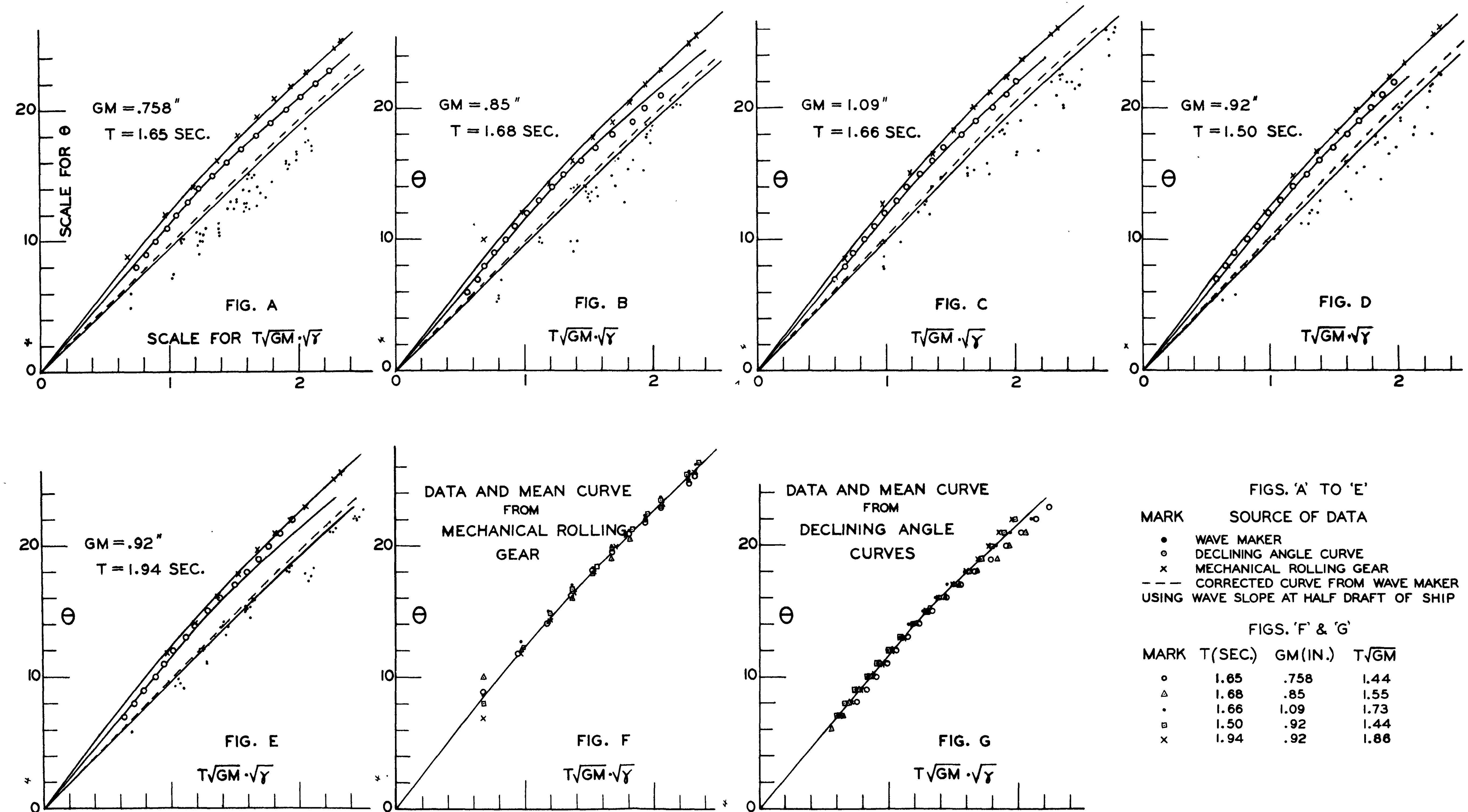

FIG. 4 DATA & FAIRED CURVES OF PREDICTED ROLLING IN SYNCHRONOUS WAVES FOR MODEL 3399 WITH BARE HULL

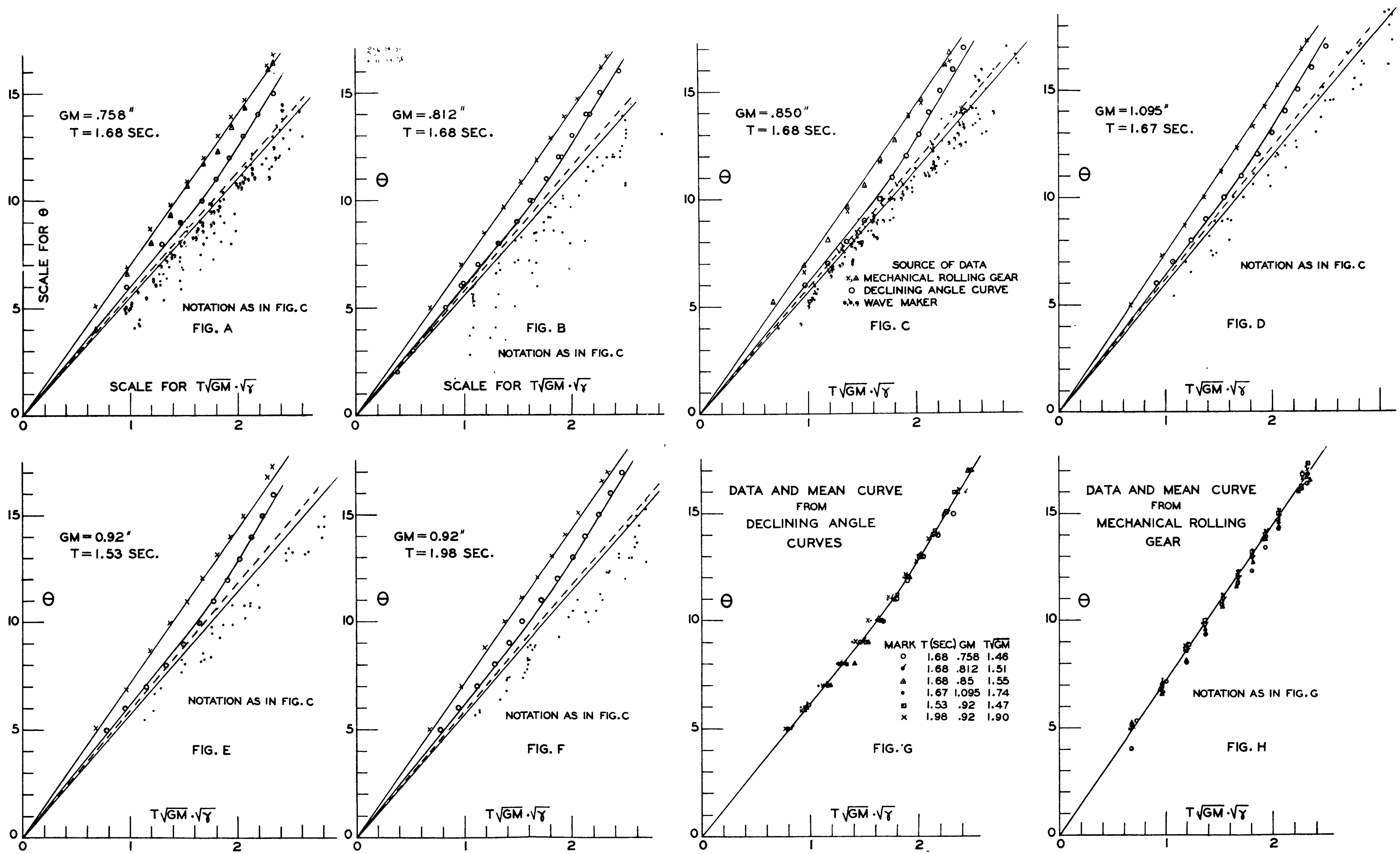

FIG. 5 DATA & FAIRED CURVES OF PREDICTED ROLLING IN SYNCHRONOUS WAVES FOR MODEL 3399 WITH COMBINATION BILGE AND DOCKING KEELS

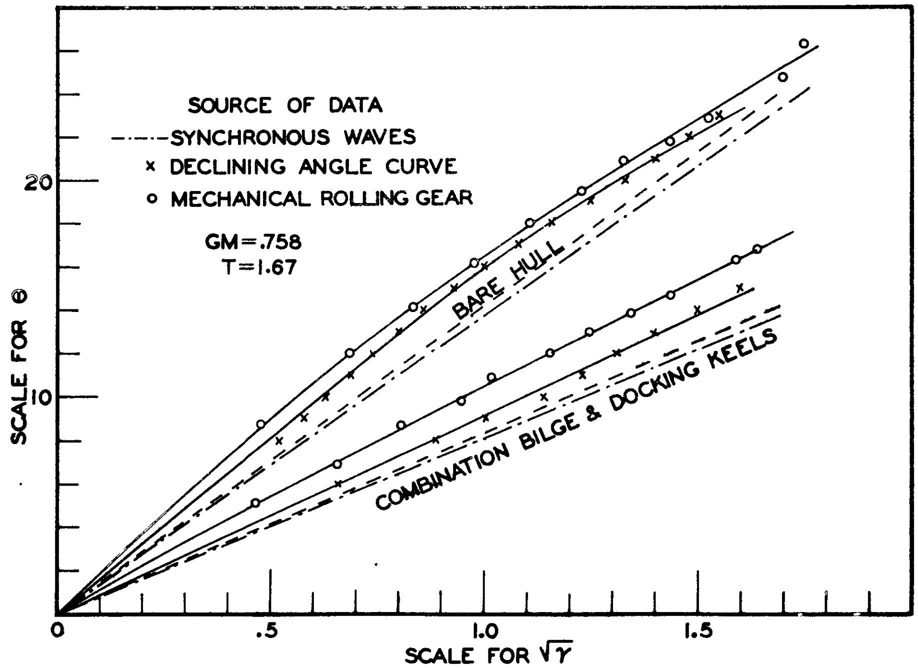

FIG. 6 PREDICTED ROLLING IN SYNCHRONOUS WAVES FOR MODEL 3399 OF A PROPOSED BATTLESHIP

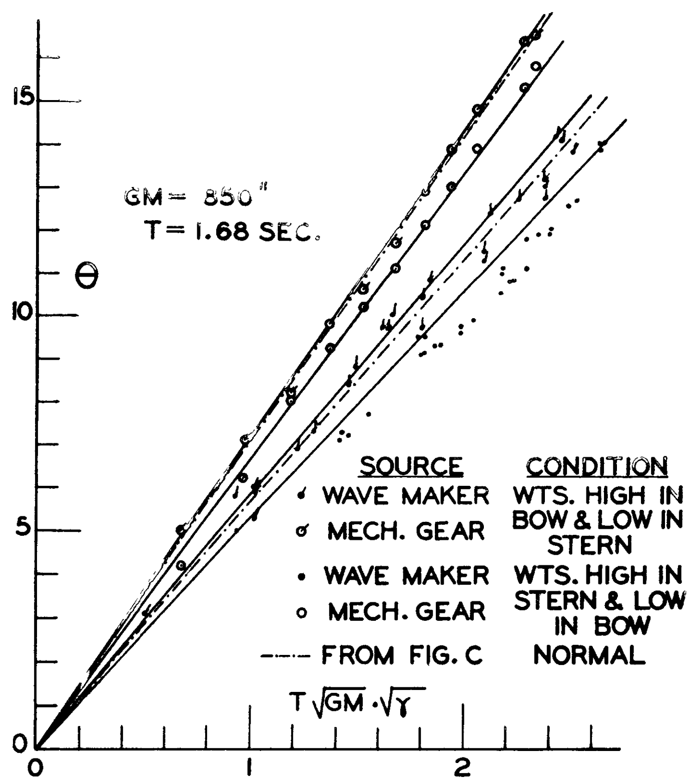

FIG. 7. EFFECT OF UNSYMMETRICAL LONGITUDINAL LOADING ON MODEL ROLLING.

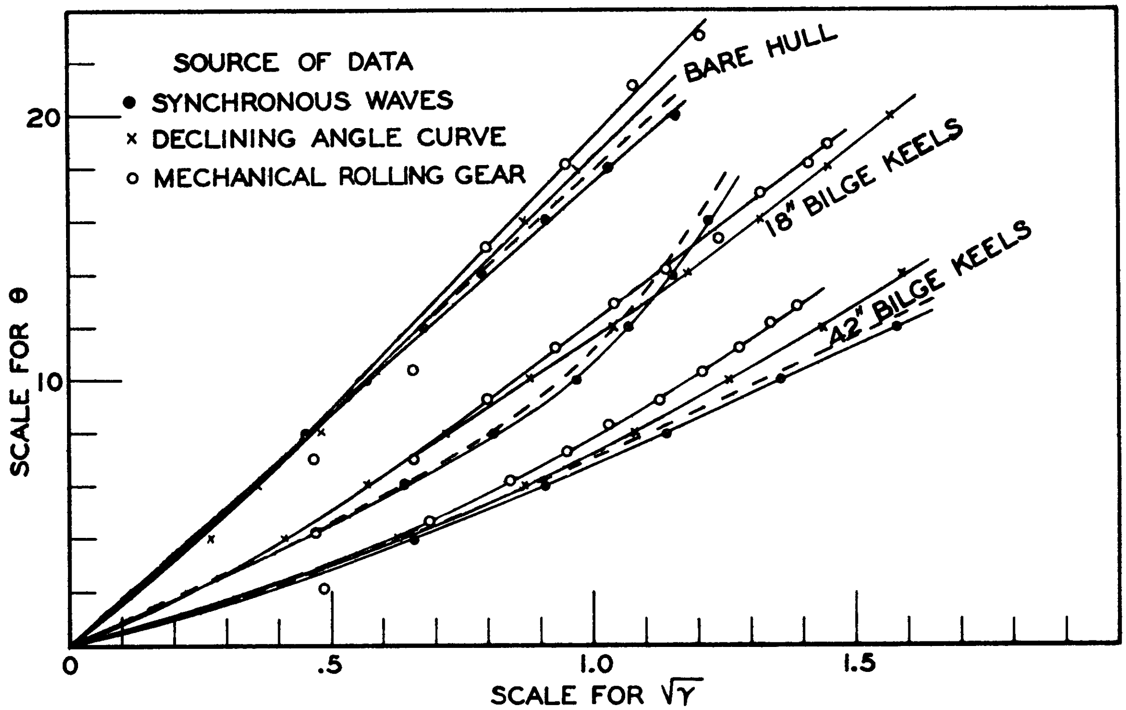

FIG. 8 PREDICTED ROLLING IN SYNCHRONOUS WAVES FOR MODEL 3454 OF THE U.S.S. LEXINGTON

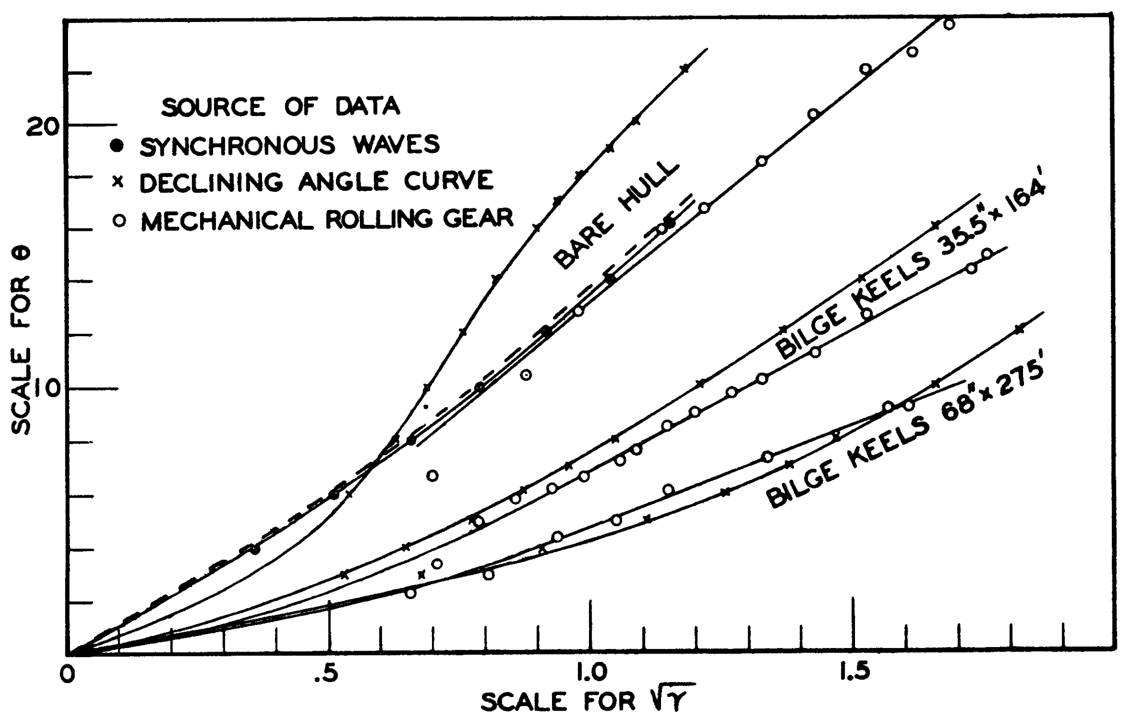

FIG. 9 PREDICTED ROLLING IN SYNCHRONOUS WAVES FOR MODEL 3454 LOADED AS THE CONTE DI SAVOIA