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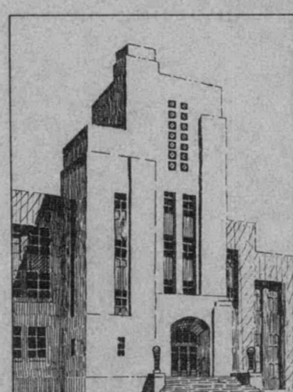
THE DAVID W. TAYLOR MODEL BASIN

UNITED STATES NAVY

THEORETICAL AND EXPERIMENTAL INVESTIGATION OF THE SHAFT-RESTRAINING BLOCK

BY J. d'H. HORD



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REPORT 497

NAVY DEPARTMENT

DAVID TAYLOR MODEL BASIN

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THE DAVID TAYLOR MODEL BASIN

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PERSONNEL

The work of developing the restraining block from ideas originating with Professor F.M. Lewis, of the Massachusetts Institute of Technology, was done by the Kingsbury Machine Works of Philadelphia under the general supervision of the Design Superintendents of the Navy Yard, New York, and the Navy Yard, Philadelphia, and of the Technical Director of the David W. Taylor Model Basin.

The theoretical treatment in this report was developed by Lieutenant Commander J. Ormondroyd, USNR, and J.d'H. Hord. The model work was carried out by J.d'H. Hord and J.V. Coombe with the assistance of Edgar Berdahl and Maxwell Elliott, all members of the David W. Taylor Model Basin Staff, under the supervision of Lieutenant Commander Ormondroyd and Lieutenant Commander A.G. Mumma, USN. The report is the work of Mr. Hord. The digest was written by Captain H.E. Saunders, USN.

DIGEST

During the early builders' trials on certain new vessels of large size and moderately high power, excessive longitudinal vibrations were encountered in the propeller and line shafting, the reduction gears, and the turbines. The general arrangement and relative position of the component parts of a typical propelling unit are as indicated in Figure 1.

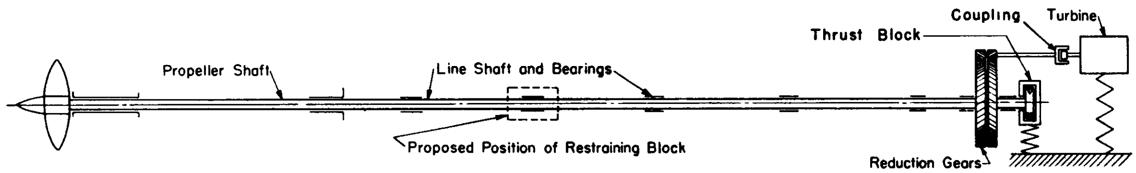

Figure 1 - Schematic Arrangement of Propelling Unit

The reduction gears and their bearings, and the thrust block, were all carried by the reduction gear case, which was in turn supported by a rather high foundation on the inner bottom. As the elastic nature of the gear and turbine foundations plays a prominent role in all the vibration calculations, they are shown as flexible rather than as rigid members, even though the gear foundation carried the full propeller thrust.

After a preliminary study of the situation, it soon became apparent that the vibration was of propeller-blade frequency and that it was caused by elastic vibration, in an axial direction, in the system formed by the propeller, the line shafting, the main reduction gearing, the turbines, and the condenser.

Because of the peculiar arrangement of the turbines and gears, in which there was an underslung condenser just forward of the reduction gears and reduction-gear foundations, it was not possible to limit the longitudinal movement of the gears and turbines to an acceptable amount by bracing and stiffening the reduction-gear foundations. Because of the length of the propeller and line shafting it was likewise not possible to make any change in the elastic characteristics of the shafting except possibly to make the shaft solid instead of hollow, a modification which would not necessarily have produced the desired result.

One of the remedies proposed for the situation was to install in the propeller or line shafts, in a position as close as practicable to the propeller, a special form of vibration damper called a shaft-restraining block. The function of a block of this kind is to absorb the blade-frequency forces and to prevent the shaft from vibrating longitudinally under the influence of these relatively high-frequency alternating forces. At the same time it permits the shaft to move slowly in a longitudinal direction so that the propeller thrust is taken by the thrust bearing at the forward end of the shaft and so that slow thermal expansion due to differential temperature variations in the shafting and in the ship structure is allowed for. In terms of vibration parlance, the function of this restraining block is to attenuate the high-frequency vibrations considerably but to produce little or no effect on the low-frequency vibrations or the steady longitudinal displacements.

A design of full-scale restraining block was begun and carried through with the cooperation of all the activities concerned as explained previously under Personnel. Two working models were constructed at the David Taylor Model Basin, one primarily to check the hydraulic theory and the other to check the general serviceability of the design prepared for the full-scale ship installation.

A simplified form of shaft-restraining block is shown in Figure 2. The device consists of a combination of shaft and piston, free to move longitudinally and rotating within a cylinder which is fixed in position. The chambers at the ends of the cylinder and the clearance space around the piston are filled with a viscous fluid.* If longitudinal motions of shaft and piston occur, as shown at x_0, this fluid is forced from one end chamber to the other through the radial clearance c_1. In this way a resistance to longitudinal displacements of shaft and piston is produced which varies as the axial velocity of the piston, and large forces are developed to resist the high-frequency movements.

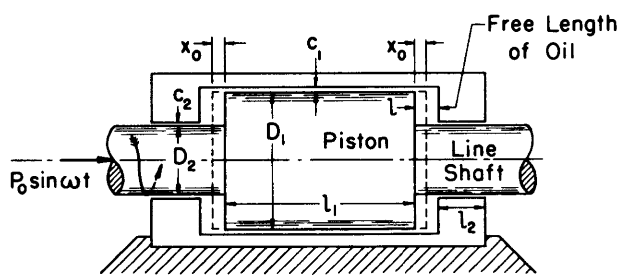

Figure 2 - Simplified Longitudinal Section of Shaft-Restraining Block

The longitudinal movement in either direction from the mean position is shown as x_0.

In the shipboard installation the piston is rigidly attached to the line shaft and the cylinder is rigidly fixed to the hull near the propeller, thus by-passing to the hull structure the alternating forces produced by the propeller, but allowing the steady or low-frequency thrusts, such as those due to temperature changes or speed changes, to be transmitted to the main thrust bearing in the machinery space.

From the theory of viscosity, and from other known characteristics of the mechanical system and the viscous fluid in the restraining block, a

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* For the model tests and for the ship installation, this fluid was lubricating oil of the same grade as that used in the line shaft steady bearings.

theory has been developed, as outlined on pages 3 to 8 of the report, by which the damping coefficient of a shaft-restraining block can be calculated for any set of assumed conditions.

A 0.23-scale hydraulic model was built at the Taylor Model Basin to check the theory thus developed. As shown in Figure 4 on pages 8 and 9 of the report, it consisted of a shaft and piston rotating in a cylinder, representing the ship restraining block. The longitudinal alternating force representing that developed by the ship propeller blades was produced by a motor-driven eccentric on the end of a long flexible rod attached to the shaft. Gages and other devices were fitted to record the frequency, longitudinal amplitude, pressure and the like.

The average value of the damping constant on this model was about 82 per cent of the value computed theoretically. This was considered a good check, considering the effect of eccentricity of the piston with respect to the cylinder.

The final design of piston for the full-scale restraining block embodied two half-collars, as shown in Figure 9, held together over the smooth shaft by a row of high-strength clamping bolts on each side. To determine whether the assumed coefficient of friction of 0.1 between the collars and the shaft could be expected in service, a 0.15-scale clamping model was built and subjected to static and alternating loads as described in Appendix 8, pages 45 to 56. These tests, combined with subsequent tests of a 0.3-scale model, indicated that the design should prove adequate in service.

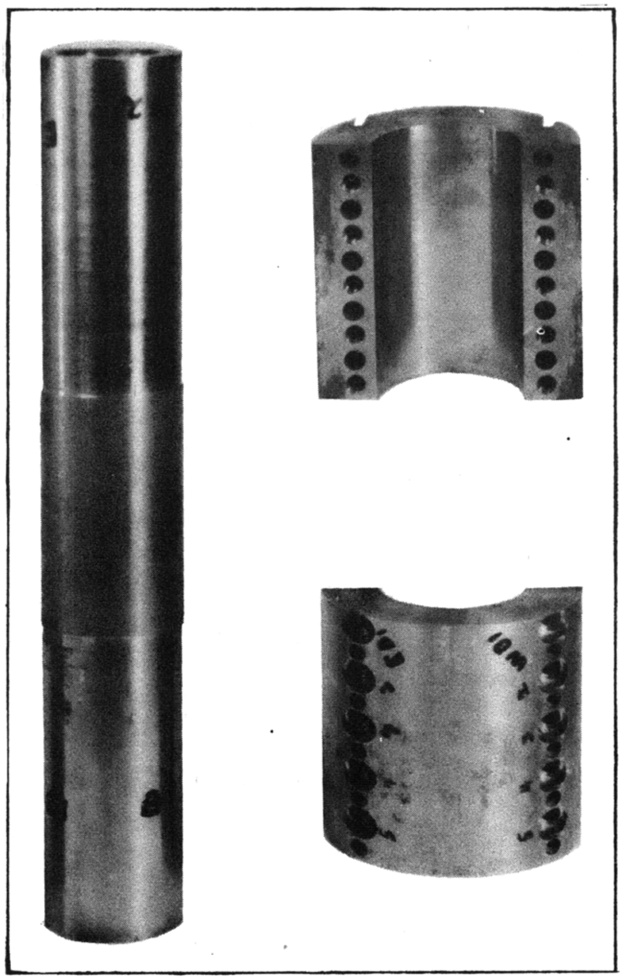

Figure 9 - Disassembled 0.15-Scale Clamping Model

At the left is the hollow model shaft, with the enlargement for the original steady bearing journal in the center of this shaft. The half-collars are at the right.

Because of the extreme novelty of the device, and the skepticism expressed in some quarters during the design, that the block would not function satisfactorily as both a vibration damper and a steady bearing, the Taylor Model Basin undertook to build a 0.3-scale working model and made arrangements to test it under conditions simulating those to be expected on shipboard. The schematic diagram of the entire model is shown in Figure 10 on page 14, and a longitudinal section through one of the units in Figure 12.

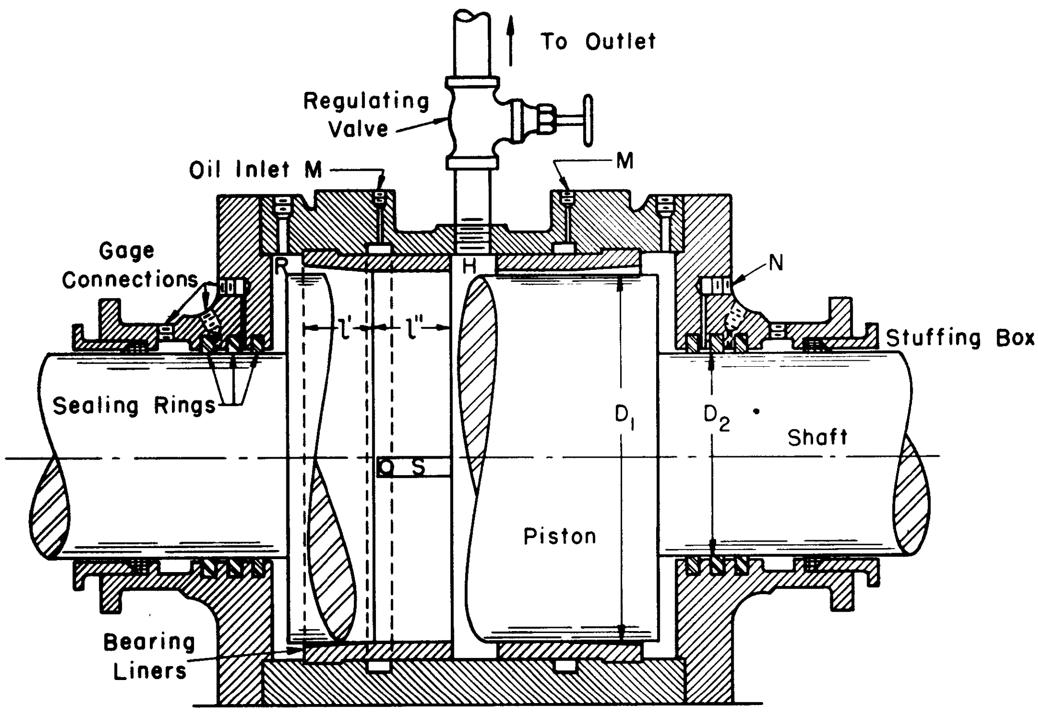

Figure 12 - Longitudinal Section through Model Shaft-Restraining Block

The block was symmetrical longitudinally, and except for exaggerated tapers and clearances, was of the exact proportions shown. The bearing liners and sealing rings were in halves, fastened together with screws.

After considerable experimenting with clearances, materials for bearing liners, oil pressures, and methods of introducing, routing and cooling the oil, as described on pages 16 to 18 of the report, final tests with the 0.3-scale model indicated that the performance was reasonably close to predictions, and that the full-scale ship installation would, if the design clearances and other features were acceptable, have fulfilled the purpose for which the blocks were to have been installed.*

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* Before the parts for the full-scale installations had been completed, the use of propellers with a different number of blades reduced the longitudinal vibrations to acceptable values and all work on the restraining-block installation was discontinued.

A number of separate but associated investigations made in connection with this project are reported in the appendices and are summarized here:

Appendix 1. Specifications prepared at the outset of the project for the shaft-restraining blocks and for the ship installations.

Appendix 2. Discussion of assumptions used in the development of the hydraulic theory.

Appendix 3. Discussion of the effect of compressibility in the fluid used in the block.

Appendix 4. Effect of piston eccentricity in the bearing liners and taper at the outer ends of these liners.

Appendix 5. Damping-constant correction for the effect of the slots in the bearing liners, used to feed oil to the bearing areas.

Appendix 6. Effect of end leakage through the sealing rings around the shafts.

Appendix 7. Use of electrical analogy methods for determining the effectiveness of a restraining block.

Appendix 8. Description and tests of the 0.15-scale and the 0.3-scale clamping models.

Finally, the following recommendations are made for future designs:

1. Thorough and adequate provision must be made for venting air from all parts of the system. If practicable, a length of strong glass tubing should be incorporated in the vent so that air, if present, can be directly observed.

2. As a safety factor, the damping constant actually needed should be trebled for design purposes because of the difficulty of keeping the system entirely free of air.

3. Provision must be made for cooling and de-aerating the oil. The latter requirement necessitates a relatively large sump or settling tank.

4. The damping constant varies inversely as the cube of the radial clearance between piston and cylinder; the clearance can be considerably affected by such factors as expansion due to temperature change, pre-straining of parts during assembly, and other causes. Hence some method of checking the clearance after assembly and at working temperature should be provided. This can readily be accomplished by inserting micrometer depth gages in different parts of the cylinder. The holes cut for that purpose would have to be plugged against the high pressure during operation.

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THEORETICAL AND EXPERIMENTAL INVESTIGATION OF THE SHAFT-RESTRAINING BLOCK

ABSTRACT

The report describes conditions under which excessive longitudinal vibrations of shafting and of propelling machinery have been encountered in ships, and outlines the development of a shaft-restraining block to inhibit these vibrations. This block is intended to be inserted in the line shafting between the propeller and the reduction gears, and to replace one of the steady bearings.

The theory of such a device is developed in several phases. By this theory it is possible to compute the damping coefficient of the restraining block for any given set of conditions.

A series of 0.15-scale, 0.23-scale and 0.3-scale models were built to check the theory, to confirm design assumptions and to check the use of the new device as a steady bearing as well as a vibration damper. These models are described in detail, and the results of an extensive series of tests are described and analyzed.

Relatively simple design rules are given for future installations of this kind and the precautions to be considered in the development of details are set forth, with descriptions of and references to one full-scale design which was developed but not installed.

A series of eight appendices discusses numerous phases of the theory and the mechanical design, with a discussion of the practicability of determining the effectiveness of a restraining-block installation by the use of electrical analogy.

INTRODUCTION

During the early builders' trials on certain new vessels of large size and moderately high power, excessive longitudinal vibrations were encountered in the propeller and line shafting, the reduction gears, and the turbines. These vibrations involved high-pressure and high-temperature steam lines as well as the machinery, and they were so severe that the engineering staffs of the builders and of the vessels considered it wise not to attempt to develop full power on the propelling machinery.

After a preliminary study of the situation, it soon became apparent that the vibration was of propeller-blade frequency and that it was caused by elastic vibration, in an axial direction, in the system formed by the propeller, the line shafting, the main reduction gearing, the turbines, and the condenser. The general arrangement and relative position of the component parts of a typical propelling unit are as indicated in Figure 1.

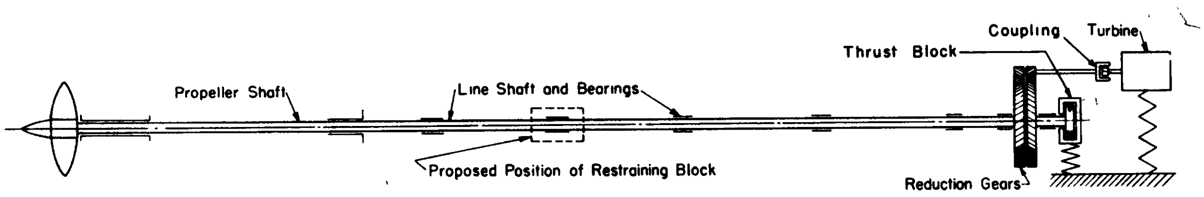

Figure 1 - Schematic Arrangement of Propelling Unit

The reduction gears and their bearings, and the thrust block, were all carried by the reduction gear case, which was in turn supported by a rather high foundation on the inner bottom. As the elastic nature of the gear and turbine foundations plays a prominent role in all the vibration calculations, they are shown as flexible rather than as rigid members, even though the gear foundation carried the full propeller thrust.

Because of the particular arrangement of the turbines and gears, in which there was an underslung condenser just forward of the reduction gears and reduction-gear foundations, it was not possible to limit the longitudinal movement of the gears and turbines to an acceptable amount by bracing and stiffening the reduction-gear foundations. Because of the length of the propeller and line shafting it was likewise not possible to make any change in the elastic characteristics of the shafting except possibly to make it solid instead of hollow, a modification which would not necessarily have produced the desired result. It became imperative, therefore, to develop some other method of eliminating or reducing the vibration, on the assumption that it was not possible to eliminate the forces developed in the propeller, which caused this vibration.

One of the remedies proposed for the situation was to install in the propeller or line shafts, in a position as close as practicable to the propeller, a special form of vibration damper called a shaft-restraining block. The function of a block of this kind is to absorb the blade-frequency forces and to prevent the shaft from vibrating longitudinally under the influence of these relatively high-frequency alternating forces, while at the same time it permits the shaft to move slowly in a longitudinal direction so that the propeller thrust is taken by the thrust bearing at the forward end of the shaft and so that slow thermal expansion due to differential temperature variations in the shafting and in the ship structure is allowed for. In terms of vibration parlance, the function of this restraining block is to attenuate the high-frequency vibrations considerably but to produce little or no effect on the low-frequency vibrations or the steady longitudinal displacements.

A search of the technical literature failed to reveal any theory directly applicable to the design of such a restraining block or a mention or description of any similar or equivalent installation that had previously been made, on board ship or elsewhere.

A design of full-scale restraining block was begun and carried through with the cooperation of all the activities concerned as explained previously under Personnel. Two working models were constructed at the David Taylor Model Basin, one primarily to check the hydraulic theory and the other to check the general serviceability of the design prepared for the full-scale ship installation.

DESIGN SPECIFICATIONS

To make possible a workable installation of a shaft-restraining block in a ship already built and in operation, it became necessary to solve simultaneously several rather complicated problems. The first was to determine the piston areas, piston and shaft clearances, chamber volumes and other design features of an entirely new device. The second was to apply on a length of existing line shafting, which had solid flanges at both ends, a collar which would act as the piston in the restraining block. The third was to embody in the restraining block one or more line shaft steady bearings, because for practical reasons it was found necessary to place the restraining block at a point in the shaft occupied by one of the original steady bearings.

In view of the foregoing, rather extensive specifications were developed, which are set down in full in Appendix 1.

The first two paragraphs of these specifications are quoted here as an introduction to the development of the theory:

"1. The primary function of the shaft-restraining block shall be to maintain, in a fixed longitudinal position relative to the ship, that portion of the main propeller shaft which is rotating within the block, under the influence of a periodically varying thrust force as applied by the ship's propeller to the shaft abaft the block.

"2. The restraining block is intended to form an artificial axial node in the propeller shaft at its point of installation and to act as the equivalent of an infinite mass fixed to the shaft at that point, so that the shaft sections forward and aft of the block, together with their attached masses, may be considered as independent vibratory systems."

DEVELOPMENT OF GENERAL THEORY

A simplified form of shaft-restraining block is shown in Figure 2. The device consists of a combination of shaft and piston, free to move longitudinally and rotating within a cylinder which is fixed in position. The chambers at the ends of the cylinder and the clearance space around the piston

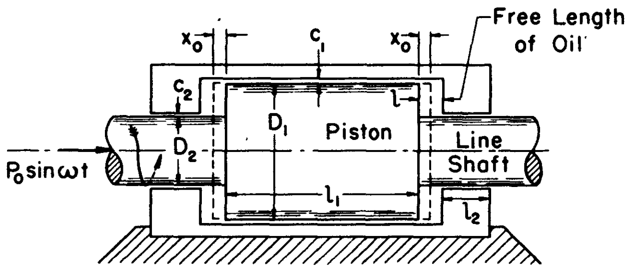

Figure 2 - Simplified Longitudinal Section of Shaft-Restraining Block

The longitudinal movement in either direction from the mean position is shown as x_0.

are filled with a viscous fluid.* If longitudinal motions of shaft and piston occur, as shown at x_0, this fluid is forced from one end chamber to the other through the radial clearance c_1. In this way a resistance to longitudinal displacements of shaft and piston is produced which varies as the axial velocity of the piston, and large forces are developed to resist the high-frequency movements.

In the shipboard installation the piston is rigidly attached to the line shaft and the cylinder is rigidly fixed to the hull near the propeller, thus by-passing to the hull structure the alternating forces produced by the propeller, but allowing the steady or low-frequency thrusts, such as those due to temperature changes or speed changes, to be transmitted to the main thrust bearing in the machinery space.

For development of the theory, the following assumptions are made

1. Longitudinal sliding friction is negligible
2. No cavitation occurs in the fluid
3. The structure is completely rigid
4. Acceleration forces are negligible
5. The clearances c_1 and c_2 are small compared to D_1, D_2, l_1, and l_2
6. Leakage through the stuffing box clearance c_2 is negligible.

For a detailed discussion of these assumptions, see Appendix 2, pages 28 to 29.

An expression may be derived for the amplitude x_0 of the residual motion, in terms of the force amplitude P_0 applied, the circular frequency ω** of this force, the physical dimensions shown in Figure 2, and the physical properties of the fluid used.

To define the absolute viscosity of a fluid we consider two surfaces of area A separated by fluid of thickness h. A force F must be applied to one of the surfaces in order to move it relative to the other with a

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* For the model tests and for the ship installation, this fluid was lubricating oil of the same grade as that used in the line shaft steady bearings.

** The circular frequency of a simple harmonic motion is 2π times the frequency in cycles per second.

velocity V. The absolute viscosity μ is then defined in terms of these quantities as

$$\mu = \frac{Fh}{AV}$$

Using pound, inch, and second units, μ has the dimensions of pound-second per square inch. If we consider what goes on between surfaces in the fluid which are separated by elementary distances dh and in which the velocities differ from each other by amounts dV, the expression becomes

$$\mu = \frac{F}{A}\frac{dh}{dV}$$

or

$$F = \mu A \frac{dV}{dh}$$

It must be noted that refers to the velocity of the fluid in inches per second.*

Figure 3 shows an enlarged section of the radial clearance space c. Assuming now that the relative axial motion between the piston and the cylinder is negligible compared to the fluid velocities, the following observations are obvious:

1. The fluid velocity will be a maximum at the middle radius of the clearance space,

2. The fluid velocity will be zero at the cylinder surface and at the piston surface.

A parabolic velocity distribution throughout the clearance space conforms to these limiting conditions and is so drawn in Figure 3, with the mental reservation that its parabolic shape must be proved. Let p be the pressure difference between the chambers at the two ends of the cylinder. The force F acting on the annular segment whose section is shaded in Figure 3 is given by

$$F = pD_1\pi h$$

where D_1 is the diameter of the piston. Since there is no relative motion between adjacent layers of fluid at $h = 0$, this force is also equal to

$$F = \mu A \frac{dV}{dh} = pD_1\pi h$$

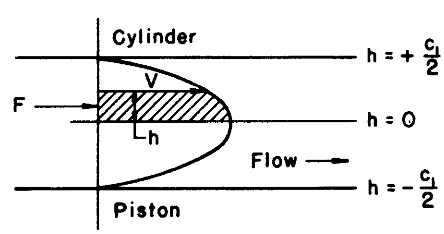


Figure 3 - Enlarged Section of Radial Clearance c_1

* Later the symbol Q will be used for the quantity of flow in cubic inches per second.

where $A = \pi D_1 l_1$, Figure 2. Since V decreases as h increases, dV/dh is negative. Therefore

$$dV = \frac{-pD_1\pi h}{\pi D_1 l_1 \mu}\, dh = \frac{-ph}{l_1\mu}\, dh$$

and

$$V = \frac{-ph^2}{2l_1\mu} + K_1$$

Since at $h = \frac{c_1}{2}$, $V = 0$,

$$K_1 = \frac{pc_1^2}{8l_1\mu}$$

and $V = \frac{p}{l_1\mu}\left(\frac{c_1^2}{8} - \frac{h^2}{2}\right)$ inches per second

It will be noted that the parabolic velocity distribution previously assumed is thus proved correct.

Now let Q be the quantity of flow through the entire clearance c_1 in cubic inches per second.

Then

$$dQ = \pi D_1 V\, dh$$

$$= \frac{p\pi D_1}{l_1\mu}\left(\frac{c_1^2}{8} - \frac{h^2}{2}\right) dh$$

$$Q = \frac{p\pi D_1}{l_1\mu}\int_{-\frac{c_1}{2}}^{+\frac{c_1}{2}}\left(\frac{c_1^2}{8} - \frac{h^2}{2}\right) dh$$

$$Q = \frac{p\pi D_1 c_1^3}{12\mu l_1} \text{ cubic inches per second}$$

Now let $\frac{\pi D_1 c_1^3}{12\mu l_1}$ be equal to S, which is thus a physical constant characterizing any given restraining block, including the oil in it, and let

$$Q = pS$$

It is shown in Appendix 3, pages 30 to 32, that the compressibility of the fluid may be neglected if the impressed frequency is less than 1000 cycles per minute. On this basis the rate of transfer of oil from one end of the piston to the other equals the instantaneous displacement rate of the piston. Therefore

$$Q = pS = A_1 v$$

where A_1 is the end area of the piston in square inches

v is the instantaneous axial velocity of the piston in inches per second.

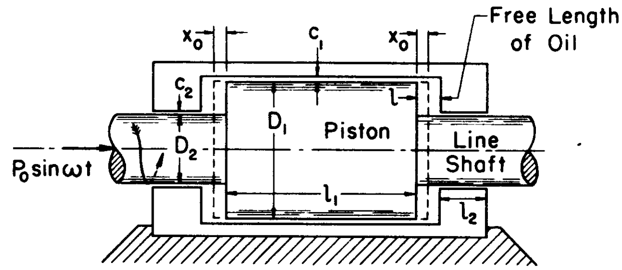


Figure 2 - Simplified Longitudinal Section of Shaft-Restraining Block

The total force acting on the piston is pA_1, therefore

$$pA_1S = A_1^2 v = PS$$

where P is the instantaneous force on the piston in pounds.

$$P = \frac{A_1^2}{S} v \text{ pounds}$$

P is therefore similar to a viscous damping force and can be written

$$P = C v \text{ pounds}$$

where $C = \dfrac{A_1^2}{S} = \dfrac{12\mu l_1 A_1^2}{\pi D_1 c_1^3}$ pound-seconds per inch.

When the axial motion x of the piston is simple harmonic motion, in which

$$x = x_0 \sin \omega t$$

the total force P is also simple harmonic

$$P = C\omega x_0 \cos\omega t$$

The amplitude of force is $P_0 = C\omega x_0$ and $x_0 = \dfrac{P_0}{C\omega}$

For convenience the foregoing symbols are recapitulated as follows:

A_1 is the end area of the piston in square inches

D_1 is the diameter of the piston in inches

l_1 is the length of the piston in inches

μ is the absolute viscosity of the oil in pound-seconds per square inch

S is $\dfrac{\pi D_1 c_1^3}{12\mu l_1}$ in cubic inches per second per unit pressure

C is $\dfrac{A_1^2}{S}$, the restraining block damping constant in pound-seconds per inch

P is the instantaneous force applied to the piston in pounds

P_0 is the amplitude of an applied harmonic force in pounds

p is the instantaneous pressure difference between the ends of the cylinder in pounds per square inch

p_0 is the amplitude of harmonic pressure difference in pounds per square inch

x is the instantaneous longitudinal displacement of the piston from the center position in inches

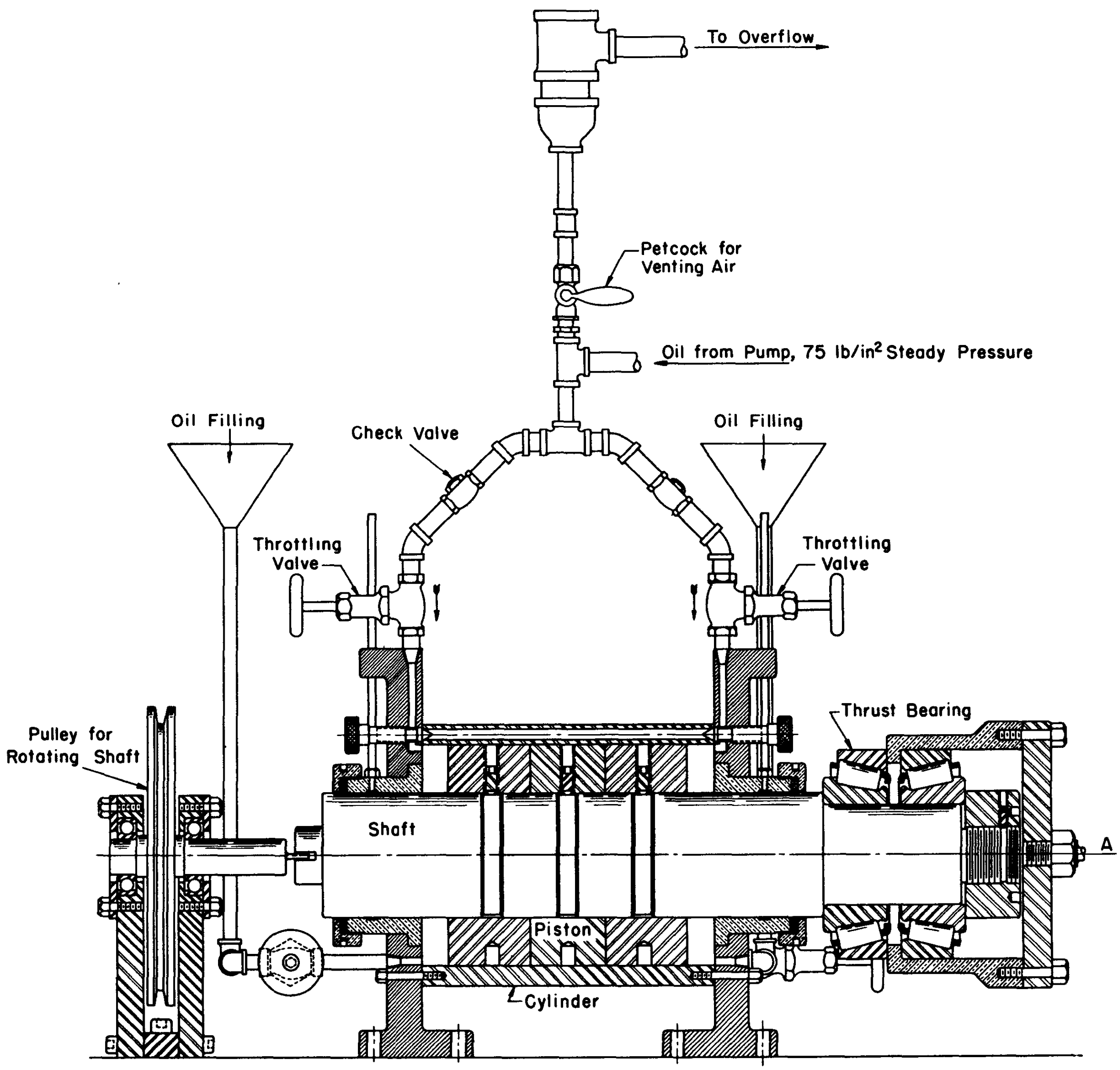
x_0 is the amplitude of harmonic displacement in inches

$\omega = 2\pi f$ is the circular frequency of the applied force in radians per second

$f = \frac{\omega}{2\pi}$ is the frequency of the applied force in cycles per second

Q is the instantaneous rate of oil flow along the clearance space around the piston in cubic inches per second

Q_0 is the amplitude of oil flow in harmonic piston motion in cubic inches per second.



DESCRIPTION AND TESTS OF 0.23-SCALE HYDRAULIC MODEL

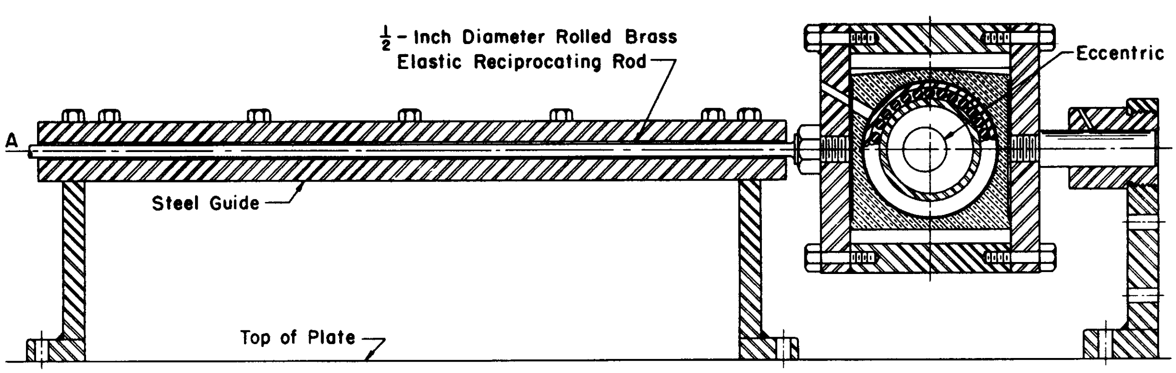
A 0.23-scale hydraulic model was built to check the theory, as developed in the preceding section. A cross section of the model is shown in Figure 4. The longitudinal alternating force representing that developed by the ship propeller blades was produced by rotating the eccentric with a 1-HP variable-speed shunt motor.

In the original setup, provision was made to rotate the shaft and piston by the pulley shown at the left of the figure. This necessitated the use of a double-row thrust bearing to impress the alternating force from the elastic rod; the latter was fixed to the eccentric and could not rotate. It was found that the operation of the model as a restraining block was essentially the same whether the shaft and piston rotated or not, so the provisions for rotation were discarded and the thrust bearing was tightened up to eliminate the lost motion inherent in an anti-friction bearing of this kind.

As described in Appendix 2, a high oil pressure had to be maintained in the cylinder during operation to prevent cavitation or to avoid sucking air into the system. This was accomplished by the inlet pipes and check valves indicated in Figure 4. The check valves were important to prevent the alternating pressure pulses from being transmitted through the high-pressure inlets from one end of the cylinder to the other. The fluid used in this model was a mixture of kerosene and lubricating oil having an absolute viscosity of 8.02 centipoises, or 1.16×10^{-6} pound-seconds per square inch, at 20 degrees centigrade.

Figure 4 - Longitudinal Cross Section of 0.23-Scale Model Restraining Block

This assembly, shown on facing pages 8 and 9, is joined at points A.



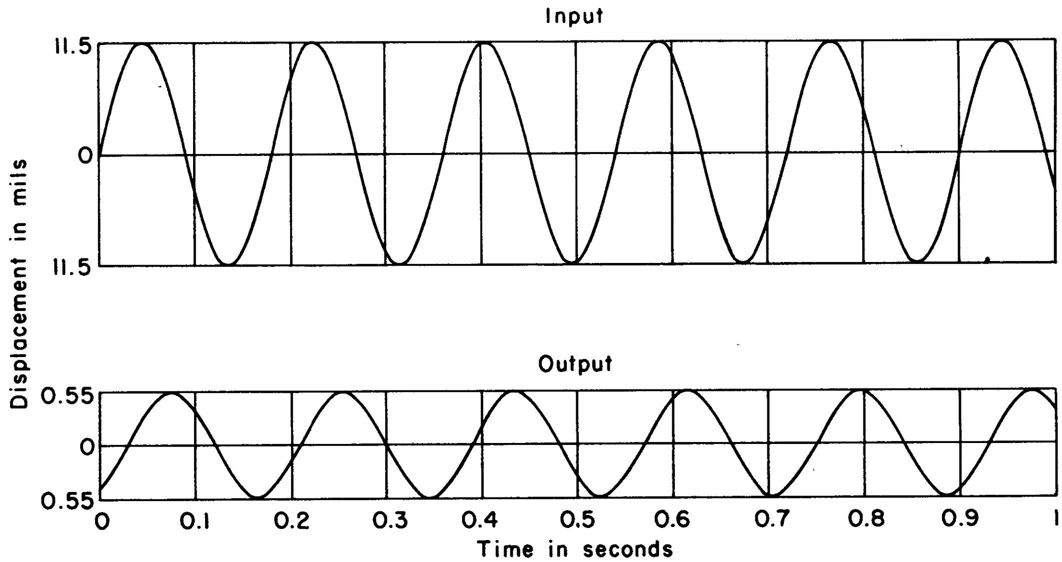


Figure 5 - 0.23-Scale Model Input and Output Displacements

Input, Double Amplitude = 0.023 inch P_0 = 1150 pounds Speed = 325 cycles per minute

The elastic reciprocating rod used had a spring constant of 100,000 pounds per inch. Since the axial motion of the piston was small compared to the motion impressed on the far end of the rod, the eccentricity could be used as a measure of the force amplitude exerted.

Frequency was obtained by tachometer measurements. Input and output motions were obtained by photographing dial gages mounted on the housing of the structure. A curve from the data thus obtained is shown in Figure 5. From these data were plotted the curves in Figures 6, 7, and 8, showing residual motion and impressed frequency for three different force amplitudes. The damping constant thus obtained was not quite constant for different frequencies. Table 1 shows the extent of this variation.

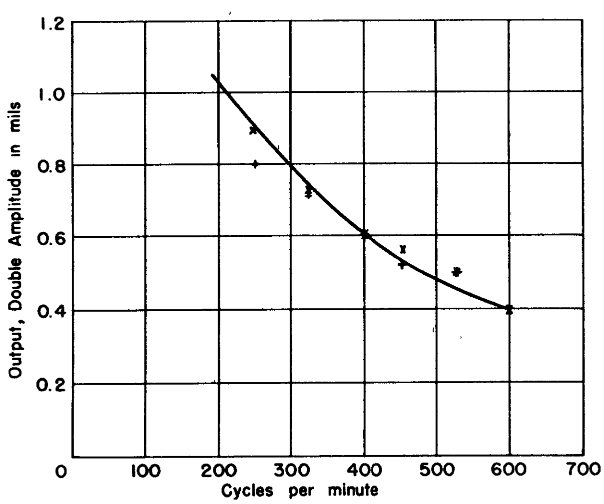


Figure 6 - 0.23-Scale Model Output Double Amplitude and Frequency

Input, Double Amplitude = 0.016 inch
P_0 = 800 pounds

Using the formula for C developed in the theory, page 7

$$C = \frac{A_1^2}{S} = \frac{12\mu l_1 A_1^2}{D_1 \pi c_1^3}$$

and substituting the values shown in Table 2,

(text continued on page 12)

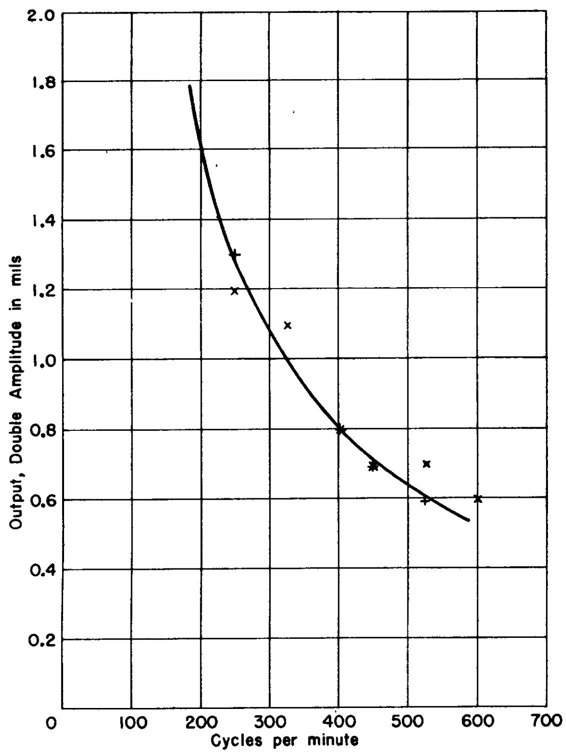


Figure 7 - 0.23-Scale Model Output Double Amplitude and Frequency

Input, Double Amplitude = 0.023 inch

P_0 = 1150 pounds

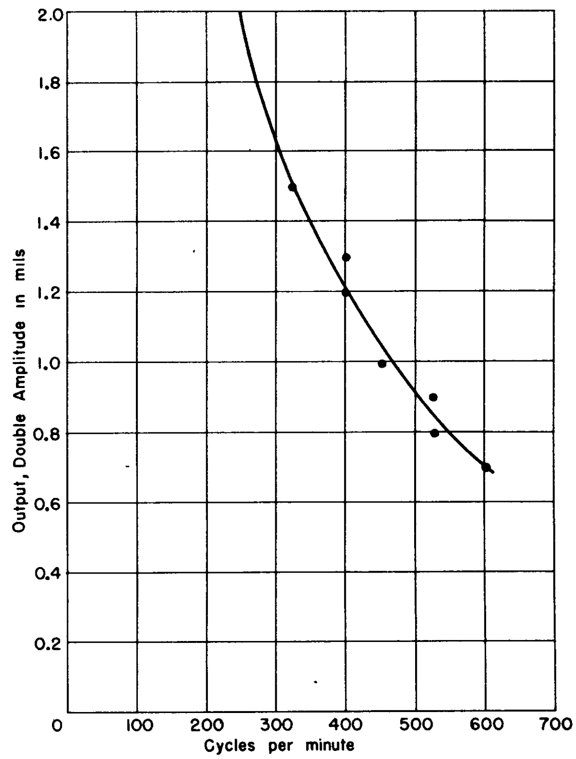


Figure 8 - 0.23-Scale Model Output Double Amplitude and Frequency

Input, Double Amplitude = 0.032 inch

P_0 = 1600 pounds

TABLE 1

Values of Damping Constant C obtained from Data on Figures 6, 7, and 8 by the relation $C = \frac{P_0}{\omega x_0}$, in units of pound-seconds per inch

Cycles per minute	P_0 = 800 pounds	P_0 = 1150 pounds	P_0 = 1600 pounds
250	67900	67400	61100
500	63600	68600	66500

TABLE 2

Values of Important Constants for the 0.23-Scale Model

A_1	= 33.5 square inches
c_1	= 0.004 inch
D_1	= 7.9 inches
l_1	= 8.37 inches
μ	= 0.0802 poise at 20° C = 1.16×10^{-6} lb-sec/in^2

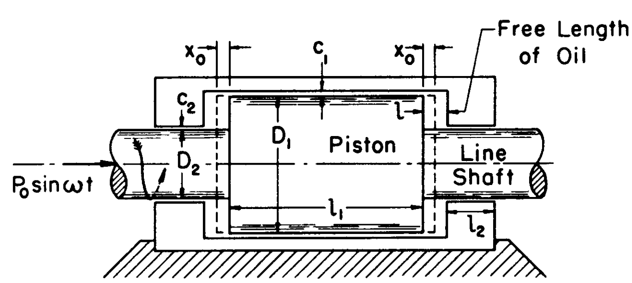


Figure 2 - Simplified Longitudinal Section of Shaft-Restraining Block

$$C = \frac{12 \times 1.16 \times 10^{-6} \times 8.37 \times 33.5^2}{7.9 \times \pi \times 4^3 \times 10^{-9}}$$

$= 82{,}000$ pound-seconds per inch

The average measured value is about 82 per cent of the value computed theoretically. This is a fairly good check, considering the effect of possible eccentricity of the piston with respect to the cylinder. As is shown in Appendix 4, if the piston were as eccentric as possible, i.e., rubbing against one side of the cylinder, the damping constant would be reduced to only 40 per cent of the value to be expected with concentric alignment.

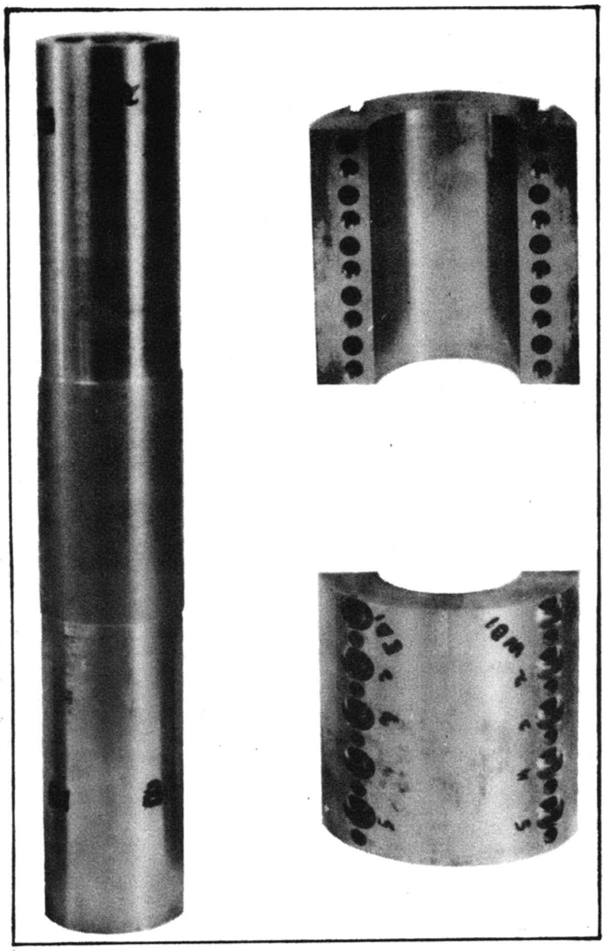


Figure 9 - Disassembled 0.15-Scale Clamping Model

At the left is the hollow model shaft, with the enlargement for the original steady bearing journal in the center of this shaft. The half-collars are at the right.

DESCRIPTION AND TESTS OF CLAMPING MODELS

A difficult mechanical problem presented itself in the necessity for attaching to the existing line shaft in the ship a heavy piston or collar which would take the thrust forces to be expected, of the order of 500,000 pounds maximum, without working loose on the shaft. The problem was further aggravated by the fact that the piston had to be fixed to the shaft in the restricted shaft alley, with only such modifications to the shaft as could be made while it was in place.

The various schemes proposed are outlined in detail in Appendix 8, from which it will be noted that the only one considered likely to succeed was to use two half-collars, tightly bolted together over the enlarged section of the shaft at one of the steady bearings by a multiplicity of high-strength bolts which could be set up under a very high initial tension.

As the approved method involved also the clamping of these half-collars over a smooth shaft, with no grooves or keying devices, the question immediately arose as to the permissible coefficient of friction under these conditions. To obtain advance information on this feature there was built a 0.15-scale clamping model, as shown in Figure 9, which upon test under static and alternating loads* showed that the assumed coefficient of friction of 0.1 for smooth steel on smooth steel, dry, could be expected in service.**

Later a double clamping model was built to 0.3-scale, which was tested for change in shape and other features, as described in Appendix 8. This larger model was then used as the rotating element in the double restraining-block model, the construction and tests of which are described subsequently in this report.

DESCRIPTION AND TESTS OF 0.3-SCALE WORKING MODEL

This was built and set up for test at the Taylor Model Basin as a working scale model of the unit to be installed in the ship; construction on it was carried along as fast as preparation of the working drawings for the ship installation would permit. In addition to serving as a restraining block or damper, it was intended also to be used as a steady bearing to support the weight of the adjacent line shafting. In fact, during the development period on this device, there was considerably more skepticism expressed about its satisfactory operation as a steady bearing than about its usefulness as a means of eliminating longitudinal vibration in the ship shafts.

As the model shaft could not conveniently be made long enough to produce a unit steady-bearing pressure equal to that in the full-scale installation,† the shaft was loaded with two heavy drums or pulleys, one on each side of the model restraining block.

Because of the extremely heavy thrust loads which would have to be imposed on the shaft, and their vibratory nature, it was considered unwise to use an anti-friction bearing for applying the simulated propeller thrust forces. The model was therefore built with two identical shaft-restraining

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* Tests of the clamping models are described in Appendix 8.

** Actually it is possible under these conditions to obtain a coefficient of friction much higher than 0.1.

† About 81 pounds per square inch of projected bearing area. The problem of fluid cooling was therefore important, since excessively lowered viscosity due to increased temperature not only would adversely affect the behavior of the device as a restraining block, but would probably result in galling the bearing liners inside the casings.

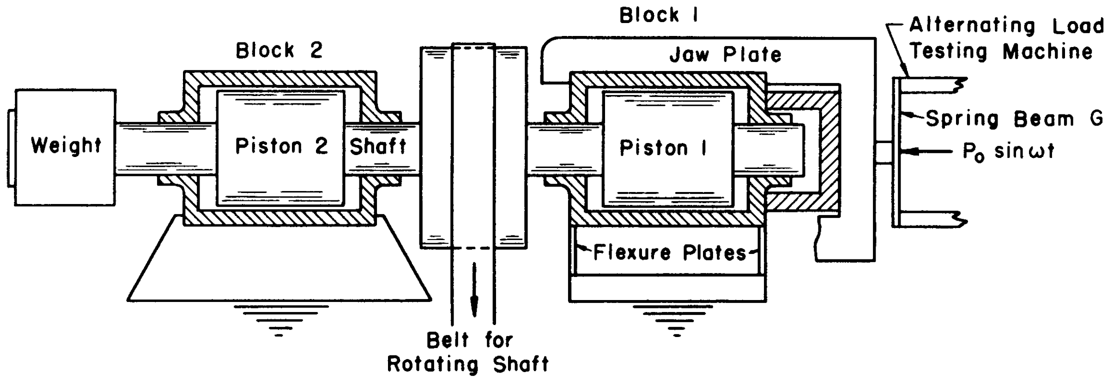

Figure 10 - Diagram of Parts Composing Test Assembly of 0.3-Scale Model of Shaft-Restraining Block

Block 2 is fixed in position on the frame of the testing machine. Block 1 is mounted on thin flexure plates so that it is held in proper position axially but can move back and forth parallel to the axis of the shaft. The heavy weight and the large solid pulley alongside Block 2 produce the full-scale unit bearing load on the piston of that block. The spring beam *G* is attached to the two reciprocating arms of the testing machine and to double jaws which clamp around the casing of Block 1.

blocks in tandem, as shown in the diagrammatic sketch, Figure 10. The right-hand unit, Block 1, was attached to the 150,000-pound alternating load testing machine* and acted as a thrust bearing to transmit the alternating forces to the left-hand unit, Block 2. The latter served as the restraining block proper, and the efficiency of the design was obtained by measuring the alternating oil pressures and the amount of longitudinal motion of the shaft relative to the casing of Block 2 for a given force amplitude and frequency.

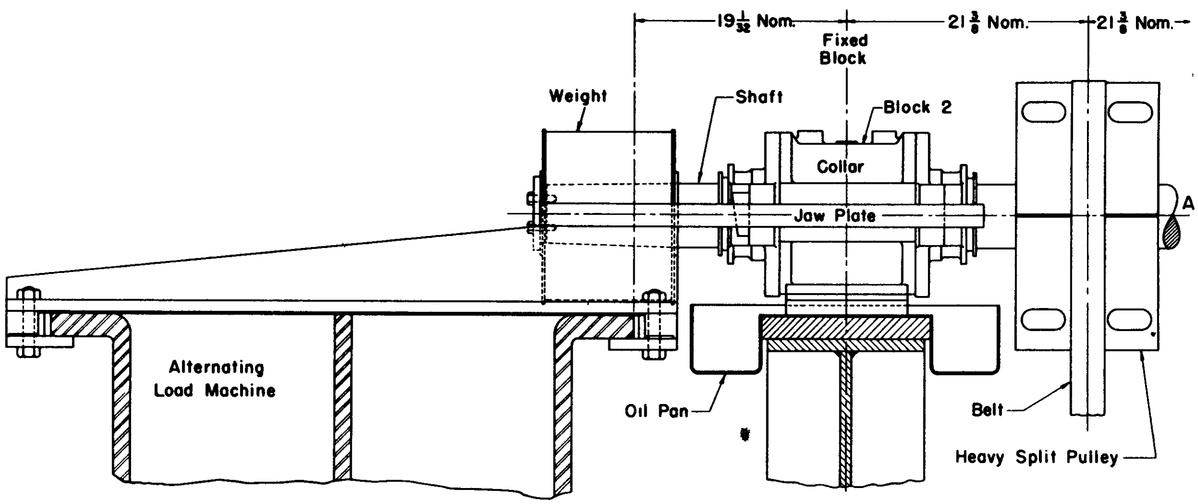

Figure 11 - Longitudinal Elevation of 0.3-Scale Model Restraining Block

This assembly, shown on facing pages 14 and 15, is joined at points A.

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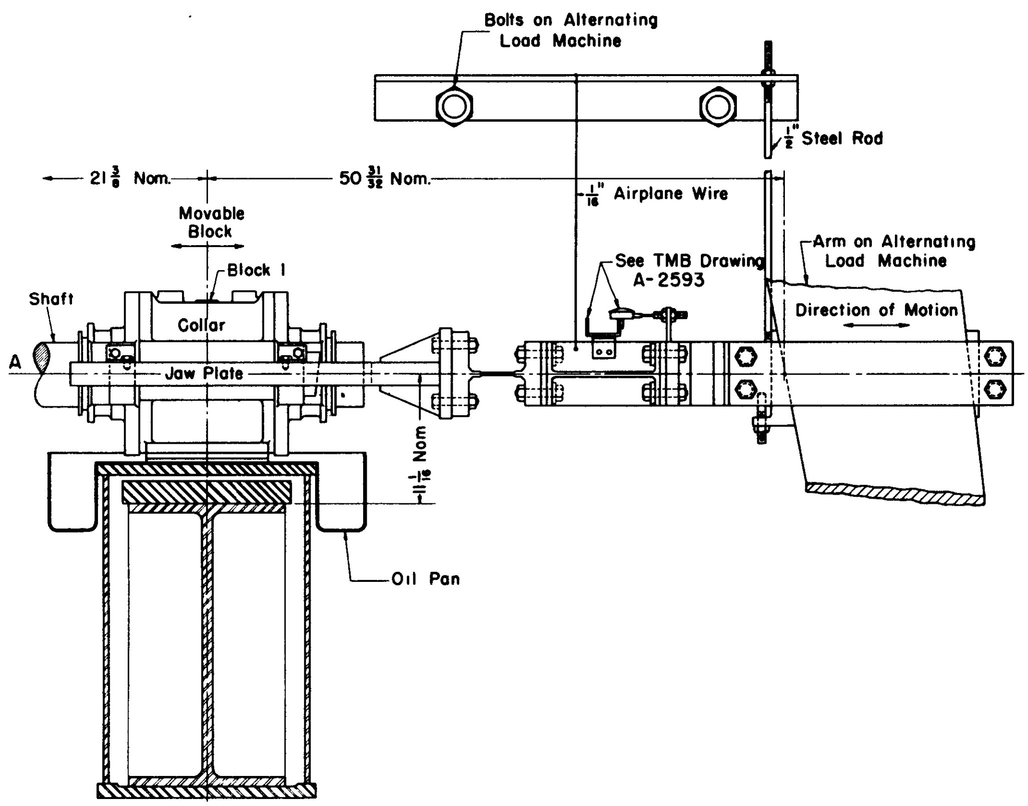
* For a general description of this machine, see Appendix 2 of TMB Report 493.

The alternating load from the testing machine was transmitted from the loading arms to the movable unit, Block 1, by a transverse spring beam *G*, as shown in Figure 10. This was an I-beam free to flex in the center, with a straight edge and a dial gage attached to it for indicating its deflection. It had a spring constant of 365,000 pounds per inch, and served as the load-measuring device, as the restraining-block assembly was attached to the testing machine at a point where the loads could not be measured by the mechanism built into the machine. The maximum load that could be applied was 45,000 pounds in each direction. The maximum loading rate was 250 cycles per minute; this was considerably less than that to be expected on the ship installation. Because a device of this kind always gives greater damping, all other things being equal, at higher frequencies, this feature introduced a considerable factor of safety, as it were, favorable to the full-scale unit.

The common shaft for the two model blocks was rotated by an electric motor and belt, as shown in Figures 10 and 11, to simulate the rotation of the line shaft in the ship. Although the maximum speed of rotation was increased to double that on the ship, about 400 RPM, the rubbing speed in the steady bearings was only about 60 per cent of that to be expected at full speed on the ship. As Block 1 was supported on flexure plates which permitted



a slight vertical motion as the block was moved horizontally, and as the whole test installation weaved and worked an appreciable amount under heavy loads, the model afforded an excellent opportunity to prove the steady bearings under conditions fully as severe as were to be expected in service.

Figure 12, which is to scale except for exaggerated tapers and clearances, shows the essential differences between the ideal case used in developing the theory and the 0.3-scale model. The cylinder around the piston is formed of two separate liners with an open space at midlength of the

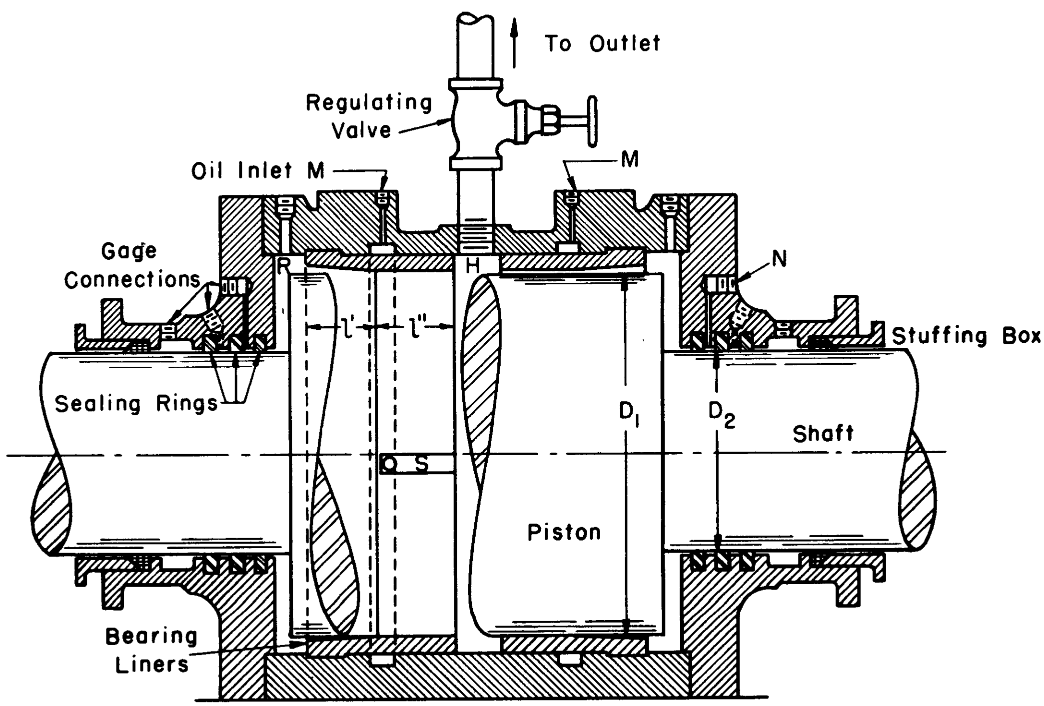


Figure 12 - Longitudinal Section through Model Shaft-Restraining Block

The block was symmetrical longitudinally, and both model blocks were constructed from the same plans. The bearing liners and sealing rings were in halves, fastened together with screws.

casing. The outer ends of each liner, for about half the length, were tapered to prevent binding of the piston or collar in the bearings in the event of misalignment of either the shaft or the bearing.

The bearing liners for the full-scale units were designed as steel castings, with babbitt linings; steel was used to avoid the effects of differential expansion in the pistons, liners and casings. As the model liners were too thin to line them satisfactorily with anti-friction metal with the equipment at hand, they were first made of cast iron, likewise to avoid the effects of differential expansion in the cast-iron housings. However, these liners galled upon starting the shaft, because the load could not be removed while an oil film was being established in the bearing. They were later replaced by bronze liners.

A great deal of thought was given to the proper method of introducing and routing the oil, as it was realized at the outset that the shaft sealing rings could by no means be made oil tight and that the unit would develop heat and would have to be cooled. Finally, it would be a complete failure if the end pressure chambers were not kept constantly filled with solid oil, free of air or oil vapor bubbles, and if the volume of leakage oil through the clearance spaces around the piston were not limited to a certain small amount.

The best method of accomplishing this important result was to maintain a positive oil pressure in each chamber at all times. This meant that a reasonably high steady pressure had to be maintained in the chambers and that it should be made easier for fresh oil to find its way into the chambers than to leak out of them. The oil supply system finally adopted embodied a constant supply of cooled oil at high pressure, at midlength of the piston, sufficient to provide bearing oil, a constant supply to the end chambers through the clearance spaces around the piston, and a constant small leakoff through the shaft sealing rings.

The lubricant used was Navy Specification 2190 oil, having an absolute viscosity, μ, of 4.65×10^{-6} pound-seconds per square inch at 150 degrees fahrenheit. It was introduced at the two inlets M, Figure 12, under a pressure of 200 pounds per square inch. It then went through an annular groove cut in the cylinder and through two holes in the liners to two slots S on opposite sides of the piston. These slots connected with the space H between the liners. From here the oil went through a throttling valve to the outlet, thence to the pump and back through the system. The slots S served to distribute oil along the length l'' of the rotating piston which was in contact with the liners, and this oil served to cool the bearings. The amount of flow could be controlled by means of the outlet valve.

The combination of the length l' in which the liner was tapered, and the slot S for introducing the bearing oil, both made necessary because of the steady-bearing feature, left little length remaining for the small clearance region between the piston and the liners; so little, in fact, as to apparently vitiate the restraining-block action by providing a low resistance by-pass for oil flow between the pressure chambers R. Nevertheless, the desired result was achieved.

The building up of a high-pressure oil region in the space H and the clearances l'' forced oil out into the chambers R and kept them full for damping purposes. A certain amount of oil leaked past the three sealing rings at each end. During part of the tests, oil was introduced under pressure between the first and second sealing rings from the chamber, to assist in keeping these chambers full of solid oil.

The principal dimensions of the 0.3-scale model blocks were as follows:

$D_1 = 10.5$ inches	$l'' = 2.2$ inches
$D_2 = 5.73$ inches	$A_1 = \frac{\pi}{4}(10.5^2 - 5.73^2)$
$l' = 2.0$ inches	$= 60.6$ square inches

Throughout the length l'' the nominal radial clearance on the model was 0.004 inch. Throughout the tapered portion l' the nominal radial clearance increased from 0.004 inch to 0.006 inch.

On the basis of these dimensions, without considering any corrections and assuming no taper in the length l', the formula for damping constant C can be applied as follows: (Subscript of letter C indicates that the value is not final)

$$C_A = \frac{A_1^2}{S} = \frac{12\mu l_1 A_1^2}{D_1 \pi c_1^3} = \frac{12\mu\,(2l' + 2l'')A_1^2}{D_1 \pi c_1^3}$$

$$C_A = \frac{12 \times 3.44 \times 10^{-6}\ (2 \times 2 + 2 \times 2.2)\ 60.6^2}{10.5\pi \times 4^3 \times 10^{-9}}$$

$C_A = 603{,}000$ pound-seconds per inch (with no corrections included).

The corrections that must be applied to this figure are as follows:

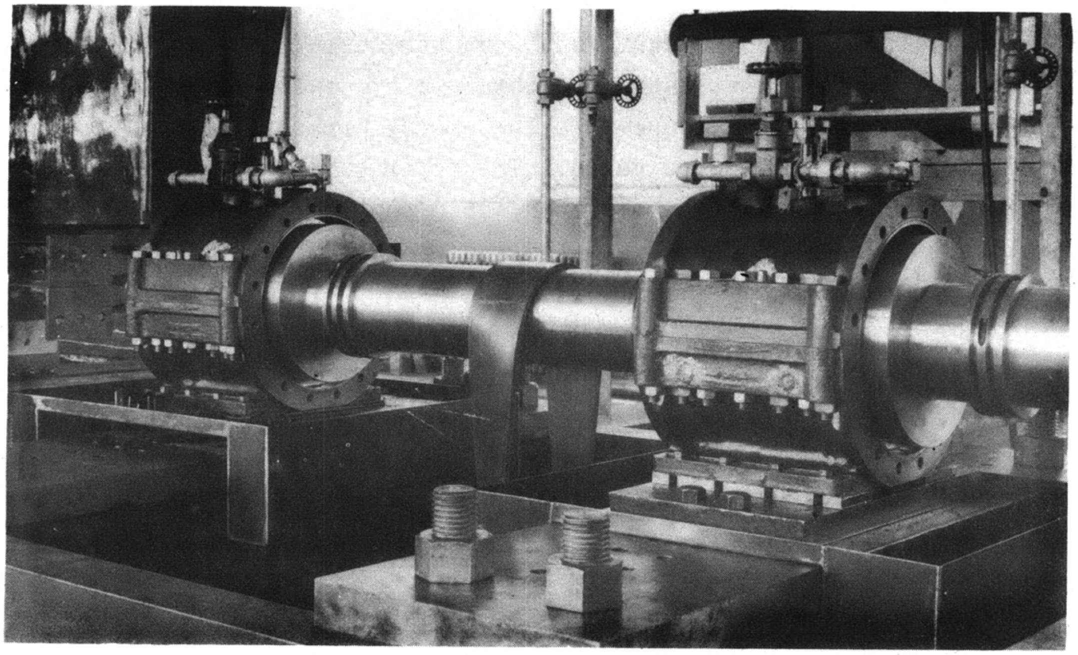
1. Since the piston rests practically on the bottom of the bearing liners, as shown in Figure 12, the radial clearance throughout l'' varies from 0.000 inch or more at the bottom to 0.008 inch at the top. Throughout the length l' this eccentricity effect and the effect of taper co-exist and must be considered simultaneously. In Appendix 4 it is shown that these corrections in the lengths l' and l'' together reduce the damping constant C_A from 603,000 pound-seconds per inch to $C_B = 200{,}000$ pound-seconds per inch.

2. The presence of the slots S introduces another correction, reducing $C_B = 200{,}000$ pound-seconds per inch to $C_C = 187{,}000$ pound-seconds per inch. See Appendix 5.

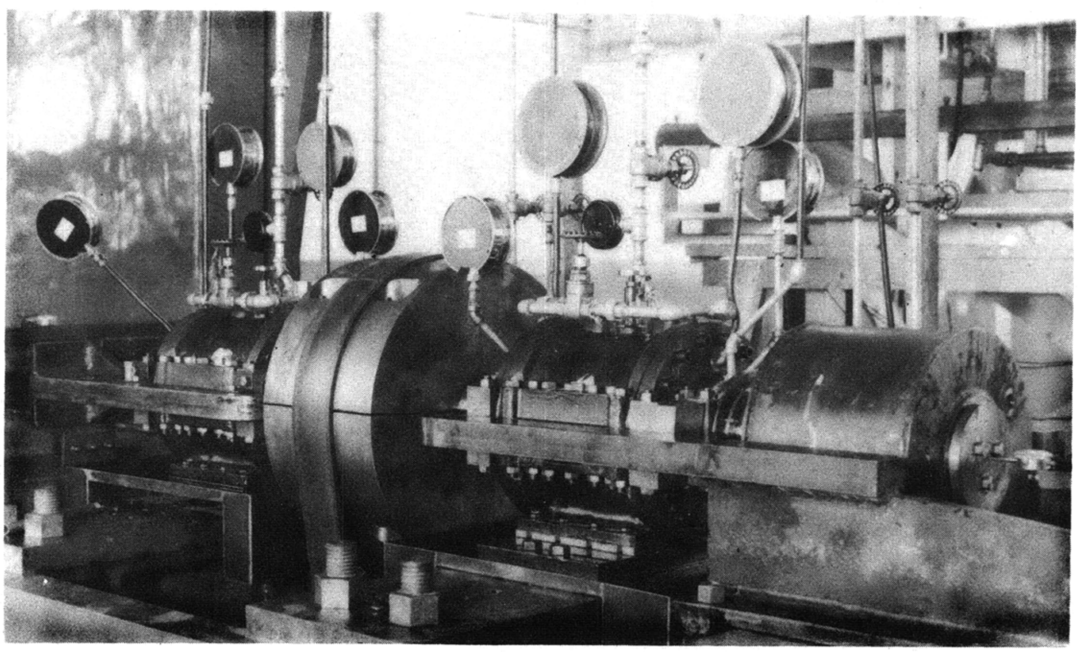
3. Alternating leakage through the sealing rings reduces $C_C = 187{,}000$ pound-seconds per inch to the final theoretical value for damping constant, $C = 175{,}000$ pound-seconds per inch; see Appendix 6. The last two corrections are of minor importance compared to the first correction.

TEST RESULTS FROM 0.3-SCALE MODEL

Figures 13 and 14 are photographs of the 0.3-scale model, showing the extra weights which were added to the rotating parts, the oil piping, and



The end covers of the chambers are removed, and the shaft is moved out of position toward the observer, to show the collars. The three sealing rings are shown on the shaft near each block.



The entire assembly is in place, as during a test. In the foreground is the fixed model block, with the rotating weight on the near side and the heavy pulley on the far side; in the background is the movable block attached to the reciprocating arms of the testing machine. Note the heavy jaw plates around each block just below the shaft center.

Figures 13 and 14 - General Views of 0.3-Scale Restraining-Block Model in Alternating Load Testing Machine various auxiliary gages and equipment that were used for obtaining preliminary data. The final test data were obtained by taking moving pictures of the instruments shown in Figure 15. The steady pressure maintained was measured by Bourdon tube pressure gages. The force amplitude was obtained by multiplying the end area of the piston, 60.6 square inches, by the amplitude of the pressure difference between the end pressure chambers, as obtained from the De Juhasz gage.

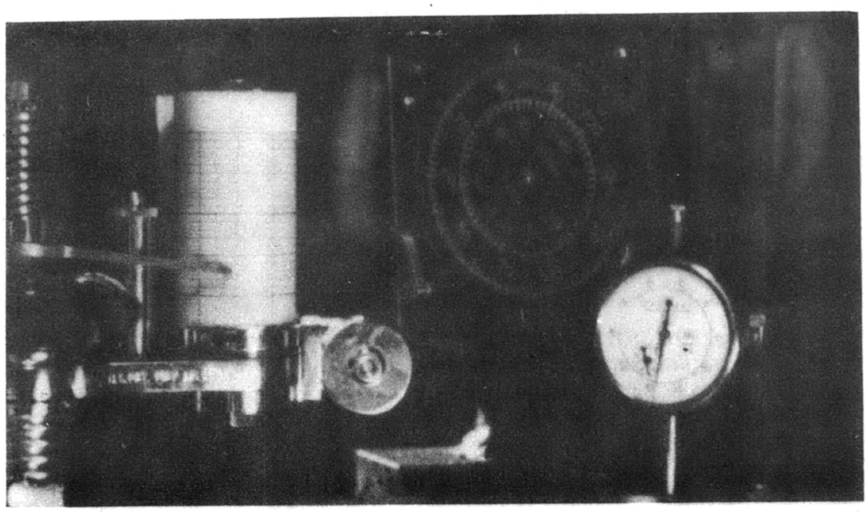


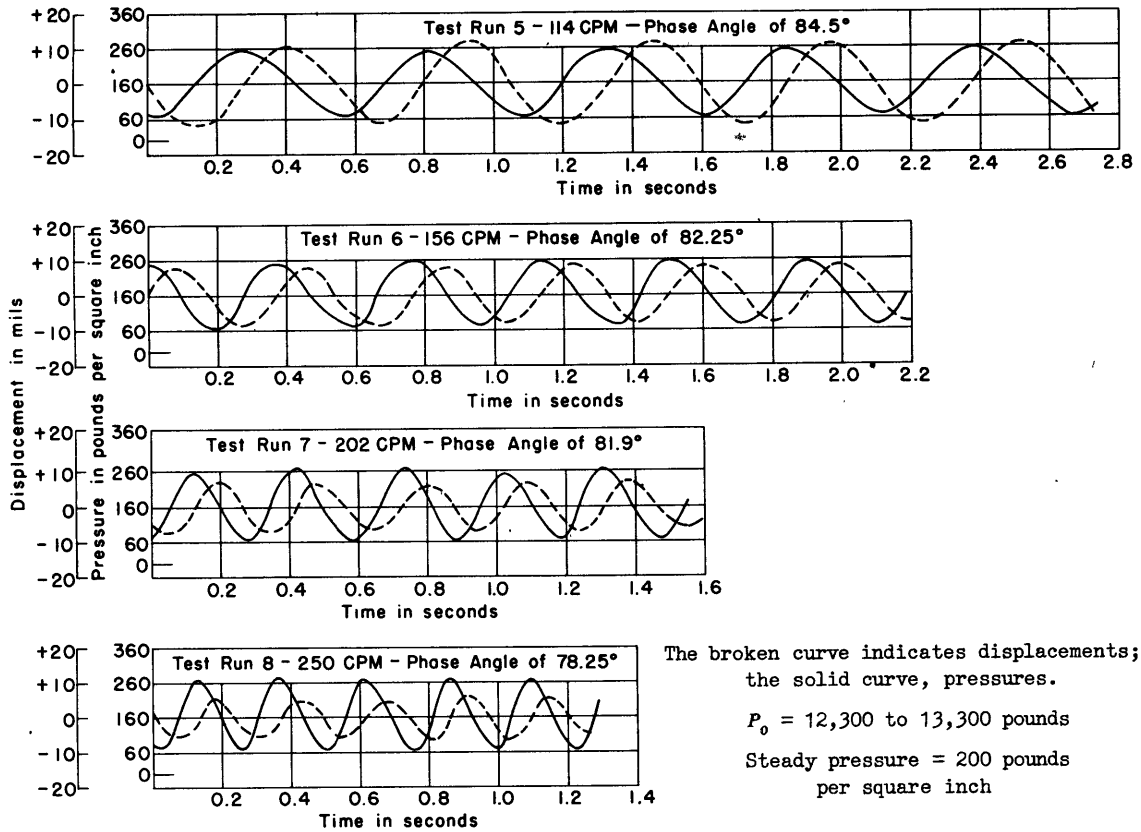
Figure 15 - Motion Picture Record of Gage Readings

These instruments are, from left to right, a De Juhasz Diesel Engine Indicator, a clock reading to 0.01 second, and a dial micrometer reading to 0.001 inch.

The moving picture data when plotted show continuous records of displacement of the shaft relative to Block 2 and of oil pressure in the west end of Cylinder 2, both against time as obtained from the clock. Figure 16 shows such data for four different frequencies. The force amplitude throughout this run varied between 12,300 and 13,300 pounds. Table 3 is compiled from these data, plus additional pressure data for the east end of the cylinder.

The observed damping constant C is considerably less than the theoretical value of 175,000 calculated in the preceding section. This is probably due to a small amount of air being trapped in the oil. The following considerations substantiate this belief:

1. The angle ϕ between pressure and motion is smaller than would be expected; see Appendix 3. This could be caused by a considerably reduced effective bulk modulus for the oil, as would be the case were air trapped in the oil.



The broken curve indicates displacements; the solid curve, pressures.

P_0 = 12,300 to 13,300 pounds

Steady pressure = 200 pounds per square inch

Figure 16 - Displacement-Time Curves for 0.3-Scale Model and Pressure-Time Data for West End of Cylinder 2

TABLE 3

Values of Damping Constant obtained from the Relation $C = \frac{P_0}{\omega x_0}$ from Tests of the 0.3-Scale Model

Steady Pressure, 200 pounds per square inch

Frequency CPM	P_0 pounds	p_0' pounds per square inch east end	p_0' pounds per square inch west end	ϕ degrees	x_0 inches	C_N pound-seconds per inch
114	12300	103	100	84.50	0.0115	89400
156	12400	110	95	82.25	0.0085	89300
202	13300	120	100	81.90	0.0065	97500
250	13300	120	100	78.25	0.0055	92300

x_0, the amplitude of residual motion; p_0', the amplitude of pressures; and ϕ, the phase angle between x and p', against actuation frequency in cycles per minute.

2. Close observation of Figure 16 shows distortion of the sinusoids, such as would be caused by a non-linear compressibility of the fluid used. Air by itself has a logarithmically variable compressibility, and its mixture with oil would hence produce a variable compressibility for the resultant fluid.

3. Considerably better results were obtained, but not recorded photographically, the day before the installation had to be dismantled. At this time an amplitude of 0.0035 inch was obtained at 250 cycles per minute and 13,000 pounds force amplitude. These values give 145,000 pound-seconds per inch for *C*, which is in much better agreement with the theoretical value.

Considerable trouble was experienced in making the installation perform well as a steady bearing. In the early attempts, when cast-iron liners were used, a 24-hour run showed considerable galling on the liner and piston. Bearing bronze liners were then installed and another 24-hour test was started. During a night shift of this run the unit stuck at the end of 13 hours. The next morning, however, it started up again satisfactorily and was then run for a total of 35 hours. No galling was observed upon dismantling, so the cause of the shut-down at 13 hours remained undetermined.

FULL-SCALE DESIGN

While the model tests were underway, the development of the full-scale design was proceeding. Although the full-scale installation was not made, for reasons given elsewhere in this report, the final plans are listed here for ready reference in the future.

BuShips Plan	Title	Date
BB55-S43-41, Alt. 2	Assembly of Shaft-Restraining Block for 19 1/8-inch Shaft.	26 July 1941
BB55-S43-42, Alt. 3	Shaft-Restraining Block for 19 1/8-inch Shaft, Details - Journal Shell, Collar, Seal Ring.	24 July 1941
BB55-S43-43, Alt. 3	Shaft-Restraining Block for 19 1/8-inch Shaft, End Cover Details.	26 July 1941
BB55-S43-44, Alt. 3	Detail of Chamber.	26 July 1941
BB55-S43-45, Alt. 2	Shaft-Restraining Block for 19 1/8-inch Shaft, Details of Oil Slinger and Oil Leak-Off Housing.	July 1941
BB55-S43-46, Alt. 2	Shaft-Restraining Block for 19 1/8-inch Shaft, Foundation Bolting and Chocks.	July 1941

An article on pages 42 to 48 of the Bulletin of Information Number 6, published by the Bureau of Ships under date of 1 April 1942, describes tests made at the Navy Yard, Philadelphia, to develop the procedure necessary to secure adherence of the babbitt lining in the wrought-steel bearing shells of the full-scale shaft-restraining blocks.

CONCLUSIONS AND RECOMMENDATIONS

The model results obtained indicate that a full-scale installation would give ample damping on board ship. Assuming the same oil viscosity in the model and in the full-scale restraining block, it is possible to predict the full-scale damping constant from the model tests. The damping constant of the model is

$$C_{\text{model}} = \frac{12\,\mu\, l_1 A_1^2}{\pi D_1 c_1^3}$$

The full-size damper is n times larger in all dimensions and will use the same oil at the same temperature. Therefore its damping constant will be

$$C_{\text{ship}} = \frac{12\mu\,(n l_1)(n^4 A_1^2)}{\pi(nD_1)(n^3 c_1^3)} = nC_{\text{model}}$$

For the 0.3-scale model, $n = 1/0.3 = 10/3$. Using the most pessimistic observed value of C, 89,400 pound-seconds per inch for the 0.3-scale model, a full-scale damping constant of 298,000 pound-seconds per inch is indicated by similitude. Reference to Figure 21, Appendix 7, shows that one-third of this value is sufficient to reduce the vibration to one-eighth its natural amplitude at the frequency corresponding to resonance in the first mode of motion, and to one-fourth the natural amplitude in the second mode of motion.

The restraining-block design theory here developed gives reliable results only if air has been thoroughly eliminated from the oil and its further exclusion is permanently assured. Since these conditions are difficult to maintain in practice, it is recommended that a safety factor of about 3 be used in the design of any full-scale equipment contemplated in the future. That is, taking into consideration the corrections discussed in the Appendices, the final value of C thus obtained should be three times the value actually desired for a ship or machinery installation.

The following recommendations are made for future designs:

1. Thorough and adequate provision must be made for venting air from all parts of the system. If practicable, a length of strong glass tubing should be incorporated in the vent so that air, if present, can be directly observed.

2. As a safety factor, the damping constant actually needed should be trebled for design purposes because of the difficulty of keeping the system entirely free of air.

3. Provision must be made for cooling and de-aerating the oil. The latter requirement necessitates a relatively large sump or settling tank.

4. The damping constant varies inversely as the cube of the radial clearance between piston and cylinder; the clearance can be considerably affected by such factors as expansion due to temperature change, pre-straining of parts during assembly, and other causes. Hence some method of checking the clearance after assembly and at working temperature should be provided. This can readily be accomplished by inserting micrometer depth gages in different parts of the cylinder. The holes cut for this purpose would have to be plugged against the high pressure during operation.

APPENDIX 1

SPECIFICATIONS FOR SHAFT-RESTRAINING BLOCKS AND SHIP INSTALLATION

1. The primary function of the shaft-restraining block shall be to maintain, in a fixed longitudinal position relative to the ship, that portion of the main propeller shaft which is rotating within the block, under the influence of a periodically varying thrust force as applied by the ship's propeller to the shaft abaft the block.

2. The restraining block is intended to form an artificial axial node in the propeller shaft at its point of installation and to act as the equivalent of an infinite mass fixed to the shaft at that point, so that the shaft sections forward and aft of the block, together with their attached masses, may be considered as independent vibratory systems.

3. There shall be combined with the block a pair of spring or steady bearings which shall maintain the clearances around the piston portion of the block and retain the oil in the restraining chambers.

4. The shaft-restraining block shall be built to operate with a steel propeller shaft approximately 19.125 inches outside diameter, having attached to it a fixed steel collar or piston, with an outside diameter of approximately 33 inches.* The fore and aft length of the collar shall be sufficient to prevent objectionable deflections of the collar under the influence of loads in either direction equal to that specified in the paragraph following, and to prevent excessive leakage of oil from one side to the other around the periphery. This thickness shall be at least 6 inches and preferably 8 inches.

5. The shaft-restraining block and all its parts, together with the attachment of the block to the ship structure and the adjacent parts of that structure, shall be designed to take thrust loads for short periods (as when executing emergency astern) equal to the full thrust load which can be developed by the propeller, with a margin for alternating load, totaling 500,000 pounds. Under these and other conditions specified, the oil pressure in any part of the block shall not exceed 1000 pounds per square inch.

6. The shaft-restraining block shall be designed to fulfill its primary and secondary functions under the influence of alternating variations in thrust equal to 150,000 pounds, reckoned alternately below and above the mean thrust value. This shall be accomplished at an axial frequency as low as 400 cycles per minute (the low frequency is here the limiting one because leakage will be greater at low than at high frequencies).

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* Later changed to 35 inches.

7. At a limiting low axial frequency of 400 cycles per minute and an alternating thrust load of 100,000 pounds, reckoned above and below the mean thrust value, the permissible movement of the shaft longitudinally within the housing shall not exceed 0.002 inch in either direction (a total of 0.004 inch). The movement of the shaft longitudinally with respect to adjacent ship structure which is not subjected to severe loading shall not in any case exceed 0.004 inch in either direction (a total of 0.008 inch).

8. The restraining thrust block housing shall be attached to the ship structure by members in the approximate plane of the shaft centerline or by pairs of members disposed symmetrically on either side of that plane, so that when longitudinal forces are applied by the shaft to the block, the latter will not be forced out of proper position with respect to the shaft.

9. The final securing of the restraining block to the structural members shall be through the medium of keys or distance pieces so placed that the axial loads through these parts are transmitted directly, without the possibility of movement because of long holding-down bolts or tilting keys.

10. The shaft-restraining block shall permit a longitudinal shaft movement or float of one (1) inch in either direction from mid-position without metallic contact of the sides of the collar with the sides of the chambers.

11. Surfaces of the restraining chambers and of the shaft collar which do not form oil clearance spaces shall be reasonably smooth, to prevent undue churning of the oil when the shaft is rotating.

12. The shaft-restraining block shall function satisfactorily as a special spring or steady bearing at all shaft speeds up to and including 210 RPM. If necessary, a special cooling system shall be installed to keep the bearing oil and the restraining oil down to acceptable limiting temperatures. The shaft-restraining block shall function as such and as a special spring or steady bearing with any type of Navy contract lubricating oil that may be used in the regular spring bearings in the shaft alleys.

13. The oil in the ahead and astern restraining chambers of the block shall be circulated through those chambers and through the spring bearings at either end of the block by a shaft-driven or a motor-driven pump which will maintain a pressure in the chambers of approximately 200 pounds per square inch. This pressure shall be sufficient in any case to maintain a positive gage pressure in each chamber under any specified operating condition of the restraining block.

14. The entire shaft-restraining block shall be so designed and constructed that it will remain completely filled with oil even though the

circulating pump is out of commission. A head of oil of one (1) foot above the top of the restraining chambers should be sufficient for this purpose.

15. All parts of this system shall be capable of withstanding maximum oil pressures of 1000 pounds per square inch.

16. The spring bearings at the ends of each restraining block* shall be fitted with separate shells lined with low-friction bearing metal. It shall be possible to open up the restraining block and to renew or repair the bearing shells without disturbing the shaft in the ship.

17. The collar for each restraining block shall be mounted rigidly on the shaft by a suitable method (to be developed) in such manner as to transmit the maximum load to the collar without the latter working loose on the shaft. The method of attachment shall, however, avoid setting up high localized stresses in the shaft. Grooving of the outside of the shaft section in way of the block shall be limited to a maximum depth of 0.25 inch and shall be performed in a manner approved by the Bureau of Ships.

18. The necessary connections shall be provided in the restraining chambers for applying pressure, circulating and by-passing the oil, and for taking pressure readings.

* In the final design, the spring or steady bearings were incorporated in the block proper. They consisted of two bearing liners in the chamber and the piston as the journal. The arrangement is shown by Figure 12 on page 16.

APPENDIX 2

DISCUSSION OF ASSUMPTIONS USED FOR DEVELOPMENT OF THEORY

1. Sliding friction is negligible.

The total weight of the moving parts in the 0.23-scale hydraulic model is about 300 pounds. If the coefficient of friction is assumed as 0.05, which is probably too high, the sliding friction is 15 pounds. This is about 2 per cent of 800 pounds, the smallest force amplitude used during tests.

2. No cavitation occurs in the fluid.

This will not be true unless a pressure greater than the pressure p_0 is maintained in the unit at all times. If cavitation is allowed to occur, the damping constant C is reduced to less than half of its value with no cavitation, since the pressure difference available for driving the fluid is reduced and the resultant mixture of air and oil destroys the incompressibility of the damping fluid.

In practice, unless a steady pressure is maintained, the device will suck air from outside, and the resulting mixture of oil and air will give highly unreliable results. In both models discussed in this report, steady pressure was maintained in the systems by auxiliary pumps.

3. The structure is completely rigid.

This cannot be achieved in practice, of course, so the complete mechanical system, including the compliance* of the supporting members, must be worked out in each application. Measurements taken on the models were relative to the cylinder housing.

4. Acceleration forces are negligible.

From the test results on the 0.23-scale hydraulic model at 600 cycles per minute, the amplitude of residual motion x_0 = 0.00035 inch when P_0 = 1600 pounds. Taking the weight of moving parts as 300 pounds, the accelerational force amplitude is

$$m\omega^2 x_0 = \frac{300}{387} \times 0.35 \times 10^{-3} \times \left(\frac{600}{60}\, 2\pi\right)^2 = 1.07 \text{ pound}$$

and hence may be neglected when compared to 1600 pounds.

A similar calculation for the 0.3-scale working model shows a maximum acceleration force of 55 pounds compared to about 10,000 pounds applied force amplitude at 250 CPM.

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* "Compliance" in the structural sense is defined as the reciprocal of "spring constant" as used in analysis of vibrating systems. It is expressed in terms of the deflection produced by unit load, and is here used in preference to "rigidity," which is load required to produce unit deflection.

5. c_1 and c_2 are small compared to D_1, D_2, l_1, and l_2.

	0.23-Scale Model inches	0.3-Scale Model inches
c_1	= 0.004	0.004 to 0.006
c_2	≅ 0.001	0.001 to 0.002 (approximately)
D_1	= 7.9	10.5
D_2	= 4.42	5.73
l_1	= 8.37	8.4
l_2	= 2.75	1.3 (approximately)

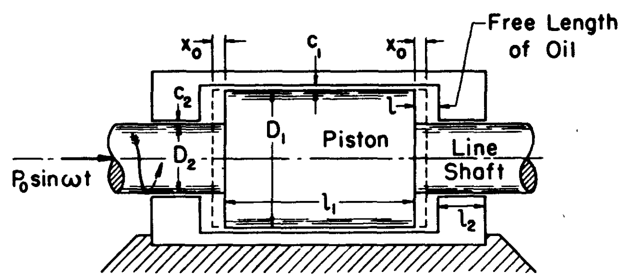

Figure 2 - Simplified Longitudinal Section of Shaft-Restraining Block

6. Leakage through c_2 is negligible.

In the case of the 0.23-scale hydraulic model, S can be calculated for the c_2 leakage paths in the same way that it was for the c_1 paths; see pages 7 and 8.

$$S = \frac{D_1 \pi c_1^3}{12 \mu l_1} = \frac{7.9\pi \times 4^3 \times 10^{-9}}{12 \times 1.16 \times 10^{-6} \times 8.37} = 0.0136$$

$$S_2 = \frac{D_2 \pi c_2^3}{12 \mu l_2} = \frac{4.42 \times 1^3 \times 10^{-9}}{12 \times 1.16 \times 10^{-6} \times 2.75} = 0.000362$$

Since $Q = pS$, the rate of flow through c_2 is much smaller than that through c_1 and may therefore be neglected in the case of the 0.23-scale model. The situation is somewhat different in the case of the 0.3-scale model and is discussed in Appendix 5.

APPENDIX 3
EFFECT OF FLUID COMPRESSIBILITY

The fluid has been considered incompressible in the development of design formulas in the "Development of General Theory," pages 3 to 8. The conditions under which this assumption is valid will now be investigated.

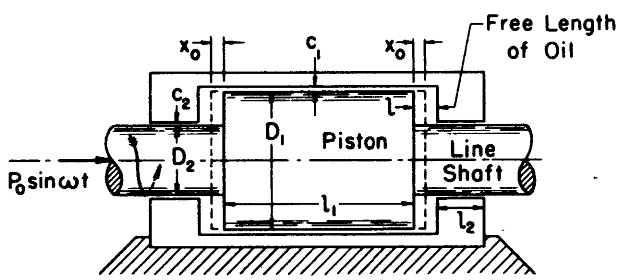

Figure 2 - Simplified Longitudinal Section of Shaft-Restraining Block

If there is no flow through the radial clearance, that is, if $c_1 = 0$, a piston movement dx decreases the volume of oil in one end of the cylinder and increases it in the other end, by an amount $A_1 dx$, where A_1 is the end area of the piston. Noting that l is the free length of oil in the end clearance chambers, Figure 2, the pressure change thus caused in each end clearance is

$$dp_1 = M_B \frac{A_1 dx}{A_1 l} \quad \text{when } c_1 = 0$$

In this equation M_B is the bulk modulus of the fluid used and has the dimensions of pounds per square inch.

In an actual case $c_1 \neq 0$, of course, and hence a volume of fluid passes through c_1 in a time dt equal to Qdt. The net change in volume in the end chambers is then $A_1 dx - Qdt$, and the resultant pressure change in each end of the cylinder is

$$dp_1 = M_B \frac{A_1 dx - Qdt}{A_1 l} \quad \text{when } c_1 \neq 0$$

Since an equal pressure change occurs in the opposite end of the cylinder, the total pressure change $dp = 2dp_1$ or

$$dp = \frac{2M_B}{A_1 l}(A_1 dx - Qdt)$$

A sinusoidally varying force $P = P_0 \sin \omega t$ has already been assumed in Figure 2. Since sliding friction and acceleration forces are being neglected, the restraint exerted by the restraining block must be produced by the pressure in the end chambers and hence must be in phase with this pressure. This means that the pressure difference $p = p_R - p_L$ must vary sinusoidally and must be in phase with the impressed force, $P_0 \sin \omega t$. A force to the right in Figure 2 is considered positive. The subscripts R and L indicate pressures associated with the right and left pressure chambers.

The residual piston motion will be assumed sinusoidal but of unknown amplitude and phase relation with respect to the pressure. To evaluate these unknowns, let

$$x = M \sin \omega t + N \cos \omega t$$

$$dx = (\omega M \cos \omega t - \omega N \sin \omega t)\, dt$$

Since

$$p = p_0 \sin \omega t$$

$$dp = \omega p_0 \cos \omega t \; dt$$

Substituting these values in the equation

$$dp = \frac{2M_B}{A_1 l} (A_1 dx - Q\, dt)$$

$$\omega p_0 \cos \omega t \; dt = B(A_1 \omega M \cos \omega t - A_1 \omega N \sin \omega t)\, dt - BS p_0 \sin \omega t \; dt$$

where

$$B = \frac{2M_B}{A_1 l} \text{ and } Sp = Q$$

Canceling dt's and equating coefficients of sine and cosine terms:

$$0 = -BA_1 \omega N - BSp_0$$

$$\omega p_0 = BA_1 \omega M$$

whence

$$N = \frac{-Sp_0}{A_1 \omega} \quad \text{and} \quad M = \frac{p_0}{BA_1}$$

and

$$x = \frac{p_0}{BA_1} \sin \omega t - \frac{p_0 S}{A_1 \omega} \cos \omega t = \frac{p_0}{A_1} \sqrt{\frac{1}{B^2} + \frac{S^2}{\omega^2}} \; \sin\left(\omega t - \tan^{-1} \frac{BS}{\omega}\right)$$

which shows that the residual motion has an amplitude of

$$x_0 = \frac{p_0}{A_1} \sqrt{\frac{1}{B^2} + \frac{S^2}{\omega^2}} = \frac{P_0}{A_1^2} \sqrt{\frac{1}{B^2} + \frac{S^2}{\omega^2}} = \frac{P_0 \sqrt{B^2 S^2 + \omega^2}}{A_1^2 B \omega}$$

or

$$x_0 \omega \frac{A_1^2 B}{\sqrt{B^2 S^2 + \omega^2}} = P_0$$

From this equation the equivalent damping constant C' is given by

$$C' = \frac{A_1^2 B}{\sqrt{B^2S^2 + \omega^2}}$$

which approaches

$$C = \frac{A_1^2}{S} \text{ as } \omega \to 0$$

This is the formula derived on page 6 of the report.

The angle $\phi = \tan^{-1} \frac{BS}{\omega} = \frac{2M_B \pi D_1 c_1^3}{\omega A_1 l \; 12\mu l}$ is the phase angle between the residual motion and the pressure in the end chambers.

It will be noticed that C' is not a true damping constant, in that it includes the restraining effect due to the spring effect of the fluid used.

To determine the frequency at which a 10 per cent error would be introduced by neglecting this correction, let

$$0.9 \frac{A_1^2}{S} = \frac{A_1^2 B}{\sqrt{B^2S^2 + \omega_1^2}}$$

$$0.81(B^2S^2 + \omega_1^2) = B^2S^2$$

$$\omega_1 = \sqrt{0.19}\; BS \quad \text{for 10 per cent error.}$$

Using M_B = 300,000 pounds per square inch, the product BS equals 245 for the 0.23-scale model and 400 for the 0.3-scale model. So:

$\omega_1 = \sqrt{0.19} \times 245 = 107$ radians per second = 1020 CPM for the 0.23-scale model, and

$\omega_1 = \sqrt{0.19} \times 400 = 174$ radians per second = 1660 CPM for the 0.3-scale model.

Since the highest frequency considered in the tests is 600 CPM, the correction may be neglected.

At the highest frequency used for the 0.3-scale model (= 250 CPM) $\phi = \tan^{-1} \frac{BS}{\omega} = \tan^{-1} \frac{400 \times 60}{250 \times 2\pi} = 86.25$ degrees (compare observed values, Table 3, page 21.)

APPENDIX 4
EFFECT OF PISTON ECCENTRICITY AND TAPER

Figure 17 is a diagrammatic cross section taken along the length l'', showing the eccentricity caused by the piston bearing on the bottoms of the liners. γ'' is the actual clearance at any angle θ measured up from the bottom center of the piston, where $\gamma'' = 0$. Since the nominal clearance, 0.004 inch, is very small compared to the diameter, 10.5 inches, the actual clearance γ'' is given very closely by the expression

$$\gamma'' = 0.004\ (1 - \cos\theta)$$

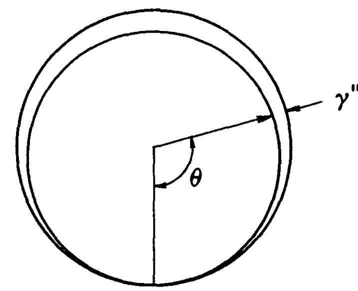

Figure 17 - Diagram showing Eccentricity of Piston

To find an effective clearance c'' to use for this length l'' of the unit, the root-mean-cube* value of γ'' against θ must be obtained, since it is the cube of the clearance that appears in the expression for the damping constant

$$C = \frac{12\mu l_1 A_1^2}{D_1 \pi c_1^3}$$

Therefore c'' is given by the expression

$$c'' = \sqrt[3]{\frac{\int_0^{2\pi} (\gamma'')^3\, d\theta}{2\pi}} = \sqrt[3]{\frac{\int_0^{\pi} (\gamma'')^3\, d\theta}{\pi}}$$

$$= 0.004\ \sqrt[3]{\frac{\int_0^{\pi} (1 - \cos\theta)^3\, d\theta}{\pi}}$$

$$= 0.004\ \sqrt[3]{1 + \frac{3}{2}}$$

$$= 0.00543 \text{ inch}$$

Using this value c'' for the clearance, a value for the damping constant C'' produced by the two lengths l'' can be calculated as follows:

$$C'' = \frac{12\mu\, (2l'') A_1^2}{D_1 \pi (c'')^3}$$

$$= \frac{12 \times 3.44 \times 10^{-6}\ (2 \times 2.2) \times 60.6^2}{10.5 \times \pi \times 5.43^3 \times 10^{-9}}$$

$$= 126{,}000 \text{ pound-seconds per inch}$$

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* By root-mean-cube is meant the cube root of the average of the cubes.

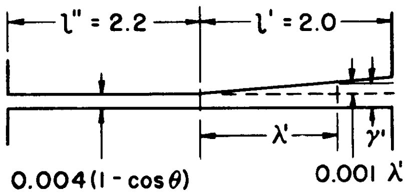


Figure 18 - Longitudinal Cross Section showing Clearance

Throughout the tapered length l' the eccentricity just considered must be taken into account as well as the taper, whose nominal value varies from 0.004 to 0.006 inch. Figure 18 shows a longitudinal cross section of the clearance. The angle θ has the same meaning as in Figure 17, but the section is taken at a point where $\theta \neq 0$.

At any point λ', θ along the tapered length l', the actual clearance, γ' is given by the expression

$$\gamma' = 0.001\ [4(1 - \cos\theta) + \lambda']$$

To find the equivalent clearance c' to use for this tapered length, the root-mean-cube value of γ' must be obtained against both θ and λ' as variables. That is:

$$c' = \sqrt[3]{\frac{\int_0^{\pi}\int_0^{2} (\gamma')^3\, d\theta\, d\lambda'}{2\pi}}$$

$$c' = 0.001 \sqrt[3]{\frac{\int_0^{\pi}\int_0^{2} [4(1 - \cos\theta) + \lambda']^3\, d\theta\, d\lambda'}{2\pi}}$$

$$c' = 0.001 \sqrt[3]{250}$$

$$= 0.0063$$

Using this value c' for the clearance, a value for the damping constant produced by the lengths l' can be calculated as follows:

$$C' = \frac{12\mu\,(2l')A_1^2}{D_1 \pi (c')^3}$$

$$= \frac{12 \times 3.44 \times 10^{-6}\ (2 \times 2.0) \times 60.6^2}{10.5 \times \pi \times 6.3^3 \times 10^{-9}}$$

$= 73{,}500$ pound-seconds per inch

The damping constant C_B, due to both of these effects is equal to

$$C_B = C'' + C'$$

$$= 126{,}000 + 73{,}500$$

$\cong 200{,}000$ pound-seconds per inch

APPENDIX 5

DAMPING-CONSTANT CORRECTION FOR SLOTS IN CYLINDER LINERS

The damping-constant correction due to the presence of the longitudinal slots S, Figure 12, will now be considered. Figure 19 shows a development of the clearance surface in one end of the cylinder. There is another such clearance surface at the other end, separated from the first by the annular slot H, Figure 12. There are thus four slots S altogether, one parallel pair in series with another parallel pair. This combination may be seen to offer the same resistance to flow as one slot, so calculations will be made as though the four slots were replaced by one slot. The same is true of the regions near the slots, such as $\overline{KA}$ in Figure 19.

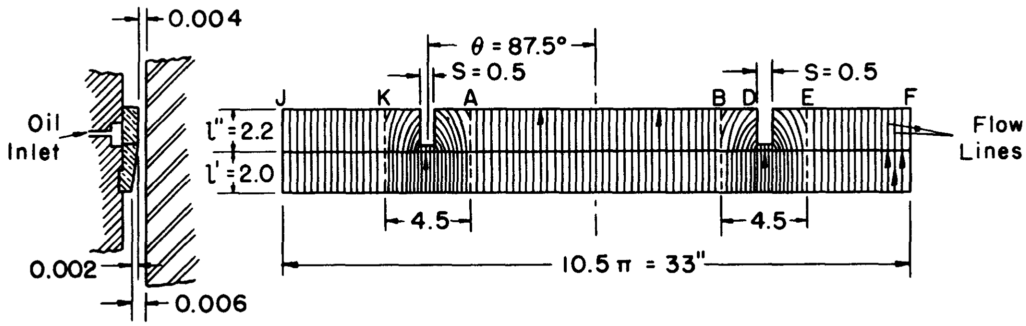


Figure 19 - Diagram showing Section through Bearing Liner and Development of Clearance Space

The dimensions given are in inches for the 0.3-scale model.

The formula for S is

$$S = \frac{\pi D_1 c_1^3}{12 \mu l_1}$$

This applies to the complete cylinder, where the peripheral distance under consideration equals the entire circumference, $= \pi D_1$. If only a part of the periphery is considered, such as $\overline{KA}$, the formula becomes

$$S = \frac{\overline{KA}\, c_1^3}{12 \mu l_1}$$

Proper substitutions must also be made for the radial clearance, c_1, and for the axial length, l_1.

In Appendix 4 the damping constant C_B was found to be 200,000 pound-seconds per inch. Recalling that $S = \frac{A_1^2}{C}$,

$$S_B = \frac{60.6^2}{200{,}000} = 18.4 \times 10^{-3}$$

The first step is now to find what part of S_B is produced by the region $\overline{KA}$. This part will be called S_Q and when subtracted from S_B will leave a remainder S_R. A corrected value, S_S, will then be calculated for S_Q and added to S_R, giving a value S_C which takes the slot correction into consideration. Since $S = \frac{Q}{p}$, it will be noted that it is proportional to quantity of flow, and hence two parallel paths, each characterized by S, will have a resultant $2S$. If the paths were in series, their resultant would then be 1/2 S.

In Figure 19, the symbols θ, l', and l'' have the same significance as in Figures 17 and 18, Appendix 4. The slot is thus seen to be located 87.5 degrees around the cylinder periphery from the bottom of the cylinder. The peripheral distance for which correction is to be made is 4.5 inches including the width of the slot itself, which is 0.5 inch. Small error and considerable simplification of the work is introduced by considering the radial clearance for the length l'' constant between K and A and equal to that calculated by the formula $c_e'' = 0.004\ (1 - \cos\theta)$ for a point midway between K and A. Since θ = 87.5 degrees at this point

$$c_e'' = 0.004\ (1 - \cos 87.5°) = 0.0038 \text{ inch}$$

Hence

$$S_Q'' = \frac{\overline{KA}\,(c_e'')^3}{12\mu l''}$$

$$= \frac{4.5 \times 3.8^3 \times 10^{-9}}{12 \times 3.44 \times 10^{-6} \times 2.2}$$

$$= 2.72 \times 10^{-3}$$

For the axial length l', the effective clearance c_e' may be found from the formula given in Appendix 4, placing θ = a constant = 87.5 degrees.

$$c_e' = \sqrt[3]{\frac{\int_0^{l'} (c_e'' + 0.001\,\lambda')^3\, d\lambda'}{l'}}$$

$$= 0.001 \sqrt[3]{\frac{\int_0^{2} (3.8 + \lambda')\, d\lambda'}{2}}$$

$$= 0.00425 \text{ inch}$$

Hence

$$S_Q' = \frac{\overline{KA}\,(c_e')^3}{12\mu l'}$$

$$= \frac{4.5 \times 4.25^3 \times 10^{-9}}{12 \times 3.44 \times 10^{-6} \times 2}$$

$$= 4.19 \times 10^{-3}$$

Since S_Q' and S_Q'' are in series

$$S_Q = \frac{1}{\frac{1}{S_Q'} + \frac{1}{S_Q''}}$$

$$= \frac{10^{-3}}{\frac{1}{4.19} + \frac{1}{2.72}}$$

$$= 1.62 \times 10^{-3}$$

and

$$S_R = S_B - S_Q$$

$$= (18.4 - 1.62)\ 10^{-3}$$

$$= 16.78 \times 10^{-3}$$

The corrected value S_S will also be computed in two parts. The part directly in line with the slot will be called S_x, and comprises a region of length 2.1 inches and width 0.5 inch. The other part will be called S_y and will refer to the rest of the region $\overline{KA}$. S_x may be calculated easily as

$$S_x = \frac{0.5 \times 4.25^3 \times 10^{-9}}{12 \times 3.44 \times 10^{-6} \times 2.1}$$

$$= 0.443 \times 10^{-3}$$

To calculate S_y, the major assumption regards the length of flow. All the flow lines shown in Figure 19 traverse a distance $l' = 2.0$ inches to begin with. Throughout the axial distance l'' some flow lines travel the entire distance, 2.2 inches, and others, near the slot, only about 0.1 inch. Since there is obviously a greater flow density near the slot than at a distance from it, these shorter paths should be given more weight in arriving at an average. Therefore the assumption is made that the average length of flow in the region under consideration is about 40 per cent of $l'' = 0.88$ inch. Hence the average length of flow to be used in calculating S_y is 2.0 + 0.88 inches = 2.88 inches. To determine what effective clearance to use, the clearances c_e' and c_e'' may be combined as follows, weighting them according to the path length over which they apply.

$$c_e = 0.001 \sqrt[3]{\frac{2.0 \times 4.25^3 + 0.88 \times 3.8^3}{2.0 + 0.88}}$$

$$= 0.00412 \text{ inch}$$

Hence

$$S_y = \frac{(4.5 - 0.5) \times 4.12^3 \times 10^{-9}}{12 \times 3.44 \times 10^{-6} \times 2.88}$$

$$= 2.35 \times 10^{-3}$$

$$S_S = S_x + S_y$$

$$= (0.443 + 2.35)\,10^{-3}$$

$$= 2.79 \times 10^{-3}$$

and

$$S_C = S_R + S_S$$

$$= (16.78 + 2.79)\,10^{-3} = 19.57 \times 10^{-3}$$

So

$$C_C = \frac{A_1^2}{S_C} = \frac{60.6^2}{19.57 \times 10^{-3}} = 187{,}000 \text{ pound-seconds per inch}$$

APPENDIX 6
EFFECT OF END LEAKAGE

In the 0.3-scale model the end pressure chambers were closed with sealing rings; see Figure 12, page 16. In view of the difficulty of getting accurate clearance measurements between the sealing rings and the shaft, particularly when hot, an actual experimental observation was taken of the rate of oil leakage. After the oil had reached a steady temperature, the alternating force was removed. The piping which introduced oil under high pressure at the points N was removed. High oil pressure, 200 pounds per square inch, was maintained at M, and the oil flowing from points N during one minute was collected and measured. After conversion to pound-inch-second units, the total leakage for both ends was 0.0057 cubic inch per second per pound per square inch of pressure. This value has the same dimensions as the constant S developed in the theory, inches^5 per pound-second, and will be called S_l in the following. Assuming an equal distribution of leakage between the two ends, each end will be characterized by $1/2\ S_l$.

Insofar as the alternating component of the pressure is concerned, the pressure amplitude at each end is one-half that between the two ends, hence the quantity of oil escaping per second through each end leakage is given by

$$Q_l = \frac{1}{2} p \times \frac{1}{2} S_l = \frac{1}{4} p S_l$$

The quantity of oil transferred per second from one end of the cylinder to the other, based on the value $S_C = 19.57 \times 10^{-3}$ (as determined in Appendix 5) is

$$Q_C = p S_C$$

The total volume rate of flow is equal to the piston displacement rate which is, in turn, equal to the sum of Q_C and Q_l

$$\begin{aligned} Q &= Q_C + Q_l \\ &= pS_C + \frac{1}{4} pS_l \\ &= p\left(S_C + \frac{1}{4} S_l\right) \end{aligned}$$

The final value of S is thus given by

$$\begin{aligned} S &= S_C + \frac{1}{4} S_l \\ &= \left(19.57 + \frac{5.7}{4}\right) \times 10^{-3} \\ &= 21.0 \times 10^{-3} \end{aligned}$$

and hence the final value for the damping constant is given by

$$C = \frac{A_1^2}{S} = \frac{60.6^2}{21 \times 10^{-3}} = 175{,}000 \text{ pound-seconds per inch}$$

Subsequent to the end leakage tests outlined in the foregoing the 0.3-scale model was operated with all the oil outlets leading to packing ring spaces closed. This forced the end leakage through the three sealing rings and the soft packing, and reduced the end leakage to about one-third or one-quarter of the amount recorded in the foregoing. The restraining action of the block was noticeably better under these conditions.

APPENDIX 7

USE OF ELECTRICAL ANALOGY METHODS FOR DETERMINING EFFECTIVENESS OF RESTRAINING BLOCK

This Appendix is added to the report for purposes of record, though it has nothing to do with the design of restraining blocks as such.

The question of how much effect any given proposed parameter such as mass, resilience, or resistance will have on any given mechanical system can be determined by the use of differential equations fairly readily if results at only one frequency are desired. The problem becomes quite laborious, however, if a complete curve of the effect throughout a large frequency range is desired, or if the effect of many different values of the parameter or parameters in question must be determined.

By setting up an electrical analogy of the mechanical system the work involved is very greatly simplified. Table 4 shows the analogous mechanical and electrical parameters.

TABLE 4

Equivalent Mechanical and Electrical Parameters

Mechanical Parameter			Equivalent Electrical Parameter		
Name	Symbol	Unit	Name	Symbol	Unit
Force	P	pound	Electromotive Force	E	volt
Displacement	x	inch	Charge	q	coulomb
Velocity	$\dot{x}$	inch per second	Current	I	ampere
Mass	M	pound-second^2 per inch	Inductance	L	henry
Compliance	$\frac{1}{k} = C_M$	inch per pound	Capacitance	C	farad
Mechanical Resistance	B	pound-second per inch	Resistance	R	ohm

It will be noticed that the symbol B has been substituted for the usual symbol C that is used in mechanical language for the damping constant. This was done to avoid confusion with the symbols for compliance and capacity.

Since mechanical vibrations are usually of relatively low frequency compared to electrical phenomena, it is often desirable to carry out the electrical analogy work in a different range of frequencies so that a commercial electrical oscillator may be used to provide the variable frequency. First the equivalent quantities C_1, R_1, and L_1 must be determined for the original frequency f_1 from Table 4. Then by use of Table 5 the equivalent parameters C_2, R_2 and L_2 at the new frequency, f_2 can be obtained. In this table $n = \frac{f_2}{f_1}$. Other conversion tables could of course be devised. This particular one was used in this case because it involved no change in q, which is equivalent to the mechanical amplitude.

TABLE 5

Conditions Satisfied in Electrical Analogy Test

$$
\begin{aligned}
f_2 &= n^1 f_1 & \qquad C_2 &= n^0 C_1 \\
I_2 &= n^1 I_1 & \qquad R_2 &= n^{-1} R_1 \\
q_2 &= n^0 q_1 & \qquad L_2 &= n^{-2} L_1 \\
E_2 &= n^0 E_1 & &
\end{aligned}
$$

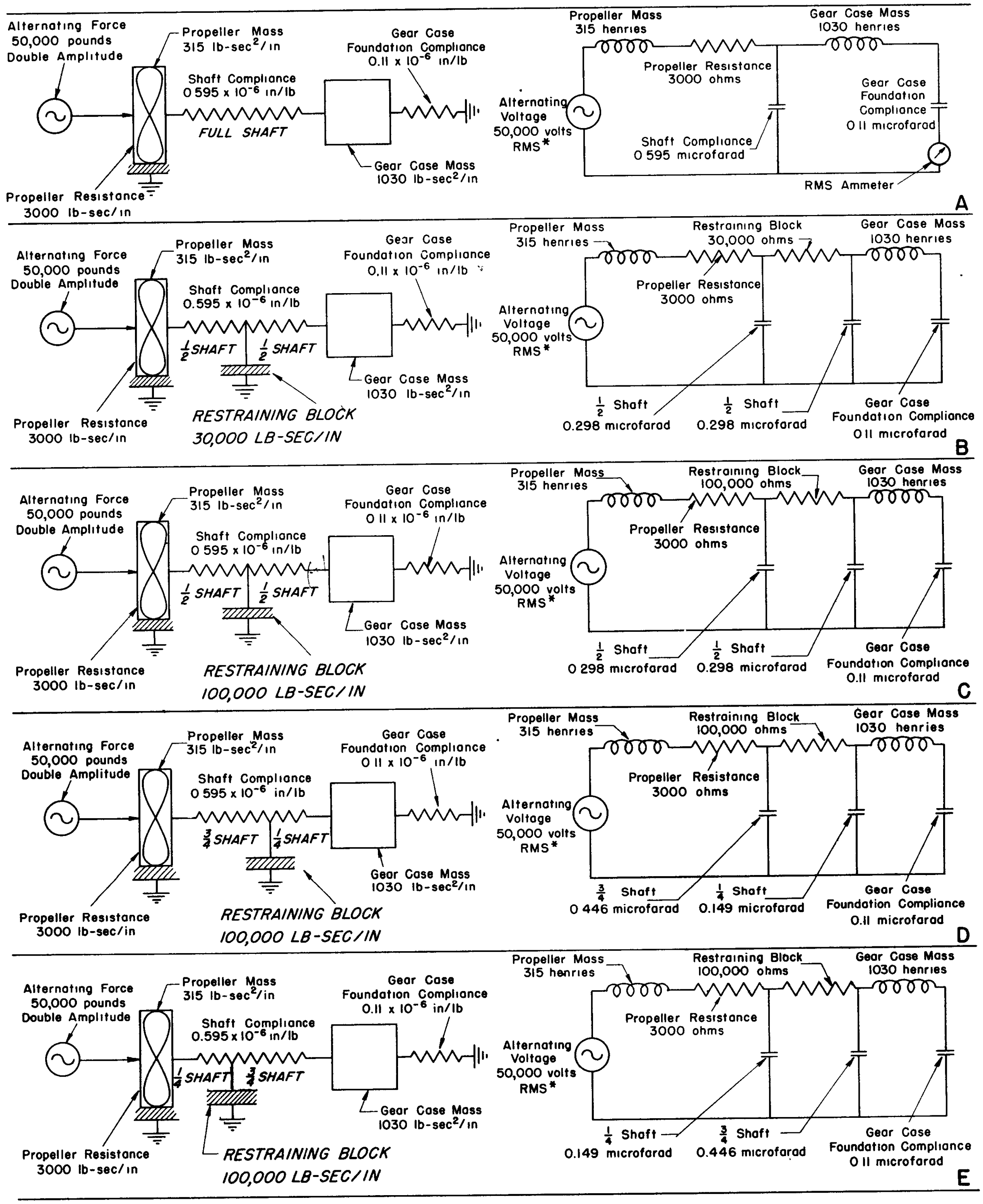
In Table 5 $f_2 = n^1 f_1$ and $q_2 = n^0 q_1$ are arbitrarily chosen relationships. All the other relationships are necessary physical consequences of the two arbitrarily chosen relationships.

In the investigation of the effectiveness of the restraining block, the propeller, shaft, gear case, and gear-case foundation were treated as a two-body system. Figure 20, Case A, shows this mechanical system and its electrical analogy. The restraining-block damping is not shown in this case, and the natural damping, aside from that associated with the propeller, is neglected.

Obviously 50,000 volts is too great a voltage to obtain from a commercial oscillator, so 10 volts was used instead. This meant that the indicated values of q had to be multiplied by the factor 50,000/10 = 5,000 to get the true values. The data obtained with the ammeter as shown in Figure 20, Case A, will give the double amplitude of the gear-case displacement when put into the equation:

$$2x_0 = \frac{I}{2\pi f} \times 5000 \text{ inches}$$

The range of vibration frequencies was too low, 240 to 1200 cycles per minute, or 4 to 20 cycles per second. Therefore a frequency conversion was made, using Table 5, so that the data could be taken between 40 and 200 cycles per second, where $n = 10$. The results of this run are shown in Figure 21, Curve A. The peak value in the first mode of motion is in good agreement with that obtained from pallograph readings taken on board one of the vessels mentioned in the report, indicating that the assumptions made are valid.



* Since coulomb values are obtained from readings on an ammeter reading RMS amperes, if the RMS voltage is the same numerically as the double force amplitude, then the RMS coulombs, i.e., RMS amperes divided by $2\pi f$, will have the same numerical value as the double displacement amplitude.

Figure 20 - Mechanical and Equivalent Electrical Circuits

The conditions which varied in the different runs are indicated by italic lettering.

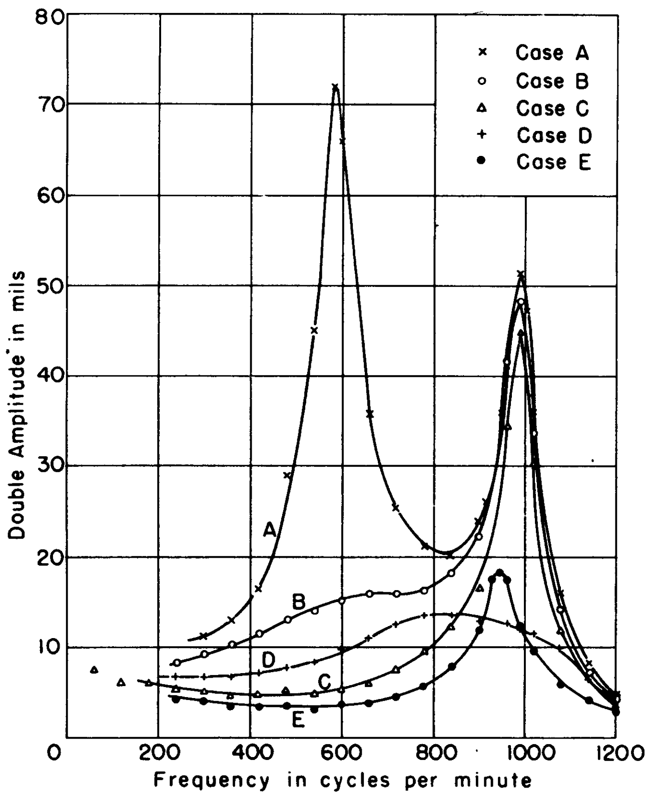


Figure 21 - Effectiveness of Restraining Block against Frequency for Different Locations and Damping Constants

The data were obtained by electrical analogy.

Runs were then made under the following conditions:

Case B. Damping constant = 30,000 pound-seconds per inch, located in the center of the shaft. See Figure 20, Case B, and Figure 21, Curve B.

Case C. Damping constant = 100,000 pound-seconds per inch, located in the center of the shaft. See Figure 20, Case C, and Figure 21, Curve C.

Case D. Damping constant = 100,000 pound-seconds per inch, located one-quarter shaft length from the gear case. See Figure 20, Case D, and Figure 21, Curve D.

Case E. Damping constant = 100,000 pound-seconds per inch, located one-quarter shaft length from the propeller. See Figure 20, Case E, and Figure 21, Curve E.

Inspection of Figure 21 shows that Cases B and C in the foregoing attenuate the first mode considerably, but have very little effect on the second mode. This is to be expected, since in these cases the restraining block would be located in a node as regards the second mode. Cases D and E are seen to attenuate fairly well the entire frequency range explored.

It would not be feasible to install the gear for Case E on board ship due to lack of room, but that for Case D could be installed. This would mean that the restraining block would be at the bottom of the access ladder to the shaft alley.

It should be noted that the maximum damping used in these runs, 100,000 pound-seconds per inch, is only one-third of that to be expected, as a pessimistic estimate, from the full-scale restraining block contemplated.

APPENDIX 8

DESCRIPTION AND TESTS OF CLAMPING MODELS

GENERAL

Various methods were proposed for attaching the piston or collar of the restraining block to the line shaft, having in mind that the existing line shaft had to be used, with only such changes to it as could be made in the shaft alley of the ship:

(a) Making the collar in halves, grooving the shaft in way of the enlarged portion at the selected steady bearing, recessing the inside of the collar to fit the grooves, and welding the two halves together around the shaft, using the shrinkage in the weld to obtain the equivalent of shrinking on a solid ring.

(b) Making the collar in halves, with each half integral with a long sleeve extension, welding the half-collars together and not grooving the shaft. Clamping to the shaft would be accomplished by bolting the sleeve extensions tightly together over the shaft.

(c) Grooving the shaft as in (a), fitting over the shaft a pair of shallow half-rings, recessed to fit the grooves, and then shrinking on over the half-rings a solid ring, which had previously been made up in halves, welded together over some convenient part of the shaft, then heated up and shrunk over the inner half-rings.

(d) Making two pairs of split collars, and clamping them together in pairs by bolts perpendicular to the shaft over two pairs of grooves in the shaft; then clamping the two collars together, with a small gap left between, by using bolts parallel to the shaft, thus tightening the collars on the shafts by both transverse and longitudinal clamping action.

(e) Making a single long collar in halves, long enough to extend about 2 inches beyond each end of the present enlarged portion in way of the steady bearing, then clamping the two halves firmly on the shaft by two rows of bolts, with nuts and heads recessed. No grooves were to be cut in the shaft, and the collars were to hold by friction only, with the enlarged portion of the shaft acting as a preventer.

Scheme (a), welding on two half-collars, was abandoned because of the difficulty of fitting grooves and recesses and the liability of springing the shaft after weld-shrinking the collar on. Scheme (b), using a bolted clamp at a distance from the collar, was abandoned because it was felt that the clamping should be at the collar, to avoid the elasticity of the intermediate section. Scheme (c), using a welded shrink ring over two shallow inside half-rings, was abandoned because it was feared that the outside ring, when welded, would not have a truly circular hole, and there would be no way of truing up this hole after welding the two halves of the ring together. Scheme (d) was eliminated because it was desired to avoid grooving the shaft.

It was decided to adopt Scheme (e), for the following reasons:

1. It appeared possible to work out a final design of collar which could be held to the shaft only by friction, thus obviating all the many difficulties involved in grooving the shaft, and in fitting and lapping heavy collar pieces accurately to these grooves in the confined spaces of the shaft alleys.

2. Reasonably accurate fitting could be obtained at the ends of the enlarged portion of the shaft, and these shoulders would then act as preventers to keep the collar from sliding on the shaft.

3. If, in the course of the design and construction, it was considered that a few grooves should be cut in the enlarged section of the shaft, this would still be feasible.

4. The collar could be made of the same material as the shaft, because no welding or heating would be necessary. The use of a high-strength material would probably be necessary because of the high bearing loads at the ends of the enlarged journal on the shaft, and because of the high stresses in the threads, if clamping studs were used.

5. The collar would be clamped mechanically over a relatively long section of shaft; this would reduce to a minimum the liability of a sprung shaft due to welding on or shrinking on relatively short collars which would require grooving the shaft.

6. The collar could always be tightened on the shaft, or removed and replaced, if for any reason repairs or changes were found necessary.

7. The clearance space for the oil between the restraining-block chamber and the collar would be materially lengthened.

The next problem in the design involved a method of mechanically clamping the two halves of the collar over the existing line shaft in the ship without any modifications to the shaft other than truing it up in the vicinity of the collar position. To be sure, the presence of an enlargement on the shaft, projecting 1/8 inch from the outer surface, which formed the journal of the original steady bearing, might have been used as a means of locking the collar in position endwise on the shaft. However, the same considerations which led to the abandonment of grooving the collar and the shaft, i.e., the difficulty of fitting the heavy parts of the collar to the shaft in the close confines of the shaft alley, led to the rejection of the journal projection as a means of holding the collar in position.

This left friction between the surfaces of the shaft and the collar as the only acceptable method of holding the two parts firmly together.

With an assumed maximum coefficient of friction of 0.1, for smooth steel surfaces free of oil, and a maximum longitudinal load of 500,000 pounds in either direction,* it was necessary to exert a total clamping force of not less than 1,600,000 pounds or not more than 2,500,000 pounds** between the two halves of the collar and the line shaft on the ship.

Problems were involved here for which no precedent could be found, so steps were taken to build two models for checking the various features of the final design. These models were built to meet, in their respective scales, the following specifications which had been developed for the ship installation:

(a) The shaft-restraining block shall be built to operate with a section of the existing line shafting. There shall be securely attached to the shaft a fixed steel collar in halves, with an outside diameter of approximately 35 inches. The fore-and-aft length of the collar shall be sufficient to provide adequate bearing area when the collar is used as the journal of the steady bearing, and to overlap the existing enlarged section of the shaft. The bearing shells (of the steady bearing) shall be sufficiently long to prevent excessive leakage of oil from each restraining chamber to the leak-off space around the periphery of the collar.

(b) The shaft-restraining block and all its parts, together with the attachment of the block to the ship structure and the adjacent parts of that structure, shall be capable of taking thrust loads in either direction for short periods equal to the full thrust load which can be developed by the propeller, 350,000 pounds, plus a margin of 150,000 pounds for alternating thrust, or a total of 500,000 pounds. Under these and other conditions specified, the oil pressure in any part of the block shall not exceed 1000 pounds per square inch.

(c) The shaft-restraining block shall be designed to fulfill its primary and secondary functions under the influence of alternating variations in thrust equal to 150,000 pounds, reckoned alternately below and above the mean thrust value. This shall be accomplished at an axial frequency as low as 400 cycles per minute.†

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* See Appendix 1, page 25.

** See the "Notes on Friction Clamping of Split Collars to Shaft," beginning on page 56.

† A specification frequency less than that which had produced objectionable vibration on the ships was used here as a factor of safety because oil leakage past the collar would be greater at low than at high frequencies.

The design for the full-scale clamp bolts was developed by the General Electric Company and was based upon the clamp bolt designs used by that company for securing together the upper and lower halves of high-pressure steam turbine casings. The clamp bolts were of special alloy steel and were provided with fine threads, engaging threads cut directly into the clamping collar, as shown in Figure 22.

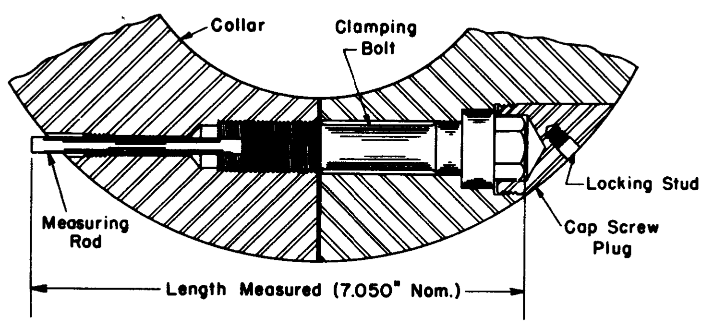

Figure 22 - Arrangement of Clamping Bolt, Plug, and Measuring Rod used in 0.3-Scale Clamping Model

The measuring rod shown at the left was employed to determine the elongation of the bolt, and the stress in it, when clamping the collar to the shaft. The hole for the measuring rod was later plugged in the manner shown for the recess over the bolt head.

Each full-scale bolt was to be drilled throughout its length so that when setting it up the bolt could be expanded by passing steam through it. The hole also permitted an overall measurement of the increase in length of the bolt, and the resulting stress in it, when it was finally set up. By heating the bolt and stretching it artificially, it could be set up with a given initial tension without the necessity of applying a high torque to tighten it in the customary manner. By measuring its increase in length when set up over its original length it was possible to determine quite accurately the stress in the bolt when it cooled and when all parts were finally assembled.

The first clamping model was made from material immediately available at the Taylor Model Basin so that the scale ratio had to be established at the rather low figure of 0.15. As a bolt of the General Electric design could not be used on such a small scale, it was necessary to substitute solid bolts. To clamp together the half-collars shown in Figure 9, twenty 3/8-inch by 2-inch Allen socket-head cap screws were used. These were found upon test to have an ultimate tensile strength of about 14,000 pounds or about two and a half times the designed clamping load in each bolt.* The bolts were threaded to the National Coarse Thread Standard, so that the fine threads which the ship design called for were not incorporated in the clamping model. The details of this 0.15-scale model are shown on TMB plan S-401, Alt. I.

DETERMINATION OF CLAMPING-BOLT LOADS

It was important that the bolts in the 0.3-scale clamping model be

* See the "Notes on Friction Clamping of Split Collars to Shaft," beginning on page 56.

set up to given loads, so that the total clamping force on the two model half-collars would be proportional, by the square of the scale ratio, to the total force to be employed on the ship. The ends of the model clamping bolts were therefore ground off smooth to permit measurement of their overall elongation by micrometer. Holes were drilled through to the outside of the collar under the threaded ends of the bolts, and special plugs were applied to the threaded ends and to the heads of the bolts to permit measurement of the increase in their length by micrometer as the bolts were set up. The general scheme is indicated in Figure 22.

Several of the model bolts were mounted in a special fixture corresponding to the half-collars, set up in a testing machine, and their elongation under the given load, 5600 pounds,* was determined. When the clamping bolts were placed in the collar and set up, they were tightened consecutively around the collar until the desired increase in length was attained on each bolt.

Before initial assembly, it was found that the 0.3-scale model half-collars had closed in about 0.0005 inch across their open ends as a result of the splitting operation, and it became necessary to press the halves of the collar into place on the shaft. The shaft and collar were cleaned carefully with alcohol, to remove all oil and dirt, the lower half of the collar was placed on the bed of a testing machine, the shaft was laid in it, and the upper half of the collar was placed on top of the shaft. The head of the machine was then brought down against the upper half-collar, to force it into place. A maximum load of 7000 pounds was required for this operation, so that for the first of the tests described, the clamping of the collar on the shaft was somewhat greater than that caused by the bolt loads alone.

CHANGE IN SHAPE OF COLLARS AND SHAFT WHEN CLAMPING

Measurements of the gap thickness between the two halves of the collar were made to determine the thickness of the liners to be inserted during the process of finish-boring the collars for the ship installation. The final design had to provide for stretching or deformation of the collars in a circumferential direction, and for a finished gap of the minimum practicable thickness in the assembled collar, without risking actual metal-to-metal contact of the two halves of the collar in the plane of the joint.

TESTS OF 0.15-SCALE MODEL

For the first series of tests, the model was placed in the alternating load testing machine, about as shown in Figure 23. Dial micrometers were

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* See the "Notes on Friction Clamping of Split Collars to Shaft," beginning on page 56.

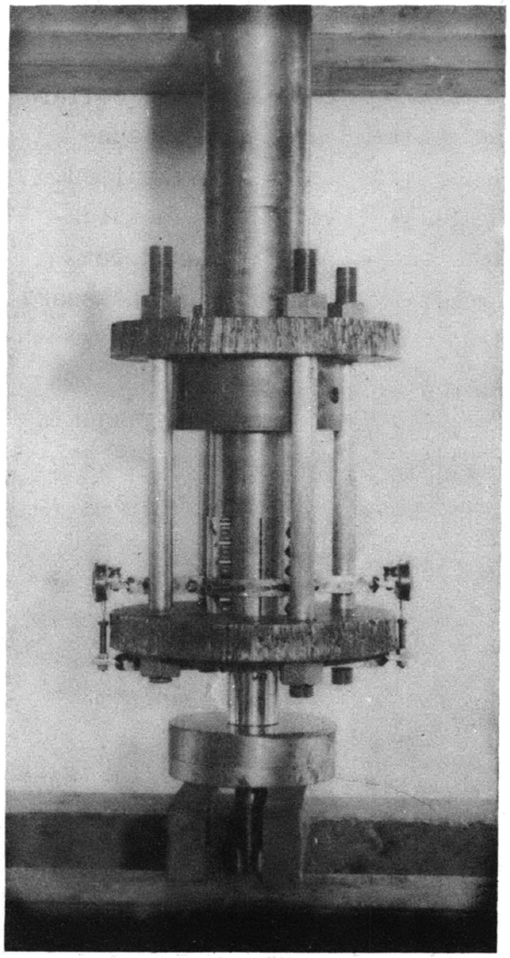

Figure 23 - General View of 0.15-Scale Clamping Model Under Test

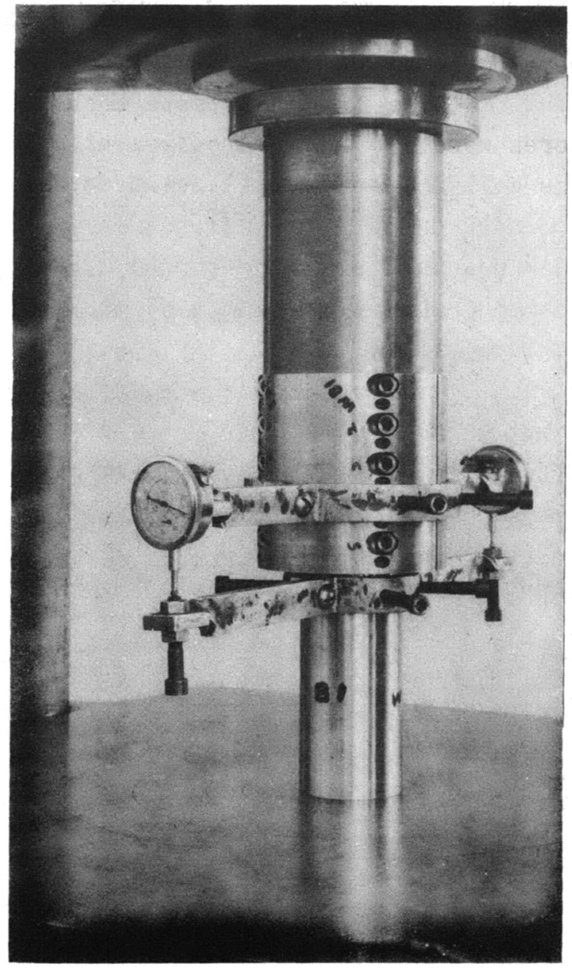

Figure 24 - Arrangement of 0.15-Scale Model for Static Test

placed with their plungers against the lower plate of the pulling linkage to measure the slip.* There was no definite indication of slip at static loads up to and including 11,000 pounds, corresponding to the maximum test load for the model, but when the load was unintentionally increased to 25,000 pounds, there was a slippage of about 0.004 inch.

After these preliminary static tests, the machine was adjusted to produce a load on the model of 3000 pounds in each direction. The machine was then run at the rate of 250 cycles per minute for 21,134 cycles. At various times during the run the dials were engaged and their movement was noted. There were no indications of additional slippage under the alternating loads.

After the test the model was removed from the machine, the clamping bolt lengths were remeasured, and the model was taken apart and carefully

* This was an earlier and slightly different arrangement from that shown in Figure 23.

examined. There was no indication of slipping or galling; a few bright spots were noted on the contact surfaces.

The foregoing tests were considered inconclusive because the collar bores were not truly circular, of the same shape as the shaft, and because the collar had definitely slipped before the alternating loads were applied. The half-collars were accordingly bolted together with shims of known thickness and the bore was ground out to within 0.0001 inch of the measured outside diameter of the shaft. During reassembly of the collars it was necessary to apply a load of only 1500 pounds, instead of the 7000 pounds previously used.

The model was then placed in the 30,000-pound static testing machine as shown in Figure 24. Loads increasing gradually up to a maximum of 15,000 pounds were applied without producing slip which could be detected on dial micrometers reading to 0.0001 inch. The model was again disassembled and examined. No changes in appearance were noted.

During the second series of tests the use of emery dust was proposed for increasing the coefficient of friction. Before reassembling for the third series of tests, a small amount of Number 220 emery powder was dusted uniformly on the inner surfaces of the half-collars. The model was then put together in the same manner as for the two previous tests, except that this time the halves were drawn together only by use of the bolts.

It was again placed in the static testing machine and loads were applied in increments up to a maximum of 30,000 pounds without producing measurable slip. Before each application of increasing load, the model was entirely unloaded. It was then placed in the alternating load machine as shown in Figure 23 and a load of 8000 pounds was applied in each direction at the rate of 250 cycles per minute for a total of 17,459 cycles. As there was no indication of slippage, the load was increased to 15,000 pounds in each direction and applied at the same rate for 13,498 additional cycles.

The model was then removed and set up in the static testing machine, and loads up to 30,000 pounds were applied without producing measurable slip.

The model was finally taken apart and examined. There was no change except some roughening of the contact surfaces due to scoring by the emery particles.

With emery dust inside the model collar, the final gap between the two halves when the collar was bolted up was 0.0035 inch less than the thickness of the liners clamped between the halves during grinding of the bore. The decrease in gap expected in the ship installation would be in proportion to the scale ratio, namely, about 0.023 inch.

It was found that the thickness of the gap remained sensibly uniform throughout its length and width as the collars were squeezed together by the clamping bolts.

Both collars and shaft were massive with respect to the clamping bolts, and the contraction in the gap may appear large in proportion to other dimensions. However, a very considerable amount of metal had been removed from the two half-collars in way of the clamping bolts, and probably the greater part of the deformation occurred in this region. It is of interest to note that the shaft contracted in diameter in a plane at right angles to the bolts.

CONSTRUCTION AND TESTS OF 0.3-SCALE MODEL

Following these tests a new clamping model was built to a scale of 0.3, following the same general design as the smaller model but with half-collars that were bored accurately after splitting, conforming to the procedure which was to be used for the ship installation. The arrangement was the same as for the smaller model, except that the half-collars were somewhat more elaborate, as shown in Figure 25, and clamping bolts were made similar to those for the full-scale design except that they were solid; see Figure 22.*

The halves of both collars were mounted easily on the model shaft without exerting any external pressure. When the collar members were in place on the shaft, the gaps were open about 0.020 inch, corresponding to twice the liner thickness; these gaps were easily closed to about 0.011 inch by setting up the clamping bolts hand tight.

As was done for the 0.15-scale clamping model, sample clamping bolts were loaded in a testing machine to the designed bolt load of 22,500 pounds** and the extension in bolt length corresponding to this load was determined to be 0.0050 inch. It is estimated that in collar 1-2, the final bolt load was less than 1 per cent larger than the designed load; in collar 3-4 it was about 5 per cent larger.

Measurements of the final gaps in the collars, taken after assembly on the shaft, showed the following:

(a) The maximum gap closure, at midlength of collar 1-2, was 0.006 inch, corresponding to 0.020 inch on the ship installation. If 0.025-inch liners were used when boring the ship collars, the final gap would be about 0.005 inch.

(b) The minimum gap closure, at the ends of the collars, was 0.0015 inch. If 0.025-inch liners were used when boring the ship collars, this would leave a resulting gap of 0.020 inch, which was considered much too large.

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* The details of these parts are shown on TMB plans A-2562, Alt. I, and A-2555, Alt. II.

** See the "Notes on Friction Clamping of Split Collars to Shaft," beginning on page 56.

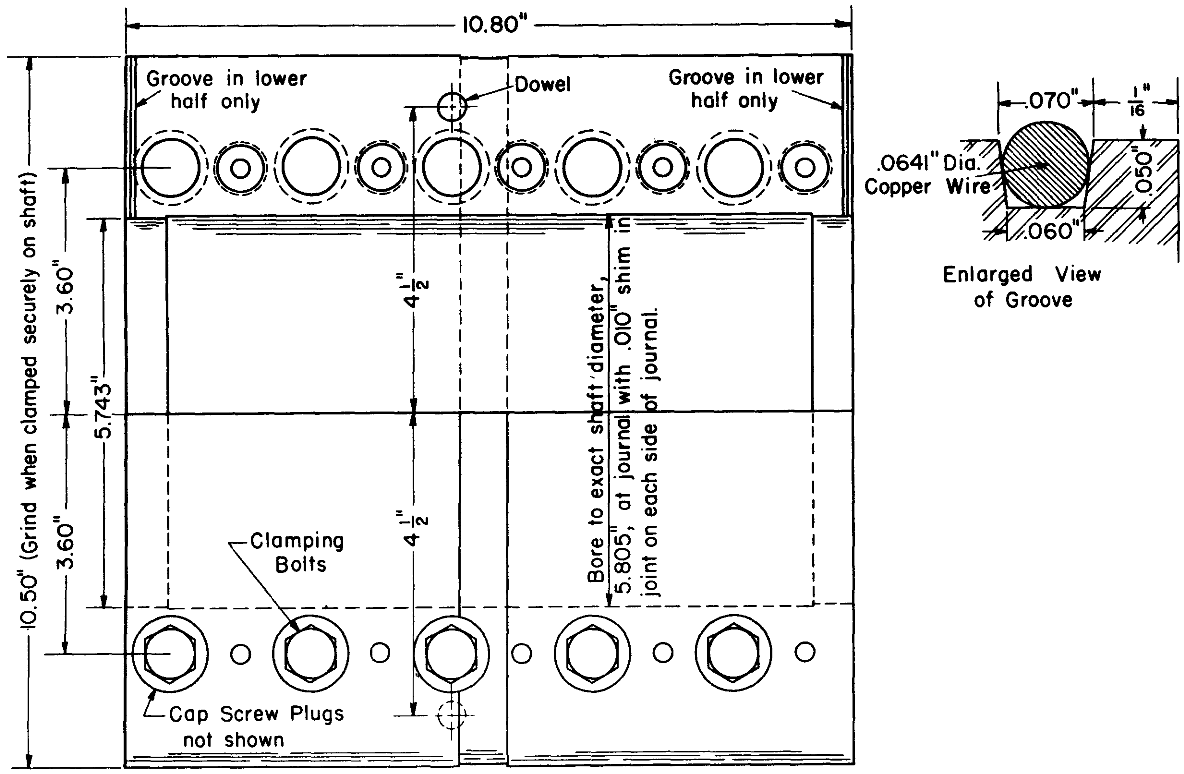

Figure 25 - Details of Half-Collars for 0.3-Scale Model

The upper portion of the half-section shows the parting face of one of the half-collars with the holes for the clamping bolts and the copper wire seals for preventing oil leakage from one end to the other. In the final ship design a corresponding copper wire seal was run longitudinally just inside the outer edge. The lower portion of the half-section is a view of the outside of the collar before the holes were plugged. The copper sealing wire is shown before it was squeezed into its groove by the clamping action.

(c) There were definite indications that the outside of the final gap, away from the shaft, would be smaller than the inside of the gap, next to the shaft. For the model this difference was about 0.0010 to 0.0015 inch, corresponding to about 0.003 or 0.004 inch on the ship, across the radial length of the gap.

(d) The gap in each collar on the model was from 0.003 to 0.0025 inch narrower at midlength than at the ends. This was accounted for by the fact that a unit length of the shaft in way of the midlength of the collar was subject to the full clamping load of the bolts for that length, whereas unit shaft lengths at the ends were subject to clamping only under the ends of the collar, and were free beyond the collar.

It was considered possible to equalize the gap opening by a reduction in clamping load of the bolts, carrying this progressively from each end toward midlength of the collar. This procedure would, however, have resulted in higher shaft stresses under the ends of the collar than under its midlength, a condition which was definitely unacceptable in a shaft subject to fluctuating stresses, and it was accordingly not recommended.

If it is assumed that the shaft under the midlength of the collar is definitely contracted in diameter with respect to the shaft under the ends of the collar,* there appears to be no objection to this condition provided the shaft runs true and remains true after the collars have been clamped upon it. As a check on this feature the 0.3-scale model shaft was swung in the lathe after the collar had been clamped, and dial micrometers were applied at various points along the shaft as it rotated. The run-out at any point along the shaft was definitely less than 0.001 inch;** this would correspond to about 0.002-inch run-out in 27 feet of shaft on the ship.

Measurements of shaft diameters taken before assembly of the 0.3-scale model were set down only to the nearest thousandth of an inch whereas those taken after assembly were read to the nearest ten-thousandth of an inch. It is therefore not possible to make extremely accurate comparisons to determine what, if any, changes in circular section of the shaft occurred beyond the ends of the collars, but it is apparent from the measurements made that there was a slight shortening of the shaft diameters parallel to the clamping bolts and a slight elongation of those diameters perpendicular to the bolts. The magnitude of these differences is less than 0.0005 inch on the model, corresponding to less than 0.0002 inch on the ship. The differences in the two shaft diameters disappear at about one and one-half inches from the ends of the collars, corresponding to about 5 inches on the ship.

MISCELLANEOUS

After consideration of the possibility of grit finding its way into the bearings, it was decided not to use emery powder on the ship collars; it was accordingly omitted when making up the 0.3-scale collars on the model shaft. Shaft surfaces and collar bores were cleaned carefully with alcohol and assembled metal-to-metal.

If the gap between the half-collars on the ship installation were found to be larger than that contemplated, this gap would form an objectionable leakage path for the oil from one end of the collar to the other. Because the two bearing shells were to be bored with a slight taper in the outer ends, this leakage path would exist, regardless of the copper-wire

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* There was no provision on the model for determining this.

** Maximum less minimum reading of the dial for one complete revolution of the shaft.

gaskets inserted radially between the halves of the collars at their ends, as shown in Figure 25. It was therefore considered necessary that longitudinal copper-wire gaskets also be used.

CONCLUSIONS

1. In the first series of model tests, without emery dust, static loads of 15,000 pounds did not produce slip, nor did 21,134 cycles of an alternating load of 3000 pounds. These corresponded to full-scale static loads of 680,000 pounds and full-scale alternating loads of 150,000 pounds. The specifications required loads of 500,000 pounds and 150,000 pounds respectively.

2. When the friction clamping was supplemented with a thin coating of emery dust, static loads of 30,000 pounds and alternating loads of 15,000 pounds did not produce slip. These corresponded to full-scale loads of 1,360,000 pounds and 680,000 pounds respectively.

3. The closing in of the slots at the sides of the collar due to tightening of the bolts amounted to 0.0035 inch on the model.* The corresponding amount for the ship was about 0.023 inch.

4. The clamping pressure of the collar caused a contraction in the diameter of the shaft, coincident with the plane of the joint in the collar, of about 0.0001 inch to 0.0002 inch. The diameter at right angles to this plane lengthened by the same amount.

5. The design of split and clamped collar for the ship restraining block was considered satisfactory, if the surfaces of both shaft and collar were smooth and accurately machined. The hole in the collar and the outside of the shaft should be as nearly size and size as machining tolerances would permit.

6. If emery dust were used between the contact surfaces an appreciable factor of safety in the design would be introduced.

RECOMMENDATIONS

Based upon the model tests, it was recommended

(a) That no major reduction in the liner thickness of 0.025 inch be made in boring the half-collars for the ship installation. In any event, the

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* This is the difference between the thickness of the shims inserted between the half-collars while the bore was ground and the gap between the half-collars when they were assembled tightly on the shaft, with emery dust.

gap opening for the clamped ship collars should be not less than 0.002 inch at any point. A final gap opening not exceeding 0.005 inch was considered acceptable.

(b) That longitudinal soft copper-wire seals be fitted longitudinally as well as radially in each collar gap, outside of the clamping bolts, and as near as practicable to the finished surfaces of the collars.

NOTES ON FRICTION CLAMPING OF SPLIT COLLARS TO SHAFT

In the haste of working up the original half-collar design, when the solution presented in this report was desperately needed, the total clamping force between the two halves of the collar was calculated by the simple operation of dividing the assumed coefficient of friction, 0.1, into the assumed longitudinal force, 500,000 pounds. The answer was 5,000,000 pounds, or 250,000 pounds divided between each of 20 clamping bolts. For the 0.15-scale and 0.3-scale models, the individual bolt loads then became 5600 pounds and 22,500 pounds respectively, as stated on pages 48, 49 and 52.

Simple reasoning, set forth in the paragraphs following, shows that these clamping loads were two to three times too large, but as they were actually used in the design and the tests, they have not been corrected in the text of this Appendix, except on page 47.

Dry or Coulomb friction on absolutely smooth surfaces is supposed to be independent of pressure between the rubbing surfaces and independent of the velocity of rubbing. This is the way it is defined and used in engineering calculations.

Actually, no mating surfaces are ever smooth. Little ridges in each surface mesh with each other and in order to slide, these ridges must be sheared off or climbed over. As to the effect of velocity, it is usual to differentiate between static and dynamic friction.

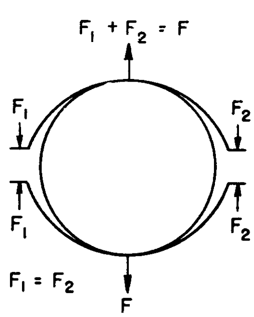

Figure 26

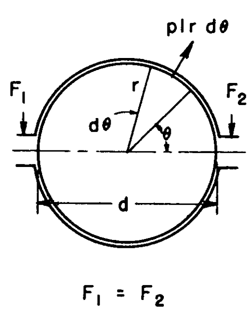

Figure 27

However, under the assumptions of dry friction it is possible to make calculations for the friction force necessary to hold a given load.

Suppose the situation is as shown in Figure 26, where the half-collars are slightly open at the sides. The total longitudinal frictional force f is μF (at the top) + μF (at the bottom), whence $2\mu F = f$. If f is assumed to be 500,000 pounds and μ is 0.1,

$$F = \frac{f}{2\mu} = \frac{500,000}{0.2} = 2,500,000 \text{ pounds}$$

When the fit of the two halves is snug, as shown in Figure 27, the situation is somewhat different. The clamping force on the mating surfaces now takes the form of a uniformly distributed pressure.

The force on an element of mating surface is $plr\,d\theta$. The vertical component is $plr\,d\theta\,\sin\theta$. The total clamping force

$$F_1 + F_2 = plr\int_0^{\pi} \sin\theta\ d\theta = plr\,[-\cos\theta]_0^{\pi} = 2plr = pld$$

$$F_1 = plr \qquad F_2 = plr \qquad F = F_1 + F_2 = pld$$

The total longitudinal frictional force is

$$\mu plr\int_0^{2\pi} d\theta = 2\pi\mu plr = \pi\mu pld = f$$

or

$$\pi\mu F = f$$

If f is again 500,000 pounds and $\mu = 0.1$,

$$F = \frac{f}{\mu\pi} = \frac{500{,}000}{(0.1)\pi} = 1{,}590{,}000 \text{ pounds}$$