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STRAIN MEASUREMENTS ON A HALF-SCALE MODEL OF THE CIRCULAR PRESSURE HULL OF A SUBMARINE

BY

CHARLES TRILLING AND DWIGHT F. WINDENBURG

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OUTLINE

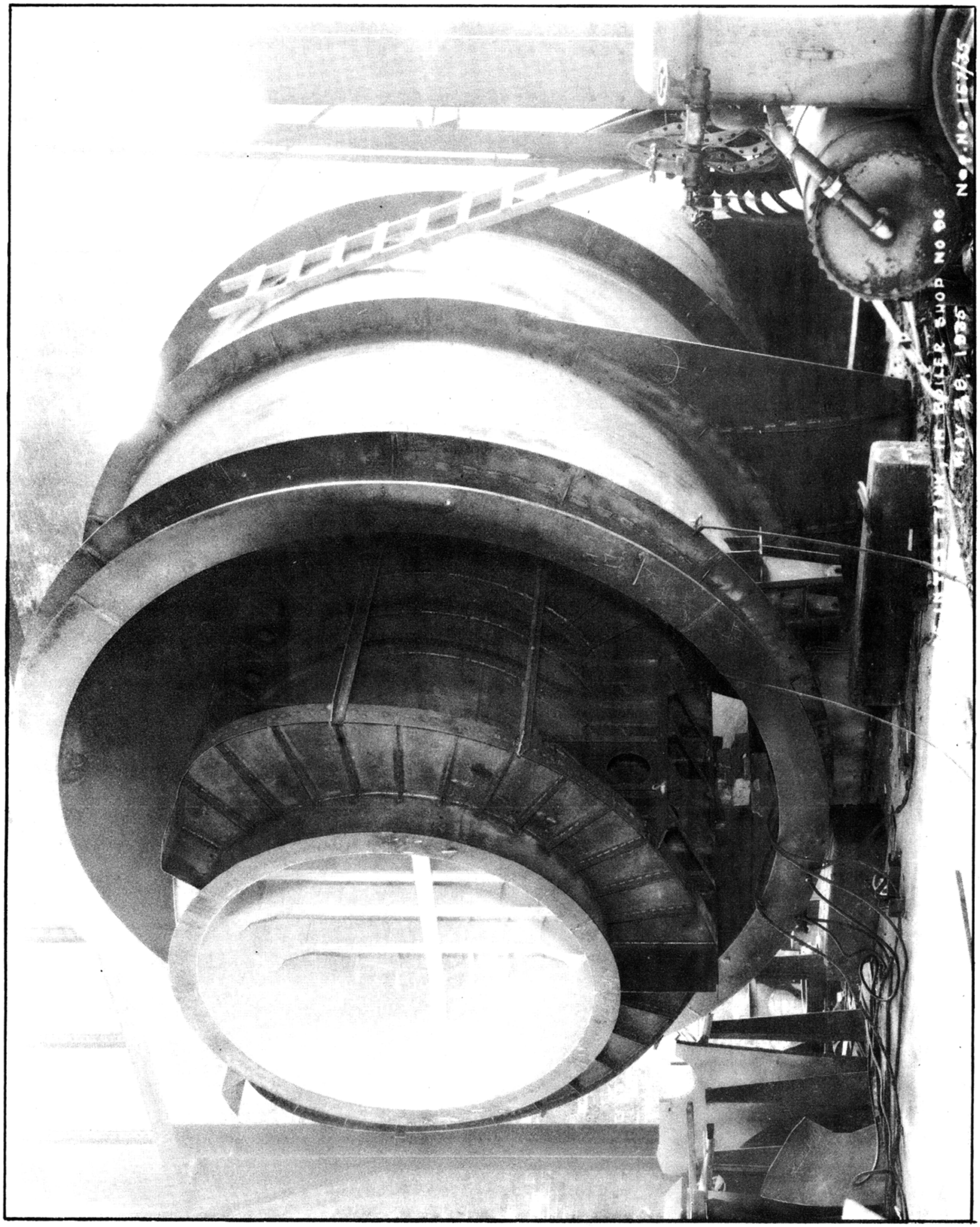

Fig 1

STRAIN MEASUREMENTS ON A HALF-SCALE MODEL OF THE CIRCULAR PRESSURE HULL OF A SUBMARINE

By Charles Trilling and Dwight F. Windenburg

A half-scale model of the circular pressure hull of the submarine USS PLUNGER (SS179) was built and tested at the Navy Yard, Portsmouth, N.H., in September 1935. The model and the tank in which it was tested are shown in Fig 1. A detailed description of the model and of the test is given in a report+ by the Portsmouth Navy Yard.

A number of strain gages were mounted in various positions on the model, and a considerable amount of strain data was obtained. Deflection measurements also were made.

This report summarizes the data and compares measured strains with theoretical strains.

The theory involved is briefly discussed.

THEORY

Von Sanden++ has made a thorough stress analysis of a thin cylindrical shell reinforced by transverse circumferential stiffening rings and subjected to internal or external pressure. A partial translation of this work was made by Hovgaard.+++ For comparison with experiment a strain analysis is desirable since strains, not stresses, were measured in the Portsmouth model test. Equations for the various strains can be deduced from the results in von Sanden's article, and in the discussion to follow such an extension of the theory is made. It is necessary to refer to the original work in order to follow the discussion completely.

The notation of U. S. Experimental Model Basin Report No. 396, p 2, is used, additional symbols being defined as they are introduced. Use is made of the coordinate system adopted by von Sanden, in which x, y, z, denote the axial, radial, and tangential directions respectively, the origin lying in the neutral surface of the shell.

Consider now a tube subjected to an internal pressure, p_i, which produces both radial and end loading. Corresponding to the four types of stresses described by von Sanden there are the four following types of strains:

+ "Submarines SS176 to 181, Half-Scale Model I; Report of tests of a Half-Scale Model of the Cylindrical Inner Hull, of Half-Scale Models of Main Bulkheads, and of a Full-Scale Bureau Type Bulkhead." Forwarded to the Bureau of Construction and Repair 25 March 1936. Fig 1, as well as Fig 6, 7, and 8 at the end, are reproduced from this Portsmouth Report.

++ K. von Sanden and K. Günther, "Ueber das Festigkeitsproblem querversteifter Hohlzylinder unter allseitig gleichmässigem Aussendruck," Werft und Reederei, No. 8, 1920, pp 163–168; No. 9, 1920, pp 189–198; No. 10, 1920, pp 216–221; No. 17, 1921, pp 505–510.

+++ William Hovgaard, Memoranda No. 88, 20 December 1921, and No. 94, 9 September 1922, to the Bureau of Construction and Repair, U.S. Navy Department.

(1) The tangential membrane strain. From von Sanden's Eq (B), this strain is simply

$$\epsilon_z^* = 2 \frac{y^*}{D} \tag{1}$$

where y^* is the radial deflection of the neutral axis of the shell. The symbols y^* and ϵ_z^* are used by von Sanden. The subscript z has no connection with the z direction.

(2) The axial or longitudinal membrane strain. This strain is given by the equation following von Sanden's Eq (80), viz.

$$\epsilon_z'^* = \frac{1}{2}(1 - \mu^2) \frac{p_i}{2 E t/D} - \mu \epsilon_z^* \tag{2}$$

(3) The tangential bending strain. This strain, which is justifiably neglected by von Sanden, can be shown to be (see Appendix I)

$$\epsilon_b'^* = \pm \frac{t}{D} \epsilon_z^* \tag{3}$$

For ordinary tubes this strain is very small in comparison with ϵ_z^*

(4) The axial or longitudinal bending strain. From elementary beam theory (compare von Sanden's Eq (A_3) and (B)) this strain is

$$\epsilon_b^* = \pm \frac{t}{2} \frac{d^2y}{dx^2} \tag{4}$$

With the plus sign, Eq (4) gives the strain on the inner surface of the shell; with the minus sign, the equation applies to the outer surface.+ The double sign is preserved throughout the discussion, the upper sign corresponding always to the inner surface.

The total tangential and longitudinal strains, denoted by ϵ and ϵ' respectively, are then

$$\epsilon = (\epsilon_z^* + \epsilon_b'^*) \tag{5}$$

$$\epsilon' = (\epsilon_z'^* + \epsilon_b^*) \tag{6}$$

+ This can be readily verified by the following considerations: In the region near midspan $\frac{d^2y}{dx^2}$ is negative since the deflection curve of a longitudinal element of the shell is concave inward and the y axis is positive outward. With the plus sign, Eq (4) gives then a negative value for the strain in this region. It is on the inner surface of the shell that the longitudinal bending strain is negative (compression), and hence Eq (4) with a plus sign applies to this surface.

All strains can now be determined from the value of the radial deflection y^*. This quantity depends not only upon the dimensions of the shell but upon the dimensions of the stiffening ring as well, since the deflection of the shell is related to the expansion of the stiffener. The evaluation of y^* was performed by von Sanden, and there remains merely to substitute his expression for y^* (Eq (84), p 219) in the preceding equations for the strains. The expression is

$$y^* = \frac{1}{4}\ \frac{D^2 p_i}{t\ E}\left[1 - \frac{\mu}{2} - \frac{(1 - \frac{\mu}{2}) - B}{(1 + \beta)(\mathrm{Sinh}\ \theta + \sin\theta)}\ f(x)\right] \tag{7}$$

where

$$\begin{aligned} f(x) = {} & \mathrm{Sinh}\ \theta\frac{x}{L}\cos\theta(1 - \frac{x}{L}) + \mathrm{Cosh}\ \theta\frac{x}{L}\sin\theta(1 - \frac{x}{L}) \\ & + \mathrm{Sinh}\ \theta(1 - \frac{x}{L})\cos\theta\frac{x}{L} + \mathrm{Cosh}\ \theta(1 - \frac{x}{L})\sin\theta\frac{x}{L} \end{aligned} \tag{8}$$

The second derivative of f(x) is needed. It is given by

$$\begin{aligned} \frac{-f''(x)}{2(\theta/L)^2} = \varphi(x) = {} & \mathrm{Sinh}\ \theta\frac{x}{L}\cos\theta(1 - \frac{x}{L}) - \mathrm{Cosh}\ \theta\frac{x}{L}\sin\theta(1 - \frac{x}{L}) \\ & + \mathrm{Sinh}\ \theta(1 - \frac{x}{L})\cos\theta\frac{x}{L} - \mathrm{Cosh}\ \theta(1 - \frac{x}{L})\sin\theta\frac{x}{L} \end{aligned} \tag{9}$$

Using Eq (7) we obtain for the total tangential and longitudinal strains the respective values+

$$\epsilon = \frac{p_i}{2\ E\ t/D}\left[(1 - \frac{\mu}{2}) - \frac{(1 - \frac{\mu}{2}) - B}{(1 + \beta)(\mathrm{Sinh}\ \theta + \sin\theta)}\ f(x)\right](1 \pm t/D) \tag{10}$$

$$\begin{aligned} \epsilon' = \frac{p_i}{2\ E\ t/D}\Bigg[& (\frac{1}{2} - \mu) + \frac{\mu(1 - \frac{\mu}{2} - B)}{(1 + \beta)(\mathrm{Sinh}\ \theta + \sin\theta)}\ f(x) \\ & \pm\sqrt{3(1 - \mu^2)}\ \frac{(1 - \frac{\mu}{2}) - B}{(1 + \beta)(\mathrm{Sinh}\ \theta + \sin\theta)}\ \varphi(x)\Bigg] \end{aligned} \tag{11}$$

+ In an Italian article by Francesco Viterbo, "The Strength of Hollow Cylinders Reinforced by Transverse Stiffeners and Subjected to Uniform External Pressure (Strength of Submarine Hulls), L'Ingegnere, vol IV; No. 7, July 1930, pp 446–456; No. 8, August 1930, pp 531–540, equations for the strains are developed which should be compared with Eq (10) and (11). These equations are: Eq (53), p 533 and Eq (59), p 534. Viterbo and von Sanden approach the problem in slightly different manners, and their stress equations show slight but non-essential differences. Likewise, Viterbo's strain equations and those developed in this report have slight but non-essential differences.

If the analysis were made for an external pressure, p, instead of for an internal pressure, p_i, equations identical to the preceding equations would be obtained except that p_i would be replaced by $-p$ thruout. It is seen then from Eq (10) and (11) that the strains in a cylindrical shell subjected to external pressure are the same as the strains in one subjected to internal pressure except for a difference in sign. Using the value $\mu = 0.3$, and the abbreviations

$$S = 100 \; \frac{(1 - \frac{\mu}{2}) - B}{(1 + \beta)(\text{Sinh}\ \theta + \sin\theta)} = \frac{85 - 100\ B}{(1 + \beta)(\text{Sinh}\ \theta + \sin\theta)}$$

$$F(x) = 0.3 \left[f(x) \pm 5.51\, \varphi(x) \right] \tag{12}$$

we can express the strains due to an external pressure, p, by the equations

$$-10^5 \epsilon = \frac{p}{20 \times 10^{-6}\ E\ (100\ t/D)} \left[85 - S\ f(x) \right] (1 \pm t/D) \tag{13}$$

$$-10^5 \epsilon' = \frac{p}{20 \times 10^{-6}\ E\ (100\ t/D)} \left[20 + S\ F(x) \right] \tag{14}$$

The values of the dimensions used in making theoretical computations for the Portsmouth Model are given in Appendix II. In addition to the standard or so-called heavy (H) frames, two other types of lighter frames, called the medium (M) and the light (L), were used on the model. Values of S computed by Eq (12) are given in Appendix II for frames H and L.

A table of values of f(x) and the functions F(x) for various values of x/L is given in Appendix III. This table will facilitate the use of Eq (13) and (14); it is restricted, however, to the particular value $\theta = 5$, the value determined in Appendix II for the Portsmouth Model.

<u>Longitudinal restraint</u> was offered in the Portsmouth Model by the keel and tank tops. In Appendix IV it is shown that the effect of this restraint is to modify Eq (14) as follows:

$$-10^5 \epsilon' = \frac{p}{20 \times 10^{-6}\ E\ (100\ t/D)} \left[13 + S\ F(x) \right] \tag{14'}$$

With $E = 29 \times 10^6$ lb per sq in., and $t/D = 0.0033$, the value given in Appendix II, the factor before the brackets in Eq (13), (14), and (14') becomes unity for an external pressure

$$p = 191 \text{ lb per sq in.}$$

Theoretical strain distributions along a longitudinal element of the shell at this pressure, as given by Eq (13) and (14'), are shown by the solid curves in Fig 2 and 3 for frames H and L respectively of the Portsmouth Model. The pressure

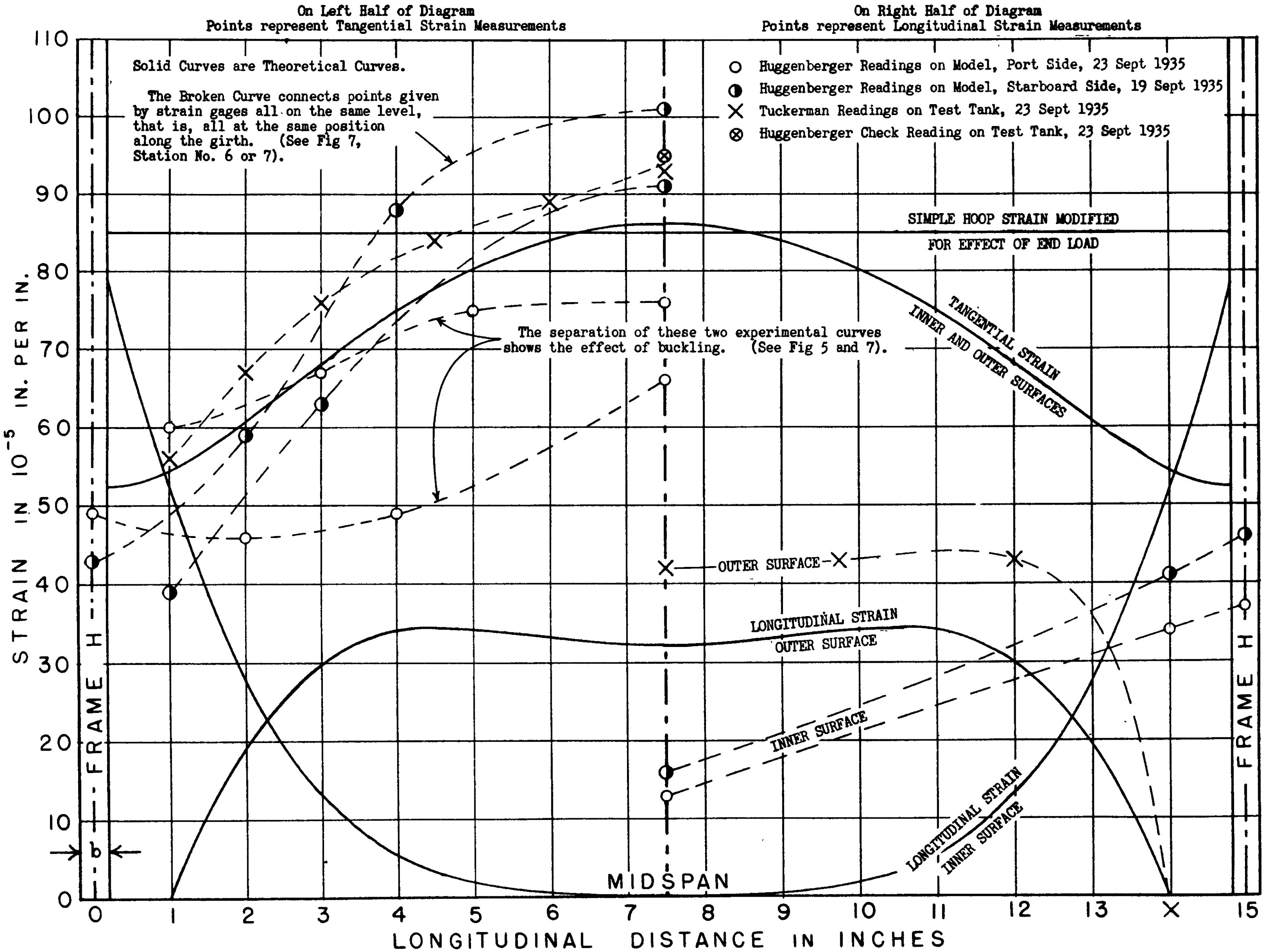

Fig 2 Longitudinal Variation of Strain between Heavy Frames at a Pressure of 191 lb per sq in.

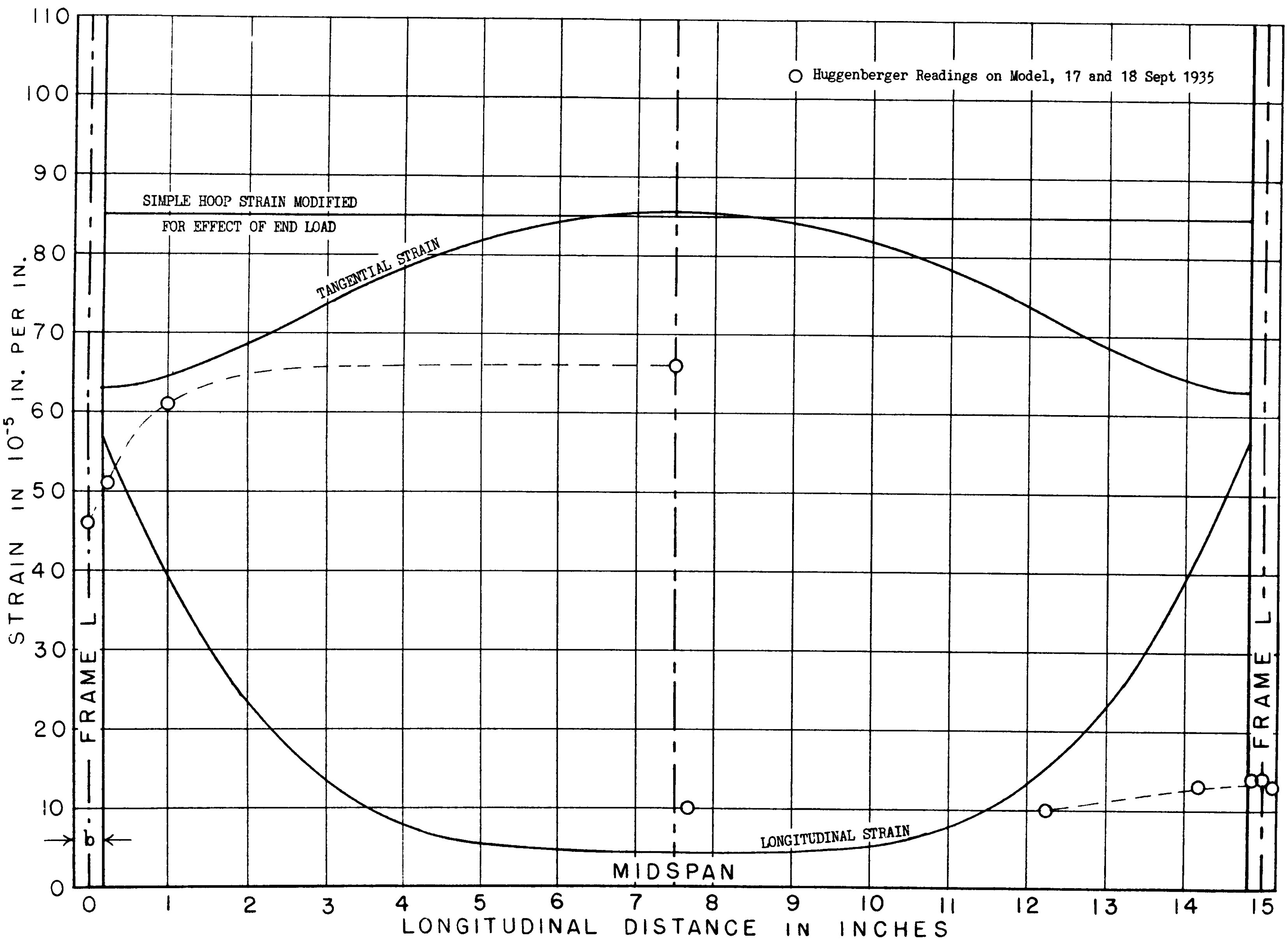

Fig 3 Longitudinal Variation of Strain between Light Frames at a Pressure of 191 lb per sq in.

191 lb per sq in. has no particular significance; it is merely a convenient pressure at which to compare theoretical and experimental strains.

Strain measurements were made not only on the model but also on the outside tank in which the model was tested (see Fig 1 and 8). This test tank is an exact duplicate of the pressure hull of the full-size submarine USS PLUNGER (SS179) in the region of the measurements; its frames conform to the heavy frames of the half-scale model. The theoretical curves in Fig 2, therefore, should apply to the test tank (reduced to half size for comparison) as well as to the model (with due regard for the signs of the strains). There is a slight exception, however. Since there is no longitudinal restraint on the test tank, its longitudinal strains should be given by Eq (14), not Eq (14'). Actually, the curve of longitudinal strain on the outer surface of the shell in Fig 2 was plotted with values obtained from Eq (14) and so applies only to the test tank, whereas the curve of longitudinal strain on the inner surface was plotted by Eq (14') and so applies only to the model. This was done because strain measurements, which later are to be compared with the theoretical values, were obtained only on the outer surface of the test tank and only on the inner surface of the model. The two curves for longitudinal strain in Fig 2 are widely different in shape because of the large effect of longitudinal bending.

Tangential strain, which is principally membrane strain, is practically the same on each side of the plating. Moreover, it is not appreciably affected by longitudinal restraint. Consequently, the curve of tangential strain in Fig 2, obtained from Eq (13) — with the $(1 \pm t/D)$ factor ignored, — applies to both test tank and model. This means that the measurements of tangential strain on test tank and model should agree except for the difference in sign.

EXPERIMENTAL APPARATUS

Thirty strain gages, each operating on a one inch base length, were used in various different positions on the Portsmouth Model. Twenty of these were Huggenberger tensometers, and ten were Tuckerman optical strain gages. The strain gages and their mounts can be seen in the photographs, Fig 6, 7, and 8 at the end of the report.

As seen in the photographs, the Tuckerman mount used consisted of a forked sheet of metal. The prongs at one end pressed against the two extending lugs of the gage; a toe at the other end pressed against the plating. The fork and gage were held to the structure by means of a bridge or yoke welded to the plating. This mount proved to be unsatisfactory. The toe of the forked piece, acting somewhat as a third knife edge, seemed at times to influence the strain gage readings and make them all too high. The danger was realized from the first, but brief preliminary tests indicated that the mount was sufficiently elastic to allow motion of the toe without disturbance of the gage. Probably in most instances this was the case and the readings were accurate, but the reliability of all Tuckerman readings might well be open to question. Improved mounts, which correct these difficulties, and which possess other advantages, have since been developed.

The deflection apparatus can be seen in Fig 7.

EXPERIMENTAL RESULTS

A large amount of deflection data was taken. Several typical deflection curves are shown in Fig 4. The data are of little value in confirming the correctness of theoretical deflections because they are so greatly influenced by buckling. The curves do show, however, that the shell deflects much more with respect to heavy stiffening rings than with respect to light ones. This indicates that the longitudinal bending strain is greater at a heavy stiffener than at a light one, a fact predicted by theory and confirmed by the strain data.

The experimental values of strain obtained from both model and test tank in the Portsmouth test are shown in the diagrams, Fig 2 and 3. Tangential strains are all plotted on the left half of each diagram; longitudinal strains on the right half. The circles represent Huggenberger readings and the crosses Tuckerman readings. Strain values at the different pressures are reduced to strain per unit pressure and averaged. From these average values of strain, the strains for 191 lb per sq in. pressure are computed and plotted.

In part experimental and theoretical results are in fair agreement; in part they are in wide disagreement. The theoretical tangential strain distribution seems to be verified by experiment. Measurements of tangential strain on the outside tank follow the theoretical curve fairly closely. These measurements were all made with Tuckerman strain gages, however, and in view of the defects noted in the mount, they cannot be regarded as conclusive. Measurements of tangential strain on the model roughly confirm the theoretical curve. A rough check of theory is all that can be expected from these strain measurements since they include additional bending strains due to buckling. In order to eliminate the influence of buckling it is necessary to determine strain at the neutral surface of the shell. This can be accomplished by taking strain readings either on both surfaces of the plating or on one side of the plating at different distances from the neutral surface (by the use of buttons or small pieces of steel welded to the plating).

The measurements of the tangential strain on the model have another use. A plot showing the variation of tangential strain along the girth of a particular transverse section, as shown in Fig 5, clearly depicts the formation of lobes in the buckling process. As seen in Fig 7, the measurements extend over about a quadrant of the circumference.

The theoretical longitudinal strain distribution is not verified by experiment. There is a wide and striking disagreement between the two, especially near

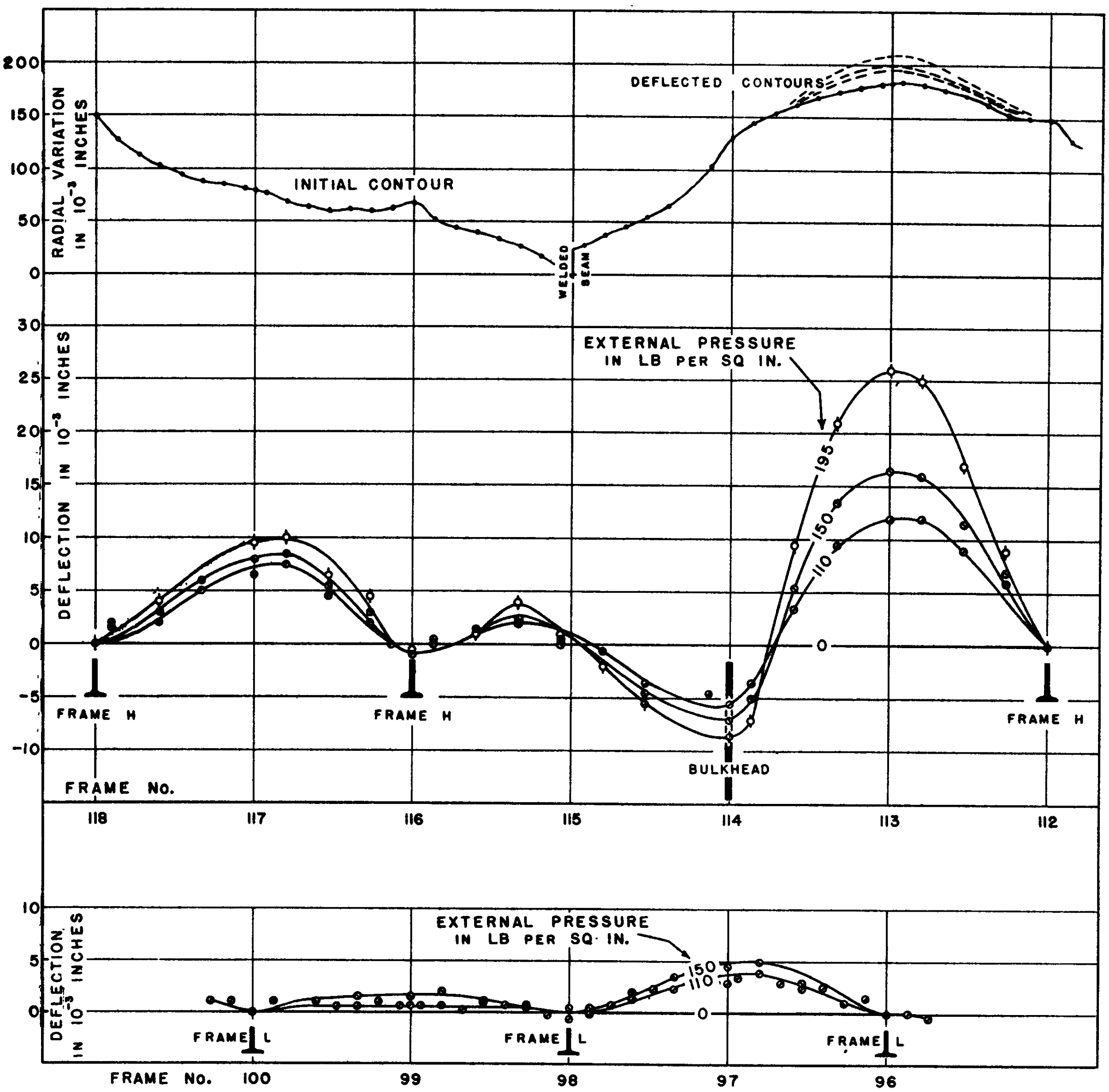

Fig 4 Longitudinal Deflection Curves of Shell
With respect to Frames

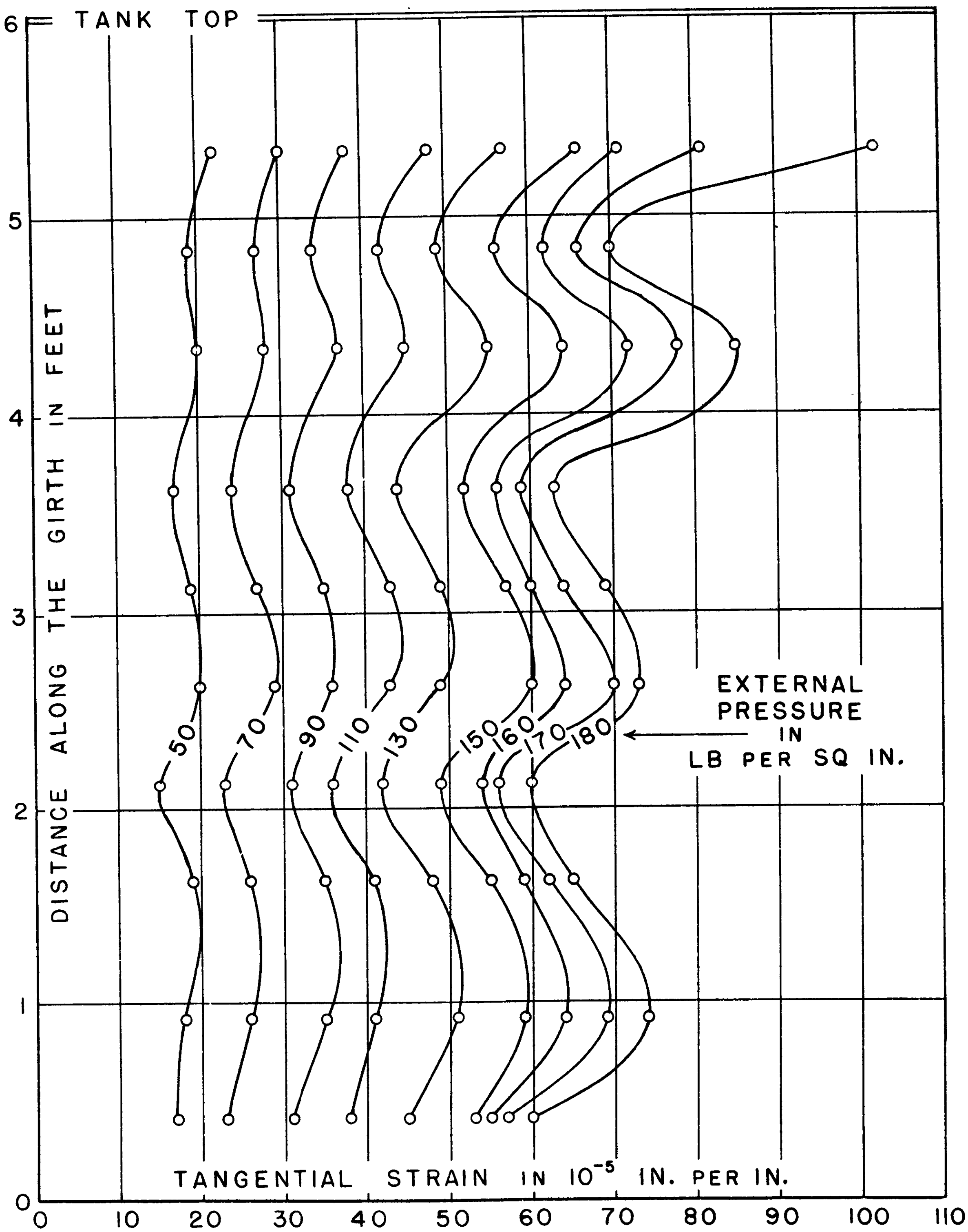

Fig 5 Variation of Midspan Tangential Strain along the Girth
(See Fig 7)

the frame. The large increase in longitudinal strain in the region of the frame, predicted by theory, is not borne out by the experimental results. This is shown by the data for all types of frames on the model, but particularly so in the case of the light frames - on which, incidently, the most longitudinal strain gage data were taken. As seen in Fig 3 the longitudinal strains in the shell between the light frames are nearly uniform along the entire distance from frame to frame.

In U. S. Experimental Model Basin Report No. 396 the general conclusion was reached that tangential stress, not longitudinal stress, is determinative in the collapse of a thin-walled vessel subjected to external pressure. The strain data now furnish an additional reason for disregarding longitudinal stress; it is less than theory predicts. Although the data indicate only that the longitudinal strain is small compared to the theoretical value, it is reasonable to conclude that the same holds for the longitudinal stress.

The von Sanden theory has been carefully and critically examined in an attempt to ascertain why this theory fails to predict the measured values of longitudinal strain in the shell near the frame. No adequate or satisfying explanation was found. Only negative conclusions could be drawn, that is, it was determined only that certain factors were <u>not</u> responsible for the observed discrepancy. A discussion of these factors follows since it might shed some light on the problem. For convenience, the discussion is confined to stresses rather than strains, but really it is applicable to both.

Von Sanden treats a longitudinal element of the shell as a built-in or encastré beam (better, as a series of such beams) whereas actually it resembles more nearly a continuous beam - especially when the width of the frame in contact with the shell is small, as in the Portsmouth Model. Because of the local deformation of the frame itself, and because only one surface of the shell is in contact with and restrained by the frame, the slope of the shell-beam is zero only at the center of the frame and not at its edges as assumed by von Sanden (Eq (8), p 195). This departure in the shell-beam from the end conditions of a built-in beam, however, does not invalidate the von Sanden theory; a consideration of a uniformly loaded continuous beam with fixed ends (the end conditions are not important if the beam has many supports) reveals the fact that the portion of the beam between two adjacent supports is identical in all respects to a built-in beam. Similarly, (outside of the small change of replacing L by the frame spacing, L') von Sanden's stress analysis is substantially correct and holds for the shell-beam considered either as a built-in beam or as a continuous beam.

Attention is next directed to the Portsmouth Model to see if there exist in it any extraneous conditions not represented in the theory. There are three possibilities: (1) initial stresses, (2) irregularities, and (3) restraints.

Initial stresses may have been introduced during the fabrication of the Portsmouth Model. Welding the frames may have created initial tensile stresses.

The preloading of the model before strain measurements were made may have left initial bending stresses in the shell at the frame. These conditions would alter the longitudinal stress in the shell. However, since strain gages indicate only variations in stress and not absolute values, initial stresses should not affect the strain gage readings. Hence initial stresses cannot be the cause of the discrepancies in the strain data.

Longitudinal elements of the shell of the Portsmouth Model showed (see Fig 4) marked deviations from flatness. These and other irregularities were introduced during the fabrication of the model, preloading, etc., as in the case of initial stresses. Such irregularities may have some influence on both the stresses and the strains in the shell; they should, however, introduce only random errors, they cannot be responsible for the consistently low values obtained for the longitudinal strains in the regions near the frames.

The effect of longitudinal restraints in the Portsmouth Model are discussed under THEORY and in Appendix IV. These restraints cause a slight reduction in the theoretical values of longitudinal strain, but this reduction is far from enough to bring the theoretical and the experimental values into agreement.

It is believed that there is no inaccuracy in the data of sufficient magnitude to account for the large discrepancy between theory and experiment in regard to the longitudinal strain.

These negative conclusions may have some utility, but the low values of the measured longitudinal strain remain unexplained.

CONCLUSION

The Portsmouth Model test indicates the following conclusions:

The theoretical stress analysis of a thin-walled cylinder reinforced by transverse stiffening rings and subjected to either external or internal pressure made by von Sanden is, in general, correct.

The theory fails, however, to predict the true state of stress in the shell in the vicinity of the stiffener. The actual longitudinal stress in the shell where it bends over the edge of the stiffener is probably considerably less than the theoretical stress.

The conclusion reached in U. S. Experimental Model Basin Report No. 396 that tangential, not longitudinal, stress is determinative in the collapse of a thin-walled vessel subjected to external pressure is reaffirmed.

ACKNOWLEDGMENT

Acknowledgment is due Mr. Charles D. Anderson of the Bureau of Construction and Repair for many valuable suggestions and assistance in obtaining data. Acknowledgment is due also to the officers and personnel of the Navy Yard, Portsmouth, N. H. whose cooperation made the acquisition of these data possible.

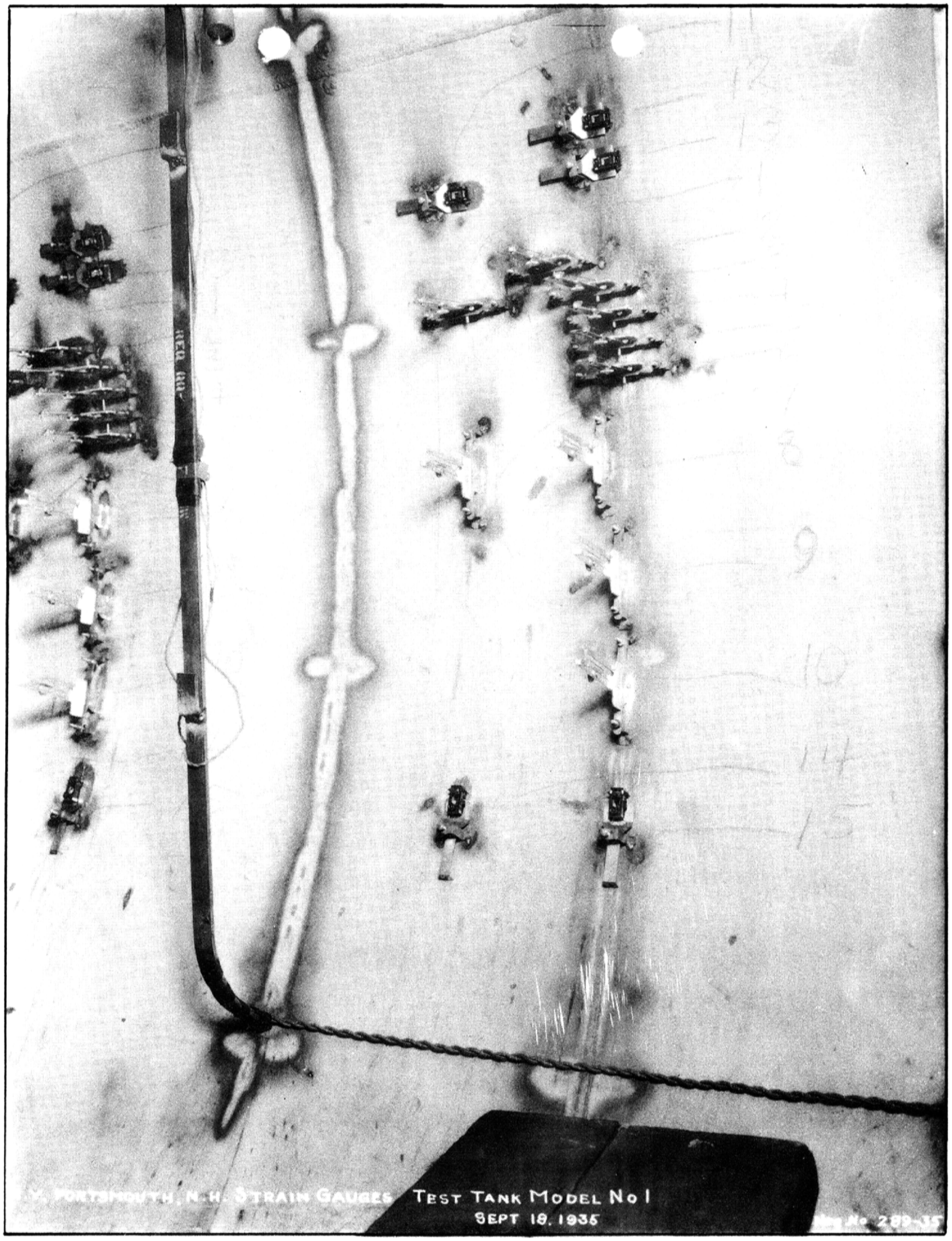
PORTSMOUTH, N.H. STRAIN GAUGES TEST TANK MODEL No I
SEPT 18, 1935
No. 289-35

Fig 6


PORTSMOUTH, N.H. STRAIN GAUGES, TEST TANK MODEL No 1 SEPT 24 1936

Fig 7

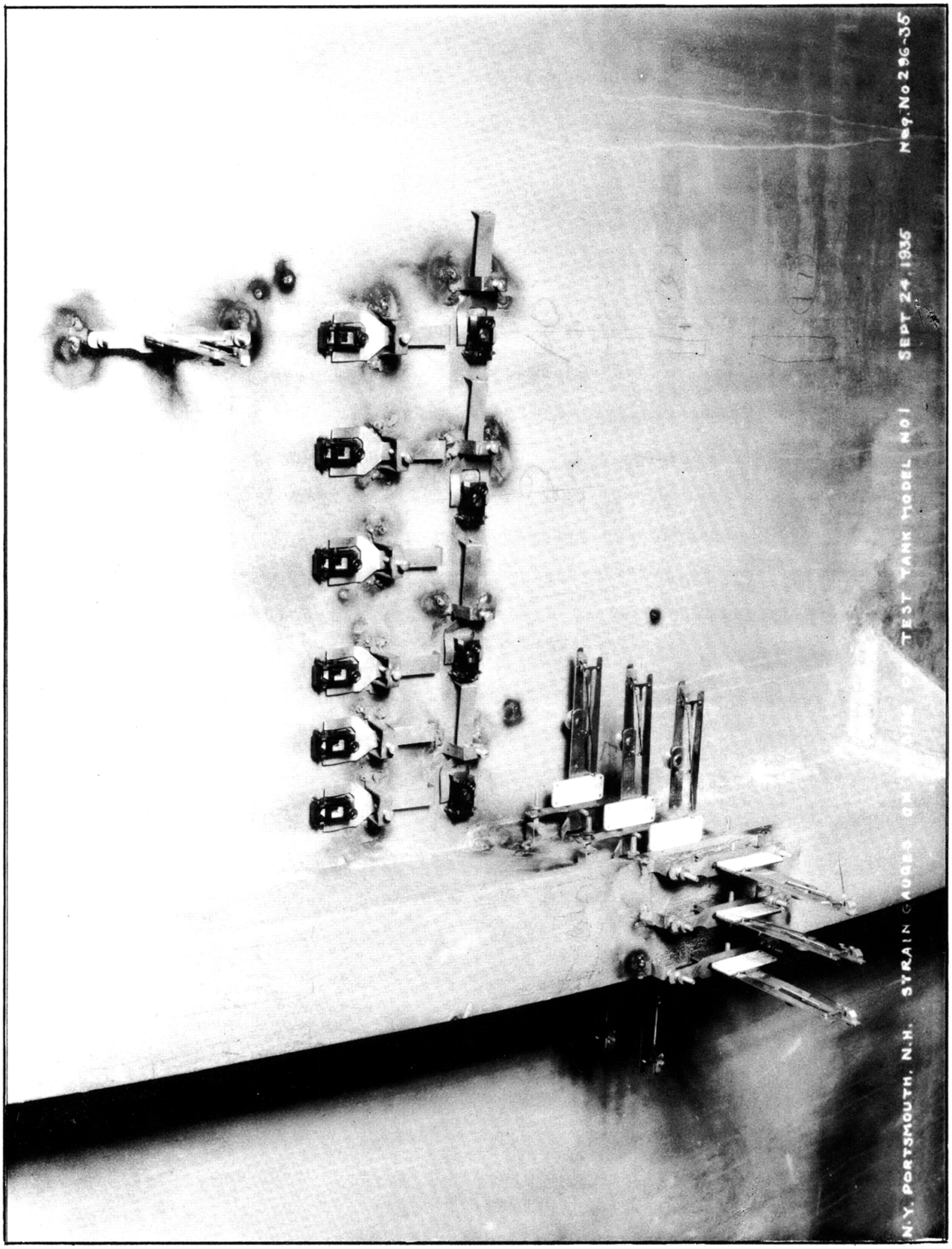
N.Y. PORTSMOUTH, N.H. STRAIN GAUGES ON
TEST TANK MODEL NO 1
SEPT 24, 1935
Neg. No 296-35

Fig 8

APPENDIX I

DETERMINATION OF THE TANGENTIAL BENDING STRAIN

When a thin cylindrical ring of diameter $D = 2r$ and thickness $t = 2h$ undergoes a radial extension of amount y due to an internal pressure, the tangential or circumferential hoop strain is

$$\epsilon_b' = \frac{y}{r} = \frac{2y}{D}$$

The self-explanatory diagram, Fig 9, shows the initial position of the cross-section of a cylindrical shell, and its position after an extension due to internal pressure has taken place. Considering the portion of the shell contained within the arc subtending a small angle $d\theta$, we note the following strains:

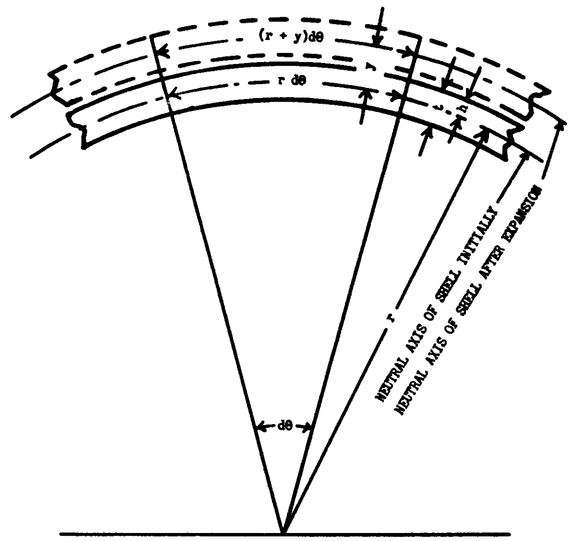

Fig 9

surface	original lengths of arcs	final lengths of arcs	elongation	strain
inner	$(r - h)\, d\theta$	$(r - h + y)\, d\theta$	$y\, d\theta$	$\frac{y}{r - h}$
neutral	$r\, d\theta$	$(r + y)\, d\theta$	$y\, d\theta$	$\frac{y}{r}$
outer	$(r + h)\, d\theta$	$(r + h + y)\, d\theta$	$y\, d\theta$	$\frac{y}{r + h}$

From this it is seen that the bending strains are

Inner Surface:

$$\epsilon_b' = \frac{y}{r - h} - \frac{y}{r} = \frac{h\,y}{r(r - h)}$$

Outer Surface:

$$\epsilon_b' = \frac{y}{r + h} - \frac{y}{r} = -\frac{h\,y}{r(r + h)}$$

Since h is very small in comparison with r, these strains are nearly equal. Hence the following common tangential bending strain can be assumed

$$\epsilon_b' = \pm \frac{h\,y}{r^2} = \pm \frac{t}{D}\,\epsilon_z$$

Von Sanden, in his Eq (B), sets $1/\rho'$, the change in tangential curvature, equal to zero, and thereby implies that the tangential bending strain is zero. Since from the preceding equation ϵ_b' is very small in comparison with ϵ_z, and since the change in curvature, y/r^2, is likewise insignificant, these assumptions of von Sanden are entirely justified. However, his deduction (p 193) that because the circular form of the cross section is, "by symmetry," preserved when the pressure is increased, the tangential curvature must likewise remain unchanged, is not really correct reasoning.

APPENDIX II

DIMENSIONS OF THE PORTSMOUTH MODEL.
Determination of S by Eq (12).

Outside Diameter,	O.D.	= 93 in.
Frame Spacing,	L'	= 15 in.
Plating:	Weight	= 12½ lb per sq ft
	t	= 0.306 in.
Frames: Contact Width,	b	= 3/8 in.
Area,	A	= 1.246 sq in. (Type H)
		= 0.698 sq in. (Type L)

$t/D = 0.0033$ $\quad B = 0.0845$ (Type H)

$L/D = 0.158$ $\quad = 0.1414$ (Type L)

$\beta = 15.74\, B$

$\theta = 5.00$ $\quad = 1.330$ (Type H)

$N = 1.009$ $\quad = 2.226$ (Type L)

$\sinh\theta + \sin\theta = 73.24$

$S = 0.448$ (Type H)

$S = 0.300$ (Type L)

APPENDIX III

TABLE OF VALUES OF STRAIN FUNCTIONS

Based on $\theta = 5$.

x	f(x)	$F_1(x)$	$F_2(x)$
0.00 L	73.24	146.21	
.02 L	72.54	122.78	
.05 L	69.27	91.41	– 49.85
.10 L	59.86	49.21	– 13.30
.20 L	35.79	– 2.22	23.70
.30 L	14.88	– 22.86	31.79
.40 L	1.93	– 28.27	29.42
.50 L	– 2.35	– 28.86	27.45

$$F_1(x) \equiv 0.3\left[f(x) + 5.51\,\varphi(x)\right]$$

$$F_2(x) \equiv 0.3\left[f(x) - 5.51\,\varphi(x)\right]$$

Note: All strain functions are symmetrical with respect to midspan, that is

$$f(x) = f(L - x), \text{ etc.}$$

APPENDIX IV

LONGITUDINAL RESTRAINT

Longitudinal restraint in a pressure vessel may arise from longitudinal stiffeners of any sort, axial thrust bars, etc. If the cross-sectional area, A_L, of the longitudinal members resisting end load is small, that is of the order of the cross-sectional area, πDt, of the pressure vessel itself, then practically the only effect of the presence of these members is to reduce the longitudinal membrane stress from

$$\sigma_z{'}^{*} = \frac{p \cdot \frac{1}{4}\pi D^2}{\pi Dt} = \frac{p}{4}\frac{D}{t} \tag{IV-a}$$

the value given by von Sanden in his Eq (C_1*), to

$$\sigma_z{'}^{*} = \frac{p \cdot \frac{1}{4}\pi D^2}{\pi Dt + A_L} = \frac{1}{1 + \frac{A_L}{\pi Dt}} \frac{p}{4}\frac{D}{t} \tag{IV-b}$$

This changes the longitudinal membrane strain from the value given in Eq (2) to

$$\epsilon_z{'}^{*} = \frac{1}{1 + \frac{A_L}{\pi Dt}} \tfrac{1}{2}(1 - \mu^2) \frac{p_i}{2\,E\,t/D} - \mu\epsilon_z^{*} \tag{IV-c}$$

By substituting Eq (IV-c), instead of Eq (2), into Eq (6) it is found that the term $(\frac{1}{2} - \mu) = 0.2$ in Eq (11) is replaced by

$$\frac{1}{1 + \frac{A_L}{\pi Dt}} \tfrac{1}{2}(1 - \mu^2) - \mu(1 - \tfrac{\mu}{2}) = \frac{0.455}{1 + \frac{A_L}{\pi Dt}} - 0.255 \tag{IV-d}$$

The cross-sectional areas of the longitudinal members in the Portsmouth Model were computed to be:

2 Tank Tops	(23 in. of 7.5 lb plate):	8.5 sq in.
Keel web	(24 in. of 10.2 lb plate):	6.0 sq in.
Keel flange	(6 in. of 20.0 lb plate):	2.9 sq in.
		A_L = 17.4 sq in.

$$\frac{A_L}{\pi Dt} = \frac{17.4}{89.4} = 0.195$$

Substituting this result in the expression (IV-d) we find the value of the expression is 0.13 in place of the uncorrected value 0.2. With this correction it is readily seen that Eq (14) is changed to Eq (14').