THE DAVID W. TAYLOR MODEL BASIN

UNITED STATES NAVY

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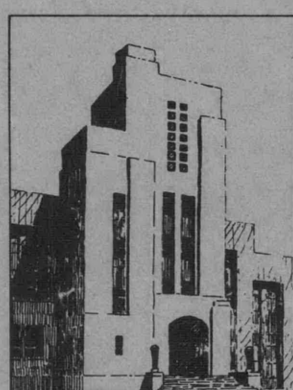
EFFECTS OF UNDERWATER EXPLOSIONS
GENERAL CONSIDERATIONS

BY PROF. E. H. KENNARD



SEPTEMBER 1942

REPORT 489

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THE DAVID W. TAYLOR MODEL BASIN

BUREAU OF SHIPS

NAVY DEPARTMENT

WASHINGTON, D.C.

REPORT 489

EFFECTS OF UNDERWATER EXPLOSIONS
GENERAL CONSIDERATIONS

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THE DAVID TAYLOR MODEL BASIN

PERSONNEL

This report was written by Professor E.H. Kennard after extensive discussion with Commander W.P. Roop, USN. Dr. M.A. Greenfield also gave valuable assistance.

DIGEST

The damage suffered by a given underwater structure depends principally upon

(a) the kind and quantity of the explosive

(b) the nature and dimensions of the structure

(c) the distance between the explosive and the structure.

The object of the analysis in this report is to take a first step toward setting up a damage* function expressing the damage in terms of parameters referring to the three factors (a), (b), and (c).

Various gages, resembling service structures more or less remotely, such as crusher plugs or thin diaphragms, have been used extensively to furnish a measure of the action of explosives, but they do not represent these structures sufficiently well to give an accurate indication of damage, and there is much doubt as to the quantities which these gages actually measure.

This analysis is based upon the concept that damage is to be correlated with the energy absorbed by the structure; specifically, it is to be represented by the energy absorbed by unit of mass of the resisting or protecting plate.

To free the analysis from too great dependence upon arbitrary values, a brief study of the energy available in the water for damaging a submerged structure is presented. This is supplemented by curves, Figure 2 on page 15, showing the relation between pressure, time, and distance from the charge, developed from the best existing information. The oscillatory nature of the explosive disturbance in the water involves further complications, as presented in various parts of the report.

By an extension of the treatment in Taylor Model Basin Report 480, a preliminary damage function is developed by analysis of the shock wave in water incident on a thin plate or piston with air or a vacuum behind it. Assuming a shock wave which rises instantaneously to a peak value and decreases exponentially with time, expressions are developed for the momentum I and energy E imparted to a plate facing the shock wave, and to a mass m_w, which represents a mass of water which, moving at a suitable velocity, possesses the same amounts of momentum and kinetic energy as exist in the shock wave per unit area of the wave front.

The resulting damage function takes the form of

$$\Omega_d = C_d \frac{E}{m} \text{ or } \Omega_d = K_d \frac{I^2}{m^2}$$

where Ω_d is the energy per unit mass, absorbed by and remaining in the deformed or damaged plate, E and I are the energy and momentum in unit area of the shock wave, respectively, m is the mass per unit area of the protecting plate, and C_d and K_d are

* The term "damage" used in this report refers to permanent deformation which, in the case of a ship, would require correction to place the structure back in its original condition. It does not necessarily imply that plates or structural parts are fractured.

dimensionless coefficients. The mass that is to be included in calculating Ω_d is to be suitably chosen with a view to the comparisons that are to be made.

With sufficient data from underwater explosion experiments to determine either E or I, and with values of either C_d or K_d from tests of typical structures, it would be possible to determine the amount of energy which a given structure would absorb under any set of specified conditions. The damage function Ω_d then becomes a tangible numerical value of the energy absorbed for the mass (or weight) per unit area of the protecting structure under the given conditions, and a direct measure of its protective value. The lower the damage function, the better the structure.

The pressure-time-distance relationships of the explosive action, and the damage functions, are presented here, not as final developments, but as a reference framework upon which experimental data can be hung for observation and analysis, as gages and indicators for current and future experimental work, and as signposts for future development and research.

EFFECTS OF UNDERWATER EXPLOSIONS
GENERAL CONSIDERATIONS

ABSTRACT

In preparation for the interpretation of tests now being made, general considerations are presented relative to explosive action upon a steel structure.

As a measure of damage,* the energy absorbed by the structure per pound of its weight is adopted. The relation between the energy absorbed and quantities characterizing the incident shock wave is discussed. As a suggestive case, the incidence of a wave upon a free steel plate is considered.

Damage functions are developed in the form of expressions in which the momentum and the energy absorbed per weight of unit area of the protecting structure are modified by certain dimensionless coefficients. It is expected that present and future tests of structures under explosive load will furnish experimental values for these coefficients.

Existing data and estimates as to the pressure field produced by an underwater explosion, and as to the period of oscillation of the gas globe, are summarized.

INTRODUCTION

In continuation of observations on the shock wave produced by the explosion of small charges (1),** tests are being made at the David Taylor Model Basin to determine the damage* caused to an elementary steel structure by an explosion at a short distance. The purpose of these tests is to obtain methods of estimating the amount of such damage and to clarify thought about the relationships involved, especially with reference to scale effects. The results of the tests themselves will be presented subsequently. The present report contains general considerations relative to the interpretation of the tests.

It is to be emphasized at the outset that the considerations advanced in this report are essentially tentative and preliminary in character. Observations already made indicate that much remains to be learned as to the manner in which damage is produced, and the analysis will probably require extensive further modification and development before it can be made to produce practical rules for use in the design of protective structures for ships.

MODE OF MEASUREMENT OF DAMAGE

The damage suffered by a given underwater structure depends principally upon the kind and quantity of the explosive, the nature and dimensions of the structure,

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* The term "damage" used in this report refers to permanent deformation which in the case of a ship, would require correction to restore the structure to its original condition. It does not necessarily imply that plates or structural parts are fractured.

** Numbers in parentheses indicate references on page 17 of this report.

and the distance between explosive and structure. The final object of analysis will be to set up a damage function expressing the damage in terms of parameters referring to the three factors named. The first requisite is a suitable method for the measurement of damage.

Various gages for measurement of damage have been devised, resembling service structures more or less remotely, and their indications have been accepted as a measure of the damaging power of explosive action. There are two strong objections to this procedure. First, damage is not properly a function of the explosive action alone, but must include parameters derived from the structure. In the use of the gages mentioned, this difficulty is met by conceiving the explosive field as possessing a supply of destructive power which is poured into the structure up to the limit of the structure's capacity. It is considered better, however, to regard the structure as an essential member of a given disposition or array and to calculate the damage directly, without making use of the fiction that damage can exist separately. The parameters referring to the structure are then introduced directly, and the whole calculation refers to a specific type of structure.

The second objection is more general. The use of a given gage, especially a crusher gage, leaves doubt even as to the dimensions of the quantity that it measures. Furthermore, doubt exists as to the identity of the quantity with which damage is to be correlated. In view of these uncertainties, it is better to set up an analytical concept as a basis for the measurement of damage and to test its validity by the examination of actual results, rather than to leave the definition of damage to the vagaries of a gage of arbitrary characteristics.

This is not to say that a gage which gives consistent results should be rejected because its readings have not been referred to absolute standards. But interpretation of gage readings in dimensional terms is necessary if the readings are to be used in any application of the law of similitude. It is only on the most completely empirical basis that crusher gages subject only to static calibration have valid application.

The concept chosen for the present purpose is that damage is to be correlated with the energy absorbed by the structure. In order to bring this concept into harmony with Hopkinson's rule (2),* which has been found to possess a rough validity, the energy will be referred to unit mass of the structure. The following quantities**

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* Hopkinson's rule has been stated briefly as follows: "The damage inflicted on a given structure by a given charge at a given distance will be reproduced to scale if the linear dimensions of the charge and structure and the distance between them are all increased or diminished in the same ratio." Even the case enclosing the charge and all minor features such as rivets, reinforcements and fastenings, should be reproduced to scale.

** The following notation is employed throughout the report:

c is the speed of sound in water, in inches per second.

l_w is the equivalent length of water projectile in inches.

(Continued on next page)

involving energy will be defined and employed:

E, the total energy flux in the water across unit area, at a given distance from the center of the explosion, expressed in inch-pounds per square inch. This refers to the whole sum of the energy flowing outward from the explosive into the water;

E_1, a part of E such as that carried by the first shock wave;

U_d, the energy absorbed and permanently retained in the structure, and thus associated with damage, per unit of area, expressed in inch-pounds per square inch;

U_s, the sum of U_d and the elastic energy that is also present in the structure per unit area at the moment of maximum deformation, expressed in the same units;

U_t, the kinetic energy in the structure per unit area;

Ω_d, the energy permanently retained in the structure per unit of its mass;

Ω_s, the sum of Ω_d and the maximum elastic energy per unit mass.

If m denotes mass per square inch of the structure,

$$\Omega_d = \frac{U_d}{m}, \quad \Omega_s = \frac{U_s}{m}$$

The relation between these various energies, as descriptive of successive phases of the entire explosive action, is further illustrated in Table 1.

The "area" referred to in some of the definitions is the area of the surface of the structure, supposed to be a plane, upon which the water acts.

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m is the mass per unit area of solid surface, expressed in absolute units in formulas but often cited in pounds per square inch.

m_w is the equivalent mass of water projectile per unit area of wave front; units as for m.

p is the pressure in the water, in pounds per square inch.

r is the radial distance from center of charge, in inches.

t is the time in seconds.

u is the particle velocity in the water, in inches per second.

u_w is the velocity of equivalent water projectile, in inches per second.

$z = \frac{m_w}{2m}$; $z = \frac{\rho c}{\alpha m}$ for an exponential wave.

I is the momentum per unit area carried by the incident pressure wave, in pound-seconds per square inch.

E is the total energy flux in the water across unit area, in inch-pounds per square inch.

E_1 is a part of E such as that carried by the first shock wave.

Q_w is the total explosive energy imparted to the water, in inch-pounds.

U_d is the energy permanently retained in a structure per unit of area, in inch-pounds per square inch.

U_s is the sum of U_d and the maximum elastic energy stored in the structure per unit area.

U_t is the kinetic energy in the structure per unit area.

W is the weight of the charge in pounds.

α is a constant; unit $= \frac{1}{\text{sec}}$.

ρ is the density of water in absolute units per cubic inch.

$\Omega_d = \frac{U_d}{m}$ and is the energy permanently retained in a structure per unit of its mass.

$\Omega_s = \frac{U_s}{m}$ and is the sum of Ω_d and the maximum elastic energy stored in the structure per unit of its mass.

TABLE 1

Diagram to identify Symbols for Successive Shifts of Energy

Time	Energy in Field	Energy in Structure
Before detonation	Zero	Zero
Detonation complete	Zero	Zero
Before first shock wave reaches structure	$E\,(= E_1 + \ldots)$	Zero
Structure attains maximum deformation, or it ruptures	E'(reflected from structure)	$\begin{cases} (U_s + U_t) < E \\ \Omega_s + \ldots \end{cases}$
After action has ceased	Zero	$\begin{cases} U_d \\ \Omega_d \end{cases}$

The quantity E calls for comment. The total energy imparted to the water is

$$Q_w = 4\pi r^2 E$$

in terms of the value of E at r inches from the center of the explosion; the factor $4\pi r^2$ represents the area of the sphere formed by the shock front. This quantity Q_w is not the same as the heat of combustion of the explosive, which is evolved in a reaction to which oxygen is supplied; but the heat of combustion sets an upper limit to the value of Q_w. Nor is Q_w necessarily the same as the heat of detonation in a closed or constant-volume container. For in such a container chemical equilibrium is reached and maintained as the gas cools, whereas in the rapid, almost adiabatic expansion of a detonated explosive the chemical reaction is likely to be left incomplete.

In the case of TNT, the heat of combustion is 60 x 10^6 inch-pounds per pound; for the heat of detonation at constant volume, a round number adopted elsewhere (3) is 1000 foot-tons or 27 x 10^6 inch-pounds per pound of explosive. No adequate experimental evidence exists as to the value of Q_w. H. Jones (4) has calculated from chemical data the values of pressure and volume for the expanding gases from TNT exploded under water; an integration based on these values gives Q_w = 15.9 x 10^6 inch-pounds per pound for expansion to atmospheric pressure. The corresponding value of E, at a distance of r inches from a charge of W pounds of TNT, is

$$E = 1.26 \times 10^6 \frac{W}{r^2} \text{ inch-pounds per square inch} \qquad [1]$$

in which 1.26 = $15.9/4\pi$, where 4π represents the area of a sphere of radius one inch. Thus at a distance of one foot from a charge of one pound (r^2 = 144)

$$E = 8750 \text{ inch-pounds per square inch.}$$

A further complication arises from the fact that the interaction of the exploded material with the surrounding water has been found to be oscillatory in character. Only part of Q_w, possibly a fifth,* appears to be carried by the first shock wave; this part of Q_w, divided by $4\pi r^2$, is denoted here by E_1. During collapse of the

* See top of page 14 following.

gas bubble, following its first expansion, energy flows from the water inward, and, on re-expansion of the bubble, further portions are carried off in succession by secondary shock waves. For the most part, however, this complication will be ignored in the present report and attention will be given mainly to the overall action of the water on the structure without special regard to the times at which successive parts of the action occur.

In any case, measurement of E is not necessary for the study of damage. The principal significance of E is that it sets an upper limit to the energy available for causing damage. The energy absorbed by the structure, which in the present tests is a thin circular diaphragm loaded short of rupture, is to be calculated from the effects produced in the structure. This energy can be divided into two parts; in dealing with structures of various types the distinction between them may be of importance. The energy that is spent in plastically deforming or rupturing the material is represented here by U_d or Ω_d. This energy is directly related to the damage produced. While the material is flowing plastically, however, it will contain also a certain amount of energy of elastic deformation. The total energy, including the energy of elastic deformation at its maximum possible value, is represented here by U_s or Ω_s.

In dimensions, Ω_s or Ω_d is a specific energy. Viewed as the equivalent of U_s/m or U_d/m, Ω is specific both with respect to area of the surface through which the energy moves, and with respect to the mass density of the structure to which the energy is delivered. Energy measured in inch-pounds is referred to square inches of surface and to the mass per square inch of the structure. In all formulas, unless otherwise specified, absolute units of mass are to be understood. Considering that mass in absolute units is expressed in pounds weight divided by the acceleration of gravity, it will be found that the dimensions of Ω_s or Ω_d are those of velocity squared. This may be interpreted physically either as the factor by which mass per square inch of the structure is to be multiplied to obtain the desired value of energy per square inch, or as the square of the velocity that the structure would have if the energy were all kinetic energy. Numerical values of Ω_s or Ω_d, on the other hand, will be calculated by dividing the energy in inch-pounds per square inch by the weight in pounds per square inch; the unit for such numbers will be specified briefly as inch-pounds per pound.

It should be possible to calculate both Ω_d and Ω_s from measurements of the permanent effects produced in the structure. While the structure is undergoing deformation, it will also contain a considerable amount of kinetic energy. This energy is more difficult to calculate, however; and in any case a rough correspondence is to be anticipated between the kinetic energy and the plastic and elastic energies. In the end, the kinetic energy, and also a large part of the elastic energy, are either converted into other forms within the structure or returned to the water by means of a reflected wave.

Either Ω_d or Ω_s, denoting energy per pound, might be adopted as a measure of damage. The choice between them may be left open for the present. Energy per pound is preferable to energy per unit area as a measure of damage because in similar structures any given degree of damage, such as the rupture of a given member, should occur at the same value of the energy absorbed per pound, regardless of the size of the structure, whereas the energy per unit area would vary in proportion to the linear dimensions. In dealing with relations between the structure and the water, on the other hand, the energy per unit area, U_d or U_s, is the more convenient quantity to use.

It must be recognized, however, that a certain degree of ambiguity may arise in regard to the mass that is to be included in calculating U_d or Ω_d. It is assumed that in practical cases this ambiguity will be resolved in whatever manner seems most appropriate in view of the comparisons that are to be made between various values of U_d or Ω_d.

THE DAMAGE FUNCTION

We now turn to the development of an expression or damage function by which Ω_s or Ω_d may be calculated from data referring to the explosion and to the design of the structure. In doing this two guiding ideas will be followed:

First, existing knowledge as to the nature of the pressure field of an underwater explosion, as outlined elsewhere (2), must serve as a starting point in preliminary definition of a damage function.

Second, the tentative functions thus found may be tested for validity by numerical comparison with results of observation on simple diaphragms. In this way a normal damage function may be obtained, with a satisfactory degree of empirical validity.

Beyond this point the task becomes that of devising and demonstrating specific structural assemblies with improved resistance. Their relative merit may be expressed in terms of a ratio of comparison with the normal single diaphragm.

A clue to a suitable form of the damage function is furnished by the analysis of a shock wave in water incident on a thin plate or piston with air or vacuum beyond it. For this purpose the treatment in Taylor Model Basin Report 480 needs to be carried a little further.

The plate is assumed to be so thin that delay in transmission of stress through it is negligible. To simplify the mathematical treatment, a shock wave of exponential form is assumed, the pressure p rising suddenly to a maximum value p_0 pounds per square inch and thereafter being given by the formula

$$p = p_0 e^{-\alpha t}$$

where t is the time and α a constant factor expressing the rate of decline of pressure from the initial peak p_0. Its dimension is that of a reciprocal time. Such a wave bears a resemblance to actual shock waves due to brisant explosives. The impulse I and energy E per unit area of the wave front in such a wave are found to be

$$I = \int p\,dt = \frac{p_0}{\alpha} \text{ pound-second per square inch} \qquad [2a]$$

$$E^* = \frac{1}{\rho c}\int p^2 dt = \frac{1}{2}\frac{p_0^2}{\rho c \alpha} = \frac{1}{2}\frac{\alpha}{\rho c} I^2 \text{ inch-pound per square inch,} \qquad [2b]$$

where ρ is the density of water and c the speed of sound in it.**

The velocity given to the plate by such a wave falling at normal incidence upon it, as given on page 25 of TMB Report 480, is

$$v = \frac{p_0}{\alpha m}\cdot\frac{2(e^{-\alpha t} - e^{-x\alpha t})}{x - 1} \qquad [3]$$

x here is an abbreviation for $\rho c/\alpha m$, and m is the mass of the plate per unit of its area.

This velocity is zero at the instant of arrival of the shock wave at the plate, but it rapidly builds up with an initial acceleration of $2p_0/m$. The difference between the exponential terms gives for a time a positive and increasing velocity, until the time t_s at which it reaches a maximum, v_s. This is found in the usual way, by equating dv/dt to zero, to be

$$v_s = \frac{p_0}{\alpha m}\cdot 2x^{\frac{x}{1-x}} \text{ at a time } t_s = \frac{1}{\alpha}\ln x^{\frac{1}{x-1}}$$

The corresponding maximum values of the momentum and of the energy of the plate, per unit of its area, are denoted by I_s and T_s, respectively. They can be expressed as follows in terms of I and E:

$$I_s = mv_s = I\cdot 2\cdot x^{\frac{x}{1-x}} \qquad [4]$$

$$T_s = \frac{1}{2}mv_s^2 = 2x^{\frac{2x}{1-x}}\cdot\frac{I^2}{m} = 4x^{\frac{1+x}{1-x}}E \qquad [5]$$

The behavior of the plate after it has reached this maximum velocity is open to some question. The exponential expression in Equation [3] indicates that the velocity eventually reaches a zero value, when both terms become zero separately. In the

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* See Equation [5] on page 39 of TMB Report 480. Care is required in distinguishing the nomenclature used in the present report from that in TMB Report 480.

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	Density ρ		Specific Impedance ρc
	pounds weight per cubic inch	$\frac{\text{mass units}}{\text{in}^3} \times 10^6$	
Fresh water	0.0361	93.6	5.3
Sea water	0.0370	96	5.57

The speed of sound is taken to be 56.5×10^3 inches per second in fresh water, 58×10^3 inches per second in sea water.
Mass units of density have received no accepted name; they have the dimensions of pounds per cubic inch ÷ inches per second squared. The dimensions of specific acoustic impedance ρc (radiation resistance) are pounds per square inch ÷ inches per second.

meantime the decrease implies a means for applying a negative acceleration, and this involves tensile action in the water. Assuming that this can occur, the velocity diminishes, at a ratio depending on the value of x, until the total travel reaches the value

$$2p_0/\alpha\rho c$$

If cavitation occurs in the water, however, the plate will retain part or all of the velocity imparted to it; if no external agency acts upon the plate, it will continue to move forward indefinitely.

It is seen that the value $x = 1$ has no singular properties as might be supposed at first sight. This is fortunate as the values actually occurring are of the order of unity.

The parameter x is of special significance in these developments. Equation [3] may for convenience be rewritten

$$v = \frac{2p_0}{\alpha m} \cdot \frac{q - q^x}{x - 1} \qquad [3a]$$

and some attention to the actual numerical values will clarify matters.

p_0 for close explosions of 300-pound charges may rise to several thousand pounds per square inch over large areas.

α for a 300-pound charge is of the order of the reciprocal of one millisecond.

m in the typical case of 25-pound steel plate, thickness 5/8 inch, is 0.175 pound per square inch or 0.175/386 = 0.00045 mass units per square inch.

q is a fractional function of time, diminishing as t increases from the value unity at $t = 0$.

The parameter x may also be interpreted physically in a way that throws a clarifying light on the whole process by which momentum and energy are transferred from the water to the plate. Since x is a dimensionless ratio, it is easy to see that the expression $\rho c/\alpha$ figures as a mass which stands in the ratio x to the mass m per unit area of the plate. Dimensionally ρc is a specific mechanical impedance, force per unit velocity, referred to unit area, pounds per inch square divided by inches per second. α is a reciprocal time. Mass per unit area is measured in absolute units, pound-second^2 per inch^3.

The mass that is represented by $\rho c/\alpha$, or, preferably, by m_w, where

$$m_w = \frac{2\rho c}{\alpha} \qquad [6]$$

can be given a simple physical significance. The term m_w thus defined represents that mass of water which, moving at a suitable velocity u_w, would possess the same amounts of momentum and of kinetic energy as exist in the shock wave per unit of area on the wave front. This mass can be visualized as a column of water having a cross section of one square inch and a certain length l_w. The length l_w may be regarded as the equivalent length of the wave; in actual cases, the wave is effectively limited to a spherical shell of thickness about equal to l_w. For a shock wave that has a finite thickness and is of uniform intensity throughout its thickness, m_w and u_w represent, respectively, the mass per unit area and the particle velocity of the actual water which at any given moment contains the wave.

The shock wave can thus be regarded as roughly equivalent to an aqueous projectile of mass m_w moving at speed u_w, but with an important difference. Whereas a solid projectile carries kinetic energy only, a shock wave carries potential energy or energy of compression as well. If the pressure is low enough so that acoustic theory applies (for example, if the pressure is less than 10,000 pounds per square inch), the potential energy is equal in amount to the kinetic energy. In such cases the shock wave carries twice as much total energy in proportion to its momentum as does the solid projectile. A shock wave might be compared to a projectile containing a compressed spring or perhaps potential explosive energy that is released upon impact.

The general formulas for m_w, u_w and l_w, for a shock wave of acoustic type carrying momentum I and energy E per unit area of the wave front, are obtained by writing, since only half of E is kinetic,

$$m_w u_w = I, \qquad \frac{1}{2} m_w u_w^2 = \frac{1}{2} E,$$

whence

$$m_w = \frac{I^2}{E}, \qquad u_w = \frac{E}{I}, \qquad l_w = \frac{m_w}{\rho} \qquad [7a,b,c]$$

In these equations m_w is understood to be in absolute units. For convenience, however, numerical values of m_w will be given in pounds of weight per square inch; to convert such values to absolute units, they must be divided by the acceleration of gravity or 386 inches per second per second.

For the exponential wave previously described, Equation [7a] becomes, in view of [2a,b], $m_w = 2\rho c/\alpha$, as stated in [6]; and the factor x may be written

$$x = \frac{\rho c}{\alpha m} = \frac{1}{2} \frac{m_w}{m}$$

The factor 2 in these formulas arises from the requirement that the kinetic energy of the equivalent projectile shall represent the kinetic energy alone of the water, rather than its total energy.

Analytical considerations and observational results indicate that m_w and l_w should vary only with the weight of the charge; they are proportional, at least approximately, to the cube root of the weight. The velocity u_w, likewise, varies as the cube root of the charge, but in addition it falls off in inverse proportion to the distance from the charge, or somewhat more rapidly at short distances.

If the kinetic energy given to the plate is now regarded as analogous to the maximum energy absorbed by a structure of more general type, the form of Equation [5] leads to a tentative suggestion as to the type of damage function that may be found useful in other cases. For [5] can be rewritten thus:

$$\frac{T_s}{m} = 4x^{\frac{1+x}{1-x}}\frac{E}{m}$$

or

$$\frac{T_s}{m} = 2x^{\frac{2x}{1-x}}\frac{I^2}{m^2}$$

in which T_s/m represents the maximum energy of the plate per unit of its mass. Thus the form of Equation [5] suggests that we write, for application to structures in general,

$$\Omega_s = C_s\frac{E}{m}, \qquad \Omega_d = C_d\frac{E}{m} \qquad [8a,b]$$

or

$$\Omega_s = K_s\frac{I^2}{m^2}, \qquad \Omega_d = K_d\frac{I^2}{m^2} \qquad [9a,b]$$

K or C standing in each case for a dimensionless coefficient. In these formulas, both Ω and m must be expressed in terms of absolute units of mass, E in inch-pounds per square inch, and I in pound-seconds per square inch. As an alternative, if Ω_s and Ω_d are expressed in inch-pounds per pound of weight and m in pounds of weight per square inch, Equations [8a,b] remain unaltered, but Equations [9a,b] become

$$\Omega_s = K_s\frac{gI^2}{m^2}, \qquad \Omega_d = K_d\frac{gI^2}{m^2} \qquad [10a,b]$$

in which g = 386 inches per second per second.

In the case of the thin plate and the exponential wave, if we identify Ω_s or Ω_d with T_s/m, comparison of Equations [8a] and [9a] with (5) shows that

$$C_s = 4x^{\frac{1+x}{1-x}}, \qquad K_s = 2x^{\frac{2x}{1-x}} \qquad [11a,b]$$

where $x = m_w/(2m)$. Thus in this case K and C are definite functions of the quantity $x = \frac{m_w}{2m}$; these functions are plotted in Figure 1.* In general, it is to be expected that the value of the coefficient K or C will vary both with the design of the structure and with the distribution of the incident pressure in time. For shock waves having roughly an exponential form, however, and for a given type of structure, it may be expected that the principal variation of K and C will be as a function of m_w/m, as for the thin plate. Thus it should be possible to write, at least to a first approximation

$$K_s = f\left(\frac{m_w}{m}\right)$$

and similarly for the other coefficients, f denoting a function whose form does not change as the linear scale of the structure or the size of the charge is varied.

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* In calculating values of x, absolute units may be used for m_w and m, or both of these quantities may be in pounds of weight per square inch.

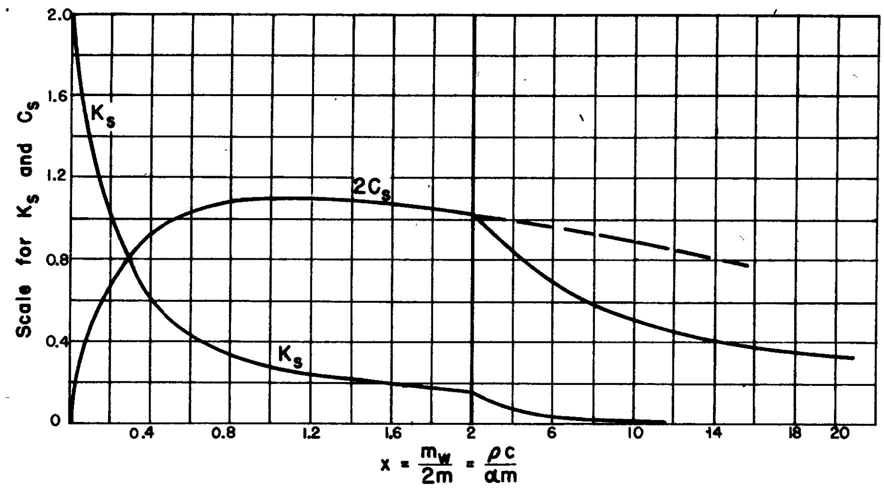


Figure 1 - Damage Coefficient for Thin Plate and Exponential Wave

For convenience a double scale of abscissas is used. The broken line shows the course of C_s if the left-hand scale is continued.

A damage function, to be satisfactory, should associate the same measure of damage with a given degree of relative damage in similar structures of different sizes. For example, the damage function should be the same for a 1/8-inch circular diaphragm one foot in diameter with a spherical dish 1 inch deep as for a 1/2-inch diaphragm four feet in diameter with a spherical dish 4 inches deep. Either [8b] or [9b] accomplishes this, provided Hopkinson's rule is valid. The uniform change of scale contemplated in this rule implies a change of m and m_w in the same ratio and, at corresponding distances, of E and I as well; and in such a change the relative distribution of pressure in the wave should be unaffected. C_d and K_d, therefore, should depend only upon $m_w/2m$. Thus Ω_d, like the relative degree of damage, is unaltered.

The choice between the use of Equations [8a,b] and Equations [9a,b], will depend upon considerations of convenience. A presumption in favor of the coefficient C, as in Equations [8a,b], is raised by the example of the thin plate and exponential wave, which was treated previously, beginning on page 6. For this case, a glance at Figure 1 shows that K_s varies much more rapidly with m_w/m than does C_s. Referring to the examples in Table 2 derived from Figure 1, K_s varies from a value of 2 for any very heavy plate (or large m) to 0.01 for the 1-inch plate (x = 9.7). On the other hand, C_s, although it is zero for an infinitely heavy plate which reflects the wave entirely, rises to a maximum value of 0.54 at m_w = 2 or x = 1, and then sinks to only 0.25 for x = 9.7.

For structures of other types the values of K and C may be expected to be considerably different, but it seems likely that a similar contrast between their behavior as functions of $m_w/2m$ will occur generally. Since rapid variation of the dimensionless coefficient is a disadvantage, Equations [8a,b] connecting damage with energy in the water seem likely to be superior to Equations [9a,b] or [10a,b], which

TABLE 2

Plate thickness, inches	Infinite	1/8	1
Pounds of TNT	Finite	1/16	300
m_w,* lbs/in^2	Finite	0.32	5.4
m, mass/unit area	Infinite	0.035	0.278
$x = m_w/2m$	0	4.6	9.7
$K_s = 2x^{\frac{2x}{1-x}}$	2	0.04	0.01
$C_s = 4x^{\frac{1+x}{1-x}}$	0	0.37	0.25

connect damage with impulse in the water. For the present, however, both types of formula will be retained.

It is hoped that formulas of the type under discussion may be found to be applicable to structures of all types, provided the explosive center is not too close, and that methods may be developed for the calculation of the coefficient C or K. Near the source of the explosion, however, it is to be expected that more complicated formulas, perhaps containing an additional term, will be necessary.

It may be found useful, furthermore, to divide the motion of the water roughly into a short A-phase, in which compression of the water plays a prominent part, and a longer B-phase, in which the water behaves nearly as an incompressible liquid. It is not yet clear how much of the damage produced on actual structures is due to the first high-pressure phase or shock wave, and how much is due to the following low-pressure phase. Furthermore, the effectiveness of the secondary impulses due to later collapses of the gas globe remains to be elucidated. In view of these uncertainties, it is left open for the present whether E and I in Equations [8a,b] to [10a,b] should be defined with reference to the shock-wave alone or in some more general manner.

THE EXPLOSIVE DISTURBANCE IN THE WATER

Some observational data pertaining to shock waves are given in Table 3, with accompanying explanations; impulse and energy are denoted here by I_1 and E_1, respectively, to indicate that the values given refer to the primary shock wave alone.

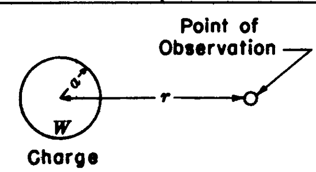
It is to be noted that $I_1\,r/W^{\frac{2}{3}}$ and $E_1\,r^2/W$ are only roughly constant, showing a tendency to increase as W decreases. This variation may be due to imperfections in the observational methods. If single values are to be chosen for checking purposes, with no attention paid to the kind of explosive, the following are suggested:

* In obtaining these numbers the value of m_w is calculated from Equation [7a] or Equation [6] in terms of the assumed form of the shock wave.

$$I_1 = 22\frac{W^{\frac{2}{3}}}{r}, \qquad E_1 = 230{,}000\frac{W}{r^2} \qquad [12a,b]$$

where I_1 is in pound-seconds per square inch, r in inches, W in pounds, E_1 in inch-pounds per square inch.

TABLE 3
Shock-Wave Magnitudes

Charge W pounds	a inches	r inches	I_1 lb-sec. per in²	$\frac{I_1 r}{W^{\frac{2}{3}}}$	E_1 inch-pounds per in²	$\frac{E_1 r^2}{W}$
1900 (50/50)+	19.8	1098	2.90*	20.8	278	1.76 x 10⁵
1000 TNT	16.0	900	2.18*	19.6	316	2.56
300 TNT	10.7	600	1.63*	21.8	190	2.28
300 (40/60)+	10.7	600	1.73*	23.2	193	2.32
40 (40/60)+	5.5	306	1.07*	28.0	120	2.81
0.048 TNT	0.58	18	0.21**	29		
0.034 TNT	0.52	18	0.20** 0.07†	33 12		
Detonator [0.0013 TNT (?)]	0.18	18	0.016** 0.02†	25 30		

W is the size of the charge, in pounds weight.

+ The symbols (40/60) and (50/50) denote amatol.

a is the radius of the charge if it is assumed to be spherical and to have specific gravity 1.6.

r is distance from center of charge to point of observation in inches.

* The values of I_1 (impulse per unit area) and E_1 (energy per unit area) in the first five lines were obtained by integration from some of Hilliar's curves (6) with a small addition for the supposed missing "tails" of the curves.

** The values of I_1 for small charges are from the work at the David Taylor Model Basin. The first one is calculated from a hoop-strain reading; the others are from data given in Reference (1). The hoop-strain values given on page 6 of Reference (1) were multiplied by $\pi/2$, corresponding to the assumption of a sinusoidal ballistic throw, and then divided by 0.92 or 1.32 respectively, to allow roughly for reflection effects.

† The values 0.07, 0.02 are from pressure-gage readings.

A few values calculated from the formulas just given, for r = 12 inches, also values of m_w and of u_w estimated from Hilliar's data and reduced in proportion to $W^{\frac{1}{3}}$, are given in Table 4. Under E are listed values of the energy flux as calculated, not from shock-wave observations, but from the estimated value of the total energy given to the water, that is, from Equation [1] or

$$E = \frac{15.9}{4\pi} \times 10^6 \frac{W}{r^2} = 1.26 \times 10^6 \frac{W}{r^2}$$

TABLE 4

Charge W	I_1 pound-sec. per in²	E_1 inch-pounds per in²	E inch-pounds per in²	m_w pounds per in²	u_w inches per sec.
1 pound	1.83	1600	8750	0.81	875
1 ounce	0.29	100	550	0.32	350
0.5 ounce	0.18	50	275	0.26	280
0.02 ounce (detonator)	0.021	2	11	0.087	95

It will be noted that E is 5 or 6 times larger than E_1. Presumably this difference is chiefly due to the fact that the measurements upon which E_1 is based covered only the primary shock-wave, whereas the larger part of the energy emitted by the explosive is to be found in the secondary waves. Similar comments may be found to apply to impulse values; the quantities tabulated under I_1 will remain uncertain until a clearer understanding of the structure of the pressure pulse is achieved.

The existing data, however, are inadequate both in extent and in reliability. The measurements made in England by the piezo-electric method yield values of the pressure 20 to 30 per cent higher than Hilliar's, but they are subject to some uncertainty, especially because the gages used were rather large. As a tentative procedure, these measurements have not been utilized here.

Concerning the pressure during later phases of the process, after the pressure has sunk below a fifth of its peak value, little is known. Evidence to be presented later indicates that under certain circumstances more damage may be done by the low-pressure phase of the pressure wave than by the initial high-pressure phase. In order to obtain a rough idea of what might be expected during the later phases, a rough estimate of the entire pressure field as a function of time was made. Hilliar's measurements, combined with certain of Penney's results (5), were used for the shock wave proper; values of the pressure at later times were then obtained by introducing a plausible time lag or retardation into results calculated on the assumption of incompressible water. It was assumed that a sixth of the total available energy would be carried away by the high-pressure shock wave and that another sixth would be spent in heating the water; thus two-thirds would remain in the form of kinetic energy in the water that is rushing outward from the gas globe.

Part of the pressure values thus obtained, adjusted to various weights of charge according to the accepted principle of similitude, are plotted in Figure 2. These values must be regarded as entirely tentative and subject to correction, but until better values become available the plot may be useful for purposes of orientation.

A striking feature brought out by the plot is that, in spite of the extreme rapidity with which the pressure in the globe of exploded gas sinks to very low values,

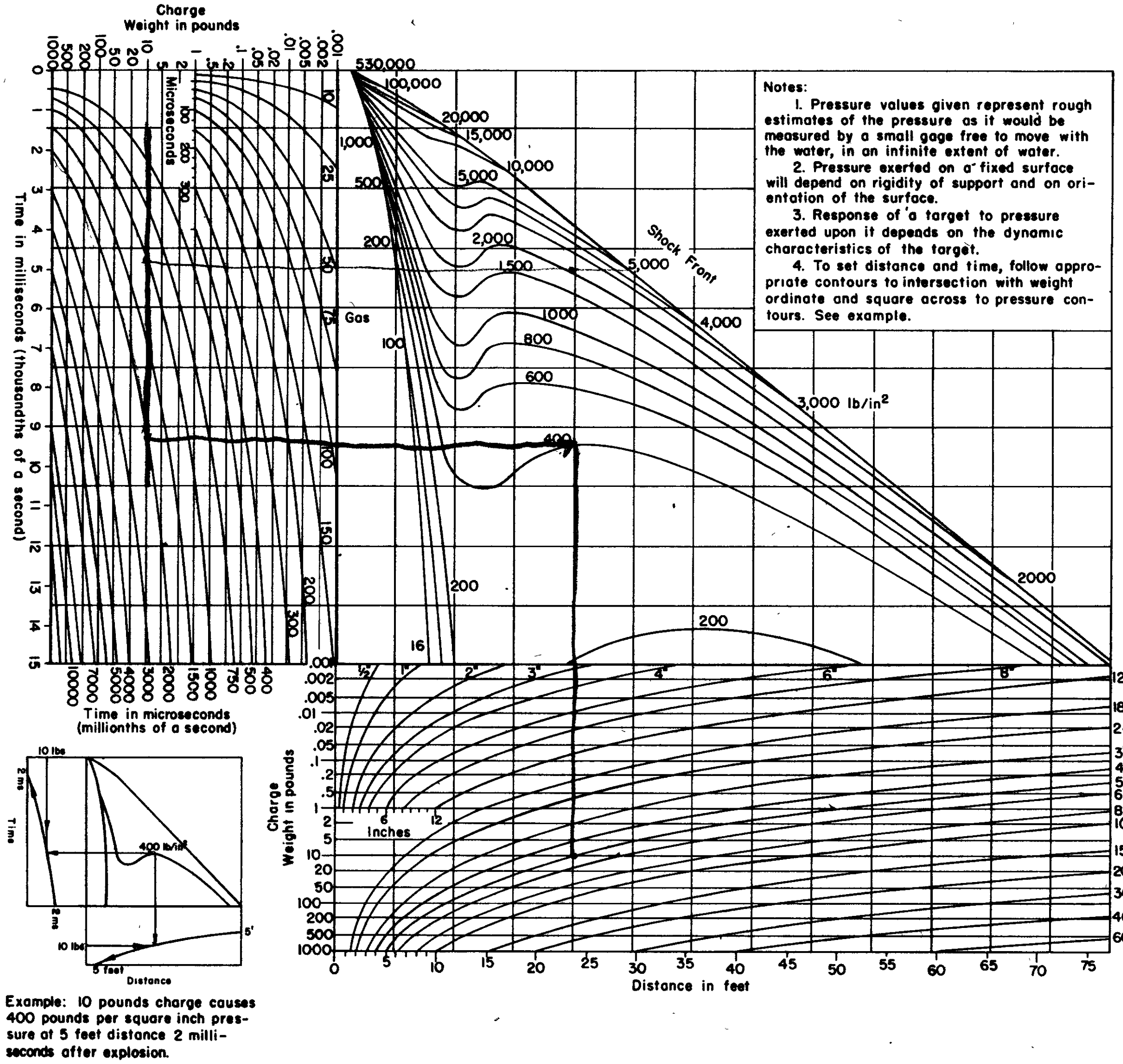


Figure 2* - Pressure due to Explosion of TNT under Water

Contours represent pressure in pounds per square inch. This chart is based on tentative data and is subject to correction. A hydrostatic pressure of one atmosphere is assumed.

high pressures continue to exist, not only in the shock wave proper, but also in the water at a moderate distance from the gas globe. For example, 5 milliseconds after detonation of a 300-pound sphere of TNT, the pressure in the gas has sunk from an initial value of over 500,000 pounds per square inch to 100, the radius of the globe being then about 4 1/2 feet; but at a distance of 8 feet from the center the pressure in the water is 1000 pounds per square inch. Beyond this point the pressure is seen

* The David Taylor Model Basin can furnish larger copies of this chart upon request.

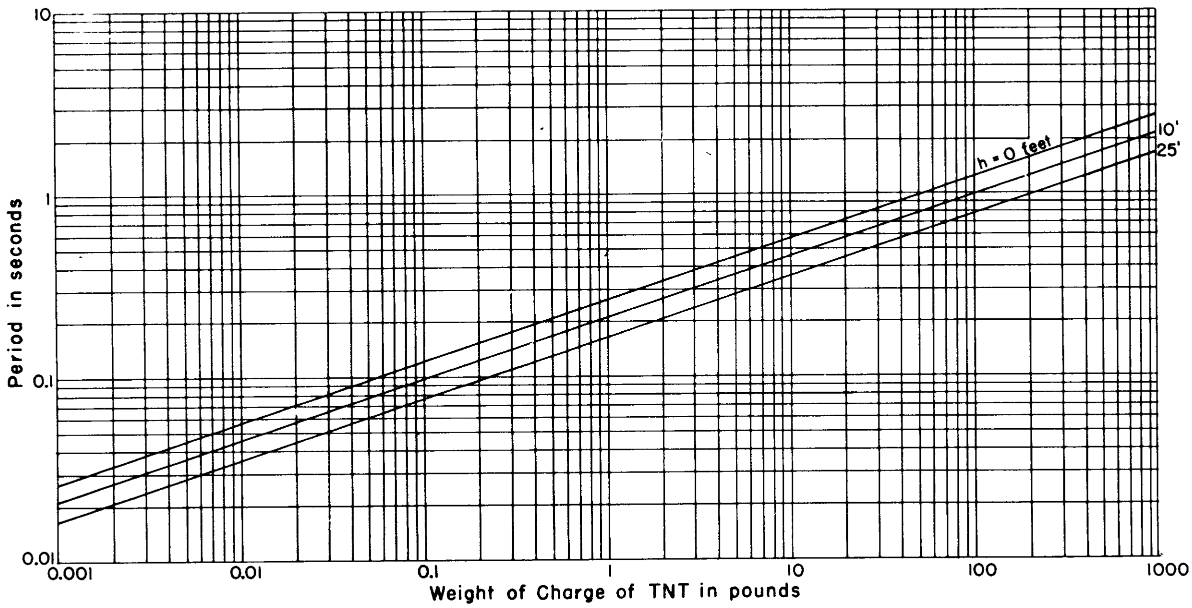


Figure 3 - Period of Oscillations of the Gas Globe

The period of large amplitude oscillations of a gas globe due to an exploded charge of TNT under water (at a depth of *h* feet) is plotted as a function of the weight of that charge. It is assumed that during the first expansion one third the total energy is lost by radiation and heating effects. The curves are valid for $r_2/r_1 \geqq 10$ where r_2 is the maximum and r_1 is the minimum globe radius.

to dip slightly before rising to a value of nearly 4,000 at the shock front, which has traveled to a distance of about 25 feet.

The high pressure thus brought into evidence near the gas globe can be ascribed to a pumping action of the outrushing water and is intimately associated with the expansion of the gas. As the water near the gas globe rushes outward, it finds itself moving across the surface of a sphere of continually increasing radius. For this reason, although the linear velocity of the water continually decreases, the rate of outflow as measured in units of volume per second increases. The water in outlying layers must, therefore, be accelerated outward in order to accommodate the increasing outflow from within; and these outlying layers, reacting because of inertia upon the inner layers of water, generate the pressure under discussion.

The plot in Figure 2 covers only the interval of time during which considerable pressures occur at points between the advancing shock wave and the gas globe. Subsequent occurrences are of some interest, however. The initial phase is followed by a much longer interval during which the outrushing water is gradually brought to rest by the surrounding hydrostatic pressure. At its moment of arrest, the gas globe should have a relatively large diameter (perhaps 50 feet for a charge of 300 pounds), and its pressure should be negligible. During the subsequent contraction of the gas, the surrounding water will recover most of the kinetic energy that it lost during the expansion. As the radius of the gas becomes small and its pressure rises again to high values, the inrushing water is very sharply checked and then as quickly reversed in its motion. As a consequence, a second pressure wave of considerable intensity is sent out. Several such oscillations may occur, each emitting a fresh wave of pressure, provided the spherical symmetry of the process remains undisturbed.

The period of such oscillations is readily estimated* and the formulas have been confirmed by observations made in England and elsewhere. For convenience of reference a plot showing the period as a function of the weight of the charge is given in Figure 3, based on the approximate formula for oscillations of large amplitude.

REFERENCES

(1) "Torpedo-Protection Systems - Preliminary Data on the Underwater Shock Wave," TMB CONFIDENTIAL Test Report, December 1941.

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(3) Bureau of Ships CONFIDENTIAL "Report of Test of 1934 Half-Scale Torpedo Protection Caissons," by Lt. H.A. Schade (CC) USN, May 1935.

(4) "The Pressure-Volume Relations and the Chemical Constitution of the Products of Detonation in TNT during Adiabatic Expansion," by H. Jones, R.C. 212, 1941.

(5) "The Pressure-Time Curve for Underwater Explosions," by Dr. W.G. Penney, Ministry of Home Security, Civil Defense Research Committee, England, R.C. 142, November 1940.

(6) "Experiments on the Pressure Wave thrown out by Submarine Explosions," by H.W. Hilliar, Research Experiment 142/19, 1919.

* See, for example, Taylor Model Basin Report 480, page 14.