UNITED STATES

EXPERIMENTAL MODEL BASIN

NAVY YARD, WASHINGTON, D.C.

DRAFT OF PROPOSED
A.S.M.E. BOILER CONSTRUCTION CODE
RULES FOR CONSTRUCTION OF
UNFIRED PRESSURE VESSELS SUBJECTED TO EXTERNAL PRESSURE

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DRAFT OF PROPOSED

A.S.M.E. BOILER CONSTRUCTION CODE

RULES FOR CONSTRUCTION OF

UNFIRED PRESSURE VESSELS SUBJECTED TO EXTERNAL PRESSURE

PREFACE

The proposed code contained in this report was prepared by the technica staff of the U.S. Experimental Model Basin at the request of Mr. Wm. D. Halsey, Chairman, Special Research Committee of the American Society of Mechanical Engine on the Strength of Vessels Subjected to External Pressure.

The form of the proposed code is based upon the present code for Unfire Pressure Vessels, Section VIII of the A.S.M.E. Boiler Construction Code; the sub stance of the code, particularly that portion relating to design, is based upon a available theoretical treatments of the subject and upon extensive model experime conducted during recent years.

The proposed code is intended for discussion and preliminary applicatio only; for this reason blank pages have been left for the making of notes and the drafting of changes by interested parties.

U.S. Experimental Model Basin,
Navy Yard, Washington, D. C.

July, 1932. Report No. 329.

PART I

DRAFT OF PROPOSED

A.S.M.E. BOILER CONSTRUCTION CODE

SECTION . . . . .

RULES FOR CONSTRUCTION OF

UNFIRED VESSELS SUBJECTED TO EXTERNAL PRESSURE

GENERAL

1. The Rules in this Section apply to unfired vessels subjected to external pressure, having a diameter in excess of 3 in. (U-1)*.

The term vessels in this Section shall apply to all constructions, including pipes and tubes 1 **.

CORROSIVE SUBSTANCES

2. All pressure vessels which are to contain substances having a corrosive action upon the metal of which the vessel is constructed, shall be designed for the pressure they are to withstand, and the thickness of all parts subject to corrosion should be increased by a uniform amount to safeguard against early rejection (U-11).

MATERIAL

3. All materials used in the construction of the important parts of vessels subjected to external pressure, for which specifications are given in Section II of this Code, shall conform to such requirements (U-12). 2

4. (a) Plates for any part of a riveted vessel required to resist stress produced by external pressure shall be of flange- or firebox-quality steel conforming with Specifications S-1 for Steel Boiler Plate and S-2 for Steel Plates of Flange Quality for Forge Welding of Section II of the Code, except as provided in (b) hereunder 2.

* Numbers in parentheses are the corresponding paragraph numbers of A.S.M.E. Boiler Construction Code, Section VIII, UNFIRED PRESSURE VESSELS.

** Numbers in circles correspond to the paragraph numbers in Part II.

If desired, both flange and firebox steel of lower tensile strength than specified may be used for an entire vessel or part thereof, the desired tensile limits to be specified with a range of 10,000 lb. per sq. in. All such steel shall conform to the Specifications for Steel Boiler Plate (3).

(b) Steel plates for any part of a pressure vessel which is to be constructed with other than riveted joints shall be of the quality specified for the particular kind of joint used (U-13).

5. For resistance to crushing of steel plate, the maximum allowable working stress shall be 19,000 lb. per sq. in. of cross-sectional area (U-15).

6. In computing the maximum allowable stress on rivets in shear, the following values in pounds per square inch of the cross-sectional area of the rivet shank shall be used:

Iron rivets in single shear	7,600
Iron rivets in double shear	15,200
Steel rivets in single shear	8,800
Steel rivets in double shear	17,600

The cross-sectional area used in the computations shall be that of the rivet shank after driving (U-16).

CONSTRUCTION

7. The maximum allowable working pressure is the maximum external or differential pressure at which a pressure vessel may be operated (U-19).

Where the term "maximum allowable working pressure" is used it refers to excess of external over internal pressure in pounds per square inch.

8. The maximum allowable working pressure on the shell of a pressure vessel shall be determined by application of the rules quoted hereunder for vessels of each class or type (U-20).

DESIGN

9. For design purposes, vessels shall be divided generally into two class-

es and three types, depending upon the ratio of thickness of shell to diameter of shell, and relative spacing of supports and stiffeners.

CLASS 1. Vessels having a ratio of thickness to outside diameter of 0.04 or greater, which invariably collapse by yielding or crushing of the material (failure of the material in pure compression).

CLASS 2. Vessels having a ratio of thickness to outside diameter of less than 0.04, in which collapse may take place by failure of the material in compression, by instability or buckling of the shell, or both.

The three types may be defined briefly as follows:

(a) Long pipes and tubes whose length exceeds the "critical length"* as determined by the diameter, thickness of material, etc.

(b) Intermediate vessels, whose length between supports is less than the critical length but greater than 0.5 diameter.

(c) Vessels with closely spaced stiffeners, whose unsupported length between stiffeners is less than 0.5 diameter.

10. Symbols. The following notation shall apply to the equations throughout this code:

p = collapsing pressure, lb. per sq. in.

E = modulus of elasticity in tension, lb. per sq. in.

$\frac{1}{m} = \sigma$ = Poisson's ratio.

$S_{(yield)}$ = yield point of material, lb. per sq. in.

$S_{(prop)}$ = proportional limit of material, lb. per sq. in.

$S_{(ult)}$ = ultimate strength of material, lb. per sq. in.

t = minimum thickness of shell plating in weakest course, in.

L = unsupported length of tube between transverse stiffeners, in. (see Chart V)

C = length of tube between centers of transverse stiffeners, in. (see Chart V)

d = diameter of tube to neutral axis of shell, in.

D = outside diameter of tube, in.

$2r$ = inside diameter of tube, in.

* The critical length of a vessel is defined as the maximum length between circumferential supports for which collapse is influenced by length.

A = cross-sectional area of stiffening ring, sq. in.

b = width of strip of shell plating acting with flange of stiffening ring (see Chart V)

f = width of flange of stiffening ring, in.

w = web thickness of stiffening ring, in.

n = number of lobes in any one circumferential belt into which shell collapses (4).

l = length of one lobe = $\frac{\pi d}{n}$

e = maximum variation in radius within the region of one lobe, as measured from the geometrical center of the shell section, i.e., maximum radius minus minimum radius.

11. Design formulas. The relation between calculated collapsing pressure, thickness of shell, diameter, and length between circumferential supports shall be determined by the following formulas, where the notation is as previously described:

CLASS 1. Vessels having a ratio of thickness to outside diameter 0.04 or greater (5).

$$p = 2 \times S_{(yield)} \left(\frac{t}{D}\right)\left(1 - \frac{t}{D}\right) = 55{,}000 \frac{t}{D}\left(1 - \frac{t}{D}\right) \quad \ldots\ldots\ldots \quad (1)$$

This formula is illustrated graphically in Chart I.

CLASS 2. Vessels having a ratio of thickness to outside diameter of less than 0.04

(a) whose length exceeds the critical length (6).

Case 1. $p = 50.2 \times 10^6 \left(\frac{t}{D}\right)^3$ .... (2)

for $\frac{t}{D}$ equal to or less than 0.023.

Case 2. $p = 86{,}670 \frac{t}{D} - 1386$ .... (3)

for $\frac{t}{D}$ greater than 0.023 but less than 0.04

These formulas, with others listed under (b) below, are illustrated graphically in Chart II.

(b) whose length between supports is less than the critical length but greater than 0.5 diameter. When the outside diameter is 24 in. and less, Case 1. $p = 73.1 \times 10^6 \frac{(t/D)^{5/2}}{L/D}$ . . . . (4)

for stresses equal to or less than the proportional limit.

Case 2. $p = 2 \times S_{(yield)} (\frac{t}{D})$ . . . . (5)

for stresses equal to the yield point.

When the outside diameter is greater than 24 in. (7).

Case 3. $y = \frac{1-\sigma^2}{n^2+\frac{\alpha^2}{2}-1}\left[\left(\frac{\alpha^2}{\alpha^2+n^2}\right)^2 + x\left\{(n^2+\alpha^2)+2\mu_1 n^2+\mu_2\right\}\right]$ . . . . (6)

where $\mu_1 = 1 + 1.65\rho + 0.455\rho^2$, $\mu_2 = 1 + 1.3\rho - 1.39\rho^2$

$y = \frac{p}{2\ t/d}\ \frac{1-\sigma^2}{E}$, $x = \frac{1}{3}(\frac{t}{d})^2$, $\alpha = \frac{\pi d}{2L}$, $\rho = \frac{\alpha^2}{n^2+\alpha^2}$ and p, σ, n, L, t, d, E, have the meanings given in the list of symbols, Part I, Par.10, and the stress is equal to or less than the proportional limit.

Case 4. $p = 2 \times S_{(yield)}(\frac{t}{d})$ for stresses equal to the yield point.

These formulas are illustrated graphically in Chart III.

(c) whose length between stiffeners is less than 0.5 diameter (7).

Case 1. $p = \frac{E}{n^2+\alpha^2/2}(\frac{t}{d})\left[\frac{2\alpha^4}{(n^2+\alpha^2)^2} + 0.73(n^2+\alpha^2)^2(\frac{t}{d})^2\right]$ . . . . (7)

for stresses equal to or less than the proportional limit,

where the terms have the meanings given for formula (6) above.

Case 2. $p = \frac{2\ S_{(yield)}\ t/d}{\frac{1}{2} + 1.815(0.85 - \frac{bt}{A+bt})\frac{K}{1+\beta}}$

where: $\beta = \frac{2Nt}{\gamma(A+bt)}$, $\gamma = \frac{1.815}{\sqrt{dt}}$, $K = \frac{\sinh\gamma L - \sin\gamma L}{\sinh\gamma L + \sin\gamma L}$,

$N = \frac{\cosh\gamma L - \cos\gamma L}{\sinh\gamma L + \sin\gamma L}$, p, $S_{(yield)}$, t, d, b, A, L, have the meanings given in the list of symbols, Part I, Par.10, and the stress reaches the yield point. (7)

These formulas are illustrated graphically in Chart III.

12. Division between Classes. The division between classes of vessels, based upon ratio of thickness to outside diameter, as defined in the proposed code, is predicated upon the use of steel having a yield point of 27,500 lb. per sq. in. If the yield point is higher, the ratio of thickness to outside diameter separating any two classes will be raised.

13. Diameter. For the design of all pipes and tubes in Class 1 and in Class 2, types (a) and (b), the outside diameter, D, shall be used as a variable in

Case 3. $$p = \frac{75.4 \times 10^6 \left(\frac{t}{d}\right)^{5/2}}{\frac{L}{d} - 0.46\sqrt{\frac{t}{d}}} \quad \ldots\ldots\ldots\ldots\ldots \quad (B)$$

NOTE: Formula (B) was developed after this code was printed and should be substituted for formulas (6) and (7). See paragraph (7-1), part II, opposite page 23.

Case 1. $$p = \frac{75.4 \times 10^6 \left(\frac{t}{d}\right)^{5/2}}{\frac{L}{d} - 0.46\sqrt{\frac{t}{d}}} \quad \ldots\ldots\ldots\ldots\ldots \quad (B)$$

harts I and II. For the design of pressure vessels in Class II, type (c), the iameter, d, measured to the middle of the shell, shall be used as a variable in hart III. 8

14. Type of Collapse by Instability. The number of lobes, n, into which the hell will divide itself at collapse shall be obtained directly from the formula 6) for any combination of thickness, length and diameter. The lobe length is calulated from the relation, lobe length, $l = \frac{\pi d}{n}$.

The number of bulges, counting those in and out from the original neutral axis f the shell, is twice the number of lobes.

The number of lobes, n, may be found by inspection from Chart IV 4.

15. Effect of Variations in Shell Thickness. Variations in actual collapsing ressure due to mill variations in thickness (or weight) of plating from the nomnal thickness of plating used for the strength calculations may be neglected proided the factor of safety is at least 2.5. 9.

16. Unsupported Length between Stiffening Rings. The unsupported length of hell L used for the determination of the variable $\frac{L}{D}$ (or $\frac{L}{d}$) in the design of ressure vessels subject to collapse by instability shall be measured as follows:-

a) When welded stiffening rings are used, either outside or inside the shell, nd when riveted stiffening rings are used inside the shell, the length, L, shall e measured from the edge of the shell flange of a stiffening ring to the near dge of the flange of the next adjacent ring.

b) When riveted stiffening rings are used outside the shell, the length L shall e measured from the center line of the rivets in the shell flange of a stiffening ing to the center line of the nearest row of rivets in the flange of the next adacent ring.

The dimensions of L, C, D, and d for various kinds of pipes, tubes, and pressre vessels are illustrated in Chart V.

17. Concentricity. Cylindrical pressure vessels shall have transverse secions as nearly circular in form as manufacturing tolerances will permit. 10

18. Eccentricity. Departures from circular form in the transverse sections

of pressure vessels shall be measured from the geometrical axis of the vessel, as nearly as that point can be determined. Variations from circular form shall be measured as differences between the maximum and the minimum radii, (not diameters) and shall be expressed as fractions of the shell thickness by use of the term $\frac{e}{t}$.

(a) The eccentricity, e, measured in the entire circumference, shall not exceed the value given by the formula:-

$$\frac{e}{t} = 0.7 \quad \ldots \ldots \ldots \ldots \ldots (9)$$

nor shall the eccentricity, e, measured in the circumferential length of any one lobe 4 (see Chart IV) exceed the value given by the formula:- 11

$$\frac{e}{t} = \frac{10}{n(1000\ t/d)} \quad \ldots \ldots \ldots \ldots \ldots (10)$$

where e is the maximum variation in radius within the region of one lobe (Max. radius minus min. radius), expressed in in.; n, t, d, have the meanings given in the list of symbols, Part I, Par. 10.

(b) When the final eccentricity in a pressure vessel unavoidably exceeds the permissible value of $\frac{e}{t}$, the calculated collapsing pressure shall be reduced by half the difference between the measured and the computed values of the term $\frac{e}{t}$. 12

$$P_{(actual)} = P_{(calc)} \left[1 - \frac{\frac{e}{t}\text{ actual} - \frac{e}{t}\text{ permissible}}{2} \right] \quad . . (11)$$

(c) When the shell thickness of a pressure vessel is, for reasons of fabrication or otherwise (except for corrosion), in excess of the thickness determined by the rules given herein, the variation from circular section will be permitted to exceed the permissible value in accordance with the following formulas:— 13 where

e is measured in the entire circumference, $\left(\frac{e}{t}\right)_{actual} = 2\,\frac{p_c - p_a}{p_c} + 0.7 \quad \ldots \ldots (12)$

where e is measured in length of one lobe, $\left(\frac{e}{t}\right)_{actual} = 2\,\frac{p_c - p_a}{p_c} + \frac{10}{n\ (1000\ \frac{t}{d})} \quad \ldots (12a)$

where p_c is the collapsing pressure in lb. per sq. in. given by the applicable formula for the weight of plating used,

p_a is the actual collapsing pressure required in lb. per sq. in., calculated from the expression $p_a = p_w$ x factor of safety (p_w=working pressure)

n, t, and d have the meanings given in the list of symbols, Part I, Par. 10.

(d) This formula for allowable eccentricity shall apply only to tanks and pressure vessels whose length between supports does not exceed two (2) diameters and shall not apply to pies or tubes less than 24 in. in diameter. (14)

STIFFENING RINGS

19. Transverse stiffening rings or frames shall be so designed that the moment of inertia of the compound section consisting of the cross-sectional area of the ring, together with the area of a strip of adjacent shell plating of approximately the same width, b, as the frame flange (see Chart V), is equal to that given by the relation:– (15)

$$I = \frac{0.046\, d^3\, C\, p}{E} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (13)$$

where I is the total moment of inertia of the compound frame section about the centroidal axis of the section, in in.^{4} (see Chart V)

C is the center to center distance between stiffening rings in in.

p, d, E have the meanings given in the list of symbols, Part I, Par. 10.

20. The moment of inertia of the compound section of the stiffening ring shall be determined by the formula:– – (16)

$$I = (I_1 + A\, d_1^2) + \left(\frac{bt^3}{12} + btd_2^2 \right) \quad \ldots \ldots \ldots \ldots \quad (14)$$

where I is as defined for formula (13),

I_1 is the moment of inertia of the stiffening ring alone, about its neutral axis parallel to the axis of the tube or vessel,

d_1 is the distance between the neutral axis of the stiffening ring alone and the neutral axis of the compound section, in. (16)

d_2 is the distance between the middle of the shell and the neutral axis of the compound section, in. (16)

A, b, t have the meaning given in Part I, Par. 10.

21. Numerous acceptable types of stiffening rings are illustrated in Chart V, with the values of b to be used for each.

22. (a) Design of Stiffening Rings for Short Frame Spacing. In the design of pressure vessels with short frame spacings, where $\frac{L}{d}$ is equal to less than 0.25, the cross-sectional area of the stiffening rings shall bear the following relation

to the required moment of inertia obtained from formula (13): 17

$$A, \text{ shall be equal to or less than } \sqrt{I} \quad \ldots\ldots\ldots \quad (15)$$

where the symbols are as described for formula (14).

(b) When frames of greater area are necessary, for reasons of fabrication or otherwise, or when heavy bulkheads are used, as in the ends of pressure vessels, the stresses in the adjacent shell plating will be increased. To compensate for this, the spacing of the stiffening rings adjacent to the heavy frames or bulkheads shall be reduced to the value given by the formula

$$L_{decreased} = \frac{4 + \sqrt{\frac{I}{A}}}{5} L \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

where L is the length between flanges of stiffening rings of cross- sectional area, A, not greater than $\sqrt{I}$, and A and I have the meanings given in formula (14). 18

CIRCUMFERENTIAL STIFFENING, LOCAL STIFFENING and SUPPORTS.

23. The structural members listed hereunder may be reckoned effective in providing circumferential stiffening, for the purposes of calculation or design:

(a) Ends, solid or closed, whether flat, dished (concave or convex) or hemisperical, provided the intersection of end and shell lies in a plane normal to the principal axis of the vessel.

(b) Internal platforms, rings, trays, and flats, provided they fit against the shell and are substantially continuous. Gaps shall not exceed one quarter (1/4) the calculated lobe length. 4

24. It shall be borne in mind, in the design of the vessels subjected to external pressure, that any variation in shape of scantlings which disturbs locally the normal elastic lobe formation will in general weaken the structure and lower the collapsing pressure. In no case can an increased collapsing pressure be expected from any such variation.

25. Likewise, any concentrated stiffening which holds one part of the shell to its initial shape or position and permits an adjacent part to deform under load, produces a relative displacement or eccentricity between the two parts which will

hasten collapse.

26. Local support of the shell at any point by means of radial stays or struts shall not be permitted, unless these stays or struts are attached to a circumferential stiffening ring.

CONCENTRATED LOADS on the SHELL

27. The use of a portion of the shell of a pressure vessel as a foundation for the carrying of miscellaneous weights will be permitted if calculations show that the combined local stresses are not escessive and if the requirements as to local stiffening are observed.

FACTOR of SAFETY

28. The factor of safety shall be the ratio between the maximum allowable working pressure and the calculated collapsing pressure.

(a) For pipes and tubes working under external pressure in excess of atmospheric, the factor of safety shall be at least 5. (19)

(b) For pipes and tubes working under a partial or complete vacuum, subjected to atmospheric external pressure only, the factor of safety shall be at least 4. (20)

(c) For large pressure vessels, which come within the range of $\frac{L}{d} = 0.10$ to 5.0 and $p = 15$ to 1000 lb. per sq. in., the factor of safety shall be at least 3, provided the eccentricity allowed in paragraph (18) is not exceeded. (21)

(d) For large pressure vessels, working under partial or complete vacuum, subjected to atmospheric external pressure only, the factor of safety shall be at least 2.5, provided the eccentricity allowed in paragraph (18) is not exceeded. (20)

All calculations shall include, and factors of safety shall be based upon ultimate or maximum collapsing pressure. (22)

JOINTS

29. The joints of pressure vessels, if of riveted construction, shall conform to the requirements of Pars.. . . . . (U-21).

30. Pressure vessels may be fabricated by means of fusion or forge welding or brazing, providing the rules governing the method adopted and as given in Pars.. .

. . . . are followed (U-22).

31. Pressure vessels may be fabricated by means of fusion welding under the rules given in Pars.. . . . . provided the construction is in accordance with the requirements for material and design as required by this code and the fusion welding process conforms to the specifications for the welding indicated for each class of vessel (U-23).

32. Pressure vessels may be fabricated by means of forge welding when the rules given in Pars.. . . . . are followed (U-24).

33. Pressure vessels for use at any temperature not exceeding 406 deg.fahr. may be fabricated by means of the brazing process when the rules given in Pars.. . . . . are followed (U-25).

34. The strength of circumferential joints of pressure vessels, the heads of which are not stayed by tubes or through braces, shall be at least 50 (?) per cent of that required for the longitudinal joints of the same structure. (U-29,a).

RIVETED JOINTS

35. The riveted longitudinal joints of a shell which is greater than 3/8 in. but less than 3/4 in. in thickness shall be of butt construction, with either single or double straps; when the shell exceeds 3/4 in. in thickness, the riveted longitudinal joints shall be of butt-and-double-strap construction 23. Adjacent edges of plates shall be fitted as nearly metal to metal as practicable (U-30).

36. The longitudinal joints of a shell not more than 3/8 in. in thickness, with the exception given below, may be of lap-riveted construction; 24 but the maximum allowable working pressure of such construction shall not exceed 200 (?) lb. per sq. in. for pipes and tubes of diameter 24 in. or less, nor 150 (?) lb. for vessels of diameter more than 24 in.

37. When a vessel is used for a purpose that makes it necessary to provide in its construction for extraordinary wear, corrosion or other deterioration in service and plates of greater thickness are used than would otherwise be required, the longitudinal joints of shells exceeding 48 in. in diameter may be lap-riveted if the following conditions are met:

The operating pressure shall not exceed 50 lb. per sq. in.

The plate thickness shall be at least 1.8 times the required plate thickness.

Tell-tale holes 1/8 to 1/4 in. in diameter shall be drilled to a depth of at least 60 per cent of the required plated thickness in those surfaces opposite the surfaces subjected to wear or other deterioration, with the spacing of the tell-tale holes not over 2 ft. apart (U-31).

38. The spherical portion of vessels of any diameter which are wholly spherical or partly hemispherical, may be constructed with lap joints provided that if the plate thickness exceeds 3/8 in., the several spherical sections of plate shall be hot pressed to the proper radius of curvature. When the vessel cannot be completed in the shop, the whole structure shall be carefully and completely fitted up ready for riveting before shipment (U-31).

39. Butt straps and the ends of shell plates forming the longitudinal joints shall be rolled or formed to the proper curvature by pressure and not by blows (U-32).

40. Rivet and Staybolt Holes. (a) All holes for rivets or staybolts in plates butt-straps, heads, braces, and lugs shall be drilled; or they may be punched at least 1/8 in. less than full diameter for material not over 5/16 in. in thickness and at least 1/4 in. less than full diameter for material over 5/16 in.
(b) Such holes shall not be punched in material more than 5/8 in. in thickness.
(c) For final drilling or reaming the holes to full diameter, the parts shall be firmly bolted in position by tack bolts, inserted in every second or third hole.
(d) The finished holes shall be true, clean and concentric (U-34).

41. Rivets shall be of sufficient length to fill the rivet holes completely and form heads at least equal in strength to the bodies of the rivets. Forms of finished rivet heads that will be acceptable are shown in Fig.. . . . . (U-35).

WELDED JOINTS

42. The longitudinal joints of pressure vessels may be of the double-welded butt-type for thicknesses of 3/4 in. or less, or of the double-welded lap-type,

or of the single-welded butt type for thicknesses of 3/8 in. or less. 25 If of the lap type the throat dimension of each of the welds shall not be less than 5/8 t, where t represents the thickness of the plate. Both edges of the lap shall be welded and the surface overlap shall not be less than 4 t. (U-73, a).

43. Circumferential joints of pressure vessels shall be of the double-welded butt type, except for thicknesses of 5/8 in. or less, in which case they may be of the single-welded butt type. Circumferential joints on vessels of shell thickness of 3/8 in. or less may be of the butt or lap type. The details of all these joints shall conform to the requirements for longitudinal joints, as given in Par. 42.

TYPE of LONGITUDINAL and CIRCUMFERENTIAL JOINTS

44. (a) No reduction need be made nor shall any increase in collapsing pressure be allowed for the use of longitudinal and circumferential joints of strength equal to or greater than that specified herein, regardless of type of construction. 26

(b) A circumferential riveted or welded joint, of 2-ply or 3-ply construction, shall not be considered as a strengthening member and no allowance shall be made for it when determining the spacing and scantlings of stiffening rings or frames. 27

FLAT HEADS

45. The thickness required in unstayed flat heads, which are unpierced and are rigidly fixed and supported at their bounding edges by riveted or bolted attachments to shells or side plates, shall be calculated by the following formula:

$$t = a\sqrt{\frac{0.162p}{S}}$$

where t = thickness of plate in head, in.

a = diameter, or short side of area, measured to the center of the inside row of rivets or bolts, in.

p = maximum allowable working pressure, lb. per sq. in.

S = allowable unit working stress, lb. per sq. in.

$S = \frac{TS}{5}$

TS = ultimate tensile strength, lb. per sq. in., stamped on the plates as provided for in the specifications for the material (U-39).

DISHED HEADS

(NOTE: - In the light of recent developments and experimental work, Pars. U-36, U-37, and U-38 of Section VIII of the A.S.M.E. BOILER CODE could be revised. As there has been no opportunity for this work, the paragraphs may be allowed to apply to the present code with special emphasis upon the following portions).

46. The radius to which the head is dished shall not be greater than the diameter of the shell to which the head is attached.

Where two radii are used, the longer shall be taken as the value of L in the formula.

No head shall be of lesser thickness than that required for a seamless shell of the same diameter.

Unstayed dished heads with the pressure on the convex side shall have a maximum allowable working pressure equal to 60 (?) per cent of that for heads of the same dimensions with the pressure on the concave side. 28

If a dished head is formed with a flattened spot or surface, the diameter of the flat spot shall not exceed that allowable for flat heads as given by the formula in Par. 45.

47. The corner radius of an unstayed dished head measured on the concave side of the head shall not be less than 3 times the thickness of the material in the head; but in no case less than 6 per cent of the diameter of the shell. In no case shall the thinning down at the corner radius of the knuckle of any dished head due to the process of forming exceed 10 per cent of the thickness required by the formula in Par.. . . . . (U-38).

SUPPORTS

48. All vessels must be so supported as to distribute properly the stresses due to the weight of the vessel and contents and to prevent the vessel from sagging out of shape (U-60).

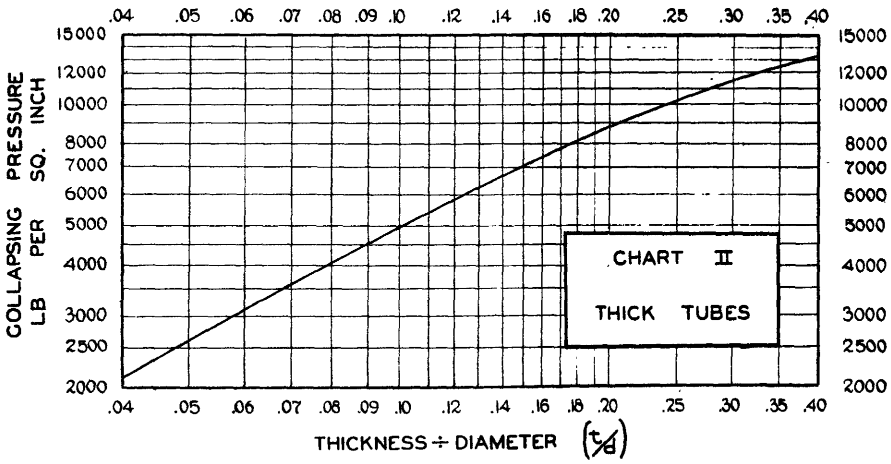
Lugs or brackets when used to support a vessel shall be properly fitted to the surfaces to which they are attached. The shearing or crushing stresses on material used for attaching the lugs or brackets to the vessels shall not exceed 40 per cent of the maximum allowable working stresses given in Pars.. . . . . .

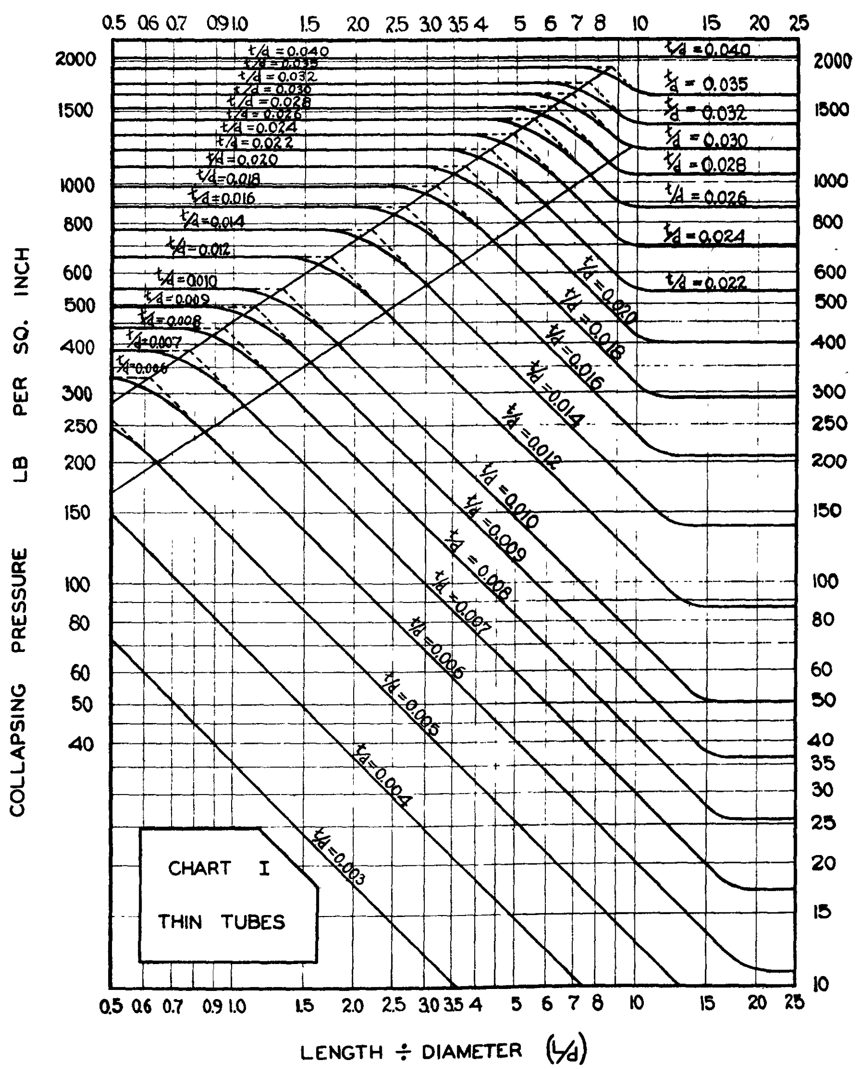
49. Vessels over 12 in. in diameter subject to interior corrosion must be so arranged that the interior and exterior of the vessel may be inspected. In the case of vertical cylindrical vessels subject to corrosion, the bottom head, if dished, must have the convex side downward to insure complete drainage (U-62).

MISCELLANEOUS

50. Where no rules are given and it is impossible to calculate with a reasonable degree of accuracy the strength of a pressure vessel or any part thereof, a full-sized sample shall be built by the manufacturer and tested in accordance with the standard practice for making a hydrostatic test on a pressure part to determine the maximum allowable working pressure, as given in the Appendix, or in such other manner as the Committee may prescribe (U-51).

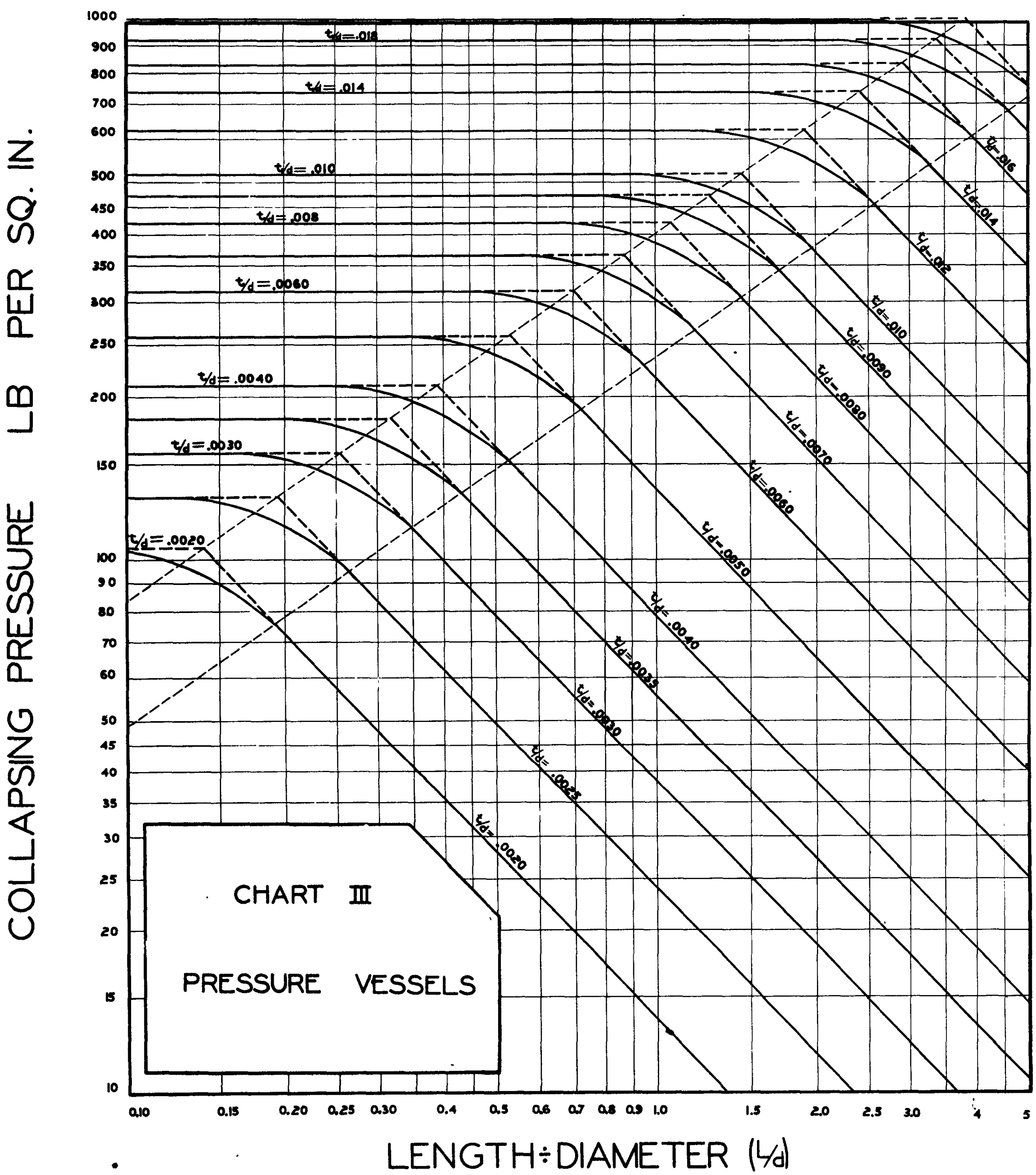
COLLAPSING PRESSURE FOR COMMERCIAL STEEL PIPES AND TUBES
OUTSIDE DIAMETER 24 INCHES AND LESS





COLLAPSING PRESSURE FOR STEEL CYLINDRICAL PRESSURE VESSELS

OUTSIDE DIAMETER GREATER THAN 24 INCHES



NUMBER OF LOBES "n" INTO WHICH A TUBE WILL COLLAPSE WHEN SUBJECTED TO UNIFORM RADIAL AND AXIAL EXTERNAL PRESSURE

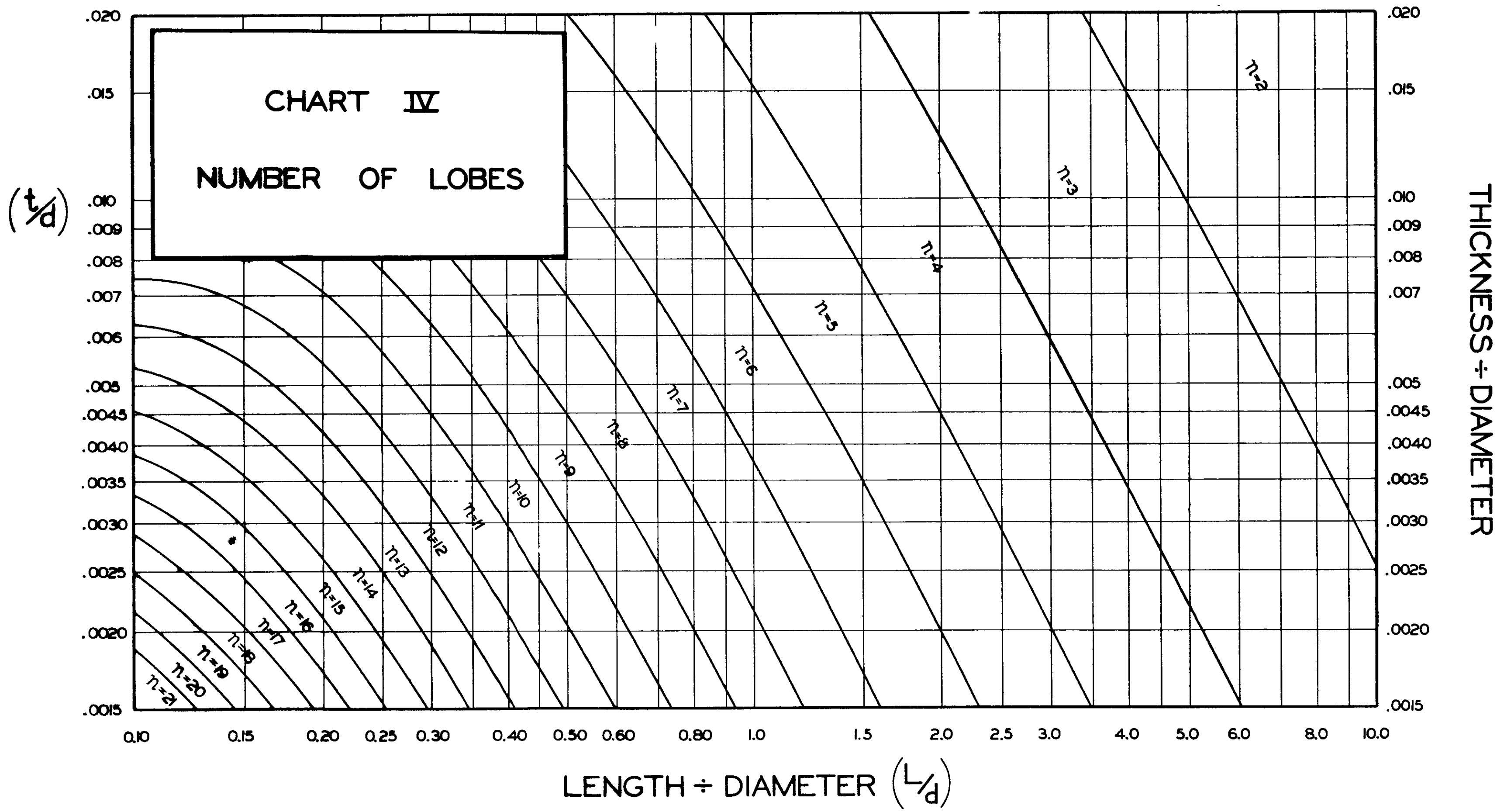
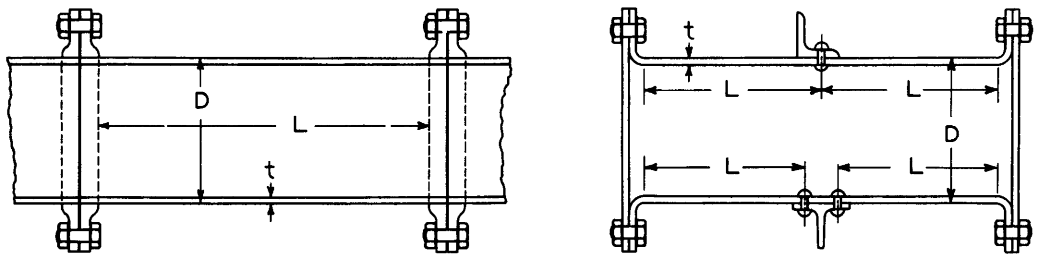
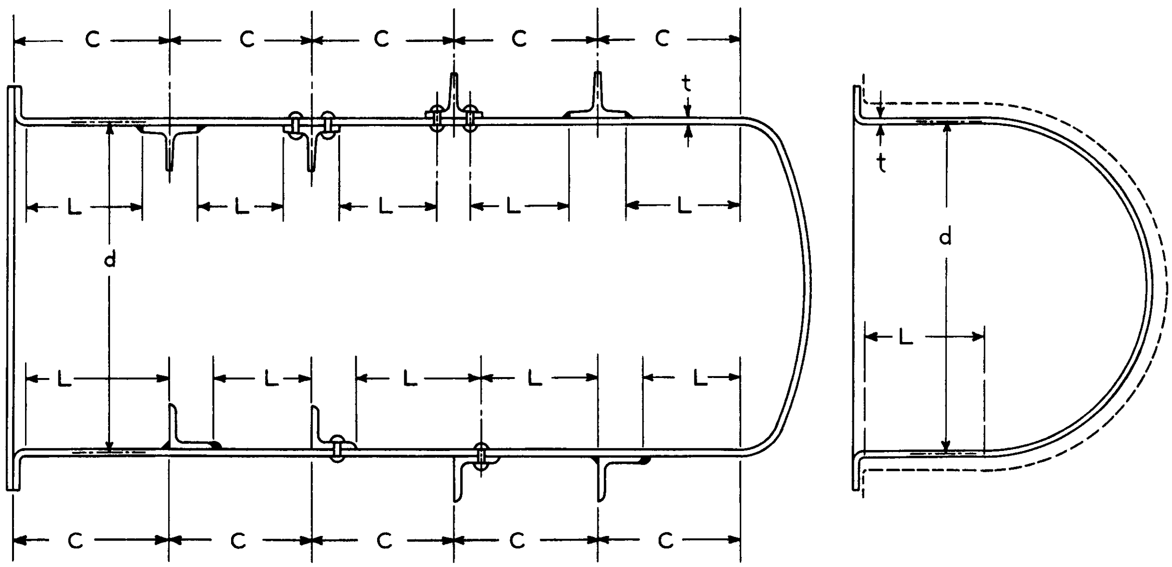


CHART V

DIAGRAMMATIC REPRESENTATION OF VARIABLES FOR DESIGN OF PIPES AND TUBES OUTSIDE DIAMETER 24 INCHES AND LESS



DIAGRAMMATIC REPRESENTATION OF VARIABLES FOR DESIGN OF PRESSURE VESSELS OUTSIDE DIAMETER GREATER THAN 24 INCHES



FORMS OF STIFFENING RINGS

OUTSIDE OR INSIDE TYPE

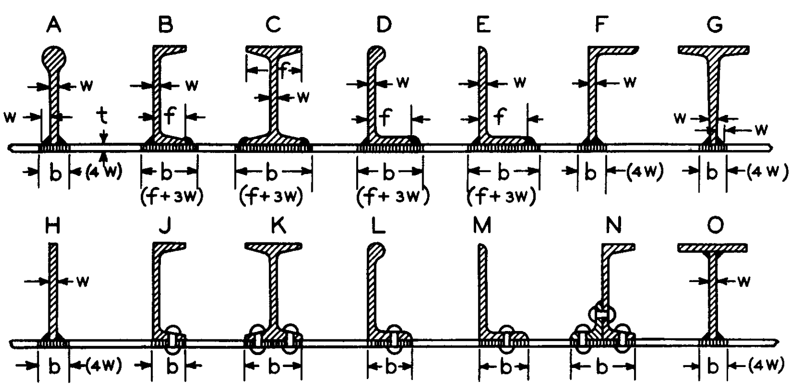
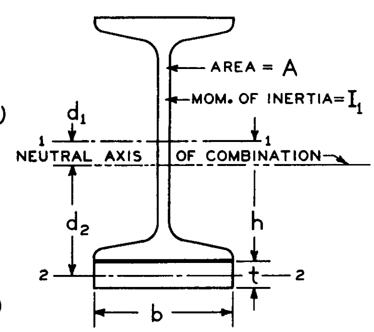


DIAGRAM FOR OBTAINING MOMENT OF INERTIA OF A COMPOUND SECTION



PART II

EXPLANATORY NOTES

accompanying DRAFT OF PROPOSED A.S.M.E. BOILER CONSTRUCTION CODE

SECTION. . . . . .

RULES FOR CONSTRUCTION OF

UNFIRED VESSELS SUBJECTED TO EXTERNAL PRESSURE

also comments on adaptation of paragraphs from present A.S.M.E. CODE "Rules for Construction of Unfired Pressure Vessels", Sect. VIII.

NOTE:- The paragraph numbers herein correspond to the numbers in circles in the proposed draft of the new code.

(1) Tubes and pipes having a diameter of less than 3 in. belong generally in the category of tubes in heaters and boilers, where the scantlings are largely governed by considerations of rolling, expanding, corrosion, and the like.

(2) All curves in Charts I, II, III, and IV of this code are based upon the following physical characteristics of the steel composing the pipes, tubes, and vessels.

Ultimate tensile strength	55,000 lb. per sq. in.
Yield point	27,500 lb. per sq. in.
Proportional limit	20,000 lb. per sq. in.
Young's modulus	29,000,000 lb. per sq. in
Poisson's ratio	0.3

(3) The code must apply also to vessels constructed of materials having various values of Young's modulus, such as bronze, nickel, copper-nickel alloys, etc. Paragraphs 3 to 6 incl., of the present codes, cover steel or ferrous materials only.

(4) Chart IV, giving the number of lobes into which a pressure vessel will

collapse when it fails by instability, was computed from formula (6) (See Note 7). This formula was employed in preparing Chart III.

A lobe is made up of two adjacent bulges, one in and one out.

5 The following formula was used in constructing Chart I:

$$p = 2\,S_{(yield)}\frac{t}{D}\left(1 - \frac{t}{D}\right) = 55{,}000\frac{t}{D}\left(1 - \frac{t}{D}\right) \quad \ldots\ldots\ldots \quad (1)$$

This formula was derived directly from the formula for the maximum compressive stress in *thick* tubes (Timoshenko: Applied Elasticity, p. 252, or any text on elasticity),

$$S_{(yield)} = \frac{2\,p\,(D/2)^2}{(D/2)^2 - r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1a)$$

where r is internal radius and

D/2 is external radius, by substituting $\frac{t}{D} = \frac{D/2 - r}{D}$.

Example 4, Part III, shows the use of Chart I.

NOTE: – The values of collapsing pressure, p, in Chart I are based on a yield point, $S_{(yield)}$ of 27,500 lb. per sq. in.. The thickness required for a given collapsing pressure, using a different yield point S', corresponds on Chart I to the thickness for a pressure p' where

$$p' = p\,\frac{S}{S'} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1b)$$

Chart I cannot be entered indiscriminately with a reduced pressure, p', since an increased yield point increases the range of Chart II and the tube may fail by instability. Hence, the value of t/D obtained from Chart I must be substituted in formula (3). If the pressure thus obtained is greater than the required collapsing pressure, failure will not occur by instability, and the t/D value is safe. However, if the pressure calculated by formula (3) is less than the required collapsing pressure, a greater value of t/D must be used. Formula (3) has been tested only up to values of t/D = 0.085. However, the curve of collapsing pressure against t/D is a straight line in this region and can undoubtedly be extrapolated to values of t/D = 0.12 or possibly t/D = 0.15. Formula (4) is inapplicable in this region.

Example 5, Part III, shows the use of Chart I for a steel pipe with yield point greater than 27,500 lb. per sq. in..

(6) The following formulas were used in constructing Chart II:

(a) Stewart-Carman formula (Trans. A.S.M.E., Vol. 27, 1906, p.795, and Univ. of Ill. Eng. Expt. Sta. Bull., No. 5, June, 1906, p. 21):

$$p = 50.2 \times 10^6 \left(\frac{t}{D}\right)^3 \quad \ldots \quad (2)$$

This formula is used for values of L/D greater than $\frac{73.1}{50.2}\sqrt{\frac{D}{t}}$ and values of $\frac{t}{D}$ equal to or less than 0.023.

(b) Stewart's formula "B" (Trans. A.S.M.E., Vol. 27, 1906, p. 793)

$$p = 86{,}670\ t/D - 1386 \quad \ldots \quad (3)$$

This formula is used for values of L/D greater than 9 and values of t/D equal to or greater than 0.023.

(c) Southwell's hyperbola (Phil. Mag., Jan. 1915, p. 70):

$$p = \frac{32}{9} E \frac{t^2}{LD} \sqrt[4]{\frac{\pi^4/16}{36} \left(\frac{m^2}{m^2-1}\right)^3 \left(\frac{t}{D}\right)^2} \quad \ldots \quad (4a)$$

When the values for E and m for Boiler Code Steel are substituted, we have

$$p = 73.1 \times 10^6 \frac{(t/D)^{5/2}}{L/D} \quad \ldots \quad (4)$$

This formula is used for values of L/D less than $\frac{73.1}{50.2}\sqrt{\frac{D}{t}}$ and for values of hoop stress, formula (5a), less than 20,000 lb. per sq. in.

(d) From the boiler, or hoop stress, formula (Seely: Resistance of Materials, p.31, or any text on elasticity)

$$S = \frac{p}{2\ t/D} \quad \ldots \quad (5a)$$

we obtain the formula $p = 2\ S_{(yield)}\ t/D$ . . . . (5)

for the maximum attainable collapsing pressure for any length.

The formula $p = 2\ S_{(prop)}\ t/D$ . . . . (5b)

determines the limit of applicability of instability formulas.

Examples 1, 2, and 3, Part III show the use of Chart II.

(7) The following formulas were used in constructing Chart III:

(a) Von Mises' formula (6) (Stodola's Festschrift, Zurich, 1929):

$$y = \frac{1-\sigma^2}{n^2+\alpha^2/2-1}\left[\left(\frac{\alpha^2}{\alpha^2+n^2}\right)^2 + x\left\{(n^2+\alpha^2)^2+2\mu_1 n^2+\mu_2\right\}\right] \quad \ldots\ldots\ldots \quad (6)$$

where

$$\mu_1 = \frac{1}{2}\left[1+(1+\sigma)\rho\right]\left[2+(1-\sigma)\rho\right] = 1+1.65\rho+0.455\rho^2$$

$$\mu_2 = (1-\sigma\rho)\left[1+(1+2\sigma)\rho-(1-\sigma^2)\left(1+\frac{1+\sigma}{1-\sigma}\rho\right)\rho^2\right]$$

$$= 1+1.3\rho-1.39\rho^2-1.417\rho^3+0.507\rho^4$$

and x, y, α, n, ρ, have the meanings given in Part I, Par.11, formula (6).

For values of n greater than 8, the number 1 may be disregarded in comparison with n^2 and the values of μ_1 and μ_2 in comparison with $(n^2+\alpha^2)^2$. Hence formula (6) reduces to

(b) $$p = \frac{E}{n^2+\alpha^2/2}\;\frac{t}{d}\left[\frac{2\alpha^4}{(n^2+\alpha^2)^2} + 0.73(n^2+\alpha^2)^2\left(\frac{t}{d}\right)^2\right] \quad \ldots\ldots\ldots \quad (7)$$

Since $\alpha = \frac{\pi d}{2L} = \frac{\pi r}{L}$, this formula becomes identical with the formula (96) quoted by von Sanden and Günther (Werft und Reederei, 1920, Heft 10, p.220).

These formulas are applicable to pressure vessels subjected to both radial and axial external pressure.

NOTE: The t/d curves in Chart III computed by formula (6) are not straight lines for the smaller values of n, since n is assumed to have integral values only. However, but slight error is introduced by using straight lines, and all values given in the chart are on the safe side. Since this region has not been sufficiently explored experimentally, the safest assumption is that collapsing pressures are inversely proportional to length. Moreover, the lengths affected are far in excess of the frame spacing of ordinary pressure vessels.

(c) Von Sanden and Günther's formula (92) (Werft und Reederei, 1920, Heft 10, p. 220, and 1921, Heft 17, p.508)

$$p = \frac{2\,S_{(yield)}\;t/d}{1/2 + 1.815(0.85 - bt/A+bt)\;K/1+\beta} \quad \ldots\ldots\ldots\ldots \quad (8)$$

where $\beta = \frac{2Nt}{\gamma(A+bt)}$, $\gamma = \frac{1.815}{\sqrt{dt}}$, $K = \frac{\sinh\gamma L - \sin\gamma L}{\sinh\gamma L + \sin\gamma L}$,

$$N = \frac{\cosh\gamma L - \cos\gamma L}{\sinh\gamma L + \sin\gamma L}$$

(7-1) Formula (7), obtained from formula (6) by the approximations already described, may be written:

$$y = \frac{1}{n^2 + \frac{\alpha^2}{2}}\left[\frac{\alpha^4}{(n^2+\alpha^2)^2}(1-\sigma^2) + (n^2+\alpha^2)^2 x\right] \quad \ldots\ldots\ldots\ldots \text{(7a)}$$

To determine the value of n which will give a minimum value of y for any given α and x, we differentiate with respect to n, equate to zero and solve for n. Differentiating thus, we obtain the equation:

$$(n^2+\alpha^2)^5 x - (n^2+\alpha^2)^4\alpha^2 x - 3(n^2+\alpha^2)\alpha^4(1-\sigma^2) + \alpha^6(1-\sigma^2) = 0 \quad \ldots \text{(7b)}$$

Since the second and fourth terms are of the same order of magnitude and of opposite signs, their difference is small in comparison with the other two terms. Hence, they may be neglected without appreciable error and

$$n^2 + \alpha^2 = \sqrt[4]{\frac{3(1-\sigma^2)}{x}} \quad \ldots\ldots\ldots\ldots \text{(7c)}$$

Substituting (7c) into (7b) and simplifying,

$$y = \frac{4.41\,\alpha\sqrt[4]{x^3}}{2.57 - \alpha\sqrt[4]{x}} \quad \ldots\ldots\ldots\ldots \text{(7d)}$$

Whence,

$$p = \frac{2.60\,E\left(\frac{t}{d}\right)^{5/2}}{\frac{L}{d} - 0.46\sqrt{\frac{t}{d}}} \quad \ldots\ldots\ldots\ldots \text{(A)}$$

or, when E = 29,000,000 lb. per sq. in.,

$$p = \frac{7.54 \times 10^6\left(\frac{t}{d}\right)^{5/2}}{\frac{L}{d} - 0.46\sqrt{\frac{t}{d}}} \quad \ldots\ldots\ldots\ldots \text{(B)}$$

Formulas (6) and (7) are practically identical for values of L/d = 0.1 – 0.5, while formula (A) differs from them by less than 2% in this region. For values of L/d greater than 0.5, formula (A) is a better approximation than formula (7). The maximum discrepancy between formula (A) and formula (6) is about 3% for L/d = 0.5 – 2.0. This discrepancy gradually increases for values of L/d greater than 2. However, in this region, formula (A) becomes practically identical with formula (4) and is amply safe.

It will be noted that when the second term in the denominator equals 1/2, formula (8) reduces to the boiler formula (5). In actual practice, the value of this term is seldom greater than 0.55 for closely spaced stiffening rings of reasonable area, and is frequently less than 0.5. The value 0.55 is used in computing the limiting collapsing pressures in Chart III, or

$$p = \frac{2\, S_{(yield)}\, t/d}{1.05} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8a)$$

determines the maximum attainable collapsing pressure, while

$$p = \frac{2\, S_{(prop)}\, t/d}{1.05} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8b)$$

determines the limit of applicability of formula (6).

Examples 6 and 7, Part III, show the use of Chart III and the application of formula (8). Example 8 illustrates the use of Chart III and the application of formulas (8) and (8a) to steel pressure vessels with a yield point greater than 27,500 lb. per sq. in..

(8) The diameter, D, is used with the first group as a matter of convenience and for purposes of correlation with previous tests. The diameter, d, is used for the second group to suit the theoretical formulas employed.

To simplify the procedure, D may be used in place of d in both groups with perfect safety.

(9) Mill variations in plating thickness are customarily of the order of 2 or possibly 3 per cent. The resulting variation in collapsing pressure would be of the order of 7 to 10 per cent at the most. For a vessel having a working pressure 40 per cent of the collapsing pressure (factor of safety 2.5), the use of the thinnest commercial plating of the nominal weight given in the design would reduce the collapsing pressure to 90 per cent of the designed figure, and would lower the factor of safety from 2.5 to 2.25. This is considered acceptable.

(10) Economy of material and increased factors of safety justify the use of special manufacturing processes to attain this feature, particularly when it is expected that collapse will take place by instability.

11 This equation is based on the analogous column theory where both length and eccentricity are expressed in terms of the least radius of gyration. The buckled portion of the shell can be considered as a column of rectangular cross-section and length, l, equal to one lobe length, whose least radius of gyration becomes $\frac{t}{\sqrt{12}}$. Length divided by radius of gyration becomes

$$\frac{l}{t/\sqrt{12}} = \frac{\pi d/n}{t/\sqrt{12}} = \frac{\pi\sqrt{12}}{n(t/d)} = \frac{\text{const.}}{n(t/d)}$$

Assuming that permissible eccentricity increases with length, we can write the equation,

$$\frac{e}{t} = a + \frac{b}{n(1000\ t/d)}$$

where a and b are coefficients to be determined.

Giving to e/t limiting values justified by experiment, viz. $t/d = 0.003$, $L/d = 0.15$, $e/t = 0.22$ and $t/d = 0.003$, $L/d = 2.0$, $e/t = 0.67$, we obtain $a = o$, $b = 10$. Hence

$$\left(\frac{e}{t}\right)_{permissible} = \frac{10}{n\ (1000\ t/d)} \quad \ldots\ldots \quad (10)$$

This equation allows a greater permissible e/t for long pressure vessels than for short ones, which is justified experimentally, and also allows a smaller value of e/t, although a greater absolute value of e, for thick vessels than for thin ones. However, the values of the coefficients, a and b, are quite arbitrary. Sufficient experimental data with which to develop a final and definite relation for allowable eccentricity are still lacking.

Example 9, Part III, shows the application of formulas (9) and (10).

12 The formula

$$p_{actual} = p_{calc}\left[1 - \frac{\left(\frac{e}{t}\right)_{actual} - \left(\frac{e}{t}\right)_{permissible}}{2}\right] \quad \ldots \quad (11)$$

represents the actual collapsing pressure in terms of the calculated collapsing pressure when the actual measured e/t exceeds the permissible value. The value of the denominator is arbitrarily set at 2, and is subject to change if additional experimental data indicate a more reliable value.

Example 10, Part III shows the application of formula (11).

(13) Formulas (12) and (12a) are obtained by solving formula (11) for $(\frac{e}{t})_{actual}$ and substituting the value of $(\frac{e}{t})_{permissible}$ obtained from formulas (9) and (10).

Example 11, Part III, shows the application of formulas (12) and (12a).

(14) This is a safety precaution, since variations cannot easily be measured with sufficient accuracy on small tubes, and the e/t values given by formula (10) for vessels with L/d greater than 2.0 are large and have not been checked experimentally.

(15) See Trans. A.S.M.E., Vol 53, 1931, p. 211.

(16) The gravity axis 1 – 1 of the section of area A (and moment of inertia I_1) will be shifted by the amount d_1 determined by the formula

$$d_1 = \frac{bt(\frac{2h + t}{2})}{A + bt} \quad \text{and} \quad d_2 = \frac{2h + t}{2} - d_1 \quad \dots\dots\dots\dots \text{(14a)}$$

where A, b, t, etc. have the meanings given in Part I, Par.10, and shown graphically in Chart V.

Hence the moment of inertia of the compound section, I, becomes

$$I = (I_1 + A\,d_1^2) + (\frac{bt^3}{12} + btd_2^2) \dots\dots\dots\dots\dots\dots \text{(14)}$$

A diagram on Chart V illustrates the method of calculating the moment of inertia of the compound section formed by the stiffening ring and the adjacent strip of shell plating.

NOTE: In cases of welding, b may have a value slightly greater than the flange width as shown in Chart V.

Example 7, Part III, shows the application of formulas (14) and (14a).

(17) A stiffening ring having excessive area will not contract to the same degree as the shell when external pressure is applied. This results in considerable relative radial displacement between ring and shell, and the consequent development of high stresses in the shell adjacent to the ring.

Example 7, Part III, shows the application of formula (15).

(18) The limiting cases of formula (16) are, first, $A = \sqrt{I}$, $L_{decreased} = L$ and, second, $A = \infty$, $L_{decreased} = 4/5\ L$.

The reduction of the frame spacing adjacent to a bulkhead to 80 per cent of the frame spacing permitted for stiffening rings having an area not greater than the square root of the required moment of inertia for the compound section is an arbitrary one but is considered amply safe.

Example 7, Part III, shows the application of formula (16).

(19) The factor of safety for pipes and tubes subjected to external pressure should be at least 5, as measurements for out-of-roundness are rarely available and special allowances are almost never made.

(20) When vessels under a partial or complete vacuum are subjected to atmospheric external pressure only, this pressure can never exceed 15 lb. per sq. in., hence an allowance for possible excess pressure may be eliminated.

(21) The factor of safety for large pressure vessels, whose collapsing pressures are obtained from Chart III, need not exceed 3 provided the rules for allowable eccentricity are followed. Experience shows that large vessels are more nearly circular (15) than small ones, but the exact degree is not now known.

(22) It is almost impossible to determine stresses experimentally in vessels under external pressure to ascertain compliance with design values or formulas but it is possible, by model or full-scale experiment, to determine the collapsing pressure by failure of the structure under external hydrostatic pressure.

(23) Single-strap riveted longitudinal joints, in which the strap is equal in thickness to the shell plate, have been found sufficiently strong and rigid for large structures; moreover, the joints are lighter and an extra ply of riveting is eliminated.

(24) The type of longitudinal joint in a vessel subjected to external pressure unquestionably has some effect upon the lobe formation and thus affects the strength of the vessel to resist collapse. Butt joints, either single or double strapped, are to be preferred to lap joints because of their symmetry, yet lapped joints have been extensively used because of their simplicity and have been found quite satisfactory in service. It is considered desirable, however, to reduce the permissible shell thickness for lap joints from 1/2 in. to 3/8 in. The marks (?) are intended to in-

dicate that the values previously specified are open to question.

25 It is considered desirable to retain the maximum thickness for double-welded lap joints as 3/8 in., the same as for riveted lap joints. Single-weld butt joints on 3/8 in. plate should be satisfactory for external pressure vessels and double-welded butt-type joints have been found satisfactory for thicknesses of 3/4 in.

26 Riveted longitudinal joints may under certain conditions be a source of weakness rather than of strength, especially if the joint is stiff and the shell plating alongside the joint is somewhat flat, due to difficulties in rolling and fabrication.

The formula, $t = \frac{pd}{2400}\left(1 + \sqrt{1 + \frac{a}{p} \quad \frac{l}{l+d}}\right) + 0.2$

where t, d, and l are in cm., p is in kg. per sq. cm., and a is an experimental coefficient as proposed by Bach in 1896, is wholly empirical. It was based upon the experiments of Fairbairn and Richards and cannot be applied to modern pressure vessels. The possible weakening effect of welded and riveted butt joints of any approved type can apparently be ignored provided the eccentricity allowed by formula (10) is not exceeded.

27 Riveted circumferential joints, especially if 3-ply (double butt straps), unquestionably provide material support to the shell in their transverse planes by reason of their rigidity. Under certain conditions, such joints might serve in lieu of regular stiffening rings, but it is difficult, in the present state of knowledge, to formulate safe rules for such practice.

28 When dished bulkheads were used on German submarines, the pressure on the convex side was limited to 40 per cent of the pressure on the concave side.

PART III

A.- EXAMPLES ILLUSTRATING the USE of CHARTS I and II in the DESIGN of COMMERCIAL STEEL PIPES and TUBES LESS THAN 24 in. in DIAMETER.

Example 1.

Given: A closed pressure vessel 24 in. long and 12 in. outside diameter.

Required: Thickness, t, for an external working pressure of 50 lb.per sq.in., using a factor of safety of 5.

Solution:

Collapsing pressure, p, = 50 x 5 = 250 lb. per sq. in.

L = 24 in., D = 12 in., L/D = 24/12 = 2

Entering Chart II with L/D = 2, and collapsing pressure = 250 lb. per sq. in., we find that t/D lies between 0.008 and 0.009. Using the upper value for additional safety, we have t/D = 0.009, whence t = 0.009 x D = 0.009 x 12 = 0.108 in..

Example 2.

Given: A pipe 20 ft. long between flanges and 6 in. outside diameter.

Required: Thickness, t, for a working pressure of 15 lb. per sq. in. (full vacuum), using a factor of safety of 5.

Solution:

Collapsing pressure, p, = 15 x 5 = 75 lb. per sq. in.

L = 20 x 12 = 240 in., D = 6 in., L/D = 240/6 = 40

It will be seen from Chart II that collapsing pressures between 50 and 100 lb. per sq. in. become independent of the length at about L/D = 15. Hence the collapsing pressure for L/D = 40 is the same for L/D = 15.

Entering Chart II with L/D = 15, and collapsing pressure = 75 lb. per sq. in., we find that t/D lies between 0.011 and 0.012. Using the upper value for additional safety, we have t/D = 0.012, whence t = 0.012 x D = 0.012 x 6 = 0.072 in..

Example 3. Given: A tube 10 ft. long and 24 in. outside diameter, reinforced by an external ring welded to the shell midway between the heads.

Required: Thickness, t, for collapsing pressure of 300 lb. per sq. in., using a factor of safety of 5.

Solution:

L becomes the distance between the frame and the head, providing the frame is strong enough to hold after the shell has collapsed, i.e., providing the moment of inertia, I, of the cross-sectional area of the stiffening ring is $I = \frac{0.046\ D^3C}{E}\ p$ as required by formula (13), Part I, Par. 19, where C is the length between frame centers, p is the collapsing pressure of the shell, and E is Young's modulus of elasticity.

Collapsing pressure, p, = 300 x 5 = 1500 lb. per sq. in.

L = C = 5 x 12 = 60 in., D = 24 in., L/D = 60/24 = 2.5

The required moment of inertia of the compound section at the stiffening ring

$$I = \frac{0.046 \times 24^3 \times 60 \times 1500}{29{,}000{,}000} = 2\ \text{in.}^4$$

Entering Chart II, with L/D = 2.5 and collapsing pressure p = 1500 lb. per sq. in., we find that t/D lies between 0.026 and 0.028. Using the upper value for additional safety we have t/D = 0.028, whence t = 0.028 x D = 0.028 x 24 = 0.672 in..

It will be noted that the collapsing pressure of 1500 lb. per sq. in. is constant for all values of L/D less than 5. Hence the tube would have the same collapsing pressure if the frame were omitted.

Example 4. Given: A pipe 10 ft. long between flanges and 10 in. outside diameter.

Required: Thickness, t, for a working pressure of 2,000 lb. per sq. in., using a factor of safety of 5.

Solution:

Collapsing pressure, p, = 2,000 x 5 = 10,000 lb. per sq. in..

It will be seen from Charts I and II that the collapsing pressure is independent of the length for all collapsing pressures above 2000 lb. per sq. in..

Entering Chart I with a collapsing pressure of 10,000 lb.per sq. in., we find that t/D = 0.24, whence t = 0.24 x D = 0.24 x 10 = 2.4 in..

This tube seems unreasonably thick. The factor of safety is likely much larger than necessary. However, Chart I is based on steel having a yield point, S, of only 27,500 lb. per sq. in. If steel with a higher yield point, S', is used, the collapsing pressure will be increased in the ratio, S'/S.

To use Chart I for a yield point of 40,000 lb. per sq. in.:

Example 5. Given: Same conditions as in example 4, except steel with a yield point, S', of 40,000 lb. per sq. in. is to be used. The reduced collapsing pressure, p', for a yield point of 27,500 lb. per sq. in., is, formula (1b), $p' = p\frac{S}{S'} = 10{,}000 \times \frac{27{,}500}{40{,}000} = 6{,}900$ lb. per sq. in.

Entering Chart I with a collapsing pressure of 6,900 lb.per sq.in., we find t/D = 0.15, whence t = 0.15 x D = 0.15 x 10 = 1.5 in..

We must now test this value of t/D for collapse by instability. By formula (3), $p = 86{,}670 \frac{t}{D} - 1386$ or

$$p = 86{,}670 \times 0.15 - 1386 = 12{,}600 \text{ lb. per sq. in.}.$$

Since this pressure is above the required collapsing pressure of 10,000 lb. per sq. in., the value of $\frac{t}{D} = 0.15$ is safe.

Although formula (3) has not been verified for values of t/D greater than 0.085, it can be used for values of t/D up to 0.12 or possibly 0.15. It is amply safe in this region when a factor of safety of 5 is used.

B.- EXAMPLES ILLUSTRATING the USE of CHART III in the DESIGN of STEEL CYLINDRICAL PRESSURE VESSELS having an OUTSIDE DIAMETER GREATER THAN 24 IN.

Example 6. Given: A pressure vessel 12 ft. long between heads and 12 ft. outside diameter (no stiffening).

Required: Thickness, t, for a working pressure of 15 lb. per sq. in.,(full vacuum) using a factor of safety of 5.

Solution:

Collapsing pressure = 15 x 5 = 75 lb. per sq. in.

$L = 12 \times 12 = 144$ in., $D = d = 12 \times 12 = 144$ in., $L/d = \frac{144}{144} = 1.0$.

Entering Chart III with L/d = 1.0 and collapsing pressure, p = 75 lb. per sq. in., we find that t/d lies between 0.0035 and 0.004. Using the upper value for additional safety, we have t/d = 0.004, whence t = 0.004 x d = 0.004 x 144 = 0.58 in..

Example 7. Given: A pressure vessel 50 ft. long and 200 in. outside diameter, closed at the ends by flat heads, with inside circumferential rings or frames.

Required: Thickness, t, best frame spacing, C, and scantlings of suitable frames for a working pressure of 50 lb. per sq. in., using a factor of safety of 4.

Solution:

Collapsing pressure, p, = 50 x 4 = 200 lb. per sq. in..

The collapsing pressure of a pressure vessel is independent of the length providing the stiffening rings have sufficient rigidity to force the shell to fail by buckling between them; that is, providing formula (13) is satisfied.

<u>Most Economical Frame Spacing.</u> It will be seen from Chart III that the collapsing pressure of 200 lb. per sq. in., is independent of L/d

for values of L/d up to L/d = 0.25. Therefore, L/d = 0.25 determines the most economical spacing.

$$L = 0.25 \times d = 0.25 \times 200 = 50 \text{ in.}.$$

Required Moment of Inertia. The required moment of inertia of the compound section consisting of the stiffening ring and a portion of shell equal in width to the flange is

$$I = \frac{0.046\ d^3 C}{E}\ p \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Since C = L + f, an approximate flange width, f, must be assumed. An I-beam is desired as a stiffening ring because of its symmetrical form. Most 8 in. to 12 in. I-beams have a flange width of 5 in. or 5-1/4 in.. Assuming the higher value, we have (see Chart V), $C = 50 + 5\frac{1}{4} = 55\frac{1}{4}$ in., whence

$$I = \frac{0.046 \times (200)^3 \times 55.25 \times 200}{29 \times 10^6} = 140 \text{ in}^4..$$

Entering Chart III with L/d = 0.25 and collapsing pressure, p, = 200 lb. per sq. in., we find, t/d = 0.0038, whence

$$t = 0.0038 \times d = 0.0038 \times 200 = 0.76 \text{ in.}.$$

Choice of Stiffening Ring. We have now only to determine a suitable stiffening ring. The requirement is that the moment of inertia of the compound section composed of the stiffening ring and a strip of plating equal in width to the frame flange (see Chart V) shall be equal to 140 in^4. and the cross-sectional area of the ring shall not exceed $\sqrt{140 \text{ in}^4.} = 11.8 \text{ in}^2$. (formula 15).

The Carnegie B-40, 9 in. x 5-1/4 in., 20.5 lb. I-beam has a moment of inertia 86.6 in^4., and an area of 6.02 in^2. The moment of inertia of the compound section is given by formula (14)

$$I = (I_1 + Ad^2{}_1) + (\frac{bt^3}{12} + btd_2{}^2) \quad \ldots\ldots\ldots (14)$$

where d_1 and d_2 are given by formula (14a). (See Chart V).

$$d_1 = \frac{5.25 \times 0.76 \times (4.5 + 0.38)}{6.02 + 5.25 \times 0.76} = 1.95 \text{ in.}.$$

$$d_2 = 4.5 + 0.38 - 1.95 = 2.93 \text{ in.}$$

Substituting in formula (14)

$$I = 86.6 + 6.02 \times (1.95)^2 + \frac{5.25 \times (0.76)^3}{12} + 5.25 \times 0.76 \times (2.93)^2 = 143.9 \text{ in}^4..$$

Hence this I-beam is sufficiently rigid for a spacing of 55-1/4 in. between centers and easily meets the condition imposed by formula (15), since the area, $A = 6.02$ in²., is considerably less than the square root of the required moment of inertia, $\sqrt{I} = 11.8$ in².

If the stiffening rings were external instead of internal, the slightly smaller value of C shown in Chart V would be used, providing the frames are riveted to the shell. If the stiffening rings are welded to the shell, the same value of L is used for both internal and external stiffening rings. However, in this region, collapsing pressure is practically independent of L.

Decrease in Frame Spacing Adjacent to Solid Ends of Pressure Vessel. It must be borne in mind that the condition imposed by formula (15) is not satisfied in the frame spacing adjacent to the heads of the pressure vessel. The area, A, of the flat heads becomes very great as compared to the required moment of inertia, I. Hence, by formula (16) the frame spacing must be decreased to 4/5 or 80% of the length permitted between frames of reasonable area.

$$L_{(\text{decreased})} = \frac{4 + 0}{5} L = \frac{4}{5} \times 50 = 40 \text{ in..}$$

This reduction is necessary to prevent collapse of the end sections at pressures below the other sections.

Check of Results by Formula (8). It is of interest to check the values just determined by use of the more accurate formula (8) which is to be used for computing collapsing pressures when failure occurs by stresses reaching the yield point of the material in the shell.

$$p = \frac{2\ S_{(yield)}\ t/d}{\frac{1}{2} + 1.815\ (0.85 - \frac{bt}{A + bt})\ \frac{K}{1+\beta}} \quad \ldots\ldots\ldots\ldots (8)$$

$t/d = 0.0038$, $t = 0.76$ in., $b = 5.25$ in., $bt = 3.99$ in².

$A = 6.02$ in²., $\gamma = 0.148$, $L = 50$ in., $\gamma L = 7.4$

$K = 1$, $N = 1$, $\beta = 1.02$

Substituting in formula (8)

$$p = \frac{2 \times 27{,}500 \times 0.0038}{\frac{1}{2} + 1.815(0.85 - \frac{3.99}{6.02 + 3.99})\ \frac{1}{1 + 1.02}} = \frac{209}{\frac{1}{2} + 0.41}$$

$= 230$ lb. per sq. in..

Heavy Stiffening Ring. Suppose the heaviest stiffening ring permitted by formula (15) had been used, i.e., $A = 11.8$ in².. This corresponds to a heavy 10 in., or light 12 in., American standard I-beam. Using the same flange width, all other values except the area, A, will be the same as above. Substituting in formula (8),

$$p = \frac{2 \times 27{,}500 \times 0.0038}{\frac{1}{2} + 1.815\left(0.85 - \frac{3.99}{11.8 + 3.99}\right)\ \frac{1}{1 + 1.02}} = \frac{209}{\frac{1}{2} + 0.54}$$

$= 201$ lb. per sq. in.

This shows that a thickness of 0.76 in. for the shell plating obtained from Chart III is sufficient, even with a stiffening ring of area $A = \sqrt{I}$. However, the B-40 I-beam gives not only a lighter and more economical but actually a stronger pressure vessel, because of the better relative proportions.

Example 8. Given: Conditions the same as in example 7, but steel with a yield point of 34,000 lb. per sq. in. to be used in the shell.

Required: Same as example 7.

Solution:

Collapsing pressure, $p = 50 \times 4 = 200$ lb. per sq. in.. $S_{(yield)} = 34{,}000$ lb. per sq. in..

Solving formula (8a) for t/d,

$$\frac{t}{d} = \frac{1.05}{2\ S_{(yield)}}\ p = \frac{1.05 \times 200}{2 \times 34{,}000} = 0.0031$$

whence, t = 0.0031 x 200 = 0.62 in..

Most Economical Frame Spacing. Chart III was drawn for $S_{(yield)}$ = 27,500 lb. per sq. in.. To adapt Chart III for $S_{(yield)}$ = 34,000 lb. per sq. in., produce the line, t/d = 0.0031 to p = 200 lb. per sq. in.. The line t/d = 0.003 can be used with sufficient accuracy. Sketching in the curved part of the t/d line tangent to p = 200 lb. per sq. in., similar to the other curves on Chart III, we find the most economical frame spacing is for L/d = 0.148 or L = 29.6 in..

Required Moment of Inertia. Assuming a flange width, f = 5 in., C = 29.6 + 5 = 34.6 in.. Hence, the required moment of inertia of the compound section becomes, formula (13)

$$I = \frac{0.046 \times (200)^3 \times 34.6 \times 200}{29 \times 10^6} = 87.7\ in^4.$$

This permits a lighter stiffening ring than used in example 7 with a 55 in. frame spacing.

Choice of Stiffening Ring. The Carnegie B-39, 8 in. by 5 in., 17.5 lb. I-beam has a moment of inertia of 57.4 in^4., and an area of 5.14 in^2. Substituting in formula (14a)

$$d_1 = \frac{5 \times 0.62(4.0 + 0.31)}{5.14 + 5 \times 0.62} = 1.62\ in.$$

$$d_2 = 4 + 0.31 - 1.62 = 2.69\ in.,$$

and substituting in formula (14)

$$I = 57.4 + 5.14 \times (1.62)^2 + \frac{5 \times (0.62)^3}{12} + 5 \times 0.62 \times (2.69)^2$$
$$= 93.4\ in^4.,$$

which is slightly above the required moment of inertia, 87.7 in^4..

Hence, the Carnegie B-39, 8 in. by 5 in., 17.5 lb. I-beam spaced 34.6 in. between centers and a shell of 0.62 in. plating with yield point of 34,000 lb. per sq. in. meets the required conditions.

Check of Results by Formula (8).

$$p = \frac{2\ S_{(yield)} \frac{t}{d}}{\frac{1}{2} + 1.815\left(0.85 - \frac{bt}{A + bt}\right) \frac{K}{1 + \beta}} \quad \ldots\ldots\ldots\ldots \quad (8)$$

$\frac{t}{d} = 0.0031$ $t = 0.62$ in. $b = 5$ in. $bt = 3.1$ in².

$A = 5.14$ in². $\gamma = 0.163$ $L = 29.6$ $\gamma L = 4.82$

$K = 1.03$ $N = 1.01$ $\beta = 0.93$

Substituting in formula (8)

$$p = \frac{2 \times 34{,}000 \times 0.0031}{\frac{1}{2} + 1.815(0.85 - \frac{3.1}{5.14 + 3.1})\frac{1.03}{1 + 0.93}} = \frac{211}{\frac{1}{2} + 0.46}$$

$= 220$ lb. per sq. in..

Heavy Stiffening Ring. Suppose the heaviest stiffening ring permitted by formula (15) had been used, i.e., $A = 9.36$ in². Using the same flange width, all other values except the area, A, will be the same as above. Substituting in formula (8)

$$p = \frac{2 \times 34{,}000 \times 0.0031}{\frac{1}{2} + 1.815(0.85 - \frac{3.1}{9.36 + 3.1})\frac{1.03}{1 + 0.93}} = \frac{211}{\frac{1}{2} + 0.58}$$

$= 195$ lb. per sq. in..

Hence, the values given above are amply safe, even though the weight of the stiffening ring be nearly doubled.

It is seen from the above results that the use of steel with a yield point of 34,000 lb. per sq. in. reduces the total weight of structure by more than 10% below that obtained by the use of boiler code steel, with a yield point of 27,500 lb. per sq. in..

Comparing the areas of longitudinal sections 55 in. long

	Yield Point, lb. per sq. in.	
	27,500	34,000
Proportional area of frame	6.02 in².	8.17 in².
Area of shell	41.80 in².	34.10 in².
Total area	47.82 in².	42.27 in².

Percentage reduction in area $= \frac{47.82 - 42.27}{47.82} = 11.6\%$.

Example 9. Given: Pressure vessel designed in example 7, that is, diameter 200 in., thickness 0.76 in., unsupported length between frames 50 in..

Required: Allowable eccentricity within the entire circumference and within the region of one lobe.

Solution:

By formula (9) the allowable value of e/t for the entire circumference is

$$\frac{e}{t} = 0.7 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

whence $e = 0.7 \times t = 0.7 \times 0.76 = 0.53$ in..

That is, the maximum and minimum radii shall not differ by more than 0.53 in..

By formula (10) the allowable value of e/t within the region of one lobe is:

$$\frac{e}{t} = \frac{10}{n(1000\ t/d)} \quad \ldots\ldots\ldots\ldots\ldots \quad (10)$$

Entering Chart IV, with L/d = 0.25 and t/d = 0.0038, we find n = 12.

Substituting in formula (10)

$$\frac{e}{t} = \frac{10}{12 \times 3.8} = 0.22$$

whence $e = 0.22 \times t = 0.22 \times 0.76 = 0.17$ in..

The length of one lobe is $l = \frac{\pi d}{n}$, or $l = \frac{\pi \times 200}{12} = 52$ in.

That is, the maximum and minimum radii in a circumferential arc of the shell 52 in. long must not differ by more than 0.17 in..

Example 10. Given: The pressure vessel described in example 7 has been built and the eccentricity measured. The maximum difference of radius within the entire circumference is 0.73 in., and within an arc 52 in. long is 0.38 in..

Required: Estimated collapsing pressure.

Solution:

By formula (11)

$$p_{(actual)} = p_{(calc)}\left[1 - \frac{(e/t)_{actual} - (e/t)_{permissible}}{2}\right] \ldots (11)$$

For entire circumference

$$p_{calc} = 230 \text{ lb. per sq. in.. Formula (8)}$$

$$\left(\frac{e}{t}\right)_{actual} = \frac{0.73}{0.76} = 0.96$$

$$\left(\frac{e}{t}\right)_{permissible} = 0.70$$

Substituting

$$p_{(actual)} = 230\left[1 - \frac{0.96 - 0.70}{2}\right] = 200 \text{ lb. per sq. in..}$$

For an arc of 52 in. (one lobe length)

$$\left(\frac{e}{t}\right)_{actual} = \frac{0.38}{0.76} = 0.50$$

$$\left(\frac{e}{t}\right)_{permissible} = 0.22 \text{ (Example 8)}$$

Substituting

$$p_{actual} = 230\left[1 - \frac{0.50 - 0.22}{2}\right] = 198 \text{ lb. per sq. in..}$$

In this particular case the pressure vessel is still safe for a collapsing pressure of 200 lb. per sq. in., as designed, although the actual collapsing pressure is 14% below the computed value.

Example 11. Given: A closed pressure vessel 50 in. long and 50 in. outside diameter. The vessel will be subjected to rough handling and it will be assumed that shell plating less than 1/4 in. thick will not be used.

Required: Allowable eccentricity for a collapsing pressure of 15 lb. per sq. in., (complete vacuum) using a factor of safety of 3.

Solution:

Collapsing pressure, p, = 15 x 3 = 45 lb. per sq. in.. Minimum t/d allowed $= \frac{0.25}{50} = 0.005$.

Entering Chart III with p = 45 lb. per sq. in., we find that without stiffening rings, (L/d = 1), t/d = 0.0032, or t = 0.16 in. Thus 1/4 in. shell plating is considerably heavier than required and greater eccentricity can be allowed in manufacture. By formulas (12) and (12a)

$$(\frac{e}{t})_{actual} = 2\,\frac{p_c - p_a}{p_c} + 0.7 \quad \ldots\ldots\ldots\ldots\ldots \quad (12)$$

$$(\frac{e}{t})_{actual} = 2\,\frac{p_c - p_a}{p_c} + \frac{10}{n(1000\ t/d)} \quad \ldots\ldots\ldots\ldots \quad (12a)$$

p_a = actual collapsing pressure required

= 15 x 3 = 45 lb. per sq. in.

p_c = calculated collapsing pressure for scantlings used,

i.e. L/d = 1, t/d = 0.005

= 135 lb. per sq. in.

n = 6 (Chart IV)

Substituting in formula (12), the maximum value of e/t allowed in the entire circumference is,

$$(\frac{e}{t})_{actual} = 2 \times \frac{135 - 45}{135} + 0.7 = 2.0$$

Whence e = 2.0 x t = 0.50 in..

Substituting in formula (12a), the maximum value of e/t allowed within the region of one lobe is,

$$(\frac{e}{t})_{actual} = 2 \times \frac{135 - 45}{135} + \frac{10}{6 \times 5.0} = 1.7$$

or, e = 1.7 x t = 0.43 in.

Thus, a total variation from circular form of 1/2 in. will be allowed in the entire circumference, and a variation of 0.43 in. will be allowed within the region of one lobe, or 26 in.