FRICTIONAL RESISTANCE OF SHIP MODELS

U.S. EXPERIMENTAL MODEL BASIN
Navy Yard, Washington, D.C.

August 1929

Report 227

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SUMMARY

Opportunity has occurred for an extensive review of frictional data, mostly hitherto unpublished, and some of which are entirely new. The numerical results of analysis of these data are presented, the methods of analysis are discussed, and the prospect of improved precision of resistance measurements is considered.

1. Table I gives frictional data from 27 different sources. Columns 5 and 6 gives the values of the constants in the formula $\frac{R}{AV^2} = a + b\ (VL)^{-1/3}$ in which R is the resistance in pounds, A is wetted surface in square feet, V speed in knots, and L. length in feet.*

The same data are presented in graphic form in Plate I.

The most significant features of these data are as follows:

(a) The dispersion of values is too high to be attributed simply to instrumental error in the measurements.

(b) The dispersion is greater at lower values of VL.

(c) Gebers' line marks the lower boundary.

(d) There is no evidence that V and L play different roles, and to this extent Reynolds' Law is satisfied.

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* To facilitate comparisons it is noted that mean temperature is 65 degrees F., Reynolds' Number is VL times 1.5×10^5. Perring's and Telfer's "Specific Resistance" is obtained by dividing by 5.535, and Kempf's "Beiwert" is obtained by dividing by 2.7675.

A circle on Plate I indicates Kempf's single value deduced from his recent measurements* of local specific resistance on a 250 ft. model. The results of his towing tests on long pipes** will be found to lie much higher.

2. An experiment similar to those of Gebers*** and Perring**** for determining the effect of the free longitudinal edge of a friction plane was made at the Experimental Model Basin in 1901 on a 30-ft. plane, but the increased resistance observed at small girth values was not regarded as significant. In view of recent attention given this subject, all the 27 cases listed in Table I have been re-examined and a correction based on girth introduced. The last four columns contain values of corrected specific resistance at speeds shown, lengths being 10, 14, 20, 30, 40 and 60 feet.

3. The method devised by Froude for estimating a ship's power requirements by means of model tests depends on separation of the total resistance into two parts one of which, the dynamic resistance, is transferable from model to ship by the law of dynamical similitude. The other part, frictional resistance, has been the subject of many experimental studies intended to determine its value for models directly and for the ship by extrapolation. The range of uncertainty as to frictional resistance of models is shown by the data here quoted. Extrapolation adds to the uncertainty

* Kempf : T I N A 1929
** Kempf : W R H, p 521, 1924
*** Gebers : Schiffbau Vol. 22
**** Perring : T I N A 1926

in full scale ships. Practice hitherto has been to adopt an exact value of frictional resistance and the success of power estimates thus made justified the procedure. In early tests the variability of frictional data appears to have been attributed to instrumental errors; certain tests on planes, very carefully made, were considered to yield a result which correctly allowed for the frictional resistance of models. Disturbing incidents occurred, however, such as total resistance less than calculated frictional values. Gebers, in Vienna, in a series of tests made with the greatest precautions, arrived at values lower than others, and asserted the principle that the lowest bonafide frictional resistance data are ipso facto the most correct.

Experience at the Experimental Model Basin has shown that Gebers' formula will lead to under-estimation of power requirements which may be attributed to under-estimation of frictional resistance at ship lengths. Plate I permits the following explanation of the difficulties.

Frictional resistance of models is inherently variable, due to instability of flow texture. Resistance of short models at low speeds tends to reduced values due to the effects of laminar flow, but these effects are capricious in their action, and have never been adequately controlled. Laminar flow is not readily recognized, and the limits of its influence are not clearly made out. Gebers' formula marks the lower limit of the region within which frictional resistance of models of length 10 ft. and up may fluctuate. Even a 20 ft. model is not long enough to make the effects of laminar flow negligibly small, but it is possible that in a 20 foot model the variability due to laminar flow is sufficiently reduced to make errors from this source negligible when the total frictional resistance is not a great part of the whole.

If these ideas are correct, it is apparent that extrapolation to lengths at which the effects of laminar flow are entirely negligible cannot be made by any formula based on data from short models in which no allowance for laminar flow is made.

To obtain a more suitable formula for extrapolation the region covered by the fluctuations should be traversed along the upper boundary and this, in effect, is what is accomplished by the procedure actually followed in analysis of the data presented.

However, it can hardly be too strongly emphasized that the risk involved in all extrapolations cannot be avoided, and that measurements in models will never yield certainty as to the frictional resistance of full-scale ships.

4. The methods used in analysis of the data presented differed from those hitherto followed at the Experimental Model Basin, which were based on the exponential formula of Froude. The two-term formula adopted is free from the restricting assumption that the resistance at infinite speed or length (if it could be ascertained) would be zero. Though it excludes the long accepted use of logarithmic coordinates in determination of the constant b

and the exponent, an equally simple graphical procedure, devised by Telfer* is available, based on a fixed exponent, but leading to determination of values for a and b. The slightness of the difference between the exponential formula and the two-term formula with fixed exponent is perhaps not generally appreciated. It is such that much more accurate data would be necessary to establish the incorrectness of either. Some pertinent mathematical considerations are appended.

In each of the 27 cases presented, values of a and b which would best represent the data, were found by statistical procedure without relying on the sort of judgment which is necessary in fitting a curve graphically to plotted spots. In all cases the necessary deductions of tare due to air resistance, appendages, and the like were made; in addition deductions for dynamic resistance of models other than planes were made by methods to be elsewhere described. The statistical methods used permitted estimate of mean error, but as the dispersion among the averages exceeded the mean errors so found, their values are omitted. No data were excluded simply on account of departure from mean values except those obtained at low values of VL, as explained below. The large numbers of runs noted were reduced to moderate numbers of corresponding values of speed and specific resistance by simple averaging, but larger numbers of runs were averaged at low than at high speed, thus partly compensating the lower precision of measurements at low speeds. Even after high speeds had thus been given

* **Telfer: T I N A 1927**

preference it was found that in many cases resistance at low speeds fell below that which would be obtained by use of the more consistent values of a and b taken from higher speeds.

From the point of view of Gebers' formula this would be explained by saying that the exponent 1/3 is too high. However it is a fact that good results are obtained with the exponent 1/3 everywhere except at the lowest speeds. The mean departures of observed from calculated resistances from a single source are of the order of 1 to 2 per cent, but the spots at the one or two lowest speeds are often 5 to 10 per cent low. For these reasons certain spots at low values of VL were excluded in finding best values of a and b. The values given in Table I will therefore not correctly reproduce observed frictional resistances at the lowest speeds of the shorter models.

5. The method of introducing girth-corrections mentioned in paragraph 2 will now be explained.

First it will be noted that although Table I gives best values of both a and b for each case separately the fluctuations of a from case to case are not very wide except at the shorter lengths. The best values in each case are chosen so as to make mean departure of observed spots from calculated values a minimum, but on account of the scattering of the spots, it is not a very sharp minimum, so that a moderately different value of a combined with an appropriate b will increase the mean departure only by a moderate amount.

As an example we may take a set of data for Gebers' 5 meter plank. This work appears to have been done with more elaborate precautions than were used elsewhere, and as Gebers gives resistances for integral speed values it is fair to assume that the published data are the result of careful fairing and cross-fairing. Resistances are given for 8 speeds which reduce to English units as shown in Columns 1 and 2 of Table II. In columns 3 to 6 are tabulated the values of a found by substituting a series of values of b. By interpolation it is found that b = 1310 will give a minimum total absolute error, and column 7 shows the corresponding value of a to be 495. But other pairs of values of a and b will give nearly as good comparison with observations.

In order to make all the data in Table I comparable with each other without making a specific choice of speed, it is necessary to make use of a single value of a and the value chosen is 450. Interpolation between columns 5 and 6 of Table II will indicate the extent of the liberty thus taken in one case. Other cases are similar except that as the scattering of spots is generally greater than in the case chosen, the minimum is still flatter and the violence done to the data correspondingly less.

The cases in Table I, and the detailed observations of Perring, Gebers, and Experimental Model Basin 1901 on edge effect were reduced to a common basis of a = 450, leaving the differences only in the value of b, and b was then plotted on reciprocal girth. Planes wholly-submerged like those of Froude, were entered with girth at half value. The data are shown in Plate II; although the scattering is considerable, a best mean value of the numberical coefficients was found by the method outlined above for a and b.

The girth correction thus determined was applied by subtracting $\frac{200}{G}$ from the value of b in column 6 of Table I. As these values are not all based on a single value of a but on those shown in column 5, this procedure is not an exact one, but in view of the uncertainties involved only an approximate correction is possible.

6. Values of R/AV^2 corrected in the way described above are shown for four speeds in Columns 8 to 11 of Table I. These already favor the high values through omission of low spots at low speeds as in Table II. But it is now proposed to go still further by assuming that the departures from uniformity at given lengths and speeds are primarily due to uncontrolled fluctuations in flow texture, and that the highest values generally correspond to the most fully developed turbulence. At each length for each speed a representative high value is chosen and these 24 values are put through the process described in Section 4 to find the best values for a and b. The values so found should lead to closer approximation to frictional resistance of full scale vessels than if the disturbing effects of laminar flow at low speeds and lengths were not eliminated.

Here are the actual choices made:

A. The highest value in each case.

B. As in A, but excluding all data not taken at the Experimental Model Basin. This is nearly equivalent to excluding Froude's data which were based on very thin planes. The 10 ft. metal planes run at the Experimental Model Basin in 1915, are also excluded; these were also thin, somewhat deficient in transverse stiffness and are known to have departed somewhat from true planes.

The results of these choices are given in Table III and the results from the Experimental Model Basin are also exhibited in Plate III.

The precision of the results is indicated in Table IV in which are summarized the mean errors associated with a series of values of b; the corresponding values of a are also given.

7. The variability of frictional data has now been known for several years, but the practice of accepting certain values as correct has been retained. This may have been justified on the ground that an average gives the most probable value, yet conclusions have sometimes been sought from models on which the number of observations was small enough so that it was by no means assured that the frictional resistance actually approximated its accepted value. An attempt to trace the effect of fluctuations was at one time made by retowing at intervals a model whose resistance curve was well established, and introducing corrections based on departures of the observed resistance of this model from its normal value. After being followed for a number of years this practice was abandoned as it did not appear to afford any substantial improvement in precision of results.

Possibly a better criterion as to fluctuations is obtained from observations at speeds low enough so that the frictional resistance is nearly equal to the total. The measurement of resistance at very low speeds is especially subject to instrumental error, however, which prevents this method from being entirely satisfactory.

The development of the idea of instability of flow texture and the demonstration of its effects now appears to point the way for a new step forward toward improved precision of resistance measurements. Such precision is attainable at present only by use of larger models at correspondingly higher speeds. Whether an acceptable means of stabilizing flow texture about smaller models can be developed remains to be seen. Larger models, of course, would imply a larger basin unless a suitable correction could be found to allow for restriction of channel.

In any event a clear recognition of the difficulty, with a reasonable hypothesis as to its cause, now gives us new confidence in endeavoring to find the remedy.

TABLE I

FRICTIONAL RESISTANCE FROM 27 SOURCES

No.	Model	L	No. Runs	$a \times 10^5$	$b \times 10^5$	Range of VL	CORRECTED VALUES OF R/AV^2*, GIRTH = 00 2 knots	5 knots	8 knots	11 knots
1	Froude FP	10'	?	554	1320	10-80	995	879	831	804
2	EMB FP 1915	10'	335	691	724	10-100	921	861	836	821
3	EMB FP 1923	10'	56	498	1245	10-100	931	817	771	743
4	2509	10'	167	625	810	10-114	878	811	784	768
5	Gebers IV	2-1/2M	?	558	970	16-120	927	757	727	711
6	Froude FP	14'	?	581	1150	14-111	918	829	794	772
7	EMB FP 1901	14'	85	450	1511	14-210	891	785	737	708
8	EMB FP 1923	14'	31	562	902	14-140	829	765	736	718
9	2505	14'	226	470	1330	14-172	866	772	728	712
10	Gebers VI	5 M	?	495	1310	30-240	927	813	767	739
11	Froude FP	20'	?	472	1560	20-160	891	781	737	710
12	EMB FP 1901	20'	257	425	1430	20-266	883	763	714	685
13	EMB FP 1915	20'	408	496	1178	20-220	803	723	690	670
14	EMB FP 1923	20'	45	485	1112	20-220	791	711	678	659
15	2501	20'	290	524	1064	20-140	816	739	708	690
16	2892	20'	35	455	1700	20-120	897	780	733	705
17	Gebers IV	(5 M & (7.5 M	?	454	1427	40-320	844	742	700	675
18	Froude FP	30'	?	413	1880	30-240	861	743	696	667
19	EMB FP 1901	30'	139	482	1459	30-300	826	735	698	677
20	2513	30'	193	452	1515	30-210	828	729	689	665
21	Gebers IX	10 M	?	349	2130	30-450	878	739	682	649
22	Perring	28'	?	464	1373	28-224	746	699	663	641
23	Froude FP	40'	?	407	1960	40-320	833	720	675	648
24	2536	40'	51	401	1822	40-280	817	708	663	637
25	Froude FP	50'	?	399	2060	50-400	803	688	646	622
26	2816	60'	68	390	1830	20-330	759	654	616	593
27	EMB FP	80'	31	484	1016	80-860	689	631	610	547

Total runs at Experimental Model Basin-2437

*** Area in Sq. Ft., Speed in Knots and Resistance in Units of .00001 pounds.**

TABLE II

GEBERS 5 M PLANK (TABLE VI) 250 MM DRAFT

$VL^{1/3}$	R/AV²*	Values of a* when b*=1000		b*=1200		b*=1400		b*=1600		b+=1310
3.171	842	527		464		401		338		429
3.996	828	578	+ 23	528	+ 11	478	0	428	- 11	500
4.572	778	559	+ 4	516	- 1	472	-6	428	- 11	492
5.033	754	555	0	516	- 1	476	-2	436	- 3	494
5.422	733	549	- 6	511	- 6	475	-3	438	- 1	491
5.761	721	548	- 7	513	- 4	478	0	443	+ 4	494
6.065	714	549	- 6	516	- 1	483	+5	450	+ 11	498
6.206	710	549	- 6	517	0	484	+6	452	+ 13	499
	Av.	555		517		478		439		495

error, absolute total - 52 ----24 ----22----54

Value at lowest VL omitted in averages.

*** See Note, Table I**

TABLE III

FRICTIONAL RESISTANCE ON 6 LENGTHS

		CHOICE A				CHOICE B		
L	V	OBS.* R/AV^2	$413+\frac{1830}{VL^{1/3}}$	ERROR	$VL^{-1/3}$	OBS.* R/AV^2	$391+\frac{1830}{VL^{1/3}}$	ERROR
10	2	995	1087	-92 0	.3685	931	1065	-134 0
	5	879	910	-31 0	.2715	817	888	- 71 0
	8	836	838	- 2	.2321	784	816	- 32 0
	11	821	795	+26	.2038	768	773	- 5
14	2	927	1016	-89 0	.3295	891	994	-103 0
	5	829	857	-28 0	.2427	785	835	- 50 0
	8	794	793	+ 1	.2076	737	771	- 34 0
	11	772	754	+18	.1865	718	732	- 14
20	2	897	948	-51 0	.2925	897	926	- 29 0
	5	781	808	-27 0	.2157	780	786	- 6
	8	737	750	-13	.1843	733	728	+ 5
	11	710	716	- 6	.1650	705	694	+ 11
30	2	878	880	- 2	.2555	828	858	- 30 0
	5	743	757	-14	.1883	735	735	0
	8	698	708	-10	.1610	698	686	+ 12
	11	677	678	- 1	.1448	677	656	+ 21
40	2	833	838	- 5	.2321	817	816	+ 1
	5	720	726	- 6	.1710	708	704	+ 4
	8	675	680	- 5	.1462	663	658	+ 5
	11	648	655	- 7	.1315	637	633	+ 4
60	2	803	795	+ 8	.2088	759	773	- 14
	5	688	686	+ 2	.1494	654	664	- 10
	8	661	647	+14	.1278	616	625	- 9
	11	642	623	+19	.1149	593	601	- 8
		Mean Error $\frac{159}{18} = 9$				Mean Error $\frac{129}{16} = 8$		

Items marked 0 omitted on account of laminar flow.

* See Note, Table 1.

TABLE IV

PRECISION OF FINAL MEAN VALUES OF A AND B

CHOICE A			CHOICE B		
b*	a*	Mean Error	b*	a*	Mean Error
1600	453	12.5	1500	445	10.9
1700	436	11.6	1700	410	8.1
1800	418	11.0	1900	375	8.0
1900	401	11.4	2100	346	12.8
2000	384	12.8			

The shallowness of the minimum in Mean Error indicates the relative uncertainty in best values of a and b.

* See Note, Table 1.

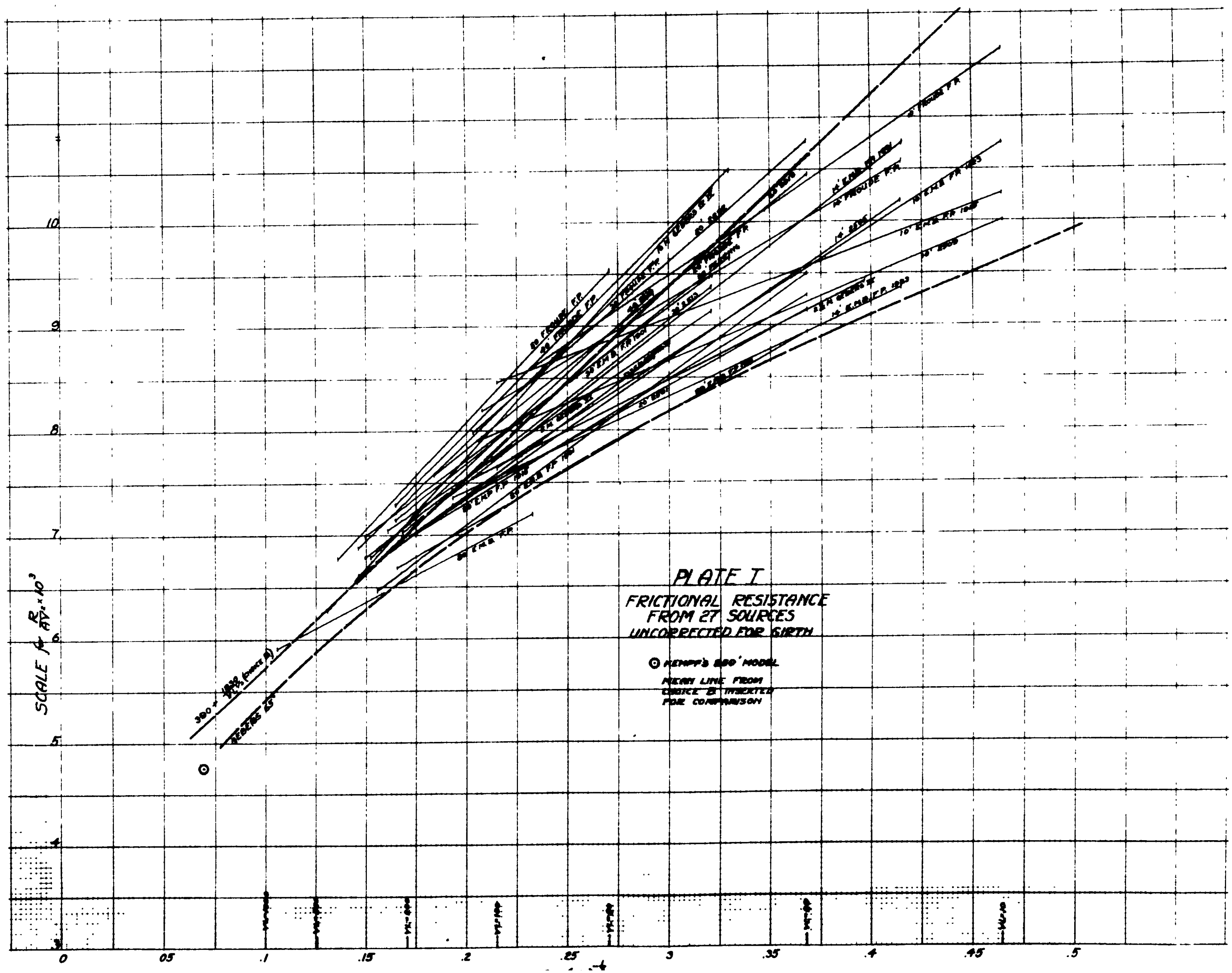
PLATE I
FRICTIONAL RESISTANCE
FROM 27 SOURCES
UNCORRECTED FOR GIRTH
KEMPF'S 250' MODEL
MEAN LINE FROM
CHOICE B INSERTED
FOR COMPARISON
SCALE

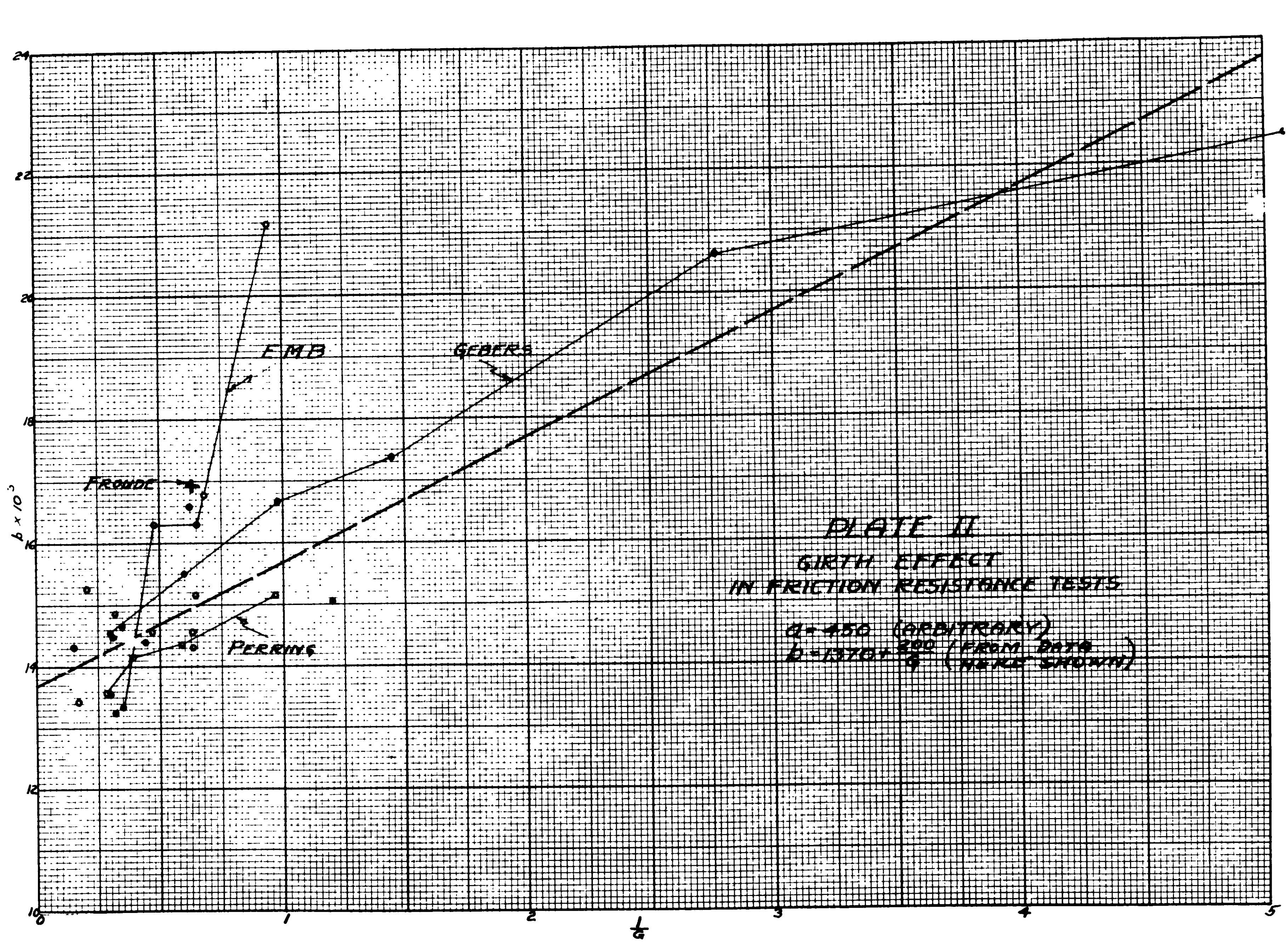

PLATE II
GIRTH EFFECT
IN FRICTION RESISTANCE TESTS
a = 450 (ARBITRARY)
b = 1370 + 200/G (FROM DATA HERE SHOWN)
EMB
GEBERS
FROUDE
PERRING
b x 10^3
1/G
24
22
20
18
16
14
12
10
0
1
2
3
4
5

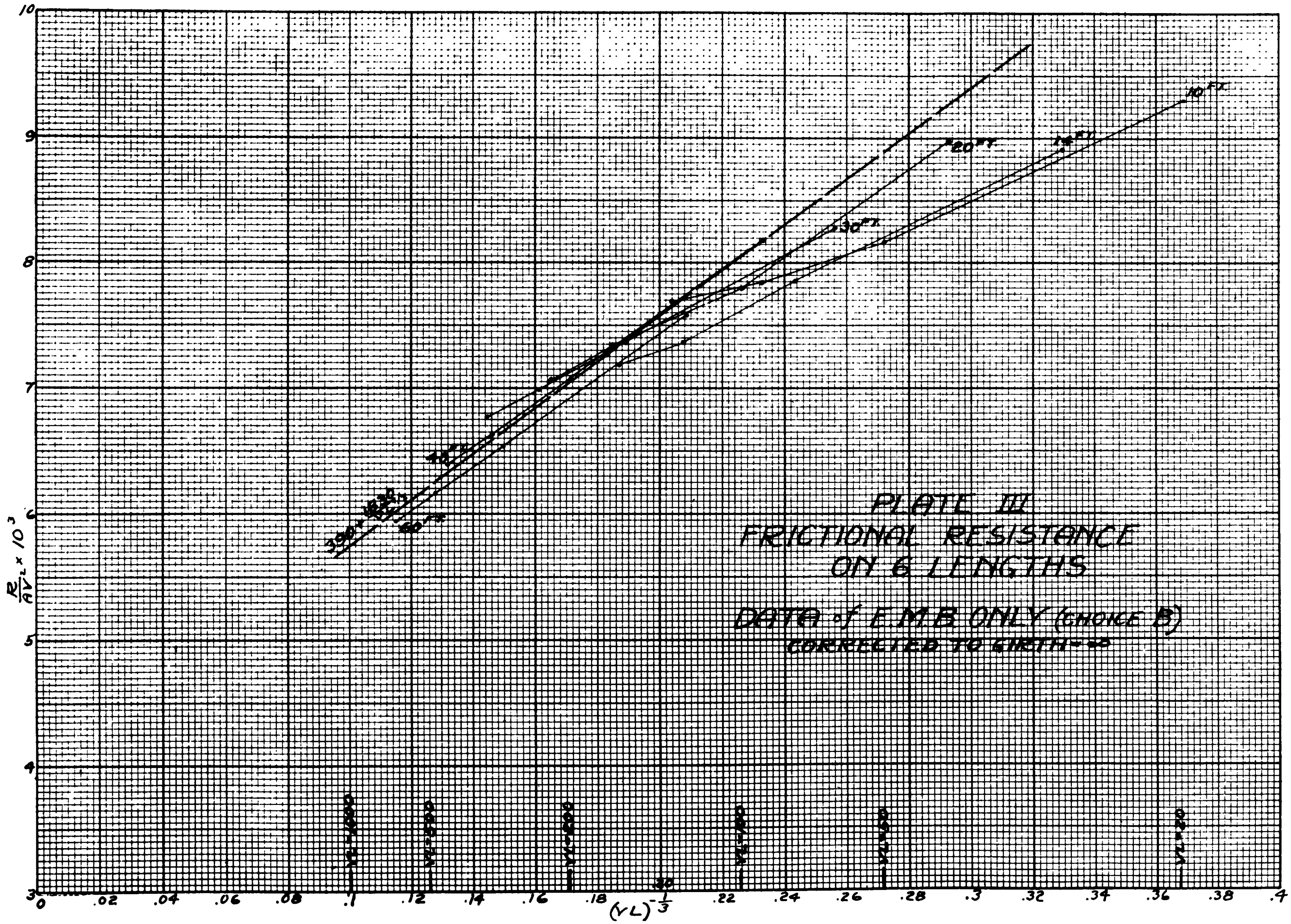
PLATE III
FRICTIONAL RESISTANCE
ON 6 LENGTHS
DATA of E.M.B. ONLY (CHOICE B)
CORRECTED TO GIRTH = ∞
10 FT.
15 FT.
20 FT.
30 FT.
40 FT.
60 FT.
$\frac{R}{AV^2} \times 10^3$
$(VL)^{-\frac{1}{3}}$
3
4
5
6
7
8
9
10
0
.02
.04
.06
.08
.1
.12
.14
.16
.18
.20
.22
.24
.26
.28
.3
.32
.34
.36
.38
.4

APPENDIX

MATHEMATICAL FORMULAE FOR FRICTIONAL DATA

The practice of fairing observed spots graphically and treating the resulting curve as though the experimental error in it were negligible has been almost universally followed, and has great convenience to recommend it. But it by no means serves to eliminate experimental error, and this alone may account for the difficulties which have been felt in attempting systematic analysis of existing data. Such regularities as may exist must be discerned through a maze of experimental errors which are more dangerour when not acknowledged.

Data on frictional resistance have generally been plotted on logarithmic coordinates and since the straight line will always be used in graphical work where possible, and the data have not positively demonstrated that anything else is necessary, the simple exponential expression used by Froude has generally been accepted as of the right type, and analysis has centered on determining appropriate values of coefficient and exponent.

Possibly it is not everywhere realized how small are the differences within experimental speed ranges between the two-term formula (with suitably adjusted constants) and the exponential formula, which is a two-term formula with zero constant term.

Let us seek the exponential formula which will most nearly match the expression $y = a + bx^{-n}$* and find the discrepancy between the two. Utilizing the expansion

$$x^n = 1 + n \log x + \frac{n^2}{2} \log^2 x + \text{etc} \;**$$

we have

$$y = a + b\,(1 - n \log x + \frac{n^2}{2} \log^2 x - \ldots\ldots + \ldots\ldots)$$

$$= (a + b)\, x^{\frac{b}{a+b} n} + \frac{1}{2} \frac{a}{b} (a + b) \left(\frac{bn}{a + b} \right)^2 \log^2 x \text{ ----}$$

and this may be written

$$y = cx^{-m} + \frac{1}{2} dm^2 \log^2 x \text{ ------}$$

where

$$c = a + b, \; d = \frac{a}{b} (a + b), \; m = \frac{b}{a + b} n.$$

x may be taken as ratio of speed to a value near the middle of the speed range covered by the data, so as to give positive and negative values of log x; and not to place undue emphasis upon values at low speeds for which experimental errors are apt to be large.

If v ranges from 2 to 12 knots, x varies from 2/5 to 5/2, log x ranges from -.9 to +.9, while m is never observed to exceed 0.2. The extreme difference between the two types of formula then depends on a/c as follows:

a/c	Maximum difference per cent
.1	.2
.2	.5
.3	.8
.4	1.3
.5	2.0
.6	3.0
.7	4.6

* x represents speed or a function of speed and y represents specific resistance.

** Natural Logarithms.

Considering the order of precision of the data it appears that greater speed ranges are necessary in order to positively distinguish between the two types of formula.

It should be especially noted that in work not extending to great range of speed, data which fit the single-term formula will leave one of the three constants in a two-term formula open for arbitrary choice. In the analysis here reported, the exponent was thus arbitrarily set a 1/3.

The two-term formula is preferable to the exponential formula for several reasons. There is some evidence which will not now be reviewed that a is not actually zero. The two-term formula leaves us free from prejudice in this connection. Then even where two constants only are utilized and the exponent is arbitrary, a and b lend themselves to comparisons rather more simply than n does and make the process of fairing by eye unnecessary. By dealing with a and b it is possible to proceed impartially to a mean to which every spot makes its statistical contribution, either equal, or weighted according to its importance.

Simple geometrical interpretations exist in both cases. In the exponential formula the exponent gives the slope on logarithmic paper, the coefficient is the intercept at x equal unity. The constant which stands alone in the two-term formula gives the intercept at x equal infinity, and the coefficient of the exponential term gives the slope of the straight line obtained by plotting $u = x^{-n}$ horizontally.

This latter procedure is better because it requires no specially ruled paper, and in particular because we are much more familiar with operations on linear than on logarithmic coordinates. Few are aware, for example, that so simple an operation as addition of a constant will alter a straight line on logarithmic paper in three ways: by displacing it vertically, by altering its general slope, and by introducing a slight curvature.

The statistical procedure for fitting a formula to a set of corresponding values of speeds and observed resistances is as follows: approximate values of c and m are first obtained, with departures of all observed from calculated values. If these departures are unsystematic, they are simply averaged to obtain an additive constant, giving

$$y = \frac{c}{x^m} + \Delta$$

the mean value of Δ being i. By writing $c' = c + i$ and $m' = \frac{c}{c+i}m$ this may be converted into $y = \frac{c'}{x^{m'}}$.

We now desire to choose values of a and b which will give an equivalent two-term formula in which $n = 1/3$. When $x = o$, $a + b = c'$; when $x = e$, $a + be^{-1/3} = c'e^{-m'}$.

Solving, we have

$$a = (c+i)\,\frac{e^{\frac{-cm}{c+i}} - e^{-1/3}}{1 - e^{-1/3}},$$

$$b = (c+i)\,\frac{1 - e^{\frac{-cm}{c+i}}}{1 - e^{-1/3}}$$

If the departures from the mean are not wholly unsystematic, the first step will be to find a simple expression for Δ, say $i + j \log x$. We now seek values of c' and m' which will make

$$\frac{c'}{x^{m'}} = \frac{c}{x^m} + i + j \log x.$$

When $x = 1$, this gives $c' = c + i$, and when $x = e$, $c'e^{-m'} = ce^{-m} + i + j$, whence

$$-m' = \log \left(\frac{c}{c+i} e^{-m} + \frac{i+j}{c+i}\right).$$

With these values of c' and m', a and b may be found as before. Only if the departures from the mean show sufficient regularity to permit Δ to be made out as a second degree function of x, does it become possible to solve for all three constants in the two-term formula.

When it is possible to be sure in advance that a fixed exponent will serve the purpose, mean values of a and b may be more simply found by choosing a series of values of b, adjusting a in each case to give zero net total error, and determining the mean absolute error corresponding to each choice of b. It is now necessary to choose a speed at which a mean value is to be taken, say v_o. Plotting mean error on $a + \frac{b}{v_o}$ the value at which the error is a minimum is found. Proceeding similarly at another speed v_1, the best value of $a + \frac{b}{v_1}$ is found. These two best values will yield best values of the constants a and b which may be accepted for all speeds if v_o and v_1 are chosen with due regard for the distribution of speeds at which observations were made.

When a number of corresponding values of a and b are given they might, of course, be averaged separately but this might lead to a pair of mean values of a and b which would require an adjusted exponent to make them fit. If the exponent is determined in advance it is necessary to obtain a mean value of one of the two constants, and by substitution of this mean value determine the appropriate value of the other constant. Since this can be done only for one speed at a time, a procedure like that described in the last paragraph remains necessary.

It is generally assumed that length has an influence on frictional resistance that is identical with that of speed. Models of different lengths are therefore compared on a basis of equal values of VL and the generalized formula becomes

$$y = a + \frac{b}{(VL)^n} .$$