MACHINE METHODS OF COMPUTATION
and
NUMERICAL ANALYSIS

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and

PROJECT WHIRLWIND
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FOREWORD

This is a combined report for the two projects at the Massachusetts Institute of Technology which are sponsored by the Office of Naval Research under Contract N5ori60.

Project on Machine Methods of Computation and Numerical Analysis

This Project is an outgrowth of the activities of the Institute Committee on Machine Methods of Computation, established in November 1950. The purpose of the Project is (1) to integrate the efforts of all the departments and groups at M.I.T. who are working with modern computing machines and their applications, and (2) to train men in the use of these machines for computation and numerical analysis.

People from several departments of the Institute are taking part in the project. In the Appendix will be found a list of the personnel active in this program.

Project Whirlwind

This Project makes use of the facilities of the Digital Computer Laboratory. The principal objective of the Project is the application of an electronic digital computer of large capacity and very high speed (Whirlwind I) to problems in mathematics, science, engineering, simulation, and control.

The Whirlwind I Computer

Whirlwind I is of the high-speed electronic digital type, in which quantities are represented as discrete numbers, and complex problems are solved by the repeated use of fundamental arithmetic and logical (i.e., control or selection) operations. Computations are executed by fractional-microsecond pulses in electronic circuits, of which the principal ones are (1) the flip-flop, a circuit containing two vacuum tubes so connected that one tube or the other is conducting, but not both; (2) the gate or coincidence circuit; (3) the magnetic-core memory, in which binary digits are stored as one of two directions of magnetic flux within ferro-magnetic cores.

Whirlwind I uses numbers of 16 binary digits (equivalent to about 5 decimal digits). This length was selected to limit the machine to a practical size, but it permits the computation of many simulation problems. Calculations requiring greater number length are handled by the use of multiple-length numbers. Rapid-access magnetic-core memory has a capacity of 32,768 binary digits. Present speed of the computer is 40,000 single-address operations per second, equivalent to about 20,000 multiplications per second.

PART I

Machine Methods of Computation and Numerical Analysis

1. GENERAL COMMENTS

In this issue a number of new projects are reported that were begun within the current academic year. Of these, two are of special interest to engineers because of their applications, though they both involve interesting mathematics and require machine methods. One continues in the same direction of work previously reported here and concerns the response of a multi-story frame building to dynamic loading. The second concerns the turbulent flow of a saturated vapor within a tube whose inside surface is below saturation temperature. A new geophysical project also following close on the heels of results in that field recently reported here concerns the interpretation of resistivity data made at the earth's surface. The physically interesting results for determining the earth's composition must be reached through equations that have not been solved analytically and are difficult to solve numerically. Added to these projects are two that arise in the field of physics. One is the computation of scattering cross sections for zero energy neutrons from a potential well with characteristics familiar to the readers of these reports. The second appearing in a comprehensive report for the first time, is on an important subject that has been under investigation for some months, the diffusion of neutrons in irregularly defined regions such as nuclear reactors. This latter subject is approached with Monte Carlo methods, a powerful technique for modern computers. Similarly, certain calculational aspects of the cascade shower problem that has been reported before find simplification with the use of Monte Carlo methods. These also are reported under a new heading.

Two projects reported not long ago are being continued into a new phase of their work. One of these in the field of physics, the study of electronic wave functions for a regular lattice, is now at the stage of investigating the approximate forms suitable for computation of a rather complicated function. This stage of the work follows the general theory and begins the numerical calculation. The second continuing project is the mathematical investigation of the general structure of numerical analysis. The objective initially outlined, to find systematic techniques for unifying the technology of programming, is brought somewhat closer with the beginnings of a programming algebra.

Aside from these two continuing projects the final results are yet to be obtained in the study of fatigue fracture and, within physics, the study of Coulomb wave functions and eigenvalues for a square well. The latter two will ultimately yield useful tables. Finally, there is the work on electron-photon cascades and energy bands in graphite yet to be completed. In the course of developing the former of these, some results useful in the theory of stochastic processes (and by now familiar to the reader) have been obtained. The graphite project has already led to many useful subroutines for the users of Whirlwind I.

Upon glancing over this work, one sees that the detailed study of applied problems leads to numerical approximation techniques, Monte Carlo techniques, programming methods, tabular data, and subroutines, all needed in the further use of computing machines.

2. GRADUATE SCHOOL RESEARCH

2.1 Index to Reports

2.2 Progress Reports

THE BASIC PROBLEM OF NUMERICAL ANALYSIS EXPRESSED IN THE LANGUAGE OF COMPUTING MACHINES

The present report is a continuation of the last one that appeared under the same title [1]. A hypothetical computing machine is described. An algebra of programming is set up for this machine and it is shown how this algebra might be used to bring a solution in a simplified sense to some of the basic problems outlined in the last report. Its structure reflects in many ways the mathematical structure of the problem being programmed and for this reason the algebra appears to be quite suggestive.

Before introducing the machine and the method of programming, it will be well to review the direction of thought. The output of a programmed problem is in almost all cases an approximation to an ideal mathematical solution. Thus a problem for a computer is not associated with one unique output but with a collection of outputs ordered according to some criterion of closeness to a desired output. In other words, a problem for a digital computer is a mapping to the real line of all sequences of instruction and parameters that constitute possible programs. Loosely speaking, it is a real function of a variable program which takes its minimum at that program giving a desired output. The limitation in using the actual minimum is the time required to carry out a program on a machine. At best we must find the minimum over the set of programs that take a reasonable time to perform. This is one form of what has been termed the basic problem of numerical analysis expressed in the language of computing machines.

The last report ended with the observation that an output for a program performed on a specified machine is not known prior to performing it. Otherwise there would be no need for the machine. Similarly, the time required for the program is not accurately known in advance. Thus, some techniques must be introduced to determine how one program compares with another from the standpoint of output even though the output of each will remain unknown. A Similar statement applies for the time. If we are to solve the minimization problem, the technique just mentioned must guide us from one program to another with some indication at each step as to how output and the time have changed.

In the present report we will simplify the situation by assuming that the output is a single number. We will introduce a programming technique that reflects the algebraic structure of the problem being solved in the following sense: Algebraic operations on the program which (within the rules of the programming algebra) leave the program algebraically identical will at the same time leave the output algebraically identical. We will say that the outputs of two programs are algebraically identical if the series of additions, subtractions, multiplications, and divisions that the two programs specify to be performed on the input data correspond to two algebraically identical forms. For instance, a machine that in one case multiplies a by x and then a by y and then adds ax+ay and in another case adds x+y and then multiplies by a will output algebraically identical results in the two cases. Algebraically identical outputs need not, of course, be numerically equivalent, because of round-off error. Having constructed a programming algebra, we will then be in a position to solve the minimization problem within the class of algebraically equivalent programs. The difference in output within this class will be determined solely by the effect of round-off error on two algebraically identical forms; and the difference in time will be determined largely by the relative mechanical efficiencies of performing the sequences of elementary operations in the orders prescribed by the two programs. Though the last analysis of comparing the changes in output and time between algebraically equivalent programs will not be carried out here, an introduction to the algebra itself will be given.

It should be noticed here as it was in the last report that certain aspects of the minimization problem for machines are completely foreign to other branches of mathematics. First, two algebraically equivalent forms cannot be considered numerically equivalent. This follows from the fact reviewed earlier that a problem for a machine is associated with an approximation. Second, a measure of time is attached to an algebraic form. This follows from the physical limitations imposed in actually performing a sequence of elementary arithmetical operations.

The physical restrictions that are imposed on a real machine by the description of performance given here will not be discussed. We will take the attitude outlined in the last report that for a mathematician a machine is completely described by (1) the set of all possible sequences of instructions and parameters that constitute meaningful programs for the machine, (2) the output for each program, (3) and the performance time for each program. We will assume in this report that there are existing machines which, with the help of interpretive routines, can carry out the instructions specified here. We will not specify how the instructions are to be fed into the machine or how they are stored in the memory. We will not specify the magnitude or form of numbers to be stored in a register or the way that numbers, in the form of input data, arrive at a register. It is assumed, however, that a single index identifies a register in the memory, that the machine destroys information contained in a register if an instruction reads new information into an occupied register, and that a register contains the number zero unless other information has been read into it.

It should be remarked here that once a set of instructions have been established for performing the four elementary arithmetical operations, then it would be logically reasonable, though perhaps impractical, to write a program as a long sequence of such instructions. Such a program would include no decision-making on the part of the machine and no modification of instructions. It would most likely be an impractical program because as many symbols would be included as there would be additions, subtractions, multiplications, and divisions and all logical decision would have to be made outside the machine. In order to economize on the length of the program and the labor that goes into it (though not on the labor of the machine), it is necessary to introduce some new symbols and conventions. Some of the symbols of the program can then serve a multiple purpose as in subroutines and others can direct the machine to logical operations. It becomes immediately obvious, however, that the degree of economy that can be obtained is quite an arbitrary thing. From the point of view of the machine, the degree of economy depends entirely on how much the machine is allowed to interpret. From the point of view of a programming algebra it depends on how many conventions and added meanings are attached to the symbols. A balance must be reached from either point of view. Thus, any algebra in which the elements of the algebra are instructions will certainly be rather arbitrary. In this report we search simply for mathematical form that has the meaning of a program and that is relatively free of complicated conventions. The form will have the meaning of a program if the symbols involved can conceivably be interpreted by a machine.

The programming algebra for a fictitious machine will be outlined in four parts as follows:

1. Instructions
2. Modification of Instructions
3. Subroutines
4. Decision-making

The first three of these parts will be included in this report and will serve as an introduction to the algebra. The last will be discussed in the next report.

Four of the basic instructions with accompanying explanation are now given. To facilitate the explanation, we use the symbol P_x to denote the number in register x.

INSTRUCTION	EXPLANATION
A_{xyz}	Add x+y. Store sum in z.
S_{xyz}	Subtract y-x. Store difference in z.
M_{xyz}	Multiply xy. Store product in z.
D_{xyz}	Divide y/x. Store ratio in z.

A modification of any of the above instructions is also an instruction. This modifying instruction acts symbolically as an operator operating on the symbol immediately to its left.

MODIFICATION OF INSTRUCTION	EXPLANATION
T_{mnp}	If preceded by A_{xyz} (or any other instruction), then change A_{xyz} to $A_{x+m\ y+n\ z+p}$.

Subroutines are combinations of one or more instructions, a pre-bracket, and a post-bracket followed by a raised integer. A sequence of bracketed instructions are to be carried out in order from left to right. If a raised integer, n, appears at the right of a bracket, the instructions appearing inside the bracket are to be repeated in order (from left to right) n times. Subroutines may appear adjacent to one another, adjacent to instructions, or within one another. A sequence of instructions and subroutines are to be performed from left to right whether or not this sequence appears within another subroutine. Examples follow.

SUBROUTINES	EXPLANATION
$(A_{144})^3$	Compute P_4+3P_1. Store in 4.
$(S_{144})^3$	Compute P_4-3P_1. Store in 4.
$(M_{144})^3$	Compute $P_4 \cdot P_1^3$. Store in 4.
$(D_{144})^3$	Compute P_4/P_1^3. Store in 4.
$(A_{144}T_{100})^3$	Compute $P_1+P_2+P_3+P_4$. Store in 4.
$(M_{144}T_{100})^3$	Compute $P_1P_2P_3P_4$. Store in 4.
$(M_{151}T_{101})^4$	Compute P_5P_k. Store in k. k=1,2,3,4.

As two simple examples of programs consider:

$$(A_{144}T_{100})^3M_{454}$$

$$(M_{151}T_{101})^4(A_{144}T_{100})^3.$$

The first program reads, compute $P_1+P_2+P_3+P_4$, compute $P_5(P_1+P_2+P_3+P_4)$, and store in 4. The second reads, compute P_5P_k, k=1,2,3,4, compute $P_5P_1+P_5P_2+P_5P_3+P_5P_4$, and store in 4. These correspond to our concept of algebraically equivalent programs and suggest the following laws relating algebraically equivalent programs.

DISTRIBUTIVE LAW FOR ADDITION

$$(A_{1nn}T_{100})^{n-1}M_{n\ n+1\ n} \equiv (M_{1\ n+1 1}T_{101})^n(A_{1nn}T_{100})^{n-1} \qquad n=2,3,4,\ldots$$

DISTRIBUTIVE LAW FOR SUBTRACTION

$$(S_{1nn}T_{100})^{n-1}M_{n\ n+1\ n} \equiv (M_{1\ n+1 1}T_{101})^n(S_{1nn}\ T_{100})^{n-1} \qquad n=2,3,4,\ldots$$

These represent two examples of the kind of rules that can be introduced for the purpose of transforming one program into another of algebraically equivalent form. A complete development of the rules for this algebra will be done elsewhere. The integer powers that indicate the number of repetitions of a subroutine can later be equated to the outputs of subroutines. We will then have the ingredients of decision-making incorporated in the algebra. Decision-making will be discussed next time.

Bayard Rankin

Reference:

[1] Machine Methods of Computation and Numerical Analysis, Quarterly Progress Report No. 15, March 15 (1955), p. 7.

AN APPLICATION OF MONTE CARLO METHODS TO NEUTRON DIFFUSION PROBLEMS

From March through September of this year, all available time was spent on problems for the M.I.T. Nuclear Reactor Project. A description of the problems studied during this time can be found under Problem No. 266 in the last two quarterly reports.

The Main problem is, as the title indicates, to try to find a reliable method to solve complicated neutron diffusion problems, e.g., a detailed nuclear reactor, portholes in research reactors, control rod effects, etc.

Several methods can be used for the solution of this type of problem, but all of them belong to two main groups:

1. The "overall type" solutions in which the behavior of large groups of neutrons is described in equations.

2. The "individual type" solution in which a limited number of neutrons is observed in their individual behavior. By taking a large enough sample, some kind of average behavior can be found.

1. The "overall type" solutions, whether differential or integral equations, have in common that analytical solutions can be obtained only for the simplest geometrical systems. In more complicated geometries, approximations of the answer can be obtained with the various spacepoint techniques, but in this way most of the advantages of the "overall type" solutions are lost: The solution can become extremely tedious, is usually trial and error and in three dimensional cases, machine solutions are required.

This type of solution is, however, extremely useful for a preliminary design when many details can be neglected, or for many simplified problems.

As soon as one investigates a space-point technique solution for three dimensional cases, several difficulties from the machine point of view become obvious. Suppose one tried to solve one of the "overall type" solution equations on a three-dimensional 10 x 10 x 10 grid system. This gives 1,000 grid points.

A possible solution of the resulting 1,000 finite-difference equations by matrix inversion gives a matrix of 10^6 elements. It is rather doubtful if a matrix solution would yield sufficiently accurate answers even if sufficient memory space were available; with the present memory, this solution is impractical.

Another possible solution would be the solution of the finite difference equations by relaxation or other iterative processes. This is a definite possibility, but convergence and round-off errors will probably pose serious problems. Besides this, the necessity of using more than one register per number in Whirlwind I will slow down the calculations considerably. This is especially the case when the use of slow magnetic tape memory is inevitable, as in the more complicated problems. Time-dependent problems will aggravate the demands on the memory even more.

In short, it is doubtful if machine time will be used most economically in these types of solutions for three-dimensional problems.

2. Therefore, the "individual type" solution, also called "Monte Carlo type" solutions, were investigated. In these solutions, many of the disadvantages cited above are not present. As no grid points are used, the memory is not cluttered with information that is seldom used. The present memory of Whirlwind I does not pose a limit in these solutions. Even the most complicated geometries can be adapted for this type solution. The round-off error is not a serious problem. Therefore, we can use single register numbers with their inherent great increase in speed of calculation. It is much easier to bring in the variation of physical constants (cross sections) with neutron energy.

Time dependent problems are far easier handled. The disadvantages are: convergence may pose problems; the solution is in many cases again trial and error. The final accuracy is limited as the probable error is proportional to the inverse of the square root of the number of neutrons used, so each problem will have a practical probable error limit below which one could go only by using disproportionate amounts of machine time.

In my opinion the Monte Carlo type solutions have great advantages over the "overall type solutions" for complicated geometries. An added advantage is the easy adaptation of the program to gamma and neutron shielding problems.

A Short Description of a Monte Carlo Type Solution for Neutron Diffusion

Assume one has a neutron and its direction cosines and velocity are known. A random number is now used to determine how far this neutron will fly. After this neutron has collided, another random number is used to determine whether the neutron is scattered, absorbed or has caused fission. If it is scattered, its new direction cosines and velocity are found by assuming isotropic scattering in the center of mass system. Now we are back at the beginning of the cycle. Various other routines are needed to check if the right constants for material and energy group are used, to process the data as they become available, etc. Some of these routines have been coded and tested, but most of them have still to be programmed. A full description of the main features of this program, especially the logical design, will be given in the next quarterly report.

Marius Troost

A MONTE CARLO TECHNIQUE FOR SUMMING OVER CONNECTED PATHS

Suppose that a number of directed paths in the plane join the points A and B as in the figure.

These paths may overlap and intersect in an arbitrary manner so long as no path intersects itself and each segment between any two nodes (intersections) has only one direction. Two paths are considered different if they are not made up of the same set of points. Assume that for each path leading from A to B there corresponds one point on the real axis and no two paths correspond to the same point. If there are N different paths, let the N real points be $T_1, T_2, \ldots, T_N$.

Suppose also that there is a function $f(T_k)$ of the points T_k and that it is desired to compute $\sum_1^N f(T_k)$. The values of both T_k and $f(T_k)$ may depend in a complicated way on the structure of the path corresponding to T_k. Such a problem may appear in many contexts. As an example see the progress report immediately following in this issue. The problem may present an extremely lengthy calculation if there are many paths. In the case when $f(T_k)$ can be assumed a smooth function of T_k, one would like to pick out a wisely selected subset of the points T_k, compute $f(T_k)$ for these points, and interpolate on the others. However, even in this case time may be consumed computing the values of T_k for each path and indexing each path with these values. In this report a Monte Carlo technique is suggested for picking a subset out of the total N of paths in such a way that the points T_k for the selected paths are dispersed at random among all points $T_k, k=1,2,\ldots,N$. It will follow that on the average there are an equal number of unselected points between each adjacent pair of selected points. With the assumption that $f(T_k)$ is a smooth function of T_k, then the values of T_k and $f(T_k)$ for the selected subset can be computed and used to provide an unbiased estimate of $\sum_1^N f(T_k)$.

In order to carry out the above idea for a Monte Carlo approximation, it will be necessary simply to construct a means of selecting any path with equal probability from the total N. Any workable procedure that does this can be repeated until a sufficiently large subset of different paths has been chosen. When N is large, it will not be practical to index each different path even though it might well be practical to associate with each node an integer indicating the number of paths leading from that node to the point B. Hence, if we can assume that the numbers of paths from each node have been counted, we will show that these numbers are sufficient to guide us according to probability laws from node to node in such a way that the path delineated has probability 1/N, that is, probability equal to that of any other path. This wandering from node to node is well suited to machine computation and does not require indexing of the paths.

The use of the numbers of paths leading from each node to B is the following. Consider that the probability laws have guided us to a node and that we are now presented with three choices of direction. There will be n_1 paths to B if we take the first choice, n_2 if we take the second, and n_3 if we take the third. The number associated with the node at which we stand is $n_1+n_2+n_3$, the number associated with the first node along the path of the first choice is n_1, and so on. The probability laws now tell us to take the first choice with probability $n_1/(n_1+n_2+n_3)$, the second with probability $n_2/(n_1+n_2+n_3)$ and the third with probability $n_3/(n_1+n_2+n_3)$. If we are guided by such laws, the effect is the same as if we index all paths and choose from this collection of indexes with equal probability.

The proof of this result is immediate. We have only to show that the probability for taking an arbitrary path is 1/N. Let the nodes along this arbitrary path be associated with the integers $m_1, m_2, m_3, \ldots, m_s$. Since this arbitrary path starts with A and ends with B, it follows that $m_1=N$ and $m_{s-1}=m_s=1$. Now, given that we are at A, the probability to go to the second node of this arbitrary path is $m_2/m_1=m_2/N$. Given that we are at the second node, the probability to go to the third is m_3/m_2 and so on. The probability for traversing the entire path is, thus,

$$(m_2/N)(m_3/m_2)\ldots(m_{s-2}/m_{s-3})(1/m_{s-2})=1/N.$$

Bayard Rankin

THE NUMBER DISTRIBUTION OF ELECTRONS AND PHOTONS IN CASCASE

The probability $P(n,m,T,t|E_o,o)$ that an electron-photon shower initiated by a primary of energy E_o (at depth o) contains n electrons, m photons and possesses total energy T at depth t has been exhibited in general terms in previous progress reports [1,2]. The quantity of interest is $\sum_T P(n,m,T,t|E_o,o) \equiv \sum f(T)$ for (n,m,t,E_o) fixed. A complete calculation would yield a set of points $T_1 < T_2 < \ldots < T_{\nu(n,m)}$ on the T axis, where $\nu(n,m)$ is the number of paths leaving from the initial state and ending at state (n,m), (cf. the grid on p. 55 [1]), and where T_k is the total energy remaining to the electron-photon system after the system has traversed the k^{th} path. The sum $\sum_T f(T)$ would then be extended over these $\nu(n,m)$ values $T_k, k=1,2,\ldots,\nu(n,m)$.

Because of the correspondence between the paths and the total energies T_k, the Monte Carlo technique developed in the report immediately preceding in this issue can be used to compute $\sum f(T)$. The technique will provide a means of selecting a subset of r paths out of the total $\nu(n,m)$ so that an unbiased estimate of $\sum f(T)$ can be made from a knowledge of $f(T_k)$ evaluated over this subset.

The following method is used to select the paths. Let $T'_1 < T'_2 \ldots < T'_r$ be the energies corresponding to the r paths. Assuming a section of path stopping at the point (n',m') of the grid, a letter is selected at random out of a collection of $\nu(n',m')$ letters, $\nu(n',m'+1)$ of which are a's and $\nu(n'+2,m'-1)$ are b's, say; here $\nu(n',m')$ denotes the number of paths connecting (n,m) and (n',m'):

$$\nu(n',m') = \nu(n',m'+1) + (n'+2,m'-1)$$

If a letter a (or b) is picked up, the state $(n',m'+1)$ (or $(n'+2,m'-1)$) is included in the path and the same process is repeated starting from $(n',m'+1)$ (or $(n'+2,m'-1)$). It follows that between two computed energies $T'_\ell, T'_{\ell+1}$ there are on the average $\frac{\nu(n,m)-r}{r+1}$ actual points T_j.

A linear interpolation is then set up:

$$f(T_j) = A(T'_\ell) + B(T'_\ell)T_j \qquad T'_\ell < T_j \le T'_{\ell+1} \text{ with}$$

$$A(T'_\ell) = \frac{f(T'_{\ell+1})T'_\ell - f(T'_\ell)T'_{\ell+1}}{T'_{\ell+1}-T'_\ell}$$

$$B(T'_\ell) = \frac{f(T'_{\ell+1})-f(T'_\ell)}{T'_{\ell+1}-T'_\ell}$$

Then $$\sum_{T'_\ell < T_j \le T'_{\ell+1}} f(T_j) = A(T'_\ell)\left[\frac{\nu(n,m)+1}{r+1}\right] + B(T'_\ell) \sum_{T'_\ell < T_j \le T'_{\ell+1}} T_j$$

Where it is assumed that

$$T_j = T'_\ell + \frac{h(T'_{\ell+1}-T'_\ell)}{[\nu(n,m)+1]/(r+1)}$$

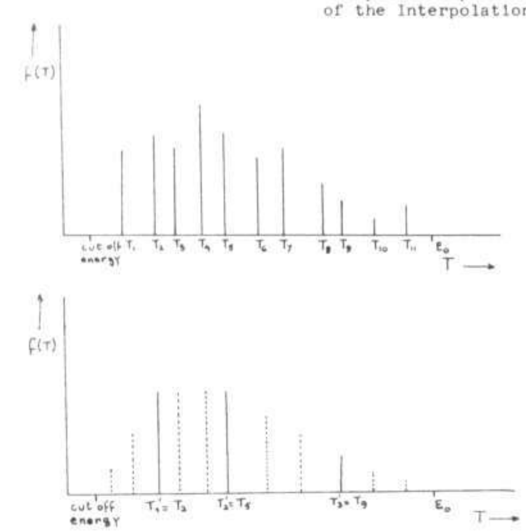
T_j being the h^{th} energy falling in the interval $(T'_\ell, T'_{\ell+1})$.

Some problems concerning the interpolation at the edges will be treated later.

The scheme provided by the grid has been converted into a form suitable for Whirlwind calculation and a program set up to compute the $\nu(n',m')$'s.

Raymond F. Stora

Graphical Representation
of the Interpolation Method



Form of results of an assumed complete calculation

$(\nu(n,m) = 11)$

Possible form of results using Monte-Carlo technique.

($r = 3$; T'_1, T'_2, T'_3, represent points chosen at random from $T_1, \ldots, T_{11}$. Dotted vertical lines represent interpolated values of which there are assumed to be $\frac{\nu - r}{r+1}$ between each pair $T'_\ell, T'_{\ell+1}$)

References:

[1] Machine Methods of Computation and Numerical Analysis, Quarterly Progress Report No. 13, September 15 (1954), p. 48.

[2] ibid. Report No. 14, December 15 (1954), p. 11.

ENERGY BANDS IN GRAPHITE

The calculation of the energy bands in graphite continues [1], the current phase involving the evaluation of one-electron three-center integrals. These integrals are being done by the method of expanding all wave functions about one coordinate center. At the core of this method lies the expression for the expansion about another center of an exponential divided by r [2],

(1) $$\frac{e^{-\kappa r'}}{r'} = \sum_{j=0}^{\infty} (2j+1)\, i_j\, k_j\, P_j(\cos\theta)$$

where the $i_j = i_j(\sigma)$ and $k_j = k_j(\sigma)$ are spherical Bessel functions of imaginary argument with σ the lesser of Ka and Kr; ρ the greater of Ka and Kr, and the coordinates are those given in Figure 1. It is noted that equation(1)follows from the scalar wave equation Green function expansion when the wave number is taken to be imaginary. Differentiation of equation 1 with respect to K yields,

(2) $$e^{-\kappa r'} = \sum_{j=0}^{\infty} (2j+1)\left[(\kappa a)\, i_j\, k_{j-1} - (\kappa r)\, i_{j+1}\, k_j\right] P_j(\cos\theta)$$

which if the expansion of an unnormalized 1s wave function. From Figure 1 the following geometric relations can be observed.

(3a) $$r' \sin\theta' = r \sin\theta$$

(3b) $$r' \cos\theta' = r \cos\theta - a$$

(3c) $$(r')^2 = r^2 + a^2 - 2ar\cos\theta$$

by multiplying equations (1) or (2) by the appropriate relations of equations (3) and using the recursion relations of the Legendre functions to reduce to a single summation, it is possible to build up the expansion of any Slater AO. Because of the importance of the i_n and k_n functions in this application and of the i_n functions in the computation of the B_n functions used in two-center integral calculations [3], the numerical procedures utilized to generate these functions will be given in detail.

In particular, the generation procedures to be given are ones which are appropriate for a high-speed digital computer. At this point it is perhaps best to list the criteria which in general apply to such computer generating programs:

1. Minimization of program storage space.
2. Minimization of program operating time.
3. Simple logical procedures which reduce possible coding errors.
4. Known accuracy limits for all regions of generation.

The first two criteria are rather obvious in that they directly apply to the clumsiness and efficiency of the final generating program. In particular, the first criterion implies that the number of auxiliary functions required be minimized, (e.g. e^x, $\frac{\sinh x}{x}$, etc.). The third criterion is especially important in that improvements in logical procedure usually enhance the first two criteria. Moreover, if one can generate functions using a minimum number of logical paths (i.e. special cases), the testing of the coding process is greatly simplified. Needless to say in a computer program all logical paths should be completely tested, but, this is, in fact, rarely possible.

The significance of this last remark becomes clearer when it is realized that the only procedure usually available for testing a logical path is an inductive test; that is, a test is made for specific parameter values and it is inferred that all other parameter values will perform similarly. The skill of minimizing this dilemma is usually correlated with the experience of the programmer and gives rise to the concept of "clean" programming. Finally, the fourth criterion involving numerical accuracy, while obviously desirable, is sometimes difficult to do and recourse often must be taken to empirical surveys of the parameter ranges in question. With these features in mind the problem of generating the i_n and k_n functions will be examined.

To establish explicitly the nature of the i_n and k_n functions, the basic properties are given:

Definitions:

(4a) $$i_n(x) = \left(\frac{\pi}{2x}\right)^{1/2} I_{n+1/2}(x) = (-i)^n j_n(ix)$$

(4b) $$k_n(x) = \frac{2}{\pi}\left(\frac{\pi}{2x}\right)^{1/2} K_{n+1/2}(x) = -(i)^n h_n^{(1)}(ix), \quad i = \sqrt{-1}$$

Integral Representations:

(5a) $$i_n(x) = \frac{(-1)^n}{2}\int_{-1}^{1} e^{-xt} P_n(t)\,dt$$

(5b) $$k_n(|x|) = \int_1^{\infty} e^{-|x|t} P_n(t)\,dt$$

Closed Forms:

(6a) $$i_n(x) = -\frac{1}{2}\left[k_n(-x) + (-1)^n k_n(x)\right]$$

(6b) $$k_n(x) = \frac{e^{-x}}{x}\sum_{j=0}^{n} \frac{(n+j)!}{j!\,(n-j)!}\left(\frac{1}{2x}\right)^j$$

Power Series:

(7) $$i_n(x) = \sum_{j=0}^{\infty} \frac{x^{n+2j}}{(2j)!!\,(2n+1+2j)!!}, \quad \text{where } m!! \equiv m(m-2)(m-4)\cdots 1 \text{ or } 2.$$

Limiting Forms:

(8a) $$i_n(x) \xrightarrow[x\to 0]{} \frac{x^n}{(2n+1)!!}\left[1 + \frac{x^2}{2(2n+3)} + \cdots\right]$$

(8b) $$k_n(x) \xrightarrow[x\to 0]{} \frac{(2n+1)!!}{(2n+1)\,x^{n+1}}\left[1 - \delta_{0n}x - \frac{x^2}{2(2n-1)} + \cdots\right]$$

(8c) $$i_n(x) \xrightarrow[x\to\infty]{} \frac{e^x}{2x}$$

(8d) $$k_n(x) \xrightarrow[x\to\infty]{} \frac{e^{-x}}{x}$$

Recurrence Formulas:

(9a) $$i_{n+1}(x) - i_{n-1}(x) = -\left(\frac{2n+1}{x}\right) i_n(x)$$

(9b) $$k_{n+1}(x) - k_{n-1}(x) = \left(\frac{2n+1}{x}\right) k_n(x)$$

Derivative Formulas:

(10a) $$i_n' = \frac{1}{2n+1}\left[n\,i_{n-1} + (n+1)\,i_{n+1}\right]$$

(10b) $$k_n' = \frac{-1}{2n+1}\left[n\,k_{n-1} + (n+1)\,k_{n+1}\right]$$

Wronskian:

(11a) $$\mathcal{W}(i_n, k_n) = -1/x^2$$

(11b) $$i_n k_{n+1} + i_{n+1} k_n = 1/x^2$$

Differential Equation: ($s_n = i_n$ or k_n)

(12a) $$s_n'' + \frac{2}{x}s_n' - \left[1 + \frac{n(n+1)}{x^2}\right]s_n = 0$$

(12b) $$(x s_n)'' - \left[1 + \frac{n(n+1)}{x^2}\right](x s_n) = 0$$

Simple Cases: ($n = -1$ defined by equations (9))

(13)
$$k_{-1} = \frac{e^{-x}}{x} \qquad i_{-1} = \frac{\cosh x}{x}$$
$$k_0 = \frac{e^{-x}}{x} \qquad i_0 = \frac{\sinh x}{x}$$
$$k_1 = \frac{e^{-x}}{x}\left[1 + \frac{1}{x}\right] \qquad i_1 = \frac{\cosh x}{x} - \frac{\sinh x}{x^2}$$
$$k_2 = \frac{e^{-x}}{x}\left[1 + \frac{3}{x} + \frac{3}{x^2}\right] \qquad i_2 = \frac{\sinh x}{x}\left[1 + \frac{3}{x^2}\right] - \frac{\cosh x}{x}\left[\frac{3}{x}\right]$$

By examination of the power series for i_n and the closed form for k_n, it is clear that i_n and k_n are positive for positive x. In addition, from equations (10) it is obvious that the i_n monotonically increase with x while the k_n monotonically decrease with x. Considered as functions of n with fixed x, the recursion formulas (9) suggest that the i_n monotonically decrease with order while the k_n monotonically increase with order. That these last relations hold can be seen from the differential equation. Consider the integration of equation 12b from near zero at $x = \delta$ to $x = \infty$. Then from the initial conditions $\delta i_n(\delta) > \delta i_{n+1}(\delta)$ and $[\delta i_n(\delta)]' > [\delta i_{n+1}(\delta)]'$ and the increasing value of $[x i_n(x)]''$ with order, it follows that the curves of $x i_n(x)$ and $x i_{n+1}(x)$ must be ever increasing without crossing points, for if there were any such points the condition $[x i_n(x)]'' < [x i_{n+1}(x)]''$ would have to be violated to reach the asymptotic value which is the same for all the i_n. Similarly integrating k_n from $x = \infty$ to $x = \delta$ the k_n are found to be monotonic in order. Thus we have

(14a) $$i_n(x) > i_{n+1}(x)$$

(14b) $$k_n(x) < k_{n+1}(x)$$

The usual physical applications require the sets $i_n(x)$ or $k_n(|x|)$ for $n=0,1,2,\ldots N$. In view of the easily demonstrated relation

(15) $$i_n(-x) = (-1)^n i_n(x)$$

there will be no loss of generality in conveniently restricting the following discussion to positive arguments. It can be immediately seen that upward recursion of the k_n and downward recursion of the i_n are the only generally accurate recurrence directions, for otherwise the numerical differencing can introduce serious loss of significant figures. Since the form of $k_{-1} = k_0$ is particularly simple, the k_n functions are thus straightforward to calculate. For the i_n functions upward recursion will also suffice in the region of $x > 4N+2$ since then $\left(\frac{2n+1}{x}\right)i_n < \frac{1}{2}i_{n-1}$; at most there will be a loss of one binary figure in each recurrence cycle, a result comparable to the inevitable truncation and round-off error. Consequently, the remainder of the discussion will consider only the determination of the set $i_n(x)$ for the case $x \leq 4N+2$, where the downward recurrence scheme must be used.

The obvious first approach to downward recurrence of the i_n would be to evaluate the power series of i_N and i_{N-1}. Examination of the series, though, shows that convergence for large x will go roughly as that of the series for

$$e^{x^2/4}$$

so clearly when $X = 4N+2$ and N becomes large, this procedure will become very poor. Instead we adopt a technique described by Miller [4] of working not with the i_n, but with the ratios

$$r_n(X) = i_{n+1}(X)/i_n(X) \tag{16}$$

for which the downward recurrence formula transforms into the numerically accurate continued fraction form

$$r_{n-1} = X/(2n+1+Xr_n) \tag{17}$$

As will be shown, the convergence of this form is so strong that it will be unnecessary to find the power series for r_{N-1}; instead it will suffice to start the recursion form (17) at a higher order n=M, setting $r_M=0$, where M is such that r_{N-1} will have the desired significant figure accuracy.

Next considering the nature of the $r_n(X)$, it follows from the relations (14a), (8a), and (8c) that

$$r_n(X) \xrightarrow[\frac{X}{2n+1}\to 0]{} \frac{X}{2n+3} \tag{18a}$$

$$r_n(X) \xrightarrow[X\to\infty]{} 1 \tag{18b}$$

$$r_n(X) < 1 \tag{18c}$$

To determine the relation between r_n and r_{n+1}, equation (17) is differentiated with respect to n to obtain

$$\begin{aligned}\frac{dr_n}{dn} &= -r_n^2\left[\frac{2}{X}+\frac{d}{dn}r_{n+1}\right]\\ &= -r_n^2\left[\frac{2}{X}-r_{n+1}^2\left\{\frac{2}{X}-r_{n+2}^2\left[\frac{2}{X}-\cdots-r_{n+k}^2\left(\frac{2}{X}-\frac{d}{dn}r_{n+k+1}\right)\right]\right\}\right]\end{aligned} \tag{19}$$

But from the expression (18a), it is seen that $\frac{d}{dn}r_{n+k+1}\xrightarrow[k\to\infty]{}0$ so that in view of the inequality (18c) it follows that each subtraction results in a positive quantity. Consequently,

$$\frac{dr_n}{dn}<0$$

or

$$r_n(X) > r_{n+1}(X) \tag{20}$$

To analyze the convergence of the continued fraction relation (17), the relative error in an approximate r_n is denoted by ϵ_n so that

$$r_{n-1}(1+\epsilon_{n-1}) = \frac{X}{2n+1+Xr_n(1+\epsilon_n)} = \frac{r_{n-1}}{1+r_{n-1}r_n\epsilon_n} \tag{21}$$

or

$$\epsilon_{n-1} \cong -r_{n-1}r_n\epsilon_n \tag{22}$$

Since the r_n are always less than unity, it is seen that the error propagation will always be "damped" when equation (17) is recursed downward. Consequently, if an error is allowed in the b^{th} significant binary figure in r_{N-1}, and $r_M=0$, then the following relation must hold to a good approximation

$$2^{-b} = \prod_{n=N}^{M} r_{n-1}r_n \tag{23}$$

In order to obtain an error formula which is more convenient to use, it is noted that because of (20),

$$b\ln 2 = -2(M_1-N+1)\ln r_{N-1}, \quad M_1>M \tag{24}$$

or even less stringently

$$b\ln 2 = -2(M_2-N)\ln r_N, \; M_2>M \tag{25}$$

Now because of relations (17) and (20) there is a value $\bar{r}_N$ such that

$$\bar{r}_N = \frac{X}{2n+1+X\bar{r}_N} \tag{26a}$$

and

$$r_{N-1}>\bar{r}_N>r_N \tag{26b}$$

Hence (25) becomes

$$b\ln 2 = -2(M_3-N)\ln\left[-\left(\frac{2N+1}{2}\right)+\sqrt{\left(\frac{2N+1}{2}\right)^2+1}\right], \; M_3>M_2 \tag{27}$$

or rewriting

$$\frac{M_3-N}{b} = \frac{\ln 2}{2\sinh^{-1}\left(\frac{2N+1}{2X}\right)} \tag{28}$$

Equation (28) is plotted in Figure 2.

It is seen that in the region of $X \leq 4N+2$, that the curve can be bounded by the line

$$A+\frac{Bx}{N+1/2} \tag{29}$$

where A = .05, B = .34. Therefore, from (28) it follows that

$$M_4 = bA+\frac{bB}{(N+1/2)}+N, \; M_4>M_3>M \tag{30}$$

Finally it is noted that it is sometimes possible to improve the last expression by asking if there exists a value of N' such that for $N'>N$, $M_4(X,N')<M_4(X,N)$, that is, is there a value of N' such that $r_{N'-1}$ will be accurate before the recursion downward is completed to r_{N-1}. Differentiation of M_4 with respect to N, along with the constraint $N'>N$ yields a final form

$$M_5 = bA+\frac{bB|X|}{N'+1/2}+N', \; M_5>M \tag{31}$$

where N' is the greater of N or $(bB|X|)^{1/2}-1/2$ and the absolute value signs on x obviously are the necessary generalization for negative x. Equation (31) is a great improvement over equation (30) for small N and is plotted in Figure 3 (for the special case of b = 30), the $N=\infty$ line corresponding to the plot of equation (30). In actual computation of M_5, it is clear that a square root of only about two or three decimal significant figures need be used. Moreover, the value of M that is required obviously must be taken to be the next higher integer than M_5.

Having seen how to obtain accurate ratios, it remains to form the i_n. The expression for

$$i_0 = \frac{\sinh X}{X}$$

could be used except that it would be inaccurate for small x (assuming that it would be generated by the already necessary exponential subroutine) and would be indeterminate for x = 0. To eliminate this problem, expression (11b) is used for n = 0

$$i_0k_1+i_1k_0 = \frac{i_0e^{-X}}{X}\left[1+\frac{1}{X}+r_0\right] = \frac{1}{X^2} \tag{32}$$

so that

$$i_0 = \frac{e^{|X|}}{1+|X|+Xr_0} \tag{33}$$

where the absolute value signs preserve numerical accuracy for negative x. Then the remaining i_n are formed by taking the product of the ratios

$$i_n = i_0\prod_{j=0}^{n-1} r_j \tag{34}$$

The foregoing scheme for generating the i_n and k_n has in practice been found to be codeable in very efficient program "loops." Moreover the procedures have the virtues of a minimum number of program forms and logical paths.

Fernando J. Corbató

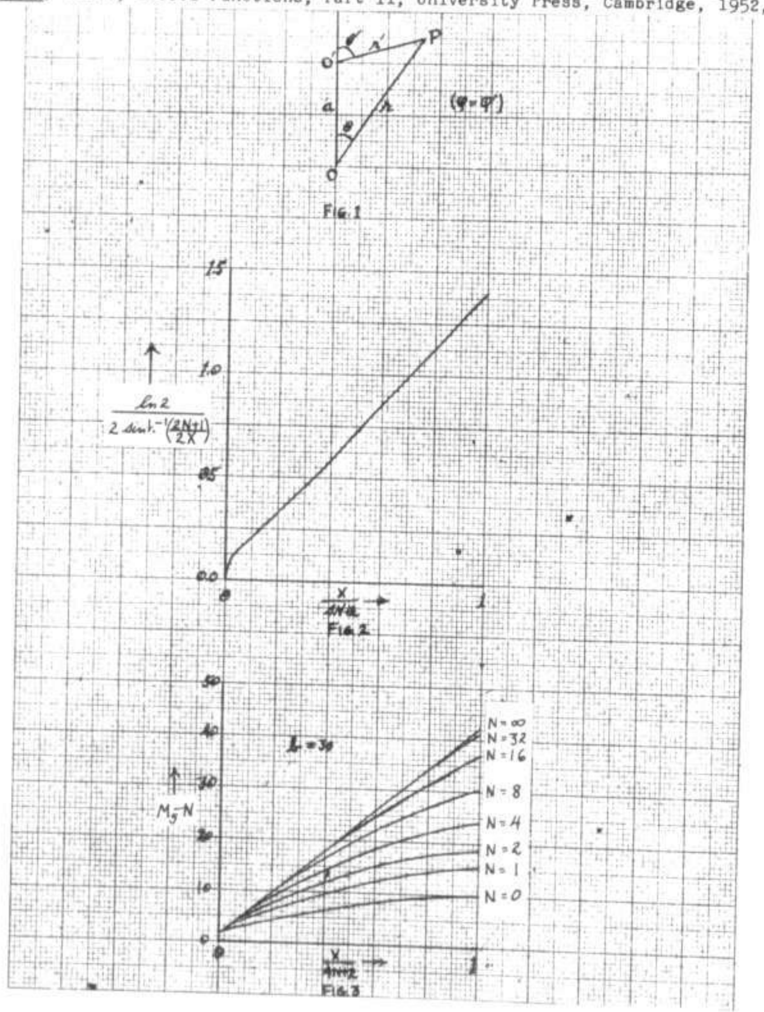
References:

[1] Machine Methods of Computation and Numerical Analysis, Quarterly Progress Report No. 16, June 15 (1955), p. 17. Also No. 17, September 15 (1955), p. 9.

[2] C. N. Watson, A Treatise on the Theory of Bessel Functions, Cambridge University Press, (London), 1952, p. 366.

[3] F. J. Corbato, "On the Computation of Auxiliary Functions for Two-center Integrals by Means of a High-speed Computer," to be published in the Journal of Chemical Physics about February 1956.

[4] J. C. P. Miller, British Association for the Advancement of Science, Mathematical Tables, Vol. X, Bessel Functions, Part II, University Press, Cambridge, 1952, p. xvi.



ATOMIC WAVE FUNCTIONS

The computations for atomic wave functions using a variational method has been completed. Values of the energies and parameters of iso-electronic sequences for lowest state configurations starting with B , C , N , O , F and Ne have been determined and a paper giving these results has been submitted to the Physical Review.

In the future, other members of the group might do further work in this subject. Now that a rather straightforward method has been found for obtaining the best parameter values for these simple analytic wave functions, we may start investigating techniques for programming the configuration interaction calculations involved when one tries to improve the original variational forms by perturbation theory. Because of the large diversity of terms involved, it will be necessary to try to set up subroutines for certain generalized expressions such as the S functions. Much work along this line has been done in the Solid State Group.

At present I am investigating the possibility of doing a Ph.D. thesis concerning certain aspects of the Bohr-Mottelson model of the nucleus.

Arnold Tubis

EIGENVALUES FOR A FINITE SPHEROIDAL SQUARE WELL

I have computed a set of eigenvalues of the Schroedinger equation with a potential well which is zero over a spheroidal region surrounding the origin and constant (positive) outside of this region. The size of the well was chosen so that its volume was always equal to that of a spherical nucleus of atomic weight 160 and radius

$R = 1.5A^{1/3} \times 10^{-13}$ cm., the well-depth being taken as 40 MEV.

Eigenvalues were computed for eccentricities of .5, .7, .8 prolate and oblate as well as for the undistorted case (see report on problem 321). A plot of these energy levels is shown on page 26. Each level is labeled at each edge of the graph by it rotational quantum number (m, in the usual spectroscopic notation), and at the center by the spectroscopic assignment of the undistorted level.

A spot check has been made of the accuracy by carrying out higher order approximations at several points. It is found that in general the accuracy is better at higher energies and smaller eccentricities, the error of the lowest (1s) level being about three per cent of its own value at $\epsilon^2 = .64$ (prolate or oblate) while that of the 2PO level is less than one-half of one per cent. In addition, it is found that the errors seem quite systematic, the values given in the graph generally being too low.

Jack L. Uretsky

References:

[1] Machine Methods of Computation and Numerical Analysis, Quarterly Progress Report No. 15, March 15 (1955), p. 19.

[2] ibid. Report No. 16, June 15 (1955), p. 21.

[3] ibid. Report No. 17, September 15 (1955), p. 7.

SCATTERING OF ZERO ENERGY NEUTRONS FROM A FINITE PROLATE SPHEROIDAL POTENTIAL WELL

Denoting the scattering cross-section by σ and the major semi-axis of the spheroid by "a," one can derive the relation

$$\sigma = 4\pi a^2(1-\Gamma)^2$$

for the zero energy limit of neutrons being scattered by a potential whose value is $-V_o$ over the spheroidal region in question and zero outside this region. The quantity Γ is given by

$$\Gamma = \sum_{n=0}^{\infty} A_n Je_{o,2n}(H,a)\, d_o(H|0,2n)$$

and the A_n's are the solutions of the infinite set of equations

$$\sum_{n=0}^{\infty} A_n d_{2K}(H|0,2n)\left[Je_{o,2n}(H,a) + \frac{a}{2K+1}\frac{\partial}{\partial a} Je_{o,2n}(H,a)\right] = \delta_{Ko}$$

The spheroidal functions and expansion coefficients are those of Morse and Feshbach (1), while

$$H^2 = \frac{2M}{\hbar^2} V_o d^2,$$ (d the interfocal distance) and K takes on all positive integer values.

A program has been prepared which computes the quantity Γ as a function of a and V_o. It was found, however, that the presently available subroutines for solving sets of linear equations were inadequate, and a new subroutine will be prepared.

Jack L. Uretsky

SPHERICAL SQUARE WELL EIGENVALUES

The discrete eigenvalue condition for the solution of Schroedinger's equation with a finite spherical square well of depth V_o and radius a may be expressed by the relations

$$\begin{cases} \frac{\partial}{\partial a} \ln\left[j_\ell(Ka) \big/ h_\ell(i\lambda a)\right] = 0 \\ K^2 + \lambda^2 = \frac{2M}{\hbar^2} V_0 \end{cases}$$

The values of K which satisfy these relations (call them $K_{\ell n}$) together with the quantities $J_\ell(K_{\ell n}a)$, $J_{\ell+1}(K_{\ell n}a), \ldots$ are often used in nuclear physics. Since it was necessary for me to prepare a whirlwind program to evaluate these quantities, in connection with the problem reported in [1], it seemed that it might be useful to prepare a table of values for general dissemination. Such a table is now in preparation.

A complete description of the method will be given in a subsequent report.

Jack L. Uretsky

Reference:

[1] Present Issue p. 19. See also references given there.

COMPUTATION OF ELECTRONIC WAVE FUNCTIONS FOR A REGULAR LATTICE

The Madelung [1]$_2$ functions $M_n^m(\bar{K},k)$ are being computed at points $|\bar{K}| = n(0.1)$ n = 0, 1,2,3,4,5 and at points $k^2 = m(0.1)$ integral m ranging from -4 to +4. It would be desirable to extend the range for both $|\bar{K}|$, k^2 and also make the mesh size smaller.

However, even for the present range and mesh size, the calculations are quite lengthy as far as machine time is concerned and it would be quite prohibitive for extended ranges and smaller intervals. Following a suggestion made by Professor Morse, we are trying to see from the available computed data whether the Madelung functions could be represented as

$$\frac{A_n^m(\bar{K})+B_n^m(\bar{K})}{1+C_n^m(\bar{K})k^2}$$ plus the subtracted large terms.

Work is under progress on this line of investigation.

Harvey Fields
Zoltan Fried

Reference:

[1] P. M. Morse, Machine Methods of Computation and Numerical Analysis, Quarterly Progress Report No. 16, June 15 (1955), p. 12.

COULOMB WAVE FUNCTIONS

The program for the irregular solution $G_o(\eta,\rho)$ mentioned in the previous progress report is nearly complete. The time of computation for one (ρ,η) is about twenty seconds. At present, we are planning to use the program only for the line $\rho = 2\eta$ and to use numerical integration of the differential equation for other values. In case the latter proves to be too inaccurate, we will extend the present program to all points on the ρ,η plane.

Aaron Temkin
Arnold Tubis

FIRST APPROXIMATION SOLUTION TO ORE BODY

For D. C. conduction in a region of arbitrary conductivities, the potential satisfies the differential equation

(1) $$\nabla^2\varphi = -q - \nabla \ln\sigma \cdot \nabla\varphi$$

where $q = q(x,y,z)$ is the source strength/unit volume and $\sigma=\sigma(x,y,z)$ is the conductivity. For a homogeneous medium σ_o and a point source Q at (x_o,y_oz_o), the potential at the point (x,y,z) is:

(2) $$\varphi(x,y,z) = \frac{Q}{4\pi\sigma_o\sqrt{[(x-x_o)^2+(y-y_o)^2+(z-z_o)^2]}}$$

In the field of applied geophysics, field measurements are made as follows: The sources, or senders, are located as shown in Figure 1 and a measured value of current applied to the earth. The receivers are placed in the same line as the senders but at varying distances from them and the potential difference measured. The apparent resistivity of the earth is calculated using equation 3 in which the effects of a pair of receivers and senders has been combined.

$$|\overline{0-m}| = R_1 \qquad |\overline{0-n}| = R_2 \qquad |\overline{1-m}| = R_3 \qquad |\overline{1-n}| = R_4$$

$$\rho_a = \frac{1}{\sigma_a}$$, Apparent resistivity

Figure 1

(3) $$\rho_a = 2\pi\frac{V}{Q}\left[\left(\frac{1}{R_1} - \frac{1}{R_3}\right) - \left(\frac{1}{R_2} - \frac{1}{R_4}\right)\right]$$

Interpretation of such resistivity data is one of the geophysical methods of determining the composition of the earth by surface measurements, and is of particular interest in prospecting for metallic mineral deposits.

Ore bodies frequently occur as shallow lenticular masses, and if a resistivity profile is taken perpendicular to the long dimension of the body the problem in three dimensions is reduced to one in two dimensions by symmetry.

In Figure 2 is represented the idealized case in which the ore body is of conductivity σ_1 and the surrounding country rock is of conductivity σ_o.

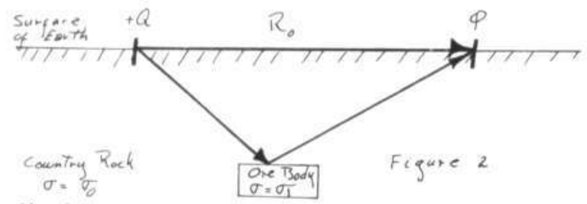


Figure 2

The solution to the differential equation (1) is given by:

$$\varphi = \iint \left(-q/\sigma_0 - \nabla \ln\sigma \cdot \nabla\varphi\right) \frac{\ln R}{\pi} \, dS$$

As a first approximation to the solution of the integral equation , we take

$$(4) \qquad \varphi_1 = -Q^1 \frac{\ln R}{\pi} \qquad\qquad Q^1 = q/\sigma_0$$

Substitution of this value of the potential into the differential equation and solving it gives the first approximate solution:

$$|\nabla \ln\sigma| = \delta(\ln \sigma_0/\sigma_1) \qquad \text{on surface of ore body}$$

$$\nabla\varphi_1 = -\frac{Q^1}{\pi}\left[\frac{\partial R_1}{\partial x_1}\bar{i} + \frac{\partial R_1}{\partial y_1}\bar{j}\right]\frac{1}{R_1}$$

Thus:

$$(5) \qquad \varphi = -\frac{Q^1}{\pi}\ln R_o + \left(\frac{Q^1}{\pi}\right)\delta(\ln \sigma_0/\sigma_1)\oint\left(\frac{\partial R_1}{\partial x_1}, \frac{\partial R_1}{\partial y_1}\right)\frac{\ln R_2}{R_1}\,dl$$

Physically the first approximation solution represents the case in which there is no interaction of the charges induced on the idealized ore body, that is to say, the value of the induced charge is determined only by the point source and is independent of the neighboring induced charges. The problem is essentially one of numerically integrating the effect of the induced charges around the four sides of the ore body. The program being developed is the general case for a variation of the parameters: length, height, depth, contrast, electrode spacing, and linear increments along the sides of the ore body as well as the number of senders and receivers. The results will be compared with previous results obtained by modeling the problem on teledeltos conducting paper.

It is hoped that this first approximation solution will agree well with the modeling experiments, and if true an investigation into the three-dimension approximation will be made. However, if the approximation is not good, it is desired to see quantitatively how good a solution it is and in which special cases. The ultimate goal at present is to set up a program which will work with actual field data and interpret it in the manner of breaking the earth into different blocks of conductivities $\sigma_1, \sigma_2 \ldots$ and from this to evaluate the possible economic value of the area under investigation.

This problem was suggested by the work being carried on in the Geophysics Department by Mr. Phillip Hallof and Mr. T. R. Madden and their assistance is greatfully acknowledged.

Norman F. Ness

RESPONSE OF A MULTI-STORY FRAME BUILDING TO DYNAMIC LOADING

Investigations utilizing programs based on a simplified and idealized structure have been satisfactorily completed. We are currently engaged in developing a more elaborate and generalized program for the determination of the dynamic behavior of any actual multi-story rectangular rigid frame building subjected to blast loading. This program is to include the effects of varying story-heights, bay widths, openings, mass distribution and loading. Results will be compared with those previously obtained to determine the adequacy

of the idealized analysis.

For the purpose of guiding development of the load portion of the new program, a program for front-face exterior and interior loads has been written and successfully run. The program for all loads is almost ready for a first run.

Ralph G. Gray

CONDENSATION IN TUBES SUBJECT TO THE EFFECTS OF VARIABLE VAPOR VELOCITY

Object:

To obtain generalized, approximate solutions to the following problems involving film condensation of vapor inside tubes:

1) A single, saturated vapor entering a vertical tube in fully-developed, turbulent, downward flow with the inside surface of the tube at a temperature below the saturation temperature of the vapor. The condensate to be in laminar flow.

2) Same as 1) except that the condensate is to be in turbulent flow.

3) Same as 1) except that the tube is to be horizontal.

4) Same as 2) except that the tube is to be horizontal.

If possible, the effect of the presence of a non-condensing gas in the entering flow to be investigated also for the four cases above.

Analysis of Case:

The case of film condensation of a saturated vapor on a vertical surface has been analyzed by Nusselt with the following conditions and assumptions:

a) The condensate is in laminar flow.

b) The velocity in the condensate is one-dimensional and not affected by shear stresses due to vapor flow.

c) The resistance to heat transfer from the vapor to the surface is totally composed of the resistance to heat conduction through the condensate layer.

d) The physical properties of the condensate are independent of temperature.

Several investigators have taken account of the frictional effects of the vapor velocity, otherwise making the same assumptions as above. However, all of these treatments of the problem are based either on a constant vapor velocity, or some sort of an average vapor velocity. The results thus obtained agree reasonably well with experimental data only when a small proportion of the vapor entering a condensing tube is condensed. The lack of agreement in other cases is probably mainly caused by the absence of adequate treatment of the variable vapor shear stress on the condensate, as the vapor velocity in the tube decreases due to condensation.

A step-wise procedure along the length of the tube has been set up to correct this difficulty. The assumptions a), c), and d) have again been made, and the condensate flow assumed to be one-dimensional.

The velocity distribution in the laminar condensate with an invariable vapor shear stress has been presented in reference [1] in the form:

$$(1) \qquad v_z = \frac{g(\rho - \rho_v)}{\mu}\left(x_o x - \frac{x^2}{2}\right) + \frac{g_o \tau_v}{\mu} x$$

and integrating, the mass flow rate per unit width in the condensate as:

$$(2) \qquad \Gamma = \int_0^{x_o} \rho v_z \, dx = g\frac{(\rho - \rho_v)}{\nu}\frac{x_o^3}{3} + \frac{g_o \tau_v}{\nu}\frac{x_o^2}{2}$$

With these equations valid for an increment Δz along the length of the tube, with the continuity equation, and with the equations for vapor shear stress on the condensate:

$$\tau_v = \frac{f \rho V^2}{8} \tag{3}$$

$$f = a\,(Re)^{-0.2} \tag{4}$$

the step-wise procedure has been established in the form of the following equations:

$$\Delta \Gamma^* = \frac{1}{4x_0^*}\,\Delta z^*$$

$$\Gamma^* = \frac{1}{3}(x_0^*)^3 + \frac{\tau_v^*}{2}(x_0^*)^2$$

$$\tau_v^* = g_0 \Psi \left(\frac{Re}{Re_1}\right)^{1.8}$$

$$\Delta\left(\frac{Re}{Re_1}\right) = -\frac{1}{4}\varphi \Delta \Gamma^*$$

where the primed quantities refer to non-dimensionalized variables:

$$\Gamma^* \equiv \frac{\Gamma}{\mu\,(1 - \rho_v/\rho)}$$

$$z^* \equiv \frac{4\,\Delta t\,C_p}{P_r\,h_{fg}'}\left(\frac{g}{\nu^2}\right)^{1/3}\frac{1}{(1 - \rho_v/\rho)}\,z$$

$$x_0^* \equiv x_0/(\nu^2/g)^{1/3}\ ; \quad \tau_v^* \equiv g_0 \tau_v / g(\rho - \rho_v)(\nu^2/g)^{1/3}$$

and φ and Ψ are defined by:

$$\varphi \equiv \left(\frac{\mu}{\mu_v}\right)(1 - \rho_v/\rho)/(Re)_i$$

$$\Psi \equiv V_i^2 a / 8g(\rho/\rho_v - 1)(\nu^2/g)^{1/3}(Re_i)^{0.2}$$

The equation open to question is equation (4). As reported by Bergelin, Kegel, Carpenter, and Gazley, the friction factor of turbulent vapor over a liquid surface depends not only on the Reynolds number of the flow, but also on a parameter $\Gamma/\rho s$. The influence of this parameter is believed to be due to the formation of surface ripples on the liquid.

Therefore, while case (2) listed in the object is being analyzed, a study is being made into the behavior and influence of these ripples.

Jukka A. Lehtinen

References:

[1] W. M. Rohsenow, J. H. Webber, A. T. Ling, "Effect of Vapor Velocity on Laminar and Turbulent Film Condensation," presented to Amer. Soc. of Mech. Engrs.

[2] A. P. Colburn, Clayton Lecture, London Discussion of Heat Transfer, July 1951.

COMPUTATION OF FATIGUE FRACTURE

A theoretical analysis of the phenomenon of fracture by fatigue is the object of the work being done by this group. For a complete analysis of fatigue fracture, three phases should be considered: the birth of the crack, the growth of the crack, and finally rupture of the specimen. Only the growth phase of the crack has been considered so far.

The assumption has been made that the growth of the crack is a function of the stress and strain fields within the specimen. Consequently, as a first step, the state of stress and the strain distribution about a crack which is not growing is of interest. The solution of such a problem is complicated by the fact that strains are not in the elastic region, and therefore, elasticity theory is invalid. This non-linearity and other considerations decree that the simplest problems to analyse are the cases of torsion of a bar with an axial crack and torsion of a circular bar with a circumferential crack. If the further simplification is made that the bar is composed of a non-workhardening material, i.e., a material

which exhibits the stress-strain characteristics in tension of Figure 1, and the Mises or Tresca yield criterion is assumed valid, the two problems may be formulated as in Figure 2 and Figure 3.

Figure 1

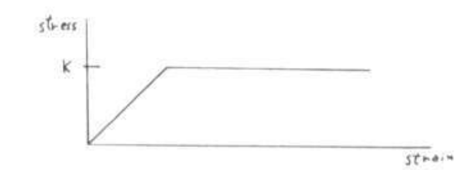


Figure 2
axial crack

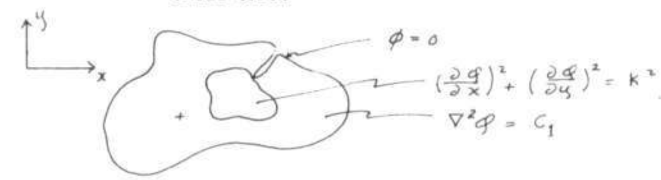
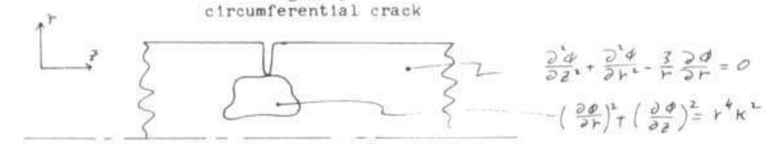
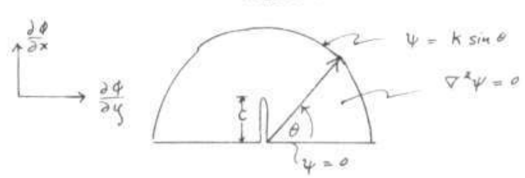


Figure 3
circumferential crack



Notice that in Figure 2 and Figure 3 the coordinates of the boundary between the elastic region and the plastic region are not known until the problem has been solved. For the case of an axial crack of small dimensions, the problem may be transformed to a boundary value problem with a known boundary as in Figure 4.

Figure 4



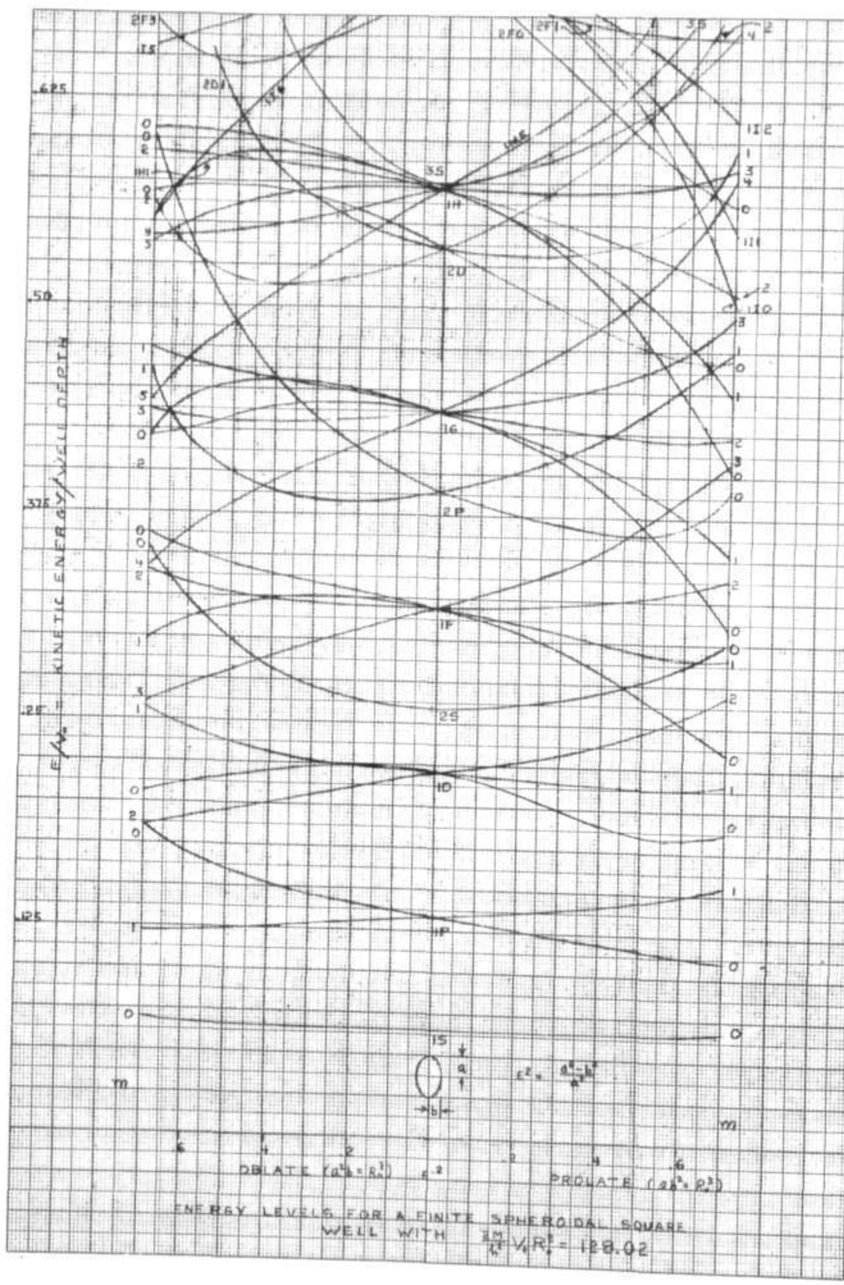
This simplified case may be solved quite easily but the transformation is not possible in the case of a crack of dimensions comparable to those of the bar.

Since the problems presented above have considered only a crack of fixed length, the solution of the large crack problem is not being undertaken until more is known about the effect of crack growth on the stress and strain distribution. This work is now under way.

Joseph B. Walsh

Reference:

[1] Machine Methods of Computation and Numerical Analysis, Quarterly Progress Report No. 17, September 15 (1955), p. 11.



PART II

Project Whirlwind

1. REVIEW AND PROBLEM INDEX

This report covers the specific period of September 19, 1955 to December 13, 1955. During this time, 85 problems made use of 328.5 hours of the 620.58 hours of Whirlwind I computer time allocated to the Scientific and Engineering Computations (S&EC) Group. The remaining 292.08 hours of the allocated time were used for terminal equipment testing and calibration, demonstrations, tape conversions for Lincoln Laboratory, and various inter-run operations not logged to specific problems.

These problems cover some 17 fields of applications. The results of 16 of the problems have been or will be included in academic theses. Of these, 13 represent doctoral theses and 3 represent master's. Thirty-one of the problems have originated from research projects sponsored at MIT by the Office of Naval Research.

Three of the problems reported on in this quarter (180, 276 and 311) have used no time during this quarter. These reports are submitted as terminating reports.

Two tables are provided as an index to the problems for which progress reports have been submitted. In the first table the problems are arranged according to the field of application, and the source and the amount of time used on WWI is given. In Table 2-II, the problems are listed according to the principal mathematical problem involved in each. In each table, the letter after the problem number indicates whether the problem is for academic credit and whether it is sponsored. The code is explained on page 30.

Field	Description	Problem Number	Min. of WWI Time	Supervisor or Programmer
Aeronautical Engineering	Transient response of aircraft structures to aerodynamic heating	236 C.	62.0	L. Schmit
	Transient temperature of a box-type beam	179 C.	32.0	L. Schmit
	Factoring high order polynomials	314 C.	26.5	V.W.Howard
	Low aspect ratio flutter	177 C.	251.8	J.Martuccelli
	Horizontal stabilizer analysis	257 C.	220.5	E.Criscione
	Postfailure response of aircraft subject to blast loading	330 C.	2.9	R. D'Amato
	Matrix iteration	331 D.	13.7	V.W.Howard
Aerophysics Research Group	Trajectory calculations for a rocket during powered flight	310 C.	29.0	J. Frigge
	Extraction of stability derivatives from flight test data	317 C.	49.0	L. Mazzola
Chemical Engineering	Transients in distillation columns	241 B,N.	240.8	S.H.Davis,Jr.
	Calculations for the MIT reactor	266 A.	68.7	M. Troost
	Critical mass calculations for cylindrical geometry	270 B.	743.7	J.R.Powell
Electrical Engineering	Optimization of alternator control systems	264 C.	16.7	J. Dennis
	Cross-correlation of blast furnace input-output data	180 B.	-----	R.G.Mills
	Diffusion equation with singularity near the boundary	325 B.	154.5	D.L.Pope
Geology and Geophysics	Geophysical data analysis	106 C.	401.7	S. Simpson
	Fourier synthesis for crystal structures	261 C.	325.3	M.J.Buerger
	Buried elastic wave source	328 B.	83.6	J. Gilbert
Hydrodynamics Lab.	Solitary wave generating cam	311 N.	-----	J. Housley
Instrumentation Laboratory	Guidance and control	239 C.	504.7	J.H.Laning,Jr.
Lincoln Laboratory	Eigenvalue problem for propagation of electromagnetic waves	193 L.	327.3	H.B.Dwight
	Ionosphere computation	259 L.	209.4	D.G.Brennan
	General raydist solution	272 L.	191.1	G.C.Sponsler
	Error analysis	312 L.	794.4	R. Bennett
	Prediction analysis	327 L.	40.1	I. Reed
Mathematics	Buckling of shallow elastic shells	275 B.	157.2	A. Ralston
Mechanical Engineering	Flow of compressible fluids (aerothermopressor)	120 B,N.	63.5	A. Erickson
	Laminar boundary layer of a steady, compressible flow in the entrance region of a tube	199 N.	29.2	T.Y.Toong
	Rolling bearings	293 C.	171.4	A. Shashaty
	Maximum size of bubbles	322 B.	19.6	P. Griffith
Metallurgy	Correlation of the martensitic transformation in stainless steel	276 B,N.	-----	F. Monkman
Meteorology	Synoptic climatology	155 N.	168.5	E. Kelley
	Investigation of the vorticity field in the general circulation of the atmosphere	226 D.	206.4	D. Cooley
Naval Supersonic Laboratory	Supersonic nozzle design	307 C.	13.2	J. Baron
Physics	An augmented plane wave method as applied to sodium	194 B,N.	490.3	M.M.Saffren
	Atomic integrals	234 N.	33.8	R.K.Nesbet
	Application of the APW method to face- and body-centered iron	253 N.	651.1	J. Wood
	Energy levels of diatomic hydrides	260 N.	482.6	A. Freeman
	Evaluation of two-centered molecular integrals	262 N.	150.3	H. Aghajanian
	Analysis of air shower data	273 N.	10.0	G. Clark
	Multiple scattering of waves from a spatial array of spherical scatterers	274 N.	156.5	P.M.Morse
	Energy levels of diatomic hydrides LiH	278 N.	324.6	G. Koster
	Augmented plane wave method as applied to crystal chromium	285 N.	317.3	M. M. Saffren
	Atomic wave functions	288 N.	832.4	R.K.Nesbet
	Relativistic atomic wave functions	304 N.	300.3	C. Schwartz
	Pure and impure KCl crystal	309 B,N.	306.7	L.P.Howland
	Determination of phase shifts from experimental cross-sections	162 N.	89.7	E. Campbell
	Analysis of cloud chamber photographs	323 N.	70.3	D.O.Caldwell
Servomechanisms Laboratory	Data reduction program, polynomial fitting	126 C.	630.0	D.T.Ross
Miscellaneous	Comparison of simplex and relaxation methods in linear programming	219.	207.6	D.N.Arden

Table 2-I Current Problems Arranged According to Field of Application

Mathematical Problem	Procedure	Problem Number
1. Matrix algebra and equations		
Root of determinantal equation	Iteration	266 A.
Linear equations	Crout's method	270 B.
Eigenvalues	Iteration	275 B.
Eigenvalues	Diagonalization	278 N.
Eigenvalues	Diagonalization	285 N.
Eigenvalues	Diagonalization	288 N.
Inversion	Crout's method	312 L.
Factorization of polynomials	Hitchcock's method	314 C.
Algebraic equations	Newton-Raphson	307 C.
Matrix equation	Crout's method	177 C.
Linear equation	Crout	317 C.
Equations	Direct evaluation	323 N.
Equations	Matrix iteration	330 C.
Equations	Matrix iteration	331 D.
Matrix inversion	Crout	332 C.
2. Ordinary differential equations		
Seven nonlinear first order	Fourth order Kutta-Gill	120 B,N.
System	Gill's method	199 N.
First order equations	Gill's method	239 C.
Nonlinear first order equations	Second order Kutta-Gill	241 B.
Second order equations	Gill's method	310 C.
3. Partial differential equations		
Second order parabolic	Finite differences	236 C.
First order system	Finite differences	226 D.
Second order parabolic partial	Finite differences	179 C.
Nonlinear partial equations	Finite differences	322 B.
Nonlinear partial equations	Finite differences	325 B.
4. Integration		
Overlap integrals	Evaluation of analytic forms	260 N.
Integration	Simpson's rule	278 N.
Interral transformation	Algebraic recursion formula	234 N.
Overlap integrals	Evaluation of analytic forms	262 N.
Integration	Trapezoidal rule	304 A.
Integration	Gauss quadrature	312 L.
Integration	Barnett and Coulson expansion method	309 B.
5. Statistics		
Multiple time series	Prediction by linear operators	106 C.
Calculation of coefficients of a multiple regression system	Inner products	155 N.
6. Transcendental equations		
Nonlinear equations	Steepest descent	264 C.
Curve fitting	Least squares	273 N.
Curve fitting	Least squares	162 N.
7. Data reduction		
Data reduction	Polynomial fitting, etc.	126 C.
8. Group theory		
Generation of projection operators	Machine generation	194 B,N.
9. Complex algebra		
Complex roots and function evaluation	Iteration	193 L.
10. Fourier series		
Fourier synthesis	Direct evaluation	261 C.
Summing series	Direct evaluation	274 N.
Fourier series	Explicit difference equations	235 B.
11. Linear programming		
Linear programming	Simplex method and relaxation method	219

Table 2-II Current Problems Arranged According to the Mathematics Involved

2. WHIRLWIND CODING AND APPLICATIONS

2.1 Introduction

Progress reports as submitted by the various programmers are presented in numerical order in Section 2.2. Since this summary report presents the combined efforts of DIC Projects 6345 and 6915, reports on problems undertaken by members of the Machine Methods of Computation (MMC) Group have been omitted from Section 2.2 of Part II to avoid duplication of Part I. For reference purposes, a list of the MMC Group problems appears on page 51.

Letters have been added to the problem numbers to indicate whether the problem is for academic credit and whether it is sponsored. The letters have the following significance.

A implies the problem is NOT for academic credit, is UNsponsored.

B implies the problem IS for academic credit, is UNsponsored.

C implies the problem is NOT for academic credit, IS sponsored.

D implies the problem IS for academic credit, IS sponsored.

N implies the problem is sponsored by the Office of Naval Research.

L implies the problem is sponsored by Lincoln Laboratory.

The absence of a letter indicates that the problem originated within the S and EC Group.

106 C. MIT SEISMIC PROJECT

During the past quarter our computational work has been centered about three research efforts. They are as follows:

1.) Testing stationarity of time series by a procedure making extensive use of matrix subroutines. We have run into difficulty with matrix inversion but hope to have this straightened out shortly.

2.) Resolving wavelet complexes by the use of inverse operators and direct spectral factorization. The spectral factorization program is also being used to generate minimum phase shift linear operators with specified amplitude (frequency) characteristics. These projects will be continued.

3.) Testing a procedure which uses a correlation averaging technique in examining for hidden reflections on multitrace seismograms - a procedure in which geometrical factors are automatically corrected for.

S.M. Simpson
Geology and Geophysics

120 B,N. THERMODYNAMIC AND DYNAMIC EFFECTS OF WATER INJECTION INTO HIGH TEMPERATURE, HIGH VELOCITY GAS STREAMS

During the past quarter, theoretical calculations were made to help with the design of a new experimental Aerothermopressor. Experimental runs will now be made to check with theoretical calculations.

A. Erickson
Mechanical Engineering Department

126 C. DATA REDUCTION

Problem 126 is a very large data reduction program for use in the Servomechanisms Laboratory. The overall problem is composed of many component sections which have been developed separately and are now being combined into complete prototype programs. Descriptions of the various component sections have appeared in past quarterly reports. After the development and testing of the prototype Whirlwind programs is completed, the programs will be re-coded for other, commercially available, large scale computers, (probably the ERA 1103, IBM 701 and IBM 704 computers), for use by interested agencies for actual data reduction at other locations. The programs are currently being developed by Douglas T. Ross, David F. McAvinn, Walter E. Weissblum and Benson H. Scheff, with the assistance of Miss Dorothy A. Hamilton, Servomechanisms Laboratory staff members. This work is sponsored by the Air Force Armament Laboratory through DIC Project 7138.

The nature of the problem requires extreme automaticity and efficiency in the actual running of the program, but also requires the presence of human operators in the computation loop for the purpose of decision making and program modification. For this reason extensive use is made of output oscilloscopes so that the computer can communicate with the human, and manual intervention registers so that the human can communicate with the computer in terms of broad ideas, while the computer is running, and have the computer program translate these ideas into the detailed steps necessary for program modification to conform to the human operator's decision. The program which does this translation and modification is called the Manual Intervention Program (MIV). The most recent version of the prototype data reduction program is called the Basic Evaluation Program.

The programs which must be operated simultaneously are much too large for core memory, and many more major sections are planned for the future. For this reason, the programs are broken up into "MIV Groups" of approximately 1500 registers and stored arbitrarily on the auxiliary and buffer drums. Each program of an MIV Group must be able to refer to certain results of other programs in other MIV Groups, so that a section of "invariant storage" is assigned and never replaced when MIV Groups are changed.

The changing of MIV Groups takes place automatically under the direction of a "Group Control Program", which also is in invariant storage. Each MIV Group has its own flad table and control is transferred from one MIV Group to another by means of a "Call Address Listing" by which the Group Control program can determine which program is desired and a "Section Address Listing" which tells the Group Control Program in which MIV Group and where within that group the program is located. The interchange of MIV Groups is then executed automatically, and the old MIV Group being "saved" only when necessary.

An involved usage of drum address assignments and preset parameters allows the use of folating addresses throughout the system of programs, and programs may be inserted or changed without impairing the elaborate interconnections.

During the past quarter the above MIV Group Control system was checked out as well as parts of the MIV program. The Basic Evaluation Program runs for long periods giving wrong answers and then encounters alarm conditions. The Mistake Diagnosis Routine described in an earlier quarterly report has been modified so that it is easier to use and over fifty break-points have been chosen for sampling by the new MDR program. The MDR results are expected to locate the program difficulties quickly and yield accuracy checks as well.

A generalized Interpolation Program is being written to increase the sampling rates of data stored in various forms at arbitrary locations on the drum. A large number of cases may be easily selected by specifying natural parameters.

D.T.Ross
Servomechanisms Laboratory

131. STAFF TRAINING, DEMONSTRATIONS, ETC.

Generalized NIM Playing Routine

Generalized NIM is a game in which a number of groups are available, in this routine,eight marked A to H. Each group is composed of a number of pieces (N_1). The game consists of each player (computer vs. human) in turn removing not more than n pieces from any one group and this from not more than m groups. The winner is the one who removes the last piece.

The game is displayed on the scope showing decimally the number of pieces in each group at the time of display.

n and N_1 (the initial number of pieces in each group) can be varied from 0 to 99 and m can be varied in the range 1 - 8 at the beginning of each game. Direct typewriter types the human win, machine win or wrong move message.

This routine is explained more fully in Digital Computer Laboratory Memorandum DCL-113.

A. Zabludowsky
Digital Computer Laboratory

Tic Tac Toe Playing Routine

The routine displays the game on the scope; the player's moves are displayed as plus signs and the computer's moves by squares. Output for "player wins", "computer wins", "draw", or "move on a previously occupied square", is by direct typewriter.

In one combination of moves the computer can be made to lose.

This routine is explained more fully in Digital Computer Laboratory Memorandum DCL-105.

A. Zabludowsky
Digital Computer Laboratory

155 N. SYNOPTIC CLIMATOLOGY

The programming and computing for the Synoptic Climatology problem has been varied and conducted on three phases of the over-all problem.

The first has been a continuation of prediction and specification of surface 5-day mean temperature anomaly from the 700-mb 5-day mean height anomaly pattern. In connection with this part of the problem it has been unnecessary to write any further programs, but has been a matter of performing computations on new input data.

The second phase has consisted of the testing of several programs for prediction of weather elements by a different approach than has been used in the past. This has necessitated programming the solution of the autocorrelation function of a particular element and the factorization of the autocorrelation spectrum. This testing has progressed satisfactorily.

For the third phase, programs are being written for the integration of models which are roughly equivalent to the earth's atmosphere. Due to certain assumptions, the models are capable of integrating by short time steps for what would correspond to months or years. The weather statistics generated by these models will be compared with observed weather statistics.

A preliminary program has been in the process of being tested.

It is expected that the work done during the future will be an attempt to bring that which has been described to completion.

The programming during this period has been done by Miss Elizabeth Kelley, Mr. Ralph Huschke and Mr. Kirk Bryan under the supervision of Professor Edward N. Lorenz, Project Supervisor, Department of Meteorology.

E. A. Kelley
Meteorology Department

162 N. ANALYSIS OF A SCATTERING EXPERIMENT IN NUCLEAR PHYSICS

Energies from .5 to 2.6 have been analyzed and various combinations of phase shifts have been found which fit satisfactorily the theoretical expression. Future plans are to explore the resonance points and to analyze higher energies.

E. Campbell
Laboratory for Nuclear Science

177 C. LOW ASPECT RATIO FLUTTER

During the past quarter, three sets of 30 simultaneous equations, each with three right hand sides, were solved. Also, one set of 36 simultaneous equations, with two right hand sides, was solved.

The multiplication of five complex matrices by five real matrices and addition of the resulting matrices was done for the following cases:

1.) three sets of 15 x 15 matrices

2.) one set of 18 x 18 matrices.

The problem has been completed and no future work is planned at present.

J.R. Martuccelli
Aeroelastic and Structures Research Lab.

179 C. TRANSIENT TEMPERATURE OF A BOX-TYPE BEAM

During this quarter two production runs were made. The peak radiant heat flux intensity was set at 4 cal/cm^2-sec.in one run and at 8 cal/cm^2-sec. in the other. Correlation of the calculated temperature responses with experimental results was generally satisfactory. At high temperatures some of the assumptions made in the mathematical formulation begin to break down and the correlation is less satisfactory; however, these discrepancies are in the direction anticipated. Correlation of the stress responses is less satisfactory The difficulty here is thought to lie in the experimental area. At temperatures in excess of 400°F the experimental measurements of strain as well as the calculation of stresses from such strain data is in question. No further calculations are being planned at this time.

L. Schmit
Aeroelastic and Structures Research Lab.

180 B. CROSSCORRELATION OF BLAST FURNACE INPUT-OUTPUT DATA

Problem 180 was initiated in connection with an undergraduate thesis in the Department of Electrical Engineering in collaboration with H.J.Scholz, Jr. A portion of the thesis project involved testing and demonstrating the use in a business application of a technique of analysis previously used only in non-business fields, such as circuit analysis and servo-systems analysis, and a portion of the time charged to Problem 180 was expended in obtaining the particular solutions required for successful completion of the thesis project in the allotted time. It was not found possible to complete the study in sufficient time to permit reporting all final results in the thesis; the project was therefore continued by the author in an effort to make a more complete evaluation of the proposed method and, if possible, to develop it further.

The result, to date, has been the evolution of a combined program which will carry out the two principal steps in the analysis, together with a number of auxiliary operations, and which will also present the result in a form permitting ready interpretation. It is expected that Problem 180 will be reopened, or a new problem initiated, in the near future to permit further study and expansion of the basic technique.

The thesis concerned itself with certain highly complex industrial processes which were observed to exhibit an overall dynamic characteristic similar, along some lines, to systems encountered in servomechanisms analysis. As a result of this apparent similarity, several different servo-system analysis methods were considered in an attempt to obtain some type of "dynamic response function" for the particular process selected as an example which would permit prediction, with a certain amount of mathematical rigor, of the behavior of the process under given "input" conditions.

It was a particular aim of the project to avoid the use of a complete internal analysis, as this was thought impractical due to the vast amount of laborious study and computation involved and also the fact that any minor variation in conditions will render such an analysis invalid.

The method which appeared to hold the most promise was that of determining, means of the techniques of crosscorrelation and Fourier transformation, a function, interpretation of which would yield information concerning the overall process. It was this method which was programmed.

In the course of the operation of the program which has been developed, the functions representing the time variation of two carefully-selected significant process variables are crosscorrelated, and the Fourier transform of the resulting correlation function is obtained. The result of this computation -- actually a frequency function or spectrum-- is characteristic of the process under study and is mathematically defined as the transfer function relating the "input" variable with the "output". (The terms "input" and "output" are used merely to conform with general practice in correlation work; the use of these terms does not indicate that the variables used are necessarily the process input and output.)

A number of problems are associated with the use of this method; two of the most obvious are the selection of variables and interpretation of results. A complete discussion of these considerations is beyond the scope of this report, but attention is invited to the thesis, in which a fairly thorough treatment of a particular case may be found. Regarding interpretation of the plotted curves which are the output of the program, much of the author's time has been expended considering this facet of the problem, and several possible avenues of further study have presented themselves. Among these are (1) the use of the present program to transform a number of sets of synthetically-prepared data which represent known situations in order to become acquainted with the general configuration of results in these controlled instances, and (2) the preparation of additional programmed routines which will permit the computer to make actual predictions based upon the computed spectrum and certain other input data. At the moment this latter possibility appears the most promising, and some work is now under way along these lines.

Nonlinearities present their usual problems in such a scheme, and they are present in profusion in the simplest industrial process. The less significant ones may be ignored, as in the servo field, but the major ones must be treated either in the preparation of the input data for the program or in the interpretation of the results. This is another area deserving of further consideration, and it is expected that an important portion of the proposed future work will be devoted to it. Reference is again made to the thesis for a discussion of a particular case.

The present work is involved with the task of generalizing the technique to include processes of a less continuous nature. It presently appears that this will not involve altering the portion of the analysis technique which utilizes the computer, but rather will require the use of auxiliary methods to assist in variable selection and interpretation of results.

It has been interesting to note the general utility of the process used in this work. The correlation-transformation operations are in general used an analytical techniques in a very wide variety of the problems amenable to high-speed computer work. For this reason an auxiliary objective of the project has been to provide the Laboratory with a library program capable of handling a major portion of these problems with little or no expenditure of computer time to develop and trouble-shoot a complex and lengthy program for each

individual application.

The program now in the Laboratory files as a result of Problem 180, although doubtless useful in some applications, falls short of the desired end in two particulars: (1) its input is limited to function-lengths of about 248 values, and (2) the scope-display type of output is probably not the most generally useful one.

It is anticipated that many of the revisions to be made in the course of continued work on the business application of this technique will also yield, as a collateral result, a correlation-transformation program having both expanded input capabilities and improved output form; it will thus be a more generally useful library program.

The prime aim of the work associated with Problem 180 has been to provide the field of Operations Research with yet another tool for rigorous analysis of extensive and highly-complex industrial processes for the purpose of predicting their behavior under varying conditions. Process-wide scope and simple and rapid computations, permissive of many repetitions with varying data, are the principal advantages of the technique over a complete internal analysis. The relative complexity of the latter is illustrated by the fact that, in the case of so simple a process as smelting iron ore (the blast furnace process), over 250 separate variables are deserving of attention.

Believing the method to be of interest and value the author has continued to build upon the general foundation laid in the thesis. The program for carrying out the computational work is now complete, at least in its preliminary form. Projected further study will probably require that a new problem be initiated to permit extension of the present program and to permit consideration of a number of possible variations on it.

R. G. Mills
Electrical Engineering Department

193 L. EIGENVALUE PROBLEM FOR THE PROPAGATION OF ELECTROMAGNETIC WAVES

A field strength versus distance curve has been completed for 6 meters wavelength, 500 and 30 foot antenna heights, and distances from 5 to 410 miles, with emphasis on the calculation of the field well beyond the horizon, in the twilight region. Similar calculations have been continued for two additional frequencies.

H. B. Dwight
Lincoln Laboratory

194 B.N. AN AUGMENTED PLANE WAVE METHOD APPLIED TO SODIUM CRYSTAL

Corrected E vs. E_o curves have been obtained. A new matrix element generation program has been written and is being tested.

Further details may be found in the Quarterly Progress Reports of the Solid State and Molecular Theory Group, MIT.

M. M. Saffren
Solid State and Molecular Theory Group

199 N. STEADY LAMINAR FLOW OF A COMPRESSIBLE FLUID IN THE ENTRANCE REGION OF A TUBE

In connection with the research on heat-transfer coefficients, recovery factors, and friction coefficients for supersonic flow of air in a tube, a theoretical investigation of the characteristics of the laminar boundary layer in the entrance region has been carried out. Partial differential equations of continuity, momentum, and energy were developed for the laminar boundary layer. These were then transformed into a series of ordinary differential equations,[1] to be solved for specific entrance Mach numbers and thermal conditions at the tube wall.

Solutions of the first three sets of the differential equations for the case of a heated tube wall and an entrance Mach number of 2.8 were obtained. Soltuions were also obtained for the case where temperature dependence of the fluid viscosity and thermal conductivity is taken into consideration. The results were satisfactory.

More solutions are to be obtained for different entrance Mach numbers and thermal conditions at the tube wall, with the viscosity and thermal conductivity of air varying with temperature.

T. Y. Toong
Mechanical Engineering Department

Footnote
1. Gill's method is used in the numerical integration of these differential equations.

219. TRANSPORTATION PROBLEM

The program has been completed including facilities for reading data tapes, printing the results in convenient form, and for selecting the amount and mode of output desired.

A problem for 9 plants and 69 demands has been solved correctly by the program in about one minute.

A fault in the coding was discovered in trying to solve a problem for 30 plants and 51 demands.

In the future, this program will be applied to some other types of operations research problems, and will be used for further study of Linear Programming techniques.

J. Dennis
Electrical Engineering Department

226 D. CIRCULATION OF THE ATMOSPHERE

Progress has been slow during this quarter because the programmers assigned to the problem have been too busy with other work to devote much time to it. The pro- gram for calculating one group of non-homogeneous terms of the differential equations is completed except for a scale factor which is being corrected. A program for calcu- lating a second group of non-homogeneous terms has been written but not completely tested.

A set of programs is being tested for partitioning matrices up to 60 x 60 and obtaining their inverses by using the 30 x 30 Crout method matrix inversion program. The results of the 60 x 60 matrix inversion will be compared with a solution by relaxation to determine which seems most satisfactory for the future work on this problem.

It is expected that during the next quarter computations will be completed for the first case for time $t = 0$ and that the iteration process will be started. It will probably be necessary to obtain additional programming assistance to achieve this.

D.S.Cooley
Geophysics Research Directorate

234 N. ATOMIC INTEGRALS

Editing programs have been developed which add the spherically symmetric component of a perturbing potential (specified by a subroutine in standard form for the appropriate radial integrals) into the central point-charge potential for atomic calculat- ions; another program transforms matrix elements of non-spherical potential components to an orthonormal basis. A standard subroutine is available for the radial integrals arising from a collection of external point charges.

R. K. Nesbet
Solid State and Molecular Theory Group

236 C. TRANSIENT RESPONSE OF AIRCRAFT STRUCTURES TO AERODYNAMIC HEATING

During the past quarter the program previously developed and reported in Summary Report No. 42 has been used to conduct various engineering studies. One set of production runs examined the influence of material properties on the response of equal weight structures of various materials subjected to the same aerodynamic heating condition. With a slight modification it was found possible to use the existing programs to examine the influence of acceleration on thermal stress response. Thus a second set of production runs, the results of which indicate the influence of acceleration on thermal stresses gen- erated were made. A further generalization of the flight history capabilities of the pro- gram is planned. To date only instantaneous change type flight histories and constant and finite level flight accelerations have been dealt with. The work carried out under problem No. 236 is reported in further detail in the following report:

"The Influence of Aerodynamic Heating on the Structural Design of High-Speed Aircraft" (CONFIDENTIAL)
"Part II The Temperature Distribution Problem" ASRL TR 55-2
"Part III An Appraisal of Three Factors which Affect Thermal Stresses" ASRL TR 55-3

The runs made under Problem No. 236 A (an academic application of this problem) were motivated by an investigation of a hypothetical manned fighter aircraft of the immediate future undertaken by Professor R. L. Bisplinghoff. One particular titanium alloy T-Section which was considered characteristic of the heavy skin multi-web structure of the lifting surface was examined in detail. The error introduced by the assumption of instan- taneous achievement of high-speed flight was assessed. The results indicate that the as- sumption of instantaneous achievement of high speed produces a substantial distortion of the time histories. However, for the particular case in question, the instantaneous assumption yields values of the peak stresses which are not excessively conservative. (10% high).

The effectiveness of a thin layer of insulation on the surface skins in attenu- ating aerodynamic heating effects was also examined. The results show that a thin layer of insulation is very effective in relieving aerodynamic heating effects in the particular case examined.

L. Schmit
Aeroelastic and Structures Research Lab.

239 C. GUIDANCE AND CONTROL

Several long runs, of a routine nature, have been made. It is expected that sporadic activity on this problem will continue through the next quarter to supplement investigations made on our own IBM 650 computer.

J. H. Laning, Jr.
Instrumentation Laboratory

241 B,N. STUDY OF TRANSIENTS IN CONTINUOUS DISTILLATION

Additional programs involving the study of the time for a column to change from one equilibrium state to another without control have been written.

S. H. Davis, Jr.
Chemical Engineering Department

245 N. THEORY OF NEUTRON REACTIONS

One case for $\delta = .2$ $\zeta = .15$ has been run. Future plans are to do another case for a different δ and ζ.

E. Campbell
Laboratory for Nuclear Science

253 N. AUGMENTED PLANE WAVE AS APPLIED TO FACE-CENTERED AND BODY-CENTERED IRON

Complete E vs E_o* curves have been obtained for the fcc structure and partial curves for the bcc structure. These curves serve to define critical functions which are fed in to a secular equation to obtain the one-electron energy bands for the two struc- tures of iron under consideration.

*See M. M. Saffren, Quarterly Progress Report, Solid State and Molecular Theory Group, MIT, October 15, 1953, p. 16.

J. H. Wood
Solid State and Molecular Theory Group

257 C. HORIZONTAL STABILIZER ANALYSIS

The problem as originally stated has been extended to include 1) a modified forcing function and 2) more degrees of freedom.

During the past quarter, investigations including unsteady motion forces in a two degree of freedom rigid body analysis (vertical translation and pitching) have been carried on. In the future, the results of the two degree of freedom studies will be in- corporated in studying the response of an airplane in both bending of the fuselage and bending of the horizontal stabilizer. The disturbance to which the airplane is subjected is a special type gust.

E. Criscione
Aeroelastic and Structures Research Lab.

260 N. ELECTRONIC ENERGY OF THE OH MOLECULE

The matrix elements for the binding energy of the OH molecule have been calculated using the integrals described in previous Summary Reports. The secular equations so formed have been solved by use of the matrix diagonalization routine written by F. J. Corbató. This gives a binding energy of about 50% of the experimentally observed value. For a complete description of the progress made on the theoretical aspects of the problem, see Quarterly Progress Report, Solid State and Molecular Theory Group, MIT, January 15, 1952, p 12.

A. Freeman
Solid State and Molecular Theory Group

261 C. FOURIER SYNTHESIS FOR CRYSTAL STRUCTURES

This problem was described in Summary Report No. 42, which covers the second quarter of 1955.

The crystal structures of Pectolite and Wollastonite, both mentioned in former reports, are principally solved by M.J.Buerger, and refinement by means of successive Fourier Syntheses is being carried on.

The structures of Livingstonite and Jamesonite are being investigated by N. Niizeke. The main projection of the former mineral is solved, while work is going on for the latter one.

The interpretation of the minimum function for Diglycinhydrochloride, mentioned in earlier reports, has resulted in a model of the structure in three dimensions, which, at the moment, is being confirmed and refined in the three main projections. This investigation is being carried out by T. Hahn.

M.J.Buerger
Geology Department

262 N. EVALUATION OF TWO-CENTER INTEGRALS

Merryman's program (204-26-171) was used to evaluate 60 hybrid integrals between 1s, 2s, $2p\sigma$ and $2p\pi$ Slater orbitals. The results agreed with the values obtained for the same integrals by Corbato's program up to four decimal places. The Whirlwind time for evaluation of these integrals was about a minute and a half per integral.

H. A. Aghajanian
Solid State and Molecular Theory Group

264 C. OPTIMIZATION OF AIRCRAFT ALTERNATOR REGULATING SYSTEM

As mentioned in the last progress report for this problem, a scheme for minimizing a function of n variables subject to constraints has been worked out. Progress this quarter has consisted of coding this procedure and proving it on the computer. The techniques for minimization and derivative evaluation developed last quarter have been incorporated in the above program.

The work reported here is under the sponsorship of the Air Force, Contract AF(616)3242, "Study of Aircraft Alternating Current Electric Power Generating Systems".

R. R. Brown
Energy Conversion Laboratory

266 A. CALCULATIONS FOR THE MIT REACTOR

This problem was described in Summary Report No. 43.

The program to calculate the reactor behavior for slow but sustained changes

in reactivity is being continued to study different reactors, all of them heavy water moderated.

All programming and coding is being done by M. Troost. The work is being carried out for the Nuclear Reactor Project.

T. Cantwell
Chemical Engineering Department

270 B. CRITICAL MASSES FOR CYLINDRICAL GEOMETRY

A program for computing and plotting radial and longitudinal fluxes (real and adjoint) for the two group cylindrical case has been written and requires only a slight amount of further debugging. Results appear very reasonable, showing marked thermal build-ing in the reflector. A three group sphere program has been written and is working satisfactorily; a three group cylinder requires more revising. Work is being carried out on a Serber-Wilson method and transport method program for a sphere. It is hoped that the latter will prove feasible and will be capable of being extended to a cylinder.

J. R. Powell
Chemical Engineering Department

273 N. ANALYSIS OF AIR SHOWER DATA

Modifications of the program have been tested using new analytical representations of $f(\Lambda)$, the lateral distribution function. The purpose of these modifications is to decrease the time required for computation. These and other modifications will be tested in the future and when a satisfactory program is completed, air shower data will be analyzed on a regular basis.

G. W. Clark
Physics Department

274 N. MULTIPLE SCATTERING OF WAVES FROM A SPATIAL ARRAY OF SPHERICAL SCATTERERS

A second set of results was obtained from the first program (tape 274-17-6).

Reprogramming to cut the time on this problem is now almost completed.

M. Karakashian
Joint Computing Group

275 B. STABILITY OF THIN SHALLOW ELASTIC SHELLS

The problem of the buckling of a uniformly loaded hyperbolic paraboloidal shell has been completed and the case of the buckling of a uniformly compressed cylindrical shell is now being studied. The problem we wish to solve is that of finding the minimum uniform axial compression which causes the buckling of a thin,shallow parabolic cylindrical shell, simply supported on all four edges and subject to certain special boundary conditions on the stresses.

The computations to find the buckling stress for a uniformly loaded hyperbolic paraboloidal shell (as described in Quarterly Progress Report No. 16 of the Committee on Machine Methods of Computation and Numerical Analysis, June, 1955, p 31) were completed during this quarter. A paper on the results of this problem is awaiting publication in the Journal of Mathematics and Physics.

A program to compute the buckling stress for a uniformly compressed cylindrical shell is in the process of being tested.

A. Ralston
Mathematics Department

276 B.N. CORRELATION OF THE MARTENSITIC TRANSFORMATION IN STAINLESS STEEL

When a stainless steel is cooled from an elevated temperature, there is the possibility of a solid state transformation, called the Martensite reaction, occurring. The presence of this Martensite is important in that it has a rather strong influence on the corrosion and mechanical properties of steels.

It had previously been established that the temperature at which Martensite forms (M_s temperature) decreases with increasing alloy content. This investigation was concerned with establishing an imperical equation that would relate M_s temperature with variations in composition of chromium, nickel and carbon.

The M_s temperatures of 49 steels were determined, these steels ranging in composition from 10 to 20 percent chromium, 5 to 13 percent nickel, and .01 to .2 percent carbon, the balance being iron. A linear relationship between composition and M_s temperature was assumed and the constants were determined using the method of least squares on the Whirlwind computer at MIT. The following equation was obtained:

$$M_s = 2160 - 66\,(\%\ Cr) - 102\,(\%\ Ni) - 2620\,(\%\ C)$$

where M_s is in degrees Fahrenheit.

This result appears satisfactory.

F.C.Monkman
F.B.Cuff, Jr.
Metallurgy Department

278 N. ENERGY LEVELS OF DIATOMIC HYDRIDES LiH

The binding energy of the lithium hydride molecule is being calculated for a series of interatomic distances; the procedure has been described in detail in preceding reports. We have completed the calculation for three interatomic distances using six configurations involving the lithium 1s, 2s, and 2p atomic orbitals, and a hydrogen 1s orbital. The spectroscopic vibrational constant as evaluated from our work is in good agreement with the observed value. The binding energy as calculated is about two-thirds of the observed value.

We shall complete these calculations for the other internuclear distances for which the integrals have been evaluated. It is planned to consider a SCF(Roothaan) calculation using programs which are available.

A.M.Karo
Solid State and Molecular Theory Group

285 N. AN AUGMENTED PLANE WAVE METHOD APPLIED TO CHROMIUM

E vs. E_0 curves have been obtained. A routine has been written and is being tested which will display the slopes at the matching radius of the interior and exterior portions of the wave function.

Further details may be found in the Quarterly Progress Reports of the Solid State and Molecular Theory Group, MIT.

M. M.Saffren
Solid State and Molecular Theory Group

288 N. ATOMIC WAVE FUNCTIONS

Calculations are being carried out on He and B. Programs for a one-center calculation on CH_4 have been tested.

R.E.Watson
R.K.Nesbet
Solid State and Molecular Theory Group

293 C. THRUST BEARINGS ANALYSIS

An iterative routine is used to solve the kinemetic equations of thrust bearings. The influence of temperature on load is also under investigation.

Acceptable results have been obtained and are being checked against experimental data. The solution of the problem with different data and the streamlining of the program are being considered.

A.J.Shashaty
Lubrication Laboratory

297 C. DIFFUSION BOUNDARY LAYER

The problem has been described in Summary Report No. 43. During the past quarter, consideration was given to thermal diffusion aspects of the problem, larger injection rates of flow, and the diffusion process for a relatively heavy gas as the injected coolant. Computations are essentially complete but for possible needs to be implied by the analysis now in progress.

J. Baron
Naval Supersonic Laboratory

304 C. RELATIVISTIC ATOMIC WAVE FUNCTIONS

The calculations described in the last Summary Report have been completed for nine atoms. The results of these calculations are being interpreted now and a thorough theoretical report of the physical problems of interest will soon be written up for publication.

I am pleased to state that the results hoped for at the start of the problem have been obtained, and , in fact, more applications of the computational results have been found than were anticipated. The problem is now terminated.

C. L. Schwartz
Physics Department

307 C. SUPERSONIC NOZZLE DESIGN

During the present quarter one additional run was completed in order to check the results in the low supersonic range.

J. Baron
Naval Supersonic Laboratory

309 B,N. PURE AND IMPURE POTASSIUM CHLORIDE CRYSTAL

The first phase of my calculation is nearing completion. This phase consists of obtaining the functions representing an expansion of certain K^+ and Cl^- atomic orbitals and associated charge densities in spherical harmonics about a center displaced from the nucleus. Most of the required functions have now been generated using programs written by Corbató and myself, and they are available on paper tape.

The second phase of my calculation will be the calculation of integrals and matrix elements using these functions. The programs for this work are now ready and tested, and the coming quarter should see the completion of this second phase. The final phase will involve the solution of secular equations, as described in my original report.

L.P.Howland
Solid State and Molecular Theory Group

310 C. TRAJECTORY CALCULATIONS FOR A ROCKET DURING POWERED FLIGHT

The following corrections have been made to the equations set forth in the last report:

$$I_{sp_i} = \frac{F_i}{m_i g_s} = \frac{C_1 \dfrac{\overline{p}_c}{p^*_{s1}} + C_2 \dfrac{p_i^*}{p^*_{s1}}}{C_3 C_4 \left[\dfrac{\overline{p}_c}{p^*_{s1}} + \dfrac{p_i^*}{p^*_{s1}}\right]}$$

where

$$C_1 = \left\{\eta_\lambda \eta_F \sqrt{\left[\frac{2k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\right]\left[1 - \left(\frac{p_e}{p_c}\right)^{\frac{k-1}{k}}\right]} + \frac{p_e}{p_c}\frac{A_e}{A_t}\right\}$$

$$C_2 = \left\{\eta_\lambda \eta_F \sqrt{\left[\frac{2k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\right]\left[1 - \left(\frac{p_e}{p_c}\right)^{\frac{k-1}{k}}\right]} + \left(\frac{p_e}{p_c} - 1\right)\frac{A_e}{A_t}\right\}$$

$$C_3 = \sqrt{k\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}$$

$$C_4 = \eta_{\dot{m}} \sqrt{\frac{g_s}{RT_c}}$$

During this quarter the complete program has been written and assembled. Initial trials of the complete program will be made early in the next quarter. All subroutines needed at this time have been written and checked out on Whirlwind.

The work is sponsored by the U.S. Air Force under Contract AF33(616)-2392.

Corrections to Values of Functions Listed Under Definition of Symbols in Summary Report No. 43

E earth's gravitational factor = 1.4070362×10^{-6} ft^3/sec^2

g_R absolute gravity at earth's surface = 32.24833 ft/sec^2

V_s velocity of satellite at earth's surface = $\sqrt{g_R R} = 25.953930 \times 10^3$ ft/sec

J.S.Prigge
Aerophysics Research Group

311 N. SOLITARY WAVE GENERATING CAM

This is the terminating report on problem 311. The problem consisted of the solution of a series of algebraic equations which describe the loci of the tool center of the numerically-controlled milling-machine of the Servomechanisms Laboratory, MIT, for cutting steel plate for cams to control the wave generator of the Hydrodynamics Laboratory, MIT. These cams are used in the Solitary Wave research project.

The form of the cam is that of two half-length sine waves superimposed on a neutral circle. The ratios of the two lengths vary from 1:15 to 1:1 for a series of eight cams. The amplitudes of all the waves are equal. The input equations to the Whirlwind computer include corrective terms to account for the angle between the radius of the cam to the point of cut and the radius of the cutting tool to the point of cut.

The output of the Whirlwind computer is binary-punched tape to be fed directly into the input of the milling-machine control. The subroutine developed by John Runyon and Arnold Siegel was used for the conversion to milling-machine coordinates. The cuts of the milling-machine are by small rectangular coordinate increments.

The programmers for this problem were Donald C. Taylor and John G. Housley of the Hydrodynamics Laboratory staff, attached to the Solitary Wave project. Technical assistance in programming was given by Arnold Siegel of the Digital Computer Laboratory staff.

A total Whirlwind time of 122.1 minutes has been expended on this problem. The cams have been cut, and are in successful operation on the Hydrodynamics Laboratory wave tank. A description of the cam design, including the Whirlwind programming, is included in the Master of Science thesis (1956) of Donald C. Taylor.

J.G.Housley
Hydrodynamics Laboratory

312 L. ERROR ANALYSIS

A program has been written and is working which provides many sets of answers for different input conditions by cycling each of the input parameters through up to ten values. The program is designed to read-in successive parameter tapes automatically, so that different modes of the cycling may be selected. Besides the answers, the program also prints-out the input conditions. A detailed description of the capabilities of this program is contained in a memorandum of December 14,1955 from R. K. Bennett

to the members of Group 312 of Lincoln Laboratory.

The results of the runs to date are being used at Lincoln Laboratory to aid in the design of equipment. They have proved very valuable in the understanding of the behavior of the errors under analysis.

This program will probably be called upon from time to time to give additional information as the present results are digested. Also, another program will probably be written in the near future which solves a more general problem.

R. K. Bennett
Lincoln Laboratory

314 C. FACTORING OF HIGH ORDER POLYNOMIALS

In any dynamic analysis of a complex system there usually results a set of coupled differential equations of "motion". This word motion is used in its broadest sense to include electrodynamics as well as mechanics. When permissible, this set of coupled differential equations are linearized and converted to algebra with the aid of the Laplace Transform resulting in the boundary value problem $[A - \lambda] = [B]$. To apply the current systems analysis techniques (frequency analysis and root locus) this matrix of coupled equations is operated on by expanding the determinants of its coefficients resulting in the performance functions of the system. This determinant expansion may be performed on the Whirlwind I or IBM computers. These $PF_{(in,out)}$ are of the form of a polynomial ratio and both the numerator and denominator must be factored to perform the frequency analysis of the system and to determine the modes of vibration.

This polynomial factorization program performs the necessary factorization on characteristic values of the problem. To proceed to the detailed studies of the root variation of closed loop systems, these performance functions may be manipulated to form additional polynomials. Techniques are available to determine these roots by systematic trial and error for various systems gains, however, with the aid of this Hitchcock polynomial factorization roots can be obtained directly. To date, 10th order polynomials have been factored in about 3 minutes, including complex roots.*

The program has a built-in check and an accuracy requirement which may have to be adjusted if roots occur too close. (After 50 trials the program stops indicating too high an accuracy is required for the given problem). There is a provision in the program to insert 1st approximations to the roots, if known, to assist in convergence.

* Ed. Note: A description of the routines available on Whirlwind I for Hitchcock's Method will be found in Digital Computer Laboratory Memorandum DCL-94-1

V. W. Howard
Aeronautical Engineering Department

317 C. EXTRACTION OF STABILITY DERIVATIVES FROM FLIGHT TEST DATA

This research program is concerned with the investigation of analytical techniques for the extraction of system parameters, such as the stability derivatives, of high speed aircraft and missiles from flight test data. A method devised by M. Shinbrot and reported in Ref. 1 is being investigated by applying it to a set of simulated responses of a high speed fighter type aircraft. The objects of this phase of the program are to determine the problems which are involved in the extraction of stability derivatives from a complex nonlinear system; to determine how accurately the stability derivatives may be calculated assuming that the form of the equations of motion, the inertial properties of the aircraft and the forcing inputs and dynamic responses of the aircraft are known exactly; and to determine with what precision and accuracy the responses and aircraft parameters have to be known in order to obtain reasonably accurate answers.

The application of Shinbrot's method involves the formation of integrals of the form:

$$\int_0^T x(t)y(t)\,dt$$

where $x(t)$ is a dynamic response or input and $y(t)$ is a known "method" function whose first n derivatives exist and whose value and the value of the first n-1 derivatives are zero at $t = 0$ and $t = T$, n being the order of the equation being considered. The method function which Shinbrot suggests is of the form $\sin^n \omega t$. The reasons for the limitations placed upon the "method" function can be readily seen by forming the integral of a term in the equation which is a derivative of a known response and integrating by parts until no derivatives of the response remain.

Thus:

$$\int_0^T y(t)\,\frac{d^n x(t)}{dt^n}\,dt = y\,\frac{d^{n-1}x}{dt^{n-1}}\Bigg|_0^T - \frac{dy}{dt}\,\frac{d^{n-2}x}{dt^{n-2}}\Bigg|_0^T + \cdots + (-1)^{n+1}\,\frac{d^{n-1}y}{dt^{n-1}}\,x\Bigg|_0^T + (-1)^n \int_0^T x\,\frac{d^n y}{dt^n}\,dt$$

It can be readily seen that if the method function has the properties defined above all terms on the right except the final integral are zero and the integral exists.

Using Filon's method (see appendix of Ref. 1) equivalent summations can be determined for the integrals having the relationship:

$$\int_0^T x(t)\,y(t)\,dt \approx \Delta t \sum_0^T \Gamma_n(y)\,x_n(t)$$

By varying the frequency ω in the method function any number of linear algebraic equations composed of such summations may be formed, which can then be solved after first using the method of least squares.

During the past quarter a program of the Z-force equation has been written for CS II. Results of this program to date indicate that an investigation of the effect of the number and spacing of frequencies used in the method function must be made. A program to accomplish this is now being written. Programs are planned for the near future which will indicate the effect of imperfect data upon the effect of imperfect data upon the effectivness of the method.

References

1. Shinbrot, M. "An Analysis of Linear and Nonlinear Dynamical Systems from Transient Response Data." NACA TN3288. December 1954.
2. Dynamic Stability Project, "First Progress Report on the Dynamic Stability of Aircraft and Missiles." Aerophysics Research Group, MIT. 15 August 1955. (CONFIDENTIAL)
3. Dynamic Stability Project, "Second Progress Report on the Dynamic Stability of Aircraft and Missiles." Aerophysics Research Group, MIT. 15 November 1955.

L. L. Mazzola
Aerophysics Research Group

322 B. MAXIMUM SIZES OF BUBBLES

The initial temperature distribution and bubble size will be assumed. The governing equation is that for heat conduction in moving medium in spherical coordinates. This is to be solved for a number of initial conditions and variation in properties.

An equation in finite difference form is to be solved, with bubble radius as a function of time wanted in each case. About 40 different combinations of temperature and properties will be wanted, each involving going through the equations about 1000 times.

The problem has been formulated and programmed during this past quarter. In the future, the program will be debugged and run.

P. Griffith
Mechanical Engineering Department

323 N. ANALYSIS OF CLOUD CHAMBER PHOTOGRAPHS

Three stereoscopic photographs are taken of each multiplate cloud chamber event, and from these the various particle tracks must be reconstructed in space. One can then determine the angles between tracks, the amount of plate material a track has passed through, the true ionization of a track from its apparent ionization, etc.

On each photograph (left = L, center = C, right = R), measurements are made of the apparent slope of each track (ϕ_L, ϕ_C, ϕ_R - restricted to -90^o to $+90^o$) and of the vertical (tan γ, restricted to $-.15$ to $+.15$) and horizontal (X'_L, X'_C, X'_R - restricted to -30 cm to + 30 cm) components of one point on that segment. In addition, one measures a linear scale factor in each photograph (s_L, s_C, s_R - restricted to 3 to 15) , and the fraction (f_i, where i = plate number, and $0 > f > 1$) of a plate which each segment appears to traverse. Since there are three solutions to the problem, weights (w_1, w_2 and w_3) are assigned to each determination. By suitable coordinate transformations involving elementary algebra, these measurements can then be converted to spatial angles and distances. Some steps in the procedure are given below.

True distances: $X_L \;\; \frac{10\,X'_L}{s_L}$, $X_C \;\; \frac{10\,X'_C}{s_C}$, $X_R \;\; \frac{10\,X'_R}{s_R}$

$$\tan\phi'_L = \frac{X_L + 87.20}{157.37} \;;\quad \tan\phi'_C = \frac{X_C}{157.37} \;;\quad \tan\phi'_R = \frac{X_R - 87.20}{157.37}$$

$\overline{X}_L = .02052 \tan\theta'_L \left(\frac{1}{\sqrt{1.56\tan^2\theta'_L + 2.56}} - 1\right)$, and analogous expressions involving θ'_C and θ'_R

$$\tan\theta_L = \tan\theta'_L + \overline{X}_L \;;\quad \tan\theta_C = \tan\theta'_C + \overline{X}_C \;;\quad \tan\theta_R = \tan\theta'_R - \overline{X}_R$$

$$y_1 = \frac{\tan\phi_L + \tan\phi_C}{\tan\theta_L + \tan\theta_C} \;;\quad y_2 = \frac{\tan\phi_C + \tan\phi_R}{\tan\theta_C + \tan\theta_R} \;;\quad y_3 \;\; \frac{\tan\phi_C + \tan\phi_R}{\tan\theta_C + \tan\theta_R}$$

$$X_1 = y_1 \tan\theta_L - \tan\phi_L \;;\quad X_2 = y_2 \tan\theta_C - \tan\phi_C \;;\quad X_3 = y_3 \tan\theta_L \tan\phi_L$$

$\tan\beta_n = \frac{X_n}{1.08\,y_n}$; $\rho_n^{\,2} = X_n^{\,2} + y_n^{\,2}$, where n = 1, 2, 3

$$\tan\alpha'_n = \frac{\rho_n}{1 - \rho_n \tan\gamma \cos\beta_n} \;;\quad \tan^2\alpha_n \;\; \tan^2\alpha_n^{1}\,(1 - .1427\cos^2\beta_n{}')$$

Depth in chamber = d = $\frac{157.37\,|X_L - X_R|}{174.40 - |X_L - X_R|}$, or if X_L or X_R = 0, d = $\frac{157.37\,|X_C - X_{L\,R}|}{87.20 - |X_C - X_{L\,R}|}$

$$\frac{\text{True ionization}}{\text{Apparent ionization}} \;\; \cos\alpha_n \Big[(1 + \tan\gamma \sin\alpha_n \cos\beta_n)^2 +$$

$$\tan^2\alpha_n \sin^2\beta_n\,(1 + \tan\theta_n \cos\beta_n)^2\Big]^{\frac{1}{2}} \Big[1 + .00271\,(d-11)\Big] = I_n$$

$$\overline{I} = \frac{W_1 I_1 + W_2 I_2 + W_3 I_3}{W_1 + W_2 + W_3}$$

Range traversed in plate i = $R_{ni} = \dfrac{\left[f_i\left(1 + \frac{d}{157.37}\right) - \frac{d}{2(157.37)} \right] 11.08}{\cos \gamma \cos \alpha_n (1 + \tan \gamma \tan \alpha_n \cos \beta_n)}$

$$\overline{R}_1 = \frac{W_1R_1 + W_2R_2 + W_3R_3}{W_1 + W_2 + W_3}$$

Later on, additional equations may be added to the program to give the angle between track segments, angles between decay planes, etc.

A program has been written and tested for the determination of the range and ionization of particles in a multiplate cloud chamber. Approximately fifty events have been analyzed with this program.

It is planned that some further use will be made of this program, and in addition, that parts will be added to the program to make it applicable to other cloud chambers, and to more complex analysis problems.

D. O. Caldwell
F. Abrams
Physics Department

325 B. DIFFUSION EQUATION

In this problem an attempt is being made to develop a suitable method for the numerical solution of the partial differential equation:

$$\frac{\partial}{\partial x}\left[K(x) \ \frac{\partial u}{\partial x}\right] = \frac{\partial u}{\partial t}$$

with the boundary conditions

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = \frac{-Q}{K(0)} \qquad \text{and} \qquad \left.\frac{\partial u}{\partial x}\right|_{x=L} = 0$$

with $U(x,0) = 0$ and one $K(x)$ $\quad \dfrac{ac}{\rho}\left[\dfrac{(x+b)}{c+a(x+b)}\right]$

where a, b, c, ρ, and Q are positive constants. The reason that this equation presents a special problem is that the left hand boundary condition is quite large since K(x) is known to have a small negative root.

The conventional difference equation approach does not converge except for an exceedingly small ratio of $\dfrac{\Delta t}{(\Delta x)^2}$. Since the terminating values of x and t are quite large in the physical problem expressed by this equation, it is necessary to find some method by which this problem can be evaluated. The solution of the above equation can be found in terms of the eigen functions of the general Sturm-Liouville Problem, and although the steady-state part of the solution is readily available, it is not always possible to find the transient or time varying portion of the solution. The first part of the method currently being tested takes advantage of the fact that the boundary conditions are satisfied by the steady-state solution. Thus, the solution of the desired problem can be formulated as a sum of the steady-state, which is known, plus the transient solution which is found from the evaluation of the original equation with "insulated" boundary conditions

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = \left.\frac{\partial u}{\partial x}\right|_{x=L} = 0$$

with the new initial distribution found from the steady-state portion. It is expected that this method should give improved accuracy near the origin where other methods fail. [1.]

The need for the solution at a few specific points has prompted an attempt to find a method which does not require the solution of every point in the xt plane as does the "conventional " technique. A generalization of the methods of ordinary differential equations using the Obreschkoff formula [2.] in a manner similar to that used by Milne[3.] for the solution of Bessel's equation, produces a step-by-step tabulation using only the points on a diagonal line of slope, $\dfrac{\Delta t}{\Delta x}$. The tabulation then appears in the following form.

(See Reference 4. for formulas used to obtain the Tabular Values.)

	x	t	U	$\partial u/\partial t$	$\partial u/\partial x$	$\partial^2 u/\partial x^2$	$\partial^2 u/\partial x \partial t$	$\partial^2 u/\partial t^2$
Predict	h	k	x	x	x	x	x	x
Correct	h	k	x	x	x	x	x	x

A program using this predictor - corrector scheme has been found to compute about 120 points a minute and could compute more if WWI code were used.

At this time the conclusions are not complete, but it appears that the first step is of use in problems with discontinuous boundary conditions, while the second is a method which can be applied in general to any partial differential equation. The second method must still be investigated for "machine" stability, but is known to have a truncation error in the order of $F^{(VI)}$ where $F^{(VI)}$ is the sixth order term in the two dimensional Taylor Series. For a further discussion of this method and the derivation of the formulas used see reference 4.

The following notation has been used:

$\delta^{(N)}$ = Nth order central difference of $U(x,t)$

$U_x = \partial u/\partial x\ (x,t)$

$U_t = \partial u/\partial t(x,t)$

$U_{t\oplus}$, U_{to}, $U_{t\ominus}$ = $\partial u/\partial t(x+\Delta x, t+\Delta t)$, $\partial u/\partial t(x,t)$ and $\partial u/\partial t(x-\Delta x, t-\Delta x)$ respectively

$h = \Delta x$

$k = \Delta t$

$F^N_{\oplus}$ The Operator $\left(h\, \partial u/\partial x(x+h, t+k) \quad k\, \partial u/\partial t(x+h, t+k)\right)^N$

$\square F^N$ truncation error of order N.

References

1. Phipps, Philip, Machine Solution of the Diffusion Equation; Thesis, MIT(1955).
2. Beck, von Eugen, Zwei Anwendugen der Obreschkoffscen Formel, Z. Angew. Math. Mech., 30, (3) 84-93 (1950).
3. Milne, W.E., A Note on the Numerical Integration of Differential Equations, J. Res. Nat. Bur. Stand. 43, 537-542 (1949).
4. Pope, D.L., A Digital Computer Method for the Solution of the Diffusion Equation; Thesis, MIT (1956).

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Digital Computer Laboratory

327 L. PREDICTION ANALYSIS

This problem is a simulation and prediction study, utilizing Monte Carlo techniques. It is the same as Problem 312 (see Quarterly Report No. 43 for a description of Problem 312) except that this is a Prediction Analysis instead of an Error Analysis.

During this quarter a program has been prepared and is being tested.

H. B. Dwight
Lincoln Laboratory

328 B. BURIED ELASTIC WAVE SOURCE

Theoretical studies of the propagation of seismic pulses are necessary for a better understanding of earthquake phenomena and earth structure. The mathematical difficulties involved in such studies have limited the number of earth models to those which are very simple. We have chosen the most simple model for this problem. It is an unlayered elastic half-space, sometimes called a semi-infinite elastic medium. Below the surface at a depth $\underline{h}$ is a line source parallel to the boundary of the half-space. One may imagine the line source to be a cylinder of very small radius. The cylinder pulsates and rotates about its axis, thereby generating elastic waves. These waves are of two types, compressional and distortional. The compressional waves are called P-waves (pressure waves) and the distortional waves are called S-waves (shear waves). The S-waves are polarized and may be separated into SV-(vertical) waves and SH-(horizontal) waves. We take the boundary surface to be horizontal. When a P-or SV-wave strikes the surface it is reflected and diffracted and a surface wave, called a Rayleigh wave, is generated. The main purpose of this problem is the investigation of the generation and propagation of Rayleigh waves.

The problem stated above is a classic one. It was first investigated by Lamb (1904) who obtained the first theoretical seismogram by approximate methods. Lamb's work was extended by Nakano (1925) who investigated the effects near the source by stationary phase integration, and distant effects by contour integration. Nakano neglected a branch line integral, however, and obtained some anomalous results. The next advancement was made by Lapwook (1949). His treatment is the best available in the literature at present. Garvin (1954) has apparently solved the problem exactly, but has published only an abstract of his work. The problem can be solved exactly, which we have done, and the remaining step is to evaluate the resultant algebraic expressions as functions of time for surface points at various distances from the source.

The horizontal and vertical displacement at twelve surface points has been found for a buried cylindrical source of P-waves. The time variation of the source is a unit step function. It has been verified that the Rayleigh wave is not attenuated along the surface, and that the S-wave arrival causes only a change in slope on the theoretical seismogram.

It is proposed to extend this work to S-wave sources and three-dimensional problems.

<u>References</u>

Garvin, W.W., 1954, Exact transient solution of the buried line source problem: GEOPHYSICS, v. 19, p 357.

Lamb, H., 1904, On the propagation of tremors over the surface of an elastic solid: Phil. Trans. Roy. Soc. London A, V. 203, p 1-43.

Lapwood, E.R., 1949, The disturbance due to a line source in a semi-infinite elastic medium: Phil. Trans. Roy. Soc. London A, v. 242, p 63-100.

Nakano, H., 1925, On the propagation of tremors over the surface of an elastic solid: Jap. Jour. Astr. and Geoph., v. 2, p 233-326.

J. F. Gilbert
S. Simpson
Geology and Geophysics Group

330 C. POSTFAILURE RESPONSE OF AIRCRAFT STRUCTURES SUBJECTED TO BLAST LOADING

The objective of this study, which was initiated by R. D'Amato of the MIT Aeroelastic and Structures Research Laboratory, is to predict the time history of the response of aircraft structural components in the "postfailure" range when subjected to blast or gust type loading. As used here, "postfailure" refers to the condition of a structure after some portion of it has experienced a marked change in structural characteristics caused by the applied load reaching a certain critical value.

When the critical value of load is reached at some location on a structure, the restoring force at this location is, in general, a nonlinear function of deflection and, as a result, the equations governing the dynamic response of the structure are also non-linear. Before a postfailure dynamic analysis is possible there must be available a knowledge of the restoring force as a function of deflection after failure. A series of static tests of some typical aircraft structural components conducted by members of the MIT Aeroelastic and Structures Research Laboratory has indicated the general behavior of the postfailure structural restoring force which can be expected from many present day aircraft structures. The results

of these static tests have been used to guide the formulation of the dynamic problem.

It is proposed to obtain the desired postfailure response by an approximate analytical solution. However, since accuracy of the analytical soltuion is unknown, a finite difference solution of the postfailure response of a nonuniform cantilever beam is to be programmed on WWI to estimate the reliability of the analytical solution.

The nonuniform cantilever beam will be represented by a dynamic model consisting of four mass points interconnected by four uniform weightless rods. The blast loading on the beam will be represented by four concentrated loads, one at each mass point, which will vary linearly with time from an initial level at time $t = 0$ to zero at time $t = 5T$, where T is the first natural period of the beam. After failure an additional coordinate or degree of freedom will be introduced. This coordinate, θ, is defined as the angle between the tangents to the deflection curve at the critical station. (Note: The failure is assumed to occur at a mathematical point.) The restoring bending moment as a function of θ is assumed to be piecewise linear, varying from a maximum level equal to the critical or failure bending moment at $\theta = 0$ to zero at $\theta = \pi/2$ radians.

The equations governing the dynamic response of the beam can be written before failure as follows:

$$q_i = \sum_{j=1}^{4} C_{ij} (F_j - m_j \ddot{q}_j) \qquad i = 1,2,3,4 \tag{1}$$

After failure the equations become:

$$q_i = \sum_{j=1}^{4} C'_{ij}(\theta) \left[F_j - m_j \ddot{q}_j\right] - C'_{i3}(\theta)\, a_1 m_3 \ddot{\theta} - C'_{i4}(\theta) a_2 m_4 \ddot{\theta} \qquad i = 1,2,3,4 \tag{2}$$

$$A_r - K_r\theta = a_1 \left[F_3 - m_3\ddot{q}_3 - a_1 m_3 \ddot{\theta}\right] + a_2 \left[F_4 - m_4\ddot{q}_4 - a_2 m_4 \ddot{\theta}\right] \tag{3}$$

where

m_j	= jth mass
C_{ij}	= deflection of the ith mass due to a unit load at the jth mass
F_j	= external force applied at the jth mass
q_i	= displacement of the ith mass
$\ddot{q}_i$	$= d^2/dt^2\ (q_i)$
θ	= angle between the tangents to the deflection curve at the failure point
$\ddot{\theta}$	$= d^2/dt^2\ (\theta)$
$C'_{ij}(\theta)$	= deflection of the ith mass due to a unit load at the jth mass as a function of θ
A_r, a_1, a_2	= constants

Finite difference approximations are made for the second derivatives as follows:

$$\ddot{q}_n = \frac{q_{n+1} - 2q_n + q_{n-1}}{\Delta t^2}$$

$$\ddot{\theta}_n = \frac{\theta_{n+1} - 2\theta_n + \theta_{n-1}}{\Delta t^2}$$

and the equations are reduced to matrix form for machine computation. It will be noticed that after failure for each time interval, a new set of influence coefficients must be computed and the resulting five equations solved simultaneously.

During this past quarter, the numerical formulation of the problem was changed slightly by substituting the following finite difference approximations for those originally used:

$$\dot{q}_n = \dot{q}_{n-1} + \frac{\Delta}{2} (\ddot{q}_n + \ddot{q}_{n-1})$$

$$q_n = q_{n-1} + \Delta \dot{q}_{n-1} + \frac{\Delta^2}{4} (\ddot{q}_n \quad \ddot{q}_{n-1})$$

During this quarter the program for this numerical solution was completed and submitted. The results have not been received at the time of writing this report.

R. D'Amato
Aeroelastic and Structures Research Lab.

331 D. MATRIX ITERATION

This problem is an extension of Problem 168, initiated by N.P.Hobbs of the Aeroelastic and Structures Research Laboratory at MIT. Hobbs' problem is described in Summary Report No. 37, which covers the first quarter of 1954.

Progress on problem 331 has not been entirely successful to date. The necessary revisions are still being worked out.

V. W. Howard
Aeroelastic and Structures Research Lab.

332 C. GAME-THEORY OPTIMIZATION OF AN INTERCEPTOR CONTROL SYSTEM

An interceptor control system consists of a device for tracking targets to be intercepted and a director which uses the tracking data to compute steering commands for interceptors.

An optimum director can be found with the aid of synthesis procedures used in the design of time-varying linear filters. Such procedures require a knowledge of the statistical properties of the following quantities:

1. Target maneuvers.
2. Noise in the target tracker.
3. Interceptor dynamics.

The designer of an interceptor control system usually has adequate knowledge of the last two of these quantities. However, the probabilities of various target maneuvers sometimes are determined by a hostile force and are therefore unknown to the designer. Indeed, since future events are concerned, any statement of their probabilities would be of questionable value.

In cases where the target is enemy-controlled, the mathematical model of the target-tracker-director-interceptor system is a special case of the game-theory model of J. von Neumann. In particular, if the target is blind (i.e., cannot see the interceptor), then the game can be regarded as a two-person game in which each person has one move, and each move involves a choice from an infinite set of alternatives. The enemy's move consists of choosing a particular form of evasive maneuver, and the defender's move consists of choosing a particular form of director. The strategy of a player is the set of probabilities of choosing particular alternatives, and a pure strategy is one in which a specific alternative is chosen with a unit probability. The game-theory problem is the problem of finding the set of strategies for each player which guarantees to him that his maximum loss (with respect to all enemy strategies) is minimized. Strategies in this set are called optimal.

In order to find a solution of the interception problem, it was first necessary to add some artificial constraints which reduced the game to finite form. In the form chosen, the number of alternatives of each player is limited to forty-six. The second step

in solving the game consists of computing the entries of the 46 x 46 payoff matrix of the reduced game. Each entry in the payoff matrix is the mean-squared miss distance produced by a different combination of pure strategies of the two players.

A program for computing the payoff matrix is being run presently on Whirlwind I. Each of the forty-six columns of the payoff matrix requires that a 19 x 19 matrix be inverted. This is being accomplished with the aid of the Crout's Method Library Subroutine. Because the output tape is not long enough to store the 46^2 numbers in the payoff matrix, it is not possible to print out the results of the computation. Instead, the results will be photographed on microfilm from a cathod-ray tube display. Approximately 27 frames will be required.

The third step in solving the interception game will be to examine the payoff matrix. If the game has a solution consisting of pure strategies for both players, then the matrix contains a saddle point which can be found by inspection. If no saddle point is found, it may be necessary to carry out further computations to determine a set of optimal mixed strategies. Iteration procedures have been developed for this purpose.

This work is being carried out by G. R. Welti, with the help of G. W. LeCompte, and Miss K. Kavanagh of the Dynamic Analysis and Control Laboratory at MIT.

G. R. Welti
Dynamic Analysis and Control Laboratory

NOTE:

Reports on the following problems, done by members of the Machine Methods of Computation Group, may be found in Section 2.2 of Part I of this report.

122 N.	COULOMB WAVE FUNCTIONS	A. Temkin Z. Fried A. Tubis
172 B,N.	ENERGY BANDS IN GRAPHITE	F.J.Corbato
203 B,N.	RESPONSE OF A MULTI-STORY FRAME BUILDING UNDER DYNAMIC LOADING	R.G.Gray
217 N.	ATOMIC WAVE FUNCTIONS AND ENERGIES	A. Tubis
231 B,N.	REACTOR RUNAWAY PREVENTION	M. Troost
235 B,N.	EIGENVALUES IN A SPHEROIDAL WELL	J.L.Uretsky
319 B,N.	ZERO ENERGY SCATTERING CROSS-SECTION OF A SPHEROIDAL WELL	J.L.Uretsky
321 B,N.	SPHEROIDAL SQUARE WELL EIGENVALUES	J.L.Uretsky
329 N.	FIRST APPROXIMATION SOLUTION ON ORE BODY	N.F.Ness

1. SYSTEMS ENGINEERING

Computer Reliability 23 September 1955 to 31 December 1955

Total computer operating time	2222	hours
Total lost time	51	hours
Percentage operating time usable	97.7%	
Average uninterrupted operating time between failures	20.6	hours
Failure incidents per 24-hour day	1.17	
Average lost time per incident	28.3	minutes
Average preventive maintenance time per day	1.75	hours

2. PUBLICATIONS

Project Whirlwind technical reports and memoranda are routinely distributed to only a restricted group known to have a particular interest in the Project and to ASTIA (Armed Services Technical Information Agency) Document Service Center, Knott Building, Dayton, Ohio. Requests for copies of individual reports should be made to ASTIA.

The following is a list of memoranda published by the Scientific and Engineering Computations Group during the past quarter.

No.	Title	Author
DCL-105	Tic Tac Toe Playing Routine	A. Zabludowsky
DCL-113	Generalized NIM Playing Routine	A. Zabludowsky
DCL-115	Function Evaluation Subroutines for Real-Time Parallel Operation	M. Watkins
DCL-117	Flad Post-Mortems and ff Tapes	M. Weinstein

3. VISITORS

Tours of the WWI installation include a showing of the film "Making Electrons Count", a computer demonstration, and an informal discussion of the major computer components. During the past quarter, 8 groups totalling 201 people visited the computer installation. Included in these groups were:

October 14	MIT Course in "Introduction to Digital Computers - Coding and Logic"
November 14	MIT Course in "Methods of Engineering Analysis"
November 17	Northeastern University Course in "Digital Computer Coding and Logic"
November 21	Eastern Simulation Council
December 8	Weeks Junior High School
December 13	Choate School
December 13	MIT Course in "Machine-Aided Analysis"
December 14	One-week MIT Course in "Management and Electronic Data Processing"

The procedure of holding Open House at the Digital Computer Laboratory on the first Tuesday of each month has continued. An additional tour was held on the 3rd Tuesday of December and will be held each month as long as there is sufficient interest. A total of 129 people attended the 4 Open House tours during this quarter, representing members and friends of the MIT community, Harvard University, John Hancock Life Insurance Company, Rand Corporation, Stone and Webster Engineering Corporation, McGraw-Hill Publishing Company, Automatic Electric Company, Sylvania Electric Corporation, Massachusetts Department of Commerce, Kyoto University in Japan, Radcliffe College, and the Columbia Broadcasting System.

During the last quarter there were also 84 individuals who made brief tours of the computer installation at different times. Represented by these individuals were: the U.S.Air Force, South African Mutual, North American Aviation, Technische Hogeschool in Delft, Holland, the U.S.Navy, Bell Telephone Laboratories, Dean Witte and Company, General Precision Laboratory, New England Confectionery Company, Radio Corporation of America, Ramo-Wooldridge Company, United Shoe Machinery Company, Sperry Rand Corporation, Datamatic Corporation, National Cash Register Company, Lockheed, Convair, Remington Rand, General Electric Corporation, Electric Boat Division, Financial Publishing Company and Indiana University.

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MACHINE METHODS OF COMPUTATION AND NUMERICAL ANALYSIS

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Harvey Fields	Physics
Zoltan Fried	Physics
Raymond F. Stora	Physics
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Jukka A. Lehtinen	Mechanical Engineering
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Marius Troost	Chemical Engineering
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PROJECT WHIRLWIND

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