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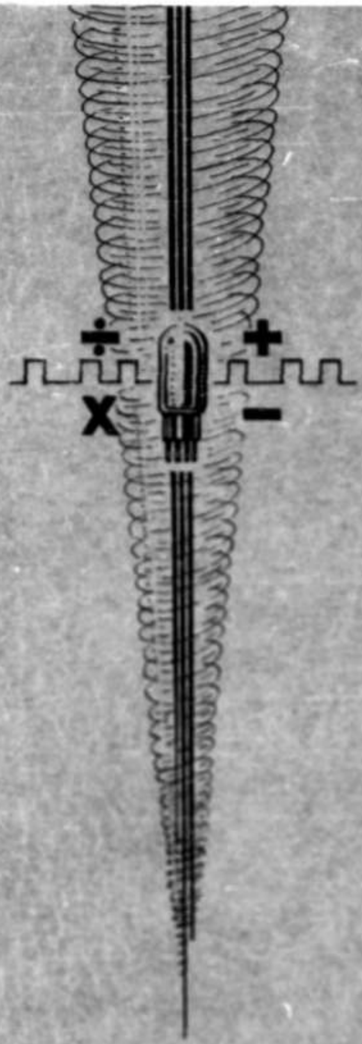

SERVOMECHANISMS LABORATORY

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

project

WHIRLWIND

SUMMARY REPORT NO. 1

APRIL 1946

COPY 80

SPECIAL DEVICES DIVISION
OFFICE OF RESEARCH AND INVENTIONS
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PROJECT
WHIRLWIND
(DEVICE RF-12)

SUMMARY REPORT NO. 1
April 1946

For the

SPECIAL DEVICES DIVISION
OFFICE OF RESEARCH AND INVENTIONS
NAVY DEPARTMENT

Issued Under the Provisions of
Letter of Intent for Contract NOa(s)7082

By the

SERVOMECHANISMS LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Cambridge, Massachusetts

Research on Project WHIRLWIND
Includes Contributions by the
Department of Aeronautical Engineering
Department of Electrical Engineering
Department of Mathematics
Department of Physics

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FORWARD

Project WHIRLWIND reports will be issued monthly or bi-monthly from the Servomechanisms Laboratory at the Massachusetts Institute of Technology as available material justifies. They are prepared for the distribution, to authorized persons, of information on high-speed digital computation.

In some cases ideas will be suggested and circuits proposed which have not been proven in practice. As tests are made, results will be recorded in later issues of the WHIRLWIND.

The most rapid and efficient progress in new aids to mathematical computation can be achieved only through cooperation and interchange of information between scientific groups engaged in this field. Comments on the ideas presented in these reports are requested from other groups as well as information on results of work at other laboratories. Suitable letters and papers concerning digital computation will, with permission of the author, be included from time to time. Where information from sources outside this laboratory is of a non-classified nature, the publication will, at the request of the author, so indicate.

Correspondence can be addressed to Project WHIRLWIND, Servomechanisms Laboratory, Building 32, Massachusetts Institute of Technology, Cambridge, Massachusetts.

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REVIEW

Project WHIRLWIND, sponsored at the Massachusetts Institute of Technology by the Special Devices Division of the Office of Research and Inventions, U. S. Navy, is a program of research, investigation, and development in the field of high-speed electronic digital computation. As outlined in Summary Report No. 1, this electronic computer is specifically aimed at the solution of aircraft stability and control problems but will also provide for the solution of certain other classes of scientific problems as well as the field of non-linear ordinary differential equations. Completion of a final computer will require several years. Summary reports will be issued from time to time indicating the latest results and trends.

When the project was organized in December 1944 plans were based on the use of analogue computation. It became apparent that analogue methods would lead to excessive complexity and that other computing techniques should be studied. High-speed electronic digital computation is now being investigated in an effort to develop components for application to the problems of immediate interest.

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SUMMARY REPORT ON CONTRACTS NOa(s)5216 AND NOa(s)7082

PART A - INTRODUCTION

1. SUMMARY

In December, 1944, the Massachusetts Institute of Technology, Division of Industrial Cooperation, undertook to develop for the Navy a generalized multi-engine flight trainer. This was an outgrowth of work during the war by the Special Devices Division of the Bureau of Aeronautics which was later transferred to the Office of Research and Inventions.

Several flight trainers had been designed for Navy aircraft. These included the PBM, the F6F, and the PB4Y2. The trainers had proven so valuable and the representation of flying conditions so satisfactory that an extension into the generalized field of aircraft simulation appeared possible. The proposed research and development at M.I.T. had two broad objectives. One was the design of equipment for generalized training purposes and the second that this flight trainer equipment should be sufficiently accurate to serve as an aircraft stability and control computer.

As a stability and control analyzer the equipment would be set up in accordance with information obtained from wind tunnel model tests of a proposed aircraft design to solve the equations of motion for the aircraft as a rigid body in space and to compute sufficient of the aerodynamic forces to predict the flying quality of the tentative design.

An aircraft cockpit complete in detail would permit a test pilot to evaluate aircraft performance, and flight instruments would respond as expected for the new aircraft design. Control forces would be computed and applied to the cockpit controls and proper sound and vibration incorporated.

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The results would be obtained not only as impressions and opinions of the test pilot but also as completely recorded data showing motion of cockpit controls and resulting response of the aircraft. To accomplish this, some 30 or more variables would be recorded.

Since human pilot reactions do determine in part the aircraft flight behavior, this analyzer has the advantage of including an actual pilot rather than synthesizing his reactions. Because human response is a factor in the problem, indicated aircraft response must occur in real time. To compute aircraft motion on a one-to-one time scale will require much higher solution rates than have before been required of mathematical computation devices. This and other problems made a survey period necessary before the start of actual computer design.

Contract NOa(s)5216 was set up to permit a study of the proposed aircraft analyzer and to study the feasibility of designing such a device. Certain obvious problems which had given trouble in the aircraft trainers were solved during the earlier phases of this contract. One such was the control force loading equipment to provide proper feel for the cockpit controls.

The heart of a device such as the aircraft analyzer is, of course the computer, which solves equations of motion in response to actions of the pilot. The usual approach to such a computer was through use of analogue computing techniques, such as have been used in differential analyzers and fire control equipment. During the first ten months of activity on this project, various analogue computing techniques were studied. These were primarily components which could be used in an analogue computing system that represented mathematical quantities by magnitudes of alternating current voltages.

As work progressed, it became apparent that an analogue-type computer for the equations of aircraft motion would be so complicated that its accuracy and performance could not be predicted. Such a computer would also lack the flexibility and ease of setup that is desirable for the aircraft analyzer problem. Although several new computer

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techniques including a high-performance servo system were developed, it seemed better that other approaches to the analyzer problem be studied.

Because the amount of computation required for the aircraft analyzer problem appears greater than that practical for analogue computers, it is desirable to consider carefully the use of high-speed electronic digital computing methods.

During the organization of Contract NOa(s)7082, consideration was given digital methods of computation in the fall of 1945, and since that time a program has been initiated to adapt high-speed electronic computing methods to the aircraft analyzer problem. A computer which will solve the aircraft stability equations will have the required capacity for many other families of scientific problems. Consequently, effort at the present time is being devoted to the study and development of general electronic digital computing techniques. Approximately a year and a half will be devoted to studying and developing these methods, by which time their adaptability to the aircraft analyzer problem can be determined.

2. ORIGIN AND PURPOSE OF THE PROJECT

2.1 BTL Flight Trainers

During the war the Bell Telephone Laboratories built several flight trainers for the Special Devices Division of O.R.I. to be used in pilot and crew instruction. Trainers were built for the PBM, the PB4Y2, and the F6F. In all these units realism and attention to detail was stressed so that an excellent illusion of flying was produced. All cockpit controls and switches were active, proper sound and vibration were provided from each engine, and solutions of flight equations were adequate to give realistic instrument readings. Motion of the cockpit was not employed, although some tests by the Special Devices Division engineers indicated its desirability. The design of special

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trainers was an expensive and time-consuming undertaking and the Special Devices Division undertook to develop a universal trainer into which constants for various types of aircraft could be set.

2.2 A Universal Flight Trainer

Such a universal flight trainer was visualized as a computer with adjustable coefficients for the equations of aircraft motion. Constants for the aircraft could then be set into the equations and the computer would provide proper flight instrument data for any type of aircraft. Simultaneously with this concept of a universal flight trainer there developed a plan for using the equipment as an aircraft analyzer.

2.3 An Aircraft Analyzer

In order that the equipment may function as an aircraft analyzer, it must predict rather than duplicate aircraft performance. In trainers, it has been the practice to adjust circuits which, in themselves, may not represent true mathematical equations, until proper behavior is achieved. An analyzer, on the other hand must take aerodynamic coefficients obtained from wind tunnel tests and use these to predict aircraft performance. The correct equations of motion must, therefore, be solved with sufficient sensitivity and accuracy to permit studies of aircraft stability and control.

2.31 Flight Stability and Control Forces

As a stability and control analyzer, the equipment would consist of a computer, a control room and auxiliary equipment, and a cockpit arranged to represent the proposed aircraft. An aircraft analyzer of the type visualized should perform three main functions.

2.311 Reduction of Experimental Testing

A considerable amount of the flight testing in the study of aerodynamic properties and control stability can be made with an analyzer of this type based upon information obtained from wind tunnel model studies. Such studies, to be sure, must be considerably

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more extensive than those usually made at the present time. The reduction in experimental test flying could greatly reduce the cost of new aircraft models and improve their operating characteristics.

2.312 Study of Unusual Aircraft Designs

An aircraft stability and control analyzer would permit the study of the more radical aircraft designs which, although perhaps entirely sound, might not be built for full-scale testing without some preliminary assurance of success.

2.313 Evaluation of Unfavorable Designs

Preliminary testing with an aircraft analyzer might prevent the unnecessary construction of aircraft models which are destined to have unsatisfactory flight characteristics.

2.32 Equations of Motion

Unlike the trainer, an aircraft analyzer will require accurate and carefully arranged equations to represent aircraft motion. Preliminary equations were set up prior to December 1944 by the staff of the Wright Brothers Wind Tunnel at M.I.T. Since that time these equations have been undergoing continuous study and change to better represent an airplane during take-off and flight conditions.

2.33 Pilot Reaction

Since the characteristics of a good airplane have never been reduced to a completely scientific basis, the evaluation of aircraft behavior depends on the reaction of aircraft pilots. It has therefore been considered necessary to obtain results not only as performance curves but also as impressions and opinions of test pilots. To make these impressions as realistic as possible, considerable attention must be given to cockpit design and the simulation of control reaction forces, noise, vibration, and motion.

2.34 Analysis and Synthesis

Not only would an aircraft analyzer permit analysis of an aircraft from model studies but, by a reverse process, coefficients

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could be changed to obtain desirable performance and the aerodynamic properties of the model redesigned to produce these coefficients.

2.35 Aircraft Criteria

The availability of an aircraft analyzer would permit more complete investigations into the characteristics of successful aircraft. With aerodynamic characteristics under control and subject to rapid change, a better appreciation of factors affecting good aircraft performance could be achieved.

2.36 Study of Automatically Controlled Flight

In addition to the study of piloted aircraft such equipment should permit studies of aircraft under the control of automatic pilots as well as studies of guided missiles.

2.4 Flexibility

An analyzer which will solve the problems outlined above will be a computer of considerably greater complexity than any now existing. At first a permanent arrangement of computing elements was contemplated in the manner originally used in the operational flight trainers. It early became apparent that such a fixed schematic would be entirely impractical because the equations to be solved were undergoing continual change and because a computer of this size and potential usefulness should not be committed to a single type of problem. Plans therefore call for as much flexibility as possible in the type of problem to be solved by the computer portion of the aircraft analyzer.

3. PRESENT STATUS

3.1 Analogue Computing Summary

The first plan for the computer to solve aircraft stability equations followed the approach previously worked out for fire control computers and differential analyzers. In analogue computation a separate computing element is required for each mathematical step involved in the problem solution. The requirement of a real time

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scale placed new and exacting specifications upon the individual computing components. In most differential analyzers variables are represented by mechanical shaft rotation where several revolutions are required to indicate the maximum excursion of the quantity represented. Since some quantities in the aircraft analyzer will traverse their entire range of operation in a fraction of a second, new high-speed computing elements would be required. A completely mechanical system did not appear practical because of the complexity, the difficulty of interconnection, and the lack of flexibility. Consequently, early studies were devoted to the development of computing elements where quantities would be represented by electrical voltages. Several circuits were developed for an alternating-current carrier type of signal representation. Because the dynamic range of many variables require a signal range that is impractical for electromechanical equipment such as potentiometers, some experimental studies were carried on with a sliding scale factor technique that will be described in the technical part of the report. A computer for solving the aircraft problem by analogue techniques would be many times the size and complexity of any differential analyzer developed thus far. As the aircraft analyzer was studied in detail and certain schematic circuits developed, it became apparent that the following factors cast doubt upon the practicality of an instrument of this type.

3.11 <u>Accuracy and Sensitivity</u>

It is doubtful that the required accuracy and sensitivity for the aircraft problem can be maintained through the use of analogue computing equipment built for the required speed of operation.

3.12 <u>Complexity</u>

The large number of computing components (See Section 4.7) would lead to a volume of equipment that would present serious maintenance and operating difficulties. Trouble shooting would be difficult and prediction of accuracy nearly impossible.

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3.13 Flexibility

The form of the equations describing aircraft motion will be subject to continual change and improvement as wind tunnel and flight test data becomes more accurate or is expressed in different terms. Modification of the equations may be from time to time desirable as studies of the methods of solution progress. To be useful, therefore, even in the solution of aircraft stability problems, a flexible computer schematic must be maintained. For an electrical analogue computer this would mean a plug-board type of interconnection which is not considered desirable for a computer having a large number of elements and requiring the dynamic range which must be associated with the aircraft analyzer equipment.

3.14 Limited Application

A computer of the analogue type would be expensive, would require a long development time, and when finished, would be of rather specialized nature and suitable only for a limited class of problems. A computer which represents the investment contemplated for the aircraft analyzer should be so designed that many other classes of scientific problems can be solved.

3.2 Digital Computation

Many of the disadvantages attributed to analogue computers can be overcome through the use of numerical analysis methods and electronic digital computation.

3.21 Advantages

3.211 Accuracy and Sensitivity

Whereas the accuracy and sensitivity of the analogue computer are limited by the mechanical and electrical tolerances associated with its physical components, the accuracy and sensitivity of a digital computer depend primarily upon the mathematical process selected, the number of digital places carried in computation, and the increments along the independent variable at which solutions are taken.

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3.212 Flexibility

A digital computer need not depend upon the physical interconnection or positioning of components for its schematic hookup. Problems are set up through the use of punched cards, tape, or some other prepared form of information. Changes in equations representing particular problems can be readily accommodated as well as making possible changes to entirely different types of scientific problems.

3.213 Predictability

Operation of the digital computer is identical except for solution time with the process followed in step-by-step numerical analysis methods. Solutions can therefore be worked out ahead of time for certain examples, using calculating machines and operators. The form and accuracy of a solution can therefore be predicted and will not depend upon physical tolerances and behavior of circuit elements in the computer so long as operation is trouble free.

3.214 Scope

The digital computer may be applied to many problems to which the analogue computer would not be applicable.

In this computer attention is being centered on applied science and engineering. Emphasis will be placed on its adaptation to dynamic systems such as aircraft, automatic fire control, and servomechanisms.

Application to algebraic equations will be studied. For the present, the solution of partial differential equations will receive only secondary interest until electronic techniques are under control.

3.22 Disadvantages of a Digital Computer

3.221 Development Time

Development time for a digital computer will be perhaps longer than for an analogue type system.

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3.222 Cost

Development cost for a digital computer will be high and rather difficult to estimate although construction cost should be lower than for the analogue system.

3.223 Parallel Operation

In order to solve the aircraft stability equations in real time, some parallel operation of electronic computing elements will probably be necessary, thereby leading to more complicated circuits than might otherwise be required.

3.23 Proposed Approach

3.231 Binary System

At the present time it is anticipated that computation will be done in the binary system of notation. For the aircraft analyzer itself this represents no difficulty since results will be derived as graphical representations and conversion from the decimal system will not be required except in setting up a problem. For other types of scientific computation input and output conversion with required recording equipment must be developed.

3.232 Serial Operation

Insofar as possible, computing operations will be done in a serial manner. The digits of a number will be transmitted consecutively over the same circuit and identified by their location in time. Some parallel operation may be required such as computation occurring simultaneously with the setup of the next step and perhaps simultaneously with certain subsidiary operations, such as interpolation for coefficients. Multiplication time will be reduced by parallel computation.

3.233 Operating Speed

At the present time operation is contemplated with the use of 1/4 microsecond video pulses to represent binary digits with a 1 megacycle repetition rate. Preliminary investigations are

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being carried on at a 100 kilocycle repetition rate and studies will be made to check the feasibility of rates above one megacycle.

3.234 Electrostatic Storage

Storage of partial solutions will be attempted by means of electrostatic storage tubes. Because of the computing speed required in problems involving real time, a storage method permitting immediate insertion and withdrawal of stored data is desirable. Storage by supersonic delay techniques may be used in certain places if advantages are demonstrated for this type of storage.

3.24 Research Problems

3.241 Storage

It is anticipated that 200,000 to 500,000 binary digits must be stored in this computer. This total represents both the program, the storage of fixed quantities, function tables and coefficients, and the storage of partial results. The two former categories can use semi-permanent storage as distinguished from high-speed electrostatic storage.

3.242 Program

Program control techniques for the computer must bedeveloped which will take information from a semi-permanent form of representation and use it for control of high-speed storage and computing processes.

3.243 Computer

Computer circuits for addition, subtraction, multiplication, and division will be required as well as perhaps a special unit for interpolation in order that solution time can be conserved. Other arithmetical and algebraic operations may be programmed in terms of these basic steps.

3.244 Checking

Checking and trouble shooting procedures must be established and built into the computer.

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3.245 Recording

Several forms of recording will be required on the computer output. Problems involving low final accuracy and particularly those representing solutions versus time can be recorded as oscillograms. Other types of problems may require card punching, automatic typewriters, highspeed photographic recording, and magnetic recording.

3.246 Signal Conversion

Projects such as the aircraft analyzer will require methods for converting mechanical shaft rotation to binary digits in order that the transition between cockpit and computer may be accomplished.

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PART B - TECHNICAL

4. AERODYNAMIC EQUATIONS

4.1 Report R-64

The aerodynamic equations as now visualized for representing the control and stability characteristics of an aeroplane in flight are given in DIC 6345 Report 64 which, along with equation schematics, is included in Appendix A. This report is the outgrowth of work by staff members of the Wright Brothers Wind Tunnel at M.I.T.

4.2 System of Axes

The general problem of solving for the motion in space involves the solution either of the equations describing the resultant motion or the equations describing components of the resultant motion along and about some system of axes. The approach to the problem has been to set up equations describing the motion along and about a 3 co-ordinate axes system called body axes with the origin at the airplane's center of gravity. The body axes very nearly coincide with the principal axes, thus the product of inertia terms are considered negligible. Motion of the airplane in space is related to the earth axes through a set of Euler angles. Thus nine simultaneous integral equations are required to determine the motion of the airplane in space and to relate it to the earth axes. The relative wind direction, found trigonometrically, determines the wind axes. See Appendix A.

4.3 Landing and Take-Off Equations

The general emperical functions which describe reactions on the airplane exclude ground reactions and complicated reactions in stalled flight. As a result, special terms for the take-off, landing, and stall representations are included in the equations. Since it is desirable to maintain a continuous solution through transition from one flight condition to another and to minimize the complexity of representation, several simplifying assumptions and several limitations on the equations have been made.

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4.31 Take-Off and Landing Representation

The take-off and landing is considered in two sections: 1) aerodynamic reactions, and 2) ground reactions.

4.311 Aerodynamic Reactions

The fact that Tc' goes to infinity at zero speed requires that those aerodynamic reactions which are a function of Tc' must be modified during the ground run. The description of the general aerodynamic data is limited to a reasonable value of Tc' and additional functions are added which describe the aerodynamic effects above the specified maximum Tc'.

4.312 Ground Reactions

The following assumptions are made for the ground run:

a) No cross wind; angle of yaw remains zero, but heading can change. Relative wind remains along intersection of planes XOZ and AOB.

b) No rolling moment occurs. Wings remain level during ground run.

c) No sideforce occurs.

d) No sideslip between wheels and ground.

e) No weight is carried by either nose or tail wheel. Weight is distributed equally on the two main wheels.

With the above assumptions, the ground reaction terms, including braking effects, can be computed and added to the equations. These terms either go to zero or cut out of the solution when the altitude is greater than zero. In the same manner, for instance, the rolling moment which remains zero during the ground run, requires that the equation for L/I_x be cut out of the continuous solution until such a time as the altitude is greater than zero.

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4.4 Stall Conditions

The representation of the stall in the analyzer will not be complete since the motion in the stall is not to be studied. Therefore, the effect desired is to provide a stall warning, such as control buffeting, to let the pilot be aware that the airplane is stalling, provide a means of control during the stalled period, and have the airplane emerge from the stall at some angle of attack and lift coefficient below the stall.

4.5 Control Forces

To make the illusion of flying as nearly complete as possible, it is necessary that control column and wheel, and rudder pedal forces have the proper feel. In addition, for a particular airplane, the forces and variation of forces should be reasonably correct for all flight conditions. Equations are set up to solve electrically for the hinge moments at the control surface hinges. If the electric impulses corresponding to the hinge moments are transformed to a force, this force could act through a system of mechanical linkages similar to the actual control linkages to give the proper control feel. In this manner, the effects of friction, cable stretch, and inertia effects of the actual airplane control system can be included. To date, no provision is made to compute the dynamic characteristics of the control surfaces.

4.6 Engine Simulation

The analyzer will be set up to simulate a four-engine airplane. It is necessary to simulate reciprocating engines, gas turbines, jet units and combinations thereof. It is hoped to find a general representation to compute thrust for any type of engine. Since the analyzer is basically to study airplane stability and control, and not airplane performance, it is not required to simulate the engines exactly; i.e., to obtain the thrust which gives the desired cruising speed, it is not necessary to have the manifold pressure and RPM exactly correct.

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4.7 Summary of Computations

The following is a summary of the number of mathematical operations indicated by the aircraft equations of Appendix A.

	A	B	C
Multiplication of two variables	105	11	149
Multiplication of a variable by a constant	103	6	127
Additions and Subtractions	179	7	207
Divisions	8	3	20
Reciprocals	9	0	9
Integrations	10	0	10
Trigonometric functions	9	0	9
Inverse Trigonometric functions	4	0	4
Square Roots	2	0	2
Non-Integral Exponentials	1	0	1

Column A includes the operations indicated by equations 1 through 56 and 80 through 90, which are the equations of motion, hinge moment equations, aerodynamic coefficient equations, instrument equations, and miscellaneous. Column B includes the operations indicated by equations 57 through 80, which are the engine equations. Column C is a total of all operations for a four engine airplane.

The following assumptions have been made in compiling this tabulation:

(a) Where two or more divisions by a function or quantity are required, the reciprocal can be found and then multiplied to reduce divisions.

(b) Reciprocal quantities which remained constant during the Analyzer operation or

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were set manually by the Flight Engineer were introduced with the assumption that the reciprocal had been calculated external to the computer.

(c) This tabulation does not count any specific operation more than once.

(d) No operation for finding values of functions of computed values from curves or function tables are included.

(e) Additions and subtractions are tabulated together since there is no essential difference in these operations.

(f) Each operation involves two quantities only, so the sum of three quantities is two additions, etc.

(g) Integral powers are called multiplications, thus $a^2 = a \cdot a$.

5. DYNAMIC RANGE AND ACCURACY

Only estimates have been thus far arrived at for the dynamic range, sensitivity and accuracy required for the computing equipment to be used in a solution of the problem of aircraft motion. Some of these estimates are given in DIC 6295 Report 49.

5.1 Pitching Angular Velocity

Flight tests show pitching velocities as great as ± 30° per second in certain necessary flight test maneuvers. Although flight test procedures no longer pay great attention to a determination of the period and damping of the phugoid or long period longitudinal oscillation, an airplane, if disturbed from its trim angle and thereafter uncontrolled, will pitch in accordance with this mode of its oscillation,and it should be reproduced for such a time as the pilot might leave the analyzer uncontrolled. This involves a very small

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angular velocity which might be as small as 0.1° per second. If such a motion is to be accurately generated, the equipment must be sensitive to a much smaller velocity change. Ideally one might wish to have sensitivity to 1/60 of this velocity or to 1/600 degree per second. Thus the ratio of maximum to minimum velocities might be as high as 18000:1.

5.2 Factor of Safety for Aircraft

The maximum value of pitching velocity of 30° per second used in the above example is based on existing aircraft and does not allow a factor of safety for future aircraft or missiles which might increase its value.

5.3 Factor of Safety in Computation

The above observations on required dynamic range of computing equipment does not include a factor of safety for significant figures lost in the computing components themselves. It is readily seen that mechanical and electrical equipment for analogue computation can hardly have a factor of safety above the dynamic range indicated in 5.1 and 5.2.

5.4 Electrical Signal Range (Analogue Computer)

Since the electronic computing components were being designed around conventional vacuum tubes and their circuits including cathode followers, a maximum A.C. signal amplitude of approximately 40 volts seemed desirable. A dynamic range of 20,000 to 1 would then require a minimum detectable signal of 0.002 volts.

Difficulties in obtaining the required dynamic range for signals are immediately apparent.

5.41 Shielding

Very careful shielding would be necessary to maintain noise levels of 1 millivolt or less on signal lines carrying maximum values of 40 volts.

5.42 Power supplies

Power supplies must be regulated to a few millivolts of ripple to prevent the introduction of extraneous signals.

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5.43 Sliding Scales

Computing elements requiring mechanical equipment such as potentiometers must have precision gear trains and, even so, must operate with a sliding scale factor based upon some assumption of permissible reduction in sensitivity with increase in signal level.

5.5 Extension of Analogue Operating Range

Since the required signal range of an analogue type computer using voltages to represent variable quantities can probably not be realized, certain alternatives must be considered. One of these is the division of a problem into several phases of investigation. For example, the aircraft take-off conditions might be studied with an expanded set of scale factors for those variables which have only limited range. Again, straight and level flight conditions could be studied with high sensitivity by setting up the analyzer specifically for this problem. Lastly, maneuvering conditions could be represented by setting up the problem for maximum range of variables. These alternatives are undesirable since they place a greater burden upon the investigators who set up a problem, because operation of the aircraft analyzer becomes more difficult, and because continuity of flight is lost along with its psychological effects upon the test pilot.

6. NON-LINEAR COEFFICIENTS

Many of the aerodynamic coefficients shown in Appendix A are non-linear and depend upon values of variables being computed by the analyzer. The following tabulation summarizes the number of these variable coefficients.

Functions of	Engines (Equations 57-79)	Hinge Moments (Equations 22-25, 40-48)	Others	Total*
One variable	2	4	15	27
Two variables	6	6	20	50
Three "	0	7	9	16
Four "	0	1	1	2
Five "	0	1	0	1

* Total for four engines.

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6.1 Examples

The following examples of non-linear coefficients are included and discussed in Appendix B;

$f_1\ (\alpha, \delta F, T_c'$ = largest term of lift coefficients

$f_4\ (\alpha, m_n)$ = term in lift coefficient caused by large mach numbers, i.e., very high speed

$f_7\ (\delta R, \gamma)$ = term in drag coefficient caused by rudder deflection

$f_{13}(\alpha, T_{c_2}' - T_{c_3}')$ = term in pitching moment coefficient caused by unsymmetrical thrust

$f_{19}(\gamma, \delta R, T_{c_{2+3}}')$ = term in yawing moment coefficient caused by rudder

6.2 Representation of Non-Linear Coefficients

Several methods of representing non-linear coefficients immediately come to mind.

6.21 Cams

Cams are perhaps the oldest form used in fire control computers. These would be too cumbersome for the aircraft analyzer because of the difficulty in cam cutting, the irregular shapes of some coefficients, the large number of variables involved in some functions, and the problem of inserting cams into the analyzer in the set-up of a new problem.

6.22 Tapered Potentiometers

Tapered potentiometers could be used for functions that did not require frequent alteration. However, since most coefficients will vary considerably from one aircraft problem to another, the use of tapered potentiometers is rather impractical.

6.23 Networks

Some studies have been made by Bell Telephone Laboratories and this group in adapting electrical resistance networks using static

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elements and potentiometers for the computation of non-linear functions. While this method shows some promise, it is difficult to set up, cumbersome to execute in practice, and lacks the flexibility which should be associated with this equipment.

6.3 Tapped Potentiometers

The use of tapped potentiometers showed the best promise for representation of variables in an analogue computer making use of electrical voltages to indicate signals. These potentiometers were planned to have several taps at which would be established voltages corresponding to the function at the tap position on the potentiometer. These voltages would be established perhaps through a punch card system and cathode follower vacuum tube circuits. Such a system can be extended to several variables. Function representation is based on the assumption that linear interpolation between points of data is satisfactory. Some tests were made with tapped potentiometers having several sliding brushes with resistance networks for smoothing curves at the points of discontinuity in slope. These methods showed promise but studies were not carried to completion.

7. ANALOGUE CIRCUITS

7.1 Alternating Voltage Carrier

Analogue computing circuits studied in the early phase of this work were based entirely on the use of alternating voltage carrier to represent variables. Work was done at 60 cycles and considerable interference was encountered between power supply ripple and signals. Phase shift between signals on the input of summing amplifiers was a problem, as could be expected. An alternating current system was initially selected because the problem seemed less formidable than those associated with direct current amplifiers and their power supply problems. A network of approximately 12 summing amplifiers was constructed to

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represent equations 1, 20, and 29 in Appendix A. This network did not involve servo-driven multiplier units but used static potentiometers at the points indicated for multiplication of two variables. Noise levels in this network were held to about 1 millivolt and the required dynamic range outlined in Section 6 were obtained although considerable difficulty was experienced. Phase shift between carriers was particularly noticeable at summing amplifiers where signal subtraction was taking place. Large quadrature and harmonic signals were found at the amplifier outputs. These might be tolerable in many cases because signals eventually fed servo systems at multipliers or integrators which are fundamentally insensitive to quadrature voltage.

7.11 Summing Amplifiers

Several types of circuits were investigated for summing A.C. signals, and a reasonably satisfactory amplifier was developed and tested in the demonstration units. The essential features consisted of bringing the signals to be added (up to five in number) in, through isolating cathode followers, summing the signals by ordinary current addition methods in a single grid resistor, and then raising this total signal to the correct voltage and impedance level through a feedback stabilized amplifier. The accuracy of summing with good matching of input and output impedances is of the order of one-fourth per cent. The dynamic range was about 1 to 30,000 from minimum to maximum signal and the output distortion was usually less than 2% total harmonics. The phase shift except for very small signals was less than one degree for 60 cycle signals. It seems probable that these summing amplifiers, with certain modifications suggested by the tests, would have been entirely satisfactory for the contemplated analogue computer.

7.12 Integrators

Two approaches to the integrator problem were investigated.

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7.121 Mechanical

A two-stage mechanical integrator using ball and disc integrators from fire control equipment were arranged in a manner similar to that indicated for the multiplier in Appendix C. Results from these units were not up to expectation, largely because of mechanical back-lash and other operating imperfections. It is anticipated that these imperfections might not be reducible to a level that would permit desired performance.

7.122 Electronic

An electronic integrator for alternating voltages was studied, based upon the use of feed-back amplifier techniques and Brown Instrument Company vibrators for the conversion of D.C. voltages to A.C. In this circuit integration was performed in a direct current R-C stage. The output condenser voltage was converted with a vibrator to 60 cycle A.C. and the resistance drop was converted to A.C. for comparison with the input signal. The difference between input and resistance drop was fed to a high gain A.C. amplifier, the output of this amplifier being converted to D.C. for operation of the integrating circuit.

Research on this integrator was likewise not carried to completion. Other promising integrators have been developed by various laboratories which would have been investigated before making a final selection of method.

7.13 Multiplication

Analogue multiplication of electrical signals was planned with potentiometers and servo units. The potentiometer slide would be driven proportional to one signal while the second signal would be applied to the potentiometer winding. To obtain the required dynamic range, a two-stage multiplication would be used where one potentiometer would set the scale factor for multiplication by the second potentiometer as shown in Appendix C. Because of the high accelerations and velocities

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experienced by the variables of the aircraft analyzer problem, it would be necessary in an analogue machine to have high-speed servo units to drive the multiplying potentiometers.

7.14 Frequency Modulated Servo

A servo system for computer operation was experimentally studied for high static accuracy and high natural frequency. This servo system operated with a polyphase, low-slip induction motor driven from a variable frequency source capable of continuous output between the limits of positive and negative (reverse phase rotation) frequency required for maximum motor speed. This servo system is discussed more fully in Appendix D.

7.2 AC versus DC Signals

Only studies of an AC carrier system for signal voltages had been conducted on this project prior to the transfer of attention to digital computation. It has not been clearly established how the disadvantages of AC carrier systems, particularly phase shift and power supply ripple, compare with the problems encountered in direct current amplifier systems. It is felt that work carried on by Columbia University using direct current amplifiers, multipliers, and integrators has been carried to a higher state of development and may well be more desirable than the AC system for computers within the size range suitable for analogue computation.

8. WIND TUNNEL TESTS

The data describing the aerodynamic reactions of a particular airplane requires extensive wind tunnel testing. In the Wright Brothers Wind Tunnel results are obtained, referred to wind axes, and are recorded manually. All results are static and no tests to determine rotary derivatives are made. In addition the hinge moment results on small scale models are inconclusive.

Extensive redesign of the balance and recording system will incorporate provisions 1) for testing for rotary derivative, 2) for automatic

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data reduction and recording and 3) for obtaining data about stability axis. Tests on large panel models of the control surfaces will provide more accurate hinge moment data, and some actual flight testing may be done for correlation work in determining dynamic characteristics.

8.1 Data for Model A

The equations of motion as now set up in Report 64 include the equations for the aerodynamic coefficients. The component functions of these coefficients has been compiled for a twin engined airplane designated Model A. These data, results of actual test for the most part, have been used in preliminary calculations and are representative both in type and quantity of what the analyzer must handle. See Report R-98. At present moment coefficients are given about body axis and force coefficients along wind axes. The equations are set up in the analyzer to transfer the force coefficients to the body axes system. It seems advisable at the present time, however, to convert all coefficients to the body axes reference before it reaches the analyzer.

9. COCKPIT DESIGN

The aircraft analyzer problem, as visualized for an analogue computer, required a cockpit having sufficient realism to provide the illusion of flight to the test pilot. Much progress in this direction has been made by the Bell Telephone Laboratories in their Operational Flight Trainer. Further improvement over these units is necessary not only in the instrument readings which depend primarily on the computer but also in the generation of control reaction forces.

9.1 Cockpit Mounting

Operational Flight Trainers have been built and operated effectively without provision for cockpit motion. Less elaborate trainers, such as those built by Link, have used cockpit motion to good advantage.

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9.11 Tilt

It is proposed that the cockpit for the aircraft analyzer be mounted to provide tilting in pitch and roll which will provide the necessary simulation of apparent change in gravity direction. With the cockpit mounted as shown in Drawing C-30053 the effects of side slip, horizontal acceleration, and pitch angle can be simulated. Cockpit angular motion is discussed more completely in 6345 Report No. R-100.

9.12 Vertical Motion

A small amount of cockpit vertical motion may be provided to divorce the cockpit from the rigidity of a fixed support and to produce the illusion of air-borne motion.

9.2 Control Force Equipment

A hydraulic force generating system was designed for the simulation of rudder, elevator, and aileron forces. This system was sensitive to forces of a fraction of a pound on the control column and could generate forces of several hundred pounds. It is described in 6345 Reports Nos. R-36, R-37, R-38, R-96, and R-99. The equipment will be redesigned into a packaged form when required for the aircraft analyzer.

9.3 Instrument Arrangement

Only tentative cockpit instrument and control column arrangements have been studied. When a final design is prepared, it will be in line with the proposal of the Navy Committee on Standardization of Aircraft Cockpits.

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APPENDIX A
to

PROJECT
WHIRLWIND
(DEVICE RF-12)

SUMMARY REPORT NO. 1
April 1946

REPORT R-64
Revision 1 - Dated April 4, 1946
on
Aircraft Stability and Control Equations

follows on the next 30 pages

- - -

NOTE: Drawings R-30011
D-30012
D-30013

show schematic arrangement of the equations in Report R-64 before Revision 1.

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6345
Report R-64,
Revision 1

SERVOMECHANISMS LABORATORY
Massachusetts Institute of Technology
Cambridge, Massachusetts

Date of Revised Report: April 4, 1946 Page 1 of 30 pages
Date of Original Report: October 31, 1945 Figures: 1-2-3-4

Subject: ASCA Equations (Revised - October 30, 1945
April 4, 1946)

References: Original Copy - J. W. Forrester's file
Authors: L. Bernbaum
J. Bicknell - both of WBWT

This report supersedes the information in Report 50 and Report 64 dated October 31, 1945. It is supplemented by Reports 15, 18, 49, 58, 62, and R-98.

It is the object of this report to include under one cover all the symbols and equations to be incorporated in the analyzer, and to describe the progress of equation development to date. Progress has carried the study to a point where equations, functions, and constants can be assigned definite numbers. Therefore, all symbols and numbers assigned in this report will be considered as official reference in the future.

The types of equations requiring solution in the analyzer are classified as follows:

1. Basic equations of motion		1 - 6
2. Equations relating wind, body, and earth axis		7 - 15
3. Auxiliary equations of motion		16 - 21
4. Control surface hinge moment equations		22 - 25
5. Auxiliary equations or aerodynamic coefficient equations		26 - 48
6. Instrument equations		49 - 56
7. Engine equations	(for one engine)	57 - 79
8. Miscellaneous		80 - 90

1, 2 The basic equations of motion and the equations relating wind, body, and earth axis have been rewritten as integral equations instead of differential equations. See equations 1 - 15 and Figs. 1, 2.

Several of the circuits require special consideration to determine when they must or must not be included in the general solution, or held in or out for a specified time. The special cases of take-off and stall are examples of unusual computing procedures. Reports describing the operations desired will be issued in the future.

3. The only change made to the auxiliary equations of motion describing the linear accelerations along wind axes is the addition of the take-off terms. It must be noted, however, that equation 17 describing the acceleration along $O\xi$ has no component of engine thrust included. The thrust component, which acts along the OX body axis is computed from the engine data and fed into the basic equation of motion describing the linear acceleration along OX.

The auxiliary equations of motion describing the angular accelerations, numbers 19, 20, 21, about body axes have had some terms added. Reduction of moments due to deflections of the airplane structure under load have been included. These reductions have been taken as functions of the horizontal tail, vertical tail, and aileron loads. Since rolling moments produced by the ailerons are already computed in equation 31, an expression is included in this equation to reduce the net aileron moment produced. The pitching and yawing moments reductions, however, cannot be handled as simply. It will be necessary to compute the moment coefficients produced by the horizontal and vertical tail, equations 38 and 39, and subtract out some percentage of these in equations 19 and 20. The percentage reductions will be constant for a given airplane.

The rotary derivatives, or moments produced by a rotation are included in equations 19, 20, 21. These terms require a separate equation to determine each derivative.

The angular momentum of the propellers is great enough to produce a measurable pitching or yawing moment when either a yawing or pitching velocity occurs. They appear in equations 19 and 20 divided by the appropriate moments of inertia.

Also included in equations 19, 20, and 21 is provision for a center of gravity shift in any direction. All moment data furnished from the wind tunnel will be about the wind tunnel balance resolving center. This point will correspond to some point on the airplane, which will be a reference point. All C. G. positions will be specified in percent of the aerodynamic chord away from the reference point. See Fig. 3.

Equation 20, for the yawing acceleration, has included the direct thrust effect due to unsymmetrical engine operation.

The diagram, Fig. 4, showing the horizontal and vertical distances of the C. G., from the point of main wheel contact, describes the moment arms which determine the moments due to the ground reaction terms. This diagram supercedes the sketch of ground angles in Report 50.

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4. Hinge moment equations 22, 23, 24 replace the original control force equations. One method to handle the control loading is to solve electrically for the hinge moments at the control surface hinges and apply these to a system of mechanical linkages similar to the actual airplane linkage. In this manner, friction, cable stretch, and inertia effects of the actual airplane control system can be included physically. Report 58 covers this subject of control loading.

5. The auxiliary equations or aerodynamic coefficient equations have been altered considerably in order to conform to the proposed point by point method of data representation. These are equations 26 - 48 and sample curves for a particular airplane representing the various functions in the auxiliary equations will be included in a future report. (See R-98.)

The auxiliary equations to determine the rotary derivatives, equations 32 - 37, have been multiplied by a non-dimensional number which includes the airplane velocity, thereby reducing by one the variables on which the rotary derivatives are dependent. Thus, if the rolling moment due to a rolling velocity $\frac{\delta C_\ell}{\delta p}$ is a function of $\alpha, \gamma, \delta F, V$ then $C_{\ell_p}\left(\frac{pb}{V}\right)$ is a function of $\alpha, \gamma, \delta F$ if C_{ℓ_p} is defined as $\left(\frac{V}{b}\right)\left(\frac{\delta C_\ell}{\delta p}\right)$.

Two types of additional functions have been added to the auxiliary equations. The first is a function of the Mach number (the ratio of airplane speed to speed of sound) included in the lift, drag, and pitching moment coefficients equations. These functions have been considered necessary for a more rigorous solution at the high speeds encountered in modern aircraft. Inclusion of a Mach number term may be considered as a correction term, and therefore the functions describing such terms will be of limited complexity; i.e., the function will be described only at relatively low angles of attack.

To include terms in the auxiliary equations as functions of the Mach number, it is necessary to continually compute the value of the Mach number; i.e., the velocity V divided by the velocity of sound V_s at the particular altitude in question. The velocity of sound as a function of altitude is given in Report 62.

The second type of functions added have been termed manual settings and can be distinguished by the symbol M_x used to describe the constant of each term. On any airplane, the data initially will be set according to results of wind tunnel tests. After operation of the analyzer, however, it may be convenient to change some parameters at will in order to determine qualitatively the manner in which the configuration of the airplane should be changed in order to obtain more desirable characteristics. These parameter changes will be simple slope changes or additions and should be readily adjustable, preferably by means of dials located on the control engineer's panel. All such adjustments will be set at zero for the initial tests.

The control engineer acts in the capacity of the flight engineer of the airplane. He creates flight problems, and handles those matters for which automatic equipment has not been included. The latter would be cowl flap settings and engine head temperatures, bomb doors, etc.

The following list of controls at the control engineer's desk is an indication of the type of control; a complete list will be given in future reports.

1. Change of mass
2. Change of horizontal C.G.
3. Change of vertical C.G.
4. Change of lateral C.G.
5. Change of moment of inertia about ox axis (I_x)
6. Change of moment of inertia about oy axis (I_y)
7. Change of moment of inertia about o_z axis (I_z)
8. Change of outboard port engine power including engine failure
9. Change of inboard port engine power including engine failure
10. Change of inboard starboard engine power including engine failure
11. Change of outboard starboard engine power including engine failure
12. Provision for gusty air
13. Change of ground friction coefficient

Certain of the above manual adjustments will entail automatic changes in other parameters. Thus, several new equations are required. They involve:

1. Changes in moments of inertia when mass is changed
2. Changes in moments of inertias when C.G. is changed
3. Change of thrust component with feathered propeller

These will be considered in detail in a future report.

Included in the aerodynamic effects which determine the resultant motion of the airplane are the stall phenomena. To continuously simulate the motion in the stall is too highly involved for the end result achieved. It has been decided, therefore, that the effect desired in the analyzer is to let the pilot be aware that the airplane is stalling, give him some means of control during the stalled period, and have the airplane emerge from the stall at some angle of attack and lift coefficient below the stall.

Some stall warning such as buffeting of the controls should appear before the stall is reached.

It is proposed to achieve this stall as follows:

1. A function of $\delta F, T_c'$ will determine the stall angle of attack. Stall occurs when $\alpha = \alpha_{stall}$.

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2. At each stall angle, a drop-off in C_L will occur depending on the severity of the stall. C_L will be held constant at its new value until such a time that α decreases, returning itself to the lift curve for normal operation.

3. At each stall angle, decrease aileron effectiveness (probably same amount for all cases) until recovery is reached.

4. At each stall angle, apply suddenly a C_n and C_ℓ proportional to the severity of the stall. These increments are to be applied for a definite time interval and removed.

During the stall, all machinery will continually compute the motion so that at recovery the airplane will be under normal operation.

Handling the take-off will be considered in detail in a future report. It is well to note, however, that several of the equations which determine the aerodynamic coefficients have T_c' included as a variable and therefore cannot be used for take-off computations because T_c' goes to infinity at zero airspeed.

The proposed method for handling this difficulty is to add data to the equations of motion based on thrust and velocity during take-off. This data will be in dimensional form, and ground reaction effects will be included. Fig. 4 supercedes the previous sketch describing the ground angles in Report 50. Certain assumptions made for the take-off run simplify the amount of data required. These assumptions are:

1. Angle of bank is zero
2. Angle of yaw equals zero
3. No side slip between wheels and ground

After take-off, the proximity of the ground affects the aerodynamic reactions, particularly the longitudinal stability. This effect is included in equation 29 for C_m which has a term added that increases the basic longitudinal stability as a function of altitude and goes to zero when $h = b$.

7. Engine simulation equations will be obtained for four reciprocating engines, for gas turbines with jets and propellers, for jet engines alone, and for jet and reciprocating units combined.

The reciprocating engines and propeller is treated as a unit. From a knowledge of the airplane velocity V, altitude h, density ρ, pressure P, and the engine control settings, the thrust is computed. T_c etc., is also computed to be supplied to the auxiliary equation functions.

The number of separate equations to be solved for each reciprocating engine to obtain the thrust output (for the present torques will be neglected) is 23, 92 equations for four engines. These equations are listed, numbers 57-79 for one engine. Their explanation and synthesis into the analyzer will be given in a later report. Additional equations for jets and gas turbines will be added at a later date.

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No account in the engine equations is taken for the fact that each engine operates at a different $\frac{V}{nD}$ when a yawing velocity occurs. The effect is to produce a damping moment due to the rate of yaw by decreasing the thrust on the engine going forward and increasing it on the engine going rearward. The yawing moment produced results then from a yawing velocity, and thus can be handled in the equation for the rotary derivative C_{n_r}. This method simplifies the engine equations somewhat by having each engine operate at the same forward speed.

It is planned that the engine circuits will include a suitable system to simulate engine noise and the accompanying vibrations.

For schematics, function chart and interconnection of equations see 6345 drawings:

R-30011
D-30012
D-30013
E-30014
D-30015

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EQUATIONS OF MOTION

ANGULAR VELOCITIES ABOUT BODY AXES

① $$q=\int_0^t\left[\frac{M}{I_Y}+\left(\frac{\bar{c}-\bar{a}}{\bar{b}}\right)rp\right]dt$$

② $$r=\int_0^t\left[\frac{N}{I_Z}+\left(\frac{\bar{a}-\bar{b}}{\bar{c}}\right)qp\right]dt$$

③ $$p=\int_0^t\left[\frac{L}{I_X}+\left(\frac{\bar{b}-\bar{c}}{\bar{a}}\right)qr\right]dt$$

LINEAR VELOCITIES ALONG BODY AXES

④ $$u=\int_0^t\left(-g\sin\theta+\frac{X'}{m}+rv-qw\right)dt$$

⑤ $$v=\int_0^t\left(g\cos\theta\ \sin\phi+\frac{Y'}{m}+pw-ru\right)dt$$

⑥ $$w=\int_0^t\left(g\cos\theta\ \cos\phi+\frac{Z'}{m}+qu-pv\right)dt$$

VELOCITY RELATIONSHIPS BETWEEN WIND AND BODY AXES

⑦ $$V=\sqrt{u^2+v^2+w^2}$$

⑧ $$\mathrm{TAN}\ \alpha=\frac{w}{u}$$

⑨ $$\mathrm{SIN}\ \gamma=-\frac{v}{V}$$

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EQUATIONS RELATING BODY AND EARTH AXES

⑩ $$\theta = \int_0^t (q \cos\varphi - r \sin\varphi)\, dt + \theta_0$$

⑪ $$\varphi = \int_0^t (p + q \sin\varphi \tan\theta + r \cos\varphi \tan\theta)\, dt$$

⑫ $$\psi = \int_0^t \left(q \frac{\sin\varphi}{\cos\theta} + r \frac{\cos\varphi}{\cos\theta}\right) dt$$

COMPOSITION OF ACCELERATIONS ALONG BODY AXES

⑬ $$\frac{X'}{m} = \frac{\Delta}{m} \cos\alpha \cos\gamma + \frac{F}{m} \sin\gamma \cos\alpha - \frac{\lambda}{m} \sin\alpha + \frac{T}{m} + \frac{TJ}{m}$$

⑭ $$\frac{Y'}{m} = -\frac{\Delta}{m} \sin\gamma + \frac{F}{m} \cos\gamma$$

⑮ $$\frac{Z'}{m} = \frac{\Delta}{m} \sin\alpha \cos\gamma + \frac{F}{m} \sin\alpha (\sin\gamma + \left(\frac{\lambda}{m}\right) \cos\alpha \underbrace{- \left(g + \frac{\lambda}{m}\right) \cos\theta}_{\text{GROUND REACTION}}$$

AUXILIARY EQUATIONS OF MOTION

⑯ $$\frac{\lambda}{m} = -C_L \frac{\rho/2\, V^2 S}{m} + \underbrace{\frac{f_a(T, V)}{m}}_{\text{GROUND*}}$$

⑰ $$\frac{\Delta}{m} = -C_D \frac{\rho/2\, V^2 S}{m} - \underbrace{\left[\mu + K_a(BP_R + BP_L)\right] \left(g + \frac{\lambda}{m}\right)}_{\text{GROUND REACTION}}$$

⑱ $$\frac{F}{m} = C_F \frac{\rho/2\, V^2 S}{m}$$

*EFFECT OF ENGINE THRUST DURING TAKE-OFF.

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⑲ $$\frac{M}{I_Y} = \left[C_m + C_{m_q}\left(\frac{qc}{V}\right) + C_{m_{\dot{\alpha}}}\left(\frac{\dot{\alpha}c}{V}\right)\right]\left(\frac{\rho v^2}{2}\right)\left(\frac{S}{m}\right)\left(\frac{c}{b}\right)\left(\frac{1}{b\bar{b}}\right)$$

$$+\underbrace{\frac{f_b(T, V, \delta E)}{\bar{b} m b^2}}_{\text{GROUND*}}\underbrace{-R_M C_{m_t}\left(\frac{\rho V^2}{2}\right)\left(\frac{S}{m}\right)\left(\frac{c}{b}\right)\left(\frac{1}{b\bar{b}}\right)}_{\text{STRUCTURAL DEFLECTION}}$$

$$+\underbrace{\left[\frac{H}{c}\times\frac{Z'}{m}-\frac{D}{c}\times\frac{X'}{m}\right]\left(\frac{c}{b}\right)\left(\frac{1}{b\bar{b}}\right)}_{\text{DISTANCE, C.G. TO REF.}}+\underbrace{\left(\frac{K_b n r}{m}\right)\left(\frac{1}{b}\right)\left(\frac{1}{b\bar{b}}\right)}_{\text{PROPELLER}}$$

$$+\underbrace{\left\{\frac{H'-H}{c}+\frac{D'-D}{c}\left[\sin\theta-\mu+K_a\left(BP_R+BP_L\right)\right]\right\}\left(g+\frac{\lambda}{m}\right)\left(\frac{c}{b}\right)\left(\frac{1}{b\bar{b}}\right)}_{\text{GROUND REACTION}}$$

⑳ $$\frac{N}{I_Z}=\left[C_n+C_{n_r}\left(\frac{rb}{V}\right)+C_{n_p}\left(\frac{pb}{V}\right)\right]\left(\frac{\rho v^2}{2}\right)\left(\frac{S}{m}\right)\left(\frac{1}{b\bar{c}}\right)+\underbrace{\frac{f_c(T, V, \delta R)}{\bar{c} m b^2}}_{\text{GROUND*}}$$

$$\underbrace{-R_N C_{n_t}\left(\frac{\rho v^2}{2}\right)\left(\frac{S}{m}\right)\left(\frac{1}{b\bar{c}}\right)}_{\text{STRUCTURAL DEFLECTION}}+\underbrace{\left[\frac{G}{c}\times\frac{X'}{m}-\frac{H}{c}\times\frac{Y'}{m}\right]\left(\frac{1}{b\bar{c}}\right)\left(\frac{c}{b}\right)}_{\text{DISTANCE, C.G. TO REF.}}$$

$$\underbrace{+\left[\frac{T_1-T_4}{m}\right]\left(\frac{y_1}{b}\right)\left(\frac{1}{b\bar{c}}\right)+\left[\frac{T_2-T_3}{m}\right]\left(\frac{y_2}{b}\right)\left(\frac{1}{b\bar{c}}\right)}_{\text{UNSYMMETRICAL THRUST}}$$

$$\underbrace{+\left(\frac{K_b n q}{m}\right)\left(\frac{1}{b}\right)\left(\frac{1}{b\bar{c}}\right)}_{\text{PROPELLER}}\underbrace{+K_d\left(BP_R-BP_L\right)\left(g+\frac{\lambda}{m}\right)\left(\frac{G'}{4c}\right)\left(\frac{c}{b}\right)\left(\frac{1}{b\bar{c}}\right)}_{\text{GROUND BRAKING}}$$

㉑ $$\frac{L}{I_X}=\left[C_l+C_{l_p}\left(\frac{pb}{V}\right)+C_{l_r}\left(\frac{rb}{V}\right)\right]\left(\frac{\rho v^2}{2}\right)\left(\frac{S}{m}\right)\left(\frac{1}{b\bar{a}}\right)$$

$$\underbrace{+\left[\frac{D}{c}\times\frac{Y'}{m}+\frac{G}{c}\times\frac{Z'}{m}\right]\left(\frac{c}{b}\right)\left(\frac{1}{b\bar{a}}\right)}_{\text{DISTANCE, C.G. TO REF.}}$$

* EFFECT OF ENGINE THRUST DURING TAKE-OFF

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HINGE MOMENT EQUATIONS

$$\text{(22)}\quad H_E = \left[K_{H_E} + K_{H_{E_q}}\left(\frac{qc}{V}\right) + K_{H_{E_{\dot\alpha}}}\left(\frac{\dot\alpha c}{V}\right)\right]\left(\frac{\rho V^2}{2}\right) + f_d(T, V, \delta E)$$

$$\text{(23)}\quad H_R = \left[K_{H_R} + K_{H_{R_r}}\left(\frac{rb}{V}\right)\right]\left(\frac{\rho V^2}{2}\right) + f_e(T, V, \delta R)$$

$$\text{(24)}\quad H_{A_L} = \left[K_{H_{AL}} + K_{H_{AL_p}}\left(\frac{pb}{V}\right)\right]\left(\frac{\rho V^2}{2}\right)$$

$$\text{(25)}\quad H_{A_R} = \left[K_{H_{AR}} + K_{H_{AR_p}}\left(\frac{pb}{V}\right)\right]\left(\frac{\rho V^2}{2}\right)$$

AUXILIARY EQUATIONS

$$\text{(26)}\quad C_L = f_1(\alpha, \delta F, \Sigma T'_c) \times f_2(\gamma) + f_3(\delta E)(1 + K_1 \times \Sigma T'_c) + f_4(\alpha, m_n)$$

$$+ M_1 - \text{STALL TERM}$$

$$\text{(27)}\quad C_D = f_5(\alpha, \delta F) + f_6(\gamma) + f_7(\delta R, \gamma) + f_8(\delta E, \alpha)$$

$$+ \underbrace{K_2 C_{f_1} + K_3 C_{f_2} + K_4 C_{f_3} + K_5 C_{f_4}}_{\text{COWL FLAPS}} + \underbrace{K_6 L_G}_{\text{LANDING GEAR}}$$

$$+ \underbrace{K_7 B_D}_{\text{BOMB DOORS}} + f_9(\alpha, m_n) + M_2$$

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㉘
$$C_F = f_{10}(\gamma, T'_{c_{2+3}}) + K_8 T'_{c_2} + K_9 T'_{c_3} + K_{10}\gamma\alpha + K_{11}\gamma\delta F$$

$$+ f_{11}(\delta R)(1 + K_{12}T'_{c_2} + K_{13}T'_{c_3}) + M_3 + M_4\gamma + M_5 \cdot \delta R$$

㉙
$$C_m = f_{12}(\alpha, \delta E, \delta F, T'_{c_{2+3}}) + f_{13}(\alpha, [T'_{c_2} - T'_{c_3}]) + K_{15}T'_{c_1}$$

$$+ K_{16}T'_{c_4} + f_{14}(\gamma, \alpha) + f_{15}(\delta R, \gamma) + K_{17}\delta E_t + K_{18}\delta A_L$$

$$+ K_{19}\delta A_R + \underbrace{K_{20}C_{f_1} + K_{21}C_{f_2} + K_{22}C_{f_3} + K_{23}C_{f_4}}_{\text{COWL FLAPS}}$$

$$+ K_{24}L_G + (K_{25} + K_{25.1}\alpha)B_D + f_{16}(\alpha, m_n) + \underbrace{f_{17}(h) \times \alpha}_{\text{GROUND EFFECT}}$$

$$+ M_6 + M_7\alpha + M_8\delta E$$

㉚
$$C_n = f_{18}(\gamma, \delta F, \alpha) + f_{19}(\gamma, \delta R, T'_{c_{2+3}}) + f_{19.1}(T'_{c_2} - T'_{c_3})$$

$$+ f_{20}(\delta A_L, \alpha) + f_{21}(\delta A_R, \alpha) + K_{26.9}(\delta R_t)$$

$$+ \underbrace{K_{27}\gamma B_D}_{\text{BOMB DOORS}} + M_9 + M_{10}\gamma + M_{11}\delta R \pm \text{STALL TERM}$$

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㉛ $$C_l = f_{23}(\gamma, \alpha, \delta F) + \left[f_{24}(\alpha, \delta A_L) + f_{25}(\alpha, \delta A_R)\right]\left(1 - \frac{V}{V_d}\right)^{K_{27.1}}$$

$$+ f_{26}(\delta R) + (K_{28} + K_{29}\gamma)T'_{c_1} + (K_{30} + K_{31}\gamma)T'_{c_2}$$

$$+ (K_{32} + K_{33}\gamma)T'_{c_3} + (K_{34} + K_{35}\gamma)T'_{c_4} + K_{36}\,\delta A_{L_t}$$

$$+ K_{37}\,\delta A_{R_t} + M_{12} + M_{13}\gamma + M_{14}\,\delta R$$

㉜ $$C_{m_q} = f_{27}(T'_{c_{2+3}})$$

㉝ $$C_{m_{\dot\alpha}} = f_{28}(T'_{c_{2+3}})$$

㉞ $$C_{n_r} = f_{29}(\alpha, \delta F) + K_{38}(T'_{c_1} + T'_{c_4}) + K_{38.1}(T'_{c_2} + T'_{c_3})$$

㉟ $$C_{n_p} = f_{30}(\alpha, \delta F)$$

㊱ $$C_{l_p} = f_{32}(\alpha, \delta F)$$

㊲ $$C_{l_r} = f_{34}(\alpha, \delta F)$$

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(38) $C_{m_t} = K_{39.1} + K_{39.2}\,\delta F + K_{39.3}\,\delta E + \left[K_{39.4} + K_{39.5} T'_{c_{2+3}}\right]\alpha$

(39) $C_{n_t} = \left[K_{39.6} + K_{39.7} T'_{c_{2+3}}\right]\gamma + \left[K_{39.8} + K_{39.9} T'_{c_{2+3}}\right]\delta R$

(40) $K_{H_E} = f_{38}(\alpha, \delta F, \delta E, T'_{c_2}, T'_{c_3}) + f_{39}(\delta E_t, \delta E, T'_{c_{2+3}})$

$+ M_{15}\alpha + M_{16}\,\delta E$

(41) $K_{H_{Eq}} = f_{40}(T'_{c_2}, T'_{c_3})$

(42) $K_{H_{E\dot{\alpha}}} = f_{41}(T'_{c_2}, T'_{c_3})$

(43) $K_{H_R} = f_{42}(\gamma, \delta R, T'_{c_2}, T'_{c_3}) + f_{43}(\delta R_t, \delta R, T'_{c_{2+3}}) f_{44}(\gamma)$

$+ f_{45}(\gamma, \alpha, \delta F) + M_{17}\gamma + M_{18}\,\delta R$

(44) $K_{H_{R\pi}} = f_{46}(T'_{c_{2+3}})$

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㊺ $$K_{H_{AL}} = f_{47}(\delta A_L, \alpha, \delta F) + f_{48}(\delta A_L, \gamma) + f_{49}(\alpha, \gamma)$$

$$+ f_{50}(\delta A_{Lt}) + M_{19}\alpha + M_{20}\,\delta A_L$$

㊻ $$K_{H_{AR}} = f_{51}(\delta A_R, \alpha, \delta F) + f_{52}(\delta A_R, \gamma) + f_{53}(\alpha, \gamma)$$

$$+ f_{54}(\delta A_{Rt}) + M_{21}\alpha + M_{22}\,\delta A_R$$

㊼ $$K_{H_{ALp}} = K_{43}$$

㊽ $$K_{H_{ARp}} = K_{44}$$

INSTRUMENT EQUATIONS

㊾ AIR SPEED METER READING, $V_c = f_{55}(\text{I.A.S.}, h)$

㊿ RATE OF CLIMB, $\text{R.C.} = u \sin\theta - v \cos\theta \sin\varphi - w \cos\theta \cos\varphi$

(51) ALTIMETER, $h = \int_0^t (\text{R.C.})\, dt$

(52) PITCH BAR OF GYRO HORIZON $= \theta$

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(53) BANK ANGLE OF GYRO HORIZON = $\sin^{-1}(\cos\theta \sin\varphi)$

(54) RATE OF TURN = r

(55) BALL BANK ANGLE, $\mu' = \tan^{-1}\frac{a_Y}{a_Z}$

(56) COMPASS HEADING = ψ

ENGINE EQUATIONS-FOR ONE INTERNAL COMBUSTION RECIPROCATING ENGINE.

(57) $Q = K_{45} N e_i$

(58) $P_2 = f_{56}(P_1, N)$

(59) $\Delta P_c = f_{57}(Q, \pi)$

(60) CHOOSE Q IN $\Delta P_c = f_{57}(Q, \pi)$ SO THAT $\frac{P_2 + \Delta P_c}{P_2} \leqq .53$

(61) $P_3 = P_2 + \Delta P_c$

(62) $\frac{P_4}{P_3} = f_{58}(N, \text{BLOWER POSITION})$

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(63) $P_4 = MP = P_3 \left(\frac{P_4}{P_3}\right)$

(64) $e_i = K_{46}\ MP$

(65) $R = f_{59}(MP, N)$

(66) $IHP = K_{47}\ Q\ R$

(67) $FHP = f_{60}(N)$

(68) $y_{a_1} = f_{60.1}\left(\frac{P_2}{P_1}\right)$

(69) $y_{a_2} = f_{60.2}\left(\frac{P_4}{P_3}\right)$

(70) $SHP_A = K_{48}\ Q\ y_{a_1}$

(71) $SHP_M = K_{48}\ Q\ y_{a_2}$

(72) $\Delta HP = f_{61}(h)$

(73) $BHP = IHP - FHP - SHP_A - SHP_M + \Delta HP$

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(74) $PHP = K_{50} C_P \, \rho \, n^3$

(75) $C_P = f_{62}(\beta, J)$

(76) $J = K_{51} \frac{V}{n}$

(77) $PHP = BHP$

(78) $C_T = f_{63}(\beta, J)$

(79) $T_1 = K_{52} C_T \, \rho \, n^2$

MISCELLANEOUS EQUATIONS

(80) $m_n = \frac{V}{V_s}$

(81) $V_s = f_{64}(h)$

(82) $\rho = f_{65}(h)$

(82a) $\rho/\rho_0 = f_{65.1}(h) = \sigma$

(82b) $I.A.S. = V\sqrt{\frac{\rho}{\rho_0}} = V\sqrt{\sigma}$

(83) $P_1 = f_{66}(h)$

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(84) $T_{c_1}' = \frac{T_1}{1/2 \rho V^2 S}$

(85) $T_{c_2}' = \frac{T_2}{1/2 \rho V^2 S}$

(86) $T_{c_3}' = \frac{T_3}{1/2 \rho V^2 S}$

(87) $T_{c_4}' = \frac{T_4}{1/2 \rho V^2 S}$

(88) $T_c' = T_{c_1}' + T_{c_2}' + T_{c_3}' + T_{c_4}'$

(89) $T_{c_{2+3}}' = T_{c_2}' + T_{c_3}'$

(90) $T = T_1 + T_2 + T_3 + T_4$

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TABLE OF SYMBOLS

Symbols for Equations of Motion

AXES

1) Earth Axes

   A - North

   B - East

   C - Down toward center of earth

2) Body Axes - Origin at airplane Center of Gravity

   X - forward along thrust line

   Y - along right wing

   Z - down perpendicular to XOY

3) Wind Axes - Origin at airplane Center of Gravity

   ξ - forward along relative wind (instantaneous flight path)

   η - toward right wing perpendicular to $O\xi$

   ζ - down perpendicular to $\xi O \eta$

FORCES

1) Along Earth Axes

   W - weight of airplane along OC

2) Along Body Axes

   X' - along OX

   Y' - along OY

   Z' - along OZ

3) Along Wind Axes

   Δ - along $O\xi$ in the negative drag direction

   F - along $O\eta$ in the positive side force direction

   λ - along $O\zeta$ in the negative lift direction

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MOMENTS Ft. Pounds

1) About Body Axes

L - Rolling moment about OX
M - Pitching moment about OY
N - Yawing moment about OZ

LINEAR VELOCITIES Feet per Second

1) Along Wind Axes

V - resultant velocity along $O\xi$

The velocities along $O\zeta$ and $O\eta$ are zero.

2) Along Body Axes

u - velocity along OX
v - velocity along OY
w - velocity along OZ

ANGULAR VELOCITIES Radians per Second

1) About Earth Axes

$\frac{d\phi}{dt}$ - about OX
$\frac{d\theta}{dt}$ - about line in intersection of plane AOB and YOZ
$\frac{d\psi}{dt}$ - about OC

2) About Body Axes

p - about OX
q - about OY
r - about OZ

3) About Wind Axes $\dot{\alpha}$ about $O\eta$

ACCELERATIONS Ft. per Second per Second

1) Along Earth Axes - g - gravity along OC

2) Along Body Axes

a_X - along OX
a_Y - along OY
a_Z - along OZ

MOMENTS OF INERTIA Pound - Feet - Second^2

1) About Body Axes

I_x - about OX = $\bar{a}mb^2$
I_Y - about OY = $\bar{b}mb^2$
I_z - about OZ = $\bar{c}mb^2$

ANGLES Degrees

θ - angle between OX and plane AOB in plane XOC
ψ - angle between OA and plane XOC in plane AOB
φ - angle between OY and plane AOB in plane YOZ
α - angle between OX and plane $\xi O \eta$ in plane XOZ
γ - angle between $O\xi$ and plane XOZ in plane $\xi O \eta$

TIME

t - seconds

SYMBOLS FOR AUXILIARY EQUATIONS

C_L - lift coefficient - static
C_D - drag coefficient - static
C_Y - side force coefficient - static
C_m - pitching moment coefficient - static
C_n - yawing moment coefficient - static
C_ℓ - rolling moment coefficient - static
C_{m_q} - pitching moment coefficient due to pitching velocity: $\left(\frac{V}{c}\right)\left(\frac{\delta C_m}{\delta q}\right)$
$C_{m_{\dot{\alpha}}}$ - pitching moment coefficient due to rate of change of angle of attack $\left(\frac{V}{c}\right)\left(\frac{\delta C_m}{\delta \dot{\alpha}}\right)$
C_{n_r} - yawing moment coefficient due to yawing velocity: $\left(\frac{V}{b}\right)\left(\frac{\delta C_n}{\delta r}\right)$
C_{n_p} - yawing moment coefficient due to rolling velocity: $\left(\frac{V}{b}\right)\left(\frac{\delta C_n}{\delta p}\right)$
C_{ℓ_p} - rolling moment coefficient due to rolling velocity: $\left(\frac{V}{b}\right)\left(\frac{\delta C_\ell}{\delta p}\right)$
C_{ℓ_r} - rolling moment coefficient due to yawing velocity: $\left(\frac{V}{b}\right)\left(\frac{\delta C_\ell}{\delta r}\right)$
F_S - stick or column force - pounds
F_R - rudder pedal force - pounds
F_W - aileron wheel force - pounds
H_E - elevator hinge moment - ft. lbs.
H_R - rudder hinge moment - ft. lbs.

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H_{A_L} - left aileron hinge moment - ft. lbs.

H_{A_R} - right aileron hinge moment - ft. lbs.

K_{H_E} - elevator hinge moment modulus - ft.^{3}

K_{H_R} - rudder hinge moment modulus - ft.^{3}

$K_{H_{AL}}$ - left aileron hinge moment modulus - ft.^{3}

$K_{H_{AR}}$ - right aileron hinge moment modulus - ft.^{3}

$K_{H_{E_q}}$ - elevator hinge moment modulus due to pitching velocity: $\left(\frac{V}{c}\right)\left(\frac{\delta K_{H_E}}{\delta q}\right)$

$K_{H_{E_{\dot{\alpha}}}}$ - elevator hinge moment modulus due to rate of change of angle of attack: $\left(\frac{V}{c}\right)\left(\frac{\delta K_{H_E}}{\delta \dot{\alpha}}\right)$

$K_{H_{R_r}}$ - rudder hinge moment modulus due to yawing velocity: $\left(\frac{V}{b}\right)\left(\frac{\delta K_{H_R}}{\delta r}\right)$

$K_{H_{AL_p}}$ - left aileron hinge moment modulus due to rolling velocity: $\left(\frac{V}{b}\right)\left(\frac{\delta K_{H_{AL}}}{\delta p}\right)$

$K_{H_{AR_p}}$ - right aileron hinge moment modulus due to rolling velocity: $\left(\frac{V}{b}\right)\left(\frac{\delta K_{H_{AR}}}{\delta p}\right)$

C_{m_t} - pitching moment coefficient contributed by horizontal tail

C_{n_t} - yawing moment coefficient contributed by vertical tail

θ_o - angle between OX and ground with plane at rest

δE - elevator angle

δR - rudder angle

δA - total aileron angle = $\delta A_L + \delta A_R$

δA_L - left aileron angle

δA_R - right aileron angle

δE_t - elevator tab angle

δR_t - rudder tab angle

δA_{L_t} - left aileron tab angle

δA_{R_t} - right aileron tab

δF - flap angle

T_1 - thrust delivered by port outboard engine - pounds

T_2 - thrust delivered by port inboard engine - pounds

T_3 - thrust delivered by starboard inboard engine - pounds

T_4 - thrust delivered by starboard outboard engine - pounds

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T - total thrust (four engines) - pounds

T_j - constant thrust from jato installation operative for a specified length of time

T'_c - total thrust coefficient (four engines)

$T'_{c_{2+3}}$ - total thrust coefficient (port and starboard inboard engines)

T'_{c_1} - thrust coefficient (port outboard engine)

T'_{c_2} - thrust coefficient (port inboard engine)

T'_{c_3} - thrust coefficient (starboard inboard engine)

T'_{c_4} - thrust coefficient (starboard outboard engine)

L_G - % landing gear deflection

B_D - % bomb door deflection

C_{f_1} - % cowl flap deflection port outboard engine

C_{f_2} - % cowl flap deflection port inboard engine

C_{f_3} - % cowl flap deflection starboard inboard engine

C_{f_4} - % cowl flap deflection starboard outboard engine

BP_L - % brake pressure - left brake

BP_R - % brake pressure - right brake

m - airplane mass - slugs

s - wing area - sq. ft.

b - span - ft.

c - mean aerodynamic chord - ft.

y_1 - distance to outboard motor ℄ along **OY** - feet

y_2 - distance to inboard motor ℄ along **OY** - feet

c.g. - center of gravity

D - distance of c.g. below reference point along **OZ** - feet

D' - distance below reference point to point of contact of main wheels and ground along **OZ** - feet

G - distance of c.g. to right of reference point along **OY** - feet

G' - main wheel tread - feet

H - distance of c.g. ahead of reference point along **OX** - feet

H' - distance forward of reference point to point of contact of main wheels and ground along **OX** - feet

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$\bar{a}$ - moment of inertia coefficient - OX axis

$\bar{b}$ - moment of inertia coefficient - OY axis

$\bar{c}$ - moment of inertia coefficient - OZ axis

ρ_0 - air density (standard sea level) - pound-sec^2 per ft.^{4}

ρ - air density (standard at altitude) - pound-sec^2 per ft.^{4}

σ - density ratio

h - altitude - ft.

V_s - velocity of sound - ft/sec

V_d - aileron divergence speed - ft/sec

m_n - mach number

μ - coefficient of ground friction

μ' - ball bank angle - degrees

R_M - correction factor to pitching moment due to airplane structural deflection

R_N - correction factor to yawing moment due to airplane structural deflection

$f(\ )$ - designates a function of the variable in ( ). In general represented by a curve or family of curves.

K_1 etc.- aerodynamic constants

M_1 etc.- manual setting constants

SYMBOLS FOR ENGINE EQUATIONS

A - displacement of engine - cubic inches

N - engine revolutions per minute

n - engine revolutions per second

Q - mass air flow through engine - pounds per hour

M - mass air flow through engine - pounds per second

π - throttle angle - degrees

ρ_i - inlet density (density at cylinder inlet) pounds - sec^2 per ft.^{4}

MP - manifold pressure - inches of mercury absolute

P_c - pressure drop through carburetor - inches of mercury absolute

P_1 - atmosphere pressure - pressure at inlet of auxiliary stage - inches H_g absolute

P_2 - pressure at exit of auxiliary stage - in H_g absolute

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P_3 - pressure at exit of carburetor - pressure at inlet to main stage of supercharger - inches H_g absolute

P_4 - pressure at exit of main stage of supercharger = MP

R - correction factor to IHP

IHP - indicated horsepower

FHP - friction horsepower

SHP_M - supercharger horsepower for main stage

SHP_A - supercharger horsepower for auxiliary stage

ΔHP - horsepower increase due to decreased back pressure

BHP - brake horsepower

PHP - propeller horsepower

G.R. - gear ratio

D_p - propeller diameter feet

J - advance diameter ratio

C_p - propeller power coefficient

C_t - propeller thrust coefficient

β - propeller blade angle - degrees

y_{a_1} - adiabatic factor for auxiliary stage

y_{a_2} - adiabatic factor for main stage

K_{48} - constant $\frac{j C'_p t_1}{550 \eta_1}$

j - mechanical equivalent of heat

t_1 - inlet temperatures to both auxiliary and main stages of superchargers - assumed to be constant - absolute 520° F.

C'_p - specific heat of air at constant pressure

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η_1 - supercharger efficiency assumed to be constant

G_s - governor setting

Compiled by: ____________________

Approved: ____________________

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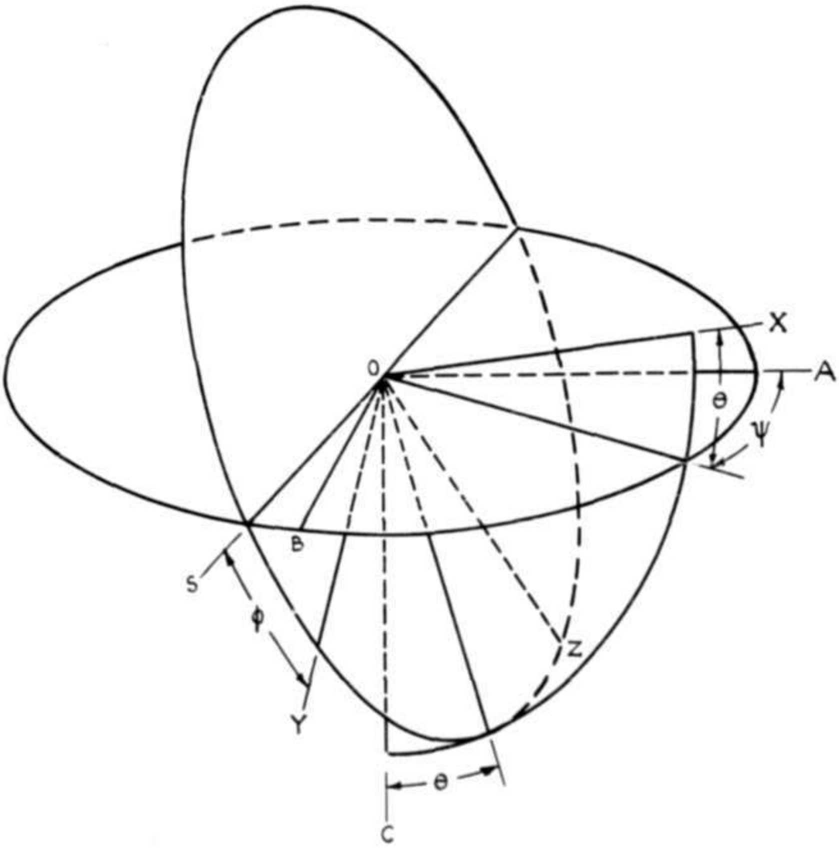
Report 64,
REVISION 1



A, B, C are earth axes,
oA toward north.

X, Y, Z are body axes,
ox toward airplane nose.

FIG. 1 - Sketch of Earth and Body Axes

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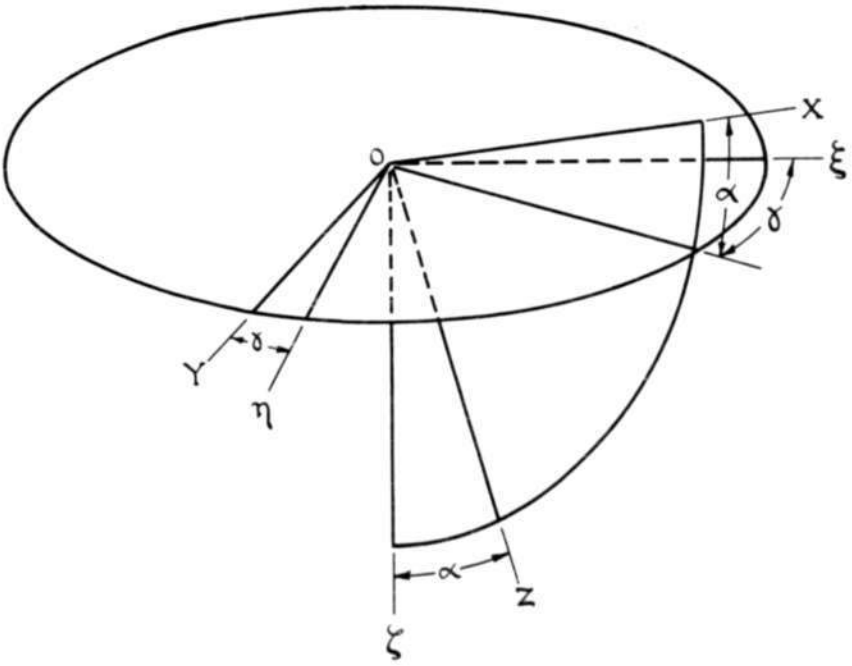

ξ, η, ζ are wind axes

X, Y, Z, are body axes

FIG. 2

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REVISION I

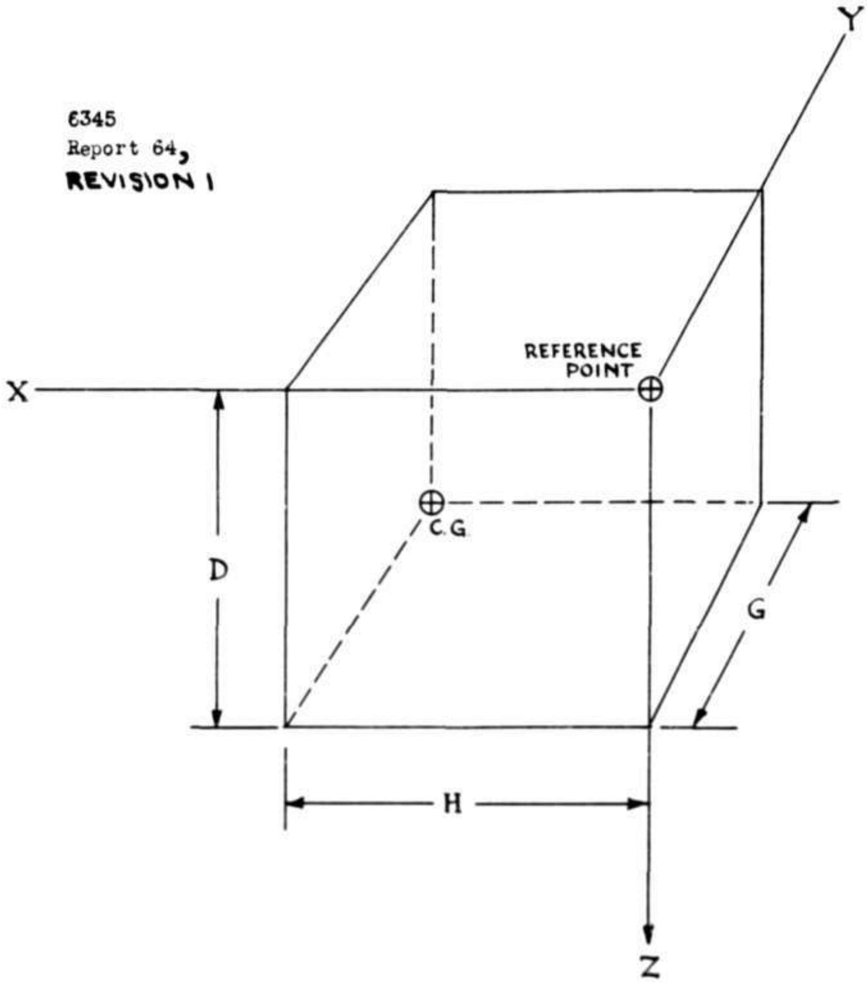

FIG. 8 - Sketch of C. G. Location

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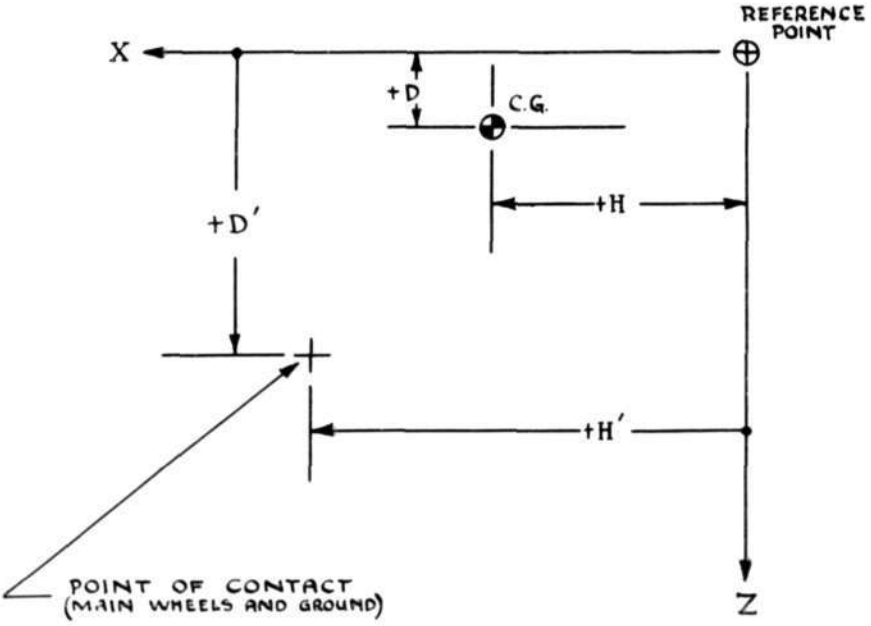

FIG. 4 - Location of Point of Contact Between Main Wheels and Ground

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APPENDIX B
to

PROJECT
WHIRLWIND
(DEVICE RF-12)

SUMMARY REPORT NO. 1
April 1946

NON-LINEAR COEFFICIENT EXAMPLES

The following non-linear coefficients are typical examples of the different types of input data. These examples were chosen as illustrations of the variety of slopes, magnitudes and inflexion points encountered in wind tunnel data.

The largest term in the expression for lift coefficient, $f_1(\alpha, \delta F, T_c')$ is shown on drawing B-38000-G for $T_c' = 0$. The curves for different flap angles, δF, are approximately parallel and straight up to the stall point. At the stall point the curves are not very critical for computations in the analyzer since the stall condition will be handled empirically and the curves will be modified at this point.

$f_4(\alpha, m_n)$, drawing A-38005-G, shows curves which cross each other. It will be noted that the cross-overs occur at high mach numbers (above 0.5) corresponding to high airplane speeds. This contribution to the lift coefficient is limited to low angles of attack since it is impossible to fly at high angles of attack and high speeds.

$f_7(\delta R, \gamma)$, drawing B-38008-G, illustrates a smoothly varying function whose curves cross at the origin. This contribution to the total drag coefficient is small or negligible in normal flight but may become as much as 50% immediately after an outboard engine failure.

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$f_{13}(\alpha, T'_{c_2} - T'_{c_3})$, drawing A-38046-G, is a function which is difficult to represent as an analytical function and, if represented by a function table, presents difficulty in interpolation in the region near the origin.

$f_{19}(\gamma, \delta R, T'_{c_{2+3}})$, drawing A-38053-G, is typical of a family of curves considerably different from the other curves discussed.

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APPENDIX C
to

PROJECT
WHIRLWIND
(DEVICE RF-12)

SUMMARY REPORT NO. 1
April 1946

TWO-STAGE POTENTIOMETER MULTIPLIER

A single-stage potentiometer multiplier is shown on Sketch 1 of the following page. Input A is supplied as an electrical voltage and input B is converted to shaft rotation by means of the servo system using a feedback potentiometer supplied with constant voltage E. The product, C. is obtained as a voltage output. For a given voltage input A, an error in C is introduced by inaccuracies in the resistance element, by back-lash or looseness in the control shaft and contact assembly and by lack of sensitivity arising from a finite number of increments in the resistance element and from the dead zone in the servo system. Since errors due to these sources are approximately constant, that is, independent of shaft angle B, while the output voltage is proportional to B, fractional errors in the output are approximately inversely proportional to B. This limitation becomes serious when one attempts to cover a wide dynamic signal range.

The two-stage multiplier extends the useful range of sensitivity by using two potentiometers in cascade. Such a system is illustrated in the second sketch. The first potentiometer slider is driven through the scale factor angle 1/K while the second set of potentiometers perform the multiplication (B')(1/K). Because of the corresponding connection of the servo feedback potentiometers, angle B' equals the scale factor K times B.

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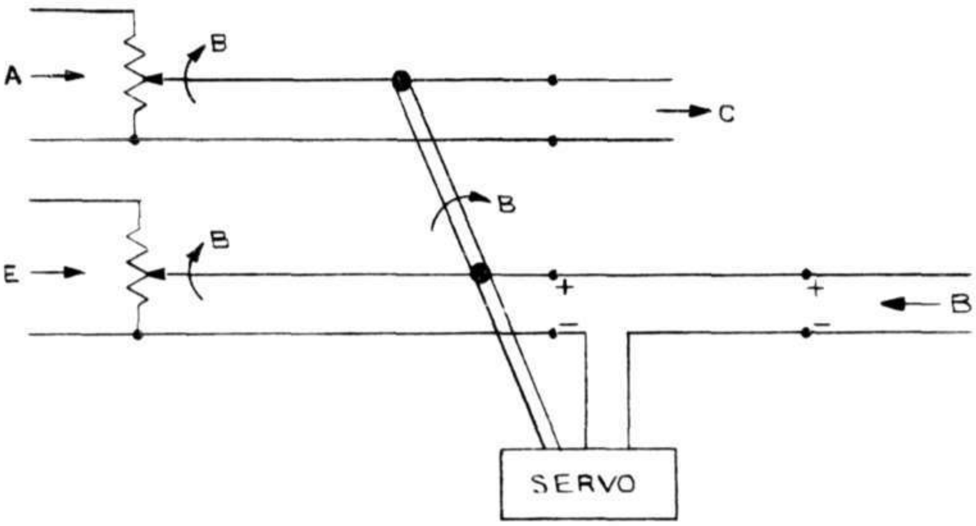
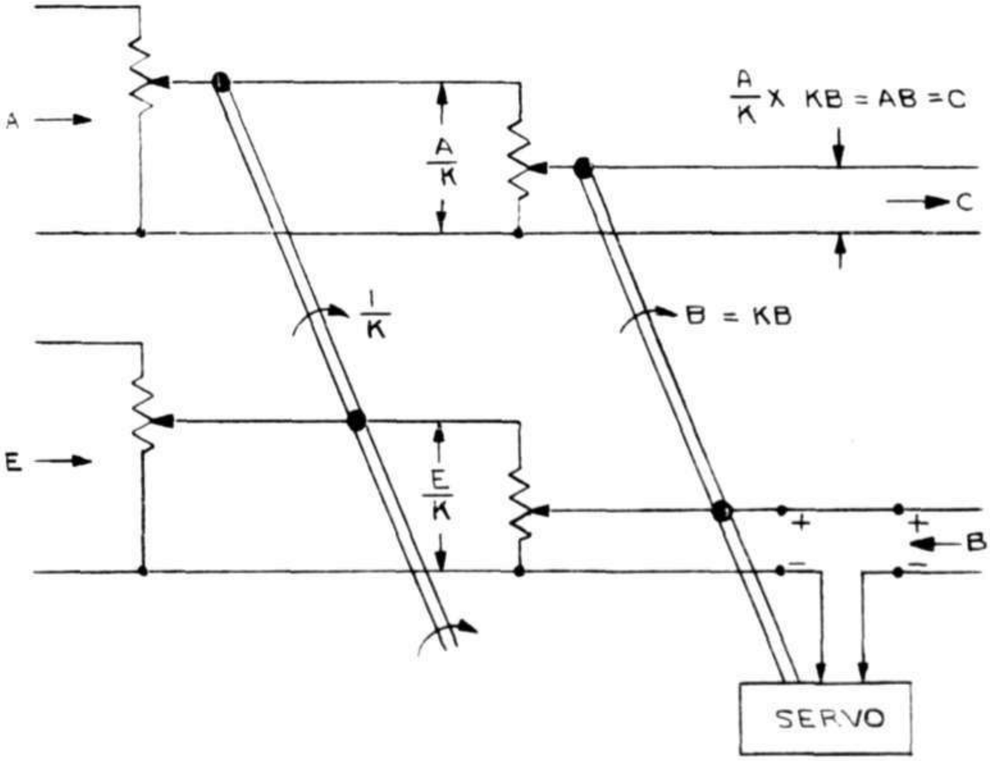


SKETCH 1



SKETCH 2

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In operation, shaft positions 1/K will be adjusted to maintain shaft position B' well above the level of minimum sensitivity and noise. Shaft position 1/K can be adjusted at the beginning of a particular computation to provide the proper scale factor for the following series of computations. However, as was planned for this analyzer, the shaft position 1/K can be continuously adjusted by a servo system loosely coupled to shaft position B'. Such coupling system must be made through potentiometers, tachometers, and limit stops to properly coordinate the operation of the two potentiometer sets.

The objective of the cascaded two-stage multiplier is primarily to increase the range of sensitivity rather than to improve accuracy.

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APPENDIX D
to
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WHIRLWIND
(DEVICE RF-12)

SUMMARY REPORT NO. 1
April 1946

FREQUENCY-MODULATED SERVO

An electro-mechanical computer requires numerous servos to drive its mechanical elements. For a real-time aircraft analyzer analogy computer, the requirements for these servos would be particularly severe.

Wherever forces and accelerations are converted from electrical signals to the mechanical positions of integrator carriages or multiplier potentiometers, servos must be used which are capable of following the rapid fluctuations possible in these variables.

The approximate specifications for the type of computing elements being considered are:

1. Peak power outputs of about 100 watts
2. Low standby losses
3. Very high static accuracy including high stiffness
4. Extremely good transient response including high maximum accelerations
5. 400 cps data

A short survey of readily available servo types discovered none that showed promise of meeting these requirements. Instead, a new type of servo was developed utilizing the desirable characteristics of a frequency controlled low slip induction motor. A block schematic of this system is shown in Drawing B-30010. A frequency controlled polyphase induction motor is desirable for the following reasons:

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1. High torque-slip ratio leads to good transient response and high accelerations and stiffness at all operating speeds.
2. Standby heating is low because rotor losses are zero at standstill. High peak power outputs over short time intervals are thus available with small motors and low continuous heat-dissipating capacity.

Supplying such a motor with varying frequencies requires considerable equipment. These frequencies must vary from a maximum in one phase rotation to a maximum in the opposite phase rotation through all frequencies in between, including zero. All circuits carrying motor power must, therefore, be able to handle direct current as well as alternating currents up to perhaps 60 cps.

This requirement is met by using a beat-frequency oscillator, the variable frequency unit of which is adjustable both above and below the frequency of the fixed frequency oscillator. The fixed frequency oscillator produces 3-phase voltages at oscillator frequency. These are beat against the single-phase variable frequency resulting in 3-phase difference frequencies.

The servo error signal is detected and the resulting d-c supplied to the grid of a reactance tube across the plate load of the variable frequency oscillator. An oscillator output frequency proportional to servo error is thus obtained.

The 3-phase difference frequencies are supplied to the thyratron power stages which supply the motor. The 3 power stages each consist of a pair of gas triodes with a-c plate voltage supplied through transformers. Which tube is to fire and over how much of a plate-voltage cycle is determined by the phase and magnitude of the oscillator output voltage supplied. The phase and magnitude of the voltage supplied to the motor winding is thus determined by the oscillator output, and the motor is supplied with 3-phase voltages of the required frequency.

The motor itself can be any polyphase induction motor but one especially designed for low inertia and low slip will provide better performance.

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An experimental servo of this type was built in a 50-watt size. It proved very satisfactory. With proper adjustment it was found possible to obtain transient decay times of the order of .01 seconds with loads comparable to integrator drives. Static errors were within the errors of the data system and static stiffness was adequate. Measured dynamic and steady state characteristics were in good agreement with expected performance.

The frequency modulated servo would be able to satisfy the more stringent servo requirements for the analogy computer. Considerable electronic equipment is needed for each servo but the amount is not excessive in view of the freedom from weight and space requirements in a computer of this type. The amount of equipment is also largely independent of servo rating; changing the size of the servo motor requires changing only the size of power stage thyratrons and transformers.

Further information as to the characteristics of the frequency-modulated servo may be found in 6345 Reports 74 through 83 listed in Appendix E.

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APPENDIX E
to
PROJECT
WHIRLWIND
(DEVICE RF-12)

SUMMARY REPORT NO. 1
April 1946

Below is a list of titles and brief descriptions of some reports which have been written during the progress of work on Contracts NOa(s)5216 and NOa(s)7082. Missing numbers in the series belong to reports which are obsolete or have been withdrawn from circulation.

Report No.	Date	
4	Jan. 16, 1945	Two-Phase Induction Motors A study of the Diehl FPF-49-5 squirrel cage two-phase induction motor as a torque motor for use in the control force loading equipment.
5	Jan. 20, 1945	Phase-Sensitive Detector A bridge-type diode phase-sensitive detector for use in converting a-c to d-c signals.
6	Jan. 7, 1945	Throttle Valve Preliminary tests of the first model designed for use in the control force loading equipment.
7	Jan. 17, 1945	Viscosity vs. Leakage Studies of a Variable Displacement Hydraulic Pump.
10	Jan. 17, 1945	Preliminary Estimates of Numerical Value Ranges For aircraft that are to be handled by the Stability Control Analyzer.

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Report No.	Date	
12	Jan. 22, 1945	Lift Coefficients Data on the nature of lift coefficients.
13	Jan. 26, 1945	Strain Gauges for Control Force Measurement Data on the Ruge-deForest Type wire strain gauge for use in control force loading equipment.
14	Jan. 30, 1945	Simulation of a Function of Several Variables This is a report written and issued originally by the Bell Telephone Laboratories.
18	Feb. 6, 1945	Aircraft Oscillations A listing of the kinds of oscillations with natural period and damping to be expected in large aircraft.
19	Feb. 7, 1945	60-Cycle Modulator Circuit An electronic modulator circuit for converting d-c to 60-cycle a-c signals.
21	Apr. 13, 1945	Control Transformers A report on laboratory tests of standard Navy control transformers.
22	Feb. 22, 1945	Preliminary Investigation of Summing Circuits For adding and subtracting 60-cycle a-c voltages.
23	Feb. 27, 1945	Precision Potentiometers Information on the Western Electric light-weight precision potentiometer.
24	Feb. 28, 1945	Lift, Drag, and Side Force Coefficients For use in the aircraft equations of motion.
26	Apr. 19, 1945	Parallel-T Networks Experimental studies of parallel-T networks for use as servo derivative networks where a 60-cycle carrier is used.

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Report No.	Date	
28	Feb. 1, 1945	Resolver - Voltage Output and Phase Shift The voltage output and phase shift of Arma Corporation electrical resolver.
29	Mar. 23, 1945	Summing Amplifiers, Electronic Second report.
30	Mar. 26, 1945	Potentiometers General comments.
33	Mar. 26, 1945	Tachometers Experimental tests of Arma and Kollsman a-c tachometers.
35	Mar. 23, 1945	Preliminary Tests of an Electro-Mechanical Integrator.
36	May 3, 1945	Description of Proposed Control Force Demonstrator
37	Apr. 28, 1945	Control Column Loading Equipment, Analysis of Component Parts
38	May 4, 1945	Control Column Loading Equipment, Stability and Gain of Proposed Closed-Cycle Schematic
41	Apr. 5, 1945	Strain Gauge Preamplifier
44	Apr. 20, 1945	Detectors, Phase-Sensitive, Full-Wave
48	May 4, 1945	Signal Calibrator
49	May 10, 1945	Numerical Ranges of Aerodynamic Constants
50	May 14, 1945	Equations of Motion, Revised with Sketches
52	May 18, 1945	Low-Pass RC Filter

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Report No.	Date	
53	June 1, 1945	Servo Amplifier Analysis
55	June 20, 1945	Noise Level Test Preliminary report.
58	Aug. 7, 1945	Control System Information Elevator, Aileron, and rudder data.
59	Aug. 21, 1945	Parallel-T as a Derivative Network Theoretical study.
61	Sept. 13, 1945	Kollsman Tachometer
62	Sept. 15, 1945	Standard Atmosphere Data Tabulation and curve.
63	Sept. 18, 1945	Conference Notes On discussion of methods to handle take-off and landing, stall representation, compressibility effects, cockpit instruments, engineer's panel, center of gravity, cockpit motion, vibration, and noise.
64	Oct. 31, 1945	Equations for Aircraft Flight Revised and expanded, April 4, 1946
65	Aug. 23, 1945	Magnetic Shielding A summary of some published works on shielding magnetic fields.
68	Oct. 1, 1945	Transformer Field Tests An investigation of the strength of the magnetic field produced by transformers under simulation conditions of normal operation.
69	Nov. 15, 1945	Phase Measurements A summary of published material on methods of measuring electrical phase angle.

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Report No.	Date	
70	Oct. 31, 1945	Response of Networks to Modulated Sinusoidal Inputs A theoretical treatment discussing transmission of modulated signals through linear networks.
71	Aug. 17, 1945	DC Voltage Regulators Circuits for a voltage regulator with an output impedance of approximately 0.01 ohm and 1 millivolt ripple.
74	Mar. 12, 1946	Thyratron Control Circuits Partial report on 60-cycle plate voltage operation for frequency modulated servo power stage.
75	Jan. 22, 1946	Frequency Modulated Servo System 3-Phase modulator stage.
77	Jan. 21, 1946	Frequency Modulated Servo System 10 kilocycle 3-phase and variable frequency oscillators.
78	Mar. 25, 1946	Frequency Modulated Servo System Report on power stage circuits.
79	Mar. 9, 1946	Frequency Modulated Servo System Design and test characteristics of the servo motor.
80	Mar. 1, 1946	Frequency Modulated Servo System Test results.
81	Mar. 7, 1946	Frequency Modulated Servo System Mathematical analysis of servo system.
82	Jan. 21, 1946	Frequency Modulated Servo System Phase-sensitive detector stage.
83	Mar. 7, 1946	Frequency Modulated Servo System Report on complete system.
84	Nov. 1, 1945	Regulated Power Supply Data on Western Electric power supply type No. CW 20 AAE, BUORD No. ESSXX 679247

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Report No.	Date	
86	Nov. 15, 1945	Noise Level Rack Tests Summary of noise level tests in analogue computing circuits.
88	Not issued	Coder for Digital Computer
89	Mar. 4, 1946	Timing Signal Generator A circuit for generating two-microsecond pulses at 125,000 pulses per second repetition rate.
90	Jan. 15, 1946	The Binary System of Numbers A description of the binary system with comments on binary and decimal equivalents, subtraction, multiplication, division, and complements.
93	Mar. 8, 1946	Rise Time of Short Coaxial Cables
94	Feb. 12, 1946	Some Psychiatric Observations on Cockpit Design by Dr. Holder of Massachusetts General Hospital
95	Dec. 7, 1945	Noise Level Test Rack Amplifier Tests
98	Mar. 7, 1946	Constants and Curves for a Twin Engine Airplane (called Model A) for Analyzer Equations. This is a set of constants and non-linear coefficients for a typical airplane as described by the equations in Report R-64 (Appendix A.)
99	Apr. 4, 1946	Design of the Mechanical System for Transmitting Control Forces from the Loading Equipment to the Cockpit Controls
100	Mar. 4, 1946	Cockpit Mounting and Actuating Mechanism Showing overhead cockpit mounting and comments on cockpit pitch, roll, and vertical motion.

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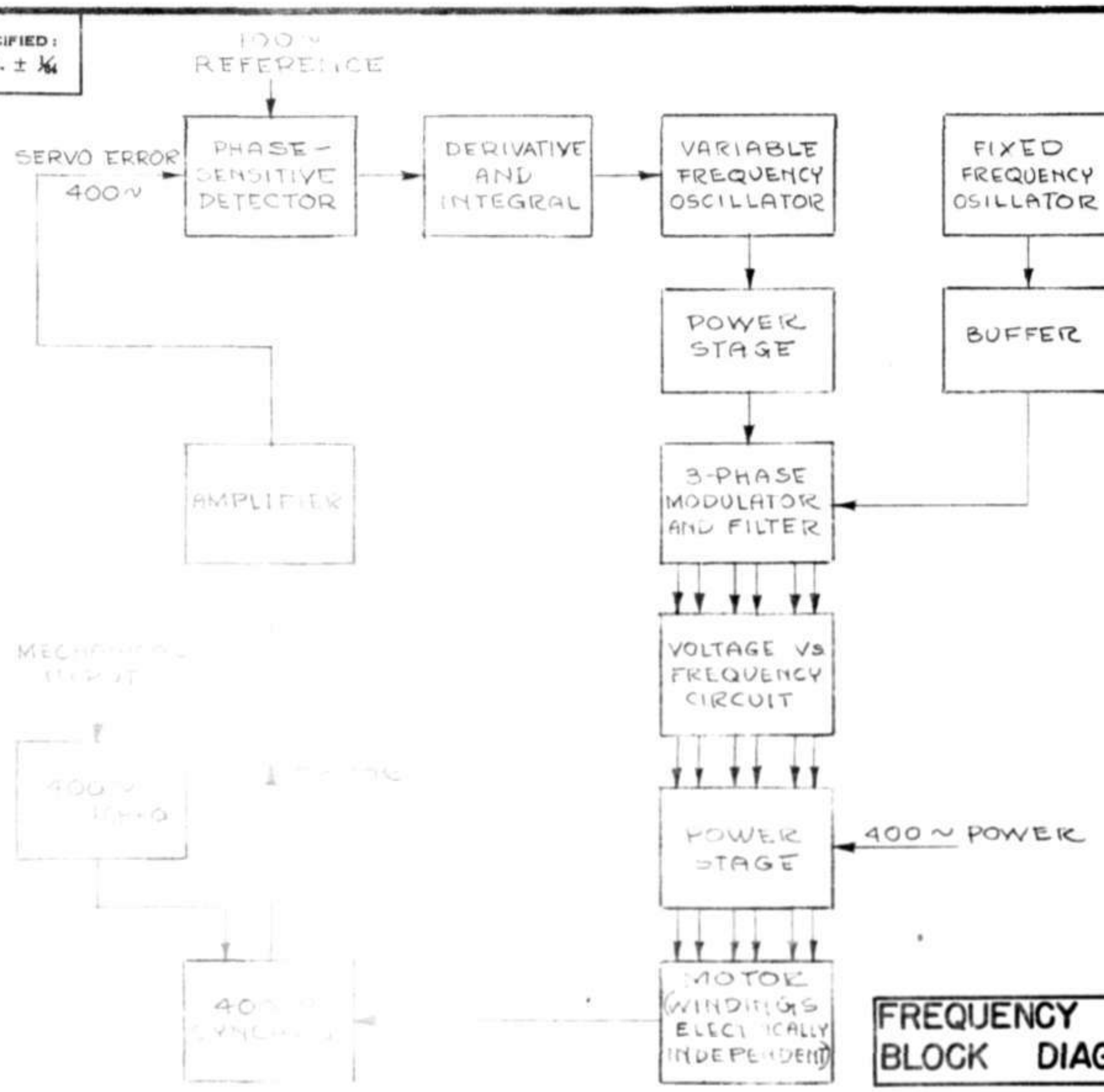
B-30010-1
WO-

Tolerances Not Otherwise Specified:
Decimal ± .005 Fractional ± 1/64



FREQUENCY SERVO BLOCK DIAGRAM

RESTRICTED

P				G				
N				F				
M				E				
L				D				
K				C				
J				B				
H				A	[illegible]	TL	[illegible]	
	WAS	APP	DATE		WAS	APP	DATE	

ITEM	MATERIAL - DESCRIPTION	PART NO.	QUAN.

SERVOMECHANISMS LABORATORY OF THE
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DIVISION OF INDUSTRIAL COOPERATION PROJECT NO. 6345

BLOCK SCHEMATIC VFS

SCALE: —
DR. [illegible]
TR. [illegible] 11-2
CK. TL 11/2/45
APP.

B-30010-1

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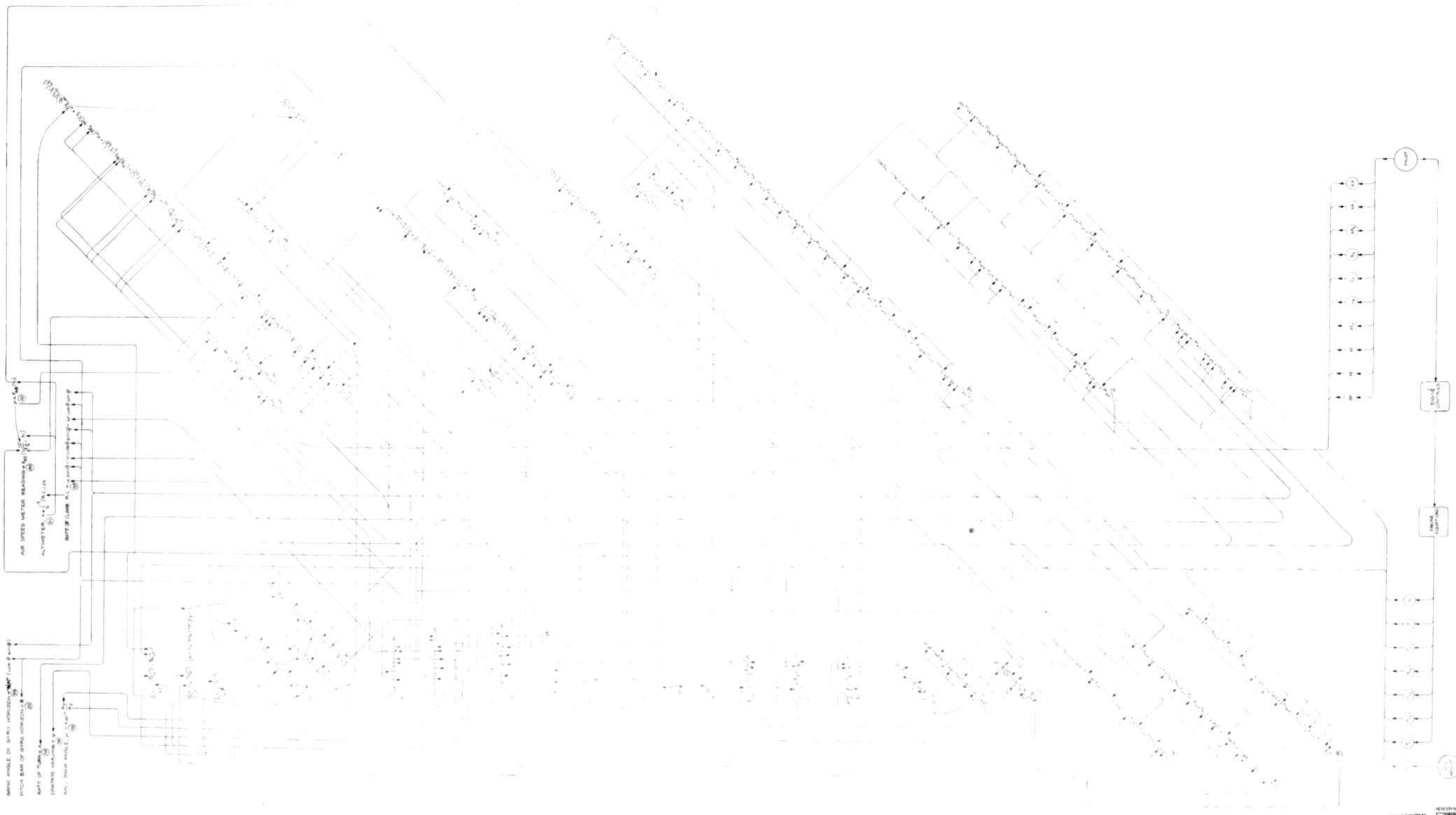

CONFIDENTIAL

AERODYNAMIC EQUATION SCHEMATIC

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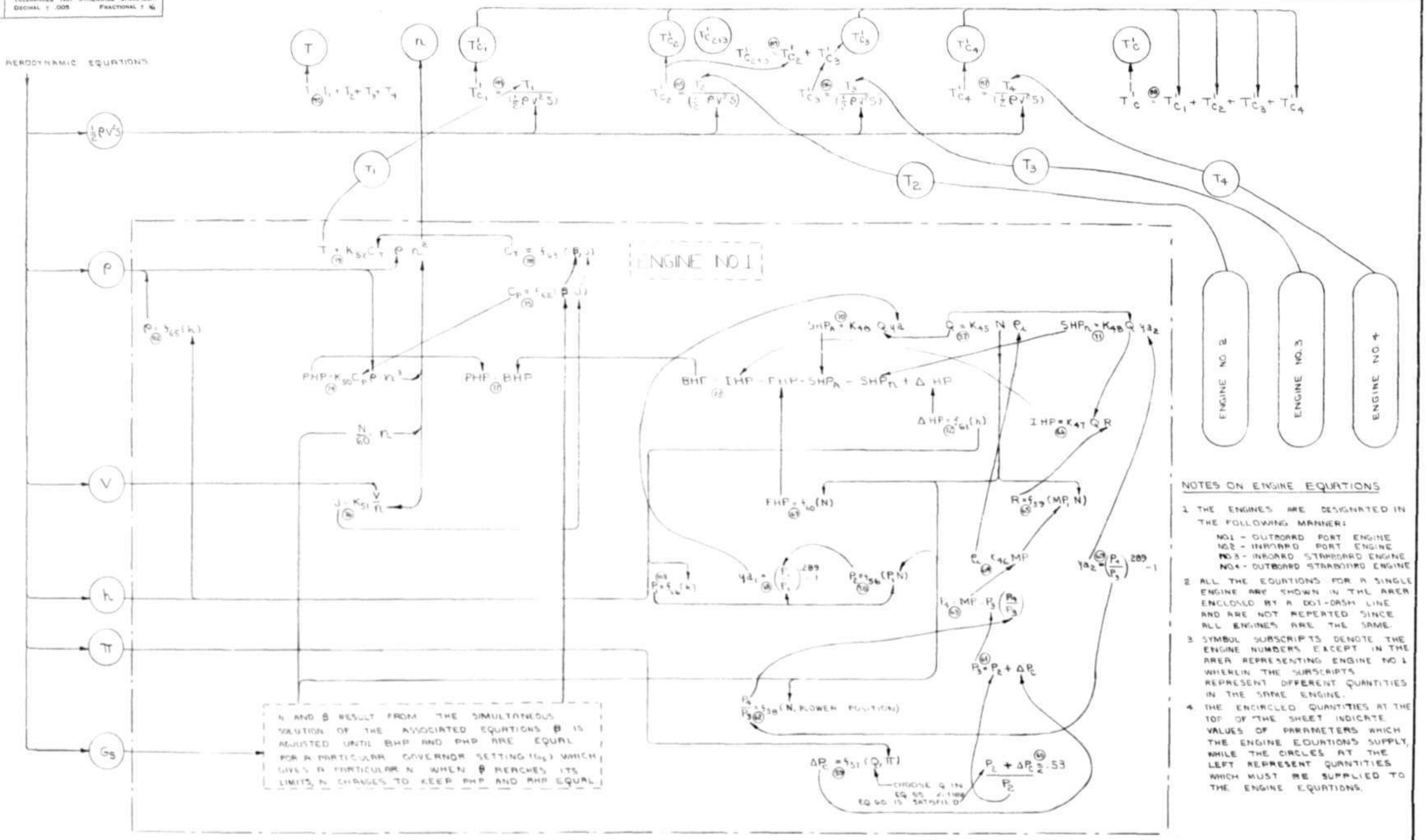
D-30012
WO-

TOLERANCES NOT OTHERWISE SPECIFIED:
DECIMAL ± .005 FRACTIONAL ± 1/64



N AND β RESULT FROM THE SIMULTANEOUS SOLUTION OF THE ASSOCIATED EQUATIONS β IS ADJUSTED UNTIL BHP AND PHP ARE EQUAL FOR A PARTICULAR GOVERNOR SETTING (G_S) WHICH GIVES A PARTICULAR N WHEN β REACHES ITS LIMITS, N CHANGES TO KEEP PHP AND BHP EQUAL

NOTES ON ENGINE EQUATIONS

1 THE ENGINES ARE DESIGNATED IN THE FOLLOWING MANNER:

NO 1 - OUTBOARD PORT ENGINE
NO 2 - INBOARD PORT ENGINE
NO 3 - INBOARD STARBOARD ENGINE
NO 4 - OUTBOARD STARBOARD ENGINE

2 ALL THE EQUATIONS FOR A SINGLE ENGINE ARE SHOWN IN THE AREA ENCLOSED BY A DOT-DASH LINE AND ARE NOT REPEATED SINCE ALL ENGINES ARE THE SAME.

3 SYMBOL SUBSCRIPTS DENOTE THE ENGINE NUMBERS EXCEPT IN THE AREA REPRESENTING ENGINE NO 1 WHEREIN THE SUBSCRIPTS REPRESENT DIFFERENT QUANTITIES IN THE SAME ENGINE.

4 THE ENCIRCLED QUANTITIES AT THE TOP OF THE SHEET INDICATE VALUES OF PARAMETERS WHICH THE ENGINE EQUATIONS SUPPLY, WHILE THE CIRCLES AT THE LEFT REPRESENT QUANTITIES WHICH MUST BE SUPPLIED TO THE ENGINE EQUATIONS.

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ENGINE EQUATIONS SCHEMATIC

SCALE:
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CH.
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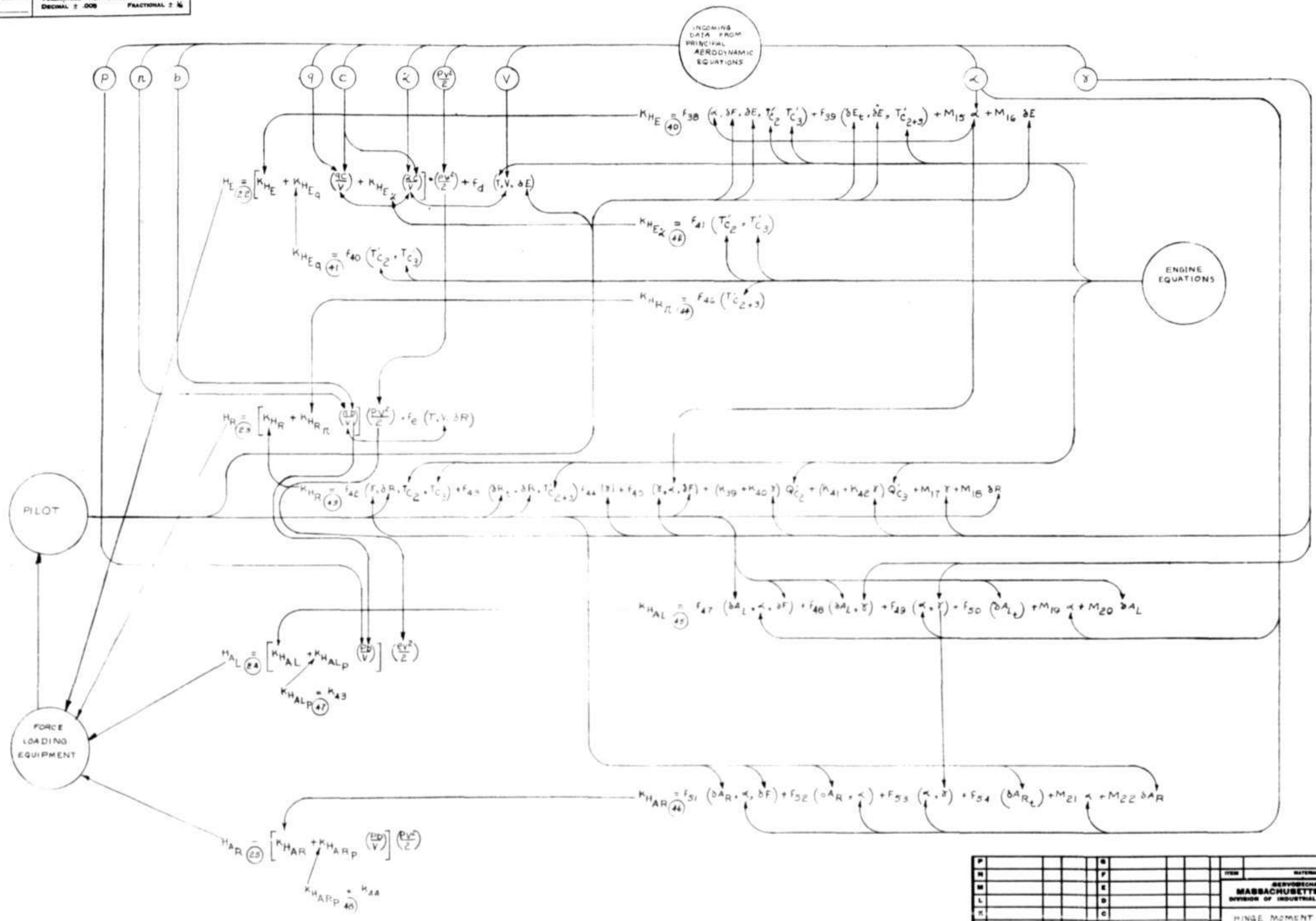

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N				F				ITEM	MATERIAL - DESCRIPTION
M				E					
L				D					
K				C					
J				B					
H				A					
	WAS	APP.	DATE		WAS	APP.	DATE		

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HINGE MOMENT EQUATIONS SCHEMATIC

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TR.
CK.
APP.

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C- 30053
WO-

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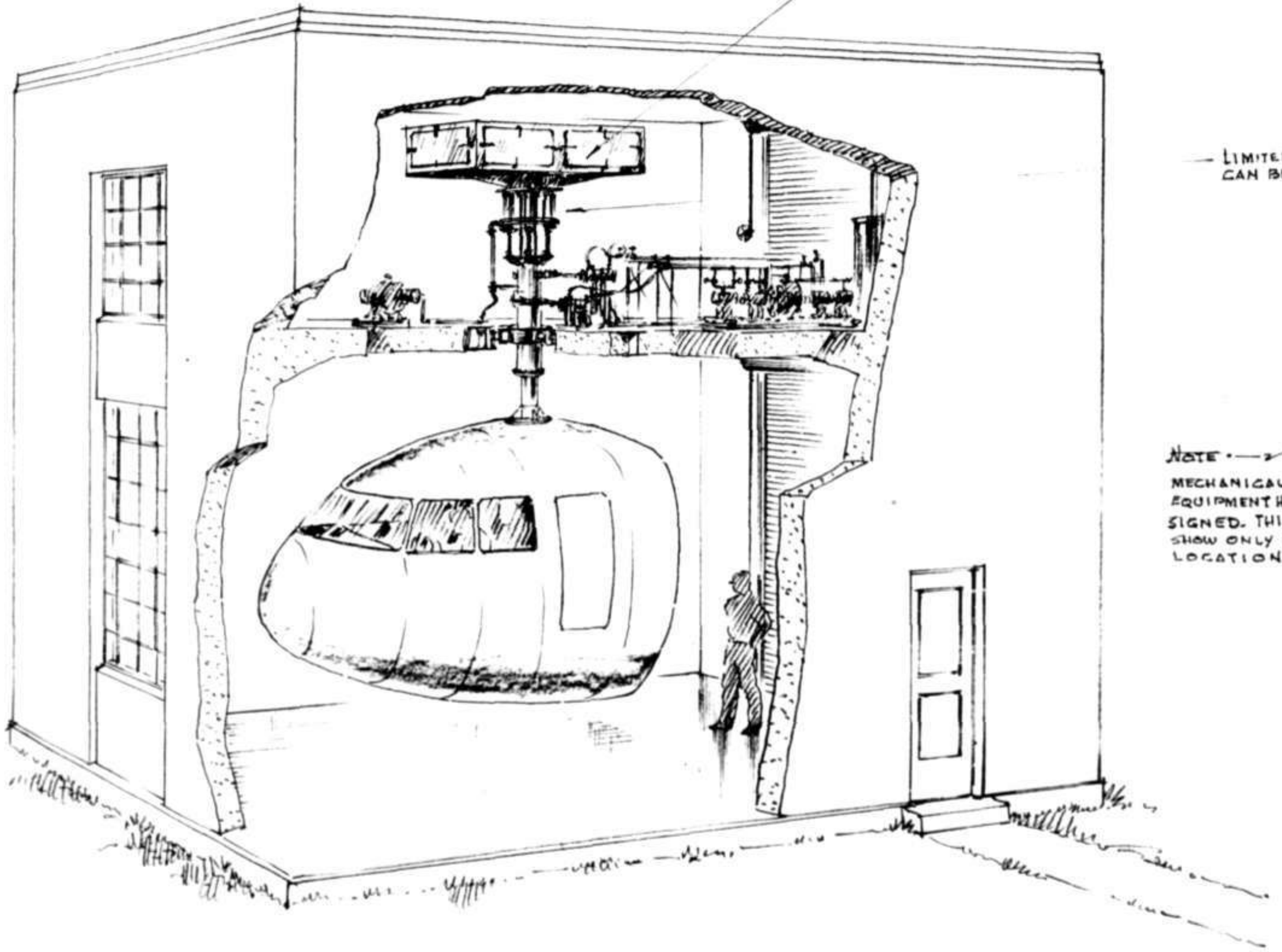

CONTROL FORCE LOADING EQUIPMENT
WILL BE LOCATED AS A COUNTERBALANCE

LIMITED VERTICAL MOTION
CAN BE PROVIDED.

NOTE:
MECHANICAL AND HYDRAULIC EQUIPMENT HAS NOT BEEN DESIGNED. THIS DRAWING IS TO SHOW ONLY EQUIPMENT LOCATION.

A·S·C·A COCKPIT MOUNTING –
SKETCH TO INDICATE OVERHEAD MOUNTING

RESTRICTED

P				G				
N				F				
M				E				
L				D				
K				C				
J				B				
H				A				
	WAS	APP.	DATE		WAS	APP.	DATE	

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C-30053

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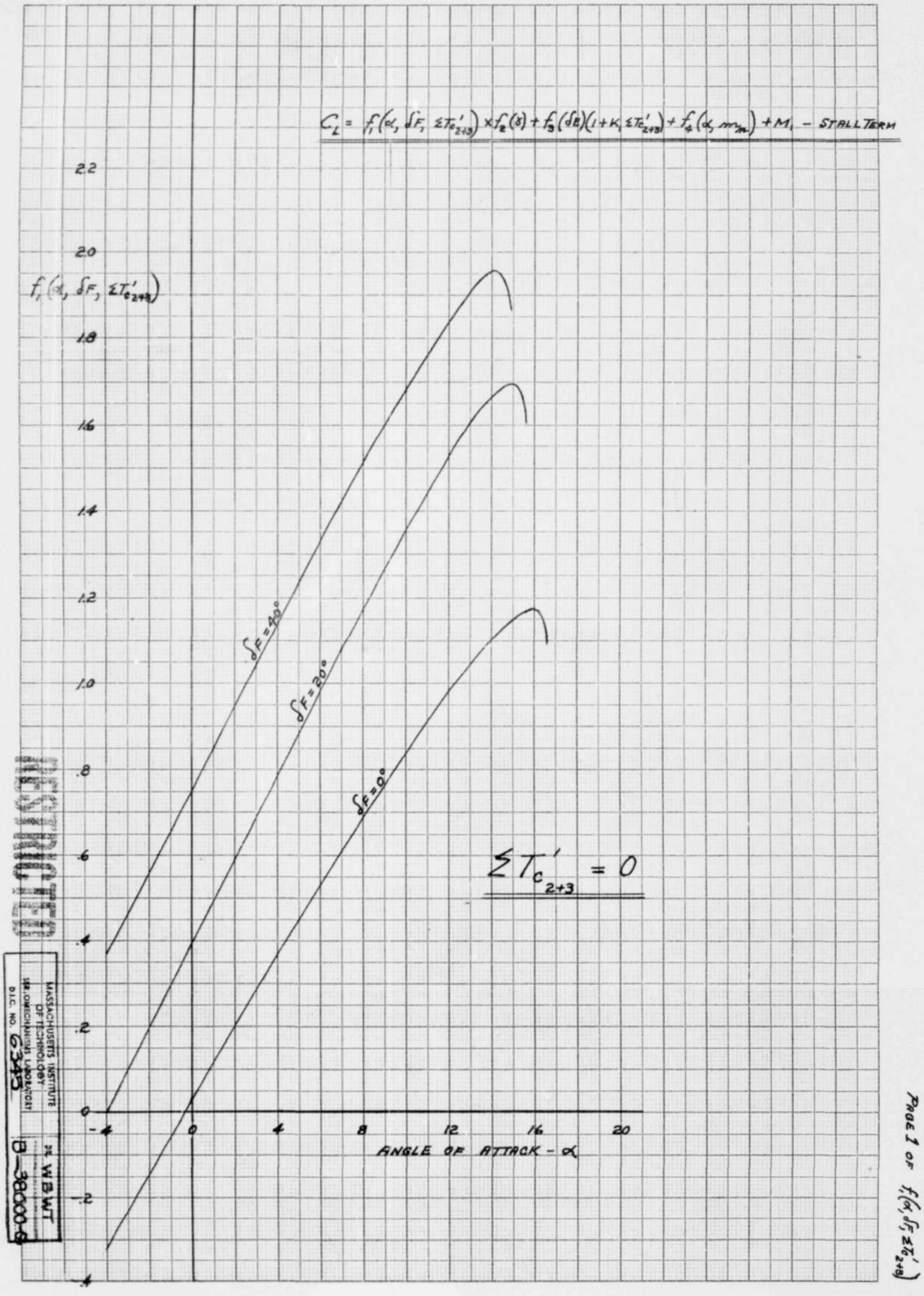
PAGE 1 OF $f_1(\alpha, \delta_F, \Sigma T'_{c_{2+3}})$



$$C_L = f_1(\alpha, \delta_F, \Sigma T'_{c_{2+3}}) \times f_2(\delta) + f_3(\delta_E)(1 + K_1 \Sigma T'_{c_{2+3}}) + f_4(\alpha, m_m) + M_1 - \text{STALL TERM}$$

$$\Sigma T'_{c_{2+3}} = 0$$

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DR. WBWT

B-38000-6

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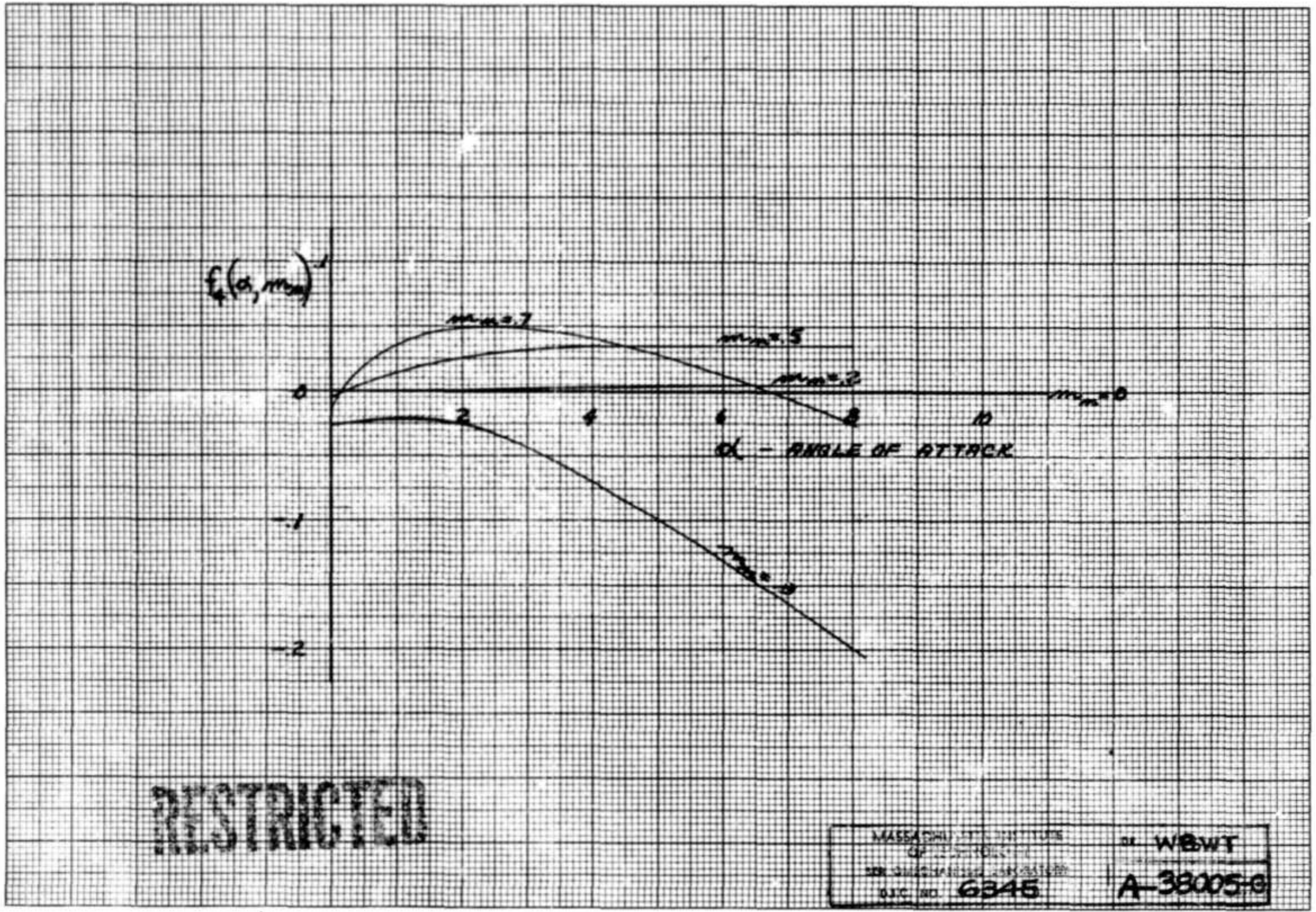
$f_4(\alpha, m_m)$



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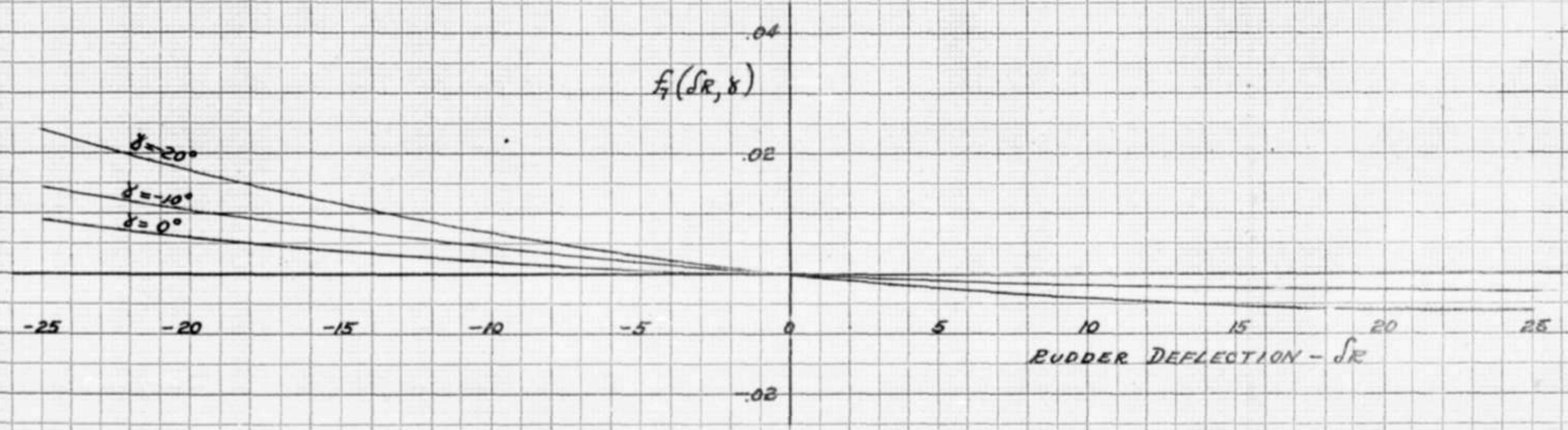
$f_7(\delta_R, \gamma)$



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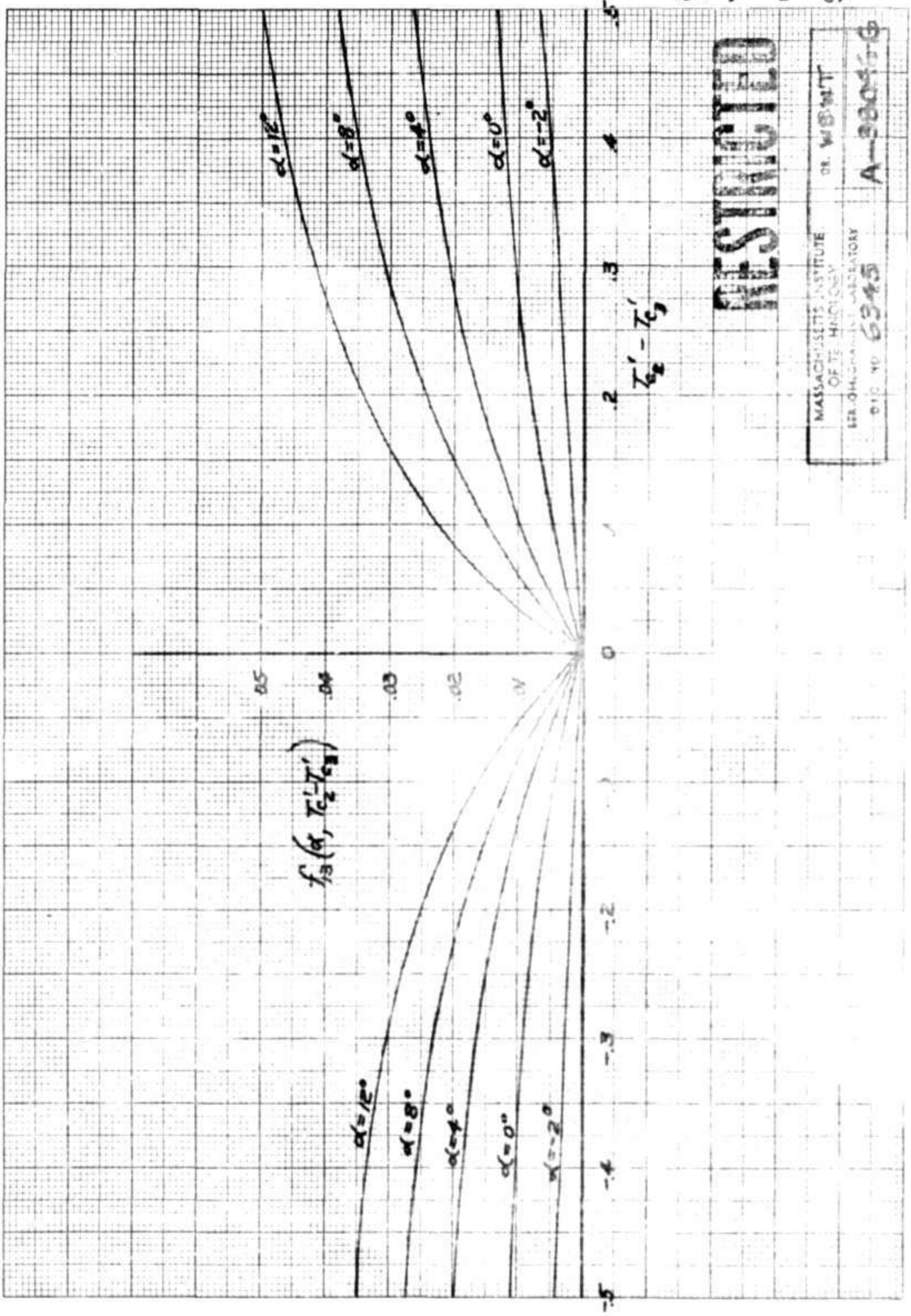
$f_{13}(\alpha, T_{c_2}' - T_{c_3}')$



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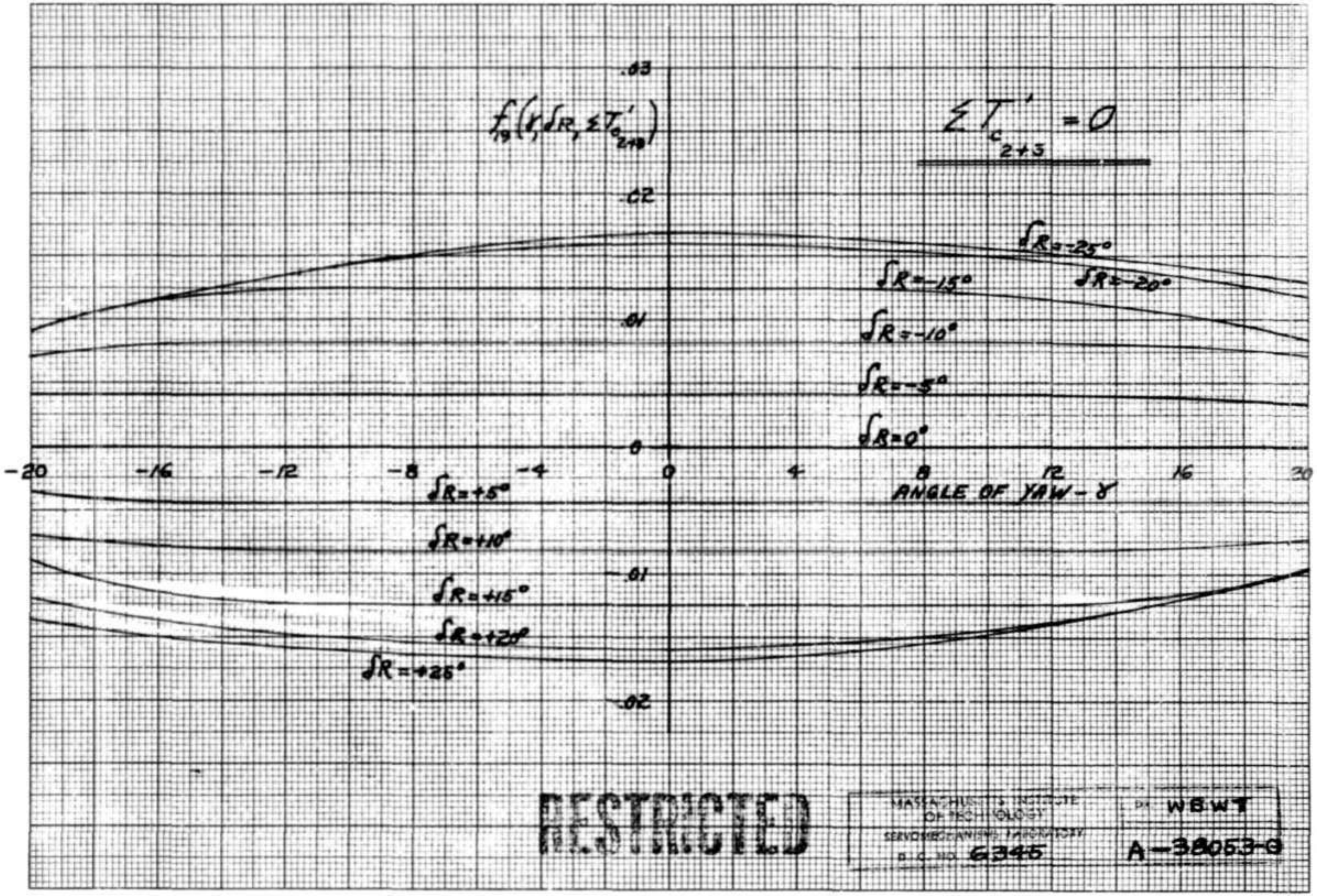

$f_{19}(\gamma, \delta R, T'_{c_{2+3}})$