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SUBJECT: STRESS EFFECTS IN FERRITES AND GENERALIZATION OF SWITCHING COEFFICIENT FOR NON-SQUARE MATERIALS

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Abstract: The hysteresis-loop squareness R_s, the switching coefficient S_w, and the threshold field H_o of a nickel-zinc ferrite are found to increase upon application of a compressive stress. The relationship between S_w and various fundamental parameters of a ferromagnetic material is based on the assumption of a square-hysteresis loop. Since the material tested does not have a square loop, a re-evaluation of the significance of S_w is made. This leads to a theory relating the distance d the Bloch walls move during switching to the magnetization of the material and the slope, S_v, of a plot of voltage output vs. applied field. A correction for the variation of d leads to the determination of new constants S_{wm} and H_{om}' which represent corrected versions of S_w and H_o, and have the physical significance previously assigned the latter terms. The calculated value of H_{om}' is verified by an independent determination of the threshold field obtained from a study of the output voltage as a function of applied field. In addition, the theory leads to a relationship between S_{wm} and S_v which holds to within the accuracy of the assumptions made. Increasing the applied compressive stress is shown to increase the value of the maximum nucleating field strength, and the increase of S_{wm} with increasing stress is shown to be primarily due to the change in d rather than the increase in the effective anisotropy or the alignment of directions of easy magnetization.

1. Introduction

It has been observed that the application of an external compressive stress to a stress-sensitive core increases the squareness of the hysteresis loop of the material and increases the time required to reverse the magnetization of the core for a given applied-field strength.

This behavior is predicted by the switching mechanism model introduced in Engineering Note E-532.[1] According to this model, the introduction of a compressive stress upon a magnetic material with a negative magnetostrictive constant produces a tendency on the part of the electron spin components to align themselves in the direction of the stress. This decreases the angular variation of the direction of easy magnetization from grain to grain in a polycrystalline sample, thereby decreasing the magnetic-pole density at the grain boundaries. As a result, the nucleation of poles of reverse magnetization at grain boundaries is inhibited. If the stress application inhibits nucleation to the point where a positive nucleating field is required to form new domains, a square hysteresis loop results. Additional stress increases this effect, and the hysteresis loop becomes squarer.

The increased switching time can be partially explained on the basis of the same model. In reference 1 a switching coefficient S_w was defined by the relationship

$$S_w = (H-H_o)\tau = \frac{I_s \Lambda d}{(\Lambda^2 + I_s^2 \gamma^2) \langle \cos \theta \rangle} \sqrt{\frac{K_{eff}}{A}} \qquad (1)$$

Since this report will deal exclusively with ferrites, which have resistivities of the order of 10^6 ohm-cm, the eddy current contribution to the switching coefficient is negligible and is ignored.

In the above equation H represents the applied magnetic field, H_o the threshold field for domain wall motion, τ the switching time, I_s the saturation magnetization, Λ the relaxation frequency based on the

1. Goodenough, J. B. and Menyuk, N., Digital Computer Laboratory, Engineering Note E-532, M.I.T., March 9, 1953.

Landau-Lifshitz formulation, γ the magneto-mechanical ratio, A the exchange constant, K_{eff} the effective anisotropy, d the maximum distance a Bloch wall moves during a magnetization reversal, and $\langle \cos \theta \rangle$ is the average value of the cosine of the angle between the applied field and the magnetization direction.

Upon application of stress, the only factors on the right-hand side which change are d, K_{eff}, and $\langle \cos \theta \rangle$, and all these quantities will increase with increasing stress. Although the change in $\langle \cos \theta \rangle$ is important in inhibiting nucleation, it is small numerically. However, if less nucleation occurs, fewer domains of reverse magnetization will take part in the switching process. The individual walls must therefore move a greater distance to switch the core. This increase in d can be appreciable. The increase in K_{eff} will be shown to be considerably smaller.

Since the increase in d is the dominant change in the right-hand side of equation 1 on increasing the stress, S_w should increase with increasing stress. This is the case, as experimental results given in section II show.

During a determination of S_w it is assumed that H_o and d remain constant throughout a sizeable region of magnetic field values. These assumptions hold for materials with square hysteresis loops, but they are not valid for materials with non-square loops. The relationships which must be used for the more general case are investigated in section III, and the physical significance of equation 1 is re-examined. The predicted consequences of this general theory of the switching mechanism is then compared with experimental evidence in section IV.

II. Experimental Results

The B-H loop and the pulse response of a Ferroxcube 103 nickel-zinc ferrite toroid was investigated as a function of applied stress. This material was chosen because it displays a strong sensitivity to stress application.

The equipment used to apply a stress to the toroid has been described by F. K. Baltzer.[2] A stainless-steel strap is wrapped around the toroid, and one end of the strap is fastened. A known force is then applied to the other end of the strap, and the compressive stress on the toroid can then be calculated.[3]

A. Hysteresis-loop squareness

The squareness of the hysteresis loop was found to increase sharply upon the application of a compressive stress. Two independent sets of measurements were taken of the maximum squareness R_s as a function of stress, and the results are given in Table I. The maximum squareness hysteresis loops for the first set of measurements are shown in Figure 1.

B. Switching Coefficient and Threshold Field

A determination was made of the switching coefficient S_w and the threshold field value H_o under various compressive stresses. The stress was varied from zero to -280×10^6 dynes/cm^2 in steps of -56 dynes/cm^2. The negative values indicate that the applied stress is compressive.

Two sets of data were taken. Measurements were first made as the stress was increased from zero and repeated by then decreasing the stress through the same values. The resultant curves of applied field vs. inverse switching time are shown in Figure 2 for the second run (decreasing compression). The resultant values of S_w and H_o taken from this set of curves and a similar set obtained from the first run are given in Table I. From these figures it can be seen that S_w does increase with increasing stress, as predicted. The single exception is the small decrease in S_w on applying -56 dynes/cm^2. The reason for this will be clarified in the next section. However, the increase in switching time with applied stress is only partially attributable to the increase in S_w. The increase in the threshold field is a far more significant factor in the switching time rise.

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2. F. K. Baltzer, Quarterly Progress Report, Lincoln Laboratory, Division 6, Digital Computer Laboratory, M.I.T., p. 69, December 15, 1952.

3. F. B. Seely, "Resistance of Materials," John Wiley and Sons, New York, p. 393, 1947.

C. Coercive Force

In view of the sharp change in the threshold field with stress, a set of static hysteresis loops were taken as functions of applied stress. As can be seen in Table I, the coercive force H_c taken from these loops remains relatively constant. Thus there is no direct correlation between the coercive force and the threshold field value.

Table I

Stress (dynes/cm^2)	Run 1 (Increasing Stress)			Run 2 (Decreasing Stress)			Coercive Force
	R_s	S_w (Oe-sec)	H_o (Oe)	R_s	S_w (Oe-sec)	H_o (Oe)	H_c (Oe)
0	0.375	1.065×10^{-6}	0.65	0.440	1.09×10^{-6}	0.65	0.80
-56×10^6	0.540	1.1×10^{-6}	1.06	0.570	1.07×10^{-6}	1.10	0.81
-112×10^6	0.633	1.26×10^{-6}	1.245	0.640	1.23×10^{-6}	1.25	0.81
-168×10^6	0.675	1.33×10^{-6}	1.33	0.665	1.43×10^{-6}	1.315	0.81
-224×10^6	0.690	1.48×10^{-6}	1.37	0.680	1.50×10^{-6}	1.385	0.82
-280×10^6	0.695			0.715	1.61×10^{-6}	1.40	0.82

III. The Switching Coefficient in Non-Square Materials

With the exception of d, all the parameters on the right-hand side of Equation 1 are constants for a particular sample, assuming no stress or temperature variations. In a material with a perfectly square hysteresis loop, all Bloch wall motion will occur at the magnetic field value $H = H_c$, and there will be no additional wall motion upon increasing the magnetic field. In this case d is also constant, and S_w is therefore a constant of the material.

Under this assumption Equation 1 is a linear equation, and it has been tested experimentally with ferrites and metals of high squareness. In all such cases the predicted linear relationship between the applied field H and the inverse switching time $1/\tau$ has been observed.

However, in a material which does not have a square hysteresis loop, the magnetization of the sample increases on increasing the external magnetic field. At the small fields involved (~1.5-2.5 Oe.) the effects of domain rotation are small, so this increase in magnetization is due to additional domain wall motion. Therefore d increases with increasing field, and the assumption of constant d is no longer valid.

Furthermore, the opposition to this additional wall motion is greater, on the average, than the opposition to the original wall motion. This is evidenced by the fact that a higher applied field is required to move the walls this additional distance. Since H_o is effectively an averaged out value of this opposition to wall motion over the entire region traversed by the walls, H_o should also increase with increasing applied field.

Thus for non-square loop materials Equation 1 no longer represents a simple linear relationship. If this equation is to be used for evaluation of the fundamental parameters of the material, the variation of d with the applied field must be found. In addition, a physically meaningful threshold field value must be obtained.

A. Determination of d = d(H)

Upon application of a step input current to the primary winding through a ferromagnetic toroid, the output voltage obtained in the secondary winding can be approximated by a Gaussian distribution function. Considering only the latter half of the output curve, and taking t = 0 when the output voltage is at its maximum value V_o,

$$\frac{d\Phi}{dt} = V_o e^{-a^2t^2} . \qquad (2)$$

Switching time is defined as the time required for the output voltage to go from 10 percent of its maximum voltage through the maximum voltage and down again to 10 percent of this value. Since Equation 2 deals with only half the output curve, this can be expressed analytically by the expression $t = \tau/2$ when $\frac{d\Phi}{dt} = V_o/10$. Therefore $t = \tau/2$ when $e^{-a^2t^2} = 0.1$.

Then $a^2 = 9.2/\tau^2$ and

$$\frac{d\Phi}{dt} = V_o e^{-9.2 t^2/\tau^2} \tag{3}$$

Upon application of a small magnetic field the magnetization reversal is almost entirely due to the motion of 180° Bloch walls. For this case it was shown in reference 1 that

$$\frac{d\Phi}{dt} = 8\pi I_s < \cos \Theta > \frac{dA_w}{dt}$$

where A_w represents the area swept out by the moving Bloch walls in a cross-sectional plane of the magnetic material.

$$\frac{dA_w}{dt} = \frac{dA_w}{d\langle r\rangle} \frac{d\langle r\rangle}{dt}$$

where $\frac{d\langle r\rangle}{dt}$ is the rate of change of the mean r, and $\frac{dA_w}{d\langle r\rangle}$ is a distribution function relating the area swept out to the mean r. In this equation $\langle r\rangle$ is the mean value of the distance the Bloch walls move in time t.

It was also shown in the same reference that

$$\frac{d\langle r\rangle}{dt} = \frac{(\Lambda^2 + I_s^2 \gamma^2) < \cos \Theta >^2 (H-H_o)}{I_s \Lambda} \sqrt{\frac{A}{K_{eff}}} .$$

Therefore

$$\frac{d\Phi}{dt} = 8\pi \chi I_s (H-H_o) < \cos \Theta > \frac{dA_w}{d\langle r\rangle} \tag{4}$$

where

$$\chi = \frac{(\Lambda^2 + I_s^2 \gamma^2) < \cos \Theta >^2}{\Lambda I_s} \sqrt{\frac{A}{K_{eff}}} \tag{5}$$

Equating (3) and (4)

$$8\pi \chi I_s (H-H_o) < \cos \Theta > \frac{dA_w}{d\langle r\rangle} = V_o e^{-9.2 t^2/\tau^2}$$

Since $\frac{d\langle r\rangle}{dt}$ is constant for a constant applied field, and $\langle r\rangle$ goes from 0 to d in time τ

$$\frac{\langle r\rangle}{t} = \frac{d}{\tau} \quad \text{or} \quad \frac{t}{\tau} = \frac{\langle r\rangle}{d}.$$

Therefore

$$\frac{dA_w}{d\langle r\rangle} = \frac{V_o \, e^{-9.2\langle r\rangle^2/d^2}}{8\pi\chi I_s(H-H_o)\langle\cos\theta\rangle} \tag{6}$$

The area in which magnetization reversal due to Bloch wall motion occurs as $\langle r\rangle$ goes from 0 to d/2 is therefore

$$A_w' = \frac{V_o}{8\pi\chi I_s(H-H_o)\langle\cos\theta\rangle}\int_0^{d/2} e^{-9.2\langle r\rangle^2/d^2}\, d\langle r\rangle \tag{7}$$

$$A_w' = \frac{V_o d}{24.3\,\pi\chi I_s(H-H_o)\langle\cos\theta\rangle} \tag{8}$$

where A_w' represents the areal change which takes place in time $t = \tau/2$.

The magnetization change is directly proportional to the area swept out by Bloch wall movement. The value of A_w can therefore be found from the relationship

$$\frac{2A_w}{A} = \frac{I}{I_s\langle\cos\theta\rangle} \tag{9}$$

where A is the total cross-sectional area of the ferromagnetic toroid. The factor 2 arises from the fact that a total magnetization reversal corresponds to a change of 2 I_s. Therefore

$$A_w = \frac{I A}{2I_s\langle\cos\theta\rangle} = \frac{V_o d}{24.3\pi\chi I_s(H-H_o)\langle\cos\theta\rangle}$$

$$d = \frac{38\chi IA\,(H-H_o)}{V_o}. \tag{10}$$

The values of χ and A are constant for a particular sample; V_o can be measured directly, H is the applied magnetic field and is known, and I = I(H) can be obtained from an initial magnetization curve. Equation (10) can therefore be used to determine the relative change of d with applied field if a physically meaningful method of finding H_o can be obtained. Since the accuracy of Equation 10 is limited by the assumption of a Gaussian-type voltage output, the numerical factor 38 cannot be taken very seriously. Thus, while this equation can be used to determine relative values of d, the use of this equation to obtain absolute value of d cannot be expected to yield much more than the proper order of magnitude.

B. Investigation of Threshold Field

Since neither S_w nor H_o are constant throughout a range of magnetic field values in a non-square loop material, an independent method of obtaining the threshold field must be found. A possible source of this information is the output voltage of the material during magnetization reversal.

If the number of domain walls which take part in a magnetization reversal is independent of the applied field in the region of interest, the coefficient of $\frac{dA_w}{d\langle r\rangle}$ in Equation 6 is constant. Then

$$\frac{V_o}{8\pi\chi I_s(H-H_o)\langle\cos\theta\rangle} = C$$

where C is a constant. This assumption essentially states that the increase in area swept out by Bloch walls upon increasing the applied field is due to additional notion of existent walls rather than the nucleation of new domains. In this case

$$H-H_o = \frac{V_o}{8\pi C\chi I_s\langle\cos\theta\rangle} = S_v V_o \qquad (11)$$

where S_v is a constant defined by the above equation. Then

$$H = S_v V_o + H_{om} \qquad (12)$$

where H_{om} represents the threshold field value obtained from a voltage output measurement.

At first glance this equation might not seem to have any particular advantage over the equation

$$H = \frac{S_w}{\tau} + H_o$$

which is directly obtainable from Equation 1. However S_w is not constant throughout a set of measurements in a non-square loop material because d is varying whereas S_v, as defined by Equation 11, does remain constant. Therefore Equation 12 successfully expresses the threshold field value in terms of known values and a constant.

If a region of applied field exists in which the change of the threshold field is small, Equation 13 will be approximately linear in this region. As will be shown in Section IV, such a region does exist. Within this region a threshold field value can be defined, and is designated H_{om}. This value is taken from the intercept of an extrapolation of the linear portion of a V_o versus H curve. The value of S_v can be obtained from the slope of the line.

As the applied field is reduced below this region, the drop in the threshold field becomes increasingly pronounced, and the equation is no longer linear. However, since S_v is known, the value of H_o can be obtained for each point. This will be done in the next section.

Substitution of Equation 11 into Equation 10 yields

$$d = 38\pi I A S_v. \tag{13}$$

The magnetization I is the only parameter on the right-hand side of Equation 13 which varies with the applied field during an S_v or S_w measurement. The variation of d over an S_w measurement is therefore directly proportional to the magnetization change.

IV. Consequences of the Theory and Experimental Evidence

Equation 13 gives d as a function of the magnetization. However, it is more useful to express d as a function of the applied field H. This can be obtained empirically from an initial magnetization curve, which gives the magnetization as a function of the applied magnetic field. Initial magnetization curves were therefore taken of the Ferroxcube 103 toroid under the stresses considered in Section II. Four of the six initial magnetization curves are shown superposed in Figure 3. The two curves not shown, for $T = -168 \times 10^6$ dynes/cm^2 and $T = -224 \times 10^6$ dynes/cm^2, were omitted for the sake of clarity. The value of the magnetization at H = 2.33 Oe. (I_m) is given in the last column of Table II.

A. S_v and H_{om} Determination

According to Equation 13, there will be a linear relationship between the applied field H and the maximum output voltage V_o if the threshold field remains constant. In order to determine if this linear relationship does hold, a curve of H vs. V_o was obtained for the Ferroxcube toroid under the various stress conditions. Two sets of data were taken, the first with increasing stress and the second with decreasing stress. The resultant curves obtained from the second run are shown in Figure 4, and the resultant values of S_v and H_{om} obtained from both sets of data are tabulated in Table II.

It can be seen in Figure 4 that for all stress conditions Equation 13 is essentially linear from the highest magnetic field applied (H = 2.33 Oe) down to H = 1.75 Oe. Measurement of the output voltage of the toroid under no stress could not be obtained for magnetic fields below 1.8 Oe. However, the relationship was linear throughout the region considered.

B. Switching Coefficient and H_o Re-evaluation with Constant d

Upon application of the maximum magnetic field (H = 2.33 Oe) during a switching coefficient determination, the Bloch walls move a distance d_m and are subject to a threshold field H_{om}'. Then, according to Equations 1 and 13,

$$S_{wm} = (H_m - H_{om}')\tau_m = B \; d_m = DI_m \qquad (14)$$

where B and D are constants, I_m is the magnetization at H_m = 2.33 Oe. as taken from the initial magnetization curve, and S_{wm} is the value of the switching coefficient at H = 2.33 Oe.

At some lower value of applied field, $H = H_a$

$$S_{wa} = (H_a - H_{oa}')\tau_a = Bd_a = DI_a . \tag{15}$$

If, upon application of H_a the walls had moved a distance d_m, a time $\tau_a' > \tau_a$ would have been required for the walls to move this greater distance. Since, in moving a distance d_m the walls would have been subjected to a threshold field H_{om}', τ_a and τ_a' are related by the equation

$$\frac{(H_a - H_{om}')\tau_a'}{(H_a - H_{oa}')\tau_a} = \frac{d_m}{d_a} = \frac{I_m}{I_a} .$$

If H_a is restricted to the region 1.75 Oe $\leqslant H_a \leqslant$ 2.33 Oe, then it has been shown that the threshold field value is essentially unchanged, or $H_{om}' \simeq H_{oa}'$. Therefore, in this region

$$\tau_a' = \tau_a \frac{d_m}{d_a} = \tau_a \frac{I_m}{I_a} \tag{16}$$

Substituting τ_a' for τ_a in Equation 15,

$$S_{wa}' = (H_a - H_{oa}')\tau_a' = Bd_m = DI_m .$$

From Equation 14 it is apparent that

$$S_{wa}' = S_{wm} .$$

Since τ_a is known for each magnetic field value H_a of an S_w measurement and $\frac{I_m}{I_a}$ can be obtained from the initial magnetization curve, the conversion of τ_a to τ_a' can be made for each point. Then in the equation

$$S_{wm} = (H - H_{om}')\tau' = \frac{I_s \Lambda d_m}{(\Lambda^2 + I_s^2\gamma^2) < \cos\theta >}\sqrt{\frac{K_{eff}}{A}} \tag{17}$$

all the terms on the right-hand side are constant. Therefore, a curve of S_{wm} vs. $\frac{1}{\tau'}$, should yield a straight line of slope S_{wm} and intercept H_{om}' in the region of constant threshold field.

The experimental points which served as the basis of the S_w measurements discussed in Section II were used for the determination of S_{wm}. The values obtained for S_{wm} and H_{om}' are given in Table II for both runs. The curves obtained from the second run are shown in Figure 5. This curve is essentially the same as Figure 2, but with $1/\tau'$ substituted for $1/\tau$ at each point. The curves of H_o vs $1/\tau'$ are continued into the non-linear region for stress conditions $T = -56 \times 10^6$ dynes/cm^2 and $T = -224 \times 10^6$ dynes/cm^2. They will be discussed further in Section IV-..

1. Significance of S_{wm}

The values of S_w as given in Table I are taken from the apparently linear portion of an H vs. $1/\tau$ curve as shown in Figure 2. However, since the produce $(H - H_o)\tau$ is varying from point to point, this apparent linearity is "accidental", since it arises from the fact that the variation of d is smooth and fairly small from point to point as compared to the value of d. However, for a non-square-loop material the effect of the variation of d over the entire range of values is appreciable, and there is a sizable difference between the values of S_w and S_{wm}. The value of S_{wm} is always smaller than S_w.

In a material with a fairly square hysteresis loop, the variation of d from point to point will be much smaller than for a non-square loop material. Therefore, the effect of this variation is much smaller, and S_w approaches S_{wm}. For the case of perfect squareness, d does not vary and S_w equals S_{wm}.

To illustrate this graphically, Figure 6 shows the variation of S_w and S_{wm} for the case of zero stress and maximum stress ($T = -280 \times 10^6$ dynes/cm^2). As can be seen from this figure, the variation of S_w and S_{wm} for zero stress is quite large ($\sim 18\%$). This corresponds to the case of low squareness ($R_s \sim 0.4$). Under maximum stress, the variation is small ($\sim 4\%$). This corresponds to the case of higher squareness ($R_s \sim 0.7$).

Thus, in non-square-loop materials, the value of S_w obtained directly from an H vs. $1/\tau$ measurement cannot be used to evaluate the parameters which appear on the right-hand side of Equation 1 unless the variation of d is taken into account.

2. Significance of H_{om}'

Since the linearity of the H vs. $1/\tau$ curve from which S_w is determined is "accidental" for non-square-loop materials, the value of H_o taken from this curve is without physical significance. It will always be lower than the threshold field value H_{om}', and the extent of its variation from H_{om}' in the non-square case is graphically illustrated in Figure 6 for Ferroxcube 103 under no stress. Although S_w and S_{wm} vary by 18 %, H_{om}' is over 40 % greater than H_o. However, as the squareness of a material increases, H_o approaches the value H_{om}'. Thus for the case of maximum stress, Figure 6 shows H_{om}' is only 2.5 % greater than H_o. In a perfectly square-loop material, $H_o = H_{om}'$.

H_{om}' represents a measure of the energy loss to surface potential in the course of moving the Bloch walls through a distance d_m.

Although H_{om} is obtained in an independent measurement of applied voltage vs. maximum voltage output, it has the same physical significance as H_{om}'. Therefore H_{om} and H_{om}' should be equal if the measurements are made under equal conditions of stress and temperature. Since the measurements were taken at the same time, with one exception, these conditions held.

The values of H_{om}' and H_{om} are given for both runs in Table II, and are in excellent agreement. The only significant difference occurs for $T = -56 \times 10^6$ dynes/cm^2, where the variation is approximately 5%. This could be an experimental error.

Table II

Run 1

(Increasing Stress)

Stress (dynes/cm^2)	I_m	$S_v(\frac{Oe\text{-}sec}{max})$	S_{wm} (Oe-sec)	H_{om} (Oe)	H_{om}' (Oe)
0	210	7.36×10^{-10}*	0.89×10^{-6}*	0.94*	0.895*
-56×10^6	221	7.66×10^{-10}	1.02×10^{-6}	1.22	1.16
-112×10^6	234	9.25×10^{-10}	1.195×10^{-6}	1.31	1.30
-168×10^6	232	9.64×10^{-10}	1.26×10^{-6}	1.39	1.38
-224×10^6	234	11.0×10^{-10}	1.415×10^{-6}	1.42	1.41
-280×10^6	235	--	--	--	--

Run 2

(Decreasing Stress)

Stress (dynes/cm^2)	I_m	$S_v(\frac{Oe\text{-}sec}{max})$	S_{wm} (Oe-sec)	H_{om} (Oe)	H_{om}' (Oe)
0	210	7.45×10^{-10}	0.92×10^{-6}	0.92	0.915
-56×10^6	221	7.5×10^{-10}	1.00×10^{-6}	1.23	1.18
-112×10^6	234	9.13×10^{-10}	1.185×10^{-6}	1.31	1.30
-168×10^6	232	9.95×10^{-10}	1.37×10^{-6}	1.39	1.37
-224×10^6	234	11.15×10^{-10}	1.43×10^{-6}	1.41	1.43
-280×10^6	235	11.65×10^{-10}	1.55×10^{-6}	1.44	1.435

* These voltage and switching time measurements were not taken at the same time. Therefore the variation of H_o' and H_{om} is not significant.

C. Limit of Validity of Assumption of Constant Number of Walls Participating in Magnetization Reversal

At any particular applied field value H_a, where the threshold field value is given by H_{oa}, Equation 1 becomes

$$(H_a - H_{oa})\tau_a = \frac{I_s \Lambda d_a}{(\Lambda^2 + I_s^2 \gamma^2) \langle \cos \theta \rangle} \sqrt{\frac{K_{eff}}{A}} \qquad (18)$$

It has been shown that a region of field values exists in which $H_{oa} = H_o'$ and is vertually constant. However, this is not true at applied-field values below 1.7 Oe. The threshold field value varies measurably from point to point below this value of applied field.

In addition, the value of d varies with applied field throughout the entire range of values. But it was shown (Equation 16) that the effect of the variation in d can be eliminated mathematically by the substitution of an "adjusted" switching time τ' for τ. In that case Equation 18 becomes

$$S_{wm} = (H_a - H_{oa})\tau_a' = \frac{I_s \Lambda d_m}{(\Lambda^2 + I_s^2 \gamma^2) \langle \cos \theta \rangle} \sqrt{\frac{K_{eff}}{A}} \qquad (19)$$

and all the terms on the right-hand side of this equation are constant during an S_{wm} measurement. Therefore, the departure of H vs $1/\tau'$ measurement from linearity at low field values is entirely due to the variation of the threshold field from point to point in this region. Since S_{wm} is obtainable from the linear portion of the H vs $1/\tau'$ curve, and H_a and $1/\tau_a'$ are known at each point, the value of the threshold field H_{oa} can be obtained for each point from Equation 19.

If an "adjusted" magnetic field value H' is introduced such that at any point a

$$H_a' - H_{om}' = H_a - H_{oa}, \qquad (20)$$

then H_a' represents the field that would have been required instead of H_a if the threshold field had not varied. Use of the adjusted field value H' instead of H eliminates the effect of the threshold field variation in much the same manner that the use of the adjusted switching time τ' eliminates the effect of the variation in d.

In Section III the assumption was made that the number of walls which participate in a magnetization reversal is independent of the applied field in the linear region. This led to the relationship

$$H - H_{om} = S_v V_o \, , \qquad (13)$$

where S_v is a constant of the material. The fact that Equations 13 and 14 are both linear in a fairly large overlapping range of applied field values tends to substantiate the validity of the assumption in this region.

However, it is of interest to determine the validity of this assumption at low field values. As noted above, the threshold field varies markedly from point to point, However, the concept of the "adjusted" magnetic field H' can be used to compensate for this effect. As shown in Table II, $H_{om}' \approx H_{om}$. Therefore, as long as the number of participating Bloch walls remains constant,

$$H' - H_{om} = S_v V_o \qquad (21)$$

will be linear. The value of H' to be substituted for each value of H is obtained from Equation 20 and the data of an S_{wm} measurement, as given in Figure 5. This adjustment has been performed for the stress conditions $T = -56 \times 10^6$ dynes/cm^2 and $T = -224 \times 10^6$ dynes/cm^2, and the resultant H' vs V_o curves for these cases are shown in Figure 7. The unadjusted H vs V_o curves are also shown to illustrate the extent of this adjustment.

It can be seen from Figure 7 that the curves of H' vs V_o are not linear for low fields. Therefore, the assumption of a constant number of participating Bloch walls does not hold in this region. However, a number of interesting conclusions can be drawn from these curves.

First, using H' instead of H in this figure apparently overcompensates for the drop in the original curve. That is, the points which originally lay below the straight line now lie above this line. This is the behavior one would expect, since the points should be above the line if the number of Bloch walls participating in the magnetization reversal is smaller than the number participating at field values in the linear region. In addition, if the number of participating walls decreases with decreasing fields, the separation between the straight line and the individual points should increase with decreasing field. This is in accord with the observed data.

Second, Figure 7 shows that the number of participating walls starts to decrease at about 1.8 Oe at a stress of -224×10^6 dynes/cm^2, whereas it remains constant to about 1.7 Oe under a stress of -56×10^6 dynes/cm^2. Since stress effects an alignment of easy magnetization directions, and this inhibits nucleation of reverse domains, the nucleation field strength should increase with increasing stress. The data indicates that under a stress of -224×10^6 dynes/cm^2 there are reverse domains which do not nucleate at fields below 1.8 Oe. However, upon reduction of the stress to -56×10^6 dynes/cm^2 the maximum nucleation field strength for participating walls has apparently been reduced to 1.7 Oe.

D. Relationship of S_{wm} and S_v

From Equations 13 and 5

$$S_v = \frac{d_m}{38 I_m A} \left\{ \frac{\Lambda I_s}{(\Lambda^2 + I_s^2 \gamma^2) \langle \cos \Theta \rangle^2} \right\} \sqrt{\frac{K_{eff}}{A}} \qquad (22)$$

Comparison of the above and Equation 17 leads to the relationship

$$S_{wm} = 38\, I_m A \langle \cos \Theta \rangle S_v \qquad (23)$$

wherein A is equal to 0.16 cm^2 for the toroid used in this experiment, and S_v is known experimentally. I_m and $\langle \cos \Theta \rangle$ are dependent upon the applied stress. The value of I_m is known for all stress conditions. The value of Θ, however, is approximately known only for zero stress,[4] in which case $\langle \cos \Theta \rangle \approx 0.834$. Upon application of stress, $\langle \cos \Theta \rangle$ will increase. However since it is limited to the values $0.834 < \langle \cos \Theta \rangle < 1.00$, the numerical variation is small.

Unfortunately, the assumption of a Gaussian distribution inherent in the numerical factor 38 can easily be in error by a factor as great as this variation in $\langle \cos \Theta \rangle$. Therefore, Equation 23 cannot be used to evaluate the angle. The equation is useful, however, as a check on the relative magnitudes obtained for S_v and S_{wm}, and for determining the extent of the error. The values of S_{wm} and $38\, I_m A S_v$, which are related by $\langle \cos \Theta \rangle$, are compared in Table III.

As can be seen, the results are not sufficiently accurate to permit an evaluation of $\langle \cos \Theta \rangle$. However, these results show that the

4. R. M. Bozorth, "Ferromagnetism," D. Van Nostrand and Co., New York, (1951).

orders of magnitude of S_v and S_{wm}, as related by Equation 22, are correct. They further indicate that the result is within 20 % of numerical accuracy. This represents a strong verification of the general relationship of d and S_v as given by Equation 17.

Table III

Stress (dynes/cm^2)	Run 1 (Increasing Stress)		Run 2 (Decreasing Stress)	
	S_{wm}	$38 I_m A S_v$	S_{wm}	$38 I_m A S_v$
0	0.89×10^{-6}	0.9×10^{-6}	0.92×10^{-6}	0.95×10^{-6}
-56×10^6	1.02×10^{-6}	1.04×10^{-6}	1.00×10^{-6}	1.02×10^{-6}
-112×10^6	1.195×10^{-6}	1.32×10^{-6}	1.185×10^{-6}	1.31×10^{-6}
-168×10^6	1.26×10^{-6}	1.37×10^{-6}	1.37×10^{-6}	1.41×10^{-6}
-224×10^6	1.415×10^{-6}	1.57×10^{-6}	1.43×10^{-6}	1.59×10^{-6}
-280×10^6	--	--	1.55×10^{-6}	1.67×10^{-6}

E. <u>Relative Effects of d and $\sqrt{K_{eff}}$ on S_{wm} with Increasing Stress</u>

S_{wm} increases with increasing stress. This increase is proportional to the increase of $\frac{d_m \sqrt{K_{eff}}}{\langle \cos \theta \rangle}$, since all other terms on the right-hand side of Equation 17 are independent of stress.

Assuming isotropic magnetostriction within the sample, the effective anisotropy is given by the relationship

$$K_{eff} = K + \frac{3}{2} \lambda T \sin^2 \varphi \simeq K + \lambda T$$

where λ is the strain $\delta\ell/\ell$, φ is the angle between the magnetization and the direction in which $\delta\ell$ is measured, K is the intrinsic anisotropy constant of the material, and T is the applied stress. Thus $\sqrt{K_{eff}}$ varies with stress according to the relationship

$$\sqrt{K_{eff}} \approx \sqrt{K + \lambda T}.$$

From Equations 5 and 13,

$$S_v \propto \frac{d_m\sqrt{K + \lambda T}}{I_m \langle \cos \theta \rangle^2}$$

where the proportionality constant is independent of stress.

Since neither d_m, K, λ, or $\langle \cos \theta \rangle$ are known, it is impossible to calculate each of these terms directly. However, with the aid of some physical considerations, an idea of the relative magnitudes of the variation of d and $K + \lambda T$ with stress can be obtained.

In order to determine the maximum value T can have, as... the increase in S_v on going from zero stress to $T = -56 \times 10^6$ dynes/c... is due only to the increase in effective anisotropy. This stress value is thereby considered too small to inhibit any of the Bloch walls which participate in magnetization under zero stress. Therefore, the increase in d_m is due only to the increased area which must be enclosed to account for the increase in I_m. In addition, this assumes the increase in $\langle \cos \theta \rangle^2$ is negligible.

Since these are extreme assumptions, the value of λT obtained will be larger than is actually the case. Under these assumptions

$$\frac{S_v(T = -56 \times 10^6)}{S_v\ (T = 0)} = 1.024 = \frac{\sqrt{K + 56\,\lambda T \times 10^6}}{K}$$

and

$$56\lambda \times 10^6 = .048\ K.$$

The value of λ is unknown for a nickel-zinc ferrite, but the most reasonable order of magnitude available is that given for nickel ferrite, $\lambda \sim 10^{-5}$. Substituting this value,

$$K \sim 10^4 \text{ dynes/cm}^2.$$

This is the expected order of magnitude, and indicates that the values given above are not unreasonable. The above values can be used to determine the ratio $\sqrt{K + \lambda T}/\sqrt{K}$ for all values of stress. Then, since

$$\frac{S_{wm}}{S_{wma}} = \frac{\sqrt{K + \lambda T}}{\sqrt{K}} \quad \frac{d_m/\langle \cos \theta \rangle}{d_{ma}/\langle \cos \theta \rangle_a}$$

where subscript a refers to the case of zero stress, the relative change in S_{wm} due to the change in $\sqrt{K_{eff}}$ and in $d_m / \langle \cos \Theta \rangle$ can be determined. The resultant ratios are listed in Table IV. The figures used in this table are based on average values for both runs.

Table IV

Stress (dynes/cm^2)	$\frac{\sqrt{K + \lambda T}}{\sqrt{K}}$	$\frac{d_m / \langle \cos \Theta \rangle}{d_{ma} / \langle \cos \Theta \rangle_a}$
0	1.00	1.00
-56×10^6	1.024	1.09
-112×10^6	1.047	1.25
-168×10^6	1.070	1.36
-224×10^6	1.092	1.44
-280×10^6	1.113	1.54

Table IV shows clearly that the increase in S_{wm} with increasing stress is due primarily to the increase in the value of d. This fact is even more striking than shown in this table since the value given the ratio $\sqrt{K + \lambda T} / \sqrt{K}$ is the maximum value this ratio can have. This makes the ratio of $d_m / \langle \cos \Theta \rangle$ as given in the third column a minimum value. Furthermore, since $\langle \cos \Theta \rangle$ increases with increasing stress, the minimum ratio of d_m / d_{ma} is necessarily greater than the values given in this column.

F. Limitations of the Experimental Results

It must be borne in mind that the experiments were limited to applied field values below 2.33 oersteds. The results obtained are therefore limited to this region, and cannot be assumed to represent the saturation magnetization values.

Since the threshold field does not change measurably in the region 1.7 Oe $\leq$ H $\leq$ 2.33 Oe, the use of the threshold field value H_{om} or H_o' can be assumed to be the true value within this region with very little error. But since $H_o = H_o$ (I) below 1.7 Oe, it is probable that the threshold field rises, albeit slowly, at higher field values. However, it will be necessary to go to fields considerably higher than 2.33 oersteds before any such change is apparent. This effect would explain the rather surprising increase of the threshold field with increasing stress.

Table I indicates no comparable rise in the coercive force. However, the magnetization of the material at 2.33 oersteds increases with increasing stress. Therefore, if $H_o = H_o(I)$, it is meaningless to compare the threshold field values of the material under varying stress conditions at constant applied field. The threshold field values should be compared at constant magnetization.

The magnetization I = 235 is obtained with a field of 2.33 Oe when the sample is under a stress of -280×10^6 dynes/cm^2. To obtain this magnetization value with no stress applied, a field of almost 5 Oe is required. Meaningful data cannot be obtained at this high a field with present equipment. Experimentation to test this hypothesis must therefore await the design of improved equipment.

V. Conclusions

The above theory permits an evaluation of the switching coefficient of materials which do not have square hysteresis loops. This theory predicts certain relationships between parameters determined by independent measurements of switching time, maximum output voltage, and initial magnetization as functions of the applied field. These relationships have been found to hold within the accuracy of the experiment and the assumptions made.

In addition, the correction made for the variation of d brings the threshold field value obtained from a switching-time measurement into agreement with the threshold field value of an output-voltage measurement.

The self-consistency of the results represent a strong validation of the theory; it also serves to verify the model of the switching mechanism presented in Engineering Note E-532, upon which this theory is ultimately based.

The experiments afford direct evidence that increasing the stress reduces the number of Bloch walls participating in magnetization reversal and increases the maximum nucleating field. Both these features are in agreement with the original switching-mechanism model. The theory and experiments also permit a separation of the effect of stress on the values of d and the effective anisotropy. This separation yields values of strain and anisotropy which are of the expected order of magnitude and shows that the increase of S_{wm} with stress is primarily due to the increase in d.

It was hoped that the S_w coefficient, as given by Equation 1, could be used for the evaluation of fundamental parameters of a polycrystalline material. Before this equation can properly be applied, however, the variation of d in a non-square-loop material must be corrected for. The theory introduced in this report affords the means of applying this correction.

Acknowledgments

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Signed Norman Menyuk
Norman Menyuk

Approved DRB
David R. Brown

NM/jk

Drawings attached:

Figure 1 - A-57195
Figure 2 - A-57196
Figure 3 - A-57197
Figure 4 - A-57198
Figure 5 - A-57199
Figure 6 - A-57200
Figure 7 - A-57201

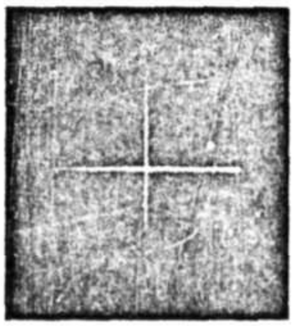
cc: Group 63 Staff
B. Lax, S. Foner, L. Gold

(a)
T = 0
$R_s = 0.375$

(b)
$T = -56 \times 10^6$ DYNES/CM^2
$R_s = 0.540$

(c)
$T = -112 \times 10^6$ DYNES/CM^2
$R_s = 0.633$

(d)
$T = -168 \times 10^6$ DYNES/CM^2
$R_s = 0.675$



(e)
$T = -224 \times 10^6$ DYNES/CM^2
$R_s = 0.690$

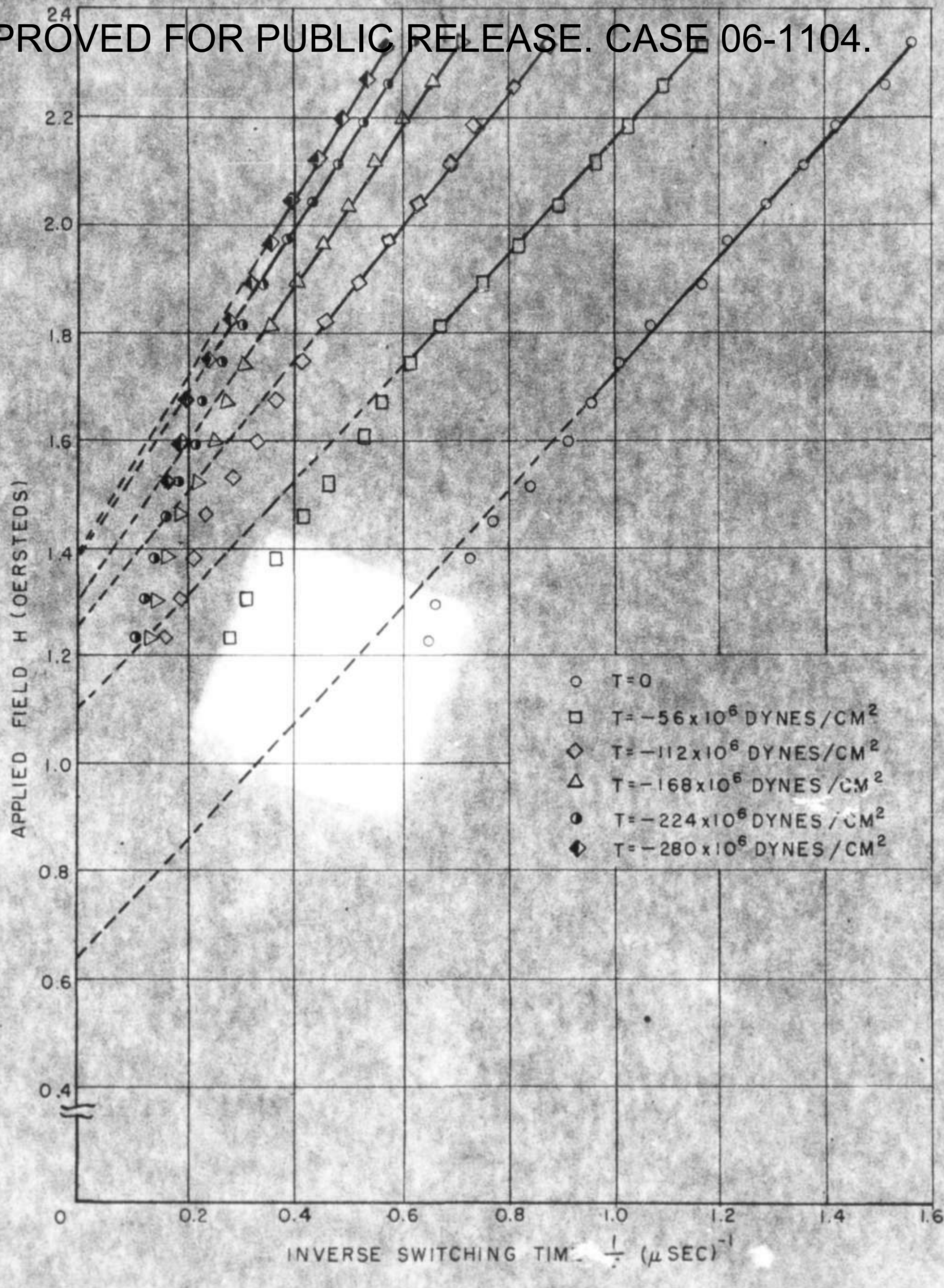
(f)
$T = -280 \times 10^6$ DYNES/CM^2
$R_s = 0.695$

FIG. 1

MAXIMUM-SQUARENESS HYSTERESIS LOOPS

OF FERROXCUBE 103 FERRITE UNDER VARIOUS STRESSES

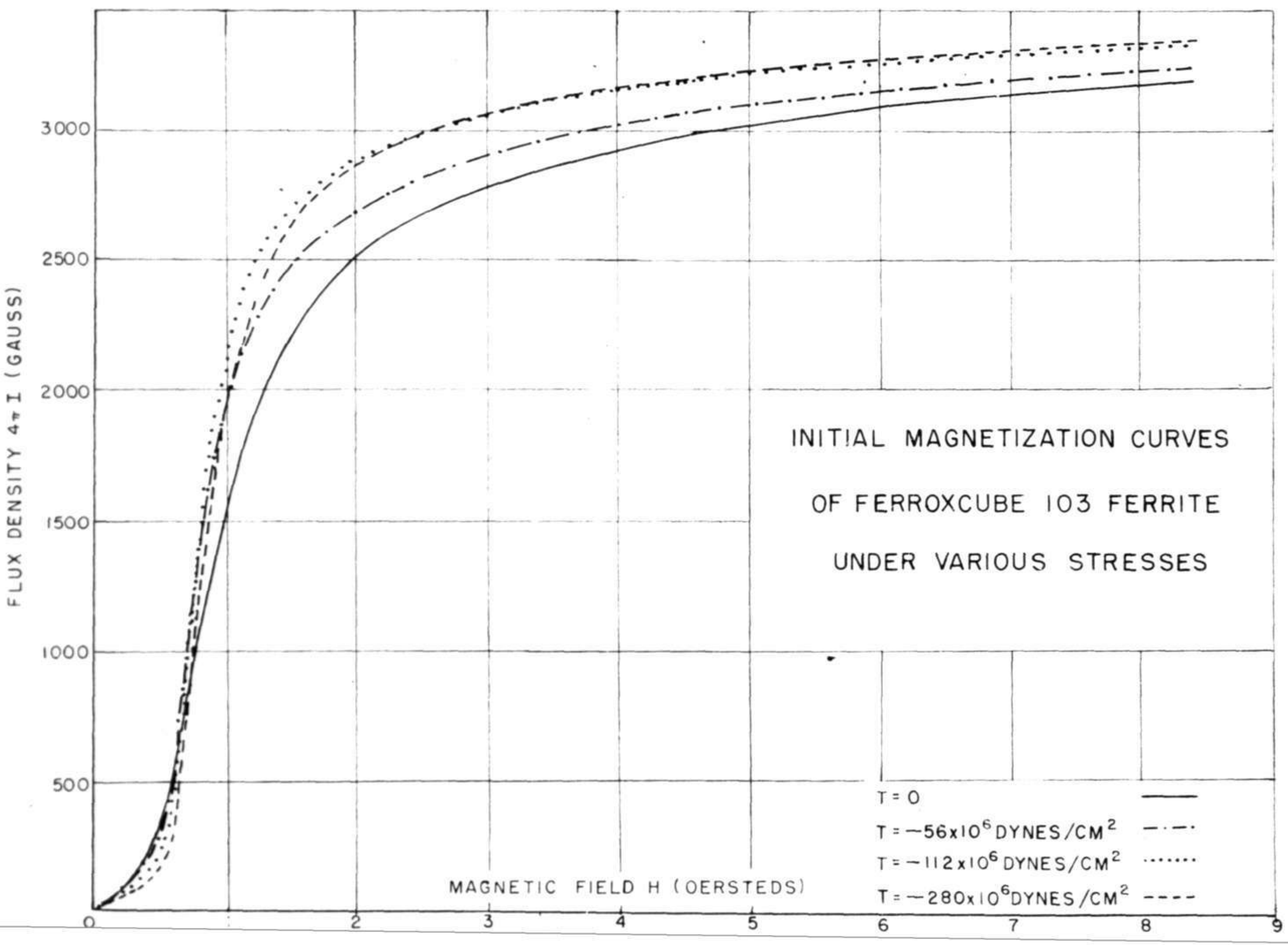
A-571.

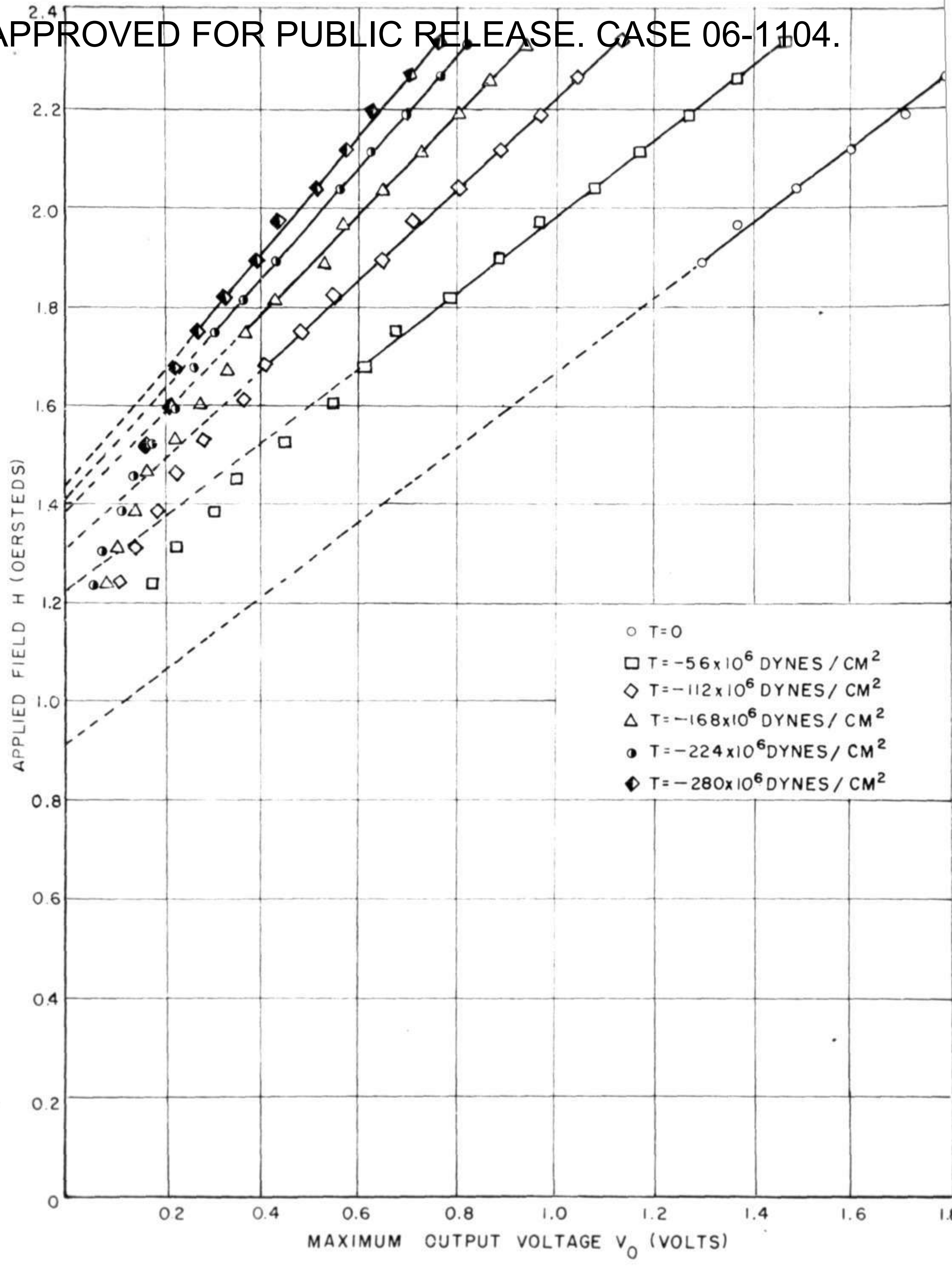


MEASUREMENTS OF THE SWITCHING COEFFICIENT AND H_0 OF FERROXCUBE 103 FERRITE UNDER VARIOUS STRESSES

A-57196
MN

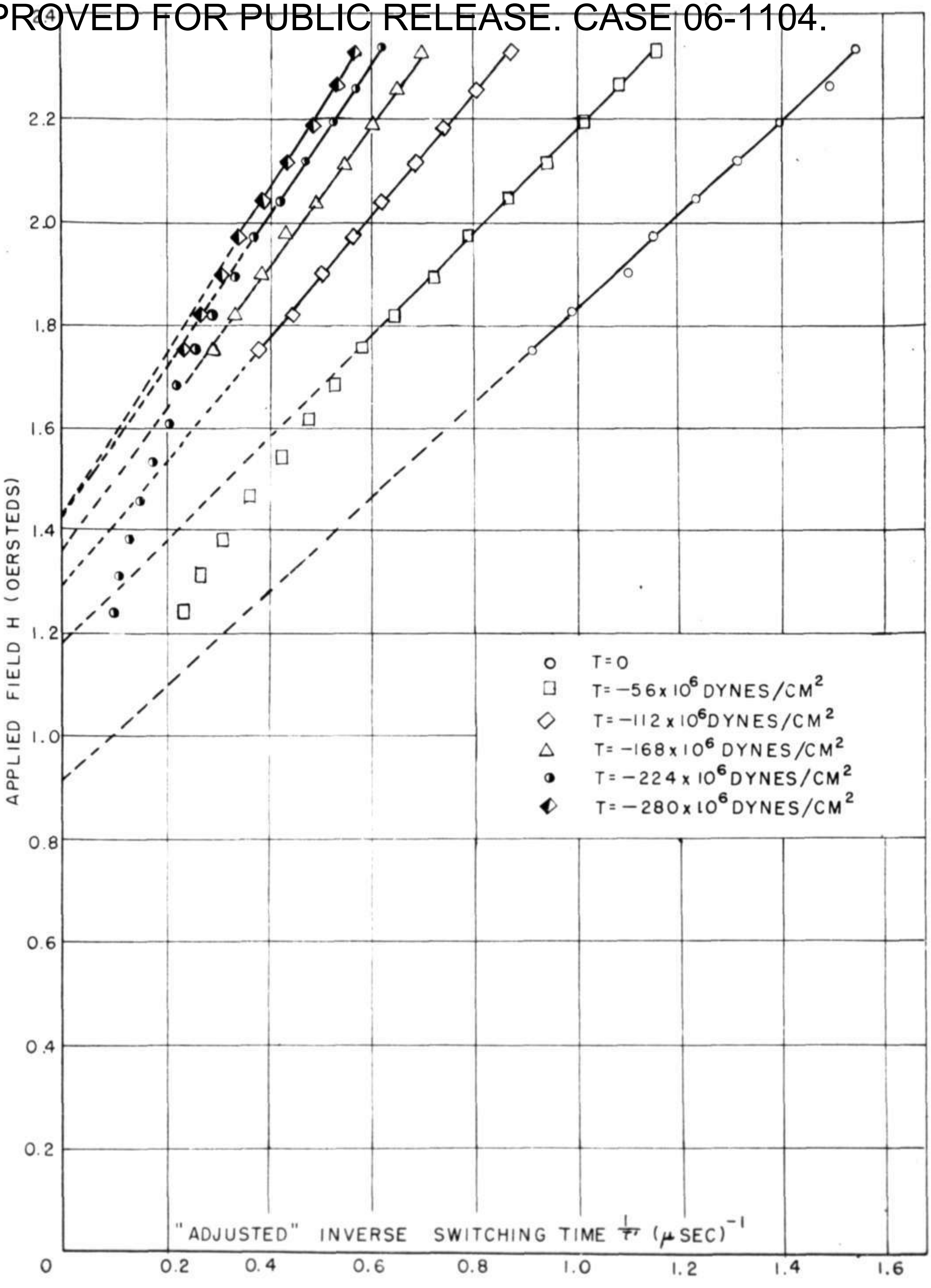
A-57197
MN


INITIAL MAGNETIZATION CURVES
OF FERROXCUBE 103 FERRITE
UNDER VARIOUS STRESSES
FLUX DENSITY 4π I (GAUSS)
MAGNETIC FIELD H (OERSTEDS)
3000
2500
2000
1500
1000
500
0
2
3
4
5
6
7
8
9
T = 0
T = −56x10^6 DYNES/CM^2
T = −112x10^6 DYNES/CM^2
T = −280x10^6 DYNES/CM^2



MEASUREMENTS OF S_V AND H_{OM} OF FERROXCUBE 103 UNDER VARIOUS STRESSES

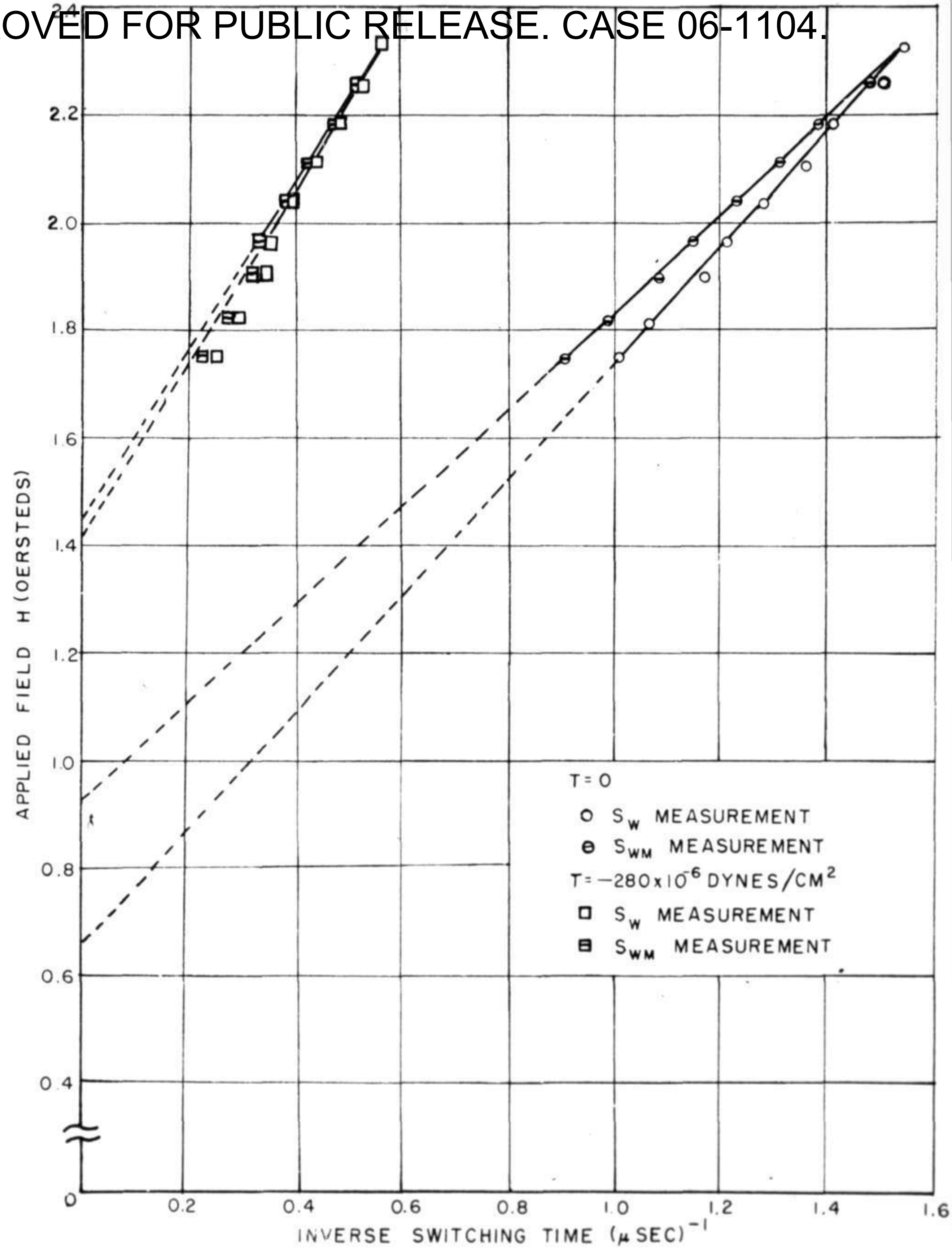
A-57198
MN



MEASUREMENTS OF S_{WM} AND H_{OM}'

OF FERROXCUBE 103 FERRITE UNDER VARIOUS STRESSES

A—57199
MN



COMPARISON OF S_W WITH S_{WM} AND H_0 WITH H_{OM}'

AT ZERO AND MAXIMUM STRESS

A-57200
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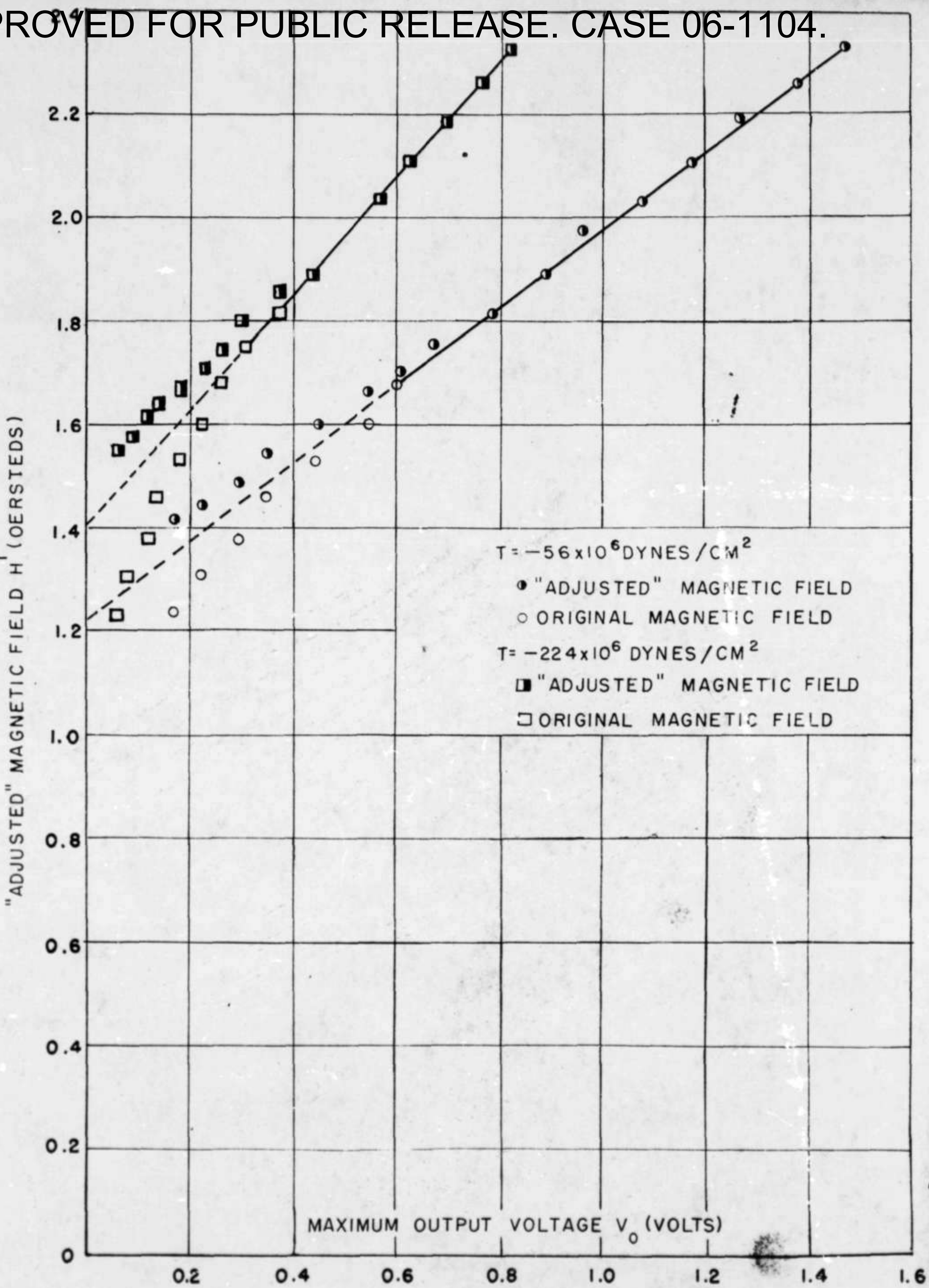

"ADJUSTED" FIELD STRENGTH VERSUS MAXIMUM OUTPUT VOLTAGE FOR FERROXCUBE 103 UNDER DIFFERENT STRESS CONDITIONS

A-57201
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