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SUBJECT: **EQUATION OF MOTION FOR A CYLINDRICAL 180° DOMAIN WALL**

To: Group 63 Staff

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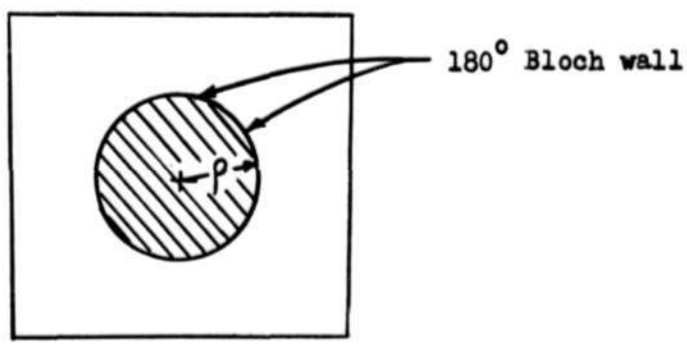
Abstract: The equation of motion for a cylindrical domain wall has been derived utilizing the Lagrangian formulation. Additional terms enter the equation of motion for a cylindrical wall that do not appear in that for a planar wall.

The motion of planar 180° domain walls have been investigated by J. K. Galt,[1] where the equation of motion is $m\ddot{z} + \beta\dot{z} + \alpha z = 2I_sH$ for a unit area of 180° Bloch Wall. m = equivalent mass per unit area, β is the damping coefficient, α is the stiffness coefficient and z is the linear displacement of the wall. I_s is the saturation magnetization and H is the applied field.

To derive the equation of motion for a cylindrical wall, the Lagrangian formulation is used. The Lagrangian $= \mathcal{L} = T - V$; where T is the kinetic energy of system and V is the total potential energy. The dissipation function is $G_{(\dot{\rho})}$ where ρ is the radial displacement of wall. The equation of motion is:

$$\frac{\partial}{\partial t}\left(\frac{\partial \mathcal{L}}{\partial \dot{\rho}}\right) + \frac{\partial G}{\partial \dot{\rho}} - \frac{\partial \mathcal{L}}{\partial \rho} = 0 .$$



Cross Section of a Unit Area of Material

Note: Saturation magnetization of shaded domain is perpendicular to paper and toward reader, and that for the unshaded domain is away from reader.

1. J. K. Galt, J. Andrus, and H. G. Hopper, "Motion of Domain Walls in Ferrite Crystals," Rev. Mod. Phys., vol. 25, p. 93, (Jan. 1953).

Wall motion is considered for a wall of the configuration shown where the domain enclosed by the wall is magnetized in opposition to that outside, and is ℓ units long. The applied field will be parallel to the domains, favoring one of the two directions, thus causing the wall to move radially in such a direction as to enlarge the domain magnetized in the direction of the applied field.

The kinetic energy is

$$T = \frac{(2\pi\rho\ell)m\dot{\rho}^2}{2}$$

The total potential energy is

$$V = E_{mutual} + E_{wall} + E_{magnetostatic}$$

$$= (1-\pi\rho^2)\ell\, 2HI_s + (2\pi\rho\ell)\sigma_w + \frac{(2\pi\rho\ell)\alpha\rho^2}{2}$$

and the dissapative function is;

$$G_{(\dot{\rho})} = \frac{(2\pi\rho\ell)\beta\dot{\rho}^2}{2}$$

where σ_w is the wall energy per unit area, all other constants being the same as those in the equation of motion used for a planar wall.

Hence substituting these values in the equation of motion;

$$\frac{\partial}{\partial t}\left(\frac{\partial \mathcal{L}}{\partial \dot{\rho}}\right) + \frac{\partial G}{\partial \dot{\rho}} - \frac{\partial \mathcal{L}}{\partial \rho} = 0$$

one obtains for the motion of a cylindrical 180° Bloch wall (per unit wall area);

$$m\ddot{\rho} + \frac{m\dot{\rho}^2}{2\rho} + \beta\dot{\rho} + \frac{3}{2}\alpha\rho + \frac{\sigma_w}{\rho} = 2HI_s$$

the equation of motion for a planar wall being

$$m\ddot{z} + \beta\dot{z} + \alpha z = 2HI_s$$

In general since m $\sim 10^{-10}$, the normal inertial term can be neglected. For low fields where $\dot{\rho}$ would not be excessively large, the additional inertial term can also be neglected. However the additional non-linear term, $\frac{\sigma_w}{\rho}$, is of the same order as the driving force and cannot be neglected in calculations involving the motion of a cylindrical wall. This term enters here since the total wall area, and thus also the total wall energy, varies as the wall moves.

Signed *Philip K. Baltzer*
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