Digital Computer Laboratory
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SUBJECT: GROUP 63 SEMINAR ON MAGNETISM, L

To: Group 63 Staff

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At this meeting the discussion of Néels article, begun at the previous meeting, was continued.

By analogy with simple ferromagnetic materials, it is to be expected that spontaneous magnetization will occur in ferrites below the Curie temperature θ_p. The observable saturation magnetization $\vec{I}_s$ will be the algebraic sum of the values of the two partial spontaneous magnetizations corresponding to the A and B sites. Since we are considering the case of anti-parallel alignment of the spins of the two sites ($\varepsilon = -1$), I_s will be equal to the arithmetic difference of the partial magnetizations. That is,

$$I_s = \left| \lambda I_{as} - \mu I_{bs} \right| \qquad \text{L-1}$$

where $\vec{I}_{as}$ and $\vec{I}_{bs}$ represent the spontaneous magnetization of the A and B sites respectively. These spontaneous magnetizations are created and maintained solely by the action of the molecular fields $\vec{h}_a$ and $\vec{h}_b$ such that $\vec{I}_{as}$ and $\vec{I}_{bs}$ are solutions of the equation

$$\left.\begin{aligned} \vec{I}_{as} &= MB_j\left[\frac{M\vec{h}_a}{RT}\right] \\ \vec{I}_{bs} &= MB_j\left[\frac{M\vec{h}_b}{RT}\right] \end{aligned}\right\} \qquad \text{L-2}$$

It should be borne in mind that the brackets in the above equations denotes "function of" rather than multiplication. Substituting the values of h_a and h_b given in equations XLIX-10 and XLIX-11, the above equations become

$$\vec{I}_{as} = MB_j \left[\frac{nM}{RT}(\alpha\lambda\vec{I}_{as} - \mu\vec{I}_{bs})\right]$$

$$\vec{I}_{bs} = MB_j \left[\frac{nM}{RT}(\beta\mu\vec{I}_{bs} - \lambda\vec{I}_{as})\right] \qquad \text{L-3}$$

The above equation will be used in determining the thermal variation of the spontaneous magnetization. In particular, we shall consider the spontaneous magnetization and its temperature coefficient at 0° absolute, and at the Curie temperature Θ_p.

At the absolute zero temperature the energy is a minimum and is given by

$$W = -\tfrac{1}{2}\lambda\vec{I}_{as}\cdot\vec{h}_a - \tfrac{1}{2}\mu\vec{I}_{bs}\cdot\vec{h}_b \qquad \text{L-4}$$

where W is the energy of a gram-ion of the material. Substituting the values of $\vec{h}_a$ and $\vec{h}_b$ given in equations XLIX-9 and XLIX-10 the above may be rewritten in the form

$$W = -\frac{n}{2}(\alpha\lambda^2 I_{as}^{\ 2} + 2\lambda\mu I_{as}I_{bs} + \beta\mu^2 I_{bs}^{\ 2}) \qquad \text{L-5}$$

I_{as} and I_{bs} are positive and have a maximum value at saturation equal to M. There are four possible solutions which will minimize the energy W. These are:

Case I. Material always paramagnetic; $\Theta_c \leq 0^\circ K$

$$I_{as} = I_{bs} = 0$$

$$W_I = 0 \qquad \text{L-6}$$

Case II. Both I_{as} and I_{bs} are equal to their maximum value.

$$I_{as} = I_{bs} = M$$

$$W_{II} = \frac{nM^2}{2} (\alpha \lambda^2 + 2 \lambda\mu + \beta\mu^2) \qquad \text{L-7}$$

Case III I_{as} has its maximum value while I_{bs} has the value which minimizes W.

$$I_{as} = M$$

$$\frac{\partial W}{\partial I_{bs}} = 0 = -n(\lambda\mu M + \beta\mu^2 I_{bs})$$

$$I_{bs} = -\frac{\lambda M}{\mu\beta}$$

$$W_{III} = -\frac{nM^2}{2} \lambda^2(\alpha - \frac{1}{\beta}) \qquad \text{L-8}$$

Case IV I_{bs} has its maximum value while I_{as} has the value which minimizes W.

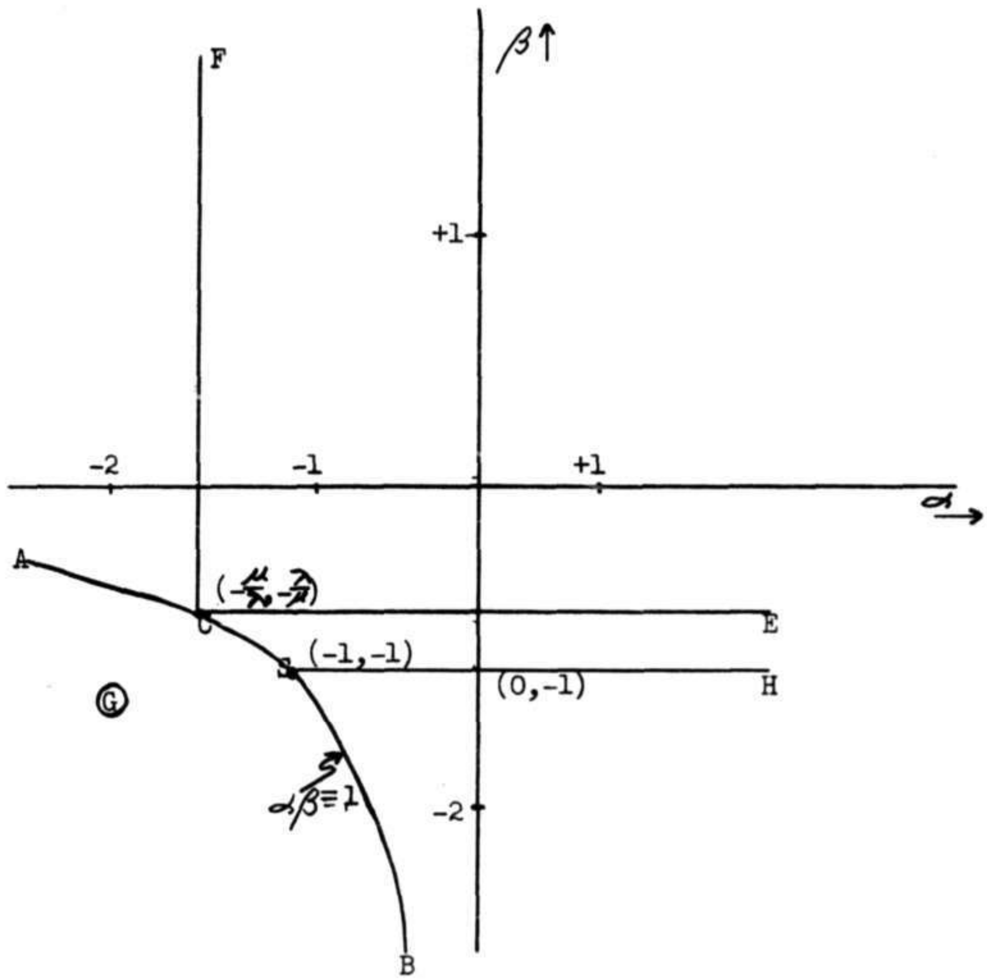
$$I_{bs} = M$$

$$\frac{\partial W}{\partial I_{as}} = 0 = -n (\alpha \lambda^2 I_{as} + \lambda\mu M)$$

$$I_{as} = -\frac{\mu M}{\lambda\alpha}$$

$$W_{IV} = -\frac{nM^2}{2}\mu^2 (\beta - \frac{1}{\alpha}) \qquad \text{L-9}$$

Let us draw coordinate axes with β as the ordinate and α as the abscissa, as in figure 108.



– PLANE WITH LINES DRAWN ASSUMING $\varepsilon = -1$ AND $\frac{\lambda}{\mu} = \frac{2}{3}$

FIGURE 108

From the equation for Θ_p given at the previous meeting (equation XLIX-18) it can be shown that $\Theta_p = 0$ when $\alpha\beta = 1$. The negative branch of this hyperbola is shown in figure 108. For $\alpha\beta > 1$ with both α and β negative, $\Theta_p < 0$ and the material is always paramagnetic. In this region of the $\alpha\beta$ curve, indicated by G, solution I holds.

In the remaining regions of the α/β plane either W_{II}, W_{III}, or W_{IV} will be the solution yielding the minimum energy condition. We must now determine the regions in which each is a minimum.

The maximum value I_{as} or I_{bs} can have is M. When both equal M, Case II holds. For case III to hold, $I_{as} = M > I_{bs}$. Therefore,

$$M > -\frac{\lambda M}{\mu\beta}$$

$$1 > -\frac{\lambda}{\mu\beta}$$

$$\beta < -\frac{\lambda}{\mu}$$

Thus, Case III is limited to the region in which $\beta < -\frac{\lambda}{\mu}$. When $\beta = -\frac{\lambda}{\mu}$, $I_{bs} = M$, which defines Case II. For $\beta -\frac{\lambda}{\mu}$, $I_{bs} > M$. Since this is impossible in view of the physical limitation of the magnetization, it is meaningless to speak of Case III in that region. Thus, in figure 108, W_{III} is limited to the region below the line CE, on which $\beta = -\frac{\lambda}{\mu}$.

Similarly, for Case IV to hold, $I_{bs} = M > I_{as}$. Therefore,

$$M > -\frac{\mu M}{\alpha\lambda}$$

$$1 > -\frac{\mu}{\alpha\lambda}$$

$$\alpha < -\frac{\mu}{\lambda}$$

Case IV is therefore limited to the region left of the line CF, on which $\alpha = -\frac{\mu}{\lambda}$.

While the above considerations limit the regions in which Cases III and IV can exist, they do not prove that these cases are energetically the most stable within the specified regions. Case III will exist in region ACF only if $W_{III} < W_{II}$ in this region, and Case IV will exist in region BCE only if $W_{IV} < W_{II}$ in this region.

In region ACF:

$$W_{III} - W_{II} = \frac{nM^2}{2}\left[\lambda^2\left(\frac{1}{\beta} - \alpha\right) + \alpha\lambda^2 + 2\lambda\mu + \beta\mu^2\right]$$

$$= \frac{nM^2}{2}\left[\frac{\lambda^2}{\beta} + 2\lambda\mu + \beta\mu^2\right]$$

$$= \frac{nM^2}{2\beta}\left[\lambda^2 + 2\beta\lambda\mu + \beta^2\mu^2\right]$$

$$= \frac{nM^2}{2\beta}\left[\lambda + \beta\mu\right]^2$$

Since n is positive, and the squared terms are positive, and β is negative in this region, $W_{III} - W_{II}$ is negative. Thus

$$W_{III} < W_{II}$$

and case III exists in region ACF.

In region BCE:

$$W_{IV} - W_{II} = \frac{nM^2}{2}\left[\mu^2\left(\frac{1}{\alpha} - \beta\right) + \alpha\lambda^2 + 2\lambda\mu + \beta\mu^2\right]$$

$$= \frac{nM^2}{2}\left[\frac{\mu^2}{\alpha} + 2\lambda\mu + \alpha\lambda^2\right]$$

$$= \frac{nM^2}{2\alpha}\left[\mu^2 + 2\alpha\lambda\mu + \alpha^2\lambda^2\right]$$

$$= \frac{nM^2}{2\alpha}\left[\mu + \alpha\lambda^2\right]$$

Since α is negative in this region

$$W_{IV} < W_{II}$$

and Case II exists in region BCE.

<u>Direction of Magnetization</u>

Upon measuring the value of the saturation magnetization in the presence of an external field only the absolute value of I_s is obtained. Since

$$|I_s| = |\lambda I_{as} - \mu I_{bs}|,$$

a study of the relative magnitudes of λI_{as} and μI_{bs} in the various regions of figure 108 will show whether the magnetization in the A or B sites predominates in these regions.

Let us adopt a sign convention wherein $\vec{I}_s$ is positive when it is parallel to $\vec{I}_{as}$, and $\vec{I}_s$ negative when antiparallel to $\vec{I}_{as}$. Then we find:

I Region ACF (Case IV)

$$I_s = -\mu M - \frac{\mu M}{\alpha} = -\mu M\left(1 + \frac{1}{\alpha}\right)$$

$$\frac{I_s}{M} = -\mu - \frac{\mu}{\lambda}.$$

Since $\alpha < -\frac{\mu}{\lambda}, \frac{1}{\alpha} > -\frac{\lambda}{\mu}$

Thus $-\frac{1}{\alpha} < \lambda$ and $-\frac{\mu}{\alpha} < \lambda$

$$\frac{I_s}{M} = -\mu - \frac{\mu}{\alpha} < -\mu + \lambda$$

If $\mu > \lambda$, $I_s < 0$, so that the net magnetization is in the direction of I_{bs}.

II Region FCE (Case II)

$$I_s = \lambda M - \mu M = M(\lambda - \mu)$$

$$\frac{I_s}{M} = \lambda - \mu < 0$$

Therefore, if $\lambda < \mu$, I_s is negative in this region also, and the net magnetization is in the direction of I_{bs}.

III Region BCE (Case III)

$$I_s = \lambda M + \frac{\lambda M}{\beta} = \lambda M(1 + \frac{1}{\beta})$$

$$\frac{I_s}{M} = \lambda (1 + \frac{1}{\beta})$$

In this region, β can be smaller or greater than -1. For the case $\beta > -1$, $\frac{1}{\beta} < -1$ and I_s is negative; for $\beta < -1$, $\frac{1}{\beta} > -1$ and I_s is positive.

The line SH in figure 108 represents the equation $\beta = 1$. Above this line I_s is therefore in the direction of I_{bs}, and below this line I_s is in the direction of I_{as}.

IV Region ACB G (Case I)

In this region both I_{as} and I_{bs} are zero.

Summarizing, I_s is in the same direction as I_{bs} throughout the region HSA, and it is in the direction of I_{as} in the region HSB.

The results obtained in this session are summarized in table I.

TABLE I

Region	Solution of Minimum Energy	$\frac{I_{bs}}{M}$	$\frac{I_{as}}{M}$	$\frac{I_s}{M}$	Direction of I_s
ACB (G)	W_I	ALWAYS PARAMAGNETIC			------
ACF	W_{IV}	$-\frac{\mu}{\lambda\alpha}$	1	$-\mu(1+\frac{1}{\alpha})$	I_{bs}
FCE	W_{II}	1	1	$\lambda-\mu$	I_{bs}
ECSH	W_{III}	1	$-\frac{\lambda}{\mu\beta}$	$\lambda(1+\frac{1}{\beta})$	I_{bs}
HSB	W_{III}	1	$-\frac{\lambda}{\mu\beta}$	$\lambda(1+\frac{1}{\beta})$	I_{as}

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