Digital Computer Laboratory
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SUBJECT: GROUP 63 SEMINAR ON MAGNETISM, XVI

To: Group 63 Staff

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The reading of Kronig's article was concluded at this session. At the previous meeting it was stated that by the free electron model it is impossible to explain the optical properties of solids. However, when we consider the model of electrons in a periodic potential, as discussed at the previous session, we find there are positions of minimum energy for the electrons. We can then treat the system as an oscillatory one, and can obtain dispersion.

Even in the model of the free electron gas, interaction between electrons and ions was assumed to occur by means of collisions, such as occurs in the kinetic theory of gases. Bloch abandoned this model and analyzed the motion of the crystal lattice in the manner first proposed by Debye. He further considered the interaction of the lattice vibration and the De Broglie* waves of the electrons in the solid.

Debye considered the metal ions as points in a three dimensional lattice. These equilibrium points are shown for a two dimensional lattice in Figure 26. For the three dimensional case one can imagine additional lattices like the one shown extending into the plane of the paper. The energy of the lattice is primarily due to vibrational motion of the lattice elements.

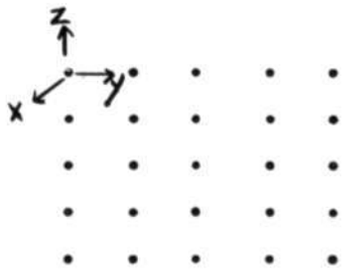

FIGURE 26

Since these elements interact, we may regard a lattice of N elements as a system containing N coupled oscillators. If the elements had only one degree of freedom, we have seen that this would lead to N modes of oscillation.

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* See Appendix III

However, since the elements have three vibrational degrees of freedom, (can oscillate in x, y, or z directions) there are 3N modes of oscillation. Since N is a very large number, there is an almost continuous range of frequencies.

Debye plotted $\frac{dN}{dW}$ versus W, obtaining a curve of the form shown in Figure 27.

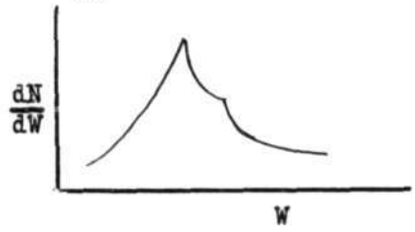

FIGURE 27

Integration over this curve yields 3N, the number of modes of vibration.

When the wave length of the vibration lattice coincides with the De Broglie wavelength of the free electrons a resonance effect occurs. In this case the electrons move through the lattice with little difficulty. A rise in temperature tends to disturb the lattice by increasing the thermal agitation of the atoms, and under these circumstances it is more difficult for electrons to pass through. One would therefore expect the conductivity of a conductor to decrease with increasing temperature; this has been verified.

In order to apply the model of the electron gas in a periodic potential in as quantitative a manner as possible, the dependence of potential on coordinates had to be determined as accurately as possible; and methods of finding energy values and wave functions of an electron had to be developed.

The first part was the simpler one; the self-consistent fields, calculated for isolated atoms or ions, could be assumed to be superposed in the metallic lattices. Thus it is essentially treated as a central field problem.

As for determining the energy values and wave functions, Wigner and Seitz pointed out that constructing a median plane between nearest neighbors for lattices of the body centered or face centered cubical type subdivides these lattices into cells. Each of these cells has an ion at the center and a polyhedral surface which does not differ very much from a sphere. (see Figure 28*).

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* See Appendix III

** Slater, J.C., The Electronic Structure of Metals, Reviews of Modern Physics, 6, October, 1934, P.225

FIGURE 28

Polyhedral cells for face centered (a) and body centered (b) cubic lattices. Dotted lines represent unit cells, containing two atoms for the body centered lattice, four for the face centered.

Using this model, one can obtain an energy value approximating the lower energy limit of the energy band occupied by the conduction electrons. This model also made it possible to explain metallic cohesion in terms of the interaction between the moving electrons and stationary ions; this is analagous to the problem of explaining the stability of molecules in terms of electrons shared by two or more atoms.

To explain ferromagnetism in metals, we must remember that many poor conductors are ferromagnetic; therefore the model chosen should not exclude systems without "free" electrons. Exchange forces, as discussed in meeting 6, do not depend on conduction electrons, but Zener claims that conducting electrons aid ferromagnetic lining up of the spins of the lattice elements. This will be discussed in detail at a later meeting.

Ferromagnetic resonance has been observed by Griffith and interpreted by Kittel. It is due to the precession of the magnetization vector ($\vec{\mu}$ - see Appendix II) about an applied constant magnetic field. The precession frequency will be approximately twice the Larmor frequency $\underline{o}$, where

$$o = \frac{eH}{4\pi mc}$$

If a periodic magnetic field is applied at right angles to the constant field, with a frequency equal to the precession frequency, resonance occurs. This leads to an anomolous behavior of the magnetic susceptibility, and is accompanied by absorption.

This effect can be explained in terms of the splitting of the energy levels in a magnetic field, corresponding to different values of $\vec{M}$, the component of $\vec{J}$ in the direction of the field. (see Appendix II).

This splitting has long been observed indirectly, and it is known as the Zeeman effect.* Figure 29 shows that the energy difference, $\underline{E}$, between the various sublevels in a split level is so small that the frequency, $\nu = \frac{E}{h}$, corresponding to a transition between sublevels of the same split level, lies in the microwave region which has been explored only recently.

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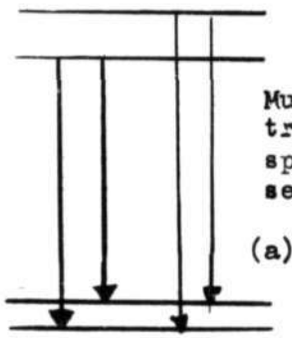
* See Herzberg, G., "Atomic Spectra and Atomic Structure," Dover Publications, New York, 1944, P. 96

When transitions occur between sublevels in different split levels, the frequency usually lies in the ultraviolet, visible, or infra-red region; and numerous lines are observed near each other. (four for the case shown in Figure 29(a)) From the distances between the lines in the spectrum, Zeeman computed the level splitting, and his results agree with the results obtained directly from microwave measurements.



Multiple lines due to transitions between split levels, as observed by Zeeman

(a)

Direct transitions between sublevels in a split level, as observed by microwave techniques

(b)

FIGURE 29

Resonance has been observed without an applied magnetic field. This may be explained in terms of the interaction of identical coupled systems which, as we have seen, leads to more than one possible mode of oscillation.

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Approved JRB

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