Digital Computer Laboratory
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SUBJECT: GROUP 63 SEMINAR ON MAGNETISM, III

To: Group 63 Staff

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In 1919, Miss Van Leeuwen showed that when classical Boltzmann statistics are applied to any dynamical system, the magnetic susceptibility is zero.

Prior to this point, Langevin had succeeded in explaining the phenomena of dia- and paramagnetism, obtaining results in general agreement with experimental evidence. However, though his approach to the problem was classical, he inadvertently assumed certain discrete values which enabled him to obtain non-zero results. These assumptions will be discussed later.

A correct approach must be based upon modern physics, but the explanations of Langevin are still of considerable interest since they provide a great deal of insight into the physical basis of dia- and paramagnetism.

Diamagnetism

Langevin explained diamagnetism in terms of orbital electron motion in an increasing magnetic field. The direction of motion of the electron is such as to cause an oppositely directed field. This means the electron moves in a circle of radius $\underline{R}$ in a plane perpendicular to the field (see Figure 2). Since we have previously established the relationship

$$\vec{\nabla} \times \vec{E} = -\frac{\mu}{c}\frac{\partial \vec{H}}{\partial t} - \frac{\vec{H}}{c}\frac{\partial \mu}{\partial t}, \qquad \text{(III - 1)}$$

and in a vacuum

$$\mu = \mu_0 \text{ and } \frac{\partial \mu}{\partial t} = 0,$$

then

$$\vec{\nabla} \times \vec{E} = -\frac{\mu_0}{c}\frac{\partial \vec{H}}{\partial t}. \qquad \text{(III - 2)}$$

Furthermore, we have Stokes' Theorem which states:

$$\oint \vec{E} \cdot d\vec{l} = \iint_S (\vec{\nabla} \times \vec{E}) \cdot d\vec{s} \qquad \text{(III - 3)}$$

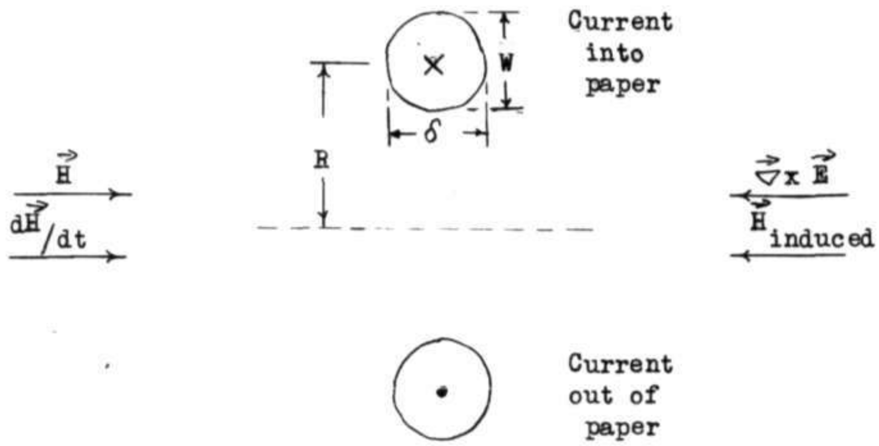

Figure 2

Substituting III - 2 into III - 3 and integrating over the configuration pictured in Figure 2, we get

$$2\pi R \cdot E = -\frac{\mu_0}{c}\frac{\partial H}{\partial t}\pi R^2 \qquad \text{(III - 4)}$$

$$E = -\frac{\mu_0 R}{2c}\frac{dH}{dt} \qquad \text{(III - 5)}$$

The equations above were obtained from dot products, and so represent a scalar relationship.

Let us define a unit vector $\hat{\imath}$ in the direction of the electron velocity at any time. Then, since electric field intensity is, by definition, the force per unit positive charge

$$\vec{F} = \hat{\imath}(-eE) = \left(\frac{e\mu_0 R}{2c}\frac{\partial H}{\partial t}\right)\hat{\imath} = m\frac{d\vec{v}}{dt}$$

$$\therefore m d\vec{v} = \left(\frac{e\mu_0 R}{2c} dH\right)\hat{\imath}$$

On integrating

$$\vec{v} = \vec{v}_0 + \left(\frac{e\mu_0 R}{2cm} H\right)\hat{\imath} \qquad \text{(III - 6)}$$

where v_0 is the constant of integration. For the special case of zero magnetic field $\vec{v} = \vec{v}_0$.

In the following discussion, we shall assume $v_0 = 0$. This essentially states that the atom has no permanent angular momentum. This must be done for the diamagnetic case since diamagnetism is effectively obliterated by a very powerful ν_o term in the case of para- and ferromagnetism.

Then for $\nu_o = 0$

$$\vec{\tau} = \rho \vec{v} = - n e \vec{v}$$

wherein ρ = charge density (charge/unit volume)

n = electron density (number of electrons/unit volume).

Letting $\vec{I}$ = Current = $\vec{\tau}$ x cross-sectional area, we shall take this area to be δ x W (see Figure 2). Then

$$I = - n e \vec{v} \delta W$$

and from equation (III - 6)

$$\vec{I} = - n e \delta W \left(\frac{e \mu_o R}{2mc} H \right) \vec{\tau}$$

$$= \frac{n e^2 \mu_o R \delta W H}{2mc} \vec{\tau} \qquad \text{(III - 7)}$$

Going back to Maxwell's Equation

$$\nabla x \vec{H} = \frac{1}{c} \left[4\pi \gamma \vec{E} + \frac{\partial D}{\partial t} \right] \qquad \text{(II - 1)}$$

and

$$\vec{\tau} = \gamma \vec{E} \qquad \text{(II - 8)}$$

for the particular case wherein $\frac{\partial \vec{D}}{\partial t} = 0$, we may combine the above to yield

$$\nabla x \vec{H}_i = \frac{4\pi \vec{\tau}}{c} \qquad \text{(III - 8)}$$

wherein H_i is the induced magnetic field--i.e., the field due to the induced current $\vec{I}_i$.

Since by Stokes Law

$$\oint \vec{H}_i \cdot d\vec{l} = \iint_S (\nabla x \vec{H}_i) \cdot ds$$

~~then~~, considering the line integral taken over the contour shown in Figure 3,

$$\oint \vec{H}_i \cdot d\vec{l} = H_i \delta$$

since the top horizontal line is taken as being infinitely far removed and H_i is zero there, while the vertical lines, being perpendicular to the field, contribute nothing. Therefore the only contribution comes from the bottom horizontal portion of the contour.

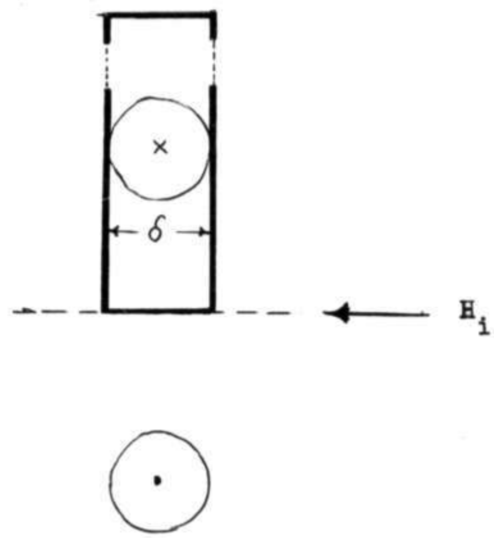

Figure 3

And, from III - 8, we have

$$\iint_S (\vec{\nabla} \times \vec{H}_i) \cdot ds = \iint_S \frac{4\pi}{c} \vec{i} \cdot ds = \frac{4\pi I}{c}$$

therefore,

$$H_i \delta = \frac{4\pi I}{c} \qquad \text{(III - 9)}$$

And, from III - 7

$$H_i \delta = -\frac{4\pi}{c} \frac{n e^2 \mu_0 \delta W R H}{2mc}$$

$$H_i = -\frac{2\pi\mu_0 n e^2 R^2}{mc^2} \left(\frac{W}{R}\right) H \qquad \text{(III - 10)}$$

Thus,

$$\chi_\mu = -\frac{2\pi\mu_0 n (eR)^2}{mc^2} \left(\frac{W}{R}\right) \qquad \text{(III - 11)}$$

Actually, the electron is not concentrated in a sharply delineated orbit, but is smeared out. Then χ_μ would be obtained by averaging the dimensionless geometric factor $\left(\frac{W}{R}\right)$ over this smeared out orbit.

It is seen, therefore, that an increasing magnetic field induces a field that is opposite to the direction of the increase. This is in accordance with the general law of nature that a stress imposed on a system causes the system to react in such a way as to produce an effect opposite to that aimed for by the original stress.

An example of this which we might mention in view of its importance in the subject of magnetism is eddy currents. A discussion of eddy currents is given in Appendix I.

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