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SUBJECT: GROUP 63 SEMINAR ON MAGNETISM V

To: Group 63

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The correct results cannot be obtained from Langevin's theory because a basic false assumption is made. Langevin assumed a continuous Boltzmann distribution of magnetic moment directions with respect to the applied field. The quantum theory shows that only certain discrete directions are permissible.

The nature of this discreteness can be shown in terms of the vector model of the atom. This is done in Appendix II.

However, let us now return to the Langevin theory of paramagnetism. Instead of considering all directions of magnetic moment possible, we will assume only certain discrete directions are possible. For a large possible number of allowable directions, we would expect the result to approach that of Langevin. This is the case. In our calculations, however, we will consider only the other extreme case wherein only two possible orientations exist. In one orientation the magnetic moment is +M ($\cos\Theta = +1$), in the other orientation it is -M ($\cos\Theta = -1$). We will still assume a Boltzmann Probability Law in determining the weighting factor to be applied to each direction. We then proceed as Langevin did to determine M, but in this case we sum over the possible directions rather than integrating over an entire range of values.

$$\overline{M} = \frac{Me^{\frac{MH}{RT}} - Me^{-\frac{MH}{RT}}}{e^{\frac{MH}{RT}} + e^{-\frac{MH}{RT}}}$$

$$\frac{\overline{M}}{M} = \operatorname{Tanh} \frac{MH}{RT} \qquad (V - 1)$$

$$= \frac{MH}{RT} - \frac{1}{3}\left(\frac{MH}{RT}\right)^3 + \frac{2}{15}\left(\frac{MH}{RT}\right)^5 + ---$$

Therefore, for $MH << RT$

$$\frac{\overline{M}}{M} \approx \frac{MH}{RT} \quad ; \quad \frac{\overline{M}}{H} = \frac{M^2}{kT}$$

$$_{\text{Molar}} = \frac{NM^2}{kT} = \frac{N^2M^2}{RT} \qquad (V - 2)$$

There are many deviations from equation V - 2 due to electron interaction. A more general equation is

$$= \frac{C}{T-\theta} \qquad (V - 3)$$

where the significance of θ is not the same for all materials. It is dimensionally a temperature and many paramagnetic materials become ferromagnetic below this point. This is not true in all cases. On the other hand, all ferromagnetic materials become paramagnetic at temperatures above θ.

Some General Paramagnetic Properties

When many atoms are close together, the various electron orbits and spins are usually so aligned as to tend to cancel out, giving no net effect.

The major exception amongst the paramagnetic materials occurs amongst the rare earth metals. Here it has been found that this is caused by the spin alignment of electrons which are deep within the atom and not as subject to external interference.

It has been found that iron vapor has a negligible magnetic effect compared to the solid case, which is the opposite of what might be expected. Further, compounds containing iron such as $FeSO_4$ are more magnetic than iron vapor. In $FeSO_4$, an electron was knocked out of iron atom, forming metallic ion. In ionic solution the extra electron was captured by $SO_4^{=}$ ion. So apparently knocking out an electron produced magnetic moment where it had not previously existed. This and the small magnetic moment of iron vapor would tend to indicate that iron ions rather than iron atoms are responsible for the huge magnetic moments found.

To understand this, let us look at iron in the solid form. The outer electrons, being very distant from the iron nucleus, are subject to relatively small influence by the nuclei. They tend to exist as free electrons forming an "electron gas" in the material. The iron is thus in ionic form with a strong magnetic moment.

Two strong paramagnetic gases are oxygen (O_2) and nitrous oxide (NO). In both cases the electrons holding the atom together have their spins aligned parallel to each other. The calculations showing this for the case of oxygen were made by Slater and Meckler on Whirlwind I.

In the case of free electrons, the paramagnetic tendency to align their spins is three times as strong as their diamagnetic tendency to move in orbits perpendicular to the applied field.

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