Digital Computer Laboratory
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SUBJECT: GROUP 63 SEMINAR ON MAGNETISM, APPENDIX VI
DERIVATION OF THE LAW OF PARAMAGNETISM IN FERRITES ASSUMING NEGATIVE INTERACTION BETWEEN SUBLATTICES.

To: Group 63 Staff

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For the special case $\mathcal{E} = -1$, equations 7, 10, 11, and 12 of meeting XLIX become

$$I = \lambda \vec{I}_a + \mu \vec{I}_b \qquad F - 1$$

$$\vec{h}_a = n\,(\alpha \lambda \vec{I}_a - \mu \vec{I}_b) \qquad F - 2$$

$$\vec{h}_b = n\,(\beta \mu \vec{I}_b - \lambda \vec{I}_a) \qquad F - 3$$

$$\vec{I}_a = \frac{C}{T}\,(\vec{H} + \vec{h}_a) \qquad F - 4$$

$$\vec{I}_b = \frac{C}{T}\,(\vec{H} + \vec{h}_b) \qquad F - 5$$

Combining equations F - 2 and F - 4 to eliminate $\vec{h}_a$

$$I_a = \frac{C}{T}\,(H + n\alpha\lambda I_a - n\mu I_b)$$

$$I_a\,(1 - \frac{nC\alpha\lambda}{T}) = \frac{CH}{T} - \frac{nC\mu}{T}\,I_b \qquad F - 6$$

Similarly, combining equations F - 3 and F - 5

$$I_b = \frac{C}{T}\left(H + n\beta\mu\, I_b - n\,\lambda I_a\right)$$

$$I_b\left(1 - \frac{nC\beta\mu}{T}\right) = \frac{CH}{T} - \frac{nC\lambda}{T}\, I_a \qquad \text{F - 7}$$

Substituting the value of I_b obtained in equation F - 7 into equation F - 6,

$$I_a\,(T - nC\alpha\lambda) = CH - nC\mu\left(\frac{CH - nC\lambda I_a}{T - nC\beta\mu}\right)$$

$$I_a\left(T - nC\alpha\lambda - \frac{n^2C^2\mu\lambda}{T - nC\beta\mu}\right) = \left(C - \frac{nC^2\mu}{T - nC\beta\mu}\right)$$

$$I_a\left[(T - nC\alpha\lambda)\,(T - nC\beta\mu) - n^2C^2\mu\lambda\right] = (CT - nC^2\beta\mu - nC^2\mu)\, H$$

$$I_a = \frac{\left[C\,(T - nC\beta\mu) - nC^2\mu\right]}{(T - nC\alpha\lambda)(T - nC\beta\mu) - n^2C^2\mu\lambda}\, H \qquad \text{F - 8}$$

Similarly, it can be shown

$$I_b = \frac{\left[C\,(T - nC\alpha\lambda) - nC^2\lambda\right]}{(T - nC\alpha\lambda)(T - nC\beta\mu) - n^2C^2\mu\lambda}\, H \qquad \text{F - 9}$$

Substituting equations F - 8 and F - 9 into equation F - 1, one obtains

$$I = \frac{\left[C\lambda T - nC^2\beta\lambda\mu - nC^2\lambda\mu + C\mu T - nC^2\alpha\lambda\mu - nC^2\lambda\mu\right]}{T^2 - nCT\,(\beta\mu + \alpha\lambda) + n^2C^2\alpha\beta\lambda\mu - n^2C^2\mu\lambda}\, H$$

$$= \frac{CT\,(\lambda + \mu) - nC^2\lambda\mu\,(\beta + \alpha + 2)}{T^2 - nC\,(\alpha\lambda + \beta\mu)\,T + n^2C^2\lambda\mu\,(\alpha\beta - 1)}$$

Therefore, since $\lambda + \mu = 1$

$$I = \frac{C\left[T - nC\lambda\mu\,(2 + \alpha + \beta)\right]}{T^2 - nC\,(\alpha\lambda + \mu\beta)\,T + n^2C^2\lambda\mu\,(\alpha\beta - 1)}\,H \qquad F - 10$$

Equation F - 10 is just equation 13 of meeting XLIX.

The inverse susceptibility $\frac{1}{\chi} = \frac{H}{I}$. Therefore

$$\frac{1}{\chi} = \frac{T^2 - nC\,(\alpha\lambda + \mu\beta)\,T + n^2C^2\lambda\mu\,(\alpha\beta - 1)}{C\,\left[T - nC\lambda\mu\,(2 + \alpha + \beta)\right]} \qquad F - 11$$

Since, for large T, $\frac{1}{\chi}$ becomes linear in T, and since there is also a singularity in $\frac{1}{\chi}$, Neel expresses this equation in the form (equation XLIX - 14)

$$\frac{1}{\chi} = \frac{T}{C} + \frac{1}{\chi_0} - \frac{\sigma}{T - \theta} \qquad F - 12$$

It is apparent that $\frac{1}{\chi} \to \infty$ as $T \to nC\lambda\mu\,(2 + \alpha + \beta)$ in F - 11 while $\frac{1}{\chi} \to \infty$ as $T \to \theta$ in F - 12. Since the right hand sides of equations F - 11 and F -12 are equivalent,

$$\theta = nC\lambda\mu\,(2 + \alpha + \beta) \qquad F - 13$$

Then, equating the right hand sides of F - 11 and F - 12,

$$\frac{T}{C} + \frac{1}{\chi_0} - \frac{\sigma}{T - \theta} = \frac{T^2 - nC\,(\alpha\lambda + \mu\beta)\,T + n^2C^2\lambda\mu\,(\alpha\beta - 1)}{C\,(T - \theta)}$$

Thus

$$T^2 + \left(\frac{C}{\chi_0} - \theta\right) T - C\left(\sigma + \frac{\theta}{\chi_0}\right) = T^2 - nC\,(\alpha\lambda + \mu\beta)\,T + n^2C^2\lambda\mu(\alpha\beta - 1)$$

Equating the coefficients of T^2, T, and the constant term

$$\Theta - \frac{C}{\chi_0} = nC\ (\alpha\lambda + \mu\beta) \qquad F - 14$$

$$C\sigma + \frac{C\Theta}{\chi_0} = - n^2C^2\lambda\mu\ (\alpha\beta - 1) \qquad F - 15$$

Substituting the value of Θ given in F - 13 into F - 14

$$nC\lambda\mu\ (2 + \alpha + \beta) - \frac{C}{\chi_0} = nC\ (\alpha\lambda + \mu\beta)$$

$$\frac{1}{\chi_0} = n\lambda\mu\ (2 + \alpha + \beta) - n\ (\alpha\lambda + \mu\beta)$$

$$= n\ (2\lambda\mu + \alpha\lambda\mu + \beta\lambda\mu - \alpha\lambda - \mu\beta)$$

$$= n\ (2\lambda\mu + \alpha\lambda(\mu - 1) + \beta\mu\ (\lambda - 1))$$

But

$$\mu - 1 = -\lambda$$

$$\lambda - 1 = -\mu$$

Therefore

$$\frac{1}{\chi_0} = n\ (2\lambda\mu - \alpha\lambda^2 - \beta\mu^2) \qquad F - 16$$

Substituting the values of Θ and $\frac{1}{\chi_0}$ into F - 15,

$$C\sigma + n^2C^2\lambda\mu\ (2 + \alpha + \beta)\ (2\lambda\mu - \alpha\lambda^2 - \beta\mu^2) = - n^2C^2\lambda\mu\ (\alpha\beta - 1)$$

$$\sigma = - n^2C\lambda\mu\ (\alpha\beta - 1) - n^2C\lambda\mu\ (2 + \alpha + \beta)\ (2\lambda\mu - \alpha\lambda^2 - \beta\mu^2)$$

$$\sigma = n^2 c \lambda\mu \left[1 - \alpha\beta - 4\lambda\mu + 2\alpha\lambda^2 + 2\beta\mu^2 - 2\alpha\lambda\mu + \alpha^2\lambda^2 + \alpha\beta\mu^2 - 2\beta\lambda\mu + \alpha\beta\lambda^2 + \beta^2\mu^2\right]$$

Let $1 = (\lambda + \mu)^2 = \lambda^2 + \mu^2 + 2\lambda\mu$

$$\beta = \alpha\beta\,(\lambda + \mu)^2 = \alpha\beta\lambda^2 + \alpha\beta\mu^2 + 2\alpha\beta\lambda\mu$$

then

$$\sigma = n^2 c \lambda\mu \left[\begin{array}{lll} \lambda^2 & + \mu^2 & + 2\lambda\mu \\ - \alpha\beta\lambda^2 & - \alpha\beta\mu^2 & - 2\alpha\beta\lambda\mu \\ + 2\alpha\lambda^2 & + 2\beta\mu^2 & - 4\lambda\mu \\ + \alpha^2\lambda^2 & + \alpha\beta\mu^2 & - 2\alpha\lambda\mu \\ + \alpha\beta\lambda^2 & + \beta^2\mu^2 & - 2\beta\lambda\mu \end{array}\right]$$

$$\sigma = n^2 c \lambda\mu \left[\lambda^2 (1 + 2\alpha + \alpha^2) + \mu^2 (1 + 2\beta + \beta^2) - 2\lambda\mu (1 + \alpha + \beta + \alpha\beta)\right]$$

$$= n^2 c \lambda\mu \left[\lambda^2 (1 + \alpha)^2 + \mu^2 (1 + \beta)^2 - 2\lambda\mu (1 + \alpha)(1 + \beta)\right]$$

$$\sigma = n^2 c \lambda\mu \left[\lambda(1 + \alpha) - \mu (1 + \beta)\right]^2 \qquad \text{F - 17}$$

Equations F - 16, F - 17, and F - 13 are identical with equations XLIX -15, 16, and 17. They therefore verify the values given at meeting XLIX for $\frac{1}{\chi_0}$, σ, and Θ consistent with equations XLIX - 13 and XLIX - 14.

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