

Memorandum M-1689

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SUBJECT: GROUP 63 SEMINAR ON MAGNETISM VI  
 To: Group 63 Staff  
 From: Arthur Loeb and Norman Menyuk  
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Suggested Reading: "Physical Theory of Ferromagnetic Domains", Charles Kittel, Review of Modern Physics, 21, October 1949, 541.

By Langevin's theory of paramagnetism we obtained as the average magnetic moment due to one spinning electron

$$\bar{M} = M(\coth a - \frac{1}{a}) \quad (\text{VI-1})$$

where  $a = \frac{\beta H}{kT}$  and  $\beta =$  magnetic moment of an electron.

When the number of possible magnetic moment directions were reduced to two, we found that the above was changed to

$$\bar{M} = M \tanh a \quad (\text{VI-2})$$

Equations VI-1 and VI-2 represent the extreme cases. In general, there will be more than two possible orientations, but less than a continuum of orientations. Thus the true result will be somewhere between the results given above.

Paramagnetic theory assumes an interaction between the magnetic dipole and the external field. No other interactions are involved since the magnetic dipoles are assumed to be independent of each other.

Now Weiss, in extending Langevin's theory to paramagnetic substances, postulated the existence of an internal field  $NI$ .

Then

$$H_e = H + NI$$

where  $H_e$  = effective magnetic field  
 $H$  = external magnetic field

and  $a$  becomes  $\frac{\beta H_e}{kT} = \frac{\beta H}{kT} + \frac{\beta NI}{kT}$

Further,

$$\frac{\bar{M}}{M} = \frac{I}{I_0}$$

where  $I$  = induced magnetization.

$I_0$  = induced magnetization at absolute zero temperature. This is true as, at absolute zero, there is no thermal agitation and all the molecular dipoles line up with the field.

Therefore, from equation VI-1, we have

$$\begin{aligned} \frac{I}{I_0} &= \coth a - \frac{1}{a} \\ &= \frac{1}{a} + \frac{a}{3} - \frac{1}{45} a^3 + \dots - \frac{1}{a} \\ &= \frac{a}{3} - \frac{a^3}{45} + \dots \end{aligned} \tag{VI-4}$$

For high values of  $T$ , this reduces to

$$\frac{I}{I_0} \approx \frac{a}{3} = \frac{\beta H}{3kT} + \frac{\beta NI}{3kT} \tag{VI-5}$$

Thus  $I$  is finite when  $H$  is zero.

If we let  $\theta = \frac{N\beta I_0}{3k}$ , then

$$\begin{aligned} \frac{I}{I_0} &= \frac{\beta H}{3kT} + \frac{I_0}{I_0 T} \\ &= \frac{\beta H}{3kT} + \frac{I/I_0}{T/\theta} \end{aligned}$$

$$\begin{aligned} \frac{I}{I_0} &= \frac{\beta H}{3kT} / (1 - \theta/T) \\ &= \frac{\beta H}{3k} / (T - \theta) = \frac{C}{T - \theta} \end{aligned} \quad (\text{VI-6})$$

Similarly, if we consider the case of only two possible orientations of the magnetic moment we have

$$\frac{I}{I_0} = \tanh a$$

and for high values of  $T$ , one may approximate

$$\frac{I}{I_0} \approx a = \frac{\beta H}{kT} + \frac{N\beta I}{kT} \quad (\text{VI-7})$$

Then define:  $\theta \equiv \frac{N\beta I}{k}$

hence

$$\begin{aligned} \frac{I}{I_0} &= \frac{\beta H}{kT} + \frac{I\theta}{I_0 T} \\ \therefore \frac{I}{I_0} &= \frac{\beta H}{k} / (T - \theta) = \frac{C'}{T - \theta} \end{aligned} \quad (\text{VI-8})$$

Thus in both cases we are led to equations of the same form.

Since the work of Weiss pre-dated the quantum theory, the equation at which he arrived by postulating an internal field is that given by equation VI-5. This is known as the Curie-Weiss Law.

The origin of the force required to produce this internal field remained unknown for many years. The internal field is much too great to be caused by purely magnetic forces, so it was quickly concluded that the molecular field must be electrostatic in nature. However, the

forces could only be explained by quantum mechanics, so the problem remained unsolved until 1928, when Heisenberg discovered the exchange forces of electrons in atoms. This force depends on the alignment of the electron spins, although the spins themselves do not appreciably interact magnetically.

In solids, and particularly in metallic crystals, the individual atoms are located close together and the orbits of their outer electrons will generally overlap. Under these circumstances one cannot even speak of the outer electrons as belonging to a particular atom, since they are moving about and being shared by neighboring atoms.

Because of a principle of a purely quantum mechanical origin, only certain spin configurations of these electrons are permissible. The configuration which will exist for any substance will, therefore, be that one of lowest energy which is permissible. In almost all cases this configuration is one in which the spins are aligned anti-parallel. In the case of iron ions and other ferromagnetic materials, the lowest energies occur with a parallel spin alignment. These are the materials for which there is a positive exchange energy.

It should be noted that the same kind of exchange interaction is used to explain the formation of stable molecules. For example, consider the interaction of two hydrogen atoms. Two energies are possible if they are a given distance apart; one corresponding to a higher energy than the other.

For the case of lower energy the spins are anti-parallel. This solution corresponds to an attraction, permitting molecular formation. This result is obtained by considering both electrons associated with both nuclei. Thus we are again dealing with an interchange interaction.

Slater and Bethe calculated the exchange energy as a function of the atomic separation divided by the diameter of the atom. The resultant curve is shown in figure 5 for the ferromagnetic range, and the position of the elements in this region are shown.

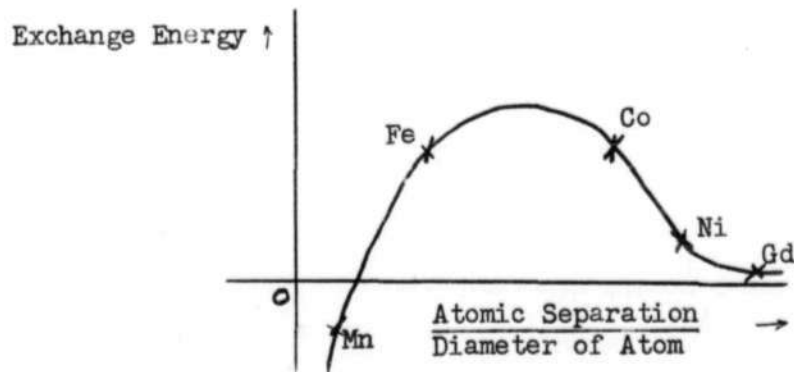


Figure 5

Although manganese is not a ferromagnetic material, and has a negative exchange energy, as shown in Figure 5, some alloys of manganese are ferromagnetic. Of particular interest are the Heusler alloys which consist of copper, aluminum and manganese. Since none of these are ordinarily ferromagnetic, the ferromagnetism of the alloy may seem somewhat surprising. However, in these alloys manganese has a greater atomic separation than in the pure solid state, and therefore falls on the curve in the region of positive exchange energy.

Gadolinium, as shown in figure 5, has a very high ratio of atomic separation to atomic diameter. As might be expected from this, gadolinium has been found to be ferromagnetic. However, its Curie point occurs at only 16°C because the exchange energy is so low as to be easily overcome by thermal disturbance.

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