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Digital Computer Laboratory
Massachusetts Institute of Technology
Cambridge, Massachusetts

SUBJECT: GROUP 63 SEMINAR ON MAGNETISM, APPENDIX V
ANALOGY BETWEEN A SET OF EIGENFUNCTIONS AND COORDINATE SYSTEMS

To: Group 63 Staff

From: Arthur L. Loeb and Norman Menyuk

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A set of functions $\psi_1, \psi_2, \ldots, \psi_n, \ldots$ is orthonormal if:

$$\int \psi_m \psi_n \, d\tau = \delta_{mn} \tag{E-1}$$

Any function can be expressed as a linear combination of all functions in a complete set of orthonormal functions:

$$\varphi = \sum_{n=1}^{\infty} a_n \psi_n \tag{E-2}$$

A complete set of orthonormal functions is a set including so many functions that no other functions orthogonal to any of the set exist.

The coefficients a_n can then be found by the expression

$$a_n = \int \psi_n^* \varphi \, d\tau \tag{E-3}$$

A set of unit vectors of a multidimensional Cartesian coordinates labeled $\hat{q}_1, \hat{q}_2, \ldots, \hat{q}_n, \ldots$ has the following properties:

$$\hat{q}_m \cdot \hat{q}_n = \delta_{mn} \tag{E-4}$$

For an ordinary three-dimensional Cartesian system the notation commonly used is $\hat{q}_1 = \hat{i}$, $\hat{q}_2 = \hat{j}$, $\hat{q}_3 = \hat{k}$ and

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$
$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

Any multi-dimensional vector $\vec{A}$ can be expressed as a linear combination of these unit vectors:

$$\vec{A} = \Sigma_n A_n \hat{q}_n \tag{E-5}$$

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and in particular in the three-dimensional case:

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

The dimensionality of a system is determined by the number of unit vectors $\hat{q}_n$ that are needed to describe any vector.

The components A_n of vector $\vec{A}$ are found by the expression

$$A_n = \hat{q}_n \cdot \vec{A} \qquad \text{(E-6)}$$

or in three dimensions:

$$\left.\begin{aligned} A_x &= \hat{i} \cdot \vec{A} = A \cos\alpha \\ A_y &= \hat{j} \cdot \vec{A} = A \cos\beta \\ A_z &= \hat{k} \cdot \vec{A} = A \cos\gamma \end{aligned}\right\} \text{ where } \alpha, \beta, \gamma, \text{ are the direction cosines of } \vec{A}$$

The analogy of equations (E-1), (E-2), and (E-3) with (E-4), (E-5), and (E-6) is at once apparent; the integral $\int \psi_n^* \varphi \, d\tau$ is analogous to the product $\hat{q}_n \cdot \vec{A}$.

Equation XXV-3 $$i\hbar \frac{\partial C_m}{\partial \tau} = \sum_n H_{mn} C_n$$

was solved by assuming $C_m = a_m e^{-i\frac{\mathcal{E}}{\hbar}\tau}$

$$\therefore \; \mathcal{E} C_m = \sum_n H_{mn} C_n \qquad \text{(E-7)}$$

If the set of C_n's is analogous to the components of a vector, then equation (E-7) is analogous to the tensor relationship between two vectors; Schroedinger's wavequation has thus turned into a matrix equation.

The problem is always to diagonalize the matrix $[H_{mn}]$, for when this is done the stationary states of the system are known. This approach is called "matrix mechanics".

Signed Arthur L. Loeb
Arthur L. Loeb

Norman Menyuk
Norman Menyuk

ALL/NM:jk
Group 62 (20)

Approved DRB
David R. Brown