MC 0241

BOX 7 FOLDER 85

Publications, 1930 - 1931

1930

Measurement of Small D. C. Potentials and Currents in High Resistance Circuits by Using Vacuum Tubes

By W. B. NOTTINGHAM

Reprinted from the March 1930 issue of the Journal of The Franklin Institute.

With Compliments
of

LEEDS & NORTHRUP COMPANY
4901 STENTON AVENUE PHILADELPHIA, PA.

LEEDS & NORTHRUP

RP-2-G 3-30

MEASUREMENT OF SMALL D.C. POTENTIALS AND CURRENTS IN HIGH RESISTANCE CIRCUITS BY USING VACUUM TUBES.

BY

W. B. NOTTINGHAM, E.E., Ph.D.,

Bartol Research Fellow.

BARTOL RESEARCH FOUNDATION

Communication No. 45.

ABSTRACT.

Part 1. The grid current and plate current characteristics of three element vacuum tubes are described and the simple equations of the D.C. amplifier are developed with special attention given to the circuit for measuring photo-electric currents. The grid current characteristic is shown to play a very important part in the equations for sensitivity.

Part 2. The second part deals with three common output circuits, namely, (1) the single tube circuit with an auxiliary battery to balance out the normal plate current; (2) the single tube bridge circuit and (3) the two tube bridge circuit. Two "universal" shunts are described for controlling the galvanometer current. One of these is a "special shunt" which controls the sensitivity accurately and maintains the damping constant while the other is the well known Ayrton shunt. The disadvantages of using this shunt are discussed.

Part 3. Complete circuits for the measurement of potentials and currents by direct deflection and null methods are discussed and some of the advantages and limitations of each are pointed out.

Part 4. Under this heading a few suggestions are given as to points of technical procedure.

PART 1. GENERAL CONSIDERATIONS.

A. Grid Current Characteristics.

a. Current components in grid circuit.

*a*1. Grid to filament leakage.

*a*2. Plate to grid leakage.

*a*3. Positive ion current to grid.

*a*4. Electron current to grid.

b. Grid impedance defined.
c. Floating potential defined.

B. Plate Current Characteristics.
a. Mutual conductance defined.
b. Plate impedance defined.
c. Tube equation for plate current.

C. Operation of Simple D.C. Amplifier.
a. Circuit equations.
b. Sensitivity to voltage.
c. Sensitivity to current.

Part 2. Special Output Circuits.

A. One Tube Circuit with Auxiliary Battery to Suppress Zero.
a. Circuit equations.
b. Sensitivity to voltage.

B. Wheatstone Bridge Circuit with Single Tube.
a. Circuit equations.
b. Conditions for "B" battery compensation.
c. Voltage sensitivity.
d. Condition for zero current in galvanometer.
e. Sensitivity of two circuits compared.

C. Bridge Circuit Using Two Vacuum Tubes.
a. Conditions for "B" battery compensation.
b. Circuit equations.
c. Condition for zero galvanometer current.
d. "Cut and try" method of adjustment.
e. Advantages of two tube circuit.
f. Conditions for filament battery compensation.

D. Summary.

E. Control of Galvanometer Deflection.
a. Special universal shunt.
b. Ayrton shunt and its limitations.
c. Summary on galvanometer control.

Part 3. Special Input Circuits.

A. Potential Measurements.
a. Case I. Where it is necessary to operate at floating potential because r_x is large.
b. Case II. Where it is possible to operate with a negative grid impedance because r_x is small.

B. Current Measurements in High Resistance Circuits.
a. Case I. Current amplifier operating at floating potential. Measurements by direct deflection.
b. Case II. Current amplifier operating at floating potential. Measurements by null method.
c. Case III. Current amplifier operating with a negative grid impedance. Measurements by direct deflection.

d. Case IV. Current amplifier operating with a negative grid impedance. Measurements by "null" method.

PART 4. TECHNICAL PROCEDURE.
A. Shielding and Insulation.
B. Structural Details.
C. Construction of High Resistances.

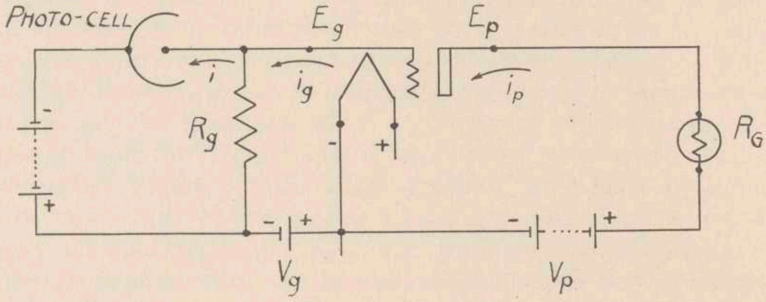
INTRODUCTION.

STUDIES requiring the measurement of small photoelectric currents have made it necessary to investigate the applicability of certain simple vacuum tube circuits to assist in the purpose. This work has been in progress some four years or more during which time a number of papers describing similar circuits have appeared. Although most of the results reported here were worked out independently of those papers, many of them are identical with those already published. Where such is the case and it is known to the writer, due acknowledgement is gladly accorded, but since there has been no exhaustive search of the literature, full acknowledgment to everyone who has reported similar results cannot be made. It is thought that there is enough new material contained in this paper to warrant its publication in spite of the fact that many of the results have already been published.

PART 1. GENERAL CONSIDERATIONS.

Of the many different types of circuits developed for the detection of small direct currents in high resistance circuits, only certain simple ones have been investigated. These depend on the observation of a change produced in the plate circuit of a three or four element vacuum tube as a result of a change in the grid potential. This change in grid potential is usually brought about by the "IR drop" produced in the grid resistance by the current being measured. When the circuit is used for potential measurements in high resistance circuits the change in grid potential is brought about directly. The simple circuit of Fig. 1 contains the essential elements. The operation of this circuit can be predicted at once from the "grid current *vs.* grid potential" and "plate current *vs.* grid potential" characteristics of the tube *taken with the operating resistance in the plate circuit.* Although theoretically it is not necessary to impose this requirement as regards the plate circuit resistance, a great simplification is brought about by it in that it is then necessary to consider only the two curves mentioned above. If this condition is *not* imposed it is necessary to work with the family of curves showing the "grid current *vs.* grid potential" relationship for a number of different plate potentials and similarly for the "plate current *vs.* grid potential" relationship.

FIG. 1.

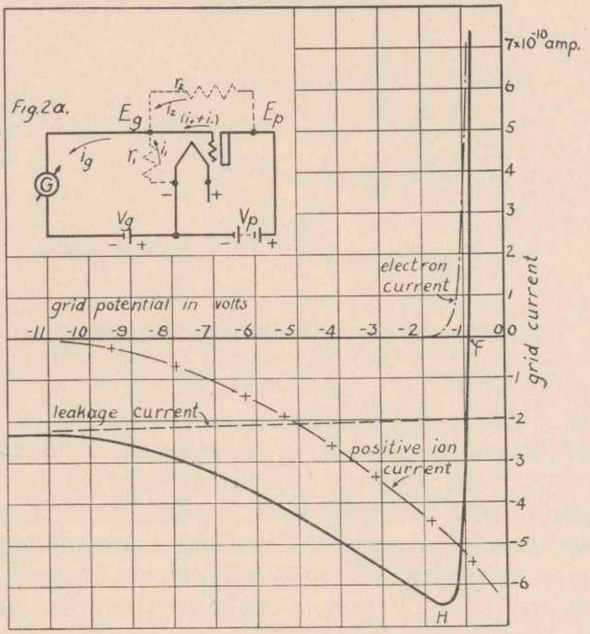


A. GRID CURRENT CHARACTERISTICS.

The grid circuit current characteristic shown by the solid line in Fig. 2 is a composite of at least four different currents. This current can be thought of as measured by means of the circuit shown in the schematic drawing Fig. 2*a*, inserted in Fig. 2.[1] The current flowing through the galvanometer for any given value of grid battery V_g and plate battery V_p is made up of the components described below. When a galvanometer of a low resistance is used the grid potential E_g will be practically equal to the "C" battery potential V_g. Since E_g will not be the same as V_g when a high resistance is introduced into the circuit, it will be well to recognize this distinction right at the beginning of this discussion. In the plate circuit there is, of course, the same sort of distinction between the "B" battery potential V_p and the plate potential E_p when the resistance in the plate circuit is not small compared with the plate impedance.

[1]An indirect method of determining the grid current which is essentially the same as that proposed by M. von Ardenne (*Jahrb. d. Drahtl. Telegr.*, *29*, 88, 1927) will be described later.

FIG. 2.



Grid current characteristics on UX-201-A vacuum tube.

a. Components of Current in Grid Circuit.

a1. Grid to filament leakage current.

The *leakage current* from the grid lead to the filament connections, socket support, etc., is

$$i_1 = -\frac{E_g}{r_1}, \tag{1}$$

where r_1 is the grid to filament insulation resistance shown symbolically in the figure and E_g is the potential of the grid when referred to the negative end of the filament as zero. The negative sign used in this equation is necessary in order that the equation be consistent with the direction of flow of the current as indicated on the diagram of Fig. 2*a*.

a2. Plate to Grid Leakage Current.

The leakage current which flows through the galvanometer due to the combined batteries V_g and V_p of Fig. 2*a* is

$$i_2 = \frac{E_p - E_g}{r_2}, \qquad (2)$$

where r_2 is the grid to plate insulation resistance symbolized by r_2 in Fig. 2*a* and E_p is the plate potential which in this case is the same as V_p.

The combined effect of these two types of leakage currents is given by

$$i_1 + i_2 = i_{12} = \frac{E_p}{r_2} - E_g\left(\frac{1}{r_1} + \frac{1}{r_2}\right). \qquad (3)$$

This leakage current is shown by the dashed line in the figure.[2]

*a*3. *Positive Ion Current.*

Since the gas can never be entirely removed during the evacuation of the tube, a certain amount of positive ion current will flow to the grid when it is at a negative potential with respect to other parts of the tube. The intensity of this positive ion current will be proportional to the number of gas molecules present and proportional to the electron current flowing from the filament to the plate, when the plate voltage is considerably higher than the ionization potential of the gas. Since the plate current depends on the grid potential, the positive ion current to the grid will depend on the grid potential.[3] This positive ion current i_+ is not a simple function of the grid potential and, therefore, can be represented only in a general way by

$$i_+ = f(E_g, E_p). \qquad (4)$$

Graphically this relationship is shown by the curve of Fig. 2 designated "positive ion current."

---

[2] If the vacuum tube is constructed with a separate grid connection as is the case with a UX 222, a few turns of copper wire wound around the tube just above the bakelite base will serve as a guard ring. The potential of this ring can be maintained at the same value as the normal operating value of the grid potential and thus reduce the effective leakage current to a very small value. This leakage in tubes with a structure similar to the UX 201-A can be reduced by cutting out the base and covering the surface of the glass with wax or sulphur.

[3] Mulder and Razek, *J. O. S. A. and R. S. I.*, **18**: 466 (1929).

a4. Electron Current to Grid.

When the grid is only slightly negative with respect to the negative end of the filament, a certain number of electrons are able to overcome the adverse negative potential between the filament and the grid because of the high initial kinetic energy of these electrons due to the high temperature of the filament. The way in which this electron current depends on the grid potential can be represented approximately by the empirical equation [4]

$$i_- = i_o \epsilon^{aE_g}, \tag{5}$$

where E_g is the value of the grid potential which must be negative with respect to the negative end of the filament for the equation to hold. a and i_o are constants depending on the temperature of the filament and possibly on the plate potential.

The graphical representation given in Fig. 2 shows the form of the function which is the thing of primary interest.

The composite of these four currents gives the "grid circuit" current

$$i_1 + i_2 + i_+ + i_- = i_g, \tag{6}$$

This is shown graphically by the solid line of Fig. 2. This question of the grid current has been gone into at some length because it plays a very important part in a number of ways in the D.C. amplifier.

b. Definition of Grid Impedance.

In the practical problems of adapting this type of D.C. amplifier to definite purposes, it is not always necessary to measure the grid current characteristic but it is *very necessary* to visualize the "*form*" of the curve in order to understand what is really going on in the input circuit. In particular, it is important to know the slope of the curve at various grid potentials, i.e., the change in grid current for a small change in grid potential.

In the course of the discussion to follow, it will be necessary to use a quantity which is equal to the reciprocal of the slope of the grid current curve at some particular point. It has

[4] M. von Ardenne, *Jahrb. d. Drahtl. Telegr.*, 29: 88 (1927).

become the accepted practice to designate the corresponding quantity in the plate circuit as the "plate impedance" of the tube. Let us therefore use the term "grid impedance" (Z_g), and let it be related to the slope of the grid current curve at any particular point by the equation

$$\frac{1}{Z_g} = \text{slope of the grid current curve} = \frac{\partial i_g}{\partial E_g}. \qquad (7)$$

From the curve we see that this "grid impedance" is positive for small negative values of E_g and is negative for larger negative values of E_g.

c. Definition of "Floating Potential."

The "floating potential,"[5] indicated by "F" in Fig. 2, is defined as the potential at which the grid current in the external circuit is *zero*. The grid impedance at this point has a definite positive value as given by the reciprocal of the slope of the tangent to the curve drawn through this point.

B. PLATE CURRENT CHARACTERISTICS.

The plate current characteristics of vacuum tubes are so well known and discussed in so many places [6] that it should not be necessary to do more than define a few of the common terms.

a. Mutual Conductance Defined.

In a vacuum tube, the mutual conductance (G_m) in the neighborhood of a given grid potential, is the slope of the "plate current *vs.* grid potential" curve taken with a *constant plate potential*. This definition expressed in equation form is

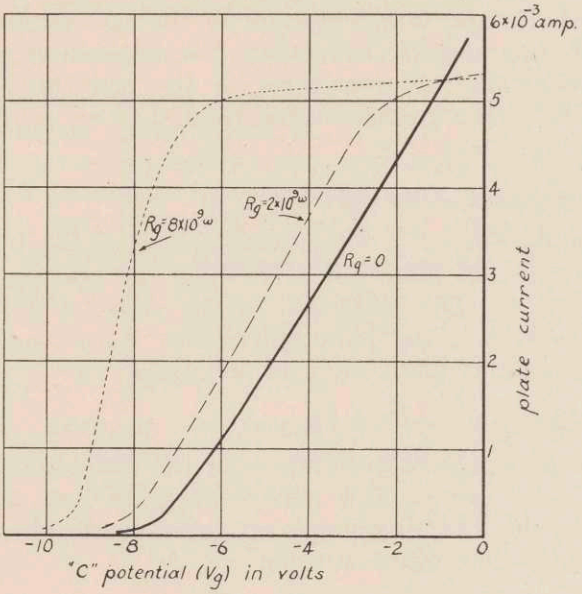
$$G_m = \left(\frac{\partial i_p}{\partial E_g}\right)_{E_p = \text{const.}}. \qquad (8)$$

---

[5] Sometimes denoted as the "potential of the free grid."

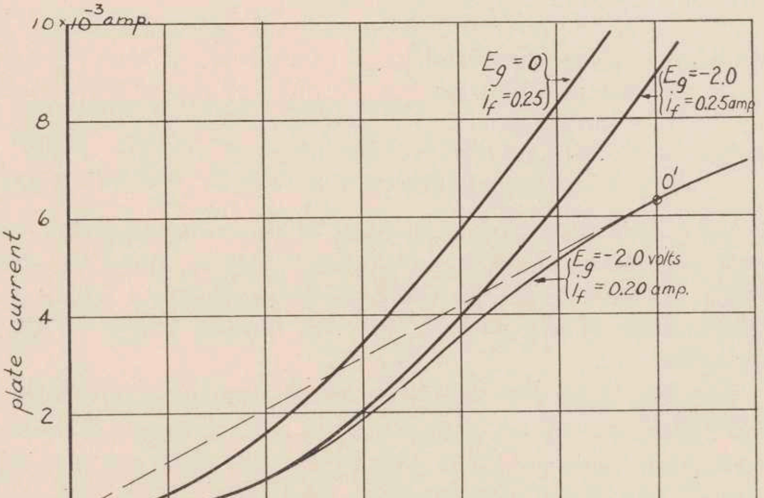
[6] Van der Bijl, "Thermionic Vacuum Tube," McGraw-Hill Book Co. Morecroft, "Principles of Radio Communication," John Wiley & Sons. The best source for practical information concerning vacuum tubes is the "Cunningham Tube Data Book" published by the E. T. Cunningham, Inc., 182 Second St., San Francisco.

FIG. 3.



6×10⁻³ amp.
5
4
3
2
1
plate current
$R_g = 8 \times 10^9 \omega$
$R_g = 2 \times 10^9 \omega$
$R_g = 0$
-10
-8
-6
-4
-2
0
"C" potential (V_g) in volts

FIG. 4.



10×10⁻³ amp.
8
6
4
2
0
plate current
$E_g = 0$
$i_f = 0.25$
$E_g = -2.0$
$i_f = 0.25$ amp
O'
$E_g = -2.0$ volts
$i_f = 0.20$ amp.
0
20
40
60
80
100
120
140
plate potential in volts

Mutual conductance so defined is dependent only on the tube characteristics and is independent of the associated circuits. The "*effective mutual conductance*," sometimes denoted by G_m', *depends* on the resistances in the associated circuits and can be either greater or less than the true mutual conductance G_m.

b. Plate Impedance Defined.

The curves given in Fig. 4 show the relationship between plate current and plate potential when the grid potential is held constant. The reciprocal of the slope of any one of these curves gives the plate impedance Z_p of the tube at the corresponding plate and grid potentials.

c. Tube Equation for Plate Current.

When it is necessary to represent the characteristics of a vacuum tube over a *very narrow range* of grid and plate potentials, the following simple equation nearly always gives a sufficiently close representation:[7]

$$i_p = \frac{1}{Z_p}(E_p + \mu E_g + E_o), \tag{9}$$

where E_p = plate potential,
E_g = grid potential,
E_o = constant,
μ = voltage amplification,
Z_p = plate impedance.

In using this very simplified form of the tube equation, we must remember that it can be used only in those cases in which E_o, μ and Z_p can be treated as constants which are independent of E_p and E_g for the limited range of their variation.

We see from the definition of the mutual conductance above that

$$\frac{\partial i_p}{\partial E_g} = G_m = \frac{\mu}{Z_p}\cdot \tag{10}$$

[7] See Van der Bijl, page 50, for a more complete discussion of the limitations of this type of equation.

C. OPERATION OF SIMPLE D.C. AMPLIFIER.

a. Circuit Equations.

We are now in a position to discuss the operation of the simple amplifier circuit of Fig. 1.

Since we are interested in the sensitivity of the amplifier we wish to know how the plate current of the tube depends on the grid potential. To compute the sensitivity, we use equation (9) and the Kirchoff Law equation for the plate circuit, which is

$$E_p = V_p - R_p i_p, \tag{11}$$

where V_p = "B" battery potential

and R_p = resistance in plate circuit including that of the battery, the galvanometer and any other resistances in the circuit.

Eliminating E_p we get

$$i_p = \frac{\frac{1}{Z_p}(V_p + \mu E_g + E_o)}{1 + \frac{R_p}{Z_p}}. \tag{12}$$

b. Sensitivity to Voltage Determined.

If we define the sensitivity S as the change in plate current due to a small change in grid potential we have

$$S \equiv \frac{\partial i_p}{\partial E_g} = \frac{\frac{\mu}{Z_p}}{1 + \frac{R_p}{Z_p}} = \frac{G_m}{1 + \frac{R_p}{Z_p}}. \tag{13}$$

The legitimacy of carrying out this variation, treating μ, Z_p and E_o as constants, is determined by the fact that the final equations and the experiments check for small variations of E_g. When R_p is very small compared with Z_p the "sensitivity" is equal to the mutual conductance G_m but if R_p is not small then the sensitivity is less than G_m because Z_p is *always* positive.

c. Sensitivity to Current Determined.

In many applications we are not interested directly in the change in plate current with a change in grid potential but in the change in plate current with a change in some current such as the photoelectric current.

From the Kirchoff Law equation for the grid circuit of Fig. 1, we have the equation

$$E_g = V_g - R_g(i_g - i) \tag{14}$$

where V_g = "C" battery potential,

R_g = external "grid leak" resistance,

i = photoelectric current.

We can perform a differentiation with respect to i in equation (14) and get

$$\frac{\partial E_g}{\partial i} = R_g - R_g \frac{\partial i_g}{\partial E_g} \times \frac{\partial E_g}{\partial i}. \tag{15}$$

Using the definition of Z_g given in equation (7) and rearranging we find

$$\frac{\partial E_g}{\partial i} = \frac{R_g}{1 + \dfrac{R_g}{Z_g}}. \tag{16}$$

This equation can also be written as

$$\frac{\partial E_g}{\partial i} = \frac{1}{\dfrac{1}{R_g} + \dfrac{1}{Z_g}}. \tag{17}$$

If R_g is small compared with Z_g then

$$\frac{\partial E_g}{\partial i} = R_g. \tag{18}$$

That is, the change in grid potential is equal to the change in "IR" drop over the grid resistance. On the other hand Z_g may be *positive* or *negative* depending on which part of the grid current characteristic we are operating. Then the change in grid potential can be either *greater* or *less* than the "IR" drop change usually expected. If Z_g is negative and R_g is adjusted to make it approach the value $|Z_g|$ then $\frac{\partial E_g}{\partial i}$ will become very great because the denominator approaches zero.

Combining (13) and (16) we get

$$\frac{\partial i_p}{\partial i} = \frac{G_m}{1 + \frac{R_p}{Z_p}} \times \frac{R_g}{1 + \frac{R_g}{Z_g}}. \qquad (19)$$

This equation gives the sensitivity as the *current amplification* of the one tube amplifier in terms of constants of the circuit. We can see from it exactly how each controllable factor enters to determine the amplifier sensitivity. For example, we see that when Z_g is *positive* and all other quantities constant except R_g, the sensitivity increases as R_g is increased but reaches an upper limit when R_g is made infinite, which is

$$\left(\frac{\partial i_p}{\partial i}\right)_{R_g=\infty} = \frac{G_m}{1 + \frac{R_p}{Z_p}} Z_g. \qquad (20)$$

It is obvious that if R_g is made infinite the Z_g which must be taken for equation (20) is that found at the *floating potential*. It, therefore, follows that when two tubes are being compared the one with the higher "input impedance" at the floating potential will have the higher sensitivity if the mutual conductances are the same. The UX-222 (screen grid tube) has the highest input impedance of any tube we have measured in our laboratory. On the average, this has been found to be over 2000 megohms.

By taking advantage of the fact that the input impedance Z_g may be negative, a much higher sensitivity to photoelectric currents is possible. All that we have to do is to adjust the "C" battery potential so that the grid potential

$$E_g = V_g - i_g R_g \qquad (21)$$

falls in the range in which Z_g is negative.

TABLE I.

Operating potential of grid.	E_g volts.	i_g amp.	G_m mho.	Z_p ohms.	R_p ohms assumed.	Z_g ohms.	R_g ohms arbitrary.	Current amplification computed by Eq. (19).
Floating..................	-0.92	0	870×10^{-6}	8600	500	200×10^{6}	∞	166,000
Near point of inflection of grid current curve.......	-3.0	5.5×10^{-10}	835×10^{-6}	9000	500	-16×10^{9}	200×10^{6}	161,000
	-3.0	5.5×10^{-10}	835×10^{-6}	9000	500	-16×10^{9}	8000×10^{6}	12,700,000

A numerical example will serve to illustrate some of the conditions to be satisfied and the advantages to be had by this method of operation. If we use the grid current and plate current characteristics taken on the UX-201-A vacuum tube which served to give the data for Figs. 2, 3, and 4, we can calculate the current amplification for the single tube amplifier under certain conditions and thus illustrate the advantage of using the negative grid impedance. Table I lists the essential constants and the corresponding current amplification obtained by using equation (19).

We see from this computation that it is easily possible to obtain a current amplification which is 76 times the greatest to be had at the floating potential by operating at a grid potential for which the grid impedance is negative.

These data which were taken from the grid current characteristic of a particular UX-201-A vacuum tube can not be taken in any sense as representative of UX-201-A tubes in general. The amount of surface leakage and the amount of residual gas left in other tubes of the same type are likely to be very different because of changes in manufacturing methods from time to time. It is also possible that tubes produced by one manufacturer will differ in these respects from those produced by another. Obviously, it would be of considerable assistance to workers in this field if data were available by which some idea could be had as to the range of variation in the grid current characteristics which are likely to be met with in the common tubes, but such data are not available.

At this stage in the discussion it will perhaps be of interest to develop the equation which shows the rate of change in sensitivity with a change in grid impedance (Z_g), in order to show some of the difficulties resulting from an attempt to get extreme sensitivity by making $R_g = |Z_g|$ when Z_g is negative. Let us write equation (16) in the following form

$$s \equiv \frac{\partial E_g}{\partial i} = \frac{R_g}{1 + \dfrac{R_g}{Z_g}}. \tag{22}$$

This equation gives s as the change in the grid potential E_g per unit change in photoelectric current. If we wish to know the rate of change in s with a change in the tube input impedance Z_g, we can differentiate equation (22) and we get

$$\frac{\partial s}{\partial Z_g} = \frac{1}{\left(1 + \frac{Z_g}{R_g}\right)^2}. \tag{23}$$

Consider first the case for which Z_g is *positive* and we operate in the region of the floating potential. Here we see that when R_g is infinite the right hand side of the equation is unity and the *change in sensitivity is directly proportional to the change in the input impedance.* If R_g is small compared with Z_g, then the right hand side of the equation is practically *zero* and the sensitivity is independent of Z_g.

In case Z_g is negative, then if R_g is small the sensitivity will be independent of Z_g. If $R_g = \frac{1}{2}|Z_g|$ the right hand member of equation (23) reduces to unity and it follows that the change in sensitivity is proportional to the change in input impedance. We can express the relation between the percentage change in s and the percentage change in Z_g by

$$\text{percent change in } s = \frac{(\text{percent change in } Z_g)}{1 + \frac{Z_g}{R_g}}. \tag{24}$$

This equation shows that if a high sensitivity is obtained by making $\frac{Z_g}{R_g} = -1.1$ as would be the case if $Z_g = -220$ megohms and $R_g = 200$ megohms, the percentage change in sensitivity will be ten times as great as the percentage change in input impedance Z_g. Therefore, if we wish to keep the sensitivity constant to within 2 per cent., the input impedance for this particular case must be constant to within 0.2 per cent. Enough has now been said about grid current characteristics for the reader to take any particular grid current *vs.* grid potential curve and determine quite accurately the best operating conditions.

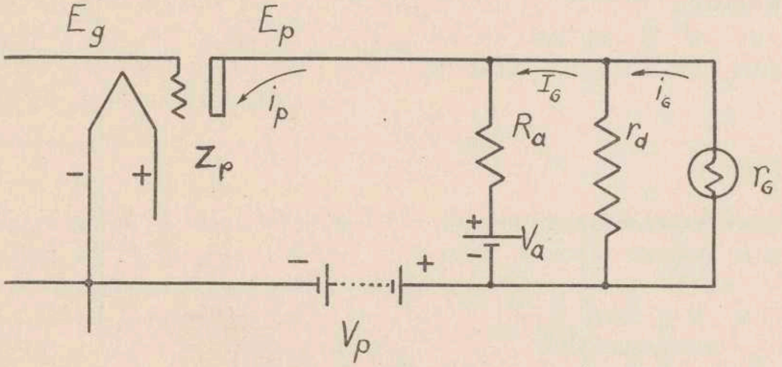
PART 2. SPECIAL OUTPUT CIRCUITS.

Practically all of the above discussion can be considered as the general circuit theory which forms the foundation for all of the amplifiers under consideration. The part which follows will show the application of this general theory to a number of specific circuits. Some of the chief advantages and limitations of each circuit will be pointed out as we progress.

A. ONE TUBE CIRCUIT WITH AUXILIARY BATTERY TO SUPPRESS THE ZERO.

In applications in which a meter or a galvanometer is used in the plate circuit to detect the changes in plate current brought about by the changes in grid potential imposed by the outside circuits, it is usually advantageous to have the meter read "zero" when there is no current in the photo-electric circuit, for instance. This result can be brought

FIG. 5.



about mechanically or electrically. If the meter is connected directly in the plate circuit, the meter will read the normal plate current. It is obvious that by turning the supporting suspension or the restoring spring through a sufficient angle the meter will read "zero." This would be a mechanical method for "suppressing the zero." There are a number of ways of bringing about this suppression of zero electrically. One of the simplest ways is to use a small auxiliary battery and a control resistance across the meter or galvanometer as shown in Fig. 5.

a. Circuit Equations.

In order to obtain equation (12) which showed the plate current as a function of grid potential, in the example worked out above it was necessary to have only two equations, (1) the tube equation and (2) the simple Kirchoff equation of the plate circuit. In the present case the procedure is exactly the same since we start again with the tube equation. In place of a single plate circuit equation we now need more than one because it is now necessary to consider more than one circuit "mesh." The following simple equations are all that are necessary.

$$i_p = \frac{1}{Z_p}(E_p + \mu E_g + E_o), \tag{25}$$

$$E_p = V_p - I_G R_G, \tag{26}$$

$$V_a = i_p R_a - I_G(R_a + R_G), \tag{27}$$

where

$$\frac{1}{R_G} = \frac{1}{r_d} + \frac{1}{r_G}, \tag{28}$$

$$i_G = I_G\left(\frac{r_d}{r_d + r_G}\right). \tag{29}$$

We can solve equations (25), (26) and (27) to get

$$I_G = \frac{\frac{1}{Z_p}(V_p + \mu E_g + E_o) - \frac{V_a}{R_a}}{1 + R_G\left(\frac{1}{Z_p} + \frac{1}{R_a}\right)}. \tag{30}$$

b. Sensitivity to Grid Voltage Changes.

The sensitivity to voltage is therefore

$$\frac{\partial I_G}{\partial E_g} = \frac{G_m}{1 + R_G\left(\frac{1}{Z_p} + \frac{1}{R_a}\right)}. \tag{31}$$

When we substitute equations (28) and (29) in (31) we get

$$\frac{\partial i_G}{\partial E_g} = \frac{G_m}{1 + r_G\left(\frac{1}{Z_p} + \frac{1}{R_a} + \frac{1}{r_d}\right)}. \tag{32}$$

In case we adjust R_a and r_d so that the galvanometer is critically damped and R_{cd} is the external critical damping resistance, then from Fig. 5 we see that

$$\frac{1}{R_{cd}} = \frac{1}{Z_p} + \frac{1}{R_a} + \frac{1}{r_d}, \qquad (33)$$

assuming that the battery resistance can be neglected.

We conclude from equations (32) and (33) that the sensitivity when the galvanometer is critically damped depends only on the mutual conductance and the ratio of the galvanometer resistance to the critical damping resistance.

Referring to equation (27), we see that the value of R_a is determined by the initial plate current, call it i_{po}, and the auxiliary battery potential V_a. By the initial plate current we mean the plate current flowing under the circuit conditions for which we want the galvanometer current to be zero. The *minimum* value of V_a, consistent with critical damping, is given by the relationship

$$V_{a\,(\text{min.})} = i_{po}\left(\frac{R_{cd}Z_p}{Z_p - R_{cd}}\right). \qquad (34)$$

Equation (34) states in analytical form, that the minimum value of V_a is equal to the "IR" drop over the lowest value of R_a which is consistent with critical damping. After V_a is determined consistent with equation (34), that is, V_a must be greater than $V_{a\,(\text{min.})}$, we have R_a given by equation (35).

$$R_a = \frac{V_a}{i_{po}}. \qquad (35)$$

From equation (33) we find

$$\frac{1}{r_d} = \frac{1}{R_{cd}} - \frac{1}{Z_p} - \frac{1}{R_a}. \qquad (36)$$

We have thus determined the constants of our circuit to give the maximum sensitivity to grid voltage consistent with critical damping using a tube of mutual conductance G_m, and a galvanometer of resistance r_G and external critical damping resistance R_{cd}. This sensitivity is

$$\frac{\partial i_G}{\partial E_g} = \frac{G_m}{1 + \frac{r_G}{R_{cd}}}. \tag{37}$$

In order to illustrate the meaning of equation (37) let us consider a numerical example in which the galvanometer has a resistance of $r_G = 500$ ohms and a critical damping resistance $R_{cd} = 2000$ ohms and a vacuum tube with a mutual conductance $G_m = 900$ micro-mhos. Then

$$\frac{\partial i_G}{\partial E_g} = \frac{900 \times 10^{-6}}{1 + \frac{500}{2000}} = 0.72 \times 10^{-3} \text{ amp. per grid volt.}$$

A simple calculation shows at once that in case the resistance of the circuit in which the unknown E.M.F. originates is 2000 ohms, the current through the galvanometer when used with the vacuum tube circuit of Fig. 5, will be only 1.44 times as great as the current which would flow through the galvanometer if it were connected directly. If the galvanometer sensitivity is 330 megohms, then the overall sensitivity of the system measured in millimeters deflection, is 238 mm. per millivolt applied to the grid.

B. WHEATSTONE BRIDGE CIRCUIT WITH SINGLE TUBE.[8]

a. Circuit Equations.

Another method of arranging the circuit so that no current flows through the galvanometer for the "zero" condition but does flow when this condition is disturbed, comes directly from the simple Wheatstone bridge circuit. In Fig. 6 we have this circuit with resistances in three of the arms and a vacuum tube in the remaining one. It is easy to show that with this circuit the galvanometer balance or even the galvanometer deflection will be *independent* of the "B" battery potential (V_p) over a certain limited range. The solution of the following simple equations gives this result. Using the vacuum tube equation (9) and the Kirchoff equations for the Wheatstone bridge we can write:

---

[8] Other single tube circuits have been discussed by Razek and Mulder, *J. O. S. A. and R. S. I.*, 18: 460 (1929).

$$i_p = \frac{1}{Z_p}(E_p + \mu E_g + E_o), \tag{9}$$

$$i_x = \frac{E_x}{R_x}, \tag{38}$$

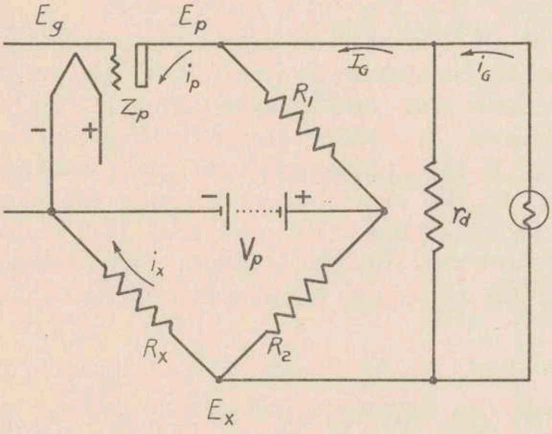
$$E_p = V_p - (i_p - I_G)R_1, \tag{39}$$

$$E_x = V_p - (i_x + I_G)R_2, \tag{40}$$

$$O = R_1(i_p - I_G) - I_G R_g - R_2(i_x + I_G). \tag{41}$$

The meaning of the symbols is given by the circuit schematic Fig. (6). The combined resistance of the galvanometer r_g and the parallel resistance r_d is R_G and is given by equation (29).

FIG. 6.



Solving for the current I_G we get:

$$I_G = \frac{(R_1R_x + R_1R_2)(E_o + \mu E_g) - (R_2Z_p - R_1R_x)V_p}{R_1Z_pR_x + R_2Z_pR_x + R_1R_2Z_p + R_1R_2R_x + R_G(Z_p + R_1)(R_x + R_2)}. \tag{42}$$

b. Plate Battery Compensation Conditions.

We see at once from equation (42) that I_G is independent of V_p if

$$R_2Z_p = R_1R_x. \tag{43}$$

In order that this be true the constants of the tube equation Z_p, μ and E_o must all be independent of E_p over the range corresponding to that over which V_p is assumed to vary. This condition can be met in most cases for changes in V_p of one to three per cent.

c. Voltage Sensitivity.

In order to compute the sensitivity of this amplifier let us take $R_1 = R_2$, then $R_x = Z_p$ and we have, remembering that I_G in this case is the current through the galvanometer and its damping resistance and that R_G is r_G in parallel with r_d,

$$\frac{\partial I_G}{\partial E_g} = \frac{G_m}{2 + R_G\left(\frac{1}{Z_p} + \frac{1}{R_1}\right)}. \tag{44}$$

d. Condition for Zero Current in Galvanometer.

It is not at once obvious from equation (42) that it is possible to have no current flowing through the galvanometer and at the same time fulfill the requirement of (43). It is possible however, by making use of the fact that for a very limited range in the "plate current *vs.* plate voltage" curve,[9] the tangent to the curve passes through the origin as is shown by the dashed line "OO'" of Fig. 4. For the particular value of E_g to which this characteristic curve belongs, call it E_g', one of the remaining factors in equation (42) is zero. That is

$$E_o + \mu E_g' = 0 \tag{45}$$

and therefore the galvanometer current will be zero when $E_g = E_g'$ and yet we shall be able to meet the requirement set down by equation (43). When E_g changes from the initial value E_g' by an amount ΔE_g the total current (I_G) in the galvanometer circuit will be given by equation (44).

The sensitivity equation (44) is expressed in terms of the

[9] The bending over of the characteristic shown in Fig. 4 is due to the fact that the filament current has been reduced to bring about a limitation of current with increasing plate potential owing to "saturation." The circuit of Fig. 6 could be modified by placing a battery in series with R_x and thus make it possible to operate on a "straighter" portion of the "plate current vs. plate potential" curve.

current flowing through the galvanometer and the damping resistance in parallel. In order to find the sensitivity in terms of the current flowing through the galvanometer we must use equations (47) and (48) below. The current i_G through the galvanometer alone is of course

$$i_G = I_G\left(\frac{r_d}{r_d + r_G}\right), \tag{46}$$

r_d = damping resistance,

r_G = galvanometer resistance.

This equation solved for I_G is

$$I_G = i_G\left(\frac{r_d + r_G}{r_d}\right). \tag{47}$$

The parallel resistance of the galvanometer and shunt is obviously

$$R_G = \frac{r_d r_G}{r_d + r_G}. \tag{48}$$

When these equations are substituted into equation (44) we get

$$S_6 = \frac{\partial i_G}{\partial E_g} = \frac{G_m}{2\left[1 + r_G\left(\frac{1}{2Z_p} + \frac{1}{2R_1} + \frac{1}{r_d}\right)\right]} \tag{49}$$

as the sensitivity equation for the circuit of Fig. 6 expressed in terms of the galvanometer current i_G. By inspection of Fig. 6 and remembering that $R_x = Z_p$ and $R_1 = R_2$ we see that the galvanometer is critically damped when

$$\frac{1}{2Z_p} + \frac{1}{2R_1} + \frac{1}{r_d} = \frac{1}{R_{cd}} \tag{50}$$

where R_{cd} is the external critical damping resistance. Making this substitution, we get

$$S_6 = \frac{G_m}{2\left(1 + \frac{r_G}{R_{cd}}\right)}. \tag{51}$$

e. Sensitivity of Circuits 5 and 6 Compared.

If we compare the sensitivity of the circuit shown in Fig. 6 and given by equation (51) with that for the circuit of Fig. 5 given by equation (37), we get

$$\frac{S_6}{S_5} = \frac{\dfrac{G_m}{2\left(1 + \dfrac{r_G}{R_{cd}}\right)}}{\dfrac{G_m}{1 + \dfrac{r_G}{R_{cd}}}} = \frac{1}{2}. \tag{52}$$

In other words we have sacrificed exactly half of our sensitivity in order to introduce the bridge circuit which, however, is superior to the simple circuit in that the zero or the deflection is independent of the "B" battery over a certain limited range of variation while in the simple circuit of Fig. 5 a change in either the "B" battery potential (V_p) or the auxiliary battery (V_a) will produce a change in the galvanometer reading.

C. WHEATSTONE BRIDGE CIRCUIT USING TWO VACUUM TUBES.

a. Conditions for "B" Battery Compensation.

The circuit shown in Fig. 6 can be modified as shown in Fig. 7 by the introduction of a vacuum tube in place of the resistance arm R_x. It will be shown below that this change in the circuit leaves the form of the equation for the sensitivity unchanged and modifies the condition for "B" battery compensation only to the extent that Z_{px} replaces R_x in equation (43). This gives

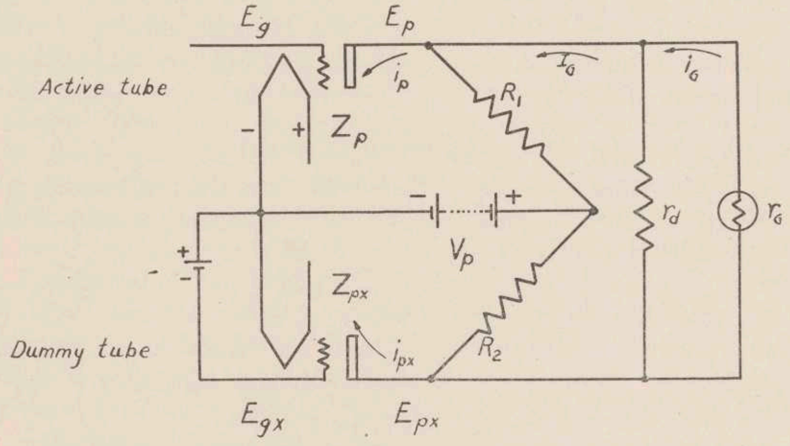
$$R_2 Z_p = R_1 Z_{px} \tag{53}$$

or

$$\frac{Z_p}{Z_{px}} = \frac{R_1}{R_2}. \tag{54}$$

Here Z_{px} is the plate impedance of the tube which has been introduced in the bridge in place of R_x. In the discussion above it was assumed that $R_1 = R_2$ and R_x was adjusted to have the value $R_x = Z_p$. That order of procedure cannot be used in this case because the value of the plate impedance Z_{px} is not so easily controlled as the value of R_x was before. It, therefore, follows that we must meet the condition required by equation (54) by adjusting $\frac{R_1}{R_2}$. Fortunately it is not necessary to know Z_p and Z_{px} exactly. All that is necessary

FIG. 7.



is that we have R_1 or R_2 adjustable and have a means of altering the "B" battery potential (V_p) through the narrow range over which we are required to get the best possible compensation.[10]

b. Circuit Equations.

In order to develop the equations which give support to the statements just made as regards independence of "B" battery (V_p), sensitivity, etc., we must start out with five equations just as we did in the one tube bridge circuit discussed above. These equations were (9), (38), (39), (40)

[10] This method must be used with a certain amount of care because it is possible to get a compensation at the end points and not have compensation for intermediate values of (V_p) owing to the curvature of the characteristic.

and (41) and the corresponding set here are the following:

$$i_p = \frac{1}{Z_p}(E_p + \mu E_g + E_o), \tag{55}$$

$$i_{px} = \frac{1}{Z_{px}}(E_{px} + \mu_x E_{gx} + E_{ox}), \tag{56}$$

$$E_p = V_p - (i_p - I_G)R_1, \tag{57}$$

$$E_{px} = V_p - (i_{px} + I_G)R_2, \tag{58}$$

$$O = R_1(i_p - I_G) - I_G R_G - R_2(i_{px} + I_G). \tag{59}$$

The meaning of the symbols is given by the circuit schematic of Fig. 7.

c. Condition for Zero Galvanometer Current.

These equations can be solved for the galvanometer current I_G just as was done before in equation (42) with the following result

$$I_G = \frac{\begin{matrix}(R_1Z_{px} + R_1R_2)(\mu E_g + E_o) \\ - (R_2Z_p + R_1R_2)(\mu_x E_{gx} + E_{ox}) \\ - (R_2Z_p - R_1Z_{px})V_p\end{matrix}}{\begin{matrix}R_1Z_pZ_{px} + R_2Z_pZ_{px} + R_1R_2Z_p + R_1R_2Z_{px} \\ + R_G(Z_p + R_1)(Z_{px} + R_2)\end{matrix}} \tag{60}$$

and the sensitivity is

$$\left(\frac{\partial I_G}{\partial E_g}\right)_7 = \frac{G_m}{2 + R_G\left(\frac{1}{Z_p} + \frac{1}{R_1}\right)}, \tag{61}$$

when $R_1 = R_2$ and $Z_p = Z_{px}$. This is the same as was found above in equation (44). When equations (47) and (48) are used so as to express the sensitivity in terms of the galvanometer current i_G, this equation obviously reduces to the same as (49) which is

$$S_7 = \left(\frac{\partial i_G}{\partial E_g}\right)_7 = \frac{G_m}{2\left[1 + r_G\left(\frac{1}{2Z_p} + \frac{1}{2R_1} + \frac{1}{r_d}\right)\right]}. \tag{62}$$

Again if the galvanometer is critically damped and R_{cd} is the external critical damping resistance we have

$$S_7 = \frac{G_m}{2\left(1 + \frac{r_G}{R_{cd}}\right)}. \tag{63}$$

From equation (60) we see that the condition for operation independent of V_p is that given above in equation (53) or (54). Assuming this condition to be satisfied, we have the further condition which we wish to impose, that there be no current flowing through the galvanometer for some definite condition in the grid circuit such as, for example, that for which there is no photoelectric current. For the arbitrary "initial" condition, the grid potential of the "active" tube will have some definite value E_g' which is determined by the circuit constants such as the grid resistance R_g, the "C" battery potential V_g, etc. (see Fig. 1). The galvanometer current can be brought to zero by properly adjusting the potential of the grid E_{gx} of the "dummy" tube. Later we shall use the phrase "balance the circuit" to indicate this process of adjusting some element of the circuit such as the grid potential E_{gx} until the current through the galvanometer is zero.

When we assume the condition, given by equation (53), to be satisfied, we see from equation (60) that the galvanometer current will be zero when the sum of the remaining two terms of the numerator of that equation is zero. This we have expressed in the equation.

$$(R_1Z_{px} + R_1R_2)(\mu E_g' + E_o) - (R_2Z_p + R_1R_2)(\mu_x E_{gx} + E_{ox}) = 0. \tag{64}$$

Referring to equation (53), which we are assuming to be satisfied, we see by adding R_1R_2 to each side of the equation that

$$R_1Z_{px} + R_1R_2 = R_2Z_p + R_1R_2. \tag{65}$$

Therefore, if equations

$$\mu E_g' + E_o = \mu_x E_{gx} + E_{ox} \tag{66}$$

and

$$R_2Z_p = R_1Z_{px} \tag{53}$$

are satisfied, the galvanometer current is zero and the system is independent of "B" battery changes over the limited range for which the "constants" of the system are independent of "B" battery.

d. "Cut and Try Method" of Adjustment.

The fact that the current through the galvanometer is zero when equations (53) and (66) are satisfied does not mean that we need to know the constants of the tubes actually used in this two tube bridge circuit any more than it did above in the one tube bridge circuit. In the single tube circuit we saw that in order to accomplish a balance of the circuit, it was necessary to operate on the part of the "plate current *vs.* plate potential" curve of Fig. 4 at O'. At this part of the characteristic the plate impedance Z_p changes rather rapidly with a change in E_p and therefore limits the range over which satisfactory "B" battery compensation can be obtained. When we use the two tube circuit we can operate over the "straight" portion of the characteristic where the change in Z_p and E_p is very small and still meet the requirements of equations (53) and (66) simultaneously. This is accomplished by adjusting the resistance ratio $\frac{R_1}{R_2}$ and the value of the grid potential on the "dummy" tube until both conditions are met. In practice this adjustment is of the "cut and try" type. That is, we arbitrarily set $R_1 = R_2$ and adjust E_{gx} to balance the circuit giving no current through the galvanometer. Then we change the "B" battery potential (V_p) by 1.5 volts or more and observe the amount and direction of the galvanometer deflection.[11] We then change R_2 by 1000 ohms,[12] for instance, rebalance the circuit by altering E_{gx} and again test for stability. If the galvanometer swings less far but in the same direction we see that we have altered the resistance ratio $\frac{R_1}{R_2}$ in the right direction but not enough. By continuing this "cut and try" method it is nearly always possible to obtain

[11] See footnote 10.

[12] If we wish to keep the galvanometer damping more nearly the same for all settings, we should alter R_1 also, keeping $(R_1 + R_2)$ constant.

perfect stabilization for moderate changes in "B" battery potential. The fact that, with the two tube circuit, we can operate on the "*straight*" part of the curve makes it possible to realize perfect stability over a wider range of "B" battery variation than can be realized in the simple one tube bridge circuit.

e. Advantages of Two Tube Circuit.

There are other advantages to the two tube circuit which deserve to be mentioned. Two of these are (1) compensation for filament voltage changes and (2) approximate compensation for tube aging. In general, the plate current through a tube gradually decreases with time when the plate and grid potentials and the filament current are held constant. It is obvious that that part of the drift in the galvanometer current which is due to this aging, will be less in the two tube circuit since we can assume that the rate of aging in two similar tubes is approximately the same.

f. Conditions for Filament Battery Compensation.[13]

If we had included filament voltage E_f as one of the variables of the tube equation we would have had the following equations instead of (55) and (56) as above.

$$i_p = \frac{1}{Z_p}(E_p + \mu E_g + E_o') + \alpha E_f, \tag{67}$$

$$i_{px} = \frac{1}{Z_{px}}(E_{px} + \mu_x E_{gx} + E_{ox}') + \alpha_x E_f. \tag{68}$$

The two new constants are defined by

$$\alpha \equiv \frac{\partial i_p}{\partial E_f}, \tag{69}$$

$$\alpha_x \equiv \frac{\partial i_{px}}{\partial E_f}. \tag{70}$$

[13] For an extended discussion of compensation methods including compensation for grid battery changes see:

C. E. Wynn-Williams, *Phil. Mag.*, **6**: 324 (1928).

J. Brentano, *Phil. Mag.*, **7**: 685 (1929).

J. M. Eglin, *J. O. S. A. and R. S. I.*, **18**: 393 (1929).

J. Razek and P. J. Mulder, *J. O. S. A. and R. S. I.*, **19**: 390 (1929).

Here again we must rely on experiment for the justification of the statement that α and α_x can be treated as constants which are independent of E_p, E_g and E_F over narrow ranges of the variables.

If we carried through the calculations to get the equation equivalent to (60), we would find the following term in the numerator along with those already discussed

$$E_f\{(R_1Z_{px} + R_1R_2)Z_p\alpha - (R_2Z_p + R_1R_2)Z_{px}\alpha_x\}. \quad (71)$$

If this term is zero then the system will be independent of filament voltage and all of the relations developed above will still hold perfectly well. Take the first, namely that required in order to have independence from "B" battery changes which was $R_1Z_{px} = R_2Z_p$ and it follows that

$$Z_p\alpha = Z_{px}\alpha_x \quad (72)$$

if the system is to be independent of filament voltage. In terms of the resistances we have

$$\frac{\alpha_x}{\alpha} = \frac{R_1}{R_2}. \quad (73)$$

By operating the two filaments in parallel but with a small variable resistance in series with one of them, the α of one of the tubes can be adjusted to meet condition (73). There are other ways of building the circuit so as to control one of the α's but the condition (73) must always be met. Again it is no more necessary to know the new constants of the tube equation than it was before. The balance is accomplished by the "cut and try" method just as before but this time we must give small arbitrary changes to the filament battery to test the compensation. In the laboratory we have found that it is possible to make the compensation so perfect that little or no deflection of the galvanometer could be measured with a filament change of as much as one or two per cent. while without the compensation a change of one-tenth of a percent caused a change in the galvanometer reading.

D. SUMMARY.

The part of this paper just completed deals with three important circuits used in single stage D.C. amplifiers. The entire attention has been devoted to the conditions in the plate circuit or the filament circuit and little or nothing has been said about the grid circuit. Many of the general principles which determine the grid circuit connections have been recorded in the first part of this paper. The detailed application of these principles will be taken up in the following section. The one tube circuit with an auxiliary battery to give a "supressed zero" was shown to be the most sensitive one stage amplifier of the three. The bridge circuits which followed were shown to have a sensitivity one-half as great as the above. With the single tube bridge circuit, it was shown that under certain definite conditions small variations in "B" battery could be compensated for. The range for which this compensation is possible was shown to be very limited. The bridge circuit with two tubes was shown to have several advantages over the single tube bridge circuit with no additional loss in sensitivity. These advantages were (1) practically perfect "B" battery compensation over a rather wide range, (2) filament battery compensation over a fairly wide range, and (3) approximate compensation for aging of the tubes.

E. CONTROL OF GALVANOMETER DEFLECTION.

a. Special Universal Shunt.

Whenever a sensitive galvanometer is used in any of the three circuits described, it is usually necessary to have some means for controlling the galvanometer response in a way which keeps the damping correct. In ordinary high resistance circuits an Ayrton shunt is suitable for this purpose. Under certain conditions an Ayrton shunt can be used here also, but as will be shown later a certain amount of sensitivity must be sacrificed in order to do this. It is possible, however, to control the current through the galvanometer in either the bridge circuit or that with the auxiliary battery and keep the damping correct without this loss in sensitivity. The following computation shows how to determine the constants of the circuit. Consider a circuit like that shown in Fig. 7 with the galvanometer replaced by the network shown in Fig. 8. When the contact is at 1, resistance "*a*" is zero and resistance "b" infinite. The sensitivity is

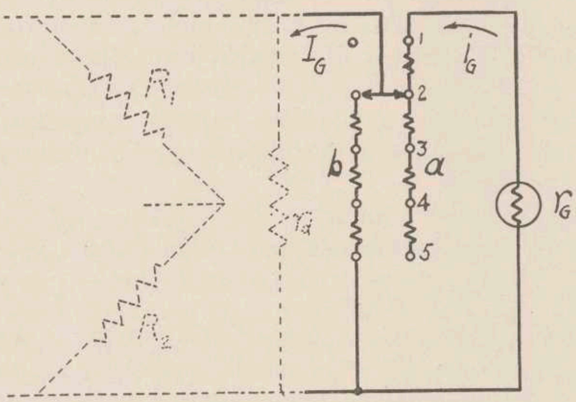
$$S_{\max} = \frac{\partial i_G}{\partial E_g} = \frac{G_m}{2\left[1 + r_G\left(\frac{1}{2Z_p} + \frac{1}{2R_1} + \frac{1}{r_d}\right)\right]}. \tag{74}$$

This result comes from equation (62) and gives the maximum sensitivity. The damping is correct when

$$\frac{1}{R_{cd}} = \frac{1}{2Z_p} + \frac{1}{2R_1} + \frac{1}{r_d}, \tag{75}$$

where R_{cd} is the external resistance for critical damping.

FIG. 8.



Circuit with special universal galvanometer shunt.

As we move the contact to successive positions, 2, 3, etc., resistances a and b have two functions to perform. They must be so proportioned that the damping will remain unchanged and the sensitivity reduced in definite amounts. If a and b are always adjusted so as to satisfy equation (76) the damping will remain constant.

$$R_{cd} = a + \frac{bR_{cd}}{b + R_{cd}}. \tag{76}$$

If S_n is the sensitivity for some combination of a and b

consistent with (76), we can define the "sensitivity ratio" n as

$$n = \frac{S_{\max}}{S_n}. \tag{77}$$

If we let I_G be the current through the galvanometer and the shunt b together, then the current i_g through the galvanometer will be

$$i_G = I_G \frac{b}{a + b + r_G}. \tag{78}$$

Since the resistance of the galvanometer along with its series and shunt resistances is

$$R_G = \frac{b(a + r_G)}{a + b + r_G}, \tag{79}$$

the sensitivity expressed in terms of I_G is the following:

$$\frac{\partial I_G}{\partial E_g} = \frac{G_m}{2\left[1 + \frac{b(a + r_G)}{a + b + r_G}\left(\frac{1}{2Z_p} + \frac{1}{2R_1} + \frac{1}{r_d}\right)\right]}. \tag{80}$$

Using equations (75) and (78) we get

$$S_n = \frac{\partial i_G}{\partial E_g} = \frac{bG_m}{2[a + b + r_G]\left[1 + \frac{b(a + r_G)}{(a + b + r_G)R_{cd}}\right]}. \tag{81}$$

From equations (74), (75), (77) and (81) we get

$$\frac{S_{\max}}{S_n} = n = \frac{R_{cd}(a + b + r_G) + b(a + r_G)}{b(R_{cd} + r_G)}. \tag{82}$$

With equations (76) and (82) it is possible to solve for a and b in terms of the damping resistance R_{cd} and the sensitivity ratio n. In this way we get

$$a = R_{cd}\left(\frac{n - 1}{n}\right), \tag{83}$$

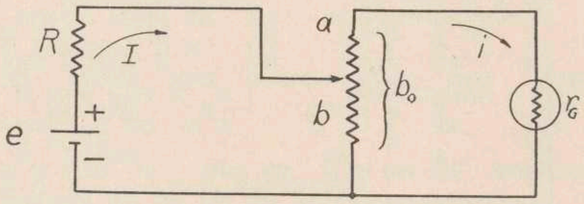
$$b = R_{cd}\left(\frac{1}{n - 1}\right). \tag{84}$$

These equations are very similar to those for the Ayrton shunt in that the values of a and b are independent of the galvanometer resistance. This is, of course, a very real advantage for it makes this shunt a "universal" shunt in exactly the same sense that the Ayrton shunt is a universal shunt. Since the sensitivity equations (32) and (49) for the one tube circuits of Figs. 5 and 6, respectively, are of the same form as equation (74), the constants of the universal shunt for these circuits are also given by equations (83) and (84).

b. Ayrton Shunt and Its Limitations.[14]

The Ayrton shunt can be used in these circuits with a small sacrifice in the maximum sensitivity under certain conditions. These conditions will now be examined somewhat in detail. It is well known that in ordinary circuits

FIG. 9.



Ayrton shunt circuit.

the Ayrton shunt does not reduce the sensitivity in the correct ratio unless the resistance of the associated circuit is considerably higher than the total resistance of the shunt. It will be recalled that the Ayrton shunt is an "L" net work similar to that shown by resistances "a" and "b" of Fig. 9.

The resistances a and b are related to the nominal sensitivity ratio N and to each other according to the following equations:

$$a = b_o\left(\frac{N-1}{N}\right), \qquad (85)$$

$$b = \frac{b_o}{N}, \qquad (86)$$

$$a + b = b_o. \qquad (87)$$

[14] Razek and Mulder, *J. O. S. A. and R. S. I.*, 18: 390 (1929), suggest using an Ayrton shunt but do not discuss its limitations.

When $N = 1$, we have $a = 0$; and $b = b_o$ which is the total resistance by which the shunt is specified in commercial catalogs. This resistance is usually determined by the external critical damping resistance of the galvanometer. From the simple equations of the circuit shown in Fig. 9 the true sensitivity ratio n corresponding to the nominal ratio N can be worked out. The galvanometer current for any setting is

$$i_G = \frac{eb}{(a + b + r_G)R + b(a + r_G)}, \tag{88}$$

and when $a = 0$ and $b = b_o$ we have

$$i_{G1} = \frac{eb_o}{(b_o + r_G)R + b_o r_G}. \tag{89}$$

Taking the ratio of i_{G1} to i_G as the definition of the true sensitivity ratio n and using equation (86) we get

$$\frac{i_{G1}}{i_G} \equiv n = N\frac{(b_o + r_G)R + b(a + r_G)}{(b_o + r_G)R + b_o r_G}. \tag{90}$$

We can now determine the accuracy with which the true sensitivity ratio is given by the nominal one by the relation

$$\frac{n - N}{n} = y, \tag{91}$$

where y when multiplied by one hundred gives the error in percent.

From equations (85), (86) and (90) we can eliminate a and b and solve for y in terms of the constants of the circuit.

$$y = \frac{Nb_o(b_o + r_G) - N^2 b_o r_G - b_o^2}{N^2 R(b_o + r_G) + Nb_o(b_o + r_G) - b_o^2}. \tag{92}$$

It is easy to show that y is zero for the values

$$N = 1 \quad \text{and} \quad N = \frac{b_o}{r_G}.$$

If $b_o > r_G$, as is usually the case, y has a positive maximum at a value of N between the two for which y is zero, namely, at

$$N = \frac{2}{1 + \dfrac{r_G}{b_o}}.$$

We see at once that this value of N for which the error is a positive maximum must lie between 1.0 and 2.0. Since the least value of N in most commercially produced Ayrton shunts is 10.0, we find the greatest error when N is large. For large values of N equation (92) reduces to

$$y = -\frac{b_o r_G}{(b_o + r_G)R}. \tag{93}$$

This equation can also be written

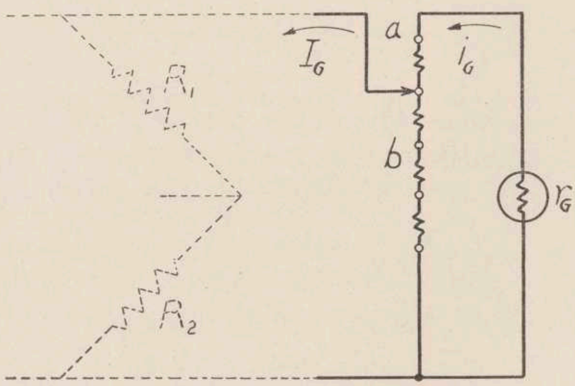
$$y = -\frac{1}{\frac{R}{b_o}\left(1 + \frac{b_o}{r_G}\right)}. \tag{94}$$

Let us take a numerical example in which

$$\frac{R}{b_o} = 2 \quad \text{and} \quad \frac{b_o}{r_G} = 4.$$

Equation (94) shows that the maximum error y will be 10 per cent. when N is large.

FIG. 10.



Circuit with Ayrton shunt for galvanometer control.

Let us consider a case similar to that discussed under the heading "Special Universal Shunt," with the difference that this time we use the Ayrton shunt as shown in Fig. 10. The damping resistance r_d is no longer needed and may therefore be dropped out. Equation (61) gives the sensitivity of this circuit where I_G is the total current through the shunt b and the galvanometer together and R_G is the resistance of $(a + r_G)$ in parallel with b. We can express the sensitivity in terms of the galvanometer current i_G by using an equation similar to (78) to eliminate I_G. Using these equations and also (87) we get the following:

$$S_n = \frac{bG_m}{2(b_o + r_G) + (a + r_G)b\left(\frac{1}{Z_p} + \frac{1}{R_1}\right)}. \qquad (95)$$

When $b = b_o$ and $a = 0$, we have the maximum sensitivity $S_{\max}$ which is

$$S_{\max} = \frac{b_o G_m}{2(b_o + r_G) + r_G b_o\left(\frac{1}{Z_p} + \frac{1}{R_1}\right)}. \qquad (96)$$

If we define the true sensitivity ratio as before

$$\frac{S_{\max}}{S_n} \equiv n \qquad (97)$$

and relate it to the nominal ratio N as before by equation (91), we get the following equation for large values of N:

$$y = \frac{1}{\frac{2Z_pR_1}{(Z_p + R_1)b_o}\left(1 + \frac{b_o}{r_G}\right)}. \qquad (98)$$

If we set

$$\frac{1}{X} = \frac{1}{2Z_p} + \frac{1}{2R_1}, \qquad (99)$$

$$y = -\frac{1}{\frac{X}{b_o}\left(1 + \frac{b_o}{r_G}\right)}. \qquad (100)$$

This is exactly of the same form as (94).

The accuracy with which the Ayrton shunt reduces the sensitivity may be of little importance to the user of an amplifier since he can determine the proper correction at any time. However, he may not be satisfied to have the time required to take readings widely different for the various sensitivities. From an inspection of the circuit in Fig. 10, we see that the damping will be greatest when $N = 1$ and the damping will be least when N is large. This effect may be minimized by making X which is defined by equation (99) as large as is conveniently possible. If we have a value of X at least twice the external damping resistance R_{cd} of the galvanometer, and an Ayrton shunt with a total resistance approximately equal to R_{cd}, then the Ayrton shunt can be used to regulate the sensitivity and the disadvantages which have been pointed out will not be found serious.

If we assume

$$X_{\min} = 2R_{cd} \tag{101}$$

and

$$b_o = R_{cd}, \tag{102}$$

equation (100) reduces to

$$y = -\frac{1}{2 + \frac{2R_{cd}}{r_G}}. \tag{103}$$

From this we see that if $\frac{R_{cd}}{r_G} = 4$, the maximum error in the Ayrton shunt sensitivity ratio, will be 10 per cent. Equations (99) and (101) serve to determine the minimum satisfactory value of the bridge resistance R_1 which is given by

$$R_{1\ \min} = \frac{R_{cd}Z_p}{Z_p - R_{cd}}. \tag{104}$$

Mention was made above that the use of the Ayrton shunt for a sensitivity control instead of the "special" universal shunt given by equations (83) and (84), usually entailed a small loss in the maximum sensitivity. It is easy to see from equation (62), that if $2Z_p$ and $2R_1$ are large compared with R_{cd} no appreciable loss in sensitivity will result from the use of the Ayrton shunt. On the other hand, if the minimum value of R_1 as defined by equation (104) is used, then the loss in sensitivity in percent will not be greater than $50\frac{r_G}{R_{cd}}$ per cent. It is so easy to construct a shunt of the kind shown in Fig. 8 that it is advisable to use this network instead of the Ayrton shunt whenever a "permanent" amplifier set-up is being built.

c. Summary on Galvanometer Control.

We have seen that there are two ways of controlling the overall sensitivity of the amplifier to grid voltage. The first required a special type of shunt in which the resistance values depend only on the external critical damping resistance and the desired sensitivity ratio. This shunt when properly arranged gives the computed sensitivity ratios and maintains the damping constant for all settings. The second method suggested depends on the use of a standard Ayrton shunt. Three minor disadvantages are accompanied by the use of this shunt which are (1) the error in the sensitivity ratio, (2) the non-uniformity of the damping, and (3) the loss in sensitivity.

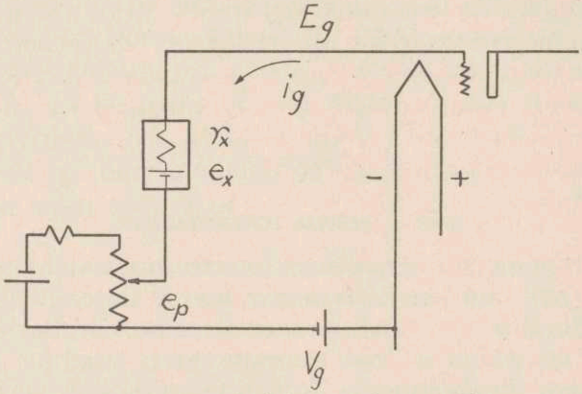
PART 3. SPECIAL INPUT CIRCUITS.

The fundamental equations for the input and the output circuits have now been developed, and in order to show how these relations are used, a few circuits will be analyzed. The circuits which we shall examine can be used for potential and current measurements in high resistance circuits and an attempt will be made to show under what conditions an amplifier can be used to advantage.

A. POTENTIAL MEASUREMENTS.

There are certain problems in which it is necessary to measure a potential originating in a very high resistance circuit. The determination of the acidity of a liquid in terms of a pH measurement with a glass electrode necessitates the measurement of a potential to within about one millivolt while the resistance of the cell may be of the order of ten to a thousand megohms. Although a quadrant electrometer with a sensitivity of 1000 mm. per volt could be used to make this measurement, a galvanometer would need to have a sensitivity of more than 10,000 megs. in order to be used satisfactorily. Where it is necessary to use an instrument which is more rugged than a quadrant electrometer, a galvanometer and an amplifier can be used to considerable advantage. The problem of designing an amplifier for this purpose divides itself in two parts. The first has to do with the input circuit of the tube and is independent of the type of output circuit used. That is, either circuit 5, 6, or 7 or any modification of these can be used without changing the underlying requirements of the input circuit. The second part has to do with the output circuit and the question of *over-all sensitivity* of the system.

FIG. 11.



Simplified input circuit for measuring potentials.

Starting with the input circuit shown in Fig. 11, we see that the following Kirchoff equation must hold.

$$e_x + e_p + V_g = E_g + r_x i_g. \tag{105}$$

Here the resistance of the potentiometer is assumed to be very small compared with r_x. In order to be able to read the unknown potential on the potentiometer directly, the potential e_p should be equal and opposite to that of the unknown e_x.

$$e_x = - e_p. \tag{106}$$

Equation (105) will then be satisfied if

$$r_x i_g = 0, \tag{107}$$

for then

$$V_g = E_g. \tag{108}$$

Of course, equation (107) does not have to be satisfied in the absolute sense. What is really necessary is that $r_x i_g$ must be small compared with the smallest value of

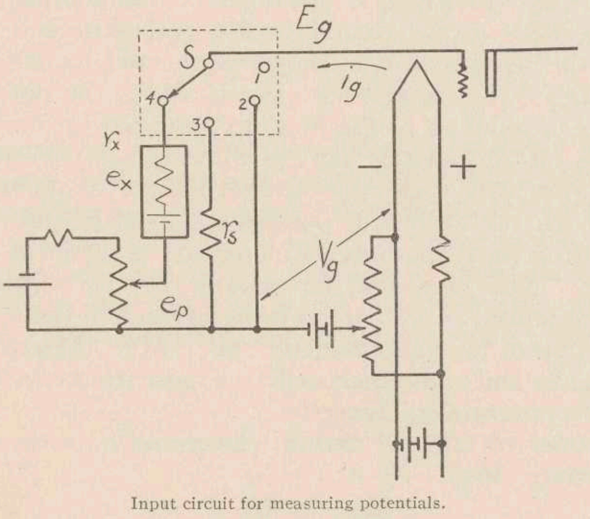
$$e_x - |e_p| = \Delta e \tag{109}$$

which we wish to measure, for example one millivolt in the case under consideration. Obviously if r_x is very large, i_g must be very small and *we must operate* very close to the floating potential where the grid current i_g can be reduced to a value as small as we need.

a. Case I. Where It is Necessary to Operate at Floating Potential because r_x is Very Large.

If r_x is so large that it is necessary to operate very close to the floating potential (this is the case with most glass

FIG. 12.



Input circuit for measuring potentials.

electrodes if the true potential e_x is to be measured[15]), then a circuit like that shown in Fig. 12 should be used. Here we have the input vacuum tube of circuit 5, 6 or 7. The problem is to measure the unknown potential e_x which originates in a cell or other device having an internal resistance

[15] W. C. Stadie, *J. Biol. Chem.*, 83: 477 (1929), maintains that only relative values of e_x are necessary for pH measurements with glass electrodes and therefore under certain conditions he does not believe that it is necessary to make $r_x i_g < \Delta e$

r_x which is very large. The procedure for operating this circuit can be outlined as follows:

1. With switch S in position (1), that is with the grid free, the circuit should be "balanced" to produce no current through the galvanometer in the plate circuit. For example if we use the circuit shown in Fig. 5, the resistance R_a should be adjusted to give no current through the galvanometer, while if we use circuit 7, the grid potential of the dummy tube should be adjusted to bring about the same result.

2. The second step in the process is to move switch S to position (2) and adjust V_g until the galvanometer again reads zero. Now we should be able to move the switch contact from (1) to (2) or back again with no detectable motion of the galvanometer coil in the plate circuit. We have now established V_g = floating potential. This test of the accuracy with which V_g is adjusted to the floating potential may be more severe than is really necessary and for that reason the resistance r_s is shown connected to one point of the switch. If r_s is at least as large as r_x, the potential V_g will be near enough to the floating potential for all practical purposes, if the switch can be moved from (2) to (3) or (3) to (2) at will, with no motion of the galvanometer coil.

3. With the switch in position (4), the potential e_p can be adjusted until the galvanometer coil is again at the zero position. Now it should be possible to move switch S to any position of (2), (3), or (4) from any one of them, without any motion of the galvanometer coil. After these tests have been made and only then will the potentiometer read the unknown potential e_x correctly.

In order to find the rate of change of E_g with e_p, let us differentiate (105).

$$\frac{\partial E_g}{\partial e_p} + r_x \frac{\partial i_g}{\partial e_p} = 1. \qquad (110)$$

Using equation (7) and rearranging we find

$$\frac{\partial E_g}{\partial e_p} = \frac{1}{1 + \dfrac{r_x}{Z_g}}. \qquad (111)$$

We see at once that Z_g should be large compared with r_x if we are to obtain good sensitivity since Z_g is always a positive number at the floating potential. We can combine equation (111) with the other sensitivity equations such as (37), (51) or (63) and we get, using (37),

$$\frac{\partial i_G}{\partial E_g} \times \frac{\partial E_g}{\partial e_p} = \frac{G_m}{1 + \frac{r_G}{R_{cd}}} \times \frac{1}{1 + \frac{r_x}{Z_g}}. \tag{112}$$

Defining the galvanometer response to current [16] as

$$C_s = \frac{\partial \theta}{\partial i_G} \tag{113}$$

(where θ is the angular deflection of the galvanometer coil), we can compute the over-all sensitivity as

$$\frac{\partial \theta}{\partial e_p} = \frac{G_m}{1 + \frac{r_G}{R_{cd}}} \times \frac{1}{1 + \frac{r_x}{Z_g}} C_s. \tag{114}$$

If θ is measured in millimeters deflection at some standard distance, the over-all sensitivity is given directly by equation (114) in millimeters per volt. This expression shows that the over-all sensitivity of the circuit is a very simple function of the tube constants, the galvanometer constants and the resistance of the device in which the potential originates. Equation (114) can be rearranged so as to collect the various factors as follows:

$$\frac{\partial \theta}{\partial e_p} = \frac{G_m}{1 + \frac{r_x}{Z_g}} \times \frac{C_s}{1 + \frac{r_G}{R_{cd}}}. \tag{115}$$

The second factor is a function of the galvanometer only, while the first factor is determined by the vacuum tube constants and cell resistance. It is obvious that if Z_g is large compared with r_x the sensitivity will be proportional to

[16] Current sensitivity (A) is usually given in microamperes per millimeter deflection. $C_s = \frac{10^6}{A}$. Galvanometer sensitivity is sometimes expressed in "megs." $C_s =$ (megohm sensitivity) 10^6.

the mutual conductance G_m and therefore where the cell resistance is quite low the vacuum tube with the highest G_m will have the highest sensitivity. The R.C.A. power tube UX-112-A is a very popular tube having a high mutual conductance. If, however, r_x is ten megohms or more the input impedance as well as the mutual conductance must be considered. Take, for example, $r_x = 50 \times 10^6$ ohms and let us compare the sensitivities which we must expect from a UX-112-A vacuum tube and a UX-222 screen grid tube. The mutual conductances of these two tubes are listed as 1600×10^{-6} mho and 400×10^{-6} mho respectively. The grid impedances, at the floating potential, of tubes of these types have been found to be 12×10^6 ohms for the UX-112-A and 3×10^9 ohms for the UX-222. Considering the first factor of equation (115) which is the part depending on the characteristics of the vacuum tube, we see that the screen grid tube will be 1.3 times as sensitive as the UX-112 and at the same time the UX-222 will be much less sensitive to fluctuations in the power supply.

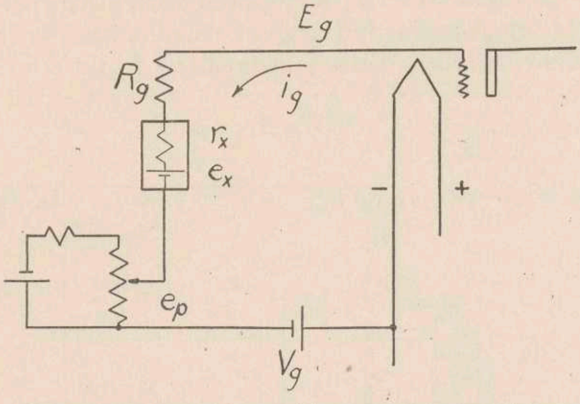
b. Case II. Where It is Possible to Operate with a Negative Grid Impedance because r_x is Small.

When the resistance of the device in which the potential originates is less than one megohm it is not always necessary to operate at the floating potential. The curve shown in Fig. 3, which gives the relation between "C" battery potential and plate current when a high resistance is used in the grid circuit, suggests that a higher sensitivity to voltage can be obtained in an amplifier if we take advantage of the fact that the grid impedance Z_g is *negative* for certain values of grid potential E_g. The use of this method of measurement presupposes that a current of the order of 10^{-8} ampere can flow through the device being measured without altering its properties. If such is not the case we must either select a vacuum tube which has a grid current small enough to cause no change in the device, or else operate near the floating potential and thus keep the current very small. Refer to Fig. 13. This is the simplified form of the circuit which we have in Fig. 14 when switch S is in position 3. The Kirchoff equation corresponding to (105) above is the following:

$$e_x + e_p + V_g = E_g + i_g(r_x + R_g). \qquad (116)$$

Here again we assume that the resistance of the potentiometer is small compared with r_x. By differentiating (116) and

FIG. 13.



Simplified input circuit for potential measurements when r_x is low.

introducing Z_g which is defined by equation (7) we are able to determine the rate of change of E_g with respect to e_p.

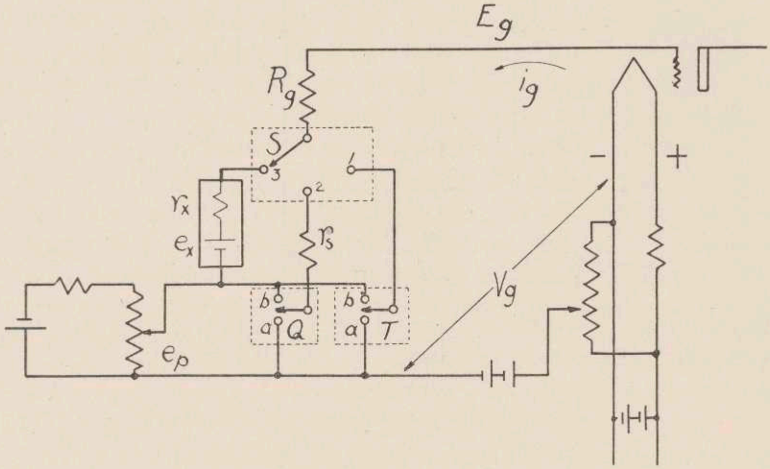
$$\frac{\partial E_g}{\partial e_p} = \frac{1}{1 + \dfrac{r_x + R_g}{Z_g}}. \tag{117}$$

With this equation before us we can turn to the complete circuit of Fig. 14 and examine its operation.

The resistance r_s should be equal to the resistance of the device r_x. The resistance R_g depends on the particular tube used and the grid potential around which we intend to operate. In order to determine the best value of "C" battery potential V_g we set switch S at 2 and switch Q at (a) and the "C" battery potential at some definite value (V_g). The plate circuit should be balanced to produce no current through the galvanometer. With the potentiometer potential e_p adjusted to some small value, throw switch Q to position b and note the galvanometer deflection. This deflection is a measure of the amplifier sensitivity for this particular value of V_g. Now take a new value of V_g and repeat the test, including the balancing of the plate circuit, until the value of V_g is found which gives the maximum sensitivity. This method of finding the best value of the "C" battery potential for some particular case is obviously a "cut and try" method.

After the best "C" battery potential V_g has been determined, the magnitude of the grid current can be measured in the following way: With S in position 2 and Q in position

FIG. 14.



Input circuit for potential measurements when r_x is low.

a the plate circuit should be balanced to give no current in the galvanometer. With switch T in position b and S in position 1, the potentiometer potential e_p should be adjusted to bring the galvanometer current to zero. Let us designate this potential by e_{po}. The potential e_{po} is now a measure of the "IR" drop brought about by the flow of grid current through resistance r_s. Since r_s is known and e_{po} is determined by the experiment, the grid current can be computed from the Ohm's law relation,

$$i_g = \frac{e_{po}}{r_s}. \tag{118}$$

Assuming that no alteration in the potential e_x results from the flow of the current i_g, we are in a position to measure e_x.

After the circuit has been balanced with S in position 2 and Q in position a, we switch S to position 3 and adjust e_p to restore the balance. This value of e_p will be equal in magnitude and opposite in sign to the unknown potential e_x.

In order to compute the over-all sensitivity for this circuit we can use equations (37), (113) and (117) to get

$$\frac{\partial\theta}{\partial i_G} \times \frac{\partial i_G}{\partial E_g} \times \frac{\partial E_g}{\partial e_p} = \frac{1}{1 + \frac{r_x + R_g}{Z_g}} \times \frac{G_m}{1 + \frac{r_G}{R_{cd}}} \times C_s. \quad (119)$$

Collecting the factors which depend on the galvanometer, we have the following:

$$\frac{\partial\theta}{\partial e_p} = \frac{G_m}{1 + \frac{r_x + R_g}{Z_g}} \times \frac{C_s}{1 + \frac{r_G}{R_{cd}}}. \quad (120)$$

Since we can operate on a part of the grid characteristic where Z_g is *negative* and we can adjust R_g to make $(r_x + R_g)$ approach the value of $|Z_g|$, the over-all sensitivity of the system can be made very great. The limit of sensitivity is set only by the difficulty of maintaining a steady "zero."

Let us consider a numerical example in which we assume that the unknown potential is of thermo-electric origin and the thermopile has a resistance of 500 ohms, and also that we use the circuit shown in Fig. 5 with a UX-112-A vacuum tube and a Leeds & Northrup 2500-*e* galvanometer. The catalog value of the constants of this galvanometer are the following: sensitivity 3×10^{-9} amp. per millimeter deflection at one meter; external critical damping resistance R_{cd} of 2000 ohms; coil resistance r_G of 500 ohms. The second factor of equation (120) can be computed from these data. We can take $G_m = 1600 \times 10^{-6}$ mho which is the rated value of the mutual conductance for the UX-112 vacuum tube and we can take the grid impedance at the point of inflection of the grid current *vs.* grid potential characteristic as $Z_g = -535$ megohms. This is the value found on a tube of this type and therefore cannot be taken as the average. It is hoped that in the near future measurements can be made on a sufficiently large number of tubes to get some indication as to the probable value of such constants. If such tests are carried out the results will be published in this Journal. We can make $(r_x + R_g) = 500$ megohms and we get for the over-all sensitivity:

$$\frac{\partial \theta}{\partial e_p} = \frac{1600 \times 10^{-6}}{1 - \frac{500}{535}} \times \frac{\frac{1}{3 \times 10^{-9}}}{1 + \frac{500}{2000}} \tag{121}$$

$$= 6.6 \times 10^{6} \text{ mm. deflection per volt.}$$

The sensitivity is thus 6.6 mm. per microvolt. Remembering that the potential being measured has been assumed to have originated in a 500 ohm thermopile, we see that the sensitivity of this amplifier galvanometer combination exceeds that which can be realized with most D'Arsonval galvanometers. Obviously, if the resistance of the thermopile can be reduced without decreasing its ability to generate potential, the amplifier cannot be used to advantage. On the other hand, if the thermopile can be made to give even higher potentials by a further increase in its resistance the use of an amplifier becomes more and more advantageous. An amplifier used as suggested above will not have a constant zero over any very long period of time, but since the measurement is made by a "null" method (using the circuit of Fig. 14), considerable drift can be permitted without seriously interfering with the measurements.

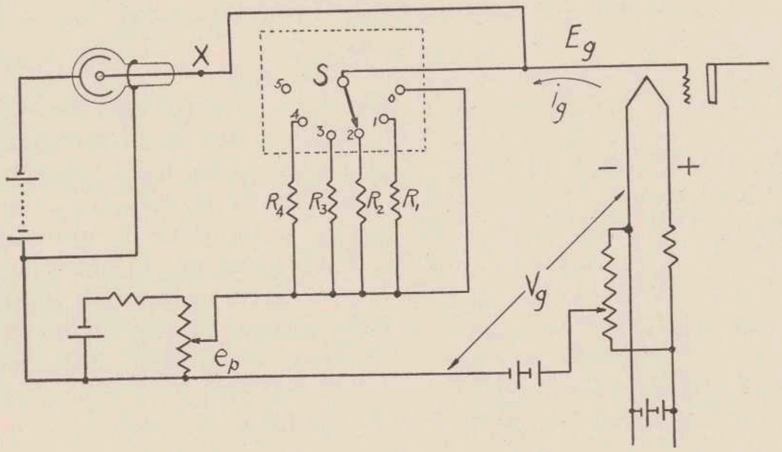
B. Current Measurements in High Resistance Circuits.

Under this head, four important circuits will be discussed. These do not begin to exhaust all of the possible circuits which might be used for current measurements but they illustrate many of the important points to be considered and show the methods by which any special circuit can be analyzed and its operation accurately predicted. Certain special requirements make it desirable to operate some amplifiers with the grid potential very close to the floating potential while in other cases we can realize a greater sensitivity by operating at a potential which is more negative than the floating potential and thus take advantage of the negative grid impedance.

a. Case I. Current Amplifier Operating at Floating Potential. Measurements by Direct Deflection.

The amplifier input circuit shown in Fig. 15 is one which has been used for some years to measure very small photo-electric currents by direct deflection. When there is no photoelectric current, the grid of this amplifier is maintained at the floating potential and the plate circuit is balanced to give no galvanometer current. Since there is no grid current in the input circuit when the grid is at the floating potential

FIG. 15.



Input circuit for photo-electric current measurements.

the "C" battery potential must be equal to the floating potential and the galvanometer zero will be independent of the resistance R_g (i.e., R_1, R_2, etc.). In other words, switch S of Fig. 15 can be moved to any position 0 to 5 without a shift in the galvanometer zero. The deflection of the galvanometer when a photoelectric current is flowing will depend on the value of R_g in a way which will be discussed below. The *only object* in operating this amplifier at the floating potential is to enable us to have a very wide range over which current measurements can be made. It may make this point more clear to illustrate it by means of a numerical example. Equation (17) shows that the change ΔE_g in grid potential brought about by the flow of a photoelectric current Δi when $R_g = \infty$, is given by

$$\Delta E_g = Z_g \Delta i. \tag{123}$$

If Z_g is 3×10^9 ohms as is the case for the UX-222 vacuum tube, and Δi is 5×10^{-15} ampere, we have a ΔE_g which is 15×10^{-6} volt. With an amplifier sensitivity of 0.4 mm. per microvolt the deflection is 6 mm. Suppose however, we now want to measure a current of 5×10^{-8} ampere. From equation (123) we would get a ΔE_g of 150 volts, which would obviously carry the grid too far from the operating potential. If, however, we use a resistance $R_g = 3 \times 10^6$ ohms, equation (17) reduces to the form:

$$\Delta E_g = R_g \Delta i. \tag{124}$$

Putting in the numerical values $R_g = 3 \times 10^6$ and $\Delta i = 5 \times 10^{-8}$ we get $\Delta E_g = 0.15$ volt which is not at all too great for the UX-222 tube when R_g is as low as we have taken it. With a sensitivity of 0.4 mm. per microvolt we have a deflection of 6000 mm., which can be reduced to 60 mm. by means of an Ayrton shunt (or its equivalent) in the plate circuit. We thus see that it is necessary to have an input control in the form of a variable resistance if we wish to measure currents of widely different magnitude and not "over load" the amplifier.

If we rewrite equation (17) in the form

$$\Delta E_g = \Delta i \left(\frac{1}{\frac{1}{R_g} + \frac{1}{Z_g}} \right) \tag{125}$$

we see the change in potential ΔE_g is equal to the "IR" drop where Δi is the current and the resistance is the external resistance R_g in parallel with the grid impedance Z_g. This assumes that Z_g is constant over the range ΔE_g. That is, we have

$$\frac{1}{R} = \frac{1}{R_g} + \frac{1}{Z_g}, \tag{126}$$

where R is the "effective" resistance which must be used to compute the change in grid potential as

$$\Delta E = \Delta i R. \tag{127}$$

It is by means of this factor R that we control the sensitivity of the input circuit.

Suppose that we wish to reduce our sensitivity and therefore our value of R to some smaller value R'. Let the ratio of these effective resistances be n.

$$\frac{R}{R'} = n. \qquad (128)$$

For any value of n and any value of R we can compute the value of R_g which is necessary from equation (126). The highest value of R is had when $R_g = \infty$ for then $R = Z_g$. Let us take the case in which

$$R' = \frac{R}{n} = \frac{Z_g}{n} = \frac{1}{\frac{1}{R_g} + \frac{1}{Z_g}}. \qquad (129)$$

Solving equation (129) for R_g we get

$$R_g = \frac{Z_g}{n - 1}. \qquad (130)$$

From this equation we can easily compute a series of values of R_g in terms of a given Z_g for any series of sensitivity ratios n_1, n_2, etc. For example with $Z_g = 3000 \times 10^6$ ohms we get

n	R_g
1	∞
3	1500×10^6
10	333
30	103
100	30

The adjustment and operation of this amplifier will now be described assuming that Fig. 15 represents the input circuit to the balanced bridge amplifier shown in Fig. 7. With switch S in position 5 and the photo-cell circuit open at X the grid potential of the dummy tube should be adjusted to bring the galvanometer to "zero." With $e_p = 0$, turn switch S to position 0 and adjust V_g until the galvanometer again returns to the "zero" position. It should now be possible to turn switch S to any position 0 to 5 without causing a deflection of the galvanometer. If this is not true it is an indication that the grid lead is not properly insulated and protected by shields and guard rings to keep out extraneous currents. If the leakage through the photoelectric cell is smaller than the least current which can be measured with this particular amplifier circuit, then the connection at X can be made without causing a shift in the "zero" as the switch is turned from 0 to 5.

The potentiometer is used to determine the sensitivity of the amplifier to grid potential. With the switch S at 0 the galvanometer deflection can be measured as a function of the potentiometer potential e_p. It may be found that the response will be linear for values of e_p as great as $\pm$ 0.5 volt or more depending on the vacuum tube used. It *does not follow* that the response to photoelectric current will also be linear for the same range of galvanometer deflection. This is due to that fact that the grid current is not a linear function of the grid potential for wide variations in grid potential. For this reason it is well to determine the grid current characteristic by turning switch S to position 1 or 2 and again measure the galvanometer deflection as a function of the potentiometer potential.

This time the potential required to cause a given galvanometer deflection will be greater by an amount Δe_p than it was when switch S was in position 0. This difference in the two potentials, is due to the grid current and is equal to the "IR" drop of this current in the resistance R_g. R_g is equal to R_2 when the switch is in position 2. If we know R_g we can find the grid current i_g corresponding to each value of e_p since

$$i_g = \frac{e_p' - e_p}{R_g} = \frac{\Delta e_p}{R_g}. \qquad (131)$$

Where e_p' and e_p are the potentiometer potentials necessary to give the same galvanometer deflection with and without the resistance R_g respectively. Since the grid potential is given by

$$V_g + e_p = E_g, \qquad (132)$$

the grid current characteristic as a function of grid potential E_g can be mapped out accurately.[17] This characteristic will usually be of the form shown by the solid line of Fig. 2.

It was stated above that even though the galvanometer response might be a linear function of the grid potential E_g, it does not follow that the galvanometer response will be a linear function of the photoelectric current. Since this non-linearity becomes greater as the resistance R_g increased, let us consider the case in which $R_g = \infty$, that is, the case when the switch is in position 5. We observe that the galvanometer deflection is a function of the light falling on the photo-cell and for each deflection of the galvanometer we can determine the grid potential from the calibration taken with the switch in position 0. With the photo-cell connected as shown the grid will become more and more negative as the light intensity is increased. The amount that it becomes negative will be determined entirely by the grid current characteristic and the photoelectric current will be exactly equal to the grid current for the corresponding grid potential as determined above. This result comes from equation (14) which may be rewritten as follows:

$$i = \frac{E_g - V_g}{R_g} + i_g, \tag{133}$$

since the first term vanishes when R is infinite. As long as the photoelectric current does not exceed the grid current at the hump of the curve at H of Fig. 2, the photoelectric current will be definitely related to the galvanometer deflection, but the moment the photoelectric current exceeds this value a sudden transition will occur and the grid will become very negative and the galvanometer deflection will increase tremendously even though the light intensity increase is very small. From equation (133), the amplifier calibration and the experimental determination of the grid current *vs.* grid potential curve, we can compute the relation between galvanometer deflection and photoelectric current.

We have gone into this question of the non-linearity of response caused by the flow of grid current at some length, because it enters in such a subtle way that it is very easily

[17] See footnote 1.

overlooked and may cause serious misinterpretations of results if it is not taken into consideration.

b. Case II. Current Amplifier Operating at Floating Potential. Measurements by "Null" Method.

If we use the potential e_p from the potentiometer in Fig. 15 to restore the galvanometer coil to its zero position, the problem of the non-linearity of response no longer plays a part. A new difficulty appears, however, owing to the fact that, as R_g is increased beyond a certain value, the percentage uncertainty in the setting of the potentiometer increases very rapidly. An analytical expression which shows this effect can be derived in the following way.

When we are using the potentiometer as suggested, the Kirchoff equation which we must use instead of (14) is the following:

$$E_g = V_g + e_p + R_g i - R_g i_g, \tag{134}$$

where V_g is constant and equal to the floating potential. When the e_p is adjusted to bring about a perfect balance of the amplifier $e_p + R_g i = 0$; $i_g = 0$ and $E_g = V_g =$ floating potential. In every practical case there is a definite change in E_g (call it δE_g) which produces the least detectable swing of the galvanometer coil. This is a function of the galvanometer sensitivity and the circuit constants. Let us give e_p an arbitrary variation which is just sufficient to produce a change δE_g in the grid potential. We find from equation (134)

$$\delta E_g = \delta e_p - R_g \frac{\partial i_g}{\partial E_g} \delta E_g. \tag{135}$$

Making use of the definition of grid impedance in equation (7) and rearranging we see that

$$\delta e_g = \delta E_p \left(1 + \frac{R_g}{Z_g} \right). \tag{136}$$

From this equation we see which factors control the uncertainty in the setting of the potentiometer and we can calculate the percentage uncertainty as follows:

$$\frac{\delta e_p}{e_p} 100 = U \text{ percent} = 100 \frac{\delta E_g}{e_p} \left(1 + \frac{R_g}{Z_g} \right). \tag{137}$$

Hence, if the sensitivity to photoelectric current is increased by increasing R_g, the percentage uncertainty in the potentiometer setting will change very little as long as $\frac{R_g}{Z_g}$ is less than unity. On the other hand, if Z_g is positive and $R_g > Z_g$, the percentage uncertainty in the potentiometer setting increases with increasing R_g practically as fast as the sensitivity and therefore accomplishes nothing.

c. Case III. Current Amplifier Operating with a Negative Grid Impedance. Measurements by Direct Deflection.

For certain applications it is unnecessary to measure currents extending over a very wide range of intensity with the minimum of adjustment. Very high sensitivity can be had by operating with a fixed resistance R_g and adjusting the "C" battery potential so as to work over that part of the grid current characteristic where the grid impedance is negative. The circuit shown in schematic in Fig. 15, can also be used for this discussion. In the previous section a method was described by which the grid current characteristic of the tube could be measured when the amplifier was balanced with the grid at the floating potential. The procedure in that case was definite because the floating potential could be determined without knowing the entire grid current characteristic. It will appear in the discussion to follow that there are certain advantages to be had if the potential V_g is adjusted so that the grid potential about which we operate is located near the point of inflection of the grid current *vs.* grid potential characteristic. Owing to the fact that the grid current characteristic depends on the conditions in the plate circuit, it is not easy to set up the procedure for determining the point of inflection. The writer has found by experience that the following method leads to satisfactory results. With switch S in position 0 and V_g adjusted to a potential which is 2.0 to 3.0 volts negative with respect to the filament, the plate circuit is balanced to give no current through the galvanometer. This is done by adjusting the potential of the grid of the dummy tube when the two tube bridge circuit is being used.

With the switch still at 0 we should determine the galvanometer deflection corresponding to a series of potentials e_{p1}, e_{p2}, etc., for both positive and negative values of e_p. If we turn the switch S to position 1, 2, 3 or 4 we can find new values of e_p which will give exactly the same series of galvanometer deflections as before. Assume that we select position 2 and let these new values be e_{p1}', e_{p2}', etc., where e_{p1}' and e_{p1} are the potentiometer potentials which give the same galvanometer deflections with the switch in position 2 and position 0 respectively. These differences $(e_{p1}' - e_{p1})$, etc., measure the "IR" drop in the resistance resulting from the flow of grid current through it. Obviously these differences divided by the resistance R_2 give the corresponding grid currents

$$\frac{e_p' - e_p}{R_2} = i_g. \tag{138}$$

We can add V_g to e_p to get E_g and plot i_g as a function of E_g. This is the grid current *vs.* grid potential characteristic. The reciprocal of the slope of the tangent to this curve drawn through the point $E_g = V_g$ will give the input impedance to the grid when operated under these particular conditions.

If this circuit is to be used for direct deflection measurements, it is important that the value of E_g around which we intend to operate be located on the straight part of the characteristic, that is, near the "point of inflection" of the grid current characteristic. In case the value V_g which was arbitrarily chosen was not close enough to this best value of E_g, a new value of V_g should be taken and the above series of measurements repeated.

In order to test the linearity of response of the system as a whole, two sets of measurements of the galvanometer deflection as a function of potentiometer potential must be made. The first of these is made with the V_g set equal to the value of E_g around which we intend to operate and with the switch at position 0 while the second is made with the switch in position 2, for instance, and with a new corresponding value of V_g. This new value of V_g is found by balancing amplifier with the switch at 0 by means of the dummy tube and then with the switch at 2, V_g is readjusted to bring the amplifier back to balance. This readjustment is necessary in order to correct for the fall in potential in the grid resistance due to the flow of grid current.

We must now measure the galvanometer deflection as a function of potentiometer potential with the resistance in the circuit and the new value of V_g. The range of galvanometer deflection over which *both* of these calibration curves are linear is a measure of range over which the amplifier as a whole will respond linearly to photoelectric current with this particular value of R_g. It is very important to make both of these tests because it is possible to have a *non-linear* response to photoelectric current and yet have one or the other of these tests indicate a linear response.

The over-all sensitivity using this input circuit and the two tube balanced bridge amplifier of Fig. 7 can be calculated from equations (17), (63) and (113).

$$\frac{\partial \theta}{\partial i_G} \times \frac{\partial i_G}{\partial E_g} \times \frac{\partial E_g}{\partial i} = \frac{G_m}{2\left(1 + \frac{r_g}{R_{cd}}\right)} \times \frac{1}{\frac{1}{R_g} + \frac{1}{Z_g}} \times C_s. \quad (139)$$

On rearranging equation (139) we get

$$\frac{\partial \theta}{\partial i} = \frac{G_m}{\frac{1}{R_g} + \frac{1}{Z_g}} \times \frac{C_s}{2\left(1 + \frac{r_G}{R_{cd}}\right)}, \quad (140)$$

in which the second factor depends only on the galvanometer constants. Since we are assuming that Z_g is negative we see at once that if R_g is less than the absolute magnitude $|Z_g|$ the sensitivity will increase as R_g approaches $|Z_g|$ and becomes infinite when $R_g = |Z_g|$. If R_g is made larger than $|Z_g|$ it will be impossible to operate the amplifier at the grid potential E_g for which $|Z_g|$ was determined.

If we put constants into equation (140) appropriate to the UX-112-A vacuum tube and the Leeds and Northrup 2500-*e* galvanometer which are $G_m = 1600 \times 10^{-6}$ mho, $Z_g = -1000 \times 10^6$ ohms, $R_g = 900 \times 10^6$ ohms, $r_g = 500$ ohms, $R_{cd} = 2000$ ohms and $C_s = 3.3 \times 10^8$ mm. per ampere at one meter, we get

$$\frac{\partial\theta}{\partial i} = 1.9 \times 10^{15} \text{ mm. per ampere.} \tag{141}$$

From this we see that one centimeter deflection corresponds to a current of 5×10^{-15} ampere. In order to determine the actual sensitivity of an amplifier of this type it is not necessary to compute it in this way. All that is necessary is to measure the voltage sensitivity $\frac{\partial\theta}{\partial e_p}$ and multiply it by the resistance R_g. The truth of this statement is at once evident from the equations for the voltage and current sensitivities when written as follows:

$$\frac{\partial\theta}{\partial e_p} = \frac{G_m}{1 + \frac{R_g}{Z_g}} \times \frac{C_s}{2\left(1 + \frac{r_G}{R_{cd}}\right)}, \tag{142}$$

$$\frac{\partial\theta}{\partial i} = \frac{R_g G_m}{1 + \frac{R_g}{Z_g}} \times \frac{C_s}{2\left(1 + \frac{r_G}{R_{cd}}\right)}. \tag{143}$$

The first equation comes from (63), (111), and (113) while the second comes from (16), (63) and (113). Substituting (142) into (143) we get

$$\frac{\partial\theta}{\partial i} = R_g \frac{\partial\theta}{\partial e}. \tag{144}$$

d. Case IV. Current Amplifier Operating with a Negative Grid Impedance. Measurements by "Null" Method.

This type of circuit is very well adapted for "null" measurements in which the potentiometer potential e_p is adjusted to keep the galvanometer coil in its zero position. The question of the linearity of response does not enter for the current being measured is given directly by

$$i = \frac{e_p}{R_g}. \tag{145}$$

From equation (137) we see that the percentage of uncertainty with which we can set the potentiometer e_p *decreases* as we increase the sensitivity by making R_g larger since Z_g has been assumed to be negative.

PART 4. TECHNICAL PROCEDURE.

A few points as to technical procedure are perhaps worth mentioning.

A. Shielding and Insulation.

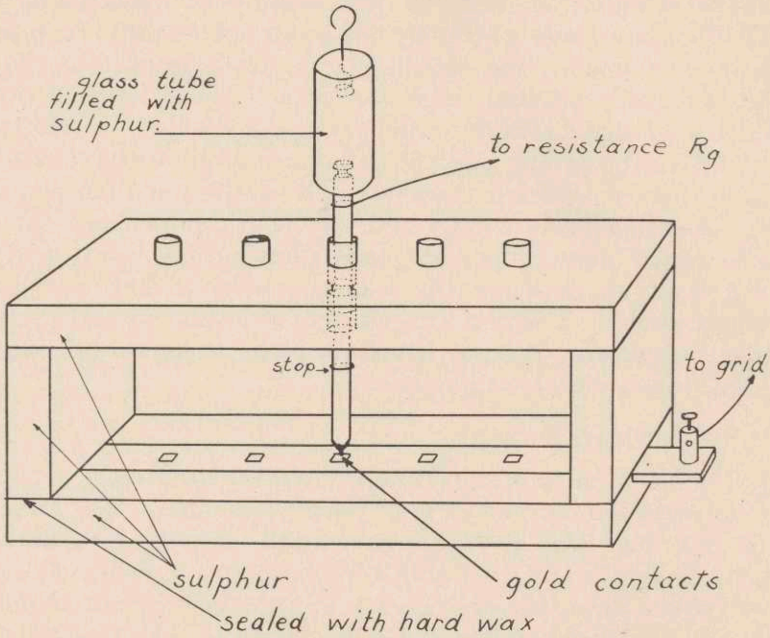
The active grid lead of these circuits should be shielded in all cases where the circuit resistances are very high. The next most important thing to be shielded is the power supply and the plate leads and lastly the control apparatus such as the potentiometer, the rheostats, etc. It is important that the grid leads on the "high resistance" side of the device which is producing the potential or current should be insulated at least to the extent that the resistance of all of the leakage paths when considered together shall not be less than about ten times the absolute magnitude of the grid impedance $|Z_g|$. It is much more important that guard rings be properly located so as to make the leakage currents flow through definite paths. The best arrangement of insulators and guard rings depends on the particular problem at hand and must be given considerable thought.

B. Structural Details.

The vacuum tube should be located as close to the source of potential or current as is physically possible. The tubes and also the grid lead should be well supported to make them as free from mechanical vibration as possible. Any switches or keys that are necessary in the grid circuit should be well insulated and mechanically certain in their operation. In the most sensitive circuits used by the author the switch S of Fig. 15 was made by imbedding a bar of brass in a cake of sulphur and allowing contacts to drop from above. These contacts were guided by brass tubes also imbedded in sulphur. Small gold tips were set in the movable contacts and small plates of gold were soldered on the brass bar at the points of contact. The sketch in Fig. 16 serves to illustrate the construction. Doubtless there are many other arrangements just as good as this one but this design is given in order to help those whose previous work has been in other fields.

If the amplifier is being constructed for high sensitivity work, great care must be taken to make sure that every junction between wires is soldered. It is even advisable to solder the leads onto the storage batteries used for the filament supply. The control rheostats should be of the best grade and the circuit should be designed with fixed series and shunt resistances associated with each rheostat so that

FIG. 16.



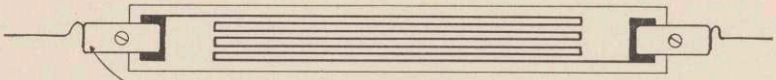
only a small proportion of the current flows through the movable contact. Resistances R_1, R_2 and r_d in the plate circuit should be "wire" resistances. It is usually advisable to insert small resistances in each filament and plate circuit so that the filament and plate currents can be measured with the help of a potentiometer and thus have all circuits undisturbed during the measurement.

C. CONSTRUCTION OF HIGH RESISTANCES.

The problem of producing high resistances of one or two megohms and more is not an easy one. Many workers in this field use xylol and alcohol solutions and recommend them highly. The writer has always used "Higgins" India ink resistances and found them perfectly satisfactory. These are made by drawing a long line in the form of a grid as shown in Fig. 17.

A heavy ink line about one millimeter wide drawn on smooth onion skin paper gives a resistance of about one megohm per centimeter. A more narrow line, of course, gives a correspondingly higher resistance per unit length. It has been found advisable to obtain the very high resistances

FIG. 17.



by drawing a very long line rather than a very narrow one.[18] In any permanent set-up these resistances should be mounted in tubes which can be filled with dry air and then sealed or else the tubes should be evacuated and sealed.

The temperature coefficients of three India ink resistances have been measured over the range 23° C. to 45° C. The results are given in the following table:

TABLE III.

$$R_t = R_{23}[1 + \alpha(t - 23)]$$

R_{23} ohms	α
0.85×10^6	-2.7×10^{-3}
750×10^6	-6.4×10^{-3}
2100×10^6	-6.4×10^{-3}

These resistances have lower temperature coefficients than the one reported by Campbell [19] for xylol-alcohol solutions which was $\alpha = 14 \times 10^{-3}$.

[18] Same experience reported by K. T. Compton and C. H. Thomas, *Phys. Rev.*, **28**: 604 (1926).

[19] N. Campbell, *Phil. Mag.*, **23**: 668 (1912).

APOLOGY.

Although this paper is one of the longest which has been printed on the use of the vacuum tube for D.C. measurements, much has been left unsaid. Many circuits even closely related to those discussed here have not been mentioned. The principal object of this paper has been to formulate in a definite way most of the underlying relationships upon which nearly all of the D.C. amplifiers depend. If these are well understood, the design and operation of an amplifier for any particular purpose becomes a problem which is definite and which can be solved by a straightforward procedure.

MEASUREMENT OF SMALL D.C. POTENTIALS AND CURRENTS IN HIGH RESISTANCE CIRCUITS BY USING VACUUM TUBES

by

W. B. NOTTINGHAM, E.E., Ph.D.

REPRINTED FROM THE JOURNAL OF THE FRANKLIN INSTITUTE,
VOL. 209, NO. 3, MARCH, 1930.

Journal
of
The Franklin Institute

Devoted to Science and the Mechanic Arts

Vol. 209 MARCH, 1930 No. 3

MEASUREMENT OF SMALL D.C. POTENTIALS AND CURRENTS IN HIGH RESISTANCE CIRCUITS BY USING VACUUM TUBES.

BY

W. B. NOTTINGHAM, E.E., Ph.D.,

Bartol Research Fellow.

BARTOL RESEARCH FOUNDATION

Communication No. 45.

ABSTRACT.

Part 1. The grid current and plate current characteristics of three element vacuum tubes are described and the simple equations of the D.C. amplifier are developed with special attention given to the circuit for measuring photo-electric currents. The grid current characteristic is shown to play a very important part in the equations for sensitivity.

Part 2. The second part deals with three common output circuits, namely, (1) the single tube circuit with an auxiliary battery to balance out the normal plate current; (2) the single tube bridge circuit and (3) the two tube bridge circuit. Two "universal" shunts are described for controlling the galvanometer current. One of these is a "special shunt" which controls the sensitivity accurately and maintains the damping constant while the other is the well known Ayrton shunt. The disadvantages of using this shunt are discussed.

Part 3. Complete circuits for the measurement of potentials and currents by direct deflection and null methods are discussed and some of the advantages and limitations of each are pointed out.

Part 4. Under this heading a few suggestions are given as to points of technical procedure.

PART 1. GENERAL CONSIDERATIONS.

A. Grid Current Characteristics.

a. Current components in grid circuit.

*a*1. Grid to filament leakage.

*a*2. Plate to grid leakage.

*a*3. Positive ion current to grid.

*a*4. Electron current to grid.

- *b.* Grid impedance defined.
- *c.* Floating potential defined.

B. Plate Current Characteristics.
- *a.* Mutual conductance defined.
- *b.* Plate impedance defined.
- *c.* Tube equation for plate current.

C. Operation of Simple D.C. Amplifier.
- *a.* Circuit equations.
- *b.* Sensitivity to voltage.
- *c.* Sensitivity to current.

PART 2. SPECIAL OUTPUT CIRCUITS.

A. One Tube Circuit with Auxiliary Battery to Suppress Zero.
- *a.* Circuit equations.
- *b.* Sensitivity to voltage.

B. Wheatstone Bridge Circuit with Single Tube.
- *a.* Circuit equations.
- *b.* Conditions for "B" battery compensation.
- *c.* Voltage sensitivity.
- *d.* Condition for zero current in galvanometer.
- *e.* Sensitivity of two circuits compared.

C. Bridge Circuit Using Two Vacuum Tubes.
- *a.* Conditions for "B" battery compensation.
- *b.* Circuit equations.
- *c.* Condition for zero galvanometer current.
- *d.* "Cut and try" method of adjustment.
- *e.* Advantages of two tube circuit.
- *f.* Conditions for filament battery compensation.

D. Summary.

E. Control of Galvanometer Deflection.
- *a.* Special universal shunt.
- *b.* Ayrton shunt and its limitations.
- *c.* Summary on galvanometer control.

PART 3. SPECIAL INPUT CIRCUITS.

A. Potential Measurements.
- *a.* Case I. Where it is necessary to operate at floating potential because r_x is large.
- *b.* Case II. Where it is possible to operate with a negative grid impedance because r_x is small.

B. Current Measurements in High Resistance Circuits.
- *a.* Case I. Current amplifier operating at floating potential. Measurements by direct deflection.
- *b.* Case II. Current amplifier operating at floating potential. Measurements by null method.
- *c.* Case III. Current amplifier operating with a negative grid impedance. Measurements by direct deflection.

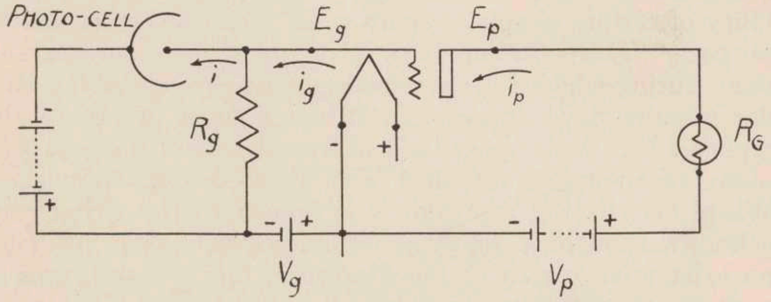
INTRODUCTION.

STUDIES requiring the measurement of small photoelectric currents have made it necessary to investigate the applicability of certain simple vacuum tube circuits to assist in the purpose. This work has been in progress some four years or more during which time a number of papers describing similar circuits have appeared. Although most of the results reported here were worked out independently of those papers, many of them are identical with those already published. Where such is the case and it is known to the writer, due acknowledgement is gladly accorded, but since there has been no exhaustive search of the literature, full acknowledgment to everyone who has reported similar results cannot be made. It is thought that there is enough new material contained in this paper to warrant its publication in spite of the fact that many of the results have already been published.

PART 1. GENERAL CONSIDERATIONS.

Of the many different types of circuits developed for the detection of small direct currents in high resistance circuits, only certain simple ones have been investigated. These depend on the observation of a change produced in the plate circuit of a three or four element vacuum tube as a result of a change in the grid potential. This change in grid potential is usually brought about by the "IR drop" produced in the grid resistance by the current being measured. When the circuit is used for potential measurements in high resistance circuits the change in grid potential is brought about directly. The simple circuit of Fig. 1 contains the essential elements. The operation of this circuit can be predicted at once from the "grid current *vs.* grid potential" and "plate current *vs.* grid potential" characteristics of the tube *taken with the operating resistance in the plate circuit.* Although theoretically it is not necessary to impose this requirement as regards the plate circuit resistance, a great simplification is brought about by it in that it is then necessary to consider only the two curves mentioned above. If this condition is *not* imposed it is necessary to work with the family of curves showing the "grid current *vs.* grid potential" relationship for a number of different plate potentials and similarly for the "plate current *vs.* grid potential" relationship.

FIG. 1.

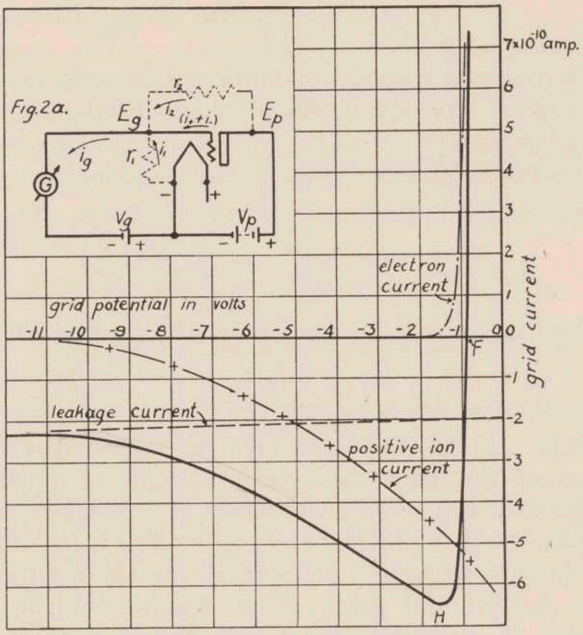


A. GRID CURRENT CHARACTERISTICS.

The grid circuit current characteristic shown by the solid line in Fig. 2 is a composite of at least four different currents. This current can be thought of as measured by means of the circuit shown in the schematic drawing Fig. 2*a*, inserted in Fig. 2.[1] The current flowing through the galvanometer for any given value of grid battery V_g and plate battery V_p is made up of the components described below. When a galvanometer of a low resistance is used the grid potential E_g will be practically equal to the "C" battery potential V_g. Since E_g will not be the same as V_g when a high resistance is introduced into the circuit, it will be well to recognize this distinction right at the beginning of this discussion. In the plate circuit there is, of course, the same sort of distinction between the "B" battery potential V_p and the plate potential E_p when the resistance in the plate circuit is not small compared with the plate impedance.

[1] An indirect method of determining the grid current which is essentially the same as that proposed by M. von Ardenne (*Jahrb. d. Drahtl. Telegr.*, *29*, 88, 1927) will be described later.

FIG. 2.



Grid current characteristics on UX-201-A vacuum tube.

a. Components of Current in Grid Circuit.

a1. Grid to filament leakage current.

The *leakage current* from the grid lead to the filament connections, socket support, etc., is

$$i_1 = -\frac{E_g}{r_1}, \tag{1}$$

where r_1 is the grid to filament insulation resistance shown symbolically in the figure and E_g is the potential of the grid when referred to the negative end of the filament as zero. The negative sign used in this equation is necessary in order that the equation be consistent with the direction of flow of the current as indicated on the diagram of Fig. 2*a*.

a2. Plate to Grid Leakage Current.

The leakage current which flows through the galvanometer due to the combined batteries V_g and V_p of Fig. 2*a* is

$$i_2 = \frac{E_p - E_g}{r_2}, \tag{2}$$

where r_2 is the grid to plate insulation resistance symbolized by r_2 in Fig. 2*a* and E_p is the plate potential which in this case is the same as V_p.

The combined effect of these two types of leakage currents is given by

$$i_1 + i_2 = i_{12} = \frac{E_p}{r_2} - E_g\left(\frac{1}{r_1} + \frac{1}{r_2}\right). \tag{3}$$

This leakage current is shown by the dashed line in the figure.[2]

*a*3. *Positive Ion Current.*

Since the gas can never be entirely removed during the evacuation of the tube, a certain amount of positive ion current will flow to the grid when it is at a negative potential with respect to other parts of the tube. The intensity of this positive ion current will be proportional to the number of gas molecules present and proportional to the electron current flowing from the filament to the plate, when the plate voltage is considerably higher than the ionization potential of the gas. Since the plate current depends on the grid potential, the positive ion current to the grid will depend on the grid potential.[3] This positive ion current i_+ is not a simple function of the grid potential and, therefore, can be represented only in a general way by

$$i_+ = f(E_g, E_p). \tag{4}$$

Graphically this relationship is shown by the curve of Fig. 2 designated "positive ion current."

---

[2] If the vacuum tube is constructed with a separate grid connection as is the case with a UX 222, a few turns of copper wire wound around the tube just above the bakelite base will serve as a guard ring. The potential of this ring can be maintained at the same value as the normal operating value of the grid potential and thus reduce the effective leakage current to a very small value. This leakage in tubes with a structure similar to the UX 201-A can be reduced by cutting out the base and covering the surface of the glass with wax or sulphur.

[3] Mulder and Razek, *J. O. S. A. and R. S. I.*, **18**: 466 (1929).

a4. Electron Current to Grid.

When the grid is only slightly negative with respect to the negative end of the filament, a certain number of electrons are able to overcome the adverse negative potential between the filament and the grid because of the high initial kinetic energy of these electrons due to the high temperature of the filament. The way in which this electron current depends on the grid potential can be represented approximately by the empirical equation [4]

$$i_- = i_o \epsilon^{aE_g}, \qquad (5)$$

where E_g is the value of the grid potential which must be negative with respect to the negative end of the filament for the equation to hold. a and i_o are constants depending on the temperature of the filament and possibly on the plate potential.

The graphical representation given in Fig. 2 shows the form of the function which is the thing of primary interest.

The composite of these four currents gives the "grid circuit" current

$$i_1 + i_2 + i_+ + i_- = i_g, \qquad (6)$$

This is shown graphically by the solid line of Fig. 2. This question of the grid current has been gone into at some length because it plays a very important part in a number of ways in the D.C. amplifier.

b. Definition of Grid Impedance.

In the practical problems of adapting this type of D.C. amplifier to definite purposes, it is not always necessary to measure the grid current characteristic but it is *very necessary* to visualize the "*form*" of the curve in order to understand what is really going on in the input circuit. In particular, it is important to know the slope of the curve at various grid potentials, i.e., the change in grid current for a small change in grid potential.

In the course of the discussion to follow, it will be necessary to use a quantity which is equal to the reciprocal of the slope of the grid current curve at some particular point. It has

[4] M. von Ardenne, *Jahrb. d. Drahtl. Telegr.*, **29**: 88 (1927).

become the accepted practice to designate the corresponding quantity in the plate circuit as the "plate impedance" of the tube. Let us therefore use the term "grid impedance" (Z_g), and let it be related to the slope of the grid current curve at any particular point by the equation

$$\frac{1}{Z_g} = \text{slope of the grid current curve} = \frac{\partial i_g}{\partial E_g}. \qquad (7)$$

From the curve we see that this "grid impedance" is positive for small negative values of E_g and is negative for larger negative values of E_g.

c. Definition of "Floating Potential."

The "floating potential,"[5] indicated by "F" in Fig. 2, is defined as the potential at which the grid current in the external circuit is *zero*. The grid impedance at this point has a definite positive value as given by the reciprocal of the slope of the tangent to the curve drawn through this point.

B. PLATE CURRENT CHARACTERISTICS.

The plate current characteristics of vacuum tubes are so well known and discussed in so many places[6] that it should not be necessary to do more than define a few of the common terms.

a. Mutual Conductance Defined.

In a vacuum tube, the mutual conductance (G_m) in the neighborhood of a given grid potential, is the slope of the "plate current *vs.* grid potential" curve taken with a *constant plate potential.* This definition expressed in equation form is

$$G_m = \left(\frac{\partial i_p}{\partial E_g}\right)_{E_p = \text{const.}}. \qquad (8)$$

---

[5] Sometimes denoted as the "potential of the free grid."

[6] Van der Bijl, "Thermionic Vacuum Tube," McGraw-Hill Book Co. Morecroft, "Principles of Radio Communication," John Wiley & Sons. The best source for practical information concerning vacuum tubes is the "Cunningham Tube Data Book" published by the E. T. Cunningham, Inc., 182 Second St., San Francisco.

FIG. 3.

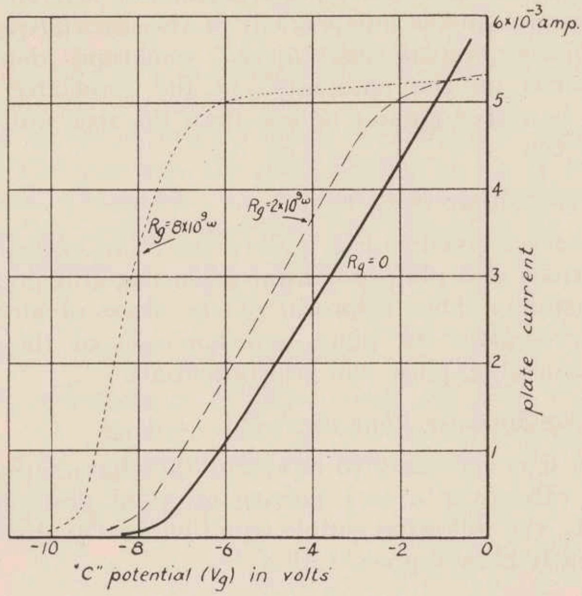
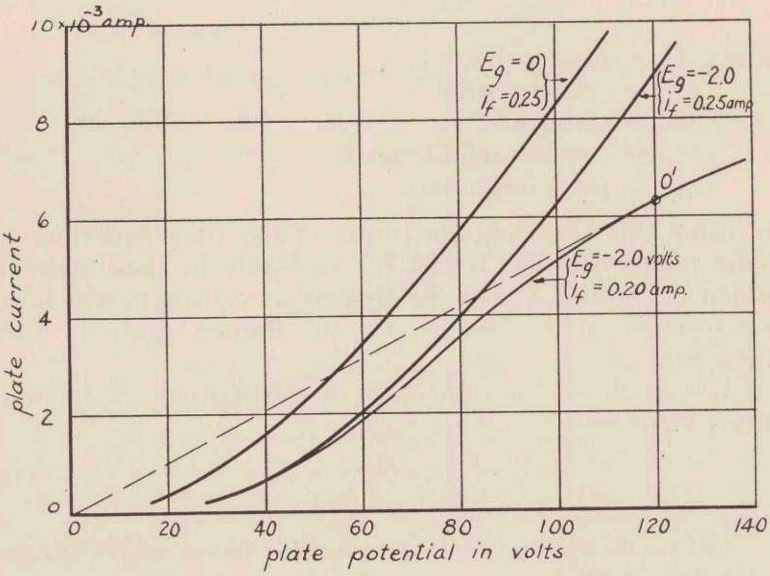

6×10⁻³ amp.
5
4
3
2
1
$R_g = 8 \times 10^9 \omega$
$R_g = 2 \times 10^9 \omega$
$R_g = 0$
plate current
-10
-8
-6
-4
-2
0
"C" potential (V_g) in volts

FIG. 4.


10×10⁻³ amp.
8
6
4
2
0
$E_g = 0$
$i_f = 0.25$
$E_g = -2.0$
$i_f = 0.25$ amp
O'
$E_g = -2.0$ volts
$I_f = 0.20$ amp.
plate current
0
20
40
60
80
100
120
140
plate potential in volts

Mutual conductance so defined is dependent only on the tube characteristics and is independent of the associated circuits. The "*effective mutual conductance*," sometimes denoted by G_m', *depends* on the resistances in the associated circuits and can be either greater or less than the true mutual conductance G_m.

b. Plate Impedance Defined.

The curves given in Fig. 4 show the relationship between plate current and plate potential when the grid potential is held constant. The reciprocal of the slope of any one of these curves gives the plate impedance Z_p of the tube at the corresponding plate and grid potentials.

c. Tube Equation for Plate Current.

When it is necessary to represent the characteristics of a vacuum tube over a *very narrow range* of grid and plate potentials, the following simple equation nearly always gives a sufficiently close representation: [7]

$$i_p = \frac{1}{Z_p}(E_p + \mu E_g + E_o), \tag{9}$$

where E_p = plate potential,
E_g = grid potential,
E_o = constant,
μ = voltage amplification,
Z_p = plate impedance.

In using this very simplified form of the tube equation, we must remember that it can be used only in those cases in which E_o, μ and Z_p can be treated as constants which are independent of E_p and E_g for the limited range of their variation.

We see from the definition of the mutual conductance above that

$$\frac{\partial i_p}{\partial E_g} = G_m = \frac{\mu}{Z_p}\,. \tag{10}$$

[7] See Van der Bijl, page 50, for a more complete discussion of the limitations of this type of equation.

C. OPERATION OF SIMPLE D.C. AMPLIFIER.

a. Circuit Equations.

We are now in a position to discuss the operation of the simple amplifier circuit of Fig. 1.

Since we are interested in the sensitivity of the amplifier we wish to know how the plate current of the tube depends on the grid potential. To compute the sensitivity, we use equation (9) and the Kirchoff Law equation for the plate circuit, which is

$$E_p = V_p - R_p i_p, \tag{11}$$

where V_p = "B" battery potential

and R_p = resistance in plate circuit including that of the battery, the galvanometer and any other resistances in the circuit.

Eliminating E_p we get

$$i_p = \frac{\frac{1}{Z_p}(V_p + \mu E_g + E_o)}{1 + \frac{R_p}{Z_p}}. \tag{12}$$

b. Sensitivity to Voltage Determined.

If we define the sensitivity S as the change in plate current due to a small change in grid potential we have

$$S \equiv \frac{\partial i_p}{\partial E_g} = \frac{\frac{\mu}{Z_p}}{1 + \frac{R_p}{Z_p}} = \frac{G_m}{1 + \frac{R_p}{Z_p}}. \tag{13}$$

The legitimacy of carrying out this variation, treating μ, Z_p and E_o as constants, is determined by the fact that the final equations and the experiments check for small variations of E_g. When R_p is very small compared with Z_p the "sensitivity" is equal to the mutual conductance G_m but if R_p is not small then the sensitivity is less than G_m because Z_p is *always* positive.

c. Sensitivity to Current Determined.

In many applications we are not interested directly in the change in plate current with a change in grid potential but in the change in plate current with a change in some current such as the photoelectric current.

From the Kirchoff Law equation for the grid circuit of Fig. 1, we have the equation

$$E_g = V_g - R_g(i_g - i) \tag{14}$$

where V_g = "C" battery potential,

R_g = external "grid leak" resistance,

i = photoelectric current.

We can perform a differentiation with respect to i in equation (14) and get

$$\frac{\partial E_g}{\partial i} = R_g - R_g \frac{\partial i_g}{\partial E_g} \times \frac{\partial E_g}{\partial i}. \tag{15}$$

Using the definition of Z_g given in equation (7) and rearranging we find

$$\frac{\partial E_g}{\partial i} = \frac{R_g}{1 + \dfrac{R_g}{Z_g}}. \tag{16}$$

This equation can also be written as

$$\frac{\partial E_g}{\partial i} = \frac{1}{\dfrac{1}{R_g} + \dfrac{1}{Z_g}}. \tag{17}$$

If R_g is small compared with Z_g then

$$\frac{\partial E_g}{\partial i} = R_g. \tag{18}$$

That is, the change in grid potential is equal to the change in "IR" drop over the grid resistance. On the other hand Z_g may be *positive* or *negative* depending on which part of the grid current characteristic we are operating. Then the change in grid potential can be either *greater* or *less* than the "IR" drop change usually expected. If Z_g is negative and R_g is adjusted to make it approach the value $|Z_g|$ then $\frac{\partial E_g}{\partial i}$ will become very great because the denominator approaches zero.

Combining (13) and (16) we get

$$\frac{\partial i_p}{\partial i} = \frac{G_m}{1 + \frac{R_p}{Z_p}} \times \frac{R_g}{1 + \frac{R_g}{Z_g}}. \tag{19}$$

This equation gives the sensitivity as the *current amplification* of the one tube amplifier in terms of constants of the circuit. We can see from it exactly how each controllable factor enters to determine the amplifier sensitivity. For example, we see that when Z_g is *positive* and all other quantities constant except R_g, the sensitivity increases as R_g is increased but reaches an upper limit when R_g is made infinite, which is

$$\left(\frac{\partial i_p}{\partial i}\right)_{R_g=\infty} = \frac{G_m}{1 + \frac{R_p}{Z_p}} Z_g. \tag{20}$$

It is obvious that if R_g is made infinite the Z_g which must be taken for equation (20) is that found at the *floating potential*. It, therefore, follows that when two tubes are being compared the one with the higher "input impedance" at the floating potential will have the higher sensitivity if the mutual conductances are the same. The UX-222 (screen grid tube) has the highest input impedance of any tube we have measured in our laboratory. On the average, this has been found to be over 2000 megohms.

By taking advantage of the fact that the input impedance Z_g may be negative, a much higher sensitivity to photoelectric currents is possible. All that we have to do is to adjust the "C" battery potential so that the grid potential

$$E_g = V_g - i_g R_g \tag{21}$$

falls in the range in which Z_g is negative.

TABLE I.

Operating potential of grid.	E_g volts.	i_g amp.	G_m mho.	Z_p ohms.	R_p ohms assumed.	Z_g ohms.	R_g ohms arbitrary.	Current amplification computed by Eq. (19).
Floating	-0.92	0	870×10^{-6}	8600	500	200×10^{6}	∞	166,000
Near point of inflection of grid current curve	-3.0	5.5×10^{-10}	835×10^{-6}	9000	500	-16×10^{9}	200×10^{6}	161,000
	-3.0	5.5×10^{-10}	835×10^{-6}	9000	500	-16×10^{9}	8000×10^{6}	12,700,000

A numerical example will serve to illustrate some of the conditions to be satisfied and the advantages to be had by this method of operation. If we use the grid current and plate current characteristics taken on the UX-201-A vacuum tube which served to give the data for Figs. 2, 3, and 4, we can calculate the current amplification for the single tube amplifier under certain conditions and thus illustrate the advantage of using the negative grid impedance. Table I lists the essential constants and the corresponding current amplification obtained by using equation (19).

We see from this computation that it is easily possible to obtain a current amplification which is 76 times the greatest to be had at the floating potential by operating at a grid potential for which the grid impedance is negative.

These data which were taken from the grid current characteristic of a particular UX-201-A vacuum tube can not be taken in any sense as representative of UX-201-A tubes in general. The amount of surface leakage and the amount of residual gas left in other tubes of the same type are likely to be very different because of changes in manufacturing methods from time to time. It is also possible that tubes produced by one manufacturer will differ in these respects from those produced by another. Obviously, it would be of considerable assistance to workers in this field if data were available by which some idea could be had as to the range of variation in the grid current characteristics which are likely to be met with in the common tubes, but such data are not available.

At this stage in the discussion it will perhaps be of interest to develop the equation which shows the rate of change in sensitivity with a change in grid impedance (Z_g), in order to show some of the difficulties resulting from an attempt to get extreme sensitivity by making $R_g = |Z_g|$ when Z_g is negative. Let us write equation (16) in the following form

$$s \equiv \frac{\partial E_g}{\partial i} = \frac{R_g}{1 + \dfrac{R_g}{Z_g}}. \qquad (22)$$

This equation gives s as the change in the grid potential E_g per unit change in photoelectric current. If we wish to know the rate of change in s with a change in the tube input impedance Z_g, we can differentiate equation (22) and we get

$$\frac{\partial s}{\partial Z_g} = \frac{1}{\left(1 + \frac{Z_g}{R_g}\right)^2}. \tag{23}$$

Consider first the case for which Z_g is *positive* and we operate in the region of the floating potential. Here we see that when R_g is infinite the right hand side of the equation is unity and the *change in sensitivity is directly proportional to the change in the input impedance.* If R_g is small compared with Z_g, then the right hand side of the equation is practically *zero* and the sensitivity is independent of Z_g.

In case Z_g is negative, then if R_g is small the sensitivity will be independent of Z_g. If $R_g = \frac{1}{2}|Z_g|$ the right hand member of equation (23) reduces to unity and it follows that the change in sensitivity is proportional to the change in input impedance. We can express the relation between the percentage change in s and the percentage change in Z_g by

$$\text{percent change in } s = \frac{(\text{percent change in } Z_g)}{1 + \frac{Z_g}{R_g}}. \tag{24}$$

This equation shows that if a high sensitivity is obtained by making $\frac{Z_g}{R_g} = -1.1$ as would be the case if $Z_g = -220$ megohms and $R_g = 200$ megohms, the percentage change in sensitivity will be ten times as great as the percentage change in input impedance Z_g. Therefore, if we wish to keep the sensitivity constant to within 2 per cent., the input impedance for this particular case must be constant to within 0.2 per cent. Enough has now been said about grid current characteristics for the reader to take any particular grid current *vs.* grid potential curve and determine quite accurately the best operating conditions.

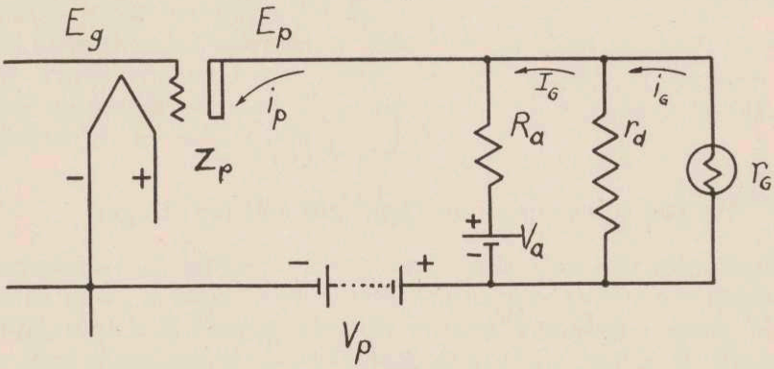
PART 2. SPECIAL OUTPUT CIRCUITS.

Practically all of the above discussion can be considered as the general circuit theory which forms the foundation for all of the amplifiers under consideration. The part which follows will show the application of this general theory to a number of specific circuits. Some of the chief advantages and limitations of each circuit will be pointed out as we progress.

A. ONE TUBE CIRCUIT WITH AUXILIARY BATTERY TO SUPPRESS THE ZERO.

In applications in which a meter or a galvanometer is used in the plate circuit to detect the changes in plate current brought about by the changes in grid potential imposed by the outside circuits, it is usually advantageous to have the meter read "zero" when there is no current in the photoelectric circuit, for instance. This result can be brought

FIG. 5.



about mechanically or electrically. If the meter is connected directly in the plate circuit, the meter will read the normal plate current. It is obvious that by turning the supporting suspension or the restoring spring through a sufficient angle the meter will read "zero." This would be a mechanical method for "suppressing the zero." There are a number of ways of bringing about this suppression of zero electrically. One of the simplest ways is to use a small auxiliary battery and a control resistance across the meter or galvanometer as shown in Fig. 5.

a. Circuit Equations.

In order to obtain equation (12) which showed the plate current as a function of grid potential, in the example worked out above it was necessary to have only two equations, (1) the tube equation and (2) the simple Kirchoff equation of the plate circuit. In the present case the procedure is exactly the same since we start again with the tube equation. In place of a single plate circuit equation we now need more than one because it is now necessary to consider more than one circuit "mesh." The following simple equations are all that are necessary.

$$i_p = \frac{1}{Z_p}(E_p + \mu E_g + E_o), \tag{25}$$

$$E_p = V_p - I_G R_G, \tag{26}$$

$$V_a = i_p R_a - I_G(R_a + R_G), \tag{27}$$

where

$$\frac{1}{R_G} = \frac{1}{r_d} + \frac{1}{r_G}, \tag{28}$$

$$i_G = I_G\left(\frac{r_d}{r_d + r_G}\right). \tag{29}$$

We can solve equations (25), (26) and (27) to get

$$I_G = \frac{\frac{1}{Z_p}(V_p + \mu E_g + E_o) - \frac{V_a}{R_a}}{1 + R_G\left(\frac{1}{Z_p} + \frac{1}{R_a}\right)}. \tag{30}$$

b. Sensitivity to Grid Voltage Changes.

The sensitivity to voltage is therefore

$$\frac{\partial I_G}{\partial E_g} = \frac{G_m}{1 + R_G\left(\frac{1}{Z_p} + \frac{1}{R_a}\right)}. \tag{31}$$

When we substitute equations (28) and (29) in (31) we get

$$\frac{\partial i_G}{\partial E_g} = \frac{G_m}{1 + r_G\left(\frac{1}{Z_p} + \frac{1}{R_a} + \frac{1}{r_d}\right)}. \tag{32}$$

In case we adjust R_a and r_d so that the galvanometer is critically damped and R_{cd} is the external critical damping resistance, then from Fig. 5 we see that

$$\frac{1}{R_{cd}} = \frac{1}{Z_p} + \frac{1}{R_a} + \frac{1}{r_d}, \tag{33}$$

assuming that the battery resistance can be neglected.

We conclude from equations (32) and (33) that the sensitivity when the galvanometer is critically damped depends only on the mutual conductance and the ratio of the galvanometer resistance to the critical damping resistance.

Referring to equation (27), we see that the value of R_a is determined by the initial plate current, call it i_{po}, and the auxiliary battery potential V_a. By the initial plate current we mean the plate current flowing under the circuit conditions for which we want the galvanometer current to be zero. The *minimum* value of V_a, consistent with critical damping, is given by the relationship

$$V_{a\,(\text{min.})} = i_{po}\left(\frac{R_{cd}Z_p}{Z_p - R_{cd}}\right). \tag{34}$$

Equation (34) states in analytical form, that the minimum value of V_a is equal to the "IR" drop over the lowest value of R_a which is consistent with critical damping. After V_a is determined consistent with equation (34), that is, V_a must be greater than $V_{a\,(\text{min.})}$, we have R_a given by equation (35).

$$R_a = \frac{V_a}{i_{po}}. \tag{35}$$

From equation (33) we find

$$\frac{1}{r_d} = \frac{1}{R_{cd}} - \frac{1}{Z_p} - \frac{1}{R_a}. \tag{36}$$

We have thus determined the constants of our circuit to give the maximum sensitivity to grid voltage consistent with critical damping using a tube of mutual conductance G_m, and a galvanometer of resistance r_G and external critical damping resistance R_{cd}. This sensitivity is

$$\frac{\partial i_G}{\partial E_g} = \frac{G_m}{1 + \frac{r_G}{R_{cd}}}. \tag{37}$$

In order to illustrate the meaning of equation (37) let us consider a numerical example in which the galvanometer has a resistance of r_G = 500 ohms and a critical damping resistance R_{cd} = 2000 ohms and a vacuum tube with a mutual conductance G_m = 900 micro-mhos. Then

$$\frac{\partial i_G}{\partial E_g} = \frac{900 \times 10^{-6}}{1 + \frac{500}{2000}} = 0.72 \times 10^{-3} \text{ amp. per grid volt.}$$

A simple calculation shows at once that in case the resistance of the circuit in which the unknown E.M.F. originates is 2000 ohms, the current through the galvanometer when used with the vacuum tube circuit of Fig. 5, will be only 1.44 times as great as the current which would flow through the galvanometer if it were connected directly. If the galvanometer sensitivity is 330 megohms, then the overall sensitivity of the system measured in millimeters deflection, is 238 mm. per millivolt applied to the grid.

B. Wheatstone Bridge Circuit with Single Tube.[8]

a. Circuit Equations.

Another method of arranging the circuit so that no current flows through the galvanometer for the "zero" condition but does flow when this condition is disturbed, comes directly from the simple Wheatstone bridge circuit. In Fig. 6 we have this circuit with resistances in three of the arms and a vacuum tube in the remaining one. It is easy to show that with this circuit the galvanometer balance or even the galvanometer deflection will be *independent* of the "B" battery potential (V_p) over a certain limited range. The solution of the following simple equations gives this result. Using the vacuum tube equation (9) and the Kirchoff equations for the Wheatstone bridge we can write:

---

[8] Other single tube circuits have been discussed by Razek and Mulder, *O. S. A. and R. S. I.*, **18**: 460 (1929).

$$i_p = \frac{1}{Z_p}(E_p + \mu E_g + E_o), \tag{9}$$

$$i_x = \frac{E_x}{R_x}, \tag{38}$$

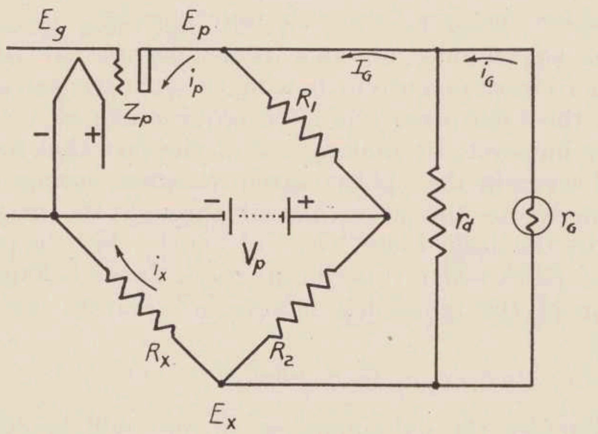
$$E_p = V_p - (i_p - I_G)R_1, \tag{39}$$

$$E_x = V_p - (i_x + I_G)R_2, \tag{40}$$

$$O = R_1(i_p - I_G) - I_G R_g - R_2(i_x + I_G). \tag{41}$$

The meaning of the symbols is given by the circuit schematic Fig. (6). The combined resistance of the galvanometer r_g and the parallel resistance r_d is R_G and is given by equation (29).

FIG. 6.



Solving for the current I_G we get:

$$I_G = \frac{(R_1R_x + R_1R_2)(E_o + \mu E_g) - (R_2Z_p - R_1R_x)V_p}{R_1Z_pR_x + R_2Z_pR_x + R_1R_2Z_p + R_1R_2R_x + R_G(Z_p + R_1)(R_x + R_2)}. \tag{42}$$

b. Plate Battery Compensation Conditions.

We see at once from equation (42) that I_G is independent of V_p if

$$R_2Z_p = R_1R_x. \tag{43}$$

In order that this be true the constants of the tube equation Z_p, μ and E_o must all be independent of E_p over the range corresponding to that over which V_p is assumed to vary. This condition can be met in most cases for changes in V_p of one to three per cent.

c. Voltage Sensitivity.

In order to compute the sensitivity of this amplifier let us take $R_1 = R_2$, then $R_x = Z_p$ and we have, remembering that I_G in this case is the current through the galvanometer and its damping resistance and that R_G is r_G in parallel with r_d,

$$\frac{\partial I_G}{\partial E_g} = \frac{G_m}{2 + R_G\left(\frac{1}{Z_p} + \frac{1}{R_1}\right)}. \tag{44}$$

d. Condition for Zero Current in Galvanometer.

It is not at once obvious from equation (42) that it is possible to have no current flowing through the galvanometer and at the same time fulfill the requirement of (43). It is possible however, by making use of the fact that for a very limited range in the "plate current *vs.* plate voltage" curve,[9] the tangent to the curve passes through the origin as is shown by the dashed line "*OO'*" of Fig. 4. For the particular value of E_g to which this characteristic curve belongs, call it E_g', one of the remaining factors in equation (42) is zero. That is

$$E_o + \mu E_g' = 0 \tag{45}$$

and therefore the galvanometer current will be zero when $E_g = E_g'$ and yet we shall be able to meet the requirement set down by equation (43). When E_g changes from the initial value E_g' by an amount ΔE_g the total current (I_G) in the galvanometer circuit will be given by equation (44).

The sensitivity equation (44) is expressed in terms of the

[9] The bending over of the characteristic shown in Fig. 4 is due to the fact that the filament current has been reduced to bring about a limitation of current with increasing plate potential owing to "saturation." The circuit of Fig. 6 could be modified by placing a battery in series with R_x and thus make it possible to operate on a "straighter" portion of the "plate current vs. plate potential" curve.

current flowing through the galvanometer and the damping resistance in parallel. In order to find the sensitivity in terms of the current flowing through the galvanometer we must use equations (47) and (48) below. The current i_G through the galvanometer alone is of course

$$i_G = I_G\left(\frac{r_d}{r_d + r_G}\right), \tag{46}$$

r_d = damping resistance,

r_G = galvanometer resistance.

This equation solved for I_G is

$$I_G = i_G\left(\frac{r_d + r_G}{r_d}\right). \tag{47}$$

The parallel resistance of the galvanometer and shunt is obviously

$$R_G = \frac{r_d r_G}{r_d + r_G}. \tag{48}$$

When these equations are substituted into equation (44) we get

$$S_6 = \frac{\partial i_G}{\partial E_g} = \frac{G_m}{2\left[1 + r_G\left(\frac{1}{2Z_p} + \frac{1}{2R_1} + \frac{1}{r_d}\right)\right]} \tag{49}$$

as the sensitivity equation for the circuit of Fig. 6 expressed in terms of the galvanometer current i_G. By inspection of Fig. 6 and remembering that $R_x = Z_p$ and $R_1 = R_2$ we see that the galvanometer is critically damped when

$$\frac{1}{2Z_p} + \frac{1}{2R_1} + \frac{1}{r_d} = \frac{1}{R_{cd}} \tag{50}$$

where R_{cd} is the external critical damping resistance. Making this substitution, we get

$$S_6 = \frac{G_m}{2\left(1 + \frac{r_G}{R_{cd}}\right)}. \tag{51}$$

e. Sensitivity of Circuits 5 and 6 Compared.

If we compare the sensitivity of the circuit shown in Fig. 6 and given by equation (51) with that for the circuit of Fig. 5 given by equation (37), we get

$$\frac{S_6}{S_5} = \frac{\dfrac{G_m}{2\left(1 + \dfrac{r_G}{R_{cd}}\right)}}{\dfrac{G_m}{1 + \dfrac{r_G}{R_{cd}}}} = \frac{1}{2}. \tag{52}$$

In other words we have sacrificed exactly half of our sensitivity in order to introduce the bridge circuit which, however, is superior to the simple circuit in that the zero or the deflection is independent of the "B" battery over a certain limited range of variation while in the simple circuit of Fig. 5 a change in either the "B" battery potential (V_p) or the auxiliary battery (V_a) will produce a change in the galvanometer reading.

C. WHEATSTONE BRIDGE CIRCUIT USING TWO VACUUM TUBES.

a. Conditions for "B" Battery Compensation.

The circuit shown in Fig. 6 can be modified as shown in Fig. 7 by the introduction of a vacuum tube in place of the resistance arm R_x. It will be shown below that this change in the circuit leaves the form of the equation for the sensitivity unchanged and modifies the condition for "B" battery compensation only to the extent that Z_{px} replaces R_x in equation (43). This gives

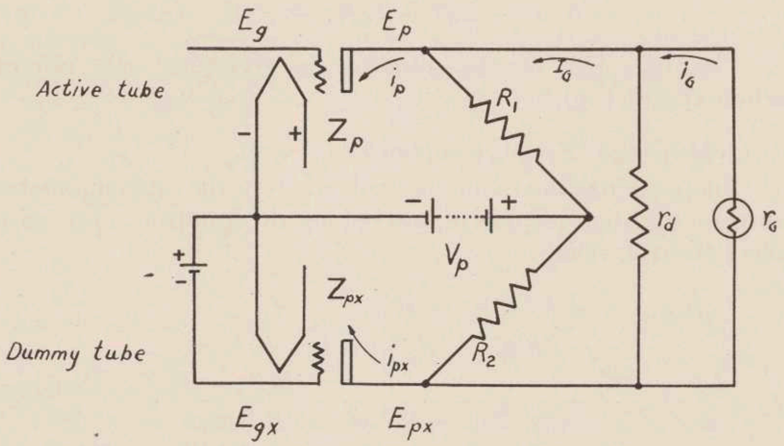
$$R_2Z_p = R_1Z_{px} \tag{53}$$

or

$$\frac{Z_p}{Z_{px}} = \frac{R_1}{R_2}. \tag{54}$$

Here Z_{px} is the plate impedance of the tube which has been introduced in the bridge in place of R_x. In the discussion above it was assumed that $R_1 = R_2$ and R_x was adjusted to have the value $R_x = Z_p$. That order of procedure cannot be used in this case because the value of the plate impedance Z_{px} is not so easily controlled as the value of R_x was before. It, therefore, follows that we must meet the condition required by equation (54) by adjusting $\frac{R_1}{R_2}$. Fortunately it is not necessary to know Z_p and Z_{px} exactly. All that is necessary

FIG. 7.



is that we have R_1 or R_2 adjustable and have a means of altering the "B" battery potential (V_p) through the narrow range over which we are required to get the best possible compensation.[10]

b. Circuit Equations.

In order to develop the equations which give support to the statements just made as regards independence of "B" battery (V_p), sensitivity, etc., we must start out with five equations just as we did in the one tube bridge circuit discussed above. These equations were (9), (38), (39), (40)

[10] This method must be used with a certain amount of care because it is possible to get a compensation at the end points and not have compensation for intermediate values of (V_p) owing to the curvature of the characteristic.

and (41) and the corresponding set here are the following:

$$i_p = \frac{1}{Z_p}(E_p + \mu E_g + E_o), \tag{55}$$

$$i_{px} = \frac{1}{Z_{px}}(E_{px} + \mu_x E_{gx} + E_{ox}), \tag{56}$$

$$E_p = V_p - (i_p - I_G)R_1, \tag{57}$$

$$E_{px} = V_p - (i_{px} + I_G)R_2, \tag{58}$$

$$O = R_1(i_p - I_G) - I_G R_G - R_2(i_{px} + I_G). \tag{59}$$

The meaning of the symbols is given by the circuit schematic of Fig. 7.

c. Condition for Zero Galvanometer Current.

These equations can be solved for the galvanometer current I_G just as was done before in equation (42) with the following result

$$I_G = \frac{(R_1 Z_{px} + R_1 R_2)(\mu E_g + E_o) - (R_2 Z_p + R_1 R_2)(\mu_x E_{gx} + E_{ox}) - (R_2 Z_p - R_1 Z_{px})V_p}{R_1 Z_p Z_{px} + R_2 Z_p Z_{px} + R_1 R_2 Z_p + R_1 R_2 Z_{px} + R_G(Z_p + R_1)(Z_{px} + R_2)} \tag{60}$$

and the sensitivity is

$$\left(\frac{\partial I_G}{\partial E_g}\right)_7 = \frac{G_m}{2 + R_G\left(\frac{1}{Z_p} + \frac{1}{R_1}\right)}, \tag{61}$$

when $R_1 = R_2$ and $Z_p = Z_{px}$. This is the same as was found above in equation (44). When equations (47) and (48) are used so as to express the sensitivity in terms of the galvanometer current i_G, this equation obviously reduces to the same as (49) which is

$$S_7 = \left(\frac{\partial i_G}{\partial E_g}\right)_7 = \frac{G_m}{2\left[1 + r_G\left(\frac{1}{2Z_p} + \frac{1}{2R_1} + \frac{1}{r_d}\right)\right]}. \tag{62}$$

Again if the galvanometer is critically damped and R_{cd} is the external critical damping resistance we have

$$S_7 = \frac{G_m}{2\left(1 + \frac{r_G}{R_{cd}}\right)}. \tag{63}$$

From equation (60) we see that the condition for operation independent of V_p is that given above in equation (53) or (54). Assuming this condition to be satisfied, we have the further condition which we wish to impose, that there be no current flowing through the galvanometer for some definite condition in the grid circuit such as, for example, that for which there is no photoelectric current. For the arbitrary "initial" condition, the grid potential of the "active" tube will have some definite value E_g' which is determined by the circuit constants such as the grid resistance R_g, the "C" battery potential V_g, etc. (see Fig. 1). The galvanometer current can be brought to zero by properly adjusting the potential of the grid E_{gx} of the "dummy" tube. Later we shall use the phrase "balance the circuit" to indicate this process of adjusting some element of the circuit such as the grid potential E_{gx} until the current through the galvanometer is zero.

When we assume the condition, given by equation (53), to be satisfied, we see from equation (60) that the galvanometer current will be zero when the sum of the remaining two terms of the numerator of that equation is zero. This we have expressed in the equation.

$$(R_1Z_{px} + R_1R_2)(\mu E_g' + E_o) - (R_2Z_p + R_1R_2)(\mu_x E_{gx} + E_{ox}) = 0. \tag{64}$$

Referring to equation (53), which we are assuming to be satisfied, we see by adding R_1R_2 to each side of the equation that

$$R_1Z_{px} + R_1R_2 = R_2Z_p + R_1R_2. \tag{65}$$

Therefore, if equations

$$\mu E_g' + E_o = \mu_x E_{gx} + E_{ox} \tag{66}$$

and

$$R_2Z_p = R_1Z_{px} \tag{53}$$

are satisfied, the galvanometer current is zero and the system is independent of "B" battery changes over the limited range for which the "constants" of the system are independent of "B" battery.

d. "Cut and Try Method" of Adjustment.

The fact that the current through the galvanometer is zero when equations (53) and (66) are satisfied does not mean that we need to know the constants of the tubes actually used in this two tube bridge circuit any more than it did above in the one tube bridge circuit. In the single tube circuit we saw that in order to accomplish a balance of the circuit, it was necessary to operate on the part of the "plate current *vs.* plate potential" curve of Fig. 4 at O'. At this part of the characteristic the plate impedance Z_p changes rather rapidly with a change in E_p and therefore limits the range over which satisfactory "B" battery compensation can be obtained. When we use the two tube circuit we can operate over the "straight" portion of the characteristic where the change in Z_p and E_p is very small and still meet the requirements of equations (53) and (66) simultaneously. This is accomplished by adjusting the resistance ratio $\frac{R_1}{R_2}$ and the value of the grid potential on the "dummy" tube until both conditions are met. In practice this adjustment is of the "cut and try" type. That is, we arbitrarily set $R_1 = R_2$ and adjust E_{gx} to balance the circuit giving no current through the galvanometer. Then we change the "B" battery potential (V_p) by 1.5 volts or more and observe the amount and direction of the galvanometer deflection.[11] We then change R_2 by 1000 ohms,[12] for instance, rebalance the circuit by altering E_{gx} and again test for stability. If the galvanometer swings less far but in the same direction we see that we have altered the resistance ratio $\frac{R_1}{R_2}$ in the right direction but not enough. By continuing this "cut and try" method it is nearly always possible to obtain

[11] See footnote 10.

[12] If we wish to keep the galvanometer damping more nearly the same for all settings, we should alter R_1 also, keeping $(R_1 + R_2)$ constant.

perfect stabilization for moderate changes in "B" battery potential. The fact that, with the two tube circuit, we can operate on the "*straight*" part of the curve makes it possible to realize perfect stability over a wider range of "B" battery variation than can be realized in the simple one tube bridge circuit.

e. Advantages of Two Tube Circuit.

There are other advantages to the two tube circuit which deserve to be mentioned. Two of these are (1) compensation for filament voltage changes and (2) approximate compensation for tube aging. In general, the plate current through a tube gradually decreases with time when the plate and grid potentials and the filament current are held constant. It is obvious that that part of the drift in the galvanometer current which is due to this aging, will be less in the two tube circuit since we can assume that the rate of aging in two similar tubes is approximately the same.

f. Conditions for Filament Battery Compensation.[13]

If we had included filament voltage E_f as one of the variables of the tube equation we would have had the following equations instead of (55) and (56) as above.

$$i_p = \frac{1}{Z_p}(E_p + \mu E_g + E_o') + \alpha E_f, \tag{67}$$

$$i_{px} = \frac{1}{Z_{px}}(E_{px} + \mu_x E_{gx} + E_{ox}') + \alpha_x E_f. \tag{68}$$

The two new constants are defined by

$$\alpha \equiv \frac{\partial i_p}{\partial E_f}, \tag{69}$$

$$\alpha_x \equiv \frac{\partial i_{px}}{\partial E_f}. \tag{70}$$

[13] For an extended discussion of compensation methods including compensation for grid battery changes see:
C. E. Wynn-Williams, *Phil. Mag.*, **6**: 324 (1928).
J. Brentano, *Phil. Mag.*, **7**: 685 (1929).
J. M. Eglin, *J. O. S. A. and R. S. I.*, **18**: 393 (1929).
J. Razek and P. J. Mulder, *J. O. S. A. and R. S. I.*, **19**: 390 (1929).

Here again we must rely on experiment for the justification of the statement that α and α_x can be treated as constants which are independent of E_p, E_g and E_F over narrow ranges of the variables.

If we carried through the calculations to get the equation equivalent to (60), we would find the following term in the numerator along with those already discussed

$$E_f\{(R_1Z_{px} + R_1R_2)Z_p\alpha - (R_2Z_p + R_1R_2)Z_{px}\alpha_x\}. \quad (71)$$

If this term is zero then the system will be independent of filament voltage and all of the relations developed above will still hold perfectly well. Take the first, namely that required in order to have independence from "B" battery changes which was $R_1Z_{px} = R_2Z_p$ and it follows that

$$Z_p\alpha = Z_{px}\alpha_x \quad (72)$$

if the system is to be independent of filament voltage. In terms of the resistances we have

$$\frac{\alpha_x}{\alpha} = \frac{R_1}{R_2}. \quad (73)$$

By operating the two filaments in parallel but with a small variable resistance in series with one of them, the α of one of the tubes can be adjusted to meet condition (73). There are other ways of building the circuit so as to control one of the α's but the condition (73) must always be met. Again it is no more necessary to know the new constants of the tube equation than it was before. The balance is accomplished by the "cut and try" method just as before but this time we must give small arbitrary changes to the filament battery to test the compensation. In the laboratory we have found that it is possible to make the compensation so perfect that little or no deflection of the galvanometer could be measured with a filament change of as much as one or two per cent. while without the compensation a change of one-tenth of a percent caused a change in the galvanometer reading.

D. SUMMARY.

The part of this paper just completed deals with three important circuits used in single stage D.C. amplifiers. The entire attention has been devoted to the conditions in the plate circuit or the filament circuit and little or nothing has been said about the grid circuit. Many of the general principles which determine the grid circuit connections have been recorded in the first part of this paper. The detailed application of these principles will be taken up in the following section. The one tube circuit with an auxiliary battery to give a "supressed zero" was shown to be the most sensitive one stage amplifier of the three. The bridge circuits which followed were shown to have a sensitivity one-half as great as the above. With the single tube bridge circuit, it was shown that under certain definite conditions small variations in "B" battery could be compensated for. The range for which this compensation is possible was shown to be very limited. The bridge circuit with two tubes was shown to have several advantages over the single tube bridge circuit with no additional loss in sensitivity. These advantages were (1) practically perfect "B" battery compensation over a rather wide range, (2) filament battery compensation over a fairly wide range, and (3) approximate compensation for aging of the tubes.

E. CONTROL OF GALVANOMETER DEFLECTION.

a. Special Universal Shunt.

Whenever a sensitive galvanometer is used in any of the three circuits described, it is usually necessary to have some means for controlling the galvanometer response in a way which keeps the damping correct. In ordinary high resistance circuits an Ayrton shunt is suitable for this purpose. Under certain conditions an Ayrton shunt can be used here also, but as will be shown later a certain amount of sensitivity must be sacrificed in order to do this. It is possible, however, to control the current through the galvanometer in either the bridge circuit or that with the auxiliary battery and keep the damping correct without this loss in sensitivity. The following computation shows how to determine the constants of the circuit. Consider a circuit like that shown in Fig. 7 with the galvanometer replaced by the network shown in Fig. 8. When the contact is at 1, resistance "*a*" is zero and resistance "b" infinite. The sensitivity is

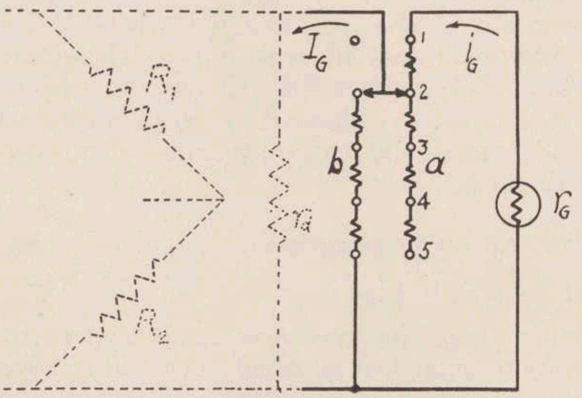
$$S_{\max} = \frac{\partial i_G}{\partial E_g} = \frac{G_m}{2\left[1 + r_G\left(\frac{1}{2Z_p} + \frac{1}{2R_1} + \frac{1}{r_d}\right)\right]}. \tag{74}$$

This result comes from equation (62) and gives the maximum sensitivity. The damping is correct when

$$\frac{1}{R_{cd}} = \frac{1}{2Z_p} + \frac{1}{2R_1} + \frac{1}{r_d}, \tag{75}$$

where R_{cd} is the external resistance for critical damping.

FIG. 8.



Circuit with special universal galvanometer shunt.

As we move the contact to successive positions, 2, 3, etc., resistances a and b have two functions to perform. They must be so proportioned that the damping will remain unchanged and the sensitivity reduced in definite amounts. If a and b are always adjusted so as to satisfy equation (76) the damping will remain constant.

$$R_{cd} = a + \frac{bR_{cd}}{b + R_{cd}}. \tag{76}$$

If S_n is the sensitivity for some combination of a and b

consistent with (76), we can define the "sensitivity ratio" n as

$$n = \frac{S_{\max}}{S_n}. \tag{77}$$

If we let I_G be the current through the galvanometer and the shunt b together, then the current i_g through the galvanometer will be

$$i_G = I_G \frac{b}{a + b + r_G}. \tag{78}$$

Since the resistance of the galvanometer along with its series and shunt resistances is

$$R_G = \frac{b(a + r_G)}{a + b + r_G}, \tag{79}$$

the sensitivity expressed in terms of I_G is the following:

$$\frac{\partial I_G}{\partial E_g} = \frac{G_m}{2\left[1 + \frac{b(a + r_G)}{a + b + r_G}\left(\frac{1}{2Z_p} + \frac{1}{2R_1} + \frac{1}{r_d}\right)\right]}. \tag{80}$$

Using equations (75) and (78) we get

$$S_n = \frac{\partial i_G}{\partial E_g} = \frac{bG_m}{2[a + b + r_G]\left[1 + \frac{b(a + r_G)}{(a + b + r_G)R_{cd}}\right]}. \tag{81}$$

From equations (74), (75), (77) and (81) we get

$$\frac{S_{\max}}{S_n} = n = \frac{R_{cd}(a + b + r_G) + b(a + r_G)}{b(R_{cd} + r_G)}. \tag{82}$$

With equations (76) and (82) it is possible to solve for a and b in terms of the damping resistance R_{cd} and the sensitivity ratio n. In this way we get

$$a = R_{cd}\left(\frac{n - 1}{n}\right), \tag{83}$$

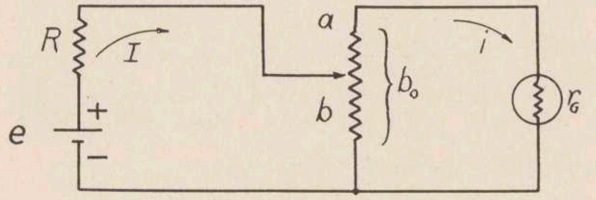
$$b = R_{cd}\left(\frac{1}{n - 1}\right). \tag{84}$$

These equations are very similar to those for the Ayrton shunt in that the values of a and b are independent of the galvanometer resistance. This is, of course, a very real advantage for it makes this shunt a "universal" shunt in exactly the same sense that the Ayrton shunt is a universal shunt. Since the sensitivity equations (32) and (49) for the one tube circuits of Figs. 5 and 6, respectively, are of the same form as equation (74), the constants of the universal shunt for these circuits are also given by equations (83) and (84).

b. Ayrton Shunt and Its Limitations.[14]

The Ayrton shunt can be used in these circuits with a small sacrifice in the maximum sensitivity under certain conditions. These conditions will now be examined somewhat in detail. It is well known that in ordinary circuits

FIG. 9.



Ayrton shunt circuit.

the Ayrton shunt does not reduce the sensitivity in the correct ratio unless the resistance of the associated circuit is considerably higher than the total resistance of the shunt. It will be recalled that the Ayrton shunt is an "L" net work similar to that shown by resistances "a" and "b" of Fig. 9.

The resistances a and b are related to the nominal sensitivity ratio N and to each other according to the following equations:

$$a = b_o\left(\frac{N - 1}{N}\right), \tag{85}$$

$$b = \frac{b_o}{N}, \tag{86}$$

$$a + b = b_o. \tag{87}$$

[14] Razek and Mulder, *J. O. S. A. and R. S. I.*, 18: 390 (1929), suggest using an Ayrton shunt but do not discuss its limitations.

When $N = 1$, we have $a = 0$; and $b = b_o$ which is the total resistance by which the shunt is specified in commercial catalogs. This resistance is usually determined by the external critical damping resistance of the galvanometer. From the simple equations of the circuit shown in Fig. 9 the true sensitivity ratio n corresponding to the nominal ratio N can be worked out. The galvanometer current for any setting is

$$i_G = \frac{eb}{(a + b + r_G)R + b(a + r_G)}, \tag{88}$$

and when $a = 0$ and $b = b_o$ we have

$$i_{G1} = \frac{eb_o}{(b_o + r_G)R + b_o r_G}. \tag{89}$$

Taking the ratio of i_{G1} to i_G as the definition of the true sensitivity ratio n and using equation (86) we get

$$\frac{i_{G1}}{i_G} \equiv n = N \frac{(b_o + r_G)R + b(a + r_G)}{(b_o + r_G)R + b_o r_G}. \tag{90}$$

We can now determine the accuracy with which the true sensitivity ratio is given by the nominal one by the relation

$$\frac{n - N}{n} = y, \tag{91}$$

where y when multiplied by one hundred gives the error in percent.

From equations (85), (86) and (90) we can eliminate a and b and solve for y in terms of the constants of the circuit.

$$y = \frac{Nb_o(b_o + r_G) - N^2 b_o r_G - b_o^2}{N^2 R(b_o + r_G) + Nb_o(b_o + r_G) - b_o^2}. \tag{92}$$

It is easy to show that y is zero for the values

$$N = 1 \quad \text{and} \quad N = \frac{b_o}{r_G}.$$

If $b_o > r_G$, as is usually the case, y has a positive maximum at a value of N between the two for which y is zero, namely, at

$$N = \frac{2}{1 + \frac{r_G}{b_o}}.$$

We see at once that this value of N for which the error is a positive maximum must lie between 1.0 and 2.0. Since the least value of N in most commercially produced Ayrton shunts is 10.0, we find the greatest error when N is large. For large values of N equation (92) reduces to

$$y = -\frac{b_o r_G}{(b_o + r_G)R}. \qquad (93)$$

This equation can also be written

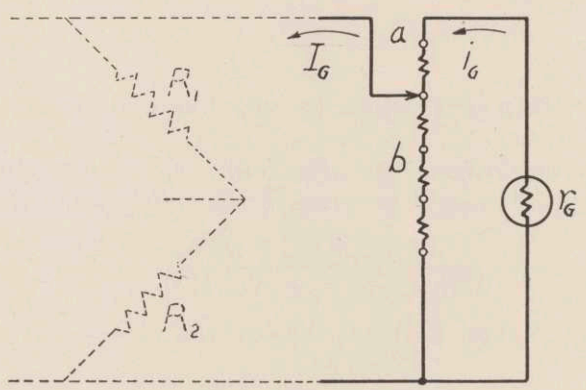
$$y = -\frac{1}{\frac{R}{b_o}\left(1 + \frac{b_o}{r_G}\right)}. \qquad (94)$$

Let us take a numerical example in which

$$\frac{R}{b_o} = 2 \quad \text{and} \quad \frac{b_o}{r_G} = 4.$$

Equation (94) shows that the maximum error y will be 10 per cent. when N is large.

FIG. 10.



Circuit with Ayrton shunt for galvanometer control.

Let us consider a case similar to that discussed under the heading "Special Universal Shunt," with the difference that this time we use the Ayrton shunt as shown in Fig. 10. The damping resistance r_d is no longer needed and may therefore be dropped out. Equation (61) gives the sensitivity of this circuit where I_G is the total current through the shunt b and the galvanometer together and R_G is the resistance of $(a + r_G)$ in parallel with b. We can express the sensitivity in terms of the galvanometer current i_G by using an equation similar to (78) to eliminate I_G. Using these equations and also (87) we get the following:

$$S_n = \frac{bG_m}{2(b_o + r_G) + (a + r_G)b\left(\frac{1}{Z_p} + \frac{1}{R_1}\right)}. \tag{95}$$

When $b = b_o$ and $a = 0$, we have the maximum sensitivity S_{max} which is

$$S_{\text{max}} = \frac{b_oG_m}{2(b_o + r_G) + r_Gb_o\left(\frac{1}{Z_p} + \frac{1}{R_1}\right)}. \tag{96}$$

If we define the true sensitivity ratio as before

$$\frac{S_{\text{max}}}{S_n} \equiv n \tag{97}$$

and relate it to the nominal ratio N as before by equation (91), we get the following equation for large values of N:

$$y = \frac{1}{\frac{2Z_pR_1}{(Z_p + R_1)b_o}\left(1 + \frac{b_o}{r_G}\right)}. \tag{98}$$

If we set

$$\frac{1}{X} = \frac{1}{2Z_p} + \frac{1}{2R_1}, \tag{99}$$

$$y = -\frac{1}{\frac{X}{b_o}\left(1 + \frac{b_o}{r_G}\right)}. \tag{100}$$

This is exactly of the same form as (94).

The accuracy with which the Ayrton shunt reduces the sensitivity may be of little importance to the user of an amplifier since he can determine the proper correction at any time. However, he may not be satisfied to have the time required to take readings widely different for the various sensitivities. From an inspection of the circuit in Fig. 10, we see that the damping will be greatest when $N = 1$ and the damping will be least when N is large. This effect may be minimized by making X which is defined by equation (99) as large as is conveniently possible. If we have a value of X at least twice the external damping resistance R_{cd} of the galvanometer, and an Ayrton shunt with a total resistance approximately equal to R_{cd}, then the Ayrton shunt can be used to regulate the sensitivity and the disadvantages which have been pointed out will not be found serious.

If we assume

$$X_{\min} = 2R_{cd} \tag{101}$$

and

$$b_o = R_{cd}, \tag{102}$$

equation (100) reduces to

$$y = -\frac{1}{2 + \dfrac{2R_{cd}}{r_G}}. \tag{103}$$

From this we see that if $\dfrac{R_{cd}}{r_G} = 4$, the maximum error in the Ayrton shunt sensitivity ratio, will be 10 per cent. Equations (99) and (101) serve to determine the minimum satisfactory value of the bridge resistance R_1 which is given by

$$R_{1\ \min} = \frac{R_{cd}Z_p}{Z_p - R_{cd}}. \tag{104}$$

Mention was made above that the use of the Ayrton shunt for a sensitivity control instead of the "special" universal shunt given by equations (83) and (84), usually entailed a small loss in the maximum sensitivity. It is easy to see from equation (62), that if $2Z_p$ and $2R_1$ are large compared with R_{cd} no appreciable loss in sensitivity will result from the use of the Ayrton shunt. On the other hand, if the minimum value of R_1 as defined by equation (104) is used, then the loss in sensitivity in percent will not be greater than $50\dfrac{r_G}{R_{cd}}$ per cent. It is so easy to construct a shunt of the kind shown in Fig. 8 that it is advisable to use this network instead of the Ayrton shunt whenever a "permanent" amplifier set-up is being built.

c. Summary on Galvanometer Control.

We have seen that there are two ways of controlling the overall sensitivity of the amplifier to grid voltage. The first required a special type of shunt in which the resistance values depend only on the external critical damping resistance and the desired sensitivity ratio. This shunt when properly arranged gives the computed sensitivity ratios and maintains the damping constant for all settings. The second method suggested depends on the use of a standard Ayrton shunt. Three minor disadvantages are accompanied by the use of this shunt which are (1) the error in the sensitivity ratio, (2) the non-uniformity of the damping, and (3) the loss in sensitivity.

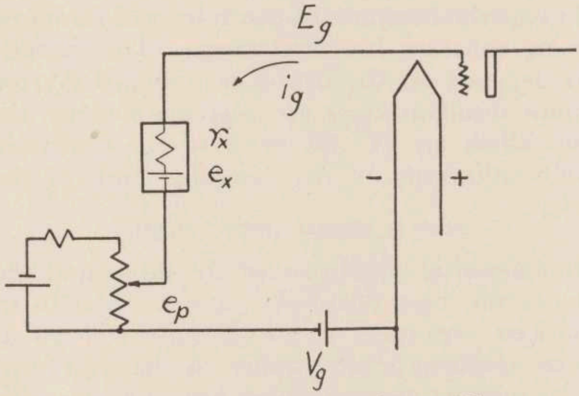
PART 3. SPECIAL INPUT CIRCUITS.

The fundamental equations for the input and the output circuits have now been developed, and in order to show how these relations are used, a few circuits will be analyzed. The circuits which we shall examine can be used for potential and current measurements in high resistance circuits and an attempt will be made to show under what conditions an amplifier can be used to advantage.

A. POTENTIAL MEASUREMENTS.

There are certain problems in which it is necessary to measure a potential originating in a very high resistance circuit. The determination of the acidity of a liquid in terms of a pH measurement with a glass electrode necessitates the measurement of a potential to within about one millivolt while the resistance of the cell may be of the order of ten to a thousand megohms. Although a quadrant electrometer with a sensitivity of 1000 mm. per volt could be used to make this measurement, a galvanometer would need to have a sensitivity of more than 10,000 megs. in order to be used satisfactorily. Where it is necessary to use an instrument which is more rugged than a quadrant electrometer, a galvanometer and an amplifier can be used to considerable advantage. The problem of designing an amplifier for this purpose divides itself in two parts. The first has to do with the input circuit of the tube and is independent of the type of output circuit used. That is, either circuit 5, 6, or 7 or any modification of these can be used without changing the underlying requirements of the input circuit. The second part has to do with the output circuit and the question of *over-all sensitivity* of the system.

Fig. 11.



Simplified input circuit for measuring potentials.

Starting with the input circuit shown in Fig. 11, we see that the following Kirchoff equation must hold.

$$e_x + e_p + V_g = E_g + r_x i_g. \tag{105}$$

Here the resistance of the potentiometer is assumed to be very small compared with r_x. In order to be able to read the unknown potential on the potentiometer directly, the potential e_p should be equal and opposite to that of the unknown e_x.

$$e_x = - e_p. \tag{106}$$

Equation (105) will then be satisfied if

$$r_x i_g = 0, \tag{107}$$

for then

$$V_g = E_g. \tag{108}$$

Of course, equation (107) does not have to be satisfied in the absolute sense. What is really necessary is that $r_x i_g$ must be small compared with the smallest value of

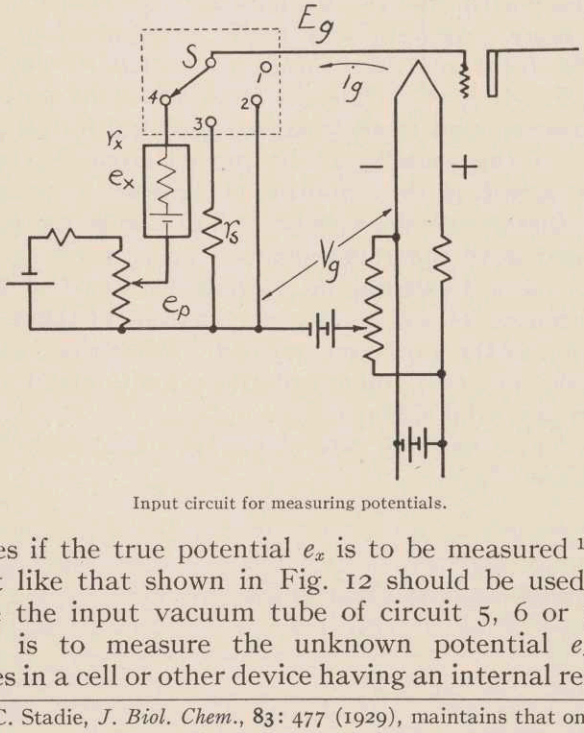
$$e_x - |e_p| = \Delta e \qquad (109)$$

which we wish to measure, for example one millivolt in the case under consideration. Obviously if r_x is very large, i_g must be very small and *we must operate* very close to the floating potential where the grid current i_g can be reduced to a value as small as we need.

a. Case I. Where It is Necessary to Operate at Floating Potential because r_x is Very Large.

If r_x is so large that it is necessary to operate very close to the floating potential (this is the case with most glass

FIG. 12.



Input circuit for measuring potentials.

electrodes if the true potential e_x is to be measured [15]), then a circuit like that shown in Fig. 12 should be used. Here we have the input vacuum tube of circuit 5, 6 or 7. The problem is to measure the unknown potential e_x which originates in a cell or other device having an internal resistance

[15] W. C. Stadie, *J. Biol. Chem.*, 83: 477 (1929), maintains that only relative values of e_x are necessary for pH measurements with glass electrodes and therefore under certain conditions he does not believe that it is necessary to make $r_x i_g < \Delta e$.

r_x which is very large. The procedure for operating this circuit can be outlined as follows:

1. With switch S in position (1), that is with the grid free, the circuit should be "balanced" to produce no current through the galvanometer in the plate circuit. For example if we use the circuit shown in Fig. 5, the resistance R_a should be adjusted to give no current through the galvanometer, while if we use circuit 7, the grid potential of the dummy tube should be adjusted to bring about the same result.

2. The second step in the process is to move switch S to position (2) and adjust V_g until the galvanometer again reads zero. Now we should be able to move the switch contact from (1) to (2) or back again with no detectable motion of the galvanometer coil in the plate circuit. We have now established V_g = floating potential. This test of the accuracy with which V_g is adjusted to the floating potential may be more severe than is really necessary and for that reason the resistance r_s is shown connected to one point of the switch. If r_s is at least as large as r_x, the potential V_g will be near enough to the floating potential for all practical purposes, if the switch can be moved from (2) to (3) or (3) to (2) at will, with no motion of the galvanometer coil.

3. With the switch in position (4), the potential e_p can be adjusted until the galvanometer coil is again at the zero position. Now it should be possible to move switch S to any position of (2), (3), or (4) from any one of them, without any motion of the galvanometer coil. After these tests have been made and only then will the potentiometer read the unknown potential e_x correctly.

In order to find the rate of change of E_g with e_p, let us differentiate (105).

$$\frac{\partial E_g}{\partial e_p} + r_x \frac{\partial i_g}{\partial e_p} = 1. \tag{110}$$

Using equation (7) and rearranging we find

$$\frac{\partial E_g}{\partial e_p} = \frac{1}{1 + \dfrac{r_x}{Z_g}}. \tag{111}$$

We see at once that Z_g should be large compared with r_x if we are to obtain good sensitivity since Z_g is always a positive number at the floating potential. We can combine equation (111) with the other sensitivity equations such as (37), (51) or (63) and we get, using (37),

$$\frac{\partial i_G}{\partial E_g} \times \frac{\partial E_g}{\partial e_p} = \frac{G_m}{1 + \frac{r_G}{R_{cd}}} \times \frac{1}{1 + \frac{r_x}{Z_g}}. \qquad (112)$$

Defining the galvanometer response to current [16] as

$$C_s = \frac{\partial \theta}{\partial i_G} \qquad (113)$$

(where θ is the angular deflection of the galvanometer coil), we can compute the over-all sensitivity as

$$\frac{\partial \theta}{\partial e_p} = \frac{G_m}{1 + \frac{r_G}{R_{cd}}} \times \frac{1}{1 + \frac{r_x}{Z_g}} C_s. \qquad (114)$$

If θ is measured in millimeters deflection at some standard distance, the over-all sensitivity is given directly by equation (114) in millimeters per volt. This expression shows that the over-all sensitivity of the circuit is a very simple function of the tube constants, the galvanometer constants and the resistance of the device in which the potential originates. Equation (114) can be rearranged so as to collect the various factors as follows:

$$\frac{\partial \theta}{\partial e_p} = \frac{G_m}{1 + \frac{r_x}{Z_g}} \times \frac{C_s}{1 + \frac{r_G}{R_{cd}}}. \qquad (115)$$

The second factor is a function of the galvanometer only, while the first factor is determined by the vacuum tube constants and cell resistance. It is obvious that if Z_g is large compared with r_x the sensitivity will be proportional to

[16] Current sensitivity (A) is usually given in microamperes per millimeter deflection. $C_s = \frac{10^6}{A}$. Galvanometer sensitivity is sometimes expressed in "megs." $C_s =$ (megohm sensitivity) 10^6.

the mutual conductance G_m and therefore where the cell resistance is quite low the vacuum tube with the highest G_m will have the highest sensitivity. The R.C.A. power tube UX-112-A is a very popular tube having a high mutual conductance. If, however, r_x is ten megohms or more the input impedance as well as the mutual conductance must be considered. Take, for example, $r_x = 50 \times 10^6$ ohms and let us compare the sensitivities which we must expect from a UX-112-A vacuum tube and a UX-222 screen grid tube. The mutual conductances of these two tubes are listed as 1600×10^{-6} mho and 400×10^{-6} mho respectively. The grid impedances, at the floating potential, of tubes of these types have been found to be 12×10^6 ohms for the UX-112-A and 3×10^9 ohms for the UX-222. Considering the first factor of equation (115) which is the part depending on the characteristics of the vacuum tube, we see that the screen grid tube will be 1.3 times as sensitive as the UX-112 and at the same time the UX-222 will be much less sensitive to fluctuations in the power supply.

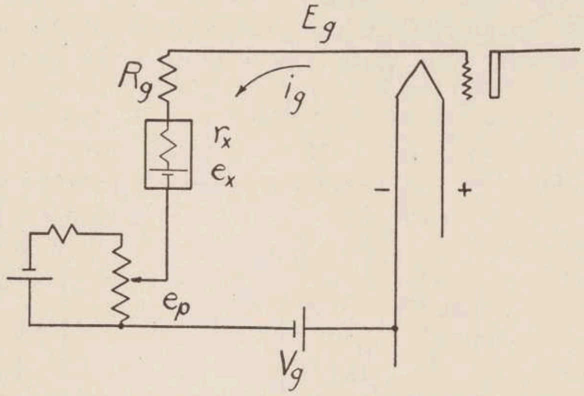
b. Case II. Where It is Possible to Operate with a Negative Grid Impedance because r_x is Small.

When the resistance of the device in which the potential originates is less than one megohm it is not always necessary to operate at the floating potential. The curve shown in Fig. 3, which gives the relation between "C" battery potential and plate current when a high resistance is used in the grid circuit, suggests that a higher sensitivity to voltage can be obtained in an amplifier if we take advantage of the fact that the grid impedance Z_g is *negative* for certain values of grid potential E_g. The use of this method of measurement presupposes that a current of the order of 10^{-8} ampere can flow through the device being measured without altering its properties. If such is not the case we must either select a vacuum tube which has a grid current small enough to cause no change in the device, or else operate near the floating potential and thus keep the current very small. Refer to Fig. 13. This is the simplified form of the circuit which we have in Fig. 14 when switch S is in position 3. The Kirchoff equation corresponding to (105) above is the following:

$$e_x + e_p + V_g = E_g + i_g(r_x + R_g). \qquad (116)$$

Here again we assume that the resistance of the potentiometer is small compared with r_x. By differentiating (116) and

FIG. 13.



Simplified input circuit for potential measurements when r_x is low.

introducing Z_g which is defined by equation (7) we are able to determine the rate of change of E_g with respect to e_p.

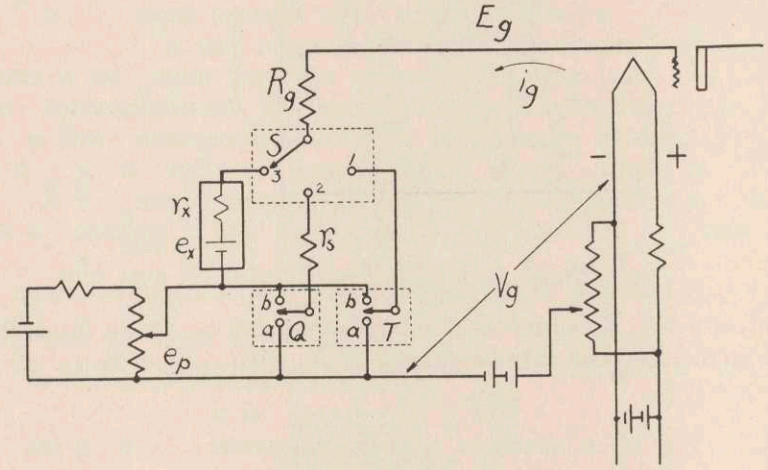
$$\frac{\partial E_g}{\partial e_p} = \frac{1}{1 + \frac{r_x + R_g}{Z_g}}. \tag{117}$$

With this equation before us we can turn to the complete circuit of Fig. 14 and examine its operation.

The resistance r_s should be equal to the resistance of the device r_x. The resistance R_g depends on the particular tube used and the grid potential around which we intend to operate. In order to determine the best value of "C" battery potential V_g we set switch S at 2 and switch Q at (*a*) and the "C" battery potential at some definite value (V_g). The plate circuit should be balanced to produce no current through the galvanometer. With the potentiometer potential e_p adjusted to some small value, throw switch Q to position *b* and note the galvanometer deflection. This deflection is a measure of the amplifier sensitivity for this particular value of V_g. Now take a new value of V_g and repeat the test, including the balancing of the plate circuit, until the value of V_g is found which gives the maximum sensitivity. This method of finding the best value of the "C" battery potential for some particular case is obviously a "cut and try" method.

After the best "C" battery potential V_g has been determined, the magnitude of the grid current can be measured in the following way: With S in position 2 and Q in position

FIG. 14.



Input circuit for potential measurements when r_x is low.

a the plate circuit should be balanced to give no current in the galvanometer. With switch T in position b and S in position 1, the potentiometer potential e_p should be adjusted to bring the galvanometer current to zero. Let us designate this potential by e_{po}. The potential e_{po} is now a measure of the "IR" drop brought about by the flow of grid current through resistance r_s. Since r_s is known and e_{po} is determined by the experiment, the grid current can be computed from the Ohm's law relation,

$$i_g = \frac{e_{po}}{r_s}. \tag{118}$$

Assuming that no alteration in the potential e_x results from the flow of the current i_g, we are in a position to measure e_x.

After the circuit has been balanced with S in position 2 and Q in position a, we switch S to position 3 and adjust e_p to restore the balance. This value of e_p will be equal in magnitude and opposite in sign to the unknown potential e_x.

In order to compute the over-all sensitivity for this circuit we can use equations (37), (113) and (117) to get

$$\frac{\partial\theta}{\partial i_G} \times \frac{\partial i_G}{\partial E_g} \times \frac{\partial E_g}{\partial e_p} = \frac{1}{1 + \frac{r_x + R_g}{Z_g}} \times \frac{G_m}{1 + \frac{r_G}{R_{cd}}} \times C_s. \quad (119)$$

Collecting the factors which depend on the galvanometer, we have the following:

$$\frac{\partial\theta}{\partial e_p} = \frac{G_m}{1 + \frac{r_x + R_g}{Z_g}} \times \frac{C_s}{1 + \frac{r_G}{R_{cd}}}. \quad (120)$$

Since we can operate on a part of the grid characteristic where Z_g is *negative* and we can adjust R_g to make $(r_x + R_g)$ approach the value of $|Z_g|$, the over-all sensitivity of the system can be made very great. The limit of sensitivity is set only by the difficulty of maintaining a steady "zero."

Let us consider a numerical example in which we assume that the unknown potential is of thermo-electric origin and the thermopile has a resistance of 500 ohms, and also that we use the circuit shown in Fig. 5 with a UX-112-A vacuum tube and a Leeds & Northrup 2500-*e* galvanometer. The catalog value of the constants of this galvanometer are the following: sensitivity 3×10^{-9} amp. per millimeter deflection at one meter; external critical damping resistance R_{cd} of 2000 ohms; coil resistance r_G of 500 ohms. The second factor of equation (120) can be computed from these data. We can take $G_m = 1600 \times 10^{-6}$ mho which is the rated value of the mutual conductance for the UX-112 vacuum tube and we can take the grid impedance at the point of inflection of the grid current *vs.* grid potential characteristic as $Z_g = -535$ megohms. This is the value found on a tube of this type and therefore cannot be taken as the average. It is hoped that in the near future measurements can be made on a sufficiently large number of tubes to get some indication as to the probable value of such constants. If such tests are carried out the results will be published in this Journal. We can make $(r_x + R_g) = 500$ megohms and we get for the over-all sensitivity:

$$\frac{\partial \theta}{\partial e_p} = \frac{1600 \times 10^{-6}}{1 - \frac{500}{535}} \times \frac{\frac{1}{3 \times 10^{-9}}}{1 + \frac{500}{2000}} \tag{121}$$

$$= 6.6 \times 10^6 \text{ mm. deflection per volt.}$$

The sensitivity is thus 6.6 mm. per microvolt. Remembering that the potential being measured has been assumed to have originated in a 500 ohm thermopile, we see that the sensitivity of this amplifier galvanometer combination exceeds that which can be realized with most D'Arsonval galvanometers. Obviously, if the resistance of the thermopile can be reduced without decreasing its ability to generate potential, the amplifier cannot be used to advantage. On the other hand, if the thermopile can be made to give even higher potentials by a further increase in its resistance the use of an amplifier becomes more and more advantageous. An amplifier used as suggested above will not have a constant zero over any very long period of time, but since the measurement is made by a "null" method (using the circuit of Fig. 14), considerable drift can be permitted without seriously interfering with the measurements.

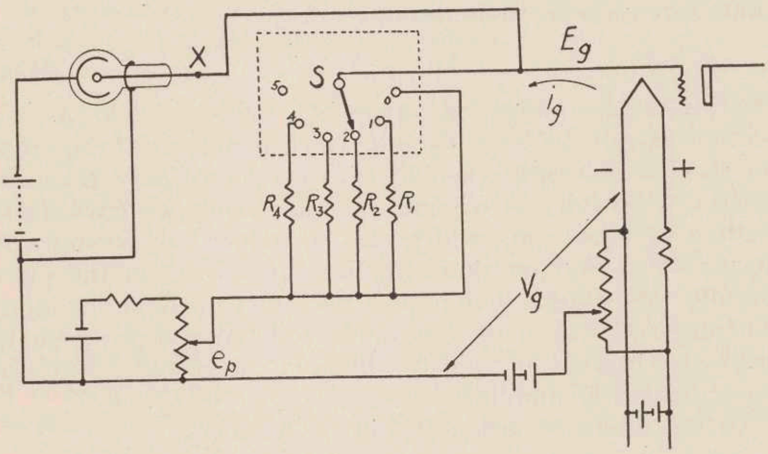
B. CURRENT MEASUREMENTS IN HIGH RESISTANCE CIRCUITS.

Under this head, four important circuits will be discussed. These do not begin to exhaust all of the possible circuits which might be used for current measurements but they illustrate many of the important points to be considered and show the methods by which any special circuit can be analyzed and its operation accurately predicted. Certain special requirements make it desirable to operate some amplifiers with the grid potential very close to the floating potential while in other cases we can realize a greater sensitivity by operating at a potential which is more negative than the floating potential and thus take advantage of the negative grid impedance.

a. Case I. Current Amplifier Operating at Floating Potential. Measurements by Direct Deflection.

The amplifier input circuit shown in Fig. 15 is one which has been used for some years to measure very small photoelectric currents by direct deflection. When there is no photoelectric current, the grid of this amplifier is maintained at the floating potential and the plate circuit is balanced to give no galvanometer current. Since there is no grid current in the input circuit when the grid is at the floating potential

FIG. 15.



Input circuit for photo-electric current measurements.

the "C" battery potential must be equal to the floating potential and the galvanometer zero will be independent of the resistance R_g (i.e., R_1, R_2, etc.). In other words, switch S of Fig. 15 can be moved to any position 0 to 5 without a shift in the galvanometer zero. The deflection of the galvanometer when a photoelectric current is flowing will depend on the value of R_g in a way which will be discussed below. The *only object* in operating this amplifier at the floating potential is to enable us to have a very wide range over which current measurements can be made. It may make this point more clear to illustrate it by means of a numerical example. Equation (17) shows that the change ΔE_g in grid potential brought about by the flow of a photoelectric current Δi when $R_g = \infty$, is given by

$$\Delta E_g = Z_g \Delta i. \tag{123}$$

If Z_g is 3×10^9 ohms as is the case for the UX-222 vacuum tube, and Δi is 5×10^{-15} ampere, we have a ΔE_g which is 15×10^{-6} volt. With an amplifier sensitivity of 0.4 mm. per microvolt the deflection is 6 mm. Suppose however, we now want to measure a current of 5×10^{-8} ampere. From equation (123) we would get a ΔE_g of 150 volts, which would obviously carry the grid too far from the operating potential. If, however, we use a resistance $R_g = 3 \times 10^6$ ohms, equation (17) reduces to the form:

$$\Delta E_g = R_g \Delta i. \tag{124}$$

Putting in the numerical values $R_g = 3 \times 10^6$ and $\Delta i = 5 \times 10^{-8}$ we get $\Delta E_g = 0.15$ volt which is not at all too great for the UX-222 tube when R_g is as low as we have taken it. With a sensitivity of 0.4 mm. per microvolt we have a deflection of 6000 mm., which can be reduced to 60 mm. by means of an Ayrton shunt (or its equivalent) in the plate circuit. We thus see that it is necessary to have an input control in the form of a variable resistance if we wish to measure currents of widely different magnitude and not "over load" the amplifier.

If we rewrite equation (17) in the form

$$\Delta E_g = \Delta i \left(\frac{1}{\frac{1}{R_g} + \frac{1}{Z_g}} \right) \tag{125}$$

we see the change in potential ΔE_g is equal to the "IR" drop where Δi is the current and the resistance is the external resistance R_g in parallel with the grid impedance Z_g. This assumes that Z_g is constant over the range ΔE_g. That is, we have

$$\frac{1}{R} = \frac{1}{R_g} + \frac{1}{Z_g}, \tag{126}$$

where R is the "effective" resistance which must be used to compute the change in grid potential as

$$\Delta E = \Delta i R. \tag{127}$$

It is by means of this factor R that we control the sensitivity of the input circuit.

Suppose that we wish to reduce our sensitivity and therefore our value of R to some smaller value R'. Let the ratio of these effective resistances be n.

$$\frac{R}{R'} = n. \tag{128}$$

For any value of n and any value of R we can compute the value of R_g which is necessary from equation (126). The highest value of R is had when $R_g = \infty$ for then $R = Z_g$. Let us take the case in which

$$R' = \frac{R}{n} = \frac{Z_g}{n} = \frac{1}{\frac{1}{R_g} + \frac{1}{Z_g}}. \tag{129}$$

Solving equation (129) for R_g we get

$$R_g = \frac{Z_g}{n - 1}. \tag{130}$$

From this equation we can easily compute a series of values of R_g in terms of a given Z_g for any series of sensitivity ratios n_1, n_2, etc. For example with $Z_g = 3000 \times 10^6$ ohms we get

n	R_g
1	∞
3	1500×10^6
10	333
30	103
100	30

The adjustment and operation of this amplifier will now be described assuming that Fig. 15 represents the input circuit to the balanced bridge amplifier shown in Fig. 7. With switch S in position 5 and the photo-cell circuit open at X the grid potential of the dummy tube should be adjusted to bring the galvanometer to "zero." With $e_p = 0$, turn switch S to position 0 and adjust V_g until the galvanometer again returns to the "zero" position. It should now be possible to turn switch S to any position 0 to 5 without causing a deflection of the galvanometer. If this is not true it is an indication that the grid lead is not properly insulated and protected by shields and guard rings to keep out extraneous currents. If the leakage through the photoelectric cell is smaller than the least current which can be measured with this particular amplifier circuit, then the connection at X can be made without causing a shift in the "zero" as the switch is turned from 0 to 5.

The potentiometer is used to determine the sensitivity of the amplifier to grid potential. With the switch S at 0 the galvanometer deflection can be measured as a function of the potentiometer potential e_p. It may be found that the response will be linear for values of e_p as great as $\pm$ 0.5 volt or more depending on the vacuum tube used. It *does not follow* that the response to photoelectric current will also be linear for the same range of galvanometer deflection. This is due to that fact that the grid current is not a linear function of the grid potential for wide variations in grid potential. For this reason it is well to determine the grid current characteristic by turning switch S to position 1 or 2 and again measure the galvanometer deflection as a function of the potentiometer potential.

This time the potential required to cause a given galvanometer deflection will be greater by an amount Δe_p than it was when switch S was in position 0. This difference in the two potentials, is due to the grid current and is equal to the "IR" drop of this current in the resistance R_g. R_g is equal to R_2 when the switch is in position 2. If we know R_g we can find the grid current i_g corresponding to each value of e_p since

$$i_g = \frac{e_p' - e_p}{R_g} = \frac{\Delta e_p}{R_g}. \qquad (131)$$

Where e_p' and e_p are the potentiometer potentials necessary to give the same galvanometer deflection with and without the resistance R_g respectively. Since the grid potential is given by

$$V_g + e_p = E_g, \qquad (132)$$

the grid current characteristic as a function of grid potential E_g can be mapped out accurately.[17] This characteristic will usually be of the form shown by the solid line of Fig. 2.

It was stated above that even though the galvanometer response might be a linear function of the grid potential E_g, it does not follow that the galvanometer response will be a linear function of the photoelectric current. Since this non-linearity becomes greater as the resistance R_g increased, let us consider the case in which $R_g = \infty$, that is, the case when the switch is in position 5. We observe that the galvanometer deflection is a function of the light falling on the photo-cell and for each deflection of the galvanometer we can determine the grid potential from the calibration taken with the switch in position 0. With the photo-cell connected as shown the grid will become more and more negative as the light intensity is increased. The amount that it becomes negative will be determined entirely by the grid current characteristic and the photoelectric current will be exactly equal to the grid current for the corresponding grid potential as determined above. This result comes from equation (14) which may be rewritten as follows:

$$i = \frac{E_g - V_g}{R_g} + i_g, \qquad (133)$$

since the first term vanishes when R is infinite. As long as the photoelectric current does not exceed the grid current at the hump of the curve at H of Fig. 2, the photoelectric current will be definitely related to the galvanometer deflection, but the moment the photoelectric current exceeds this value a sudden transition will occur and the grid will become very negative and the galvanometer deflection will increase tremendously even though the light intensity increase is very small. From equation (133), the amplifier calibration and the experimental determination of the grid current *vs.* grid potential curve, we can compute the relation between galvanometer deflection and photoelectric current.

We have gone into this question of the non-linearity of response caused by the flow of grid current at some length, because it enters in such a subtle way that it is very easily

[17] See footnote 1.

overlooked and may cause serious misinterpretations of results if it is not taken into consideration.

b. Case II. Current Amplifier Operating at Floating Potential. Measurements by "Null" Method.

If we use the potential e_p from the potentiometer in Fig. 15 to restore the galvanometer coil to its zero position, the problem of the non-linearity of response no longer plays a part. A new difficulty appears, however, owing to the fact that, as R_g is increased beyond a certain value, the percentage uncertainty in the setting of the potentiometer increases very rapidly. An analytical expression which shows this effect can be derived in the following way.

When we are using the potentiometer as suggested, the Kirchoff equation which we must use instead of (14) is the following:

$$E_g = V_g + e_p + R_g i - R_g i_g, \qquad (134)$$

where V_g is constant and equal to the floating potential. When the e_p is adjusted to bring about a perfect balance of the amplifier $e_p + R_g i = 0$; $i_g = 0$ and $E_g = V_g =$ floating potential. In every practical case there is a definite change in E_g (call it δE_g) which produces the least detectable swing of the galvanometer coil. This is a function of the galvanometer sensitivity and the circuit constants. Let us give e_p an arbitrary variation which is just sufficient to produce a change δE_g in the grid potential. We find from equation (134)

$$\delta E_g = \delta e_p - R_g \frac{\partial i_g}{\partial E_g} \delta E_g. \qquad (135)$$

Making use of the definition of grid impedance in equation (7) and rearranging we see that

$$\delta e_g = \delta E_p \left(1 + \frac{R_g}{Z_g} \right). \qquad (136)$$

From this equation we see which factors control the uncertainty in the setting of the potentiometer and we can calculate the percentage uncertainty as follows:

$$\frac{\delta e_p}{e_p} 100 = U \text{ percent} = 100 \frac{\delta E_g}{e_p} \left(1 + \frac{R_g}{Z_g} \right). \qquad (137)$$

Hence, if the sensitivity to photoelectric current is increased by increasing R_g, the percentage uncertainty in the potentiometer setting will change very little as long as $\frac{R_g}{Z_g}$ is less than unity. On the other hand, if Z_g is positive and $R_g > Z_g$, the percentage uncertainty in the potentiometer setting increases with increasing R_g practically as fast as the sensitivity and therefore accomplishes nothing.

c. Case III. Current Amplifier Operating with a Negative Grid Impedance. Measurements by Direct Deflection.

For certain applications it is unnecessary to measure currents extending over a very wide range of intensity with the minimum of adjustment. Very high sensitivity can be had by operating with a fixed resistance R_g and adjusting the "C" battery potential so as to work over that part of the grid current characteristic where the grid impedance is negative. The circuit shown in schematic in Fig. 15, can also be used for this discussion. In the previous section a method was described by which the grid current characteristic of the tube could be measured when the amplifier was balanced with the grid at the floating potential. The procedure in that case was definite because the floating potential could be determined without knowing the entire grid current characteristic. It will appear in the discussion to follow that there are certain advantages to be had if the potential V_g is adjusted so that the grid potential about which we operate is located near the point of inflection of the grid current *vs.* grid potential characteristic. Owing to the fact that the grid current characteristic depends on the conditions in the plate circuit, it is not easy to set up the procedure for determining the point of inflection. The writer has found by experience that the following method leads to satisfactory results. With switch S in position 0 and V_g adjusted to a potential which is 2.0 to 3.0 volts negative with respect to the filament, the plate circuit is balanced to give no current through the galvanometer. This is done by adjusting the potential of the grid of the dummy tube when the two tube bridge circuit is being used.

With the switch still at 0 we should determine the galvanometer deflection corresponding to a series of potentials e_{p1}, e_{p2}, etc., for both positive and negative values of e_p. If we turn the switch S to position 1, 2, 3 or 4 we can find new values of e_p which will give exactly the same series of galvanometer deflections as before. Assume that we select position 2 and let these new values be e_{p1}', e_{p2}', etc., where e_{p1}' and e_{p1} are the potentiometer potentials which give the same galvanometer deflections with the switch in position 2 and position 0 respectively. These differences $(e_{p1}' - e_{p1})$, etc., measure the "IR" drop in the resistance resulting from the flow of grid current through it. Obviously these differences divided by the resistance R_2 give the corresponding grid currents

$$\frac{e_p' - e_p}{R_2} = i_g. \tag{138}$$

We can add V_g to e_p to get E_g and plot i_g as a function of E_g. This is the grid current *vs.* grid potential characteristic. The reciprocal of the slope of the tangent to this curve drawn through the point $E_g = V_g$ will give the input impedance to the grid when operated under these particular conditions.

If this circuit is to be used for direct deflection measurements, it is important that the value of E_g around which we intend to operate be located on the straight part of the characteristic, that is, near the "point of inflection" of the grid current characteristic. In case the value V_g which was arbitrarily chosen was not close enough to this best value of E_g, a new value of V_g should be taken and the above series of measurements repeated.

In order to test the linearity of response of the system as a whole, two sets of measurements of the galvanometer deflection as a function of potentiometer potential must be made. The first of these is made with the V_g set equal to the value of E_g around which we intend to operate and with the switch at position 0 while the second is made with the switch in position 2, for instance, and with a new corresponding value of V_g. This new value of V_g is found by balancing amplifier with the switch at 0 by means of the dummy tube and then with the switch at 2, V_g is readjusted to bring the amplifier back to balance. This readjustment is necessary in order to correct for the fall in potential in the grid resistance due to the flow of grid current.

We must now measure the galvanometer deflection as a function of potentiometer potential with the resistance in the circuit and the new value of V_g. The range of galvanometer deflection over which *both* of these calibration curves are linear is a measure of range over which the amplifier as a whole will respond linearly to photoelectric current with this particular value of R_g. It is very important to make both of these tests because it is possible to have a *non-linear* response to photoelectric current and yet have one or the other of these tests indicate a linear response.

The over-all sensitivity using this input circuit and the two tube balanced bridge amplifier of Fig. 7 can be calculated from equations (17), (63) and (113).

$$\frac{\partial \theta}{\partial i_G} \times \frac{\partial i_G}{\partial E_g} \times \frac{\partial E_g}{\partial i} = \frac{G_m}{2\left(1 + \frac{r_g}{R_{cd}}\right)} \times \frac{1}{\frac{1}{R_g} + \frac{1}{Z_g}} \times C_s. \qquad (139)$$

On rearranging equation (139) we get

$$\frac{\partial \theta}{\partial i} = \frac{G_m}{\frac{1}{R_g} + \frac{1}{Z_g}} \times \frac{C_s}{2\left(1 + \frac{r_G}{R_{cd}}\right)}, \qquad (140)$$

in which the second factor depends only on the galvanometer constants. Since we are assuming that Z_g is negative we see at once that if R_g is less than the absolute magnitude $|Z_g|$ the sensitivity will increase as R_g approaches $|Z_g|$ and becomes infinite when $R_g = |Z_g|$. If R_g is made larger than $|Z_g|$ it will be impossible to operate the amplifier at the grid potential E_g for which $|Z_g|$ was determined.

If we put constants into equation (140) appropriate to the UX-112-A vacuum tube and the Leeds and Northrup 2500-*e* galvanometer which are $G_m = 1600 \times 10^{-6}$ mho, $Z_g = -1000 \times 10^6$ ohms, $R_g = 900 \times 10^6$ ohms, $r_g = 500$ ohms, $R_{cd} = 2000$ ohms and $C_s = 3.3 \times 10^8$ mm. per ampere at one meter, we get

$$\frac{\partial\theta}{\partial i} = 1.9 \times 10^{15} \text{ mm. per ampere.} \tag{141}$$

From this we see that one centimeter deflection corresponds to a current of 5×10^{-15} ampere. In order to determine the actual sensitivity of an amplifier of this type it is not necessary to compute it in this way. All that is necessary is to measure the voltage sensitivity $\frac{\partial\theta}{\partial e_p}$ and multiply it by the resistance R_g. The truth of this statement is at once evident from the equations for the voltage and current sensitivities when written as follows:

$$\frac{\partial\theta}{\partial e_p} = \frac{G_m}{1 + \frac{R_g}{Z_g}} \times \frac{C_s}{2\left(1 + \frac{r_G}{R_{cd}}\right)}, \tag{142}$$

$$\frac{\partial\theta}{\partial i} = \frac{R_g G_m}{1 + \frac{R_g}{Z_g}} \times \frac{C_s}{2\left(1 + \frac{r_G}{R_{cd}}\right)}. \tag{143}$$

The first equation comes from (63), (111), and (113) while the second comes from (16), (63) and (113). Substituting (142) into (143) we get

$$\frac{\partial\theta}{\partial i} = R_g \frac{\partial\theta}{\partial e}. \tag{144}$$

d. Case IV. Current Amplifier Operating with a Negative Grid Impedance. Measurements by "Null" Method.

This type of circuit is very well adapted for "null" measurements in which the potentiometer potential e_p is adjusted to keep the galvanometer coil in its zero position. The question of the linearity of response does not enter for the current being measured is given directly by

$$i = \frac{e_p}{R_g}. \tag{145}$$

From equation (137) we see that the percentage of uncertainty with which we can set the potentiometer e_p *decreases* as we increase the sensitivity by making R_g larger since Z_g has been assumed to be negative.

PART 4. TECHNICAL PROCEDURE.

A few points as to technical procedure are perhaps worth mentioning.

A. SHIELDING AND INSULATION.

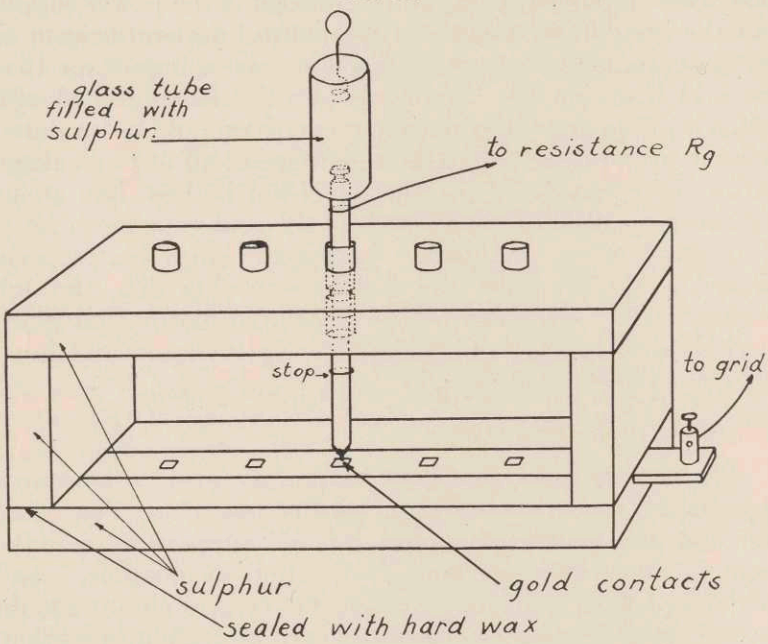
The active grid lead of these circuits should be shielded in all cases where the circuit resistances are very high. The next most important thing to be shielded is the power supply and the plate leads and lastly the control apparatus such as the potentiometer, the rheostats, etc. It is important that the grid leads on the "high resistance" side of the device which is producing the potential or current should be insulated at least to the extent that the resistance of all of the leakage paths when considered together shall not be less than about ten times the absolute magnitude of the grid impedance $|Z_g|$. It is much more important that guard rings be properly located so as to make the leakage currents flow through definite paths. The best arrangement of insulators and guard rings depends on the particular problem at hand and must be given considerable thought.

B. STRUCTURAL DETAILS.

The vacuum tube should be located as close to the source of potential or current as is physically possible. The tubes and also the grid lead should be well supported to make them as free from mechanical vibration as possible. Any switches or keys that are necessary in the grid circuit should be well insulated and mechanically certain in their operation. In the most sensitive circuits used by the author the switch *S* of Fig. 15 was made by imbedding a bar of brass in a cake of sulphur and allowing contacts to drop from above. These contacts were guided by brass tubes also imbedded in sulphur. Small gold tips were set in the movable contacts and small plates of gold were soldered on the brass bar at the points of contact. The sketch in Fig. 16 serves to illustrate the construction. Doubtless there are many other arrangements just as good as this one but this design is given in order to help those whose previous work has been in other fields.

If the amplifier is being constructed for high sensitivity work, great care must be taken to make sure that every junction between wires is soldered. It is even advisable to solder the leads onto the storage batteries used for the filament supply. The control rheostats should be of the best grade and the circuit should be designed with fixed series and shunt resistances associated with each rheostat so that

FIG. 16.



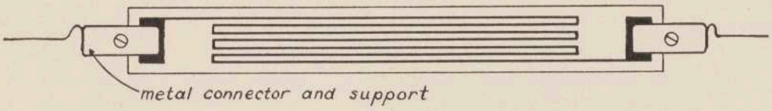
only a small proportion of the current flows through the movable contact. Resistances R_1, R_2 and r_d in the plate circuit should be "wire" resistances. It is usually advisable to insert small resistances in each filament and plate circuit so that the filament and plate currents can be measured with the help of a potentiometer and thus have all circuits undisturbed during the measurement.

C. CONSTRUCTION OF HIGH RESISTANCES.

The problem of producing high resistances of one or two megohms and more is not an easy one. Many workers in this field use xylol and alcohol solutions and recommend them highly. The writer has always used "Higgins" India ink resistances and found them perfectly satisfactory. These are made by drawing a long line in the form of a grid as shown in Fig. 17.

A heavy ink line about one millimeter wide drawn on smooth onion skin paper gives a resistance of about one megohm per centimeter. A more narrow line, of course, gives a correspondingly higher resistance per unit length. It has been found advisable to obtain the very high resistances

FIG. 17.



by drawing a very long line rather than a very narrow one.[18] In any permanent set-up these resistances should be mounted in tubes which can be filled with dry air and then sealed or else the tubes should be evacuated and sealed.

The temperature coefficients of three India ink resistances have been measured over the range 23° C. to 45° C. The results are given in the following table:

TABLE III.

$$R_t = R_{23}[1 + \alpha(t - 23)]$$

R_{23} ohms	α
0.85×10^6	-2.7×10^{-3}
750×10^6	-6.4×10^{-3}
2100×10^6	-6.4×10^{-3}

These resistances have lower temperature coefficients than the one reported by Campbell [19] for xylol-alcohol solutions which was $\alpha = 14 \times 10^{-3}$.

[18] Same experience reported by K. T. Compton and C. H. Thomas, *Phys. Rev.*, **28**: 604 (1926).

[19] N. Campbell, *Phil. Mag.*, **23**: 668 (1912).

APOLOGY.

Although this paper is one of the longest which has been printed on the use of the vacuum tube for D.C. measurements, much has been left unsaid. Many circuits even closely related to those discussed here have not been mentioned. The principal object of this paper has been to formulate in a definite way most of the underlying relationships upon which nearly all of the D.C. amplifiers depend. If these are well understood, the design and operation of an amplifier for any particular purpose becomes a problem which is definite and which can be solved by a straightforward procedure.

PRINTED IN THE U. S. A.

LANCASTER PRESS, INC.
LANCASTER, PA.