A New Equation for the Static Characteristic of the Normal Electric Arc by W. B. Nottingham 1923

A New Equation for the Static Characteristic of the Normal Electric Arc

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COMMERCIALLY the electric arc is now extensively used, and yet after more than a hundred years of study, there has been little crystallization of opinion as to certain phases of its fundamental theory. Absolute contradictions are frequently found in the literature, and premature generalizations have given rise to more disagreement among the investigators of the equation of the static characteristic of the normal[1] arc than the experimental difficulties warrant.

The static characteristic is the relation between the current (i) flowing in the arc, the length (L) of the arc, and the difference in potential (E) across the arc. This relation can be represented approximately by simple expressions of the form of the Ayrton[2] equation

$$E = a + b \cdot L + \frac{c + d \cdot L}{i} \qquad (1)$$

or the Steinmetz[3] equation

$$E = a + \frac{c\,(L + d)}{i^{0.5}} \qquad (2)$$

(a, b, c, and d are constants)

if the range in variation of the current and arc length, for which the constants are calculated, is the closely defined.

In particular, these equations are recognized as approximations, because *the difference in potential* (E) *is not a linear function of the arc length* (L), and although the difference in potential does decrease when the current increases, *the current* (i) *enters neither as the first power* (as in the Ayrton equation) *nor as the square root* (as in the Steinmetz equation). Duddell[4] was among the first to demonstrate by the use of the curve shown in Fig. 1, that, for arcs less than 15 mm. in length, the difference in potential cannot possibly be truly represented by a linear function of the arc length. As far as I know, Steinmetz[5] was the first to advance the thesis that, "while the stream voltage varies with the current, and in the opposite direction to it, its variation is much less than the inverse proportion to the current."

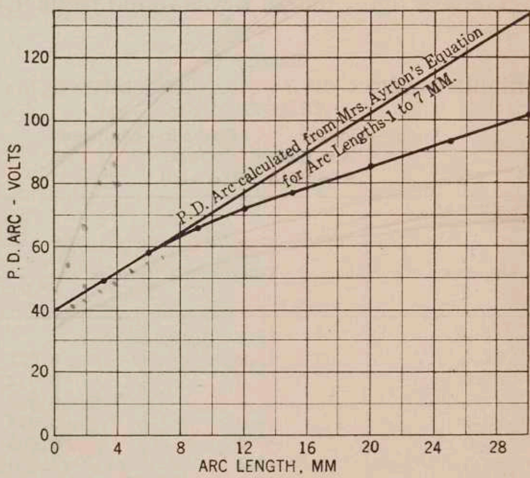

Fig. 1

This paper attempts to make these two criticisms constructive or positive instead of negative.

The Exponent of the Current is Directly Proportional to the Temperature of the Boiling Point of the Anode

If the arc length is assumed to be constant, the Ayrton and the Steinmetz equations can be written in the form of equation (3)

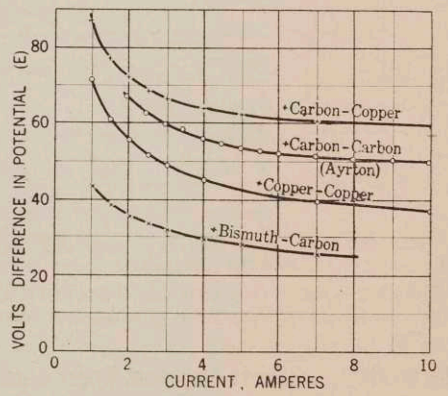

Fig. 2—Volt-Ampere Characteristics with the Arc Length Constant $L = 3.0$ mm.

$$E = A + B / i^n \qquad (3)$$

1. Each stage of the electric arc from the glow arc to the hissing arc has its own physical, electrical and chemical characteristics. Therefore any equation of the electric arc must necessarily be limited to the characteristics of one stage. The normal arc has two general requirements: (1) the current intensity must be greater than the maximum for the glow arc, and less than the minimum for the hissing arc; and (2) the arc must be free from external electrical, magnetic, atmospheric, and physical disturbances.

2. Hertha Ayrton, The Electric Arc, "The *Electrician*," Printing and Publishing Company, London.

3. C. P. Steinmetz, Trans. A. I. E. E., 1906, p. 802.
C. P. Steinmetz, "Radiation, Light, and Illumination," McGraw Hill, New York, 1909, p. 139.
C. P. Steinmetz, "Theory and Calculation of Electric Circuits," McGraw Hill, New York, 1917, p. 35.
C. P. Steinmetz, "Chem. and Met. Eng.," 22, p. 455, (1920).

4. W. Duddell, *Phil. Trans.*, 203 (A), p. 338, (1904).

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5. See foot-note 3.

in which A and B are constants dependent on the arc length and the electrode material and in which n is a constant dependent only on the electrode material. The value of n for the characteristic curves of an arc between any electrode materials can be calculated directly from curves such as those in Fig. 2. For example, the calculation of the constants of equation (**3**) for an arc 3.0 mm. long between copper electrodes, indicates that n is equal to 0.665. (See Fig. 2, Table I, and Equation (**3a**)). This study with copper electrodes was extended to include many characteristic curves for arcs ranging in length between 1.0 and 10.0 mm. The average value of n was found to be 0.670.

TABLE I

THE DETERMINATION OF n FOR AN ARC BETWEEN COPPER ELECTRODES

Anode—Copper Cathode—Copper

Arc Length 3.0 mm.

$A = 27.5 \quad B = 44.0$

i Amperes	E Volts	$n = \dfrac{\log \dfrac{44.0}{E - 27.5}}{\log i}$
1.0	71.5	
1.5	61.0	0.677
2.0	55.5	0.652
2.5	51.5	0.668
3.0	49.0	0.653
4.0	45.0	0.660
5.0	42.5	0.670
7.0	39.5	0.669
10.0	37.0	0.667

Average $n_3 = 0.665$

Average for curves of 1.0 to 10.0 mm. arc length .... $n_{ave.} = 0.670$

$$E = 27.5 + \frac{44.0}{i^{0.665}} \tag{3a}$$

It is generally agreed that n is equal to 1.0 for the characteristic curves representing the arc between

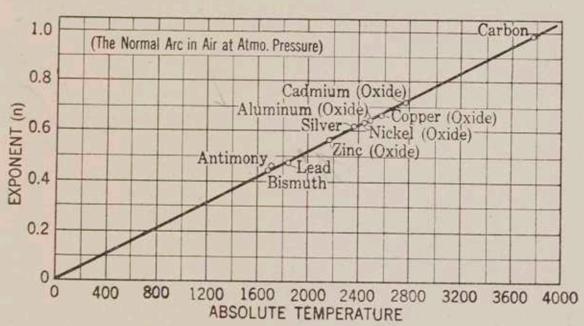

FIG. 3—THE EXPONENT (N) AND THE ABSOLUTE TEMPERATURE OF THE BOILING POINT OF THE ANODE MATERIAL

carbon electrodes[6], while it has just been demonstrated that n is 0.670 in the case of the arc between electrodes of copper. The natural question arising from these observations is: what will be the value of n characteristic of an arc between *unlike* electrodes? The outcome of the first investigation was striking; it showed n to be 0.985 when the *anode* was *carbon* and the cathode was copper. This result indicates that the anode material predominates in the determination of n.

TABLE II

THE DETERMINATION OF n FOR AN ARC BETWEEN UNLIKE ELECTRODES

Anode—Carbon Cathode—Copper

Arc Length 3.0 mm.

$A = 56.0 \quad B = 31.0$

Current Amperes	E Volts	$n = \dfrac{\log \dfrac{31.0}{E - 56.0}}{\log i}$
1.0	87.0	
1.5	77.0	0.960
2.0	72.0	0.955
2.5	68.5	0.992
3.0	66.5	0.985
4.0	64.0	0.997
5.0	62.5	0.970
7.0	60.5	0.993
10.0	59.5	0.950

Average $n_3 = 0.975$

Average for curves of 1.0 to 10.0 mm. arc length .... $n_{ave.} = 0.985$

A continuation of this investigation of arcs between unlike electrodes not only supplied convincing evidence to substantiate the above thesis, but it opened the way to the discovery of the exact property of the anode material upon which n is dependent. It is apparent from Table III that there is a definite relationship between the value of n for each element represented and its boiling point. Furthermore, if the absolute

TABLE III

THE RELATION BETWEEN n AND THE BOILING POINT OF THE ANODE

Anode	Cathode	Oxide[7] or Non Oxide on Anode	n	Abs. Temp. of Boiling pt. or Sublimation pt.	Authority
Carbon	Copper	..	0.985	3770	Van der Waals
Cadmium	Carbon	Oxide	0.720	2770	Approximation
Copper	Copper	Oxide	0.670	2580	Greenwood
Aluminum	Carbon	Al_2O_3	0.650	2480	Ruff—Schmidt
Nickel	Carbon	Oxide	0.640	2450	Hagenbach—Langbein
Silver	Carbon	Non oxide	0.624	2370	v. Wartenberg
Zinc	Carbon	Oxide	0.570	2170	Approximation
Lead	Carbon	Non oxide	0.480	1850	v. Wartenberg
Antimony	Carbon	Non oxide	0.460	1710	Greenwood
Bismuth	Carbon	Non oxide	0.445	1690	Greenwood

temperature of the boiling point for each element is plotted as the abscissa and the corresponding value of n as the ordinate in a system of coordinates, the exact nature of the relation is at once evident (See Fig. 3.):

The exponent (n) of the current (i) in the equation

$$E = A + B/i^n \tag{3}$$

is directly proportional to the absolute temperature of the

6. Calculations from Mrs. Ayrton's observed data indicate that n is slightly less than 1.0 for the arc between carbon electrodes.

7. G. Shulze, *Ann. d. Phys.* (4), 12, p. 828, (1903).

A. Hagenbach and K. Langbein, "*Archives des Sciences Physiques et Naturelles*," Vol. 5, Part 1, p. 48, (1919).

boiling point or the sublimation point, as the case may be, of the anode[8].

E is not a Linear Function of L when the Current is Constant

Both Mrs. Ayrton and Steinmetz have assumed the difference in potential (E) to be a linear function of the arc length (L) when the current is constant, but this is not even approximately true if the arc is less than 15.0 mm. in length. In spite of the fact that Duddell[9] made this criticism in 1904, as far as I know, no substitute has been offered for this much discredited assumption. A cursory examination of a typical constant current curve (which shows the relation between the difference in potential (E) across the arc and the length of the arc (L)) and its component parts will

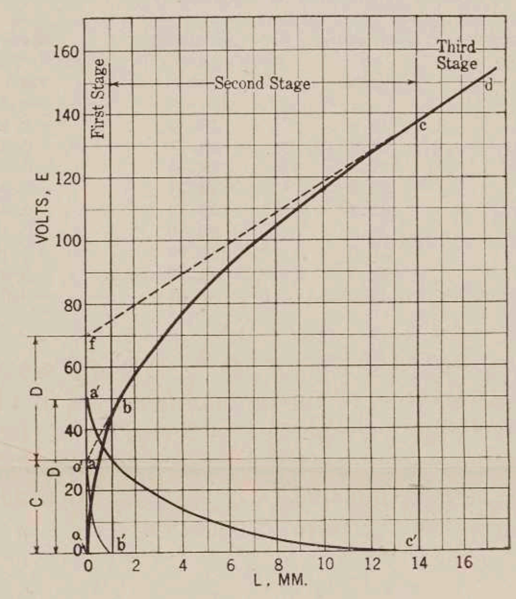

Fig. 4—A Typical Constant Current Curve and Its Component Parts

reveal the steps to be taken in the formulating of a more accurate equation to supplant the old. (See Fig. 4.)

The arc goes through three distinct stages as it increases from an infinitesimal length to 15.0 mm. or more. These changes are in turn reflected in the constant current curves, and although the dividing line between the stages is not sharp, the first modification takes place when the arc length reaches 0.7 mm. to 1.0 mm. and the second when it is 13.0 mm. to 15.0 mm.

Arcs of the third stage can be represented easily, since the constant current curve is straight, by

$$E = D + C + \gamma \cdot L \qquad (4)$$

Furthermore, curve $a'\,c'$ (the difference between the straight line $f\,d$ and the constant current curve extrapolated) is so nearly a pure exponential of the form of the equation

$$e_2 = D \cdot \epsilon^{-\delta \cdot L} \qquad (5)$$

that the second and the third stages may be approximated very closely by the equation

$$E_{23} = C + D \cdot (1 - \epsilon^{-\delta \cdot L}) + \gamma \cdot L \qquad (6)$$

Curve $o'\,b'$, which is the deviation of $a\,b\,c\,d$ from the constant current curve, can be represented by the equation

$$e_1 = C \cdot \epsilon^{-d \cdot L} \qquad (7)$$

Therefore the equation of the constant current curve taken in its entirety[10] is

$$E = C \cdot (1 - \epsilon^{-d \cdot L}) + D \cdot (1 - \epsilon^{-\delta \cdot L}) + \gamma \cdot L \qquad (8)$$

In the above equations

$C + D$ = the intercept of line $f\,d$ on the E axis,
C = the intercept of curve $o'\,b'$ on the E axis,
D = the intercept of curve $a'\,c'$ on the E axis,
γ = the slope of line $f\,d$,
$-\delta$ = the slope of the straight line represented by the equation $\log e_2 = \log D - \delta \cdot L$,
$-d$ = the slope of the straight line represented by the equation $\log e_1 = \log C - d \cdot L$.

Since equation (8) represents the curves in which the current is constant and equation (3) represents the curves in which the arc length is constant, a combination of these two equations will show the relation between the current (i) flowing in the arc, the length (L) of the arc, and the difference in potential (E) across the arc, without limitation. In other words, a new equation will have been derived to represent the static characteristic of the normal arc.

$$E = A + B / i^n \qquad (3)$$

The New Equation

In order to discover just how equations (3) and (8) should be combined, the arc between copper electrodes was investigated. Constant current curves were plotted from the observed data. (See Fig. 5.)

The first curve analyzed was the one for a current of 1.0 ampere. This analyzation resulted in the determination of the numerical values of the constants in equation (8) so that it represented this curve showing

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8. Although this law is being published at the present time, its confirmation is pending the investigation of a greater number of electrodes.

9. See footnote 4.

10. Equation (8) does not take into consideration the possibility that there may be, for each electrode material, an arcing potential below which no arc can be struck, however short it may be. This potential seems to be about 5.0 volts in the case of the arc between copper electrodes. If this observation is correct, equation (8) can be modified by the introduction of the constant ρ in the first term, so that the minimum potential for arcing is taken into consideration.

$$E = C \cdot (1 - \rho \cdot \epsilon^{-d \cdot L}) + D \cdot (1 - \epsilon^{-\delta \cdot L}) + \gamma \cdot L$$

Where:

$$E_{min} = C \cdot (1 - \rho) .$$

Column Δ_a in Table IV shows the error between the observed and the calculated values of E when the modified form of equation (8) is used. The examination of the minimum potential for arcing would offer an interesting field for additional investigation.

the relation between the arc length and the difference in potential. The substitution of the values:

$$C = 28.0, \quad D = 42.5, \quad \gamma = 5.0, \quad d = 6.0, \quad \delta = 0.373,$$

in equation (**8**) gave (**9**).

$$E = 28.0\,(1 - \epsilon^{-6 \cdot L}) + 42.5\,(1 - \epsilon^{-0.373 \cdot L}) + 5.0 \cdot L \quad (\mathbf{9})$$

TABLE IV

THE COMPARISON BETWEEN THE OBSERVED AND THE CALCULATED VALUES OF E

Electrodes—Copper; Current—1.0 Ampere

E_c Calculated with equation (**9**)

Arc Length L mm.	Observed E_0 Volts	Calculated E_c Volts	Error Δ Volts	Error[11] Δ_a Volts
0.02	6.0	3.20	+2.40	−1.50
0.04	10.0	6.20	+3.80	−0.20
0.10	17.0	14.64	+3.36	−0.44
0.20	26.0	23.60	+2.40	+0.60
0.30	31.0	31.50	−0.50	−1.30
0.50	37.0	36.32	+0.68	+0.58
0.70	42.0	40.90	+1.10	+1.10
1.0	46.0	45.97	+0.03	+0.03
2.0	60.0	60.30	+0.30	+0.30
3.0	71.5	71.60	−0.10	−0.10
4.0	81.0	81.00	0.00	0.00
5.0	89.0	88.90	+0.10	+0.10
6.0	96.0	95.90	+0.10	+0.10
7.0	102.0	102.20	−0.20	−0.20
8.0	108.0	108.20	−0.20	−0.20
10.0	120.0	119.50	+0.50	+0.50

11. See footnote 10.

The difference in potential as observed and that as calculated by equation (**9**), have been placed in parallel columns in Table IV. The error (Δ) between the observed and the calculated values, is well within that attributable to experimental variations.

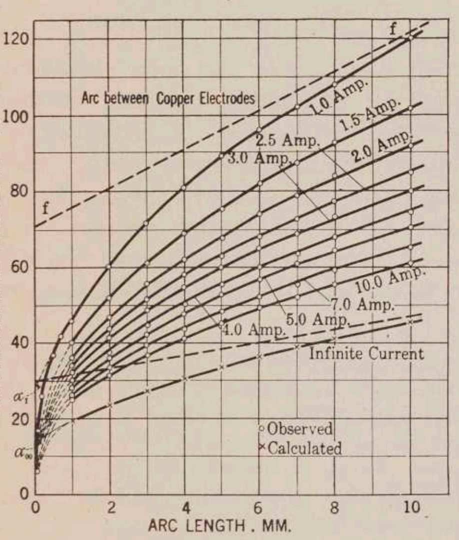

FIG. 5—VOLT-ARC LENGTH CHARACTERISTIC FOR DIFFERENT CURRENT VALUES

If the current is infinite equation (**3**) becomes

$$E = A = f\,(L) \quad (\mathbf{10})$$

Therefore, the curve showing the relation between the arc length and the constant A can be called the "infinite current" curve. (See Fig. 5.) Equation (**8**) represents this curve if the constants are assigned the following values:

$$C = 14.5, \quad D = 15.5, \quad \gamma = 1.7, \quad d = 6.0, \quad \delta = 0.235.$$

Hence, the second curve studied can be approximated very closely by equation (**11**). (See Table V.)

$$E = 14.5\,(1 - \epsilon^{-6 \cdot L}) + 15.5\,(1 - \epsilon^{-0.235 \cdot L}) + 1.7\,L \quad (\mathbf{11})$$

TABLE V

THE COMPARISON BETWEEN THE VALUES OF A AS DETERMINED FROM EQUATION (3) AND THAT CALCULATED BY EQUATION (11)

Electrodes—Copper Current—Infinite

A Determined by Equation (**3**)

A_c Calculated by Equation (**11**)

Arc Length L mm.	Determined A Volts	Calculated A_c Volts	Error Volts
1.0	19.6	19.43	+0.17
2.0	23.7	23.71	−0.01
3.0	27.5	27.48	+0.02
4.0	30.8	30.74	+0.06
5.0	33.8	33.74	+0.06
6.0	36.5	36.40	+0.10
7.0	39.0	38.90	+0.10
8.0	41.3	41.23	+0.07
10.0	45.5	45.52	−0.02

The numerical values of C, D, γ, δ, and d were also determined for each of the remaining curves in Fig. 5, and these values were plotted as functions of the current. In this way, it was discovered that d is independent of the current and that C, D, γ, and δ are functions of the current of the form of equation (**12**)

$$f\,(i) = a + \frac{b - a}{i^n} \quad (\mathbf{12})$$

In equation (**12**), a is the value of the constant when the current is infinite and b, the value when the current is 1.0 ampere. Therefore, in the case of the arc between copper electrodes, the constants of equation (**8**) may be expressed as functions of the current as follows:

$$C = 14.5 + \frac{28.0 - 14.5}{i^{0.67}} \quad (\mathbf{13})$$

$$D = 15.5 + \frac{42.5 - 15.5}{i^{0.67}} \quad (\mathbf{14})$$

$$\gamma = 1.7 + \frac{5.0 - 1.7}{i^{0.67}} \quad (\mathbf{15})$$

$$\delta = 0.235 + \frac{0.373 - 0.235}{i^{0.67}} \quad (\mathbf{16})$$

$$d = 6.0 \quad (\mathbf{17})$$

The substitution of the above functions in equation (**8**) gives (**18**), the desired final equation for the arc between copper electrodes.

$$E = \left(14.5 + \frac{28.0 - 14.5}{i^{0.67}} \right) (1 - \epsilon^{-6 \cdot L})$$

$$+ \left(15.5 + \frac{42.5 - 15.5}{i^{0.67}} \right) (1 - \epsilon^{-\left(0.235 + \frac{0.373 - 0.235}{i^{0.67}}\right) L})$$

$$+ \left(1.7 + \frac{5.0 - 1.7}{i^{0.67}} \right) L \qquad (18)$$

As an indication that equation (**18**) is the characteristic equation sought, eighty-eight calculated values of the difference in potential have been placed beside the corresponding observed values in Tables IV and VI. Tables IV and VI indicate that equation (**18**) does fit these data without systematic error. However, if apparatus had been available for the study of arcs more than 10.0 mm. in length, the constants of equation (**18**) could no doubt have been determined with a higher degree of accuracy.

and for infinite current. For the curve for 1.0 ampere, let

$$C = \alpha_1, \quad D = \beta_1, \quad \gamma = \gamma_1, \quad \delta = \delta_1, \quad \text{and} \quad d = d$$

and for the curve for infinite current, let

$$C = \alpha_\infty, \quad D = \beta_\infty, \quad \gamma = \gamma_\infty \quad \delta = \delta_\infty, \quad \text{and} \quad d = d$$

These constants incorporated in a general equation give equation (**19**).

$$E = \left(\alpha_\infty + \frac{\alpha_1 - \alpha_\infty}{i^n} \right) (1 - \epsilon^{-d \cdot L})$$

$$+ \left(\beta_\infty + \frac{\beta_1 - \beta_\infty}{i^n} \right) (1 - \epsilon^{-\left(\delta_\infty + \frac{\delta_1 - \delta_\infty}{i^n}\right) \cdot L})$$

$$+ \left(\gamma_\infty + \frac{\gamma_1 - \gamma_\infty}{i^n} \right) \cdot L \qquad (19)$$

TABLE VI

A COMPARISON BETWEEN OBSERVED AND CALCULATED VALUES OF E

Electrodes—Copper

E_0 Observed Value of E

E_c Calculated by Equation (**18**)

Arc Length L	1 mm.		2 mm.		3 mm.		4 mm.		5 mm.		6 mm.		7 mm.		8 mm.		10 mm.	
Current 1 amp.	E_0	E_c	E_0	E_c	E_0	E_c	E_0	E_c	E_0	E_c	E_0	E_c	E_0	E_c	E_0	E_c	E_0	E_c
1.5	40.0	39.73	52.0	51.81	61.0	60.83	69.0	68.64	75.5	75.34	82.0	81.65	87.5	87.30	93.0	92.43	102.0	101.82
2.0	36.0	36.23	47.0	47.11	55.5	55.36	62.0	62.04	68.0	67.87	74.0	73.70	79.0	78.75	84.0	83.33	92.0	92.30
2.5	34.0	33.73	43.5	43.74	51.5	51.38	58.0	57.74	63.0	63.29	68.5	68.40	73.0	72.85	77.5	77.13	85.0	85.30
3.0	32.5	32.15	41.5	41.38	49.0	48.58	55.0	54.69	60.0	59.84	65.0	64.75	69.0	69.00	73.5	73.13	80.0	80.47
4.0	30.0	29.98	38.0	38.21	45.0	44.88	50.5	50.52	55.5	55.56	60.0	59.90	64.0	63.85	68.0	67.65	74.5	74.60
5.0	28.5	28.51	36.5	36.26	42.5	42.38	48.0	47.83	53.0	52.46	57.0	56.75	61.0	60.70	64.5	64.18	70.5	70.64
7.0	26.5	26.58	33.5	33.71	39.5	39.41	44.0	44.19	48.5	48.54	52.5	52.55	56.0	56.10	59.5	59.43	65.5	65.60
10.0	25.5	25.15	31.5	31.61	37.0	36.89	41.5	41.44	45.5	45.52	49.5	49.18	52.5	52.45	55.5	55.53	61.0	61.46

TABLE VII

THE CONSTANTS OF EQUATION (**19**) DETERMINED FOR TEN ARC CHARACTERISTICS

Anode	Cathode	n	α_∞	α_1	β_∞	β_1	γ_∞	γ_1	δ_∞	δ_1	d
Carbon	Copper	0.985	24.5	39.0	35.0	41.4	1.25	5.0	0.535	0.535	7.0
Cadmium	Carbon	0.720	10.0	15.0	9.2	37.5	0.80	5.0	0.220	0.229	7.0
Copper	Copper	0.670	14.5	28.0	15.5	42.5	1.70	5.0	0.235	0.373	6.0
Aluminum	Carbon	0.650	8.0	17.5	6.0	35.0	0.80	5.0	0.258	0:410	5.0
Nickel	Carbon	0.640	10.0	28.0	8.0	38.0	0.90	5.0	0.166	0.270	7.0
Silver	Carbon	0.624	13.0	27.5	7.0	37.5	0.80	6.0	0.229	0.246	7.0
Zinc	Carbon	0.570	8.6	24.0	2.9	41.0	0.65	5.0	0.210	0.263	7.0
Lead	Carbon	0.480	8.2	16.5	4.8	17.5	0.80	5.0	0.180	0.214	7.0
Antimony	Carbon	0.460	12.2	24.0	4.3	19.0	0.60	5.0	0.190	0.230	7.0
Bismuth	Carbon	0.445	7.8	17.5	6.2	24.5	0.80	5.0	0.187	0.200	7.0

Although equation (**18**) is an empirical equation derived as a result of the study of the arc between copper electrodes, it can be expressed in general terms and used to represent the static characteristic of any arc. It will be remembered that n was the first constant in equation (**18**) to be determined. This determination also yielded the values of A from which the curve for infinite current was plotted. (See equation (3) and Fig. 5.) The constants of equation (**8**) were then determined for the curves in Fig. 5 for 1.0 ampere

EQUATION (19) TESTED FOR TEN ELECTRODE MATERIALS

All of the constants of equation (**19**) were calculated for the ten arc combinations studied and the resulting equations were checked graphically against the observed data. (See Table VII.)

The agreement between the observed and the calculated values was exceedingly close in every case. Therefore, *equation* (**19**) *is undoubtedly a truly general equation* *representing the static characteristics of normal arcs between any electrode materials.*

The Cathode, Anode, and Arc Falls as Comfonents of E

The three components of the difference in potential between the electrodes have long been recognized as the anode fall, the cathode fall, and the arc fall (See Fig. 6), but there is still some doubt as to the exact dependence of each on the current intensity and the arc length. The simplest of these, the arc fall, is

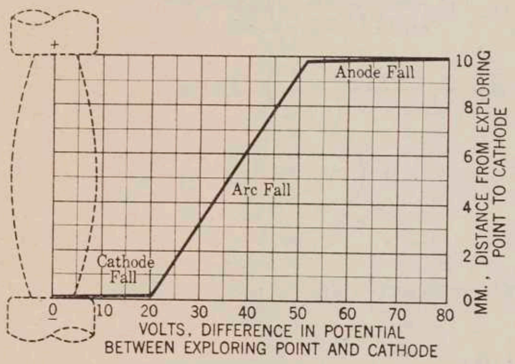

Fig. 6

generally thought to be a linear function of the arc length and a hyperbolic function of the current. If this is true, the arc fall (e_a) must be represented by the last term of equation (**19**).

$$e_a = \left(\gamma_\infty + \frac{\gamma_1 - \gamma_\infty}{i^n}\right) \cdot L \qquad (20)$$

Some studies of the cathode fall have shown it to be independent of the arc length and the current. Although it is quite impossible actually to measure the cathode fall in arcs less than 1.0 mm. in length, the fact that arcs of 0.1 mm. can be maintained on less than 20.0 volts (Grotrian's[12] value for the cathode fall alone) invalidates the theory that the cathode fall is independent of the arc length when the arcs are short. A change in the magnitude of the cathode and the anode falls might naturally be expected when the arcs are short, because these falls take place in the space a fraction of a millimeter from the surfaces of the electrodes instead of exactly at their surfaces. In addition, it is apparent from the curves in Fig. 5 that the sum of the anode fall (e_b) and the cathode fall (e_c) can be neither independent of the arc length nor a linear function of the arc length. It is a plausible assumption that the cathode fall reacts upon the anode fall as the arc length becomes shorter until finally the two falls merge, and each rapidly cancels the effect of the other. This interaction may be measured roughly by the deviation of the slope of the curves for constant current from the minimum slope of these curves. The relation between the slope and the arc length for any particular current is expressed by equation (**21**).

$$\frac{dE}{dL} = \left(\alpha_\infty + \frac{\alpha_1 - \alpha_\infty}{i^n}\right) \cdot d \cdot \epsilon^{-d \cdot L} + \left(\beta_\infty + \frac{\beta_1 - \beta_\infty}{i^n}\right) \left(\delta_\infty + \frac{\delta_1 - \delta_\infty}{i^n}\right) \cdot \epsilon^{-\left(\delta_\infty + \frac{\delta_1 - \delta_\infty}{i^n}\right) \cdot L} + \left(\gamma_\infty + \frac{\gamma_1 - \gamma_\infty}{i^n}\right) \qquad (21)$$

The curve in Fig. 7 is the graphical representation of equation (**21**) for an arc between copper electrodes when the current is 3.0 amperes.

Since the cathode fall is independent of the arc length for arcs longer than 1.0 mm., the second term of equation (**19**) must be a function of the anode fall. Therefore, this term represents either the *increase in* the anode fall as the arc length increases and the *change* in this increase as the current varies, or the *total anode fall* for any arc length and current. If the latter is the case, the first term of equation (**19**) represents the cathode fall for any current and arc length. Such conclusions would be in entire agreement with Mrs. Ayrton's[13] observations of the arc in its component parts.

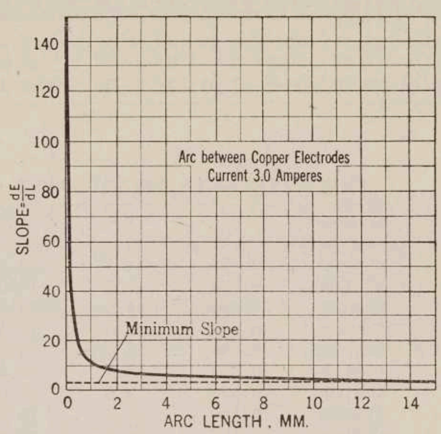

Fig. 7—A Measure of the Interaction of the Cathode and the Anode Falls

It is evident from this examination of the equations (**19**) and (**21**) that arcs of less than 15.0 mm. can not possibly be represented accurately by an equation less complex than equation (**19**). However, arcs of a length greater than 15.0 mm. can be represented by equation (**22**) which is not unlike Mrs. Ayrton's in simplicity.

$$E = A + B \cdot L + \frac{C + D \cdot L}{i^n} \qquad (22)$$

12. W. Grotrian, *Ann. d. Phy.*, 47, p. 141, (1915).

13. The Electric Arc, p. 238.

In equation (**22**),
$A = \alpha_\infty + \beta_\infty$, $B = \gamma_\infty$,
$C = \alpha_1 - \alpha_\infty + \beta_1 - \beta_\infty$, and $D = \gamma_1 - \gamma_\infty$

Although the results of this study are illuminating, they only point the way. A more extensive investigation of the arc in different atmospheres and at different pressures should be undertaken in hope that this equation, or some modification of it, will offer a method by which the arc characteristic can be studied in its component parts without the introduction of an exploring electrode. The equation might then be used to solve the many problems involving the distribution of the power consumption in the arc and the problem of determining the temperature in the arc, which now confronts the investigators of commercial arcs employed for the fixation of nitrogen, arc lighting, the electric furnace, etc.

Conclusions

1. The exponent n of the current i in the equation

$$E = A + B/i^n \qquad (3)$$

is directly proportional to the absolute temperature of the boiling point or the sublimation point, as the case may be, of the anode.

2. Equation (**19**) can be made to represent the static characteristic of the normal arc without apparent systematic error.

$$E = \left(\alpha_\infty + \frac{\alpha_1 - \alpha_\infty}{i^n} \right) (1 - \epsilon^{-d \cdot L}) + \left(\beta_\infty + \frac{\beta_1 - \beta_\infty}{i^n} \right) \left(1 - \epsilon^{-\left(\delta_\infty + \frac{\delta_1 - \delta_\infty}{i^n}\right) \cdot L} \right) + \left(\gamma_\infty + \frac{\gamma_1 - \gamma_\infty}{i^n} \right) \cdot L \qquad (19)$$

3. Arcs more than 15.0 mm. in length can be represented accurately by the equation

$$E = A + B \cdot L + \frac{C + D \cdot L}{i^n} \qquad (21)$$

I wish to acknowledge my indebtedness to the American-Scandinavian Foundation whose fellowship made this work possible, and to Professor Gustav Grandqvist of Uppsala, Sweden and Mr. P. P. Cram for their assistance in the preparation of paper.

Appendix

The Description of the Apparatus. The electrodes were 12 mm. in diameter and made of the very highest grade of materials. After each reading the electrodes were refaced to insure precision in arc length measurements and to reduce the meter fluctations to a minimum. The upper electrode was the anode except in the cases of the cadmium, antimony, bismuth and lead arcs.

Loosely fitting jackets supplied by independent water systems, were used to carry the excessive heat from the electrode supports. Readings taken with and without these jackets showed that this slight cooling had no appreciable effect on the arc characteristic.

For the longer arcs, the length was determined by the measurement of the image of the arc produced by a lens system (magnification 15 diameters) on a scale calibrated to read arc length directly in millimeters. The short arcs were adjusted by a micrometer screw.

The arc was freely open to the air at all times.

The Observed Data. The principal data from which the constants of the equations were calculated, are recorded in the following tables, *I* to *X* inclusive.

TABLE I

Anode—Carbon Cathode—Copper

Arc length *L*	1 mm.	2 mm.	3 mm.	4 mm.	5 mm.	6 mm.	7 mm.	8 mm.	10mm.
Current amperes	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*
1.0	61.0	76.0	87.0	95.5	102.5	108.5	114.5	120.0	130.0
1.5	54.0	67.5	77.0	84.5	90.0	94.5	99.0	103.0	110.5
2.0	50.5	63.0	72.0	78.5	83.5	87.5	91.0	95.0	101.0
2.5	48.5	60.5	68.5	74.5	79.5	83.0	86.5	90.0	95.5
3.0	47.0	59.0	66.5	72.5	77.0	80.0	83.0	86.0	91.5
4.0	45.3	56.5	64.0	69.5	73.5	76.5	79.5	82.0	86.5
5.0	44.2	55.5	62.5	67.5	71.5	74.5	77.0	79.5	84.0
7.0	43.0	54.0	60.5	65.5	69.0	72.0	74.2	76.5	80.0
10.0	42.0	52.5	59.5	64.0	67.5	70.0	72.5	74.5	78.0

TABLE II

Anode—Cadmium (oxide) Cathode—Carbon

Arc length *L*	1 mm.	2 mm.	3 mm.	4 mm.	5 mm.	6 mm.	7 mm.	8 mm.	10mm.
Current amperes	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*
1.0	27.5	38.5	48.5	57.5	65.5	73.0	80.0	86.5	98.0
1.5	24.0	32.5	40.5	47.5	53.5	60.0	65.0	70.5	80.0
2.0	21.5	29.0	36.5	42.5	48.0	53.0	57.0	62.0	67.5
2.5	20.5	27.0	33.5	39.0	43.5	48.0	52.0	56.0	63.0
3.0	19.5	25.5	31.0	36.5	40.5	45.0	48.5	52.0	58.0
4.0	18.0	23.5	28.5	32.0	36.5	40.5	44.0	47.0	52.5
5.0	17.5	22.5	27.0	31.0	34.0	37.5	41.0	44.0	49.0
7.0	16.3	20.5	24.5	28.0	31.5	34.0	37.0	39.0	43.5
10.0	15.5	19.5	23.0	26.5	29.0	31.5	34.0	36.0	39.5

TABLE III

Anode—Copper (oxide) Cathode—Copper

Arc length *L*	1 mm.	2 mm.	3 mm.	4 mm.	5 mm.	6 mm.	7 mm.	8 mm.	10mm.
Current amperes	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*
1.0	46.0	60.0	71.5	81.0	89.0	96.0	102.0	108.0	120.0
1.5	40.0	52.0	61.0	69.0	75.5	82.0	87.5	93.0	102.0
2.0	36.0	47.0	55.5	62.0	68.0	74.0	79.0	84.0	92.0
2.5	34.0	43.5	51.5	58.0	63.0	68.5	73.0	77.5	85.0
3.0	32.5	41.5	49.0	55.0	60.0	65.0	69.0	73.5	80.0
4.0	30.0	38.0	45.0	50.5	55.5	60.0	64.0	68.0	74.5
5.0	28.5	36.5	42.5	48.0	53.0	57.0	61.0	64.5	70.5
7.0	26.5	33.5	39.5	44.0	48.5	52.5	56.0	59.5	65.5
10.0	25.5	31.5	37.0	41.5	45.5	49.5	52.5	55.5	61.0

TABLE IV

Anode—Aluminum (oxide) Cathode—Carbon

Arc length L	1 mm.	2 mm.	3 mm.	4 mm.	5 mm.	6 mm.	7 mm.	8 mm.	10mm.
Current amperes	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*
1.0	34.0	47.0	56.0	65.5	73.0	79.5	85.5	91.0	102.0
1.5	28.0	38.5	45.5	53.0	58.5	64.0	69.0	74.0	82.0
2.0	25.0	34.0	40.0	46.0	51.0	55.5	60.0	64.0	71.5
2.5	22.5	30.5	36.0	41.5	46.0	50.0	53.5	57.0	64.0
3.0	21.0	28.5	33.0	38.0	42.5	46.0	49.5	52.5	59.0
4.0	19.0	27.5	29.5	34.0	38.0	41.0	44.0	47.0	52.0
5.0	18.0	23.5	27.5	31.0	34.5	38.0	40.5	43.0	47.5
7.0	16.0	21.0	24.6	28.0	31.0	33.0	36.0	38.0	42.0
10.0	15.0	19.0	22.5	25.0	28.0	30.0	32.0	34.0	38.0

TABLE V

Anode—Nickel (oxide) Cathode—Carbon

Arc length L	1 mm.	2 mm.	3 mm.	4 mm.	5 mm.	6 mm.	7 mm.	8 mm.	10mm.
Current amperes	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*
1.0	42.0	53.5	64.0	73.0	81.0	88.5	95.0	101.5	113.5
1.5	35.5	44.0	53.5	60.0	67.0	73.5	78.0	83.0	93.0
2.0	32.0	39.5	47.0	53.0	59.0	64.0	69.0	73.0	82.0
2.5	29.0	36.0	42.5	48.0	53.0	58.0	62.0	66.0	74.0
3.0	27.0	34.0	39.5	45.0	49.5	54.0	57.0	61.0	69.0
4.0	24.5	30.0	35.5	40.0	44.5	48.5	51.5	55.0	61.0
5.0	23.0	28.0	33.0	37.0	41.0	45.0	48.5	51.0	57.0
7.0	21.0	25.5	29.0	33.5	37.0	40.0	42.5	45.5	50.5
10.0	19.0	23.0	27.0	30.0	33.0	36.0	38.5	41.0	46.0

TABLE VI

Anode—Silver Cathode—Carbon

Arc length L	1 mm.	2 mm.	3 mm.	4 mm.	5 mm.	6 mm.	7 mm.	8 mm.	10mm.
Current amperes	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*
1.0	41.5	54.0	65.0	75.0	84.0	92.5	100.5	108.0	121.5
1.5	36.0	46.0	54.5	63.0	69.5	78.0	83.5	90.0	99.5
2.0	32.5	42.5	49.0	55.5	62.0	69.0	74.0	79.5	88.5
2.5	30.0	38.0	45.0	51.0	56.5	62.5	67.5	72.0	81.0
3.0	28.5	36.0	42.5	48.0	53.0	58.0	62.5	67.0	74.5
4.0	26.0	32.5	38.5	43.5	48.0	52.0	56.5	60.0	66.5
5.0	25.0	30.5	36.0	40.5	45.0	49.0	51.9	55.5	61.0
7.0	23.0	28.0	32.5	36.5	40.0	44.0	48.0	50.0	55.0
10.0	22.5	26.0	30.0	33.5	36.5	40.0	43.0	45.0	50.0

TABLE VII

Anode—Zinc (oxide) Cathode—Carbon

Arc length L	1 mm.	2 mm.	3 mm.	4 mm.	5 mm.	6 mm.	7 mm.	8 mm.	10mm.
Current amperes	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*
1.0	38.5	51.0	61.0	71.0	79.0	86.5	93.5	100.0	112.0
1.5	33.0	43.5	51.5	59.5	66.5	72.5	78.0	83.0	92.0
2.0	30.0	39.0	45.5	53.0	59.5	64.0	69.0	74.0	81.0
2.5	27.5	36.0	42.0	48.0	53.5	58.0	63.0	67.5	74.0
3.0	24.5	33.5	39.0	44.5	49.5	54.0	58.0	62.0	69.0
4.0	23.5	30.0	35.0	40.0	44.0	48.0	52.5	55.0	61.0
5.0	22.5	28.0	32.5	37.0	41.0	44.0	47.0	50.0	55.0
7.0	20.0	25.0	29.0	33.0	36.0	39.0	41.5	44.0	48.0
10.0	18.0	22.5	26.0	29.0	32.0	34.5	36.5	39.0	42.0

TABLE VIII

Anode—Lead Cathode—Carbon

Arc length L	1 mm.	2 mm.	3 mm.	4 mm.	5 mm.	6 mm.	7 mm.	8 mm.	10mm.
Current amperes	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*
1.0	25.0	32.5	40.0	46.5	53.0	59.0	65.0	71.0	82.0
1.5	22.5	29.0	35.0	41.0	46.0	51.0	56.0	61.5	71.0
2.0	21.0	26.5	32.0	37.5	42.5	47.0	51.0	56.0	64.5
2.5	19.5	25.0	30.0	35.0	39.5	44.0	48.0	52.0	60.0
3.0	18.8	23.5	28.5	33.0	38.5	41.5	45.0	49.0	56.0
4.0	17.5	22.5	26.5	30.5	34.5	38.0	41.5	45.5	52.0
5.0	17.0	21.0	25.0	29.0	32.5	36.0	39.5	42.5	49.0
7.0	15.8	19.5	23.5	26.5	30.0	33.0	36.0	39.0	44.5

TABLE IX

Anode—Antimony Cathode—Carbon

Arc length L	1 mm.	2 mm.	3 mm.	4 mm.	5 mm.	6 mm.	7 mm.	8 mm.	10mm.
Current amperes	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*
1.0	33.0	41.0	48.5	55.5	62.0	68.0	74.0	80.0	91.0
1.5	29.5	36.5	43.0	48.5	54.0	59.0	64.0	70.0	79.0
2.0	27.5	33.5	39.5	45.0	49.5	54.0	58.5	63.0	71.5
2.5	26.0	31.5	37.0	42.0	46.0	50.0	54.5	58.5	66.5
3.0	25.0	30.0	35.0	40.0	44.0	48.0	51.5	55.5	63.0
4.0	23.5	28.0	33.0	37.0	40.5	44.5	48.0	51.5	57.5
5.0	22.5	27.0	31.0	35.0	38.5	42.0	45.0	48.5	54.5
7.0	21.0	25.0	29.0	32.0	35.5	38.5	41.5	45.0	50.0

TABLE X

Anode—Bismuth Cathode—Carbon

Arc length L	1 mm.	2 mm.	3 mm.	4 mm.	5 mm.	6 mm.	7 mm.	8 mm.	10mm.
Current amperes	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*	*E*
1.0	27.0	35.5	43.5	51.0	58.0	64.5	71.0	77.0	88.5
1.5	24.0	31.5	38.5	45.0	50.5	56.5	62.0	67.0	78.0
2.0	22.5	29.0	35.5	41.5	47.0	51.5	57.0	61.5	71.0
2.5	21.0	27.5	33.5	38.5	44.0	48.5	52.5	57.0	66.0
3.0	19.5	26.0	32.0	37.0	41.5	46.0	50.0	54.0	62.5
4.0	19.0	24.5	29.5	34.0	38.5	42.5	46.5	50.0	57.0
5.0	18.0	23.0	28.0	32.0	36.0	40.0	43.5	47.0	54.0
7.0	16.5	21.5	25.5	29.5	33.0	37.0	40.0	43.0	49.0