THE THERMIONIC DIODE AS A HEAT-TO-ELECTRICAL POWER TRANSDUCER, undated

THE THERMIONIC DIODE AS A HEAT-TO-ELECTRICAL-POWER TRANSDUCER*

W. B. Nottingham

Massachusetts Institute of Technology, Cambridge, Massachusetts

Introduction

The thermionic diode is a heat-to-electrical-power transducer if a fraction of the heating power required to maintain the emitter at a constant temperature can be recovered as useful electrical power in the external circuit that joins the electron collecting electrode to the emitter. The two broad classes of this device are known as the high vacuum diode and the plasma diode. The main purpose of this paper is to discuss the theoretical background by which the performance characteristics of the high vacuum diode can be evaluated in terms of the three most important parameters which are: 1. the temperature of the emitter; 2. the spacing between the emitter and the collector; and 3. the work-function of the collector. It will be shown that the work-function of the emitter is not critical in most transducer designs of practical importance. The necessary detailed technical information to predict the operating characteristics of a "plasma diode" converter are not available at this time. A rather superficial examination of this diode indicates that the emitter work-function will play a far more important part, even though always secondary to that of the collector.

All of the basic information on which the theory of the high vacuum diode depends was published by W. B. Nottingham in "Thermionic Emission" [1]. In this article the treatment of the space-charge problem differs from that developed by Langmuir and others only in the choice of the basic reference used to relate the results of computation to the boundary conditions of an actual experiment. Specifically, the reference condition of importance is that associated with the coincidence of the "space-charge minimum" with the collector surface. For this idealized situation to be realized in practice, the work-function of the collector must be uniform. Failure to achieve uniformity can generally be recognized as a measurable and systematic deviation between experimental results and the predictions of the idealized theory.

Many of the details on this subject are covered in a recent article[2]. It is the purpose of this report to review some of the facts already covered and supplement them with additional data and discussion.

Current-Voltage Characteristics of the High Vacuum Converter

A diode may be constructed with plane parallel electrodes and, after suitable processing, connected in a circuit similar to that illustrated by the insert in Fig. 1. The voltmeter indicates the applied voltage V_a and the electron current is measured

---

*This work was supported in part by the U. S. Army (Signal Corps), the U. S. Air Force (Office of Scientific Research, Air Research and Development Command), and the U. S. Navy (Office of Naval Research).

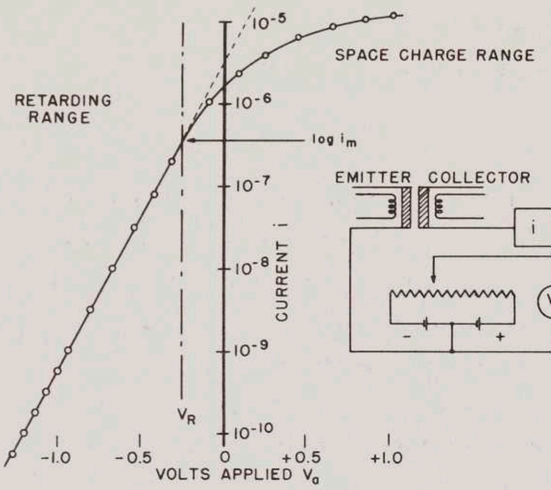


Fig. 1

Diode voltage-current data for temperature determination and onset of space-charge limitation.

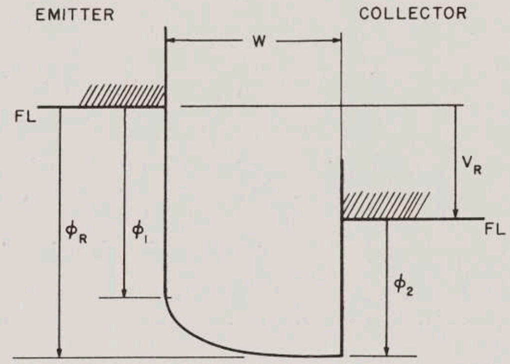


Fig. 2

Potential distribution with critical condition of zero gradient at the collector.

as a function of the applied potential. Even if the diode is constructed to have no appreciable ohmic leakage over surfaces that support the emitter and the collector, it may be necessary to correct the observed currents for any photoelectric or thermionic emission that flows in the opposite direction to the thermionically emitted electrons from the "emitter." After these corrections are made, a plot may be prepared in the manner shown in Fig. 1 to record the logarithm of the electron current as a function of the applied voltage. Over many orders of magnitude of current, its logarithm will be a linear function of the applied voltage. This is the "retarding range." At a critical value of the applied potential designated V_R, this linear plot joins tangentially to a curve characterized by the "space-charge range." It is not easy to identify this point of tangency by inspection. The plotting of the data according to a method to be described permits the observer to compare his observations with a universal curve to establish with considerable accuracy the value of V_R and the current flow at the point of tangency identified here as i_m.

Accurate data taken in the retarding range serve as the best means of determining the true electron temperature of the emitter. Equation 1 serves to define the relation between the temperature T (Kelvin scale) and the electron-volt equivalent of temperature V_T.

$$V_T = \frac{k}{q} T = \frac{T}{11,600} \qquad (1)$$

The basic equations for the relation between the current and the applied potential are given in the direct and the logarithmic forms in Eqs. 2 and 3.

$$i = i_o \, e^{-\frac{V_a}{V_T}} \tag{2}$$

$$\log_{10} i = \log_{10} i_o - \frac{V_a}{2.3\, V_T} \tag{3}$$

The constant i_o in these equations is an empirical constant and has no further significance. A plot similar to Fig. 1 permits one to establish the best straight line to represent all of the data taken in the retarding range. Any two points on this line serve to determine the electron temperature by the following equation:

$$V_T = \frac{V_2 - V_1}{2.3 \log_{10} (i_1/i_2)} = \frac{T}{11,600} \tag{4}$$

The critical condition, referred to at the applied potential V_R that divides the retarding from the space-charge regions, corresponds to a potential distribution between the emitter and the collector represented schematically by Fig. 2. Note that the work-functions of the emitter and the collector are respectively ϕ_1 and ϕ_2 and that the voltmeter reading V_R corresponds to the displacement of the Fermi level of the collector with respect to the Fermi level of the emitter and is, therefore, a directly observable quantity. If the applied potential is more negative than this amount, then even though some space charge exists between the emitter and the collector, it has no influence on the electron current which can flow across the space w.

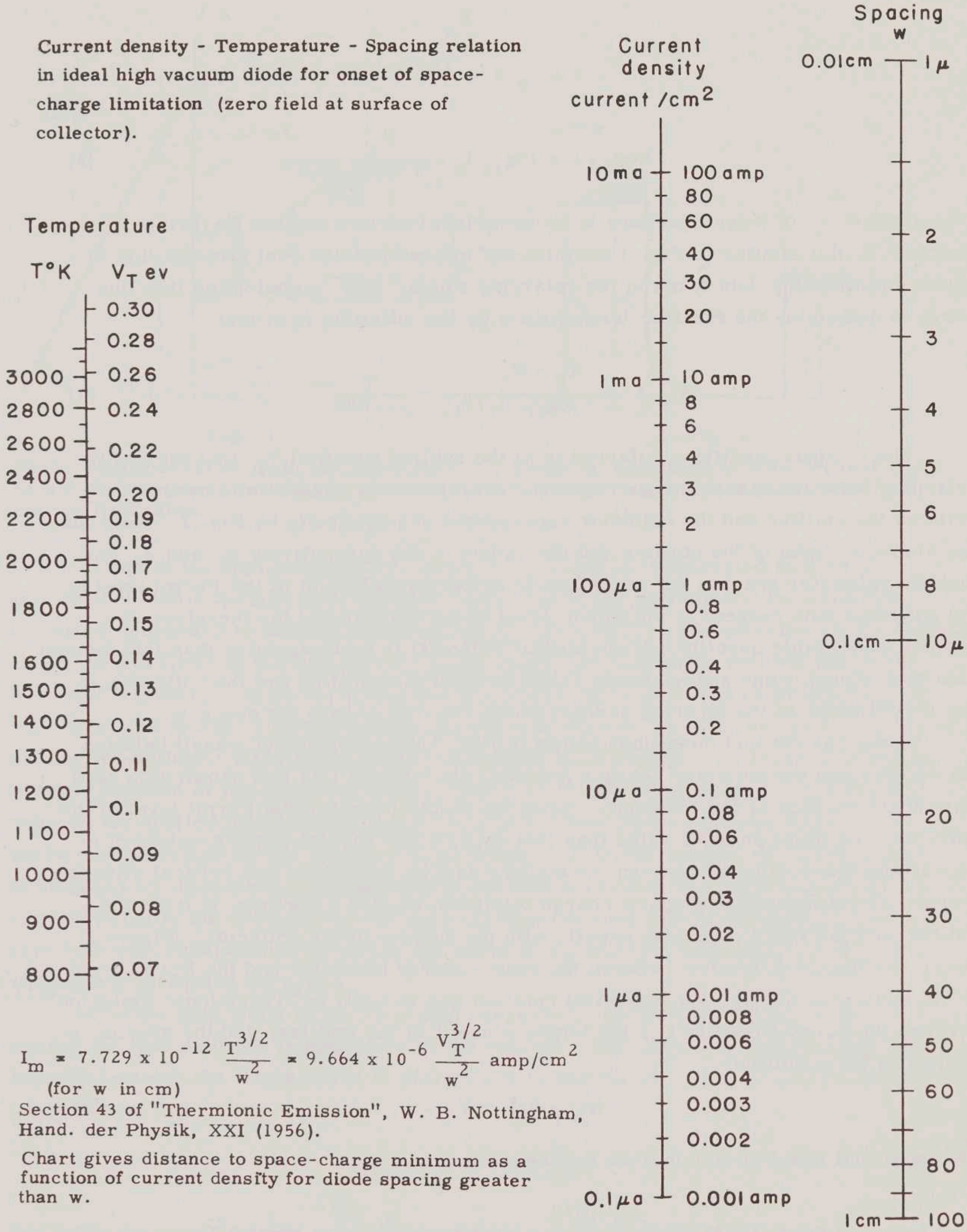
Under the critical conditions shown in Fig. 2 the electrons in transit between the emitter and the collector set up a potential distribution like that shown with zero field at the surface of the collector. Since the displacement of the Fermi level of the collector to a more positive value than this critical one results in the creation of a "space-charge minimum" between the emitter and the collector, this critical situation therefore corresponds to a space-charge minimum located a distance w from the emitter and therefore coincides exactly with the surface of the collector. Figure 2 shows the energy difference between the space-charge minimum and the Fermi level of the emitter to be ϕ_R. An important relation was derived in "Thermionic Emission"[1] between the current density I_m; the temperature T of the emitter; and the spacing w subject to the condition:

$$(\phi_R - \phi_1) > V_T \tag{5}$$

The universal equation which gives this relation is:

$$I_m = 7.729 \times 10^{-12} \frac{T^{3/2}}{w^2} = 9.664 \times 10^{-6} \frac{V_T^{3/2}}{w^2} \text{ amp/m}^2 \tag{6}$$

Current density - Temperature - Spacing relation in ideal high vacuum diode for onset of space-charge limitation (zero field at surface of collector).



$$I_m = 7.729 \times 10^{-12} \frac{T^{3/2}}{w^2} = 9.664 \times 10^{-6} \frac{V_T^{3/2}}{w^2} \text{ amp/cm}^2$$

(for w in cm)

Section 43 of "Thermionic Emission", W. B. Nottingham, Hand. der Physik, XXI (1956).

Chart gives distance to space-charge minimum as a function of current density for diode spacing greater than w.

Fig. 3

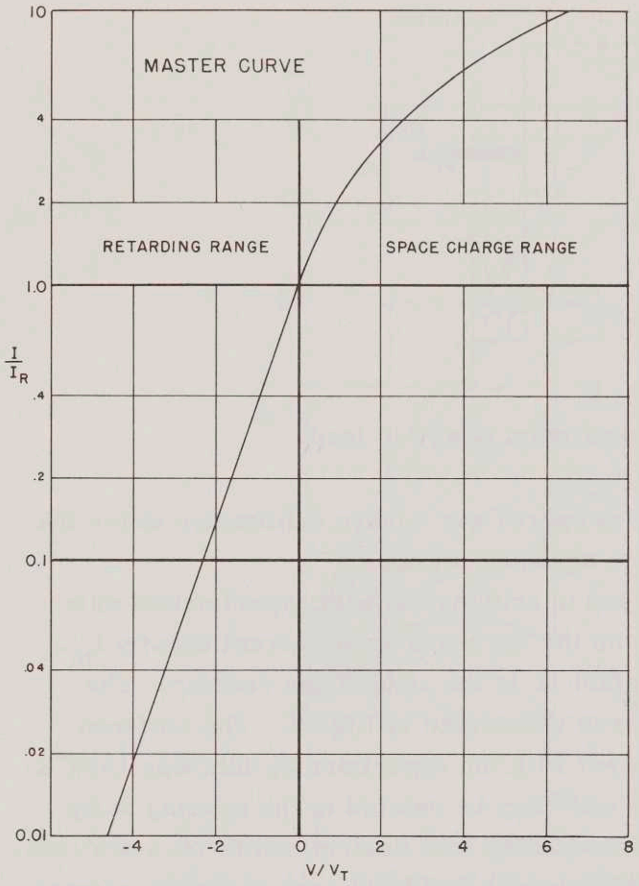


Fig. 4

Master curve used to determine onset of space-charge. See Table 8 of "Thermionic Emission."

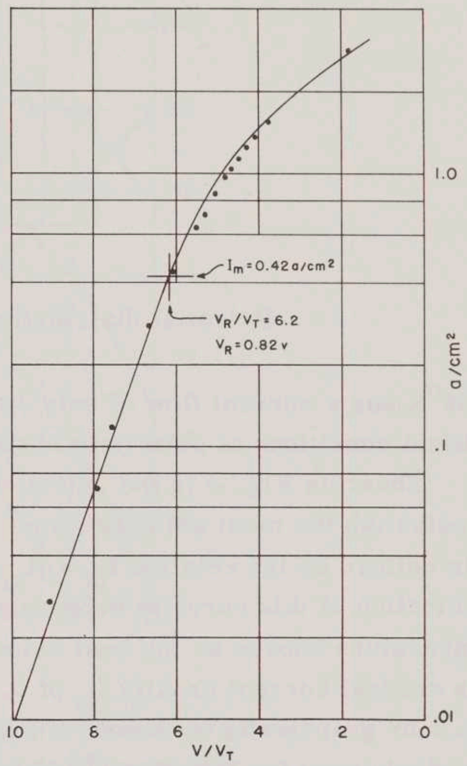


Fig. 5

Comparison theory and experiment. (Data furnished by Hatsopoulos and Kaye.) Spacing 10.5×10^{-6} m from analysis.

Since Eq. 6 is such an important one in many problems related to thermionic emission and space charge, a nomographic chart has been prepared and is presented here as Fig. 3. This chart may also be used to find the distance from the emitter to the space-charge minimum at w when it is less than the diode spacing.

The left and the right scales correspond to the temperature and the spacing respectively. Note that the right-hand side of the spacing scale expresses w in microns, whereas the left-hand side of the scale expresses it in centimeters. Since 100 microns and 0.01 centimeters are the same, it is clear that the chart is applicable to the entire range of spacing from 1 micron to 1 centimeter. The current density scale corresponds to the spacing scale in that the densities expressed in amperes per square centimeter on the right-hand side of the line correspond to the use of spacings between 1 and 100 microns. For the larger spacing up to 1 centimeter the current density units are given on the left-hand side of the scale and range from a maximum of 10 milliamperes per square centimeter down to 0.1 microampere per square centimeter. As an example of the use of this chart, note that a diode of 1 mm spacing operated at a temperature of

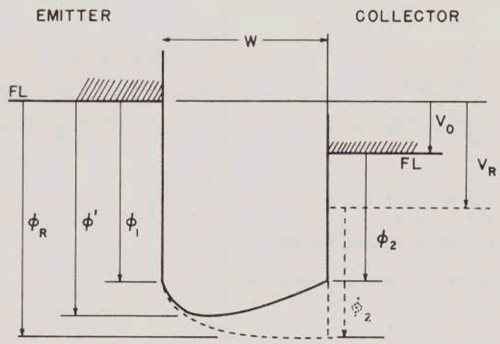


Fig. 6
Potential distribution with maximum power in load.

1160^{O}K has a current flow of only 30.6 microamperes per square centimeter under the critical conditions of zero field at the surface of the collector.

Shown in Fig. 4 is the master curve used in conjunction with experimental data to establish the most suitable choice of V_R and the corresponding current density I_m. It is defined by the relation $I_m = (i_m/a)$ in which a is the area of the emitter. The application of this curve to experimental data is illustrated in Fig. 5. The electron temperature chosen as the best value consistent with the experimental data was 1535^{O}K. The critical current density I_m of 0.42 amp/cm^2 may be related to the spacing w by Eq. 6 or graphically by means of Fig. 3. The spacing thus determined is 10.5 microns. The diode used for this study had an adjustable spacing and had been set for 10 microns. The scatter of the points in the retarding range would have been smaller if more precise control of the emitter temperature had been maintained during the period of observation.

Power to an External Load

Figure 2 illustrates a potential distribution in a transducer in which the power available to an external circuit is the product $V_R i_m$. This seldom represents the maximum power available from a high vacuum transducer. The potential distribution illustrated by Fig. 6 shows a situation in which the output voltage has been reduced from V_R to V_O. As this change in output voltage takes place, the barrier across which the emitted electrons must travel is reduced from ϕ_R to ϕ' and, subject to the condition $(\phi' - \phi_1)$ is greater than V_T, the current flow around the diode increases according to the relation

$$i = i_m \, e^{\frac{\phi_R - \phi'}{V_T}} \tag{7}$$

Since the power is the product of V_O and i, the power available increases to a maximum and finally falls to zero when V_O is zero. Space-charge theory permits one to establish a universal relation by which the power output may be computed from the parameter (V_R/V_T).

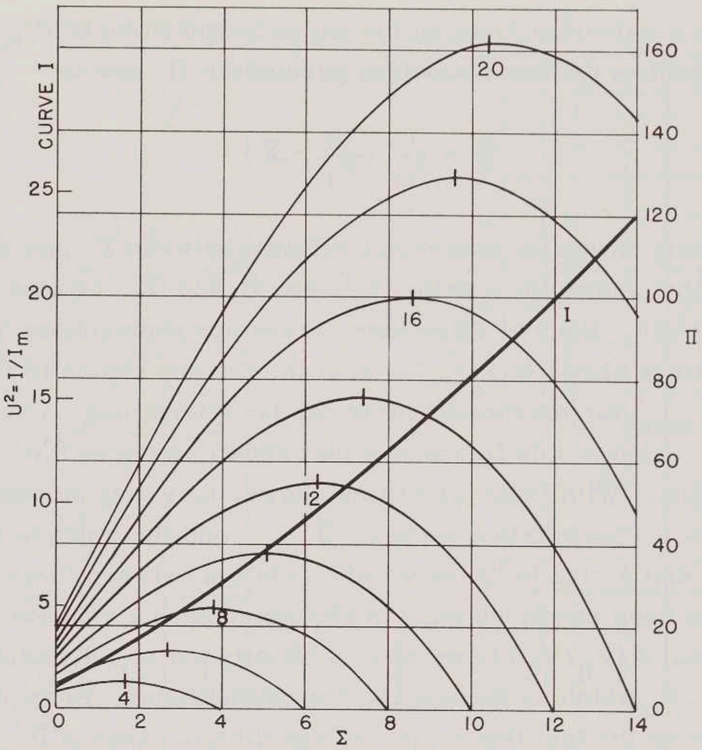


Fig. 7
Values of Π as a function of Σ for selected values of (V_R/V_T) of 4 to 20. Values of (I/I_m) of curve I from "Thermionic Emission" Table 8.

As V_O becomes smaller than V_R it is convenient to express this <u>change</u> in output voltage in units of V_T. This change is expressed by Σ defined by Eq. 8.

$$\Sigma = \frac{V_R - V_O}{V_T} \tag{8}$$

This equation rearranged to express the value of V_O is

$$V_O = V_T\left[\frac{V_R}{V_T} - \Sigma\right] \tag{9}$$

The power per unit area is the product of the current density and the output voltage and is written as follows:

$$P = IV_O = I_m V_T \left[\frac{I}{I_m}\left(\frac{V_R}{V_T} - \Sigma\right)\right] \tag{10}$$

Since Σ and the ratio (I/I_m) are uniquely related to each other as shown by Table 8 of "Thermionic Emission," the factor of Eq. 10 enclosed within the square brackets may be computed as a universal function for any selected value of (V_R/V_T). The bracketed quantity defines the dimensionless parameter Π and is

$$\Pi = \frac{I}{I_m} \left(\frac{V_R}{V_T} - \Sigma\right) \tag{11}$$

Figure 7 not only shows the numerical relation between Σ and the current ratio $U^2 = (I/I_m)$ but it also shows the relation between Π and Σ for nine different values of (V_R/V_T) from 4 to 20. Each of these nine curves has a maximum located very close to the vertical markings shown in Fig. 7. That is, for any choice of (V_R/V_T) between these values, the Σ_{max} for maximum power can be determined. Curve I of Fig. 8 shows this relation. Also in this figure is a plot shown as curve II of the current ratio U^2_{max} at the maximum. With these quantities known, they may be combined as in Eq. 11 to establish a unique relation between Π_{max} and the important parameter (V_R/V_T). The fact that a simple "square-law" relation between these quantities exists could not have been predicted analytically since such a relation is not precisely correct for all values of (V_R/V_T) from zero to infinity but is substantially correct for the range from 4 to 20, which is included by the calculations. Proof of the square-law relation depends on the fact that a plot on logarithmic paper of Π_{max} as a function of (V_R/V_T) is well represented by the straight line of slope 2 shown in Fig. 9. The equation for this straight line is:

$$\Pi_{max} = 0.385 \left(\frac{V_R}{V_T}\right)^2 \tag{12}$$

The power delivered to the external load is expressed by Eq. 13 and the combination with Eqs. 6 and 12 in this form gives Eq. 14.

$$P_{max} = I_m V_T \Pi_{max} \tag{13}$$

$$P_{max} = 3.7 \times 10^{-6} V_T^{1/2} \frac{V_R^2}{w^2} \tag{14}$$

Equation 14 is very important to the designer who wishes to obtain the maximum power since it brings out very clearly the importance of small spacing. The need for a high value of V_R is also important. Since this critical voltage V_R is strongly influenced by the work-function of the collector, it is of the utmost importance that the collector work-function be as low as possible in order to increase V_R for any given configuration and emitter temperature.

The current density at maximum power is given by

$$I_{max} = I_m \left[1 + 0.31 \left(\frac{V_R}{V_T}\right)^{4/3}\right] \tag{15}$$

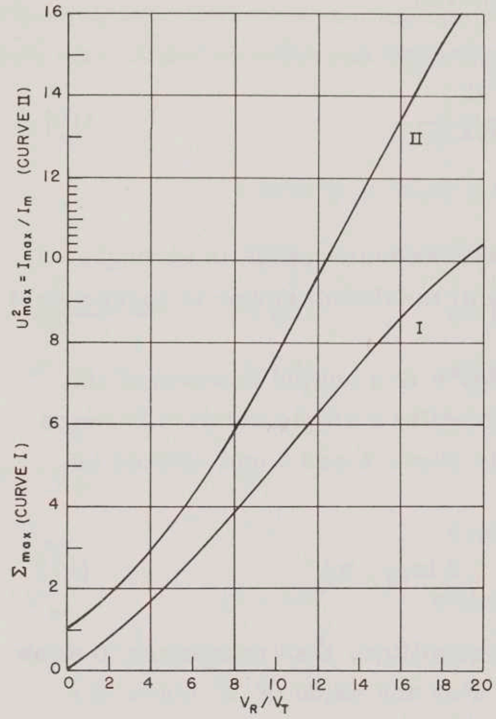


Fig. 8

Values of Σ_{max} and U^2_{max} as functions of V_R/V_T.

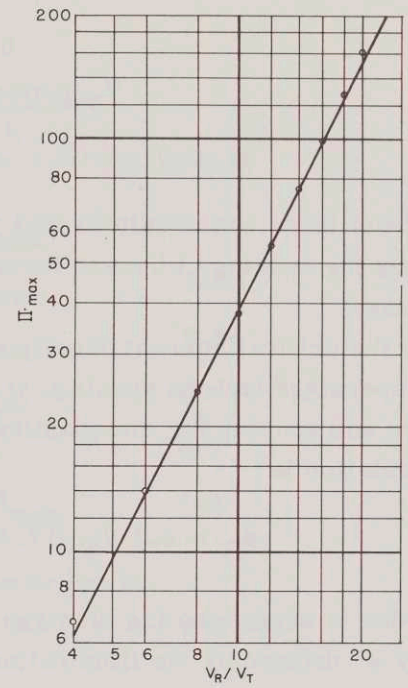


Fig. 9

Values of Π_{max} as a function of V_R/V_T. The straight line representation of the data is a square law.

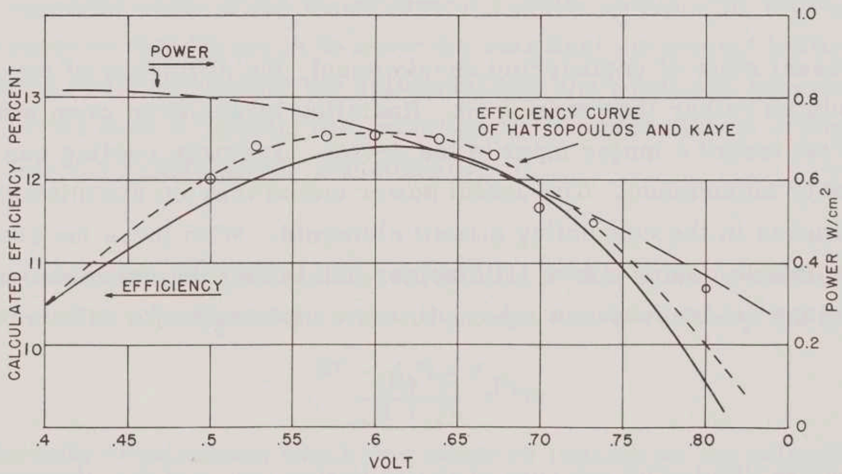


Fig. 10

Comparison between theory and experiment. (Data furnished by Hatsopoulos and Kaye.)

The output voltage at maximum power may be computed by

$$V_{out} = \frac{0.383 \left(\frac{V_R}{V_T}\right) V_R}{1 + 0.31 \left(\frac{V_R}{V_T}\right)^{4/3}} \qquad (16)$$

It follows from these two equations that the current at maximum power is strongly influenced by the spacing, whereas the output voltage at maximum power is <u>independent</u> to the spacing.

Since the critical current density as given by Eq. 6 is a unique function of the emitter temperature and the spacing, it is possible to write a single relation between temperature and spacing for the quantity illustrated in Figs. 2 and 6 and defined as ϕ_R. This relation is

$$\phi_R = 2.3\ V_T\ (17.2 + \tfrac{1}{2} \log_{10} T + 2 \log_{10} w) \qquad (17)$$

In order to determine the electron cooling of the emitter, it is necessary to know the quantity ϕ' defined by the illustration in Fig. 6. For any value of Σ there is a unique value of (I/I_m) as shown graphically in Fig. 7 and tabulated in Table 8 of "Thermionic Emission." Equation 18 serves then as the means for calculating ϕ'.

$$\phi' = \phi_R - 2.3\ V_T \log_{10} \frac{I}{I_m} \qquad (18)$$

The electron cooling per unit area is then calculated as follows:

$$P_e = I(\phi' + 2\ V_T) \qquad (19)$$

<u>Efficiency</u>

At the present state of engineering development, the efficiency of the transducer is usually calculated rather than measured. Radiation losses even when minimized as far as possible represent a major input to the device. Electron cooling can be of nearly comparable importance. The useful power output may be diminished by electrical resistance losses in the connecting circuit elements. With these uncertainties it is nearly impossible to compute true efficiencies and in fact the approach to some ideal will depend on many design features. An optimistic expression for efficiency is given by

$$\text{eff} = \frac{P_{out}}{P_r + P_e} \qquad (20)$$

The power output is divided by the sum of the radiation loss and the electron cooling. If the only radiation loss is from the emitter to the collector, and this loss is inevitable, then no practical design can be expected given efficiency greater than that described by this equation. Based on a very approximate method of calculating the minimum radiation loss, the following table has been prepared.

Calculated Properties

Diode $w = 2.54 \times 10^{-5}$ m; $\phi_2 = 1.57$ ev; Area A

T	1510	^oK	P_{max}	3470	watt/m^2
ϕ_R	2.87	volt	I_{max}	5400	a/m^2
ϕ'	2.60	volt	V_{out}	0.64	volt
V_R	1.3	volt	R_{LO}	$1.2 \times 10^{-4}/A$	ohm
(V_R/V_T)	10		T_c	1025	^oK
I_m	702	a/m^2	$(\phi_1)_{max}$	2.5	volt
P_r	10×10^4	watts/m^2	At max Power		
P_e	1.67×10^4	watts/m^2	$(eff)_o = 3.0$ per cent		

In order to compare the predictions of theory with experiment, a calculation was made to determine the efficiency of a diode spaced at 10 μ as a function of output voltage. Also the power per unit area was calculated as a function of output voltage. The efficiency was calculated according to Eq. 20 with the radiation loss figure P_r thought to be applicable by Hatsopoulos and Kaye[3] so that a comparison could be made with their experimental values. The two calculated curves are shown in Fig. 10 by the solid line for efficiency and the dashed line for power. The dotted line is a smoothed-out representation of the experimental results obtained by Hatsopoulos and Kaye. The purpose of this figure is to show the excellent agreement between theory and experiment in order to support the statement that the equations presented in this report can be relied upon to predict the performance characteristics of the high vacuum diode as a heat-to-electrical-power transducer.

Conclusions

An important consequence of this presentation is that little or nothing needs to be said about the work-function of the emitter except that covered by the inequality

$$\phi' - \phi_1 \geq V_T \qquad (21)$$

That this is the only requirement which one needs to impose on the cathode work-function is not obvious unless a careful inspection is made of Table 9 in "Thermionic Emission" in its relation to Table 8. Figures 16 and 17 of "Thermionic Emission" show the graphical representation of this fact and prove that Eq. 21 is the most important criterion as to whether or not the emitter has a sufficiently low work-function.

In the plasma transducer, heavy ions of low kinetic energy can be trapped in the potential minimum illustrated in Fig. 6. The presence of one ion can effectively cancel the space charge of many hundreds of electrons in transit. The important point is that when the ion density equals the electron density, the ion current is a very small fraction of the electron current.

It seems evident that cesium vapor is the most suitable gas available for use in a thermionic diode transducer. It has a very low ionization potential of 3.88 ev. The density of gas atoms can be controlled either by limiting the total number available or by controlling the temperature of the excess cesium present in the liquid state. Cesium adsorbs to most metallic surfaces and in general reduces their work-function. Thus, an adsorbed layer of cesium of suitable thickness can possibly give the lowest work-function collector available for this purpose. In order for the maximum current to flow across a diode in the presence of cesium vapor, partially ionized, the work-function of the emitter must be low. Such a situation would not of necessity give a good transducer because even though the current might be high, the output voltage would be low. As the effective resistance in the output circuit is increased, the voltage drop across the load increases and the current decreases. The plasma potential thus tends to follow the potential at the surface of the electron collector and the current is reduced because there is an electron "retarding potential" in the immediate neighborhood of the emitter. The surface potential of the emitter is positive with respect to the plasma and an ion retarding sheath exists there which inhibits the loss of ions to that electrode and encourages ions that might be formed from neutral atoms at the surface of the emitter to enter the plasma.

Although many details remain to be filled in, it may be stated again that as far as the operation of the transducer is concerned, the work-function of the emitter is relatively unimportant. Insofar as the emitter may at the same time be the means by which cesium atoms are converted to cesium ions, the efficiency of this conversion can be expected to be work-function sensitive. The future research in this field, therefore, should be directed toward a better understanding of the most efficient means of producing the required number of ions to maintain a plasma. That may be done at the electron emitter itself or at some auxiliary high-temperature electrode.

In spite of the low efficiency of the high vacuum transducer constructed according to presently established techniques, a development of new techniques and the introduction of cesium can very well bring the device to a state in which it may serve as an important method of converting heat to electricity.

References

1. W. B. Nottingham, "Thermionic Emission," Handbuch der Physik, Springer-Verlag, Germany, Vol. 21, 1956.
2. W. B. Nottingham, J. Appl. Phys. 30, 413 (1959); W. B. Nottingham, G. N. Hatsopoulos, and J. Kaye, J. Appl. Phys. 30, 440 (1959).
3. G. N. Hatsopoulos and J. Kaye, J. Appl. Phys. 29, 1124 (1958); Proc. IRE, 46, 1574 (1958).