A SIMPLIFIED METHOD FOR THE COMPUTATION OF THE ELECTRICAL PROPERTIES OF A CLOSE-SPACED THERMIONIC CONVERTER, W.B. NOTTINGHAM
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A Simplified Method for the Computation of the Electrical Properties of A Close-Spaced Thermionic Converter

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Abstract

The electrical properties of close-spaced thermionic diodes likely to be used as high vacuum heat to electrical energy converters may now be predicted with accuracy. This new method depends on the application of accurate equations that permit the designer to introduce all of the important factors which are: (1) the emitter temperature, (2) the diode spacing, (3) the collector work-function, and (4) the emitter work-function. Although tables and plotted curves may be used, all calculations may be carried through quickly with only a good slide rule. Results on seventeen different configurations are compared with the "exact" digital computer solutions obtained by Rittner.

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Introduction

Two publications[1] have been made by the author on the subject of electrical properties of high vacuum thermionic diodes used as heat converters. The scope of those papers was limited by the requirement that the equations be applied only to diode configurations in which the electron emitter could be approximated by one having "unlimited emission capability". For thermionic converters that must function at a maximum temperature of about 1500°K, the dispenser cathode seems to hold great promise. Rittner[2] has demonstrated, by his detailed and exact calculations applied to close spaced diode configurations, that my earlier equations predicted considerably greater power conversion than can be realized practically. It is the purpose of the present paper to call attention to the tables and computational methods outlined in Thermionic Emission[3] and show how they may be applied to the analysis of the electrical properties of close spaced thermionic converters having practical cathodes. Although the computations are somewhat involved, these methods permit the user to predict the anticipated performance of any design configuration likely to be of practical interest. The maximum error need not be more than 4 per cent with a good slide rule as the only computational tool needed.

General Outline of Procedure

Important symbols needed for this discussion are illustrated graphically in Fig. 1 which shows the electron motive diagram for a diode of actual spacing w. This diagram applies from the Fermi level of the emitter through the work-function barrier ϕ_1 just at the surface of the emitter, across the evacuated space to the surface of the collector, and then inside the collector to its Fermi level. The controllable voltage V has been adjusted to V_R so that the space-charge distribution of the electrons in transit results in exactly zero field coinciding with the surface of the collector. Under this condition the current density carried from the emitter to the collector is I_R. The following equation gives the absolute maximum value of electron current I_m that can flow across a diode of spacing w from the emitter at a temperature T under the condition of zero field at the collector.

$$I_m = 7.729 \times 10^{-12} \frac{T^{3/2}}{w^2} = 9.664 \times 10^{-6} \frac{V_T^{3/2}}{w^2}. \quad (1)$$

$$V_T = \frac{kT}{q} = \frac{T}{11600} . \quad (2)$$

In this equation the current density will be in amp/cm^2 if the distance w is expressed in centimeters. The true value of I_R is always less than I_m since the actual potential distribution in the space is that associated with a diode of augmented spacing shown in Fig. 1 as δ. The following equation relates these quantities and defines a new parameter z^2.

$$\frac{w^2}{\delta^2} = z^2 = \frac{I_R}{I_m} . \quad (3)$$

The true zero field emission capability of a cathode is determined by its temperature and the value of the true work-function ϕ_1. This emission, often known as the "saturated emission", can be computed accurately by:

$$I_o = 120\, T^2 \, e^{-\phi_1/V_T} . \quad (4)$$

These quantities may be related by the following equation

$$\frac{I_o}{I_m} = z^2 \, e^{\frac{\phi_R - \phi_1}{V_T}} . \quad (5)$$

The langmuir space-charge theory applied to the relation in Eq. (5) as presented in Sect. 43 of T.E. permits the computation of the correct value of z^2 as a function of (I_o/I_m). An abridged table of values is given here in the Appendix as Table 3 and the graphical presentation of the table is shown as Fig. 2.

Since interpolation between entries of the table is often less convenient than the use of empirical equations, a "slide rule" method of evaluating z^2 with accuracy generally better than 2 per cent is given by the following equations:

Apply only to range $\quad 1.5 < \frac{I_o}{I_m} < 100$

$$z^2 = 1 - \frac{0.840}{\tan\left[\log_{10}\frac{I_o}{I_m}\right]^n + 1.433} \tag{6}$$

$$n = 1.30 - 0.41 \log \frac{I_o}{I_m} \tag{6a}$$

Apply to range $\quad 100 < \frac{I_o}{I_m}$

$$z^2 = 1 - \frac{1.11}{\left(\frac{I_o}{I_m}\right)^{1/2}} \tag{7}$$

Procedure for the Computation of Properties

Step I: Compute the ratio (I_o/I_m) from Eqs. (1) and (4) and record the logarithm of this ratio.

Step 2: Evaluate the appropriate z^2 and use it as in Eq. (3) to determine I_R.

Step 3: The value of V_R (see Fig. 1) may be computed as follows:

$$V_R = \phi_1 + V_T \ln \frac{I_o}{I_R} - \phi_2 . \qquad (8)$$

Step 4: Evaluate the ratio of (V_R/V_T).

Figure 3 shows schematically the motive diagram for the diode delivering maximum power. If V is the voltmeter reading that indicates the separation between the Fermi level of the emitter and the Fermi level of the collector, there will be a particular value of V, namely V_O, for which the maximum power can be delivered to an external load. The following equation defines a symbol used in Table 9 of T.E.

$$S' = (V_R - V)/V_T . \qquad (9)$$

If the emitter had had unlimited emission capability a change in collector voltage ΔV relative to that required for the current I_m to flow is defined by

$$\sum = \frac{\Delta V}{V_T} . \qquad (10)$$

The collector current density I_∞ received at any value of $\sum$ is related to I_m by a universal function in which

$$\frac{I_\infty}{I_m} = U^2 = f_\infty (\sum) \qquad (11)$$

This functional relation is tabulated in Table 8 of T.E. and an abridged listing is given here as Appendix Table 4.

A similar relation may be expressed for the actual current density (I) delivered from an emitter of limited capability

$$\frac{I}{I_R} = u^2 = f(S') . \qquad (12)$$

For these purposes, emission capability is best expressed as

$$\frac{I_o}{I_R} = u_o^2 \qquad (13)$$

Thus u_o^2 is the factor which relates the zero field current from the emitter to the actual critical current I_R obtained with zero field at the collector.

Analysis shows that for all values of $u^2 < u_o^2\, e^{-1}$, that is $0.3769\, u_o^2$, the universal function $f_\infty(\Sigma) = f(S')$. The correction even under the high current demand of $u^2 = 0.3769\, u_o^2$ is less than 1 per cent. If u^2 is greater than $0.3769\, u_o^2$, an additional correction is needed as discussed later. The meter voltage V is related by

$$V = V_T \left(\frac{V_R}{V_T} - \Sigma \right) . \qquad (14)$$

The current at any value of Σ is

$$I = z^2\, I_m\, f_\infty(\Sigma) . \qquad (15)$$

The power delivered is

$$P = IV = z^2\, I_m\, V_T \left[f_\infty(\Sigma) \left(\frac{V_R}{V_T} - \Sigma \right) \right] . \qquad (16)$$

The universal function for the quantity in the brackets is

$$\Pi = U^2 \left(\frac{V_R}{V_T} - \Sigma\right) \qquad (17)$$

This relation may be evaluated as a function of Σ for any specific value of V_R/V_T. Typical evaluations are shown in Fig. 4.

For each value of (V_R/V_T) it is of special interest to know the value of Σ that gives the maximum Π.

Two methods have been used to obtain the value of Σ very close to the maximum in Π. These may be applied best to specific ranges.

Range limits: $\quad 1 < \frac{V_R}{V_T} < 20$

$$S'_{max} = \Sigma_{max} = 0.556 \left(\frac{V_R}{V_T} - 1\right). \qquad (18)$$

Range limits: $\quad 6 < \frac{V_R}{V_T} < 20$

$$\Pi_{max} = 0.385 \left(\frac{V_R}{V_T}\right)^2. \qquad (19)$$

Step 5

Range limits: $\quad 1 < \frac{V_R}{V_T} < 12.$

The value of Σ_{max} is first calculated from Eq. (18) and the value of U^2_{max} is determined from Table 4 (Appendix) by interpolation.

An alternative method good for most purposes uses the following approximate formula which is correct to better than 2 per cent up to $\Sigma = 100$.

$$U^2_{max} = 1 + \Sigma_{max} + c\,(\Sigma_{max})^m \qquad (20)$$

with Range 0.5 to 5; c = 0.08; m = 1.85   Range 4 to 15; c = 0.1; m - 1.7

Range 10 to 100; c = 0.11; m = 1.65

The voltage output at maximum power is

$$V_O = V_T\,(\frac{V_R}{V_T}) - \Sigma_{max})\,. \qquad (21)$$

The current is

$$I_{max} = I_R\,U^2_{max} \qquad (22)$$

$$\pi_{max} = U^2_{max}\,(\frac{V_R}{V_T} - \Sigma_{max}) \qquad (23)$$

$$P_{max} = I_{max}\,V_O = I_R\,V_T\,U^2_{max}\,(\frac{V_R}{V_T} - \Sigma_{max}) \qquad (24)$$

Step 6: An additional correction is needed with enitters of very low capability which depends on the use of Table 9 of T.E. The range of U^2_{max} which calls for this step is identified by:

$$U^2_{max} > 0.3769\ u^2_o\,. \qquad (25)$$

With the help of the curves shown in Fig. 5 it is possible to convert the computed value of U^2_{max} to a similar quantity directly usable in Eqs. (22) and (23) which is defined as u^2_{max}. The relation between these quantities serves to define the required multiplying factor α.

$$u^2_{max} = \alpha\,U^2_{max} \qquad (26)$$

In order to use Fig. 5, a second parameter is needed which is defined by:

$$\frac{U_{max}^2}{u_o^2} = \beta \qquad (27)$$

The method of using the curves is the following: In step 5, the value of U_{max}^2 was determined and u_o^2 serves to define the emission capability as in Eq. (13). The ratio determines β as in Eq. (27). The lines in Fig. 5 correspond to different values of β. The bottom line marked "limit" is the one which corresponds to a β of approximately 1.3. For any value of U_{max}^2 and β value between the limit of about 1.3 to 0.3769, a suitable value of α can be selected from the coordinate system. Thus for a value of U_{max}^2 of 3 and a β value of 0.7, the best value of α is 0.945. Equation (26) yields then a value of u_{max}^2 of 2.84 to be used in Eq. (22) and Eq. (23).

Discussion

There is no better way of illustrating the usefulness of these equations to the designer of a high vacuum thermionic converter than to apply them to the seventeen different converter configurations computed so accurately by Rittner[2]. The first four columns of Table 1 describe the configuration parameters set by Rittner. Columns 5 through 12 record the results of the application of the equations given here and constitute the necessary working entries required for the computation of the electrical properties of these diode configurations. Table 2 is a continuation of Table 1 with the configurations identified by number only, in which numerical values are given for the most essential factors used in the computation of the current, voltage, and power associated with operating conditions which give the maximum power. Columns 8, 10, and 12 record the values of current, voltage, and power computed by Rittner's "exact"

Table 1

Computed Electrical Properties of Diodes

Configuration Parameters Identical To Those of Rittner[2]

	1	2	3	4	5	6	7	8	9	10	11	12
	T^oK	ϕ_1	ϕ_2	w, cm	I_o	I_m	$\frac{I_o}{I_m}$	z^2	I_R	$\frac{I_o}{I_R}$	ϕ_R	V_R
1	1465	2.155	1.855	0.001	10.034	0.43339	23.152	0.801	0.3471	28.91	2.580	0.725
2	1465	2.155	2.155	0.001	10.034	0.43339	23.152	0.801	0.3471	28.91	2.580	0.425
3	1465	2.155	1.655	0.001	10.034	0.43339	23.152	0.801	0.3471	28.91	2.580	0.925
4	1465	2.155	1.855	0.0005	10.034	1.7336	5.788	0.652	1.130	8.880	2.431	0.576
5	1465	2.155	2.155	0.0005	10.034	1.7336	5.788	0.652	1.130	8.880	2.431	0.276
6	1465	2.155	1.655	0.0005	10.034	1.7336	5.788	0.652	1.130	8.880	2.431	0.776
7	1465	2.155	1.855	0.002	10.034	0.10835	92.61	0.892	0.0966	103.81	2.741	0.886
8	1465	2.155	2.155	0.002	10.034	0.10835	92.61	0.892	0.0966	103.81	2.741	0.586
9	1465	2.155	1.655	0.002	10.034	0.10835	92.61	0.892	0.0966	103.81	2.741	1.086
10	1540	2.179	1.855	0.001	21.261	0.46709	45.52	0.852	0.3980	53.42	2.707	0.852
11	1410	2.137	1.855	0.001	5.544	0.40922	13.55	0.752	0.3077	18.02	2.488	0.633
12	1350	2.117	1.855	0.001	2.764	0.38337	7.21	0.680	0.2607	10.60	2.392	0.537
13	1295	2.100	1.855	0.001	1.368	0.360194	3.798	0.594	0.214	6.393	2.307	0.452
14	1240	2.082	1.855	0.001	0.6444	0.33749	1.909	0.488	0.1647	3.913	2.228	0.373
15	1465	2.355	1.855	0.001	2.059	0.43339	4.751	0.626	0.2713	7.589	2.611	0.756
16	1465	1.855	1.855	0.001	107.985	0.43339	294.16	0.933	0.4044	267.02	2.561	0.706
17	1000	1.600	0.800	0.00254	1.0484	0.037884	27.673	0.817	0.03095	33.87	1.904	1.104

Table 2

Continuation of Computed Electrical Properties of Diodes

Summarized Results. Direct Comparison with Rittner's "exact" values

1	2	3	4	5	6	7	8	9	10	11	12
	$\frac{V_R}{V_T}$	Σ_{max}	U^2_{max}	$\frac{U^2_{max}}{u^2_o}$	u^2_{max}	I_{max}	I_{exact}	V_O	V_{Oexact}	P_{max}	P_{exact}
1	5.74	2.65	4.14	0.143		1.44	1.42	0.390	0.393	0.562	0.557
2	3.36	1.31	2.45	0.085		0.851	0.845	0.260	0.259	0.221	0.219
3	7.32	3.53	5.35	0.185		1.86	1.84	0.479	0.480	0.891	0.884
4	4.56	1.98	3.27	0.369	3.21	3.63	3.55	0.326	0.332	1.200	1.18
5	2.19	0.655	1.69	0.188		1.91	1.89	0.193	0.193	0.369	0.365
6	6.14	2.86	4.45	0.502	4.29	4.84	4.7	0.413	0.426	2.00	2.00
7	7.02	3.35	5.10	0.050		0.493	0.495	0.463	0.461	0.228	0.228
8	4.64	2.03	3.33	0.032		0.322	0.322	0.330	0.329	0.106	0.106
9	8.60	4.25	6.38	0.061		0.617	0.620	0.549	0.547	0.339	0.339
10	6.42	3.02	4.64	0.087		1.85	1.85	0.451	0.449	0.834	0.830
11	5.21	2.34	3.74	0.207		1.15	1.17	0.348	0.338	0.400	0.396
12	4.61	2.01	3.31	0.312	3.28	0.855	0.836	0.303	0.307	0.261	0.256
13	4.05	1.69	2.91	0.455	2.82	0.604	0.589	0.263	0.270	0.164	0.159
14	3.49	1.38	2.53	0.644	2.40	0.395	0.376	0.226	0.238	0.094	0.0895
15	5.99	2.79	4.33	0.571	4.16	1.129	1.09	0.404	0.417	0.473	0.454
16	5.59	2.56	4.02	0.015		1.62	1.62	0.383	0.383	0.623	0.620
17	12.81	6.63	10.12	0.299		0.310	0.308	0.532	0.534	0.166	0.164

method. Whereas columns 7, 9, and 11 record results I obtained by the approximate method outlined here. Note that in all cases the agreement is remarkably good.

This favorable result is to be contrasted with that indicated in his table which records the computations one would have made if the specified emission properties of the cathode were ignored. The basic difference between the calculations which he made dependent on my earlier approximate method appears quantitatively in column 8 of Table 1. The specified emitters yield z^2 values that range from a minimum of 0.488 to a maximum of 0.933. The "unlimited emission capability" theory offers a means of computation when the z^2 value is completely unknown and, assumed for the purpose of calculation, to be 1.0.

The conclusion to be drawn is that a designer of a thermionic converter may now set boundary conditions in terms of dimensions, temperature, and work-functions and compute very readily after a very few minutes of systematic calculation the electrical properties of such a converter with an accuracy which is far better than his ability to construct a converter that meets his specifications.

The reverse process is also important to the designer in that these equations permit him to analyze experimental results so as to use them as a means of determining the actual configuration which he has built without a destructive physical examination.

Acknowledgements

The writer is very grateful to Dr. Rittner for his careful examination of this problem and the privilege of seeing a preprint of his paper. This cooperation between workers in this field is greatly to be admired. I wish also to acknowledge the assistance of Miss Betty Campbell who checked most of the calculations presented here.

References

1. W. B. Nottingham: J. Appl. Phys. 30, 413 (1959); Report on the 19th Annual Conference on Physical Electronics, 1959, M.I.T. p. 71.

2. E. S. Rittner: J. Appl. Phys. (preceding paper). A preliminary draft of this paper was kindly made available to the writer by Dr. Rittner. His results being "exact" demonstrate the need for an improved approximate method for wider applicability.

3. W. B. Nottingham: Thermionic Emission, Handbuch der Physik, Vol. 21, 1956. Also referred to here as T.E.

Figure Captions

Fig. 1 Electron potential distribution with critical condition of zero gradient at the collector.

Fig. 2 Computational chart relating z^2 to (I_o/I_m).

Fig. 3 Potential distribution with maximum power in load.

Fig. 4 Values of Π as a function of Σ for selected values of (V_R/V_T) of 4 to 20 and curve I is (I_∞/I_m) of Table 4. Appendix.

Fig. 5 Computational chart for determining α as a function of U^2_{max} for various β values.

Appendix. Table 3

Relation of z^2 to (I_o/I_m)

Abridged from Table 4 of T.E., p. 161

I_o/I_m	z^2	I_o/I_m	z^2
1.066	0.3921	13.70	0.7538
1.263	0.4204	15.38	0.7654
1.485	0.4474	17.21	0.7751
1.735	0.4729	19.30	0.7868
2.015	0.4969	21.57	0.7957
2.329	0.5197	24.14	0.8057
2.686	0.5423	27.01	0.8156
3.086	0.5637	30.14	0.8236
3.532	0.5838	37.54	0.8398
4.029	0.6026	46.69	0.8551
4.587	0.6208	57.91	0.8684
5.221	0.6394	71.66	0.8798
5.914	0.6553	88.67	0.8913
6.699	0.6716	109.6	0.9017
7.574	0.6871	135.2	0.9113
8.549	0.7019	227.7	0.9304
9.623	0.7149	381.4	0.9454
10.834	0.7281	636.7	0.9573
12.19	0.7413		

Appendix. Table 4

Relation defined by Eq. (11)

Abridged listing from Table 8 of T.E., p. 165

Σ	U^2	Σ	U^2
0.5655	1.593	23.84	46.28
0.8424	1.901	29.09	59.80
1.279	2.408	34.31	74.10
1.650	2.858	39.49	89.09
2.306	3.690	44.65	104.8
2.900	4.483	49.80	121.2
3.731	5.648	54.93	138.3
5.025	7.573	65.16	174.1
7.443	11.506	75.36	212.2
9.747	15.60	85.53	252.3
13.10	22.08	95.68	294.2
15.28	26.61	105.8	338.2
17.44	31.30	156.4	582.4
19.59	36.15		

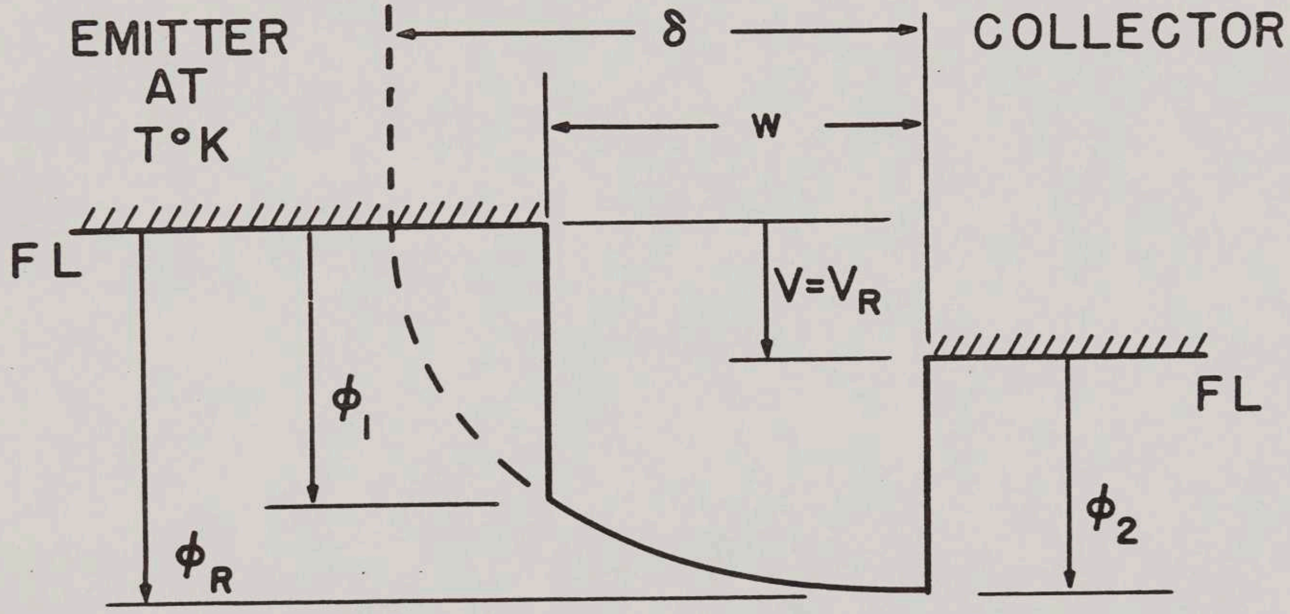


FIGURE 1

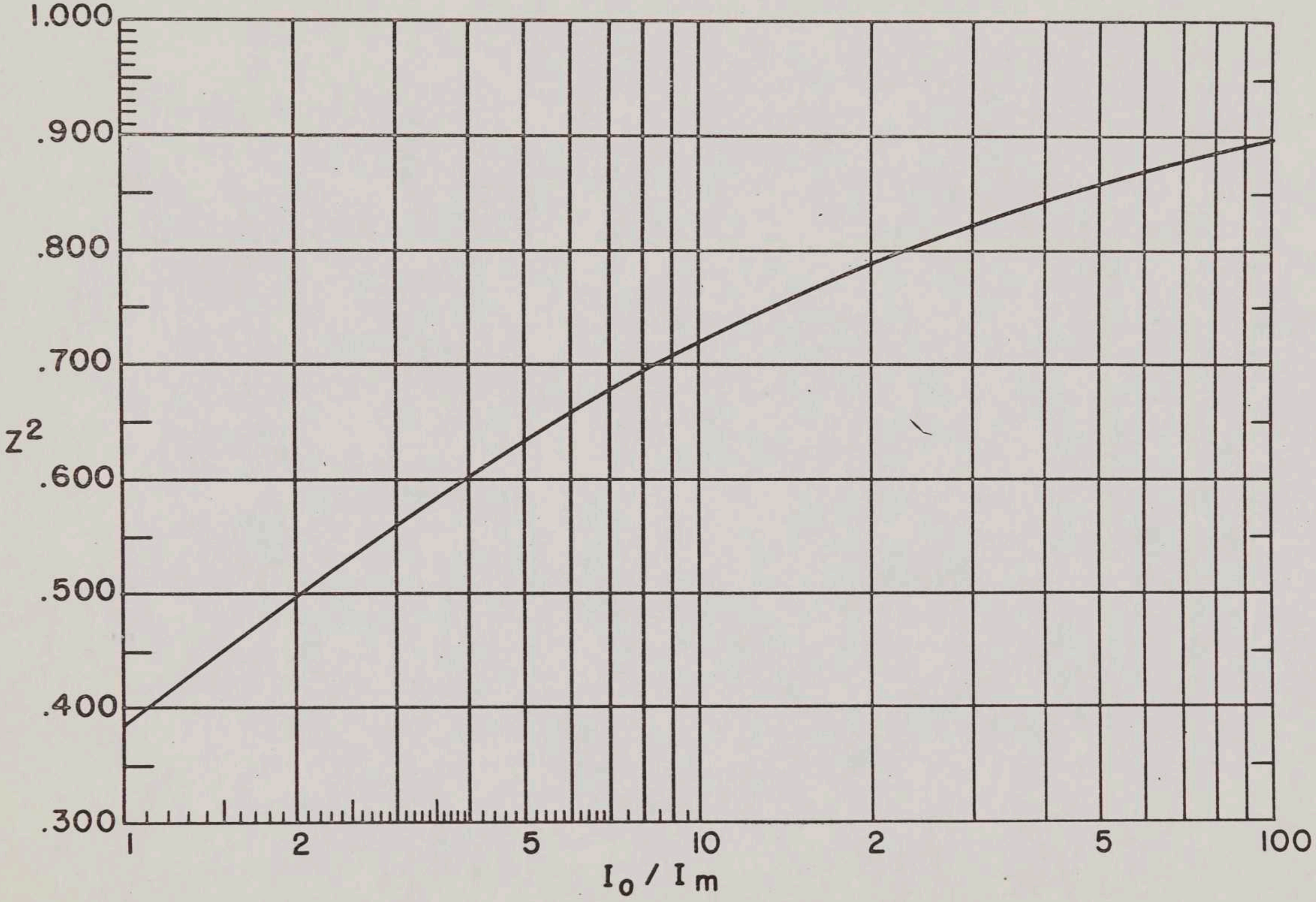


FIGURE 2

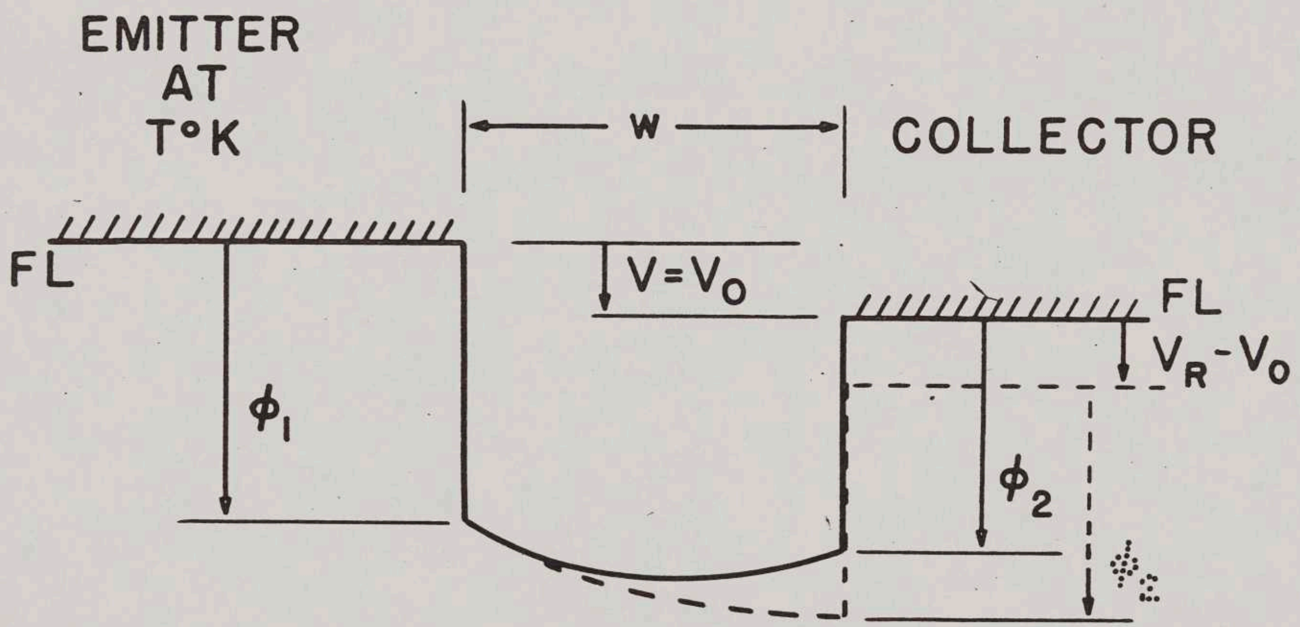


FIGURE 3

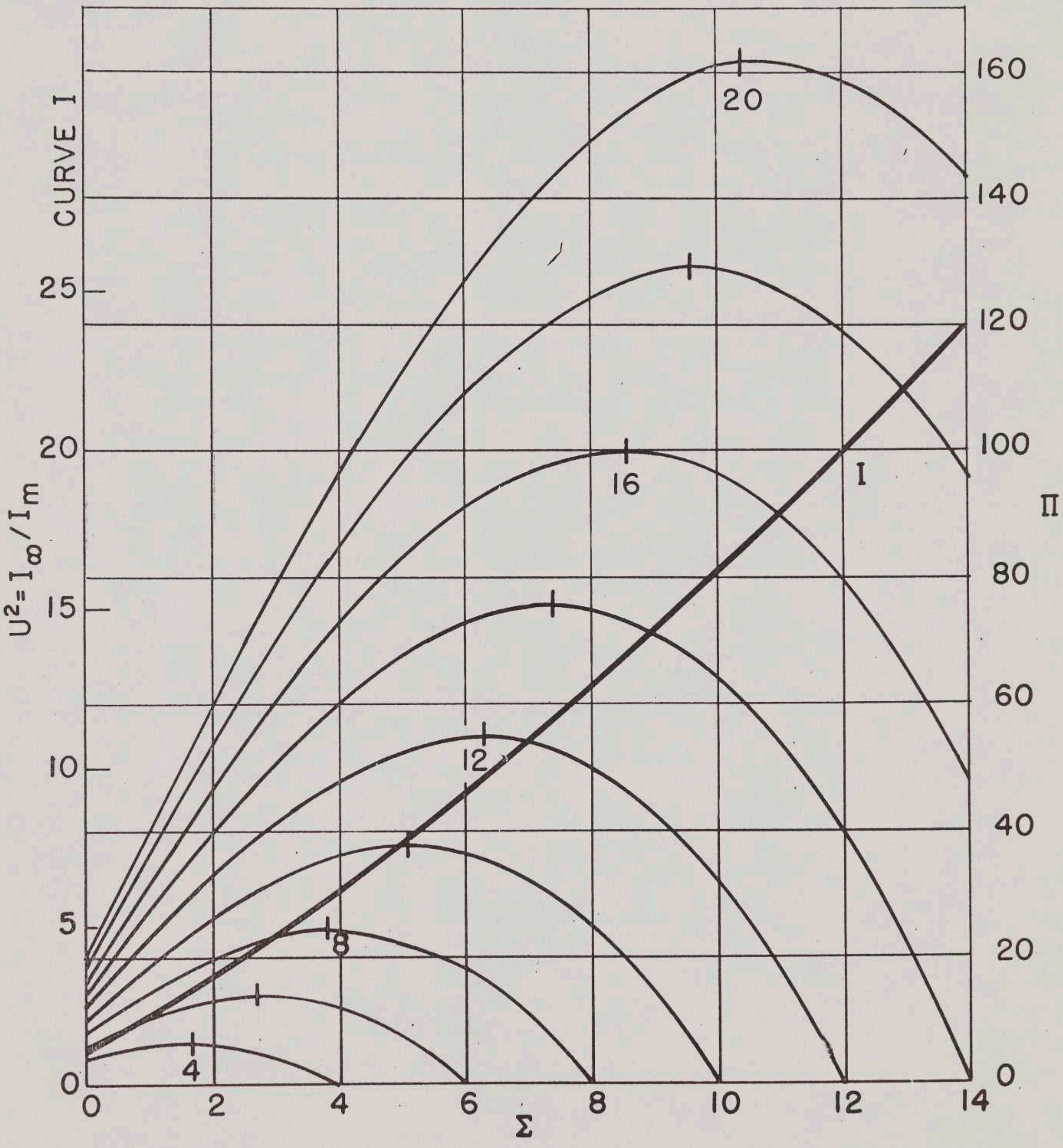
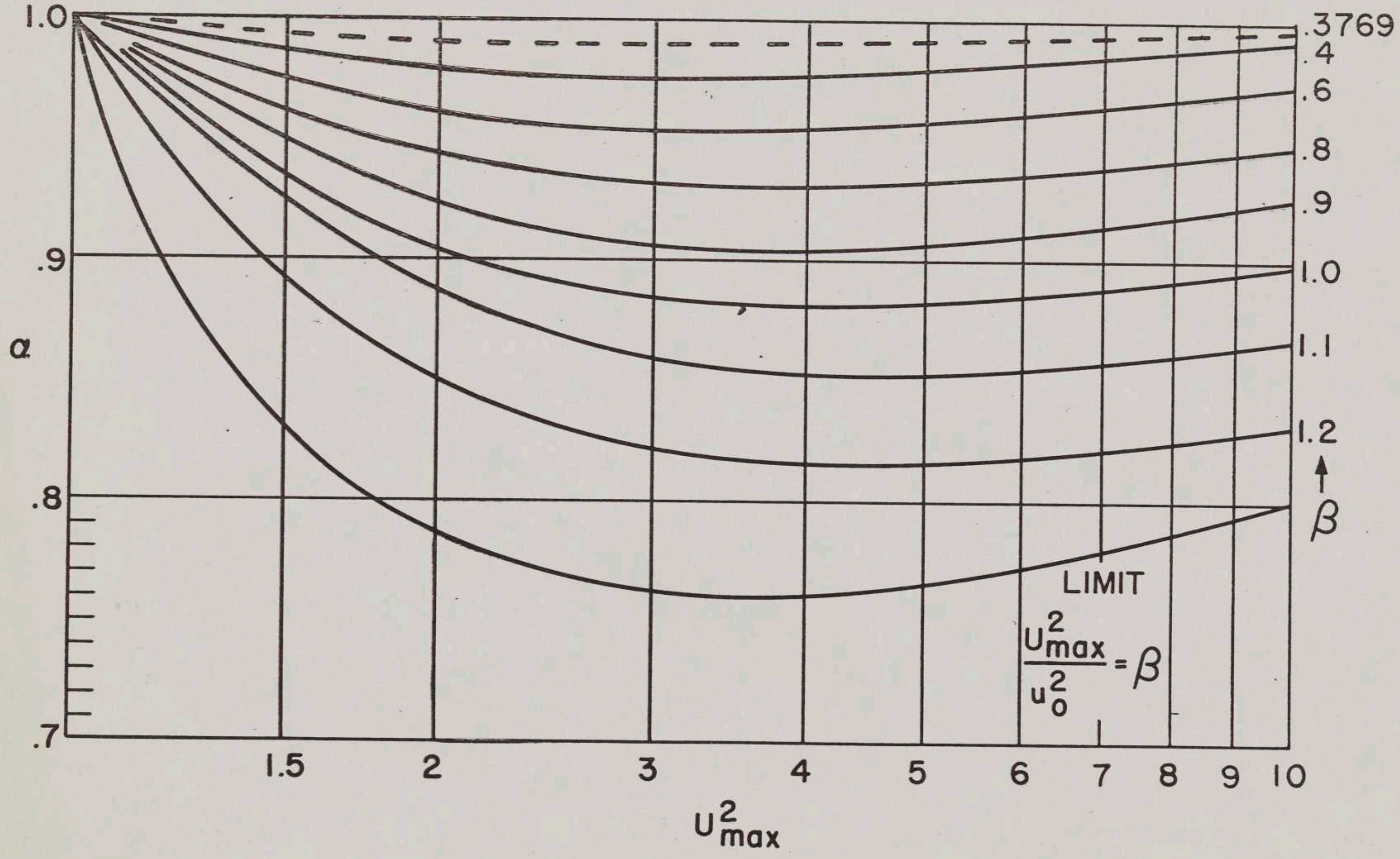

CURVE I
$U^2 = I_\infty / I_m$
0
5
10
15
20
25
0
2
4
6
8
10
12
14
Σ
0
20
40
60
80
100
120
140
160
II
4
8
12
16
20
I

FIGURE 4



1.0
.9
.8
.7
α
1.5
2
3
4
5
6
7
8
9
10
U^2_{max}
.3769
.4
.6
.8
.9
1.0
1.1
1.2
β
LIMIT
$\frac{U^2_{max}}{u^2_o} = \beta$

FIGURE 5