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THE THERMIONIC ENERGY CONVERTER

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THE THERMIONIC ENERGY CONVERTER

W. B. Nottingham

I. General Theory of the Plasma Diode Energy Converter

Abstract

The introduction of cesium into a thermionic device serves to improve its potential efficiency in three ways. It can improve both the emission capability of the emitter and the conduction properties of the space between the emitter and the collector, and it can lower the work-function of the collector. All of these features are advantageous. In some respects, the establishing of a plasma in the intervening space might be considered a most important factor, in that it permits the designer to build in a converter with realizable spacing. One of the principal points of this report is to show that under suitable conditions, a positive ion space-charge sheath forms in the immediate neighborhood of the emitter and, as a result of this space-charge situation, electrons are accelerated from the emitter into the intervening space. Here, because the concentrations of ions and electrons are practically equal, plasma oscillations develop and the energy distribution of the electrons changes from a monoenergetic one associated with an energy of the order of 0.5 volt, or more, to a quasi-Maxwell-Boltzmann distribution characterized by a temperature of 5000° K, or more. Such an electron distribution is sufficiently rich in high-energy electrons to ionize the gas and maintain it in a state of practically 100 per cent ionization. Thus a highly conducting plasma is formed to serve the purpose of permitting a high-density electron current to flow from the emitter to the collector. Cesium also serves to reduce the work-function of the collector, and it is anticipated that high efficiency will be obtained as these features are incorporated in the design and operation of the plasma diode as an energy converter.

II. A Simplified Method for the Computation of Electrical Properties of a Closed-Space Thermionic Converter

Abstract

The closer the spacing of a high-vacuum thermionic converter the less its performance is governed by space charge and the more dependence must be placed on the producer to develop superior electron emitters. Whereas the theory applicable to the converter with an emitter of unlimited emission capability is less difficult, it is nevertheless possible to work out in a relatively simple manner all of the equations that are applicable to the diode of limited emission capability. It is the purpose of this report to show these methods and to compare the results of our calculations with those published by Rittner, which are dependent on the application of a digital computer programmed to solve this type of problem and based on rigorously derived space-charge equations. The simplified method gives results that are in excellent agreement with the "exact" computations.

III. The Thermionic Diode as a Heat-to-Electrical-Power Transducer

Abstract

The theory of the vacuum diode is worked out in considerable detail, fundamentally on the basis of Langmuir's complete analysis of the flow of electrons under space-charge-limited conditions across a diode. The results of his analysis have been put into useful form that is specifically applicable to emitters of "unlimited emission capability." The results presented in this report are optimistic, in that no practical cathode achieves this distinction. Acceptable efficiency can be obtained only with spacings in the neighborhood of 10 microns.

FOREWORD

Direct conversion of heat to electricity gives sufficient promise, in its relation to the generation of power for auxiliary space applications and for the generation of power directly from nuclear energy, to attract much attention and encourage investment of time and money in its exploitation. Three papers concerning the analysis of the vacuum diode as a thermionic converter, based almost exclusively on the published contribution of this author on "Thermionic Emission" (Handbuch der Physik, Vol. 21, 1956, pp. 1-175), are made available in this report.

The first two papers applied to this study were based on the most optimistic approach that is associated with such good electron emitters, so that the details of their properties could be neglected in comparison with the limiting factors of space charge and anode work-function. Later theoretical investigations, as well as experimental studies, indicated that emitter properties cannot be neglected. The first paper was published by the Journal of Applied Physics; the second and third, applicable to the vacuum diode, were published in the Reports of the Twentieth and Nineteenth Annual Conferences on Physical Electronics held March 24-26, 1960 and March 26-28, 1959 at Massachusetts Institute of Technology.

It is generally evident that efficiency can only be attained with the vacuum diode as the result of engineering a device so as to make use of fantastically close spacings of the order of 10 microns. But the introduction of cesium into the diode could very well relieve this situation and very likely make it possible to create practical conversion units. At least 90 per cent of the development effort is now going into the cesium diode as a thermionic energy converter. For that reason, the author developed the "General Theory of the Plasma Diode Energy Converter." This treatment was first made available to interested researchers by the Thermo-Electron Engineering Corporation, and an abridged presentation was given to the 1960 Conference on Physical Electronics. It is the material of this presentation that is reproduced here.

GENERAL THEORY OF THE PLASMA DIODE ENERGY CONVERTER* **

Introduction

The new concept which is put forward in this report is that the presence of a suitable pressure of cesium atoms near a hot emitting surface results in an ionization so copious that an ion space-charge sheath can be built up at the surface. If the drop in potential over this sheath is approximately 0.5 volt or more, then the electrons are injected into the interelectrode space with an average energy greater than this. The presence of a large supply of ions in this space essentially equal in number to the electrons per unit volume results in a generation of plasma oscillations. Electrons which entered the region with a more or less monoenergetic energy distribution become randomized both in direction and in energy. With no change in the total energy content, the distribution becomes one that could be characterized as being Maxwellian with a temperature of 4000°K or more. In the presence of neutral atoms, such a distribution can generate such a copious supply of new ions that losses in ion density due to recombination and due to the sweeping of ions across the boundary sheaths can be made up.

The detailed analysis concerning these steps is carried through, equations are formulated, and methods of analysis are provided so that good approximations can be made to determine the electron injection energy directly from the known parameters of the problem. These include the work-function of the emitter, its temperature, the condensation temperature of the cesium supply, and an estimation concerning the average temperature of the gas in the space between the emitter and the collector.

In the plasma space, ionization and recombination take place. Again, the equations are formulated so that it is possible to determine the probable fraction of the cesium concentration there that is ionized. The theory shows that if electron energy distribution is characterized by a temperature of less than 3000°K, the fractional ionization is very small; whereas if it is characterized by a temperature of about 5000°K or more, the gas in equilibrium is very fully ionized. The analysis carries on to a discussion of the formation of the ion sheath at the collector. A calculation is made concerning the rate of loss of ions from the plasma over this sheath and it is compared with the rate of production of ions within the sheath at a distance of a mean-free path from the boundary. Again the critical temperature for the electrons is in the 4000 to 5000° range in that if the temperature is too low, the supply cannot

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*Refer to the Glossary of Symbols on pp. 19-20.

**This paper is based on a study supported in part by the Thermo-Electron Engineering Corporation.

be maintained and instability is likely to result.

There are three distinct regions that make up the total space from the hot emitter surface to the colder collector. These will be identified as (1) the emitter sheath; (2) the plasma space; and (3) the collector sheath. Details concerning each of these regions will be presented and the means for relating them to each other will be discussed. Very close to the surfaces of both the emitter and the collector, space-charge sheaths will generally be found. These may be either positive ion sheaths or electron sheaths. In the space region conduction will largely take place as a result of the single-stream flow of ions or electrons or both for the low pressure plasma. In the high pressure plasma a drift of electrons or ions will generally be superimposed on a random current flow of electrons or ions or both. By random current flow the implication is that a velocity distribution will exist among the charge carriers which is more or less isotropic and has superimposed upon it a drift velocity which is responsible for carrying the drift current.

General Discussion of Emitter Sheath

The emitter generally serves a dual purpose. It emits electrons from its interior and it serves as a generator of positive ions at its surface. All of the discussion that follows will be dependent on the assumption that the gas used in the diode is cesium. Any area of the emitter surface which has a true work-function a few tenths of a volt greater than the ionization potential of cesium which is 3.89 volts will serve as an area which converts practically every neutral atom which arrives at the surface to a cesium ion. If the true work-function of the surface is less than 3.89 volts, then the probability that the valence electron of the neutral atom will make a transition to an empty state among the conduction electrons of the emitter is very small. This probability decreases exponentially with the energy difference between the ionization potential and the true work-function of the emitter at the arrival point of the cesium atom. If the emitter is nonuniform, the greatest electron emission comes from low work-function areas. This could tend to build electron space charge. The high work-function areas will be the most efficient ion generators and will tend to build positive ion space charge in their neighborhood. These two regions of space charge will then tend to neutralize by the fact that the ions will become trapped in the electron space charge and the electrons trapped in the ion space charge to finally equalize the situation. Since it is impossible to generalize concerning the detailed influence of inhomogeneity, this aspect of the problem will not be treated specifically.

Figure 1 may be used to define some of the important quantities pertinent to this discussion. Here the emitter and the collector are designated and their Fermi levels indicated by "FL." On this motive diagram the difference between the Fermi levels shown as V is directly measurable as the applied voltage difference between them. The true work-function of the emitter ϕ_1 represents the energy difference between the Fermi level of the emitter and an electron at rest in the immediate neighborhood of the emitter surface that is of the order of 10^{-6} centimeters. The

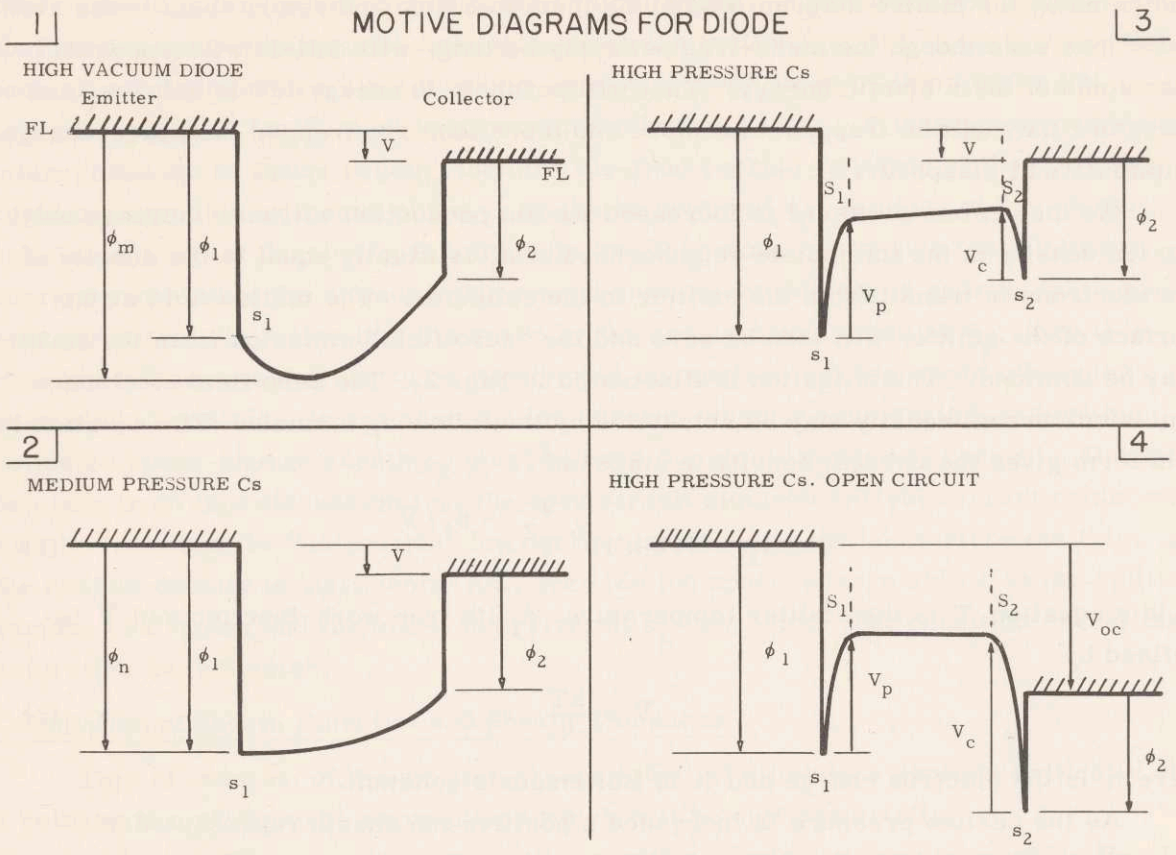


Fig. 1 Motive diagram for high vacuum diode.

Fig. 2 Motive diagram for medium pressure cesium diode.

Fig. 3 Motive diagram for high pressure cesium diode.

Fig. 4 Motive diagram for high pressure cesium diode under open circuit condition.

true work-function of the collector is defined in the same way and is represented by ϕ_2. If the electron emission is high, then an electron space-charge sheath may be expected to create a space-charge minimum which lies at ϕ_m with respect to the emitter Fermi level. Electrons emitted within the energy band ϕ_1 to ϕ_m are returned to the emitter whereas those emitted with greater energy than ϕ_m contribute to the observed diode current.

If cesium atoms are permitted to enter the space between the emitter and the collector, many of them in their random motion will collide with the emitter. If ϕ_1 is greater than 3.89, most of the atoms that arrive at the surface are ionized and will leave it as though they started at the surface point s_1. Since the ions will have an initial velocity distribution characteristic of the temperature of the emitter, they will occupy ion states within the motive diagram beginning at s_1 and move in states within the space region at energy levels higher than s_1. As long as a net electron space charge exists in front of the emitter area the ions will be accelerated toward the collector but they will not pass all the way to it since the positive surface charge

indicated by the motive diagram will bring them to a stop and return them to the emitter. Some ions even though the mean-free path may be long, will collide with atoms and lose some of their kinetic energy. That will put them in energy levels below s_1 and therefore they will be trapped. As more and more ions are trapped the space-charge minimum will disappear.

As the cesium pressure is increased the ion production will also increase until the ion density in the immediate neighborhood of s_1 is exactly equal to the density of the electrons in transit from the emitter to the collector. The motive field at the surface of the emitter will then be zero and the "zero field" emission from the emitter may be obtained. This situation is illustrated in Fig. 2. The zero field electron emission current density may be calculated by the following formula. The equation in this form gives the current density in amps/m^2.

$$I_1 = 1.2 \times 10^6 \, T^2 \, e^{-\phi_1 / \overline{V}} \tag{1}$$

In this equation T is the emitter temperature, ϕ_1 its true work-function and $\overline{V}$ is defined by

$$\overline{V} = \frac{kT}{q} \tag{2}$$

Here q is the electron charge and k is Boltzmann's constant.

As the cesium pressure is increased a positive ion sheath represented as extending over the space S_1 of Fig. 3 will develop. The change in the motive function from the surface potential to that at the end of the sheath is represented by V_p. In the space region between S_1 and the collector sheath at S_2 we can anticipate that there will be a small rise or fall in the motive function. This change is necessary because some electrons that are emitted over the surface barrier at s_1 will lose energy by collision and become trapped in the electron energy levels below the surface potential of the collector at s_2. This trapped negative charge must be compensated for by ions which are brought to rest and returned toward the emitter. Under the conditions shown, the surface potential of the collector is negative with respect to the motive function at the sheath boundary S_2. In this range the trapped electrons are repelled and the positive ions which enter the sheath at S_2 are accelerated into the collector. Again in this region the net space charge is positive.

If the applied potential V is made more negative than that shown in Fig. 3 then it is possible to set up a condition for which the ion arrival rate is exactly equal to the electron arrival rate. When we assume that there is no electron emission from the collector, then this condition of balancing currents is the "open-circuit" voltage designated as V_{oc} of Fig. 4. Under these conditions there will be nearly twice as much negative charge per unit volume in the space region as there was under the condition illustrated in Fig. 3 because there will be a double stream of electrons across this space. That change will result in a somewhat lower value of V_p to permit

more ions to enter the space region for neutralization. Again near the collector a positive ion space charge will develop because the motive function is such as to turn back all electrons with a kinetic energy associated with their motion toward the collector which is insufficient to surmount the barrier at s_2. If there were no electron interactions in the space region, then the electron current received at the collector could be computed by means of Eq. 1 by the insertion of ϕ_{oc} in place of ϕ_1. A calculation of current density by this formula would in many cases give the minimum current expected at the collector whereas if some electron energy sharing takes place because of their injection into the space by the accelerating potential V_p, then it is possible for more electrons to be received at the surface s_2 than would otherwise have been permitted. In any case the ion current which is accelerated through the collector space-charge sheath S_2 must balance the electron arrival current. It should be clear from this discussion that the open circuit electron arrival current could very well be less than the "saturation" ion current produced at the hot surface emitter. If the cesium density is sufficiently low, then the ion space-charge sheath at the emitter surface can vanish and the electron arrival at s_2 can, under that condition, equal the saturation ion emission.

Calculation of Sheath Potential and Sheath Thickness

This calculation of the emitter sheath potential difference depends critically on a combination of physical assumptions which will be outlined briefly:

Assumption 1. Electrons leave the emitter with an emission density given by Eq. 1, if the ion generation at the surface of the emitter is more than adequate to compensate the electron space charge. (See Eq. 7.)

Assumption 2. The ion production rate cannot exceed the atom arrival rate and therefore the first qualitative estimate as to whether or not there is an ion sheath at the emitter depends on a knowledge of atom arrival rate and the effective average work-function of the emitter. Equation 7 serves as an aid to determine the minimum cesium condensation temperature needed for the development of an emitter sheath.

Assumption 3. It is only after preliminary calculation that the validity of the calculation method proposed can be evaluated. Thus if the calculated sheath potential V_p is computed to be less than about $2\overline{V}$, then the approximations used become progressively poorer. The ion density at the boundary between the plasma and the sheath is taken to be equal to the electron density associated with the emission current that moves with a velocity proportional to $(V_p + \overline{V})^{1/2}$.

Assumption 4. The total atom arrival rate including both ions and neutrals at the sheath-plasma boundary is taken to be equal to the equilibrium atom emission rate at the highest temperature cesium-condensation surface. In the ideal experiment, this is taken to be the temperature of the controlled cesium supply but may be higher than this temperature in experiments in which the free flow of atoms is inhibited by small communication channels.

Assumption 5. At the sheath boundary it is assumed that an isotropic distribution of ions of the required density exists. Since in general the sheath thickness is less than the mean-free path, this implies that the ion arrival rate from the plasma is equal to the ion emission rate over the sheath barrier.

Assumption 6. As a consequence of Assumption 5, the rate of emission of neutrals from the emitter surface is set equal to the rate of arrival of neutrals from the plasma.

Assumption 7. The probability that a neutral atom will become converted upon arrival at the emitter surface will depend on the "effective" true work-function of the surface (see Eqs. 8 and 9) and its temperature.

Assumption 8. The ion annihilation probability is assumed to depend not only on the effective work-function of the emitter and its temperature but also on the kinetic energy associated with the ion as it comes in contact with the surface atoms of the emitter. The true form of this probability function is not known by experiment for the energy range involved.

In order to proceed numerically a formula is required for the relationship between the cesium condensation temperature and the cesium pressure in the immediate neighborhood of such a surface. From this relation as a starting point, the application of ideal gas laws permits the derivation of additional formulae for atom concentration, atom arrival rate, and ion emission current density when every atom is converted to an ion. These four equations are summarized as follows:

Pressure:

$$p = 2.45 \times 10^{8}\, T_{Cs}^{-1/2}\, e^{-\frac{8910}{T_{Cs}}} \text{ mm} \tag{3}$$

$$p = 2.45 \times 10^{8}\, T_{Cs}^{-1/2}\, 10^{-\frac{3870}{T_{Cs}}} \tag{3a}$$

Atom concentration:

$$N_{Cs} = 2.37 \times 10^{33}\, T_{Cs}^{-3/2}\, e^{-\frac{8910}{T_{Cs}}} \text{ atoms/m}^3 \tag{4}$$

Atom evaporation or arrival rate:

$$\mu_{Cs} = 7.48 \times 10^{33}\, T_{Cs}^{-1}\, e^{-\frac{8910}{T_{Cs}}} \text{ atoms/m}^2\text{-sec} \tag{5}$$

Maximum ion current:

$$I_{+} = 1.2 \times 10^{15}\, T_{Cs}^{-1}\, e^{-\frac{8910}{T_{Cs}}} \text{ amp/m}^2 \tag{6}$$

$$I_{+} = 1.2 \times 10^{15}\, T_{Cs}^{-1} \times 10^{-\frac{3870}{T_{Cs}}} \text{ amp/m}^2 \tag{6a}$$

Range $300 < T_{Cs} < 500$

$$I_+ = 1.11 \times 10^{12} \times 10^{-\frac{3700}{T_{Cs}}} \text{ amp/m}^2 \qquad (6b)$$

The square root of the mass ratio of the cesium ion and electron is 494. This fact combined with Eqs. 1 and 6b yields a simple formula for the computation of the minimum cesium temperature required for the neutralization of the electron emission which in turn is a function only of the true effective work-function and the temperature. The equation follows:

Minimum cesium temperature in ^{o}K:

$$T_{Cs} = \frac{3700}{8.65 + (\phi_1/2.3\,\overline{V}) - 2\log_{10} T} \qquad (7)$$

Since in many cases some cesium atoms will not be converted to ions, it is to be expected that a somewhat higher cesium temperature will be needed for the complete neutralization of electron space charge.

The expression "effective true work-function" may need some explanation. Generally speaking, the current-voltage relation observed in a plasma diode is one in which the net current around the circuit increases rapidly as the applied potential at the electron collector is made less and less negative. A typical curve then suddenly flattens off when the rate of removal of electrons from the interelectrode space is equal to the net arrival rate of electrons from the emitter. If, as a first approximation, "Assumption 5" is satisfied well enough, then the electron emission observed is the thermionic emission characteristic of the temperature of the emitter and its effective work-function. In terms of the area a expressed in meters^2; the observed current i_c expressed in amperes; and the temperature, the equation for the computation of the effective work-function is:

$$\phi_1 = 2.3\,\overline{V}\,(6.08 + \log_{10} a - \log_{10} i_c + 2\log_{10} T) \qquad (8)$$

The equation used for the ion production rate is:

$$\nu_+ = \mu_{Cs} \frac{1}{e^{-\frac{\phi_1 - V_i}{\overline{V}}} + 1} \qquad (9)$$

An examination of the researches of Taylor and Langmuir[1] shows that there exists for the tungsten used a relation between the cesium bath temperature and the temperature of the hot surface for 100 per cent ionization to take place at the surface. The relation is:

$$T \geq 3.6\,T_{Cs} \qquad (10)$$

For tungsten temperatures below that given by Eq. 10, a cesium film adsorbs on the tungsten surface to produce an average work-function lower than the ionization potential of cesium. Under this condition the ionization probability drops to less than 10 per cent. As the work-function is reduced for a given cathode temperature, Eq. 7 indicates that the minimum cesium temperature increases in order to supply enough cesium ions for space-charge neutralization. A major reduction in work-function is likely to create a marked decrease in ionization efficiency which would demand a still greater increase in atom arrival rate for neutralization. Experimental data upon which a clear picture of these phenomena can be based seems to be non-existent.

For moderately high work-function materials (4.1 or more) it is to be anticipated that the diode current at maximum power would increase with cesium condensation temperature until perfect space-charge neutralization occurred. Further increases in cesium temperature would not increase the current much as long as Eq. 10 is satisfied. Still more cesium would cause a quick rise in current as cesium adsorption takes place because of the lowering of the work-function. At the same time, there would be a fall in ion production. There are circumstances, under which in spite of this fall in ion production, space charge could be neutralized. These remarks point up the need for fundamental research in this area.

The most qualitative description of the ion sheath model implies that if the production rate of ions at the hot surface is in excess of $(I_1/494)$, a space-charge sheath of difference in potential V_p will develop over a space near the emitter surface small compared with the electron mean-free path. After injection, the electron density required to carry a specified current will depend on V_p according to the following relation:

$$n_o = 1.05 \times 10^{13} \frac{I_1}{\overline{V}^{1/2} \left(\frac{V_p}{\overline{V}} + 1\right)^{1/2}} \qquad (11)$$

It is this electron density which is set equal to the ion density at the boundary of the sheath. It is further assumed that the ion density at the emitter surface will be related to the boundary density by the Boltzmann relation as follows:

$$\ln n_s - \ln n_o = \frac{V_p}{\overline{V}} \qquad (12)$$

If V_p and n_o are known, then a Fowler relation[2] may be used to estimate the sheath thickness. This equation is:

$$x = \left(\frac{2\overline{V}\epsilon_o}{q n_o}\right)^{1/2} \arctan \left(e^{\frac{V_p}{\overline{V}}} - 1\right)^{1/2} \quad m \qquad (13)$$

Limiting value for $(V_p/\overline{V}) \gg 1$

$$x = \frac{\pi}{2}\left(\frac{2\overline{V}\epsilon_o}{q n_o}\right)^{1/2} = 1.65 \times 10^4 \left(\frac{\overline{V}}{n_o}\right)^{1/2} \quad m \tag{13a}$$

Numerical Evaluation of Sheath Potential

Two equations have been derived applicable to the estimation of the drop in potential across the emitter sheath for high work-function and low work-function emitters. The results computed by these equations cannot be said to be exact but represent more or less upper limits with the expectation that the actual sheath potential will be smaller rather than greater than the one computed. High and low work-function are defined relative to the ionization potential of cesium by the parameter u given as:

$$u = \frac{\phi_1 - V_i}{\overline{V}} \tag{14}$$

If u is positive, then the work-function is higher than the ionization potential, but if u is negative, it is lower. The equation given first applies to the positive values of u.

Case I High work-function $(u \geq 0)$

$$\frac{V_p}{\overline{V}} - \frac{1}{2}\ln\left(\frac{V_p}{\overline{V}} + 1\right) = 9.49 - \frac{8910}{T_{Cs}} + \frac{\phi_1}{\overline{V}} - \ln(1-f)^{-1} + 2\ln\overline{V}^{-1} - \ln T_{Cs} + H_1(u) \tag{15}$$

In this equation the symbol $H_1(u)$ is a function of u only and given by Eq. 16 for its exact value or Eq. 16a for an excellent approximation for the larger values of u.

$$H_1(u) = u - \ln(1 + e^{-u}) - \ln(u + S') \tag{16}$$

For $u > 2.5$

$$H_1(u) = u - \ln u - \left(1 + \frac{1}{u}\right)e^{-u} \tag{16a}$$

The series expansion represented by S' is given as follows:

$$S' = e^{-u} - \frac{1}{2}e^{-2u} + \frac{1}{3}e^{-3u} - \frac{1}{4}e^{-4u} + \ldots \tag{17}$$

In order to simplify calculations, a table of values of H(u) good for both positive and negative values of u is given in Appendix 1. A plot of this function is shown in Fig. 5.

Note that in Eq. 15 the essential specifications of the problem are found on the right-hand side of this equation. It remains only to choose the value of $(V_p/\overline{V})$ required on the left-hand side of the equation to give the same value and thus lead to the correct solution. Figure 6 has been prepared to serve as a graphical means of relating the corresponding values. The procedure is to insert all of the essential facts into the right-hand side of the equation, find its value, locate it on the lower scale of Fig. 6, and identify the corresponding solution on the upper scale.

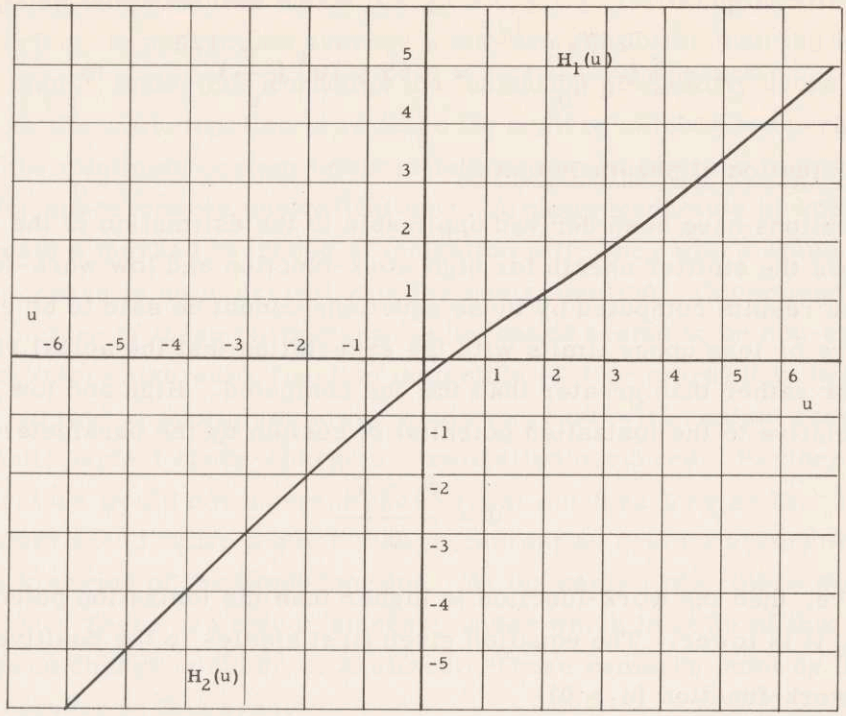
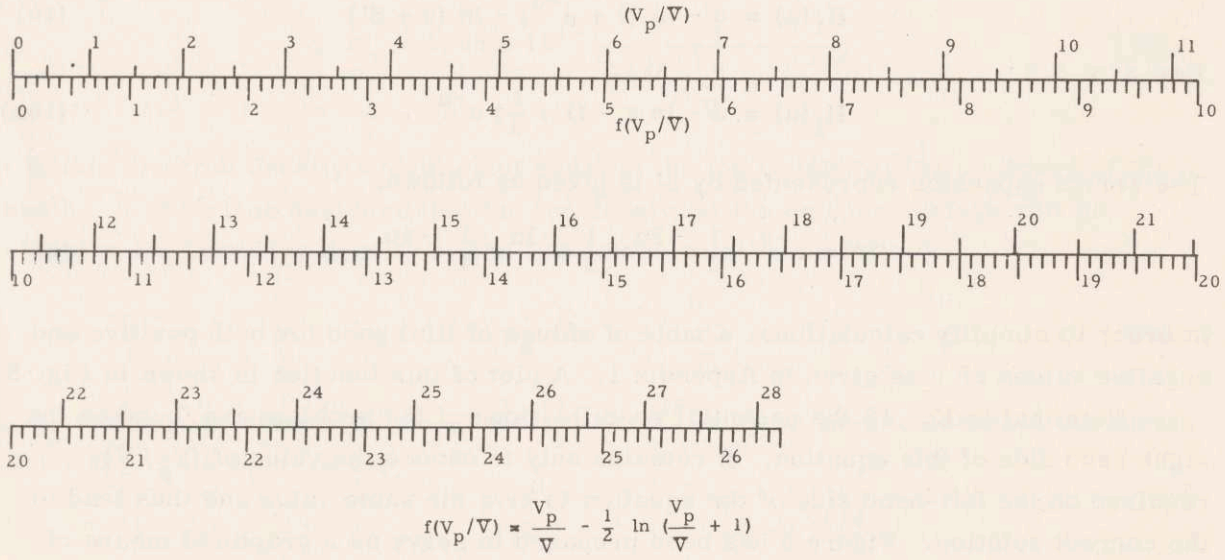


Fig. 5

Universal function needed in Eqs. 31 and 33. See Appendix 1 for tabulated values, p. 112.



$$f(V_p/\nabla) = \frac{V_p}{\nabla} - \frac{1}{2} \ln \left(\frac{V_p}{\nabla} + 1\right)$$

Fig. 6

Linear correlation chart of function used in Eqs. 31 and 33. See Appendix 2 for tabular values, p. 112.

Case II Low work-function ($u \leq 0$)

$$\frac{V_p}{\overline{V}} - \frac{1}{2} \ln \left(\frac{V_p}{\overline{V}} + 1\right) = 9.49 - \frac{8910}{T_{Cs}} + \frac{\phi_1}{\overline{V}} - \ln(1 - f)^{-1} + 2 \ln \overline{V}^{-1} - \ln T_{Cs} + H_2(u) \qquad (18)$$

$$H_2(u) = u - \ln (1 + e^u) - \ln \left(1 - \frac{1}{2} e^u + \frac{1}{3} e^{2u} - \frac{1}{4} e^{3u} + \frac{1}{5} e^{4u} - \ldots\right) \qquad (19)$$

For $u < -2$

$$H_2(u) = u - \frac{1}{2} e^u \qquad (19a)$$

Generation of a High Temperature Maxwell-Boltzmann Distribution of Electrons

Associated with this concept of electron injection across an emitter sheath is the thought that the electrons will find themselves in an ionized region between the emitter and the collector. Here it is anticipated the "plasma oscillations" will randomize the otherwise monoenergetic group of electrons. On this assumption, it is possible to compute the temperature which would be an upper bound for this group of electrons. The method assumes that the total energy of the group of electrons remains unchanged in the randomizing process. The result is:

$$T_- = 7850 (V_p + 2\overline{V}) \qquad (20)$$

$$\overline{V}_- = \frac{2}{3} (V_p + 2\overline{V}) \qquad (20a)$$

Collision Phenomena

Many plasma diode properties depend on specific properties of cesium atoms and in particular on the probability of electron collisions both elastic and inelastic, ionization probability, and radiational recombination probability. Reliable data for these atomic cross sections are almost nonexistent. The suggested values used in this section are taken in lieu of better data. By convention, collision phenomena are most often described in terms of a quantity misnamed as the "collision probability" P_c. Numerically P_c^{-1} is the average distance travelled by an electron between collisions in a gas at 1 mm mercury pressure and 0^oC. Although it is usual to express this quantity in reciprocal centimeters, reciprocal meters will be used in this report. The Brode[3] data yield values of P_c between 10^5 and 2×10^5 for the general range of electron energies of interest in this application. Since the main purpose of mean-free path calculations is to determine more or less approximate relations between mean-free paths and diode dimensions as well as sheath distances a value of $P_c = 1.4 \times 10^5$ will be used here. The general equation by which the collision mean-free path may be computed is:

$$\ln \lambda_c = \frac{8910}{T_{Cs}} + \ln T_{Cs} + \frac{1}{2} \ln T_g - \ln P_c - 24.94 \qquad (21)$$

A more convenient equation suitable for the present purposes including the chosen value of P_c is:

$$\log_{10} \lambda_c = \frac{3670}{T_{Cs}} + \frac{1}{2} \log_{10} T_g - 12.86 \qquad (22)$$

It is sometimes desirable to convert the quantity P_c or its equivalent to a corresponding cross section. This can be done by the following formula:

$$\sigma_c = \frac{P_c}{3.54 \times 10^{22}} \qquad m^2 \qquad (23)$$

In this expression the denominator is the number of atoms per cubic meter when the pressure is 1 mm and the temperature 0^oC.

Similar to P_c is the "ionization efficiency" P_i. If ionization is produced by low energy electrons upon direct collision with cesium atoms, no ions are produced for electron energies less than 3.89. Above this threshold, the efficiency rises in a complex manner. Neither theory nor experiment give the functional relation that is most suited for cesium. Experience with other atoms indicates that a linear rise with a suitably chosen coefficient is not likely to lead to a misunderstanding of the phenomenon of direct ionization. The general equation is:

$$P_i = \alpha(V_- - V_i) \qquad m^{-1} \qquad (24)$$

The estimated value of α is 3.5×10^3. This value in combination with Eq. 23 gives:

$$\sigma_i = 10^{-19} (V_- - V_i) \qquad m^2 \qquad (25)$$

The purpose served by the knowledge of P_i is the calculation of the rate of ion production per unit volume produced by a Maxwellian distribution characterized by the volt-equivalent of temperature $\overline{V}_-$. The expression obtained is:

$$\nu_+ = 1.9 \times 10^{-15} \alpha(1 - f) N n_- \overline{V}_-^{3/2} e^{-\frac{V_i}{\overline{V}_-}} \left(2 + \frac{V_i}{\overline{V}_-}\right) \qquad (26)$$

As the cesium concentration in the interelectrode space is increased more and more, it is conceivable that ionization from the resonance level could be important. Electrons with energy in excess of 1.4 v can excite cesium atoms to this level. After the atoms are excited, they may lose this energy by radiation to the outside of the discharge, or they may lose it by collisions of the second kind, that is, collisions with slow electrons. In the latter case, the density of excited atoms is almost independent of the density of the electrons over a considerable range in electron

density. Suitable experiments applicable to cesium for the evaluation of this concept have not been made. If we assume it to be true, then ionization with the addition of only 2.5 v energy can be accomplished.

The recombination rate between free electrons and positive ions may play a part in the plasma diode. Mohler[4] determined the recombination coefficient for an electron temperature of 1200°K. Theoretical calculations show that if the recombination probability is inversely proportional to the velocity of the electron and the electron energy distribution is Maxwellian, the recombination rate is independent of the electron temperature that characterizes the distribution. The recombination coefficient R can be expressed as:

$$\frac{dn_-}{dt} = -\nu_r = -R\, n_+ n_- \qquad (27)$$

The Mohler value for R is 3.4×10^{-16} m^3/sec.

Fractional Ionization in the Plasma

At the emitter sheath boundary ions are lost from the plasma region while at the same time this loss is partially or completely compensated by ion production at the hot surface. Assumption 5 above assumes an equality of this loss and gain. At the collector boundary an uncompensated loss of ions occurs since practically every ion accelerated across the collector sheath is converted into a neutral atom and returned to the space. An upper limit to the fractional ionization in the plasma space region is obtained by the computation based on the equality of the ionization rate and the recombination rate. The equation obtained in this manner is:

$$\frac{f}{1-f} = 1.96 \times 10^4 \, \overline{V}_-^{3/2} \, e^{-\frac{3.89}{\overline{V}_-}} \left(2 + \frac{3.89}{\overline{V}_-}\right) \qquad (28)$$

Curve A of Fig. 7 shows the functional relation between the fractional ionization f and the electron-volt-equivalent of temperature $\overline{V}_-$. A corresponding temperature scale is at the top of the figure. Since there is no guarantee that the probability of recombination is inversely proportional to the velocity, a corresponding curve has been calculated on the assumption that the recombination probability is inversely proportional to the electron energy. The difference between the curves is not great and the conclusion that may be drawn is that for electron temperatures less than 3000° a very small fraction of the atoms will be ionized under equilibrium conditions; whereas for electron temperatures in excess of 5000° a very large fraction of the atoms will be ionized. The actual fractional ionization will always be less than that calculated by this formula because it does not take into account the loss of ions from the plasma across the sheath boundary.

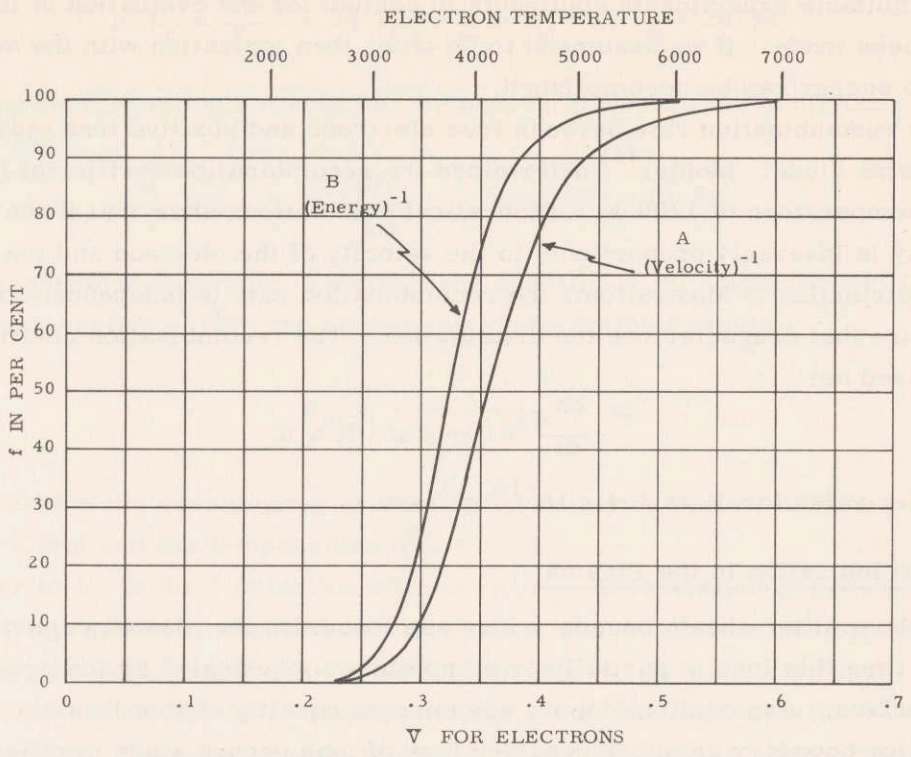


Fig. 7

Fractional ionization in plasma space, computed by balance of ionization rate versus recombination rate. Curve A for recombination inversely proportional to electron velocity. Curve B recombination probability proportional to inverse electron energy.

Collector Sheath Potential and Dimensions

It seems unlikely that a simple method will become available for an exact solution of the potential distribution and charge distribution in the collector sheath that will be adaptable to the great variety of boundary conditions likely to be encountered. This section will therefore deal with the general problem and offer equations by which approximate results can be obtained. The discussion will apply to that range in the current-voltage characteristic and to cesium pressures for which a stable mode of operation is possible. The analysis therefore applies to the range in voltage between the open circuit voltage and that associated with maximum power. Maximum efficiency is likely to be close to the condition of maximum power and will be within the voltage range included here.

Typical of the motive function applicable is the open circuit condition of Fig. 4. At the boundary between the sheath S_2 and the plasma space, equal concentrations of electrons and ions are to be found. The surface charge on the collector is strongly negative as is indicated by the potential function. In order for zero field to exist in the plasma, an excess of ions must be in transit between the sheath boundary and the collector surface to exactly balance the negative surface charge on the collector.

Thus the electron density in the sheath falls off rapidly whereas the positive ion density falls only slightly because of the acceleration of these ions. The number of ions that cross any imaginary boundary within the sheath is constant and is equal to the arrival rate of ions at the collector. The easiest approximation to use to obtain an estimate of the sheath thickness and its configuration in potential is that of the Langmuir[5] solution to the space-charge problem between the space-charge minimum and the collector. To use this solution is the equivalent of stating that the exponential decrease in electron density reduces the contribution of the electrons to the space charge sufficiently so that Langmuir's solution for a single stream of charges is adequate. It is on this basis that we visualize that the plasma potential relative to the emitter in the immediate neighborhood of the sheath boundary is practically independent of the voltage applied to the collector. Thus the ion current across to the collector will on this basis remain practically constant and be approximately equal to the random current of ions close to the boundary surface. This random current would be expressed as follows:

$$I_+ = q f_s N \left(\frac{kT_g}{2\pi M}\right)^{1/2} \tag{29}$$

$$I_+ = 5.05 \times 10^{-19} f_s N T_g^{1/2} \tag{29a}$$

In this equation f_s is the fractional ionization in the neighborhood of the sheath boundary. In a highly ionized plasma f_s approaches 1, but close to the boundary it might be better to take this fraction close to 0.5 since practically every ion that goes to the collector will return as an atom at this boundary. It will not have had an opportunity to become re-ionized until the atom stream has penetrated appreciably away from the boundary. N is the total atom concentration, and the current is expressed in amp/m^2. The calculation to follow will show that the maintenance of good plasma conditions in the immediate neighborhood of the collector sheath depends on the presence of a Maxwellian electron distribution characterized by a temperature of at least 4300°K. With temperatures as high as 5000°K the rate of production of ions within the plasma, one mean-free path away from the sheath boundary, is more than adequate to supply the ion current that flows out of this small volume of the plasma across the sheath to become neutralized at the collector surface. The following equation expresses this balance between the rate of loss of ions across the sheath and the rate of production of ions within a mean-free path.

$$N f_s \left(\frac{kT_g}{2\pi M}\right)^{1/2} = 1.9 \times 10^{-15} \alpha (1 - f_s) N^2 f_s \overline{V}_-^{3/2} e^{-\frac{V_i}{\overline{V}_-}} \left(2 + \frac{V_i}{\overline{V}_-}\right) \lambda_c \tag{30}$$

With the mean-free path expressed as in Eq. 31 and the constants suitable for cesium given just below, it is possible to solve Eq. 30 for the fractional ionization at the

surface of the collector sheath plasma space boundary.

$$\lambda_c = \frac{1}{N \sigma_c} \tag{31}$$

Use $a = 35 \times 10^2$ and $\sigma_c = 4 \times 10^{-18}$ m^2

$$f_s = 1 - \frac{1.9 \times 10^{-6} \times T_g^{1/2} \; e^{\frac{V_i}{\overline{V}_-}}}{\overline{V}_-^{3/2} \left(2 + \frac{V_i}{\overline{V}_-}\right)} \tag{32}$$

If it is assumed that the gas temperature T_g is 1500°K then the introduction of a value of $\overline{V}_-$ of 0.37 gives the result that if more than 3 per cent of the gas atoms are ionized, then the flow to the collector will exceed the generation rate within a mean-free path. In contrast, the use of $\overline{V}_- = 0.5$ results in a solution which indicates that 94 per cent of the atoms could be ionized and still the ion generation rate would equal the loss of ions across the sheath. The conclusion that one may draw is that if the conditions at the emitter are such that the plasma potential is 0.5 volt or more positive with respect to the surface potential of the emitter, then under the condition that the surface potential to the collector is equal to or more negative than that of the emitter, the ion production in the plasma will be adequate for the maintenance of a stable discharge.

Concluding Remarks

All of the relations of importance presented in this report have been tested to some extent by comparison with experiments on a quantitative basis. This does not mean that all experiments have been analyzed in terms of these relations. It is my hope to carry through a program of systematic application of this general theory of the plasma diode to all existing data and to use the theory to interpret results of new experiments which probably will become available in the near future. The most important features of this report include the concept that in the presence of a hot surface at a sufficiently high temperature and having a sufficiently high work-function, ionization of cesium can be so copious that an ion space-charge sheath will form in the immediate neighborhood of the hot surface and serve as a means of injecting electrons into the plasma space. If, after injection, the nearly monoenergetic electron distribution becomes randomized so that it can be characterized as an electron temperature of approximately 5000° or more, a fractional ionization exceeding 90 per cent can be maintained in the diode space. If the electron temperature is significantly lower than this value, the ionization of the cesium will be so slight that its function will be largely that of altering the work-functions of the conducting materials and to some extent the ions produced will neutralize space charge. The full development of an adequate

plasma that will be stable requires this mechanism of injection and energy redistribution to obtain electron temperatures far in excess of those characterized by the heated surface itself.

The numerical relations given to prove these points have depended upon the interpretation of basic ionization and collision cross section data available in the literature. A close reading of this report shows that some choices had to be made based on a personal interpretation of the existing data. Criticism of these choices backed up by better data or more conclusive arguments would be very welcome.

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1. J. B. Taylor and I. Langmuir, Phys. Rev. 44, 423 (1933).
2. R. H. Fowler, Statistical Mechanics, 2nd ed. p. 366. Cambridge Univ. Press 1936; W. B. Nottingham, "Thermionic Emission", Handbuch der Physik, Springer-Verlag, Germany, Vol. 21, 1956. See Sects. 21-24.
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4. S. L. Mohler, U. S. Bureau of Standards Jour. Research, 19, 447, 559 (1937); Research Paper RP 1036 and RP 1045.
5. W. B. Nottingham, "Thermionic Emission," Handbuch der Physik, Springer-Verlag, Germany, Vol. 21, 1956. See Sect. 25 and Table 3c for the Langmuir analyses. Equation for sheath thickness is:

$$x_c = \chi_c x_1 = \sqrt{2}\left[\frac{kT_g \epsilon_o}{q^2 f_s N}\right]^{1/2} \chi_c = 97.6\left[\frac{T_g}{f_s N}\right]^{1/2} \chi_c$$

Express $\psi_c = (V_c / \overline{V}_g)$ with $\overline{V}_g = \frac{T_g}{11600}$;

take χ_c corresponding to this ψ_c from Table 3c and solve for x_c. The actual sheath thickness will be greater than x_c.

Appendix 1

H(u) functions used in Eqs. 15 and 18. See Fig. 5 for plot.

For $u = \dfrac{\phi_1 - V_i}{\overline{V}_-} \leq 0$

$$H_2(u) = u - \ln (1 + e^u) - \ln (1 - \tfrac{1}{2} e^u + \tfrac{1}{3} e^{2u} - \tfrac{1}{4} e^{3u} + \tfrac{1}{5} e^{4u} - \dots)$$

$$u = \frac{\phi_1 - V_i}{\overline{V}_-} \geq 0$$

$$H_1(u) = u - \ln (1 + e^{-u}) - \ln (u + e^{-u} - \tfrac{1}{2} e^{-2u} + \tfrac{1}{3} e^{-3u} - \tfrac{1}{4} e^{-4u} + \dots)$$

u	H(u)	u	H(u)
- 4	- 4.018	1.0	+ 0.414
- 3.5	- 3.530	1.5	+ 0.767
- 3	- 3.050	2.0	+ 1.118
- 2.5	- 2.580	2.5	+ 1.474
- 2.0	- 2.063	3.0	+ 1.835
- 1.5	- 1.600	3.5	+ 2.208
- 1.0	- 1.152	4.0	+ 2.591
- 0.5	- 0.729	4.5	+ 2.982
0	- 0.327	5.0	+ 3.383
+ 0.5	+ 0.053		

Appendix 2

Correlation function. See Eqs. 15 and 18 and Fig. 6

$$f(V_p / \overline{V}) = \frac{V_p}{\overline{V}} - \frac{1}{2} \ln \left(\frac{V_p}{\overline{V}} + 1\right)$$

$\frac{V_p}{\overline{V}}$	$f(\frac{V_p}{\overline{V}})$	$\frac{V_p}{\overline{V}}$	$f(\frac{V_p}{\overline{V}})$
0	0	6.0	5.017
0.5	0.297	8.0	6.891
1.0	0.653	10.0	8.801
1.5	1.042	12.0	10.718
2.0	1.451	14.0	12.646
3.0	2.307	16.0	14.583
4.0	3.195	18.0	16.528
5.0	4.104	20.0	18.478
		24.0	22.410

Glossary of Symbols

General Theory of the Plasma Diode Energy Converter

a	Area of an electron emitter, Eq. 8
f	Fractional ionization in the plasma, Eq. 15
f_s	Fractional ionization near collector sheath boundary, Eq. 30
FL	Fermi level, Fig. 1
$H_1(u)$	Function of u given by Eq. 16 for positive values
$H_2(u)$	Function of u given by Eq. 19 for negative values of u
i_c	Current in amp. from an area a as in Eq. 8
I_l	Current density from an emitter in amp. per sq. meter, Eq. 1
I_+	Atom arrival rate expressed in amp. per sq. meter, Eq. 6
k	Boltzmann's constant of value 1.38×10^{-23} joule/deg. Eq. 2
K	Kelvin temperature
M	Mass of cesium atom. Value is 2.2×10^{-25} kg. Eq. 29
n_o	Number of electrons or ions per unit volume at the emitter sheath boundary in number per cubic meter, Eq. 11
n_s	Number of ions per unit volume at the emitter surface s_1, Eq. 12
n_-	Electron concentration in the plasma proper in number per cubic meter, Eq. 26
N	Total atom concentration (including ions) in number per cubic meter
N_{Cs}	Atom concentration near condensation surface in number per cubic meter, Eq. 4
p	Pressure in mm of mercury, Eq. 3
P_c	"Collision probability" expressed as reciprocal of mean-free path in gas at 1 mm pressure and 0^oC. Unit is $(meter)^{-1}$.
P_i	"Ionization efficiency" expressed as reciprocal of mean-free path of an electron for an ionizing collision at 1 mm pressure and 0^oC. Unit is $(meter)^{-1}$. Eq. 24
q	Electron charge of 1.6×10^{-19} coulomb, Eq. 2
R	Rate proportionality constant of Eq. 27
s_1	Surface of emitter, Fig. 1
s_2	Surface of collector, Fig. 1
S_1	Emitter sheath, Fig. 1
S_2	Collector sheath, Fig. 1
S'	Function of u given by Eq. 17 for positive values
t	Time in seconds, Eq. 27
T	Emitter temperature in degrees Kelvin, Eq. 1
T_{Cs}	Temperature of condensation surface of cesium in ^oK, Eq. 3
T_g	Temperature of cesium gas near sheath boundary in ^oK, Eq. 30
T_-	Electron temperature in the plasma, Eq. 20
u	Emitter work-function related to ionization potential by Eq. 14
V	Potential difference between the Fermi levels of the emitter and collector, Fig. 1
V_c	Potential difference across collector sheath, Ref. 5
V_i	Ionization potential for cesium 3.89 volt, Eq. 14

Glossary of Symbols (continued)

V_{oc}	Potential difference between Fermi levels under open-circuit condition, Fig. 4
V_p	Plasma potential relative to surface potential at s_1, Fig. 1
$\overline{V}$	Electron volt equivalent of the temperature T, Eq. 2
$\overline{V}_-$	Volt equivalent of electron temperature T_-, Eq. 20a
x	Emitter sheath thickness with sheath potential V_p and concentration at boundary of n_o, Eq. 13
x_c	Collector sheath thickness, Ref. 5
x_1	Unit distance in meters as expressed in Ref. 5
α	Coefficient used in Eq. 24
ϵ_o	Permitivity of free space of value, 8.85×10^{-12} farad/m, Eq. 13
λ_c	Collision mean-free path, Eq. 21
μ_{Cs}	Atom evaporation rate in number per sq. meter in one second, Eq. 5
ν_r	Recombination rate in number per cubic meter in a second, Eq. 27
ν_+	Ion production rate in number per sq. meter in a second
σ_i	Ionization cross section, Eq. 25
ϕ_{oc}	Surface potential of collector (open-circuit condition) relative to emitter FL, Fig. 4
ϕ_m	Space-charge minimum relative to emitter FL, Fig. 1
ϕ_1	Emitter work-function, Fig. 1
ϕ_2	Collector work-function, Fig. 1
χ_c	Sheath thickness expressed in units of x_1, Ref. 5
ψ_c	Collector sheath potential in units of $\overline{V}_g$, Ref. 5

A SIMPLIFIED METHOD FOR THE COMPUTATION OF THE ELECTRICAL PROPERTIES OF A CLOSE-SPACED THERMIONIC CONVERTER

Foreword

The electrical properties of close-spaced thermionic diodes likely to be used as high vacuum heat to electrical energy converters may be predicted with accuracy. This method depends on the application of accurate equations that permit the designer to introduce all of the important factors which are: (1) the emitter temperature, (2) the diode spacing, (3) the collector work-function, and (4) the emitter work-function. Although tables and plotted curves may be used, all calculations may be carried through quickly with only a good slide rule. Results on seventeen different configurations are compared with the "exact" digital computed solutions obtained by Rittner[2]. This paper was originally prepared for publication simultaneously with Rittner's paper. It is hope that this medium of publication will make the results of this study available to most of the workers interested in this development. Reprints will be made available on request.

Introduction

Two publications[1] have been made by the author on the subject of electrical properties of high vacuum thermionic diodes used as heat converters. The scope of those papers was limited by the requirement that the equations be applied only to diode configurations in which the electron emitter could be approximated by one having "unlimited emission capability." For thermionic converters that must function at a maximum temperature of about 1500°K, the dispenser cathode seems to hold great promise. Rittner[2] has demonstrated, by his detailed and exact calculations applied to close-spaced diode configurations having limited emission capability, that my earlier equations predicted greater power conversion than can be realized practically. It is the purpose of the present paper to call attention to the tables and computational methods outlined in Thermionic Emission[3] and show how they may be applied to the analysis of the electrical properties of close-spaced thermionic converters having practical cathodes. These methods permit the user to predict the anticipated performance of any design configuration likely to be of practical interest. The maximum error need not be more than 4 per cent with a good slide rule as the only computational tool needed.

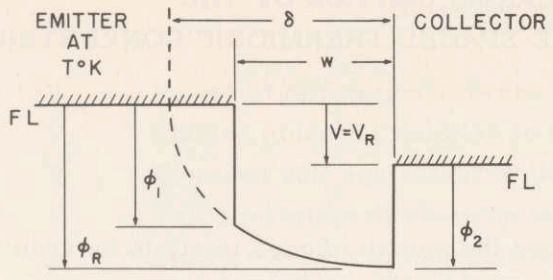


Fig. 1

Electron potential distribution with critical condition of zero gradient at the collector.

General Outline of Procedure

Important symbols needed for this discussion are illustrated graphically in Fig. 1 which shows the electron motive diagram for a diode of actual spacing w. This diagram applies from the Fermi level of the emitter through the work-function barrier ϕ_1 just at the surface of the emitter, across the evacuated space to the surface of the collector, and then inside the collector to its Fermi level. The controllable voltage V has been adjusted to V_R so that the space-charge distribution of the electrons in transit results in exactly zero field coinciding with the surface of the collector. Under this condition the current density carried from the emitter to the collector is I_R. The following equation gives the absolute maximum value of electron current I_m that can flow across a diode of spacing w from the emitter at a temperature T under the condition of zero field at the collector.

$$I_m = 7.729 \times 10^{-12} \frac{T^{3/2}}{w^2} = 9.664 \times 10^{-6} \frac{V_T^{3/2}}{w^2} . \qquad (1)$$

$$V_T = \frac{kT}{q} = \frac{T}{11600} . \qquad (2)$$

In this equation the current density will be in amp/cm^2 if the distance w is expressed in centimeters. The true value of I_R is always less than I_m since the actual potential distribution in the space is that associated with a diode of augmented spacing shown in Fig. 1 as δ. The following equation relates these quantities and defines a new parameter z^2.

$$\frac{w^2}{\delta^2} = z^2 = \frac{I_R}{I_m} . \qquad (3)$$

The true zero field emission capability of a cathode is determined by its temperature and the value of the true work-function ϕ_1. This emission, often known as the "saturated emission," can be computed accurately by:

$$I_o = 120\, T^2\, e^{-\phi_1/V_T} . \qquad (4)$$

These quantities may be related by the following equation

$$\frac{I_o}{I_m} = z^2\, e^{\frac{\phi_R - \phi_1}{V_T}} \qquad (5)$$

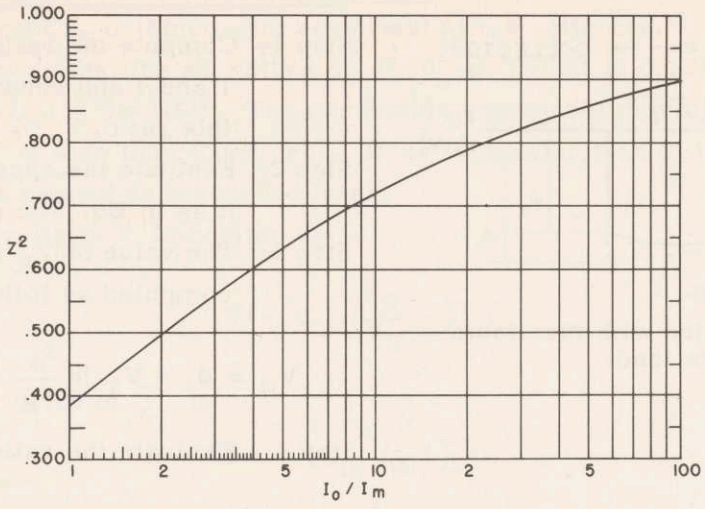


Fig. 2

Computational chart relating z^2 to (I_o/I_m).

The Langmuir space-charge theory applied to the relation in Eq. 5 as presented in Sect. 43 of T.E. permits the computation of the correct value of z^2 as a function of (I_o/I_m). An abridged table of values is given here in the Appendix as Table 3 and the graphical presentation of the table is shown as Fig. 2.

Since interpolation between entries of the table is often less convenient than the use of empirical equations, a "slide rule" method of evaluating z^2 with accuracy generally better than 2 per cent is given by the following equations:

Apply only to range $$1.5 < \frac{I_o}{I_m} < 100$$

$$z^2 = 1 - \frac{0.840}{\tan\left[\log_{10}\frac{I_o}{I_m}\right]^n + 1.433} \tag{6}$$

$$n = 1.30 - 0.41 \log_{10}\frac{I_o}{I_m} \tag{6a}$$

Apply to range $$100 < \frac{I_o}{I_m}$$

$$z^2 = 1 - \frac{1.11}{\left(\frac{I_o}{I_m}\right)^{1/2}} \tag{7}$$

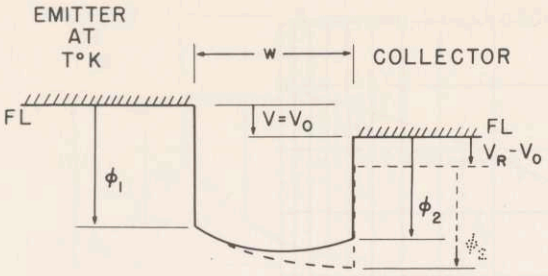


Fig. 3

Potential distribution with maximum power in load.

Procedure for the Computation of Properties

Step 1: Compute the ratio (I_o/I_m) from Eqs. 1 and 4 and record the logarithm of this ratio.

Step 2: Evaluate the appropriate z^2 and use it as in Eq. 3 to determine I_R.

Step 3: The value of V_R (see Fig. 1) may be computed as follows:

$$V_R = \phi_1 + V_T \ln \frac{I_o}{I_R} - \phi_2 \qquad (8)$$

Step 4: Evaluate the ratio of (V_R/V_T).

Figure 3 shows schematically the motive diagram for the diode delivering maximum power. If V is the voltmeter reading that indicates the separation between the Fermi level of the emitter and the Fermi level of the collector, there will be a particular value of V, namely V_O, for which the maximum power can be delivered to an external load. The following equation defines a symbol used in Table 9 of T.E.

$$S' = (V_R - V)/V_T \, . \qquad (9)$$

If the emitter had had unlimited emission capability a change in collector voltage ΔV relative to that required for the current I_m to flow is defined by

$$\sum = \frac{\Delta V}{V_T} \, . \qquad (10)$$

The collector current density I_∞ received at any value of Σ is related to I_m by a universal function in which

$$\frac{I_\infty}{I_m} = U^2 = f_\infty (\sum) \qquad (11)$$

This functional relation is tabulated in Table 8 of T.E. and an abridged listing is given here as Appendix Table 4.

A similar relation may be expressed for the actual current density (I) delivered from an emitter of limited capability

$$\frac{I}{I_R} = u^2 = f(S') \, . \qquad (12)$$

For these purposes, emission capability is best expressed as

$$\frac{I_o}{I_R} = u_o^2 \qquad (13)$$

Thus u_o^2 is the factor which relates the zero field current from the emitter to the actual critical current I_R obtained with zero field at the collector.

Analysis shows that for all values of $u^2 < u_o^2\, e^{-1}$, that is $0.3769\, u_o^2$, the universal function $f_\infty(\sum) = f(S')$. The correction even under the high current demand of $u^2 = 0.3769\, u_o^2$ is less than 1 per cent. If u^2 is greater than $0.3769\, u_o^2$, an additional correction is needed as discussed later.

The meter voltage V is related by

$$V = V_T \left(\frac{V_R}{V_T} - \sum\right). \tag{14}$$

The current at any value of $\sum$ is

$$I = z^2\, I_m\, f_\infty(\sum). \tag{15}$$

The power delivered is

$$P = IV = z^2\, I_m\, V_T \left[f_\infty(\sum)\left(\frac{V_R}{V_T} - \sum\right)\right]. \tag{16}$$

The universal function for the quantity in the brackets of Eq. 16 is

$$\Pi = U^2 \left(\frac{V_R}{V_T} - \sum\right). \tag{17}$$

This relation may be evaluated as a function of Σ for any specific value of V_R/V_T. Typical evaluations are shown in Fig. 4.

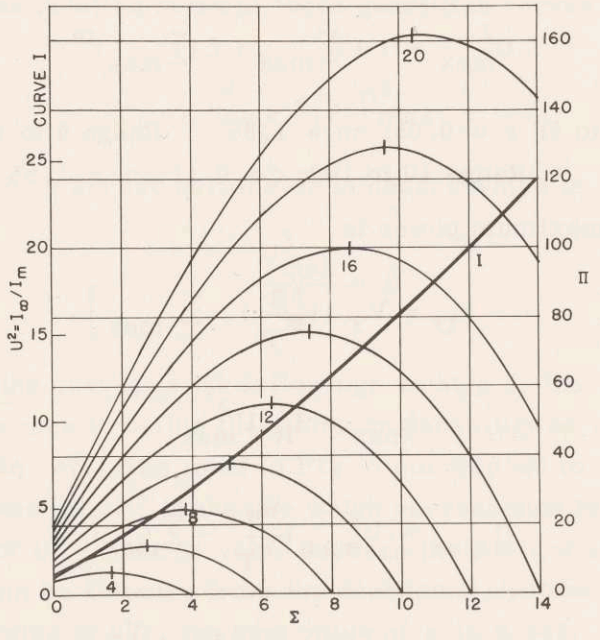


Fig. 4

Values of Π as a function of Σ for selected values of (V_R/V_T) of 4 to 20 and curve I is (I_∞/I_m) of Table 4. Appendix.

For each value of (V_R/V_T) it is of special interest to know the value of Σ that gives the maximum Π .

Two methods have been used to obtain the value of Σ very close to the maximum in Π . These may be applied best to specific ranges.

Range limits: $$1 < \frac{V_R}{V_T} < 20$$

$$S'_{max} = \Sigma_{max} = 0.556 \left(\frac{V_R}{V_T} - 1\right) . \tag{18}$$

Range limits: $$6 < \frac{V_R}{V_T} < 20$$

$$\Pi_{max} = 0.385 \left(\frac{V_R}{V_T}\right)^2 \tag{19}$$

Step 5:

Range limits: $$1 < \frac{V_R}{V_T} < 12 .$$

The value of Σ_{max} is first calculated from Eq. 18 and the value of U^2_{max} is determined from Table 4 (Appendix) by interpolation.

An alternative method good for most purposes uses the following approximate formula which is correct to better than 2 per cent up to $\Sigma = 100$.

$$U^2_{max} = 1 + \Sigma_{max} + c\,(\Sigma_{max})^m \tag{20}$$

with

Range 0.5 to 5; $c = 0.08$; $m = 1.85$ Range 4 to 15; $c = 0.1$; $m = 1.7$

Range 10 to 100; $c = 0.11$; $m = 1.65$

The voltage output at maximum power is

$$V_O = V_T \left[\left(\frac{V_R}{V_T}\right) - \Sigma_{max}\right] . \tag{21}$$

The current is

$$I_{max} = I_R\, U^2_{max} \tag{22}$$

$$\Pi_{max} = U^2_{max} \left(\frac{V_R}{V_T} - \Sigma_{max}\right) \tag{23}$$

$$P_{max} = I_{max}\, V_O = I_R\, V_T\, U^2_{max} \left(\frac{V_R}{V_T} - \Sigma_{max}\right) \tag{24}$$

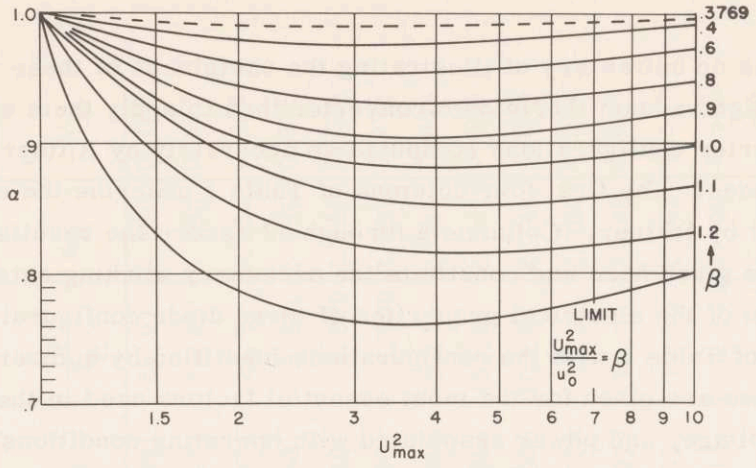


Fig. 5

Computational chart for determining α as a function of U^2_{max} for various β values.

Step 6: An additional correction is needed with emitters of very low capability which depends on the use of Table 9 of T.E. The range of U^2_{max} which calls for this step is identified by:

$$U^2_{max} > 0.3769\ u^2_o\ . \tag{25}$$

With the help of the curves shown in Fig. 5, it is possible to convert the computed value of U^2_{max} to a similar quantity directly usable in Eqs. 22 and 23 which is defined as u^2_{max}. The relation between these quantities serves to define the required multiplying factor α.

$$u^2_{max} = \alpha\ U^2_{max}\ . \tag{26}$$

In order to use Fig. 5, a second parameter is needed which is defined by:

$$\frac{U^2_{max}}{u^2_o} = \beta\ . \tag{27}$$

The method of using the curves is the following: In Step 5, the value of U^2_{max} was determined and u^2_o serves to define the emission capability as in Eq. 13. The ratio determines β as in Eq. 27. The lines in Fig. 5 correspond to different values of β. The bottom line marked "limit" is the one which corresponds to a β of approximately 1.3. For any value of U^2_{max} and β value between the limit of about 1.3 to 0.3769, a suitable value of α can be selected from the coordinate system. Thus for a value of U^2_{max} of 3 and a β value of 0.7, the best value of α is 0.945. Equation 26 yields then a value of u^2_{max} of 2.84 to be used in Eqs. 22 and 23.

Discussion

There is no better way of illustrating the usefulness of these equations to the designer of a high vacuum thermionic converter than to apply them to the seventeen different converter configurations computed so accurately by Rittner[2] for limited emission cathodes. The first four columns of Table 1 describe the configuration parameters set by Rittner. Columns 5 through 12 record the results of the application of the equations given here and constitute the necessary working entries required for the computation of the electrical properties of these diode configurations. Table 2 is a continuation of Table 1 with the configurations identified by number only, in which numerical values are given for the most essential factors used in the computation of the current, voltage, and power associated with operating conditions which give the maximum power. Columns 8, 10, and 12 record the values of current, voltage, and power computed by Rittner's "exact" method. Whereas columns 7, 9, and 11 record results I obtained by the approximate method outlined here. Note that in all cases the agreement is remarkably good.

This favorable result is to be contrasted with that indicated in his table which records the computations he made by his applying equations suitable only to unlimited emission cathodes. He ignored the fact that all of the correct equations and tables were in T.E. The basic difference between the calculations which he made dependent on my earlier approximate method appears quantitatively in column 8 of Table 1. The specified emitters yield z^2 values that range from a minimum of 0.488 to a maximum of 0.933. The "unlimited emission capability" theory offers a means of computation when the z^2 value is completely unknown and, assumed for the purpose of calculation, to be 1.0.

The conclusion to be drawn is that a designer of a thermionic converter may set boundary conditions in terms of dimensions, temperature, and work-functions and compute very readily after a very few minutes of systematic calculation the electrical properties of such a converter with an accuracy which is far better than his ability to construct a converter that meets his specifications.

The reverse process is also important to the designer in that these equations permit him to analyze experimental results so as to use them as a means of determining the actual configuration which he has built without a destructive physical examination.

Acknowledgments

The writer is very grateful to Dr. Rittner for his examination of this problem and the privilege of seeing a preprint of his paper. This cooperation between workers in this field lis greatly to be admired. I wish also to acknowledge the assistance of Miss Betty Campbell who checked most of the calculations presented here.

Appendix. Table 1

Computed Electrical Properties of Diodes

Configuration Parameters Identical To Those of Rittner[2]

	1	2	3	4	5	6	7	8	9	10	11	12
	$T^{\circ}K$	ϕ_1	ϕ_2	w, cm	I_o	I_m	$\frac{I_o}{I_m}$	z^2	I_R	$\frac{I_o}{I_R}$	ϕ_R	V_R
1	1465	2.155	1.855	0.001	10.034	0.4334	23.15	0.801	0.3471	28.91	2.580	0.725
2	1465	2.155	2.155	0.001	10.034	0.4334	23.15	0.801	0.3471	28.91	2.580	0.425
3	1465	2.155	1.655	0.001	10.034	0.4334	23.15	0.801	0.3471	28.91	2.580	0.925
4	1465	2.155	1.855	0.0005	10.034	1.7336	5.788	0.652	1.130	8.880	2.431	0.576
5	1465	2.155	2.155	0.0005	10.034	1.7336	5.788	0.652	1.130	8.880	2.431	0.276
6	1465	2.155	1.655	0.0005	10.034	1.7336	5.788	0.652	1.130	8.880	2.431	0.776
7	1465	2.155	1.855	0.002	10.034	0.1084	92.61	0.892	0.0966	103.81	2.741	0.886
8	1465	2.155	2.155	0.002	10.034	0.1084	92.61	0.892	0.0966	103.81	2.741	0.586
9	1465	2.155	1.655	0.002	10.034	0.1084	92.61	0.892	0.0966	103.81	2.741	1.086
10	1540	2.179	1.855	0.001	21.261	0.4671	45.52	0.852	0.3980	53.42	2.707	0.852
11	1410	2.137	1.855	0.001	5.544	0.4092	13.55	0.752	0.3077	18.02	2.488	0.633
12	1350	2.117	1.855	0.001	2.764	0.3834	7.21	0.680	0.2607	10.60	2.392	0.537
13	1295	2.100	1.855	0.001	1.368	0.3602	3.798	0.594	0.214	6.393	2.307	0.452
14	1240	2.082	1.855	0.001	0.6444	0.3375	1.909	0.488	0.1647	3,913	2.228	0.373
15	1465	2.355	1.855	0.001	2.059	0.4334	4.751	0.626	0.2713	7.589	2.611	0.756
16	1465	1.855	1.855	0.001	107.99	0.4334	294.16	0.933	0.4044	267.02	2.561	0.706
17	1000	1.600	0.800	0.00254	1.048	0.03788	27.67	0.817	0.03095	33.87	1.904	1.104

Appendix. Table 2

Continuation of Computed Electrical Properties of Diodes

Summarized Results. Direct Comparison with Rittner's "exact" values

1	2	3	4	5	6	7	8	9	10	11	12
	$\frac{V_R}{V_T}$	Σ_{max}	U^2_{max}	$\frac{U^2_{max}}{u^2_o}$	u^2_{max}	I_{max}	I_{exact}	V_O	V_{Oexact}	P_{max}	P_{exact}
1	5.74	2.65	4.14	0.143		1.44	1.42	0.390	0.393	0.562	0.557
2	3.36	1.31	2.45	0.085		0.851	0.845	0.260	0.259	0.221	0.219
3	7.32	3.53	5.35	0.185		1.86	1.84	0.479	0.480	0.891	0.884
4	4.56	1.98	3.27	0.369	3.21	3.63	3.55	0.326	0.332	1.200	1.18
5	2.19	0.655	1.69	0.188		1.91	1.89	0.193	0.193	0.369	0.365
6	6.14	2.86	4.45	0.502	4.29	4.84	4.7	0.413	0.426	2.00	2.00
7	7.02	3.35	5.10	0.050		0.493	0.495	0.463	0.461	0.228	0.228
8	4.64	2.03	3.33	0.032		0.322	0.322	0.330	0.329	0.106	0.106
9	8.60	4.25	6.38	0.061		0.617	0.620	0.549	0.547	0.339	0.339
10	6.42	3.02	4.64	0.087		1.85	1.85	0.451	0.449	0.834	0.830
11	5.21	2.34	3.74	0.207		1.15	1.17	0.348	0.338	0.400	0.396
12	4.61	2.01	3.31	0.312	3.28	0.855	0.836	0.303	0.307	0.261	0.256
13	4.05	1.69	2.91	0.455	2.82	0.604	0.589	0.263	0.270	0.164	0.159
14	3.49	1.38	2.53	0.644	2.40	0.395	0.376	0.226	0.238	0.094	0.0895
15	5.99	2.79	4.33	0.571	4.16	1.129	1.09	0.404	0.417	0.473	0.454
16	5.59	2.56	4.02	0.015		1.62	1.62	0.383	0.383	0.623	0.620
17	12.81	6.63	10.12	0.299		0.310	0.308	0.532	0.534	0.166	0.164

Appendix. Table 3

Relation of z^2 to (I_o/I_m)

Abridged from Table 4 of T.E., p. 161

I_o/I_m	z^2	I_o/I_m	z^2
1.066	0.3921	13.70	0.7538
1.263	0.4202	15.38	0.7654
1.485	0.4474	17.21	0.7751
1.735	0.4729	19.30	0.7868
2.105	0.4969	21.57	0.7957
2.329	0.5197	24.14	0.8057
2.686	0.5423	27.01	0.8156
3.086	0.5637	30.14	0.8236
3.532	0.5838	37.54	0.8398
4.029	0.6026	46.69	0.8551
4.587	0.6208	57.91	0.8684
5.221	0.6394	71.66	0.8798
5.914	0.6553	88.67	0.8913
6.699	0.6716	109.6	0.9017
7.574	0.6871	135.2	0.9113
8.549	0.7019	227.7	0.9304
9.623	0.7149	381.4	0.9454
10.834	0.7281	636.7	0.9573
12.19	0.7413		

Appendix. Table 4

Relation defined by Eq. 11

Abridged listing from Table 8 of T.E., p. 165

Σ	U^2	Σ	U^2
0.5655	1.593	23.84	46.28
0.8424	1.901	29.09	59.80
1.279	2.408	34.31	74.10
1.650	2.858	39.49	89.09
2.306	3.690	44.65	104.8
2.900	4.483	49.80	121.2
3.731	5.648	54.93	138.2
5.025	7.573	65.16	174.1
7.443	11.506	75.36	212.2
9.747	15.60	85.53	252.3
13.10	22.08	95.68	294.2
15.28	26.61	105.8	338.2
17.44	31.30	156.4	582.4
19.59	36.15		

References

1. W. B. Nottingham, J. Appl. Phys. 30, 413 (1959); Report on the 19th Annual Conference on Physical Electronics, 1959, M.I.T. p. 71.
2. E. S. Rittner, J. Appl. Phys. 31, 1065 (1960).
3. W. B. Nottingham, Thermionic Emission, Handbuch der Physik, Vol. 21, 1956. Also referred to here as T.E.

THE THERMIONIC DIODE AS A HEAT-TO-ELECTRICAL-POWER TRANSDUCER*

Introduction

The thermionic diode is a heat-to-electrical-power transducer if a fraction of the heating power required to maintain the emitter at a constant temperature can be recovered as useful electrical power in the external circuit that joins the electron collecting electrode to the emitter. The two broad classes of this device are known as the high vacuum diode and the plasma diode. The main purpose of this paper is to discuss the theoretical background by which the performance characteristics of the high vacuum diode can be evaluated in terms of the three most important parameters which are: 1. the temperature of the emitter; 2. the spacing between the emitter and the collector; and 3. the work-function of the collector. It will be shown that the work-function of the emitter is not critical in most transducer designs of practical importance. The necessary detailed technical information to predict the operating characteristics of a "plasma diode" converter are not available at this time. A rather superficial examination of this diode indicates that the emitter work-function will play a far more important part, even though always secondary to that of the collector.

All of the basic information on which the theory of the high vacuum diode depends was published by W. B. Nottingham in "Thermionic Emission" [1]. In this article the treatment of the space-charge problem differs from that developed by Langmuir and others only in the choice of the basic reference used to relate the results of computation to the boundary conditions of an actual experiment. Specifically, the reference condition of importance is that associated with the coincidence of the "space-charge minimum" with the collector surface. For this idealized situation to be realized in practice, the work-function of the collector must be uniform. Failure to achieve uniformity can generally be recognized as a measurable and systematic deviation between experimental results and the predictions of the idealized theory.

Many of the details on this subject are covered in a recent article[2]. It is the purpose of this report to review some of the facts already covered and supplement them with additional data and discussion.

Current-Voltage Characteristics of the High Vacuum Converter

A diode may be constructed with plane parallel electrodes and, after suitable processing, connected in a circuit similar to that illustrated by the insert in Fig. 1. The voltmeter indicates the applied voltage V_a and the electron current is measured

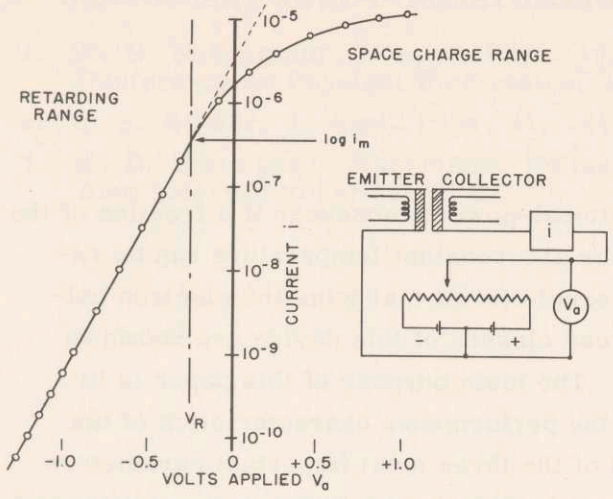


Fig. 1

Diode voltage-current data for temperature determination and onset of space-charge limitation.

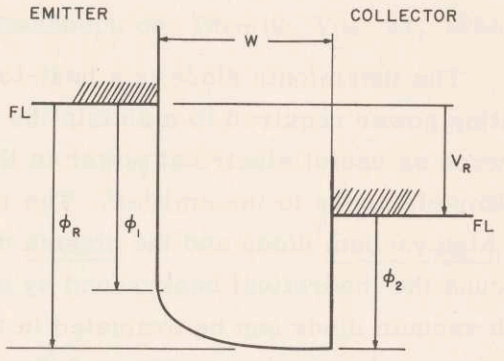


Fig. 2

Potential distribution with critical condition of zero gradient at the collector.

as a function of the applied potential. Even if the diode is constructed to have no appreciable ohmic leakage over surfaces that support the emitter and the collector, it may be necessary to correct the observed currents for any photoelectric or thermionic emission that flows in the opposite direction to the thermionically emitted electrons from the "emitter." After these corrections are made, a plot may be prepared in the manner shown in Fig. 1 to record the logarithm of the electron current as a function of the applied voltage. Over many orders of magnitude of current, its logarithm will be a linear function of the applied voltage. This is the "retarding range." At a critical value of the applied potential designated V_R, this linear plot joins tangentially to a curve characterized by the "space-charge range." It is not easy to identify this point of tangency by inspection. The plotting of the data according to a method to be described permits the observer to compare his observations with a universal curve to establish with considerable accuracy the value of V_R and the current flow at the point of tangency identified here as i_m.

Accurate data taken in the retarding range serve as the best means of determining the true electron temperature of the emitter. Equation 1 serves to define the relation between the temperature T (Kelvin scale) and the electron-volt equivalent of temperature V_T.

$$V_T = \frac{k}{q} T = \frac{T}{11,600} \qquad (1)$$

The basic equations for the relation between the current and the applied potential are given in the direct and the logarithmic forms in Eqs. 2 and 3.

$$i = i_o \, e^{-\frac{V_a}{V_T}} \tag{2}$$

$$\log_{10} i = \log_{10} i_o - \frac{V_a}{2.3\, V_T} \tag{3}$$

The constant i_o in these equations is an empirical constant and has no further significance. A plot similar to Fig. 1 permits one to establish the best straight line to represent all of the data taken in the retarding range. Any two points on this line serve to determine the electron temperature by the following equation:

$$V_T = \frac{V_2 - V_1}{2.3 \log_{10} (i_1/i_2)} = \frac{T}{11,600} \tag{4}$$

The critical condition, referred to at the applied potential V_R that divides the retarding from the space-charge regions, corresponds to a potential distribution between the emitter and the collector represented schematically by Fig. 2. Note that the work-functions of the emitter and the collector are respectively ϕ_1 and ϕ_2 and that the voltmeter reading V_R corresponds to the displacement of the Fermi level of the collector with respect to the Fermi level of the emitter and is, therefore, a directly observable quantity. If the applied potential is more negative than this amount, then even though some space charge exists between the emitter and the collector, it has no influence on the electron current which can flow across the space w.

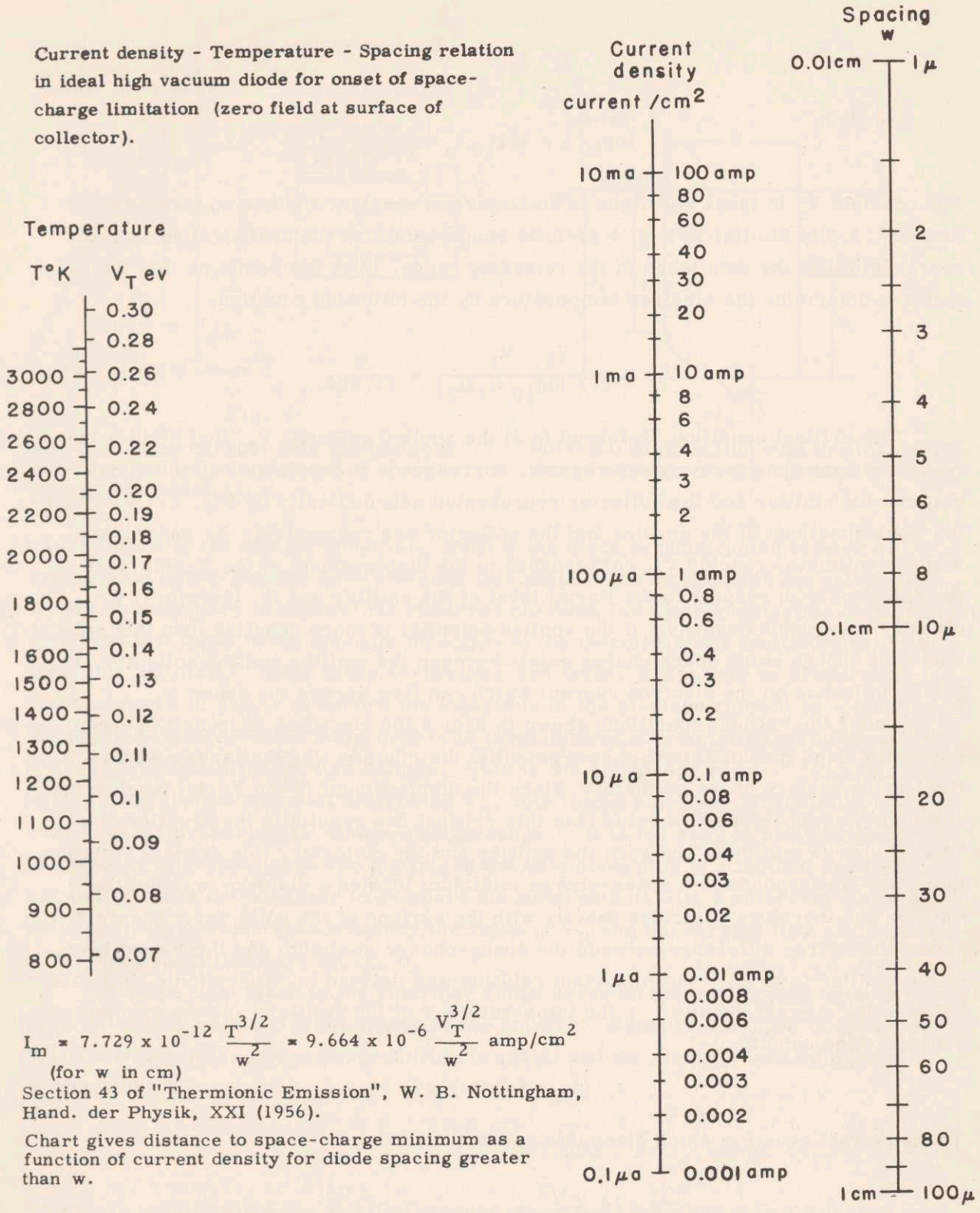
Under the critical conditions shown in Fig. 2 the electrons in transit between the emitter and the collector set up a potential distribution like that shown with zero field at the surface of the collector. Since the displacement of the Fermi level of the collector to a more positive value than this critical one results in the creation of a "space-charge minimum" between the emitter and the collector, this critical situation therefore corresponds to a space-charge minimum located a distance w from the emitter and therefore coincides exactly with the surface of the collector. Figure 2 shows the energy difference between the space-charge minimum and the Fermi level of the emitter to be ϕ_R. An important relation was derived in "Thermionic Emission"[1] between the current density I_m; the temperature T of the emitter; and the spacing w subject to the condition:

$$(\phi_R - \phi_1) > V_T \tag{5}$$

The universal equation which gives this relation is:

$$I_m = 7.729 \times 10^{-12} \frac{T^{3/2}}{w^2} = 9.664 \times 10^{-6} \frac{V_T^{3/2}}{w^2} \text{ amp/m}^2 \tag{6}$$

Current density - Temperature - Spacing relation in ideal high vacuum diode for onset of space-charge limitation (zero field at surface of collector).



$$I_m = 7.729 \times 10^{-12} \frac{T^{3/2}}{w^2} = 9.664 \times 10^{-6} \frac{V_T^{3/2}}{w^2} \text{ amp/cm}^2$$

(for w in cm)

Section 43 of "Thermionic Emission", W. B. Nottingham, Hand. der Physik, XXI (1956).

Chart gives distance to space-charge minimum as a function of current density for diode spacing greater than w.

Fig. 3

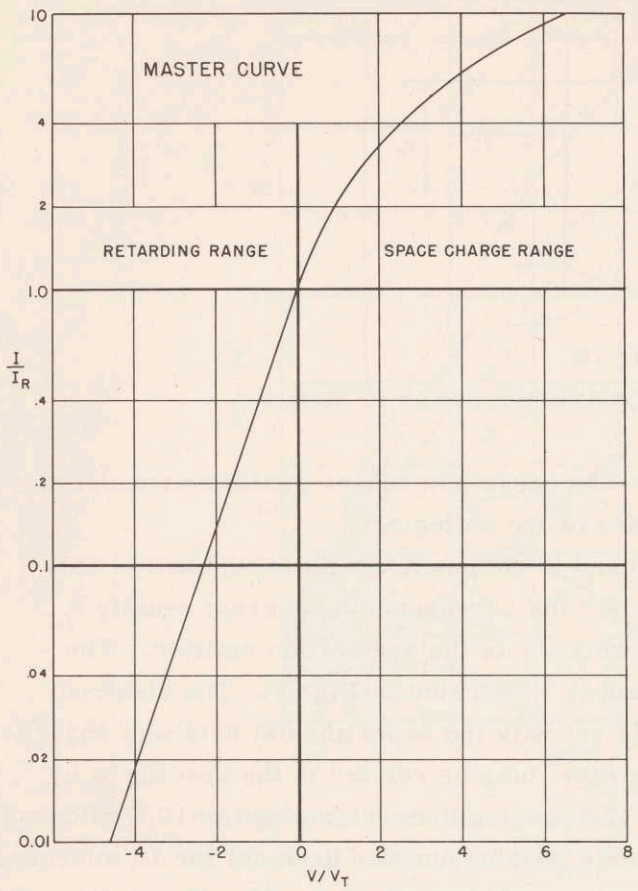


Fig. 4

Master curve used to determine onset of space-charge. See Table 8 of "Thermionic Emission."

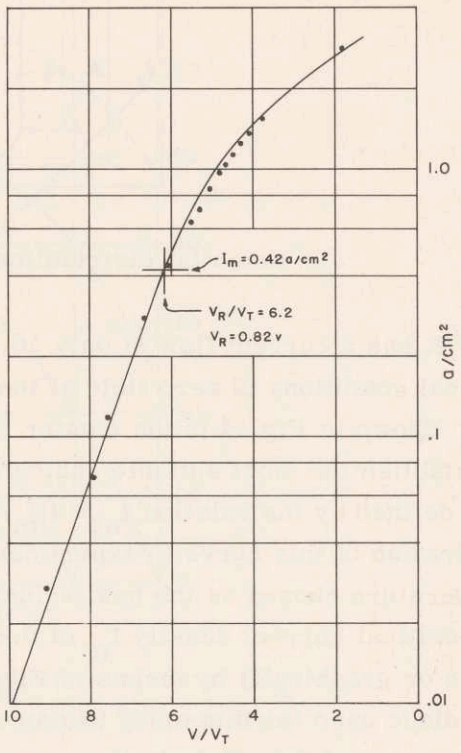


Fig. 5

Comparison theory and experiment. (Data furnished by Hatsopoulos and Kaye.) Spacing 10.5 x 10^{-6} m from analysis.

Since Eq. 6 is such an important one in many problems related to thermionic emission and space charge, a nomographic chart has been prepared and is presented here as Fig. 3. This chart may also be used to find the distance from the emitter to the space-charge minimum at w when it is less than the diode spacing.

The left and the right scales correspond to the temperature and the spacing respectively. Note that the right-hand side of the spacing scale expresses w in microns, whereas the left-hand side of the scale expresses it in centimeters. Since 100 microns and 0.01 centimeters are the same, it is clear that the chart is applicable to the entire range of spacing from 1 micron to 1 centimeter. The current density scale corresponds to the spacing scale in that the densities expressed in amperes per square centimeter on the right-hand side of the line correspond to the use of spacings between 1 and 100 microns. For the larger spacing up to 1 centimeter the current density units are given on the left-hand side of the scale and range from a maximum of 10 milliamperes per square centimeter down to 0.1 microampere per square centimeter. As an example of the use of this chart, note that a diode of 1 mm spacing operated at a temperature of

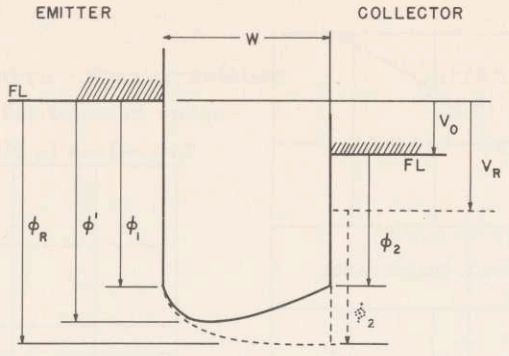


Fig. 6

Potential distribution with maximum power in load.

1160°K has a current flow of only 30.6 microamperes per square centimeter under the critical conditions of zero field at the surface of the collector.

Shown in Fig. 4 is the master curve used in conjunction with experimental data to establish the most suitable choice of V_R and the corresponding current density I_m. It is defined by the relation $I_m = (i_m/a)$ in which a is the area of the emitter. The application of this curve to experimental data is illustrated in Fig. 5. The electron temperature chosen as the best value consistent with the experimental data was 1535°K. The critical current density I_m of 0.42 amp/cm^2 may be related to the spacing w by Eq. 6 or graphically by means of Fig. 3. The spacing thus determined is 10.5 microns. The diode used for this study had an adjustable spacing and had been set for 10 microns. The scatter of the points in the retarding range would have been smaller if more precise control of the emitter temperature had been maintained during the period of observation.

Power to an External Load

Figure 2 illustrates a potential distribution in a transducer in which the power available to an external circuit is the product $V_R i_m$. This seldom represents the maximum power available from a high vacuum transducer. The potential distribution illustrated by Fig. 6 shows a situation in which the output voltage has been reduced from V_R to V_O. As this change in output voltage takes place, the barrier across which the emitted electrons must travel is reduced from ϕ_R to ϕ' and, subject to the condition $(\phi' - \phi_1)$ is greater than V_T, the current flow around the diode increases according to the relation

$$i = i_m e^{\frac{\phi_R - \phi'}{V_T}} \qquad (7)$$

Since the power is the product of V_O and i, the power available increases to a maximum and finally falls to zero when V_O is zero. Space-charge theory permits one to establish a universal relation by which the power output may be computed from the parameter (V_R/V_T).

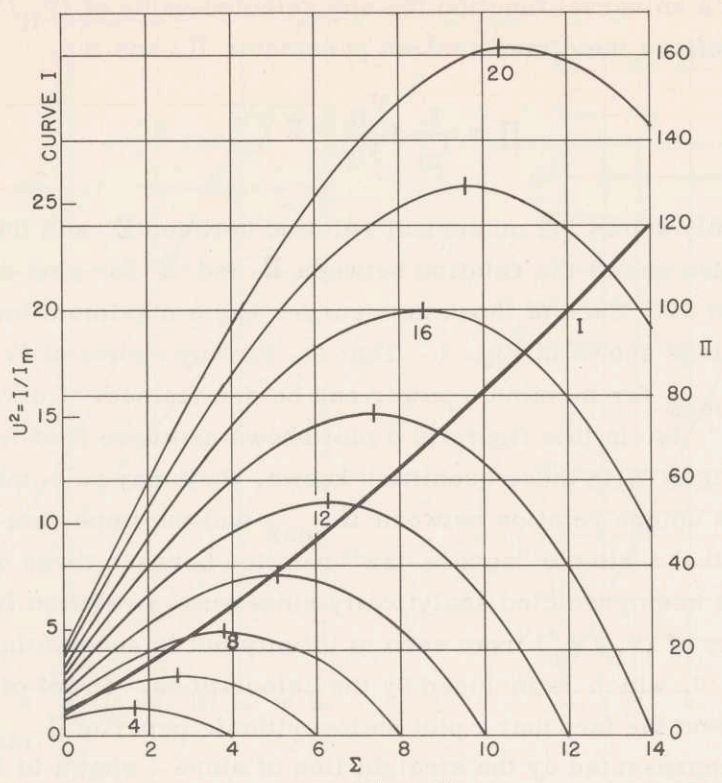


Fig. 7

Values of Π as a function of Σ for selected values of (V_R/V_T) of 4 to 20. Values of (I/I_m) of curve I from "Thermionic Emission" Table 8.

As V_O becomes smaller than V_R it is convenient to express this change in output voltage in units of V_T. This change is expressed by Σ defined by Eq. 8.

$$\Sigma = \frac{V_R - V_O}{V_T} \tag{8}$$

This equation rearranged to express the value of V_O is

$$V_O = V_T\left[\frac{V_R}{V_T} - \Sigma\right] \tag{9}$$

The power per unit area is the product of the current density and the output voltage and is written as follows:

$$P = IV_O = I_m V_T\left[\frac{I}{I_m}\left(\frac{V_R}{V_T} - \Sigma\right)\right] \tag{10}$$

Since Σ and the ratio (I/I_m) are uniquely related to each other as shown by Table 8 of "Thermionic Emission," the factor of Eq. 10 enclosed within the square brackets

may be computed as a universal function for any selected value of (V_R/V_T). The bracketed quantity defines the dimensionless parameter Π and is

$$\Pi = \frac{I}{I_m}\left(\frac{V_R}{V_T} - \Sigma\right) \tag{11}$$

Figure 7 not only shows the numerical relation between Σ and the current ratio $U^2 = (I/I_m)$ but it also shows the relation between Π and Σ for nine different values of (V_R/V_T) from 4 to 20. Each of these nine curves has a maximum located very close to the vertical markings shown in Fig. 7. That is, for any choice of (V_R/V_T) between these values, the Σ_{max} for maximum power can be determined. Curve I of Fig. 8 shows this relation. Also in this figure is a plot shown as curve II of the current ratio U^2_{max} at the maximum. With these quantities known, they may be combined as in Eq. 11 to establish a unique relation between Π_{max} and the important parameter (V_R/V_T). The fact that a simple "square-law" relation between these quantities exists could not have been predicted analytically since such a relation is not precisely correct for all values of (V_R/V_T) from zero to infinity but is substantially correct for the range from 4 to 20, which is included by the calculations. Proof of the square-law relation depends on the fact that a plot on logarithmic paper of Π_{max} as a function of (V_R/V_T) is well represented by the straight line of slope 2 shown in Fig. 9. The equation for this straight line is:

$$\Pi_{max} = 0.385\left(\frac{V_R}{V_T}\right)^2 \tag{12}$$

The power delivered to the external load is expressed by Eq. 13 and the combination with Eqs. 6 and 12 in this form gives Eq. 14.

$$P_{max} = I_m V_T \Pi_{max} \tag{13}$$

$$P_{max} = 3.7 \times 10^{-6} V_T^{1/2} \frac{V_R^2}{w^2} \tag{14}$$

Equation 14 is very important to the designer who wishes to obtain the maximum power since it brings out very clearly the importance of small spacing. The need for a high value of V_R is also important. Since this critical voltage V_R is strongly influenced by the work-function of the collector, it is of the utmost importance that the collector work-function be as low as possible in order to increase V_R for any given configuration and emitter temperature.

The current density at maximum power is given by

$$I_{max} = I_m\left[1 + 0.31\left(\frac{V_R}{V_T}\right)^{4/3}\right] \tag{15}$$

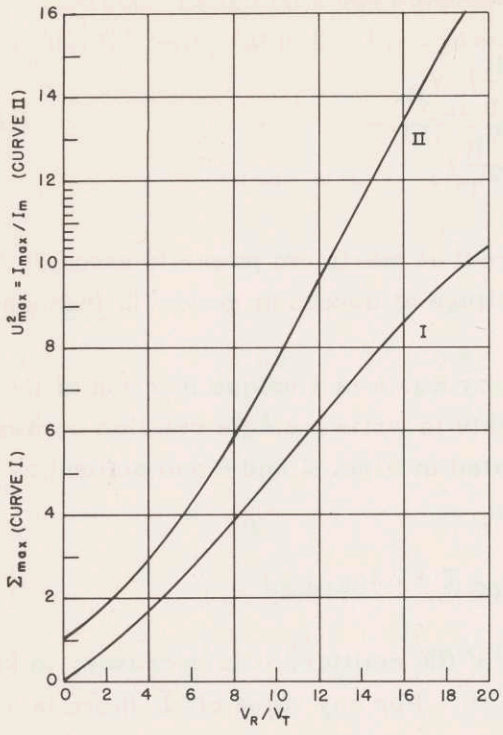


Fig. 8

Values of Σ_{max} and U^2_{max} as functions of V_R/V_T.

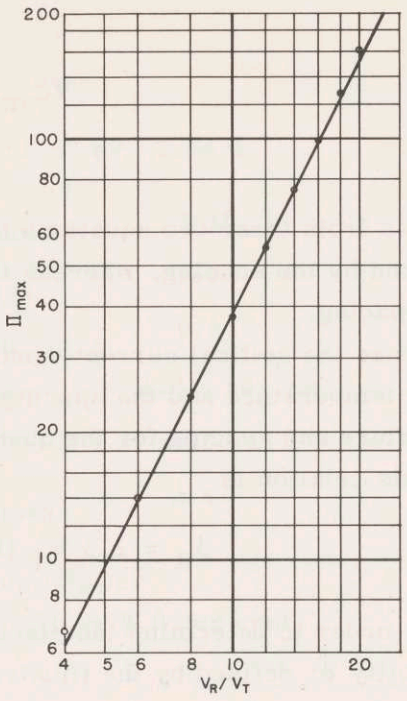


Fig. 9

Values of Π_{max} as a function of V_R/V_T. The straight line representation of the data is a square law.

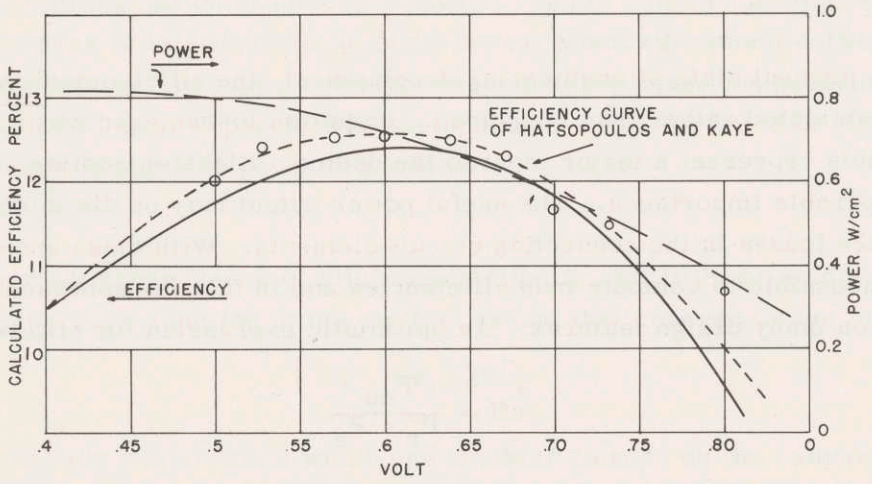


Fig. 10

Comparison between theory and experiment. (Data furnished by Hatsopoulos and Kaye.)

The output voltage at maximum power may be computed by

$$V_{out} = \frac{0.383 \left(\frac{V_R}{V_T}\right) V_R}{1 + 0.31 \left(\frac{V_R}{V_T}\right)^{4/3}} \tag{16}$$

It follows from these two equations that the current at maximum power is strongly influenced by the spacing, whereas the output voltage at maximum power is independent to the spacing.

Since the critical current density as given by Eq. 6 is a unique function of the emitter temperature and the spacing, it is possible to write a single relation between temperature and spacing for the quantity illustrated in Figs. 2 and 6 and defined as ϕ_R. This relation is

$$\phi_R = 2.3\ V_T \left(17.2 + \frac{1}{2} \log_{10} T + 2 \log_{10} w\right) \tag{17}$$

In order to determine the electron cooling of the emitter, it is necessary to know the quantity ϕ' defined by the illustration in Fig. 6. For any value of Σ there is a unique value of (I/I_m) as shown graphically in Fig. 7 and tabulated in Table 8 of "Thermionic Emission." Equation 18 serves then as the means for calculating ϕ'.

$$\phi' = \phi_R - 2.3\ V_T \log_{10} \frac{I}{I_m} \tag{18}$$

The electron cooling per unit area is then calculated as follows:

$$P_e = I(\phi' + 2\ V_T) \tag{19}$$

Efficiency

At the present state of engineering development, the efficiency of the transducer is usually calculated rather than measured. Radiation losses even when minimized as far as possible represent a major input to the device. Electron cooling can be of nearly comparable importance. The useful power output may be diminished by electrical resistance losses in the connecting circuit elements. With these uncertainties it is nearly impossible to compute true efficiencies and in fact the approach to some ideal will depend on many design features. An optimistic expression for efficiency is given by

$$\text{eff} = \frac{P_{out}}{P_r + P_e} \tag{20}$$

The power output is divided by the sum of the radiation loss and the electron cooling. If the only radiation loss is from the emitter to the collector, and this loss is inevitable, then no practical design can be expected given efficiency greater than that described by

this equation. Based on a very approximate method of calculating the minimum radiation loss, the following table has been prepared.

Calculated Properties

Diode $w = 2.54 \times 10^{-5}$ m; $\phi_2 = 1.57$ ev; Area A

T	1510	^oK	P_{max}	3470	watt/m^2
ϕ_R	2.87	volt	I_{max}	5400	a/m^2
ϕ'	2.60	volt	V_{out}	0.64	volt
V_R	1.3	volt	R_{LO}	$1.2 \times 10^{-4}/A$	ohm
(V_R/V_T)	10		T_c	1025	^oK
I_m	702	a/m^2	$(\phi_1)_{max}$	2.5	volt
P_r	10×10^4	watts/m^2	At max Power		
P_e	1.67×10^4	watts/m^2	$(eff)_o$ = 3.0 per cent		

In order to compare the predictions of theory with experiment, a calculation was made to determine the efficiency of a diode spaced at 10 μ as a function of output voltage. Also the power per unit area was calculated as a function of output voltage. The efficiency was calculated according to Eq. 20 with the radiation loss figure P_r thought to be applicable by Hatsopoulos and Kaye[3] so that a comparison could be made with their experimental values. The two calculated curves are shown in Fig. 10 by the solid line for efficiency and the dashed line for power. The dotted line is a smoothed-out representation of the experimental results obtained by Hatsopoulos and Kaye. The purpose of this figure is to show the excellent agreement between theory and experiment in order to support the statement that the equations presented in this report can be relied upon to predict the performance characteristics of the high vacuum diode as a heat-to-electrical-power transducer.

Conclusions

An important consequence of this presentation is that little or nothing needs to be said about the work-function of the emitter except that covered by the inequality

$$\phi' - \phi_1 \geq V_T \qquad (21)$$

That this is the only requirement which one needs to impose on the cathode work-function is not obvious unless a careful inspection is made of Table 9 in "Thermionic Emission" in its relation to Table 8. Figures 16 and 17 of "Thermionic Emission" show the graphical representation of this fact and prove that Eq. 21 is the most important criterion as to whether or not the emitter has a sufficiently low work-function.

In the plasma transducer, heavy ions of low kinetic energy can be trapped in the potential minimum illustrated in Fig. 6. The presence of one ion can effectively cancel the space charge of many hundreds of electrons in transit. The important point is that when the ion density equals the electron density, the ion current is a very small fraction of the electron current.

It seems evident that cesium vapor is the most suitable gas available for use in a thermionic diode transducer. It has a very low ionization potential of 3.88 ev. The density of gas atoms can be controlled either by limiting the total number available or by controlling the temperature of the excess cesium present in the liquid state. Cesium adsorbs to most metallic surfaces and in general reduces their work-function. Thus, an adsorbed layer of cesium of suitable thickness can possibly give the lowest work-function collector available for this purpose. In order for the maximum current to flow across a diode in the presence of cesium vapor, partially ionized, the work-function of the emitter must be low. Such a situation would not of necessity give a good transducer because even though the current might be high, the output voltage would be low. As the effective resistance in the output circuit is increased, the voltage drop across the load increases and the current decreases. The plasma potential thus tends to follow the potential at the surface of the electron collector and the current is reduced because there is an electron "retarding potential" in the immediate neighborhood of the emitter. The surface potential of the emitter is positive with respect to the plasma and an ion retarding sheath exists there which inhibits the loss of ions to that electrode and encourages ions that might be formed from neutral atoms at the surface of the emitter to enter the plasma.

Although many details remain to be filled in, it may be stated again that as far as the operation of the transducer is concerned, the work-function of the emitter is relatively unimportant. Insofar as the emitter may at the same time be the means by which cesium atoms are converted to cesium ions, the efficiency of this conversion can be expected to be work-function sensitive. The future research in this field, therefore, should be directed toward a better understanding of the most efficient means of producing the required number of ions to maintain a plasma. That may be done at the electron emitter itself or at some auxiliary high-temperature electrode.

In spite of the low efficiency of the high vacuum transducer constructed according to presently established techniques, a development of new techniques and the introduction of cesium can very well bring the device to a state in which it may serve as an important method of converting heat to electricity.

References

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