"Cesium Plasma Diode as a Heat-To-Electrical-Power Transducer" by W. B. Nottingham,
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# CESIUM PLASMA DIODE
# AS A
# HEAT-TO-ELECTRICAL-POWER TRANSDUCER

BY

W. B. NOTTINGHAM

## Chapter 8

# CESIUM PLASMA DIODE AS A HEAT-TO-ELECTRICAL-POWER TRANSDUCER

Wayne B. Nottingham

## Abstract

The new interest in the direct conversion of heat-to-electrical power has stimulated research in both the application of the high vacuum diode and the plasma diode to accomplish this purpose. The theory of the high vacuum diode is relatively simple and the experimental verification of the theory has been satisfactory. The plasma diode which depends on the ionization of cesium at a hot surface cannot be worked out in all of its detail at present because of the lack of certain fundamental experimental data. It is possible to make use of published results of Taylor and Langmuir and a detailed analysis of recent thermionic studies to carry the understanding of the plasma diode far enough to make a direct comparison with experiment. This analysis first involves an understanding of the phenomenon of surface ionization. General properties of a plasma and space-charge considerations control the delivery of ions to neutralize electron space charge. When applied to the experimental data available, an interesting result comes as an important simplification. Essential to the theory of the high vacuum diode is the knowledge of the emitter temperature and the diode spacing. The electrical characteristics of the plasma diode have been found to be very closely duplicated by those of a high vacuum diode characterized by an effective distance that is reduced from the actual diode spacing. This fact supports the opinion that the efficiency of the plasma diode may be tremendously improved over that of vacuum diodes of practical design.

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Chapter 6

# CESIUM PLASMA DIODE AS A HEAT-TO-ELECTRICAL POWER TRANSDUCER

Wayne B. Nottingham

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## Introduction

An analysis of the high vacuum diode as a heat-to-electrical-power transducer has been worked out in detail [1] based on theories presented in "Thermionic Emission" published some years ago[2]. In the high vacuum diode it is of the utmost importance to have the lowest possible work-function on the collector and to have the smallest practical spacing. It is anticipated that under these conditions the work-function of the emitter will be practically without influence so long as the space-charge minimum in front of the emitter is of the order of $V_T$ (the electron-volt equivalent of the temperature). Any further decrease in the emitter work-function will do neither good nor harm in controlling the usefulness of the device. In the plasma diode it is not so easy to generalize and since much of the most significant fundamental data relevant to the problem will not be available until additional research studies have been made. These remarks relevant to the plasma diode represent some of my own views on the subject in that they are being developed without the benefit of the experimental evidence needed.

In the high vacuum diode the understanding of the space charge is very important. It controls the properties of diodes that operate within the temperature limits set by available materials and the electron emission properties associated with them. The function of the positive ions in the plasma diode is to alter the effect of electron space charge. Even in the plasma diode there will be space charges near the boundaries of either the emitter or the collector or both.

Assume for the purpose of illustration that the emitter work-function is somewhat higher than the collector work-function and by some mechanism not specified for the moment exactly the right number of ions are available to reduce the space charge to zero. Assume furthermore that the spacing if sufficiently small in relation to the gas pressure so that the drop in potential over the plasma between the surface of the emitter and the collector can be neglected. This condition is represented diagramatically by Fig. 1. Here the true work-functions of the emitter and the collector are $\phi_1$ and $\phi_2$. Note particularly that these are not the "Richardson" work-functions. The current emitted over the barrier $\phi_1$ is given by the equation:

$$I_1 = 120 \times T_1^2 e^{-\frac{\phi_1}{V_T}} \quad \text{amp/cm}^2 \qquad (1)$$

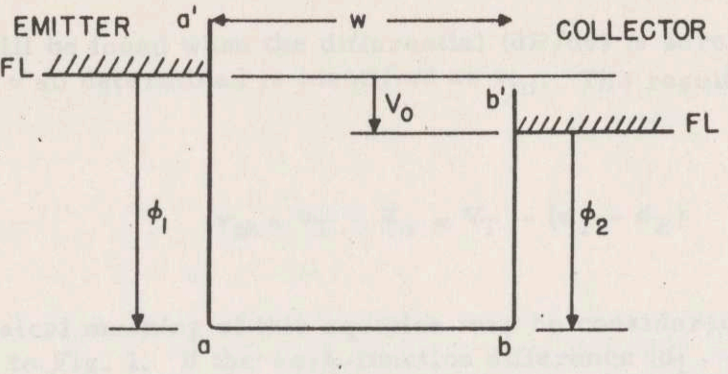

Fig. 1-Motive of an electron with zero field between a and b.

In this equation T is the temperature of the emitter and $V_T$ is its electronvolt equivalent defined by:

$$V_T = \frac{kT}{q} = \frac{T}{11,600} \tag{2}$$

The available power is the product $I_1 V_o$. The total resistance in the circuit from the receiving surface on the collector through the external load and to the emitting surface of the emitter may be expressed for a unit area as

$$R_{10} = \frac{V_o}{I_1} \tag{3}$$

If the tital resistance is less than $R_{10}$, then the current around the circuit will remain constant and the output voltage decrease, thus giving a smaller available electrical power. If the load resistance is made greater than $R_{10}$, then the output voltage increases while at the same time the current decreases. Identify this increase in voltage by the symbol v. The following equation serves to relate these quantities for the calculation of maximum power.

$$P = I_1 e^{-\frac{v}{V_T}} (V_o + v) \tag{4}$$

Equation 4 may be differentiated to obtain an expression for the deviation in the power as a function of v. A maximum of power

output will be found when the differential (dP/dv) is zero and the value of v so determined is identified as $v_m$. The result is given as

$$v_m = V_T - V_o = V_T - (\phi_1 - \phi_2) \qquad (5)$$

The physical meaning of this equation may be considered in its relation to Fig. 1. If the work-function difference $(\phi_1 - \phi_2)$ is greater than $V_T$ then $v_m$ is zero or stated in other words maximum power is delivered to the external load when the resistance is chosen to satisfy Eq. 3 and the overall output voltage is exactly the difference between the work-functions. Under the circumstances in which the receiver work-function is actually greater than the emitter work-function, the power output is a maximum when the voltage available is $V_T$. This situation is illustrated by the diagram of Fig. 2 and the total available power is expressed by

$$P_{max} = 120 \times T^2 \; e^{\frac{-\phi_2 + V_T}{V_T}} \; V_T \; w/cm^2 \qquad (6)$$

In the range of work-function difference for which $(\phi_1 - \phi_2)$ is equal to or less than $V_T$ even though $\phi_1$ is greater than $\phi_2$, maximum power will occur with an available voltage of $V_T$ and Eq. 6 applies.

These statements may be summarized as follows. For work-function differences greater $V_T$ in which the emitter work-function is larger than the collector work-function, maximum power is available at an overall voltage level equal to the difference in the work-functions, whereas in all other cases the maximum power occurs with an available voltage of $V_T$.

It will be the purpose of the following sections to discuss the problem of ion production as well as the delivery to the plasma of the ions and the current flow expected in the presence of atoms and ions.

## Ionization of Cesium at Heated Surfaces

The classic work on the ionization of cesium at heated tungsten surfaces was largely due to Langmuir and his collaborators. One of the latest in that series of researches was reported by Taylor and Langmuir[3]. This work serves as the basis of many of the deductions made here. If the arrival rate of cesium atoms for each square centimeter of a heated tungsten surface is

less than $10^{13}$ atoms per second (cesium condensation temperature $0^{o}C$), a tungsten filament at $970^{o}K$ will have a neligible surface coverage of cesium atoms and every cesium atom that arrives will leave as a cesium ion. As the arrival rate increases to $10^{18}$ atoms / sec-cm$^2$, the minimum temperature for complete conversion of atoms to ions is approximately $1475^{o}K$.

This statement should not be attributed to Taylor and Langmuir but does depend on an analysis of their data which was made available in their Fig. 18. Since no other data are available, these are used to yield a relation between the atom arrival rate $\mu_a$ expressed in atoms per second for each square centimeter and the minimum temperature for which a negligible surface film will form on tungsten. At this temperature practically every atom that arrives will leave the surface as an ion. An empirical equation has been derived to relate this minimum temperature to the atom arrival rate and is

$$T_{min} = \frac{14,100}{27.56 - \log_{10}\mu_a} \qquad (7)$$

At any given arrival rate, the production of ions is discontinuously reduced by approximately a factor of 10 if the tungsten surface is at a temperature slightly below the minimum temperature given by Eq. 7.

The ideal gas laws are used to convert cesium vapor pressure data to atom arrival data. The following formula relates atom arrival to the temperature of the liquid cesium in equilibrium with its vapor.

$$\log_{10}\mu_a = 27.48 - \frac{3900}{T_{Cs}} \qquad (8)$$

The corresponding formulae in the exponential form are given in Eqs. 9 and 10.

$$\mu_a = 3 \times 10^{27} \times 10^{-\frac{3900}{T_{Cs}}} \text{ atoms/cm}^2 \qquad (9)$$

$$\mu_a = 3 \times 10^{27}\, e^{-\frac{8980}{T_{Cs}}} \text{ atoms/cm}^2 \qquad (10)$$

It is an interesting fact that Eqs. 7 and8 may be combined to yield a very simple relation between the temperature of the liquid cesium and the minimum temperature of the heated tungsten surface at which complete ionization takes place. This relation is:

$$T_m = 3.6\ T_{Cs} \tag{11}$$

The fact that the minimum temperature at which all cesium atoms are converted to ions is linearly related to the temperature of the liquid cesium may not be a pure accident of numbers. In the broader sense of the word both the evaporation of atoms from the liquid and the evaporation from a tungsten surface may possibly involve atomic attractive forces that are not so different from each other. This would imply that their vibrational frequency before evaporation might be characterized by a force function not unlike that of a vibrating molecule. In the molecular case there is a dissociation energy whereas in the case of atomic evaporation there is the latent heat of evaporation which is dominated by a quantum state of vibrational activity which is the maximum that can be attained by the atom as it is on the border line between continued adherence to the surface and evaporation. If the equations of motion are not too dissimilar then the factor that relates these two critical temperatures could well be the ratio of the "activation" energies. Equation 8 implies an activation energy for evaporation from the liquid state that is not too far from 0.773 ev. Taylor and Langmuir deduce from their studies of cesium evaporation from tungsten that the heat of evaporation is 2.83 ev if the fraction of the surface coated is small. Note that the ratio (2.83/0.773) is 3.65 which is consistent with the empirical results presented in the form of Eq. 11.

Results of the Taylor-Langmuir experiments as they relate to the yield of ions from surfaces that have an appreciable fraction covered by adsorbed cesium atoms show an extremely small yield of ions. A detailed analysis of the experimental data available yields some interesting results. The equations to be presented fit quite well over the range of cesium pressures and tungsten surface temperatures studied by Taylor and Langmuir. The purpose in deriving empirical equations to represent their data is to permit computations to be made under conditions that involve considerable extrapolation into unexplored areas. The philosophy here is that estimates based on an extrapolation from existing data are better than pure guesses without a systematic relation to establish facts. The ion production rate $\nu_p$ expressed as the number of ions produced per second for each square centimeter area is expressed by the set

of equations that follow:

$$\log_{10} \nu_p = 54.16 - 1.113 \log_{10} \mu_a - \frac{26,600}{T} \qquad (12)$$

$$\nu_p = \frac{1.45 \times 10^{54}}{\mu_a^{1.113}} \, 10^{-\frac{26,600}{T}} \qquad (13)$$

$$\nu_p = \frac{1.45 \times 10^{54}}{\mu_a^{1.113}} \, e^{-\frac{5.28}{V_T}} \qquad (14)$$

Attention may be directed specifically to the results shown as Eq. 13. The ion yield decreases rapidly with a decrease in the temperature of the surface for the range below the temperature $T_m$ introduced in Eq. 7 and related to the liquid cesium temperature by Eq. 11. Equation 13 has no significance at temperatures higher than $T_m$ since then the ion yield is precisely equal to the atom arrival rate $\mu_a$. It is evident from this equation that at a fixed surface temperature T, the ion yield decreases almost linearly (1.113 power) with the atom arrival rate. Stated in another way this equation shows that the probability of ionization at a given surface temperature decreases with the 2.113 power of the arrival rate. The physical explanation for this result depends on the fact that at a given surface temperature the fraction of the surface covered by adsorbed cesium atoms increases as the rate of arrival of neutral atoms increases. This increase in surface coverage brings about a decrease in the average work-function of the surface. The probability that a cesium atom will evaporate as an ion instead of neutral atom is strongly influenced by the work-function of the surface. Since the ion yield decreases as the average work-function decreases, it is to be anticipated that a relation not unlike that of Eq. 13 might hold. The insertion of numbers into these equations and a comparison with the published data of Taylor and Langmuir may convince the reader that over the range of temperature and cesium arrival rate for which good data exist, the equations represent the observations. It is thought that the failure to represent the true facts over the ranges of the variables likely to be encountered in association with the plasma transducer problem will be less than an order of magnitude, that is a factor of ten. It is hoped that the error will be much less than this in many examples that are of practical interest.

## Kinetics of Ions Formed at Hot Surfaces

The reader should be reminded that when drawings are made which show energy relations of electrons in diode structures, these drawings really relate to the "motive function" of an electron. That is, lines and energy levels apply to the potential of an electron in the space of interest. Figure 1 may be used as an example. We start at the Fermi level in the emitter and the line a' - a represents the potential function of an electron in the neighborhood of the surface. The energy difference between the Fermi level and the surface potential at a is the true work-function $\phi_1$ of the emitter. In this energy diagram the surface of the emitter is joined to the surface of the collector by the line a-b and since that line is drawn horizontally it represents the potential energy of an electron in the field-free space between the emitter and the collector. The work-function of the collector is shown by $\phi_2$ as the energy difference between the Fermi level in the collector and the potential energy of an electron at its surface at the point b. The misalignment of the Fermi level of the emitter with respect to that of the collector is indicated by $V_o$. This separation of Fermi levels can be maintained only by having some auxiliary and external source of power and may be measured directly by a voltmeter connected between the emitter and the collector electrodes. The kinetic energy of an electron in this diagram is represented by the fact that the electron is in an energy level which would exist in the diagram below the horizontal line a-b. The separation between the electronic energy level and the line a-b is a direct indication of the actual kinetic energy of the electron associated with the motion across the space from a to b. This direction is taken as the x direction whereas the y and z directions are taken perpendicular to this one and therefore parallel to the planes of the emitter and the collector. We can show in this diagram only the kinetic energy associated with the motion in the x direction. It is not suitable to try to show the kinetic energy associated with the other directions in this one-dimensional diagram. Energy levels do not exist for electrons in the space between the electrodes that would be represented by electron levels above the a-b line. Such levels do exist within the conductors but not outside of them.

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This same potential diagram applies equally well to ions that might be in the space between a and b. Specifically an ion at rest would be represented as being on an energy level coincident with the line a-b, whereas one which is moving in the x direction with some kinetic energy would be occupying an energy level in this diagram above the line a-b. No positive-ion energy levels exist that would be represented by states drawn below the line a-b. These remarks are made to anticipate the discussion with reference to the energy distribution of ions created at a hot surface. One of the few sets of measurements on the ion energy distribution that exists in the literature is that of Fig. 16 of Taylor and Langmuir. Their data are reproduced as Fig. 3. The plot shows the ion current received at a coaxial collector as a function of the applied voltage with the emitter temperature held constant and the rate of arrival of neutral atoms established by a constant bath temperature of 275°K. The critical temperature for this rate of arrival of atoms is 990°K and therefore since the measurements were made at 980° K it was probable that the ion yield was less than 10 per cent of the atom arrival rate. Under this condition the average work-function of the emitter was very close to 3.4 volts. Other evidence would indicate that the work-function of the collector was very close to 2 volts. Therefore under these circumstances, the contact difference in potential was close to 1.4 volts. This fact is exhibited reasonably well by the data of Fig. 3. Under the conditions of the experiment, the heated tungsten surface undoubtedly had considerable nonuniformity in work-function, and also space-charge effects reduce the ion current at zero field. It may be assumed that these two factors account for the relatively poor saturation of the ion current. The evidence seems to be satisfactorily clear that the application of an ion retarding voltage of a fraction of a volt is sufficient to inhibit the flow of ions from the hot tungsten surface over to the collecting electrode. The slope of the semi-log plot is in agreement with the temperature of 980°K.

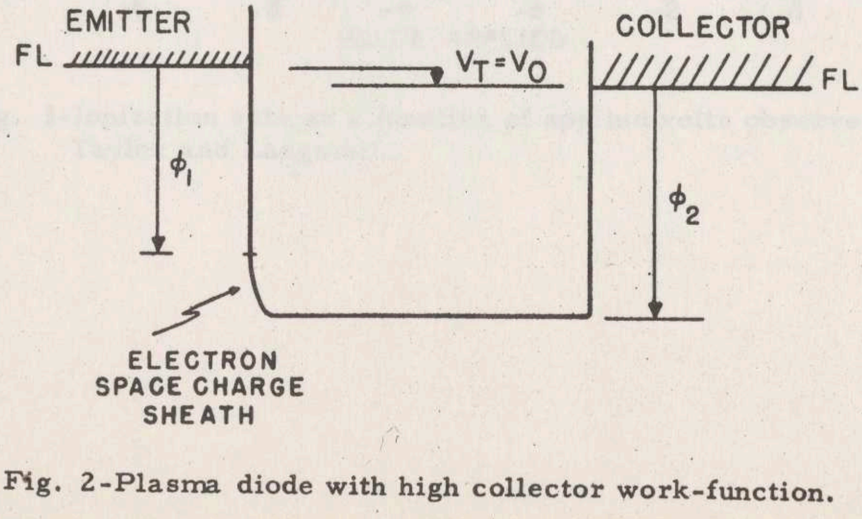

Fig. 2-Plasma diode with high collector work-function.

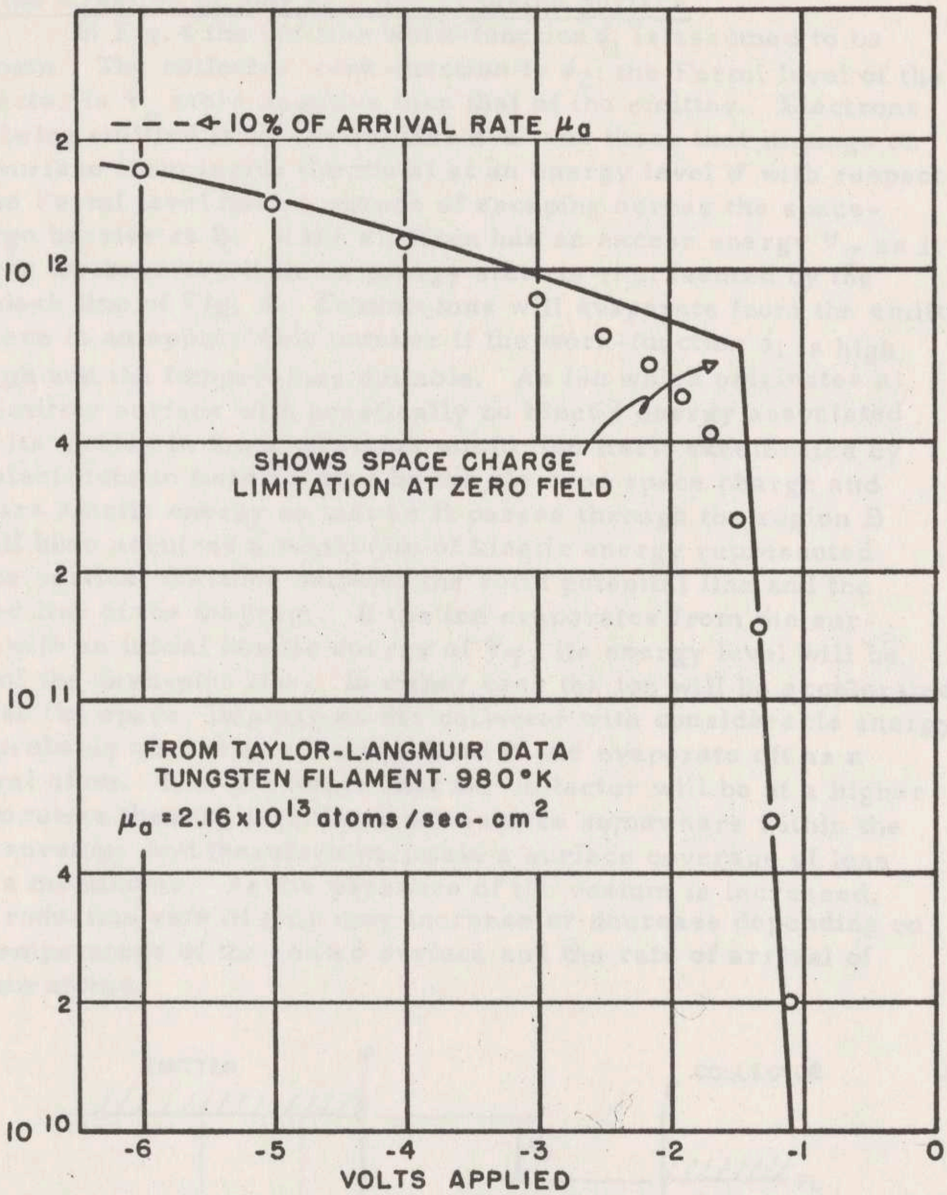

Fig. 3-Ionization rate as a function of applied volts observed by Taylor and Langmuir.

## A Generalized Discussion of the Simultaneous Emission of Electrons and the Creation of Ions at a Hot Tungsten Surface

In Fig. 4 the emitter work-function $\phi_1$ is assumed to be uniform. The collector work-function is $\phi_2$; the Fermi level of the collector is $V_o$ more negative than that of the emitter. Electrons are being emitted from the surface a-a' and those that impinge on the surface from inside the metal at an energy level $\phi'$ with respect to the Fermi level have a chance of escaping across the space-charge barrier at B. If the electron has an excess energy $V_T$ as it passes the barrier, then its energy state is represented by the dot-dash line of Fig. 4. Cesium ions will evaporate from the emitter surface in an appreciable number if the work-function $\phi_1$ is high enough and the temperature suitable. An ion which originates at the emitter surface with practically no kinetic energy associated with its motion in the x direction would find itself excelerated by the electrostatic field created by the electron space charge and acquire kinetic energy so that as it passes through the region B it will have acquired a maximum of kinetic energy represented by the vertical distance between the solid potential line and the dotted line of the diagram. If the ion evaporates from the surface with an initial kinetic energy of $V_T$, its energy level will be that of the dash-plus line. In either case the ion will be accelerated across the space, impinge on the collector with considerable energy, and probably absorb an electron from it and evaporate off as a neutral atom. It is presumed that the collector will be at a higher temperature than the liquid cesium surface somewhere within the tube envelope and therefore maintain a surface coverage of less than a monolayer. As the pressure of the cesium is increased, the production rate of ions may increase or decrease depending on the temperature of the heated surface and the rate of arrival of cesium atoms.

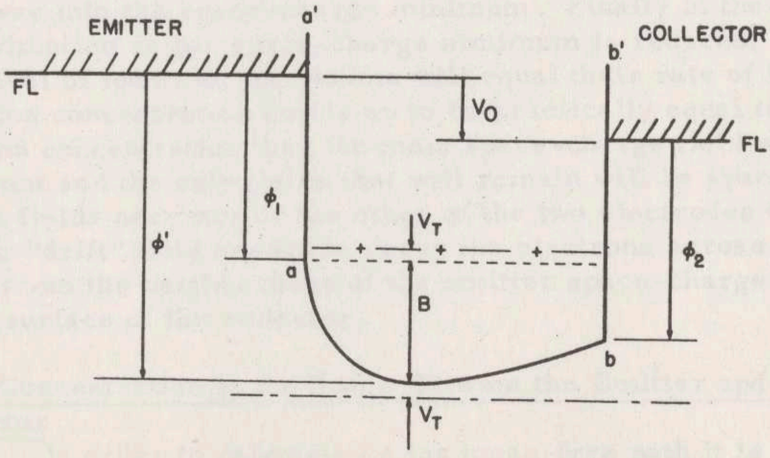

Fig. 4-Motive curve with electron space charge.

This discussion indicates that if the cathode has a uniform work-function structure, ions may be produced at the energy level a or above and will not be as effective in neutralizing space charge

as they would be if they could become trapped in the electronic space-charge potential minimum at B. As the cesium pressure is increased, there will be a greater and greater probability that the cesium ion will encounter a neutral cesium atom in such a way as to deliver energy to the atom and thus find itself in an energy level below the surface potential of the collector shown in the diagram at b.

The interaction between the neutral cesium and the ionized cesium may follow either one of two mechanisms. The cesium ion may collide with the cesium atom with an energy-sharing and momentum-conserving collision. The subsequent products then would be a slower-moving ion which might then become trapped, and a faster moving neutral atom. An alternate mechanism which also could have a good probability of occurring would be that the fast-moving ion would come into the neighborhood of a slow-moving cesium atom, absorb the electron from the atom, and continue on its way as a neutral atom with a relatively high kinetic energy. The ion thus created would be a slow-moving ion and therefore lie close to the electronic motive function and be trapped. Either of these processes would accomplish the desired result, namely, that of altering the energy level of the ion to bring it into a state that is below the energy level of the most negative surface. In the example shown in Fig. 4 this is the energy level at b.

As the cesium pressure is increased, the probability of such energy-losing collisions increases in direct proportion to the concentration of cesium atoms in the space between the two electrodes. It may therefore be of some interest to have equations by which this concentration may be estimated with reasonable accuracy. These equations in turn can be combined with an estimated "cross-section" associated with the energy-losing transition to try to determine the fraction of the ions produced that find their way into the space-charge minimum. Finally in the steady state situation as the space-charge nimimum is reduced, the rate of arrival of ions into the plasma will equal their rate of loss. If the ion concentration builds up to be practically equal to the electron concentration then the main space-charge field will be wiped out and the only fields that will remain will be space-charge fields near one or the other of the two electrodes or both and the "drift" field needed to carry the electrons across the space from the neighborhood of the emitter space-charge sheath to the surface of the collector.

## Atom Concentration in the Space Between the Emitter and the Collector

In order to calculate an ion mean-free path it is first necessary to have an approximate formula for determining the

atom density in the immediate neighborhood of liquid cesium in equilibrium with its vapor at a temperature $T_{Cs}$. This concentration is given by:

$$n_{Cs} = 2.7 \times 10^{23} e^{-\frac{8750}{T_{Cs}}} \text{ atoms}/\text{cm}^3 \qquad (15)$$

A useful form of this equation for calculation purposes is:

$$\log_{10} n_{Cs} = 23.44 - \frac{3800}{T_{Cs}} \qquad (16)$$

The fact that the exponents of Eq. 10 and Eq. 15 are different is not an error but results from the inclusion of a temperature coefficient term into the exponent. Although the above two equations serve to give the density of cesium atoms in equilibrium with the liquid, a correction should be used to relate the estimated temperature in the space between the emitter and the collector to the temperature at the cesium liquid surface.
A suitable form of this correction term is given as:

$$n_s = n_{Cs} \left( \frac{2\,T_{Cs}}{T_E + T_C} \right)^{1/2} \qquad (17)$$

This equation takes this form on the assumption that the effective temperature in the diode space may be taken as the average of the emitter temperature $T_E$ and the collector temperature $T_C$. To give some idea of the magnitude of this correction, assume for example that the emitter temperature is 1600° K and the collector temperature is 900° K when the cesium temperature is 500°K. The correction factor is then 0.63.

In order to estimate the mean-free path of an ion it is necessary to know the effective cross-section for an energy-losing collision. This quantity is undoubtedly energy-dependent and therefore any statement concerning its value should preferably be backed by experimental data not known accurately at present. For the purposes at hand, the assumption will be made the cross-section for energy-losing collisions is $2 \times 10^{-14}$ cm$^2$. This assumption is the equivalent of stzting that is a fast-moving ion comes close enough to a slow-moving atom for the center-to-center distance to be $8 \times 10^{-8}$ cm, there will be a high probability that an exchange of some sort will take place between the two in a manner to leave as a product a slower-moving ion. If the cross-section is represenatom density in the immediate neighborhood of liquid cesium in equilibrium with its vapor at a temperature $T_{Cs}$. This concentration is given by:

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This equation takes this form on the assumption that the effective temperature in the diode space, may be taken as the average of the emitter temperature $T_E$ and the collector temperature $T_C$. To give some idea of the magnitude of this correction, assume for example that the emitter temperature is 1000° K and the collector temperature is 500° K when the cesium temperature is 500°K. The correction factor is then 0.63.

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$$\lambda_+ = \frac{1}{n_s \sigma_+} = \frac{5 \times 10^{13}}{n_s} \text{ cm} \tag{18}$$

Equations 16, 17, and 18 may be combined for numberical calculations. Since the correction term in Eq. 17 is hardly great enough to be compared with the uncertainty in the collision cross-section, an equation can be written which neglects this factor and serves as a quick means of estimating the ion mean-free path in terms of the cesium-condensation temperature. The equation is:

$$\mathrm{Log}_{10} \lambda_+ = \frac{3800}{T_{Cs}} - 9.7 \tag{19}$$

A second useful form of this equation serves to determine the condensation temperature needed to approach a desired mean-free path. This equation is:

$$T_{Cs} = \frac{3800}{9.7 + \log_{10} \lambda_+} \tag{20}$$

This equation yields the result that approximately 1 mm mean-free path will be associated with the condensation temperature of $437^{\circ}$K and one-tenth of a millimeter will be the approximate mean-free path at $493^{\circ}$K.

If these calculations and estimations are valid, they may be helpful to the designer of a practical heat-to-electrical-power transducer that operates according to the plasma principle. It has been pointed out (1) that the high vacuum transducer must have a very small spacing in order to operate with satisfactory efficiency. The conclusion that one would draw from this discussion is that a small spacing could work as a definite disadvantage in the plasma transducer.

For illustration purposes, assume a spacing of 0.5 cm for a diode and a cesium pressure adjusted so that the ion-free path would be approximately half this distance or 2.5 mm. The cesium temperature would be $418^{\circ}$K. Equation 11 is used to estimate the minimum temperature of the cathode for moderately efficient ion production. This temperature is $T = 1500^{\circ}$K and is ted by $\sigma_g$, and the density of atoms, by $n_a$, then the best estimate for the mean-free path for the ions is given by:

$$\bar{\lambda}_+ = \frac{1}{n_a \sigma_g} = \frac{5 \times 10^{13}}{n_a} \text{ cm} \qquad (18)$$

Equations 16, 17, and 18 may be combined for numerical calculations. Since the correction term in Eq. 17 is hardly great enough to be compared with the uncertainty in the collision cross-section, an equation can be written which neglects this factor and serves as a quick means of estimating the ion mean-free path in terms of the cesium-condensation temperature. The equation is:

$$\log_{10} \lambda_+ = \frac{3800}{T_{Cs}} - 9.7 \qquad (19)$$

A second useful form of this equation serves to determine the condensation temperature needed to approach a desired mean-free path. This equation is:

$$T_{Cs} = \frac{3800}{9.7 + \log_{10}\lambda_+} \qquad (20)$$

This equation yields the result that approximately 1 mm mean-free path will be associated with the condensation temperature of 437°K and one-tenth of a millimeter will be the approximate mean-free path at 493°K.

If these calculations and estimations are valid, they may be helpful to the designers of a practical heat-to-electrical-power transducer that operates according to the plasma principle. It has been pointed out[1] that the high vacuum transducer must have a very small spacing in order to operate with satisfactory efficiency. The conclusion that one would draw from this discussion is that a small spacing could work as a definite disadvantage in the plasma transducer.

For illustration purposes, assume a spacing of 0.5 cm for a diode and a cesium pressure adjusted so that the ion-free path would be approximately half this distance or 2.5 mm. The cesium temperature would be 418°K. Equation 11 is used to estimate the minimum temperature of the cathode for moderately efficient ion production. This temperature is T = 1500°K and is a reasonable one for a dispenser-type cathode. If its average work-function is approximately 2.5 ev, then the available electron emission would be about 1 amp/cm$^2$. Assume that a space-charge sheath near the surface of the emitter added an additional 0.4 volt step to make the effective value of $\phi'$ shown in Fig. 4 equal to 2.9 ev. This would reduce the available electron emission a factor of 20 and bring it to 0.05 amp/cm$^2$. This current density would be 25,000 times the current density that would have been available from the same cathode in a high vacuum diode of this spacing when the space-charge minimum coincided with the collector. Again with this cathode operating at this temperature, the current density of 0.05 amp. cm$^2$ implies that the location of the space-charge minimum, would be approximately 30 microns from the emitter. The point in mentioning these figures is to lead the way toward an evaluation of the production rate of ions needed to hold the space-charge minimum at this point and give a very small potential difference between the minimum point at B and the electron collector surface at b. The next section will attempt to evaluate these factors.

## Ion and Electron Evaporation from a Nonuniform Cathode

The motive diagram shown in Fig. 4 is a one-dimensional diagram suitable for an emitter and a collector, each of which has a uniform surface work-function. In many practical examples of emitters this situation does not represent the facts. Specifically for pure tungsten the variation of work-function even without the adsorption of films may range from 4.3 to 5.3 ev. Adsorbed films may result in still wider differences. Details concerning dispenser cathodes are very uncertain. Specifically one may picture a work-function structructure of a dispenser cathode as one in which there are many islands or wells of strong electron emission to be found at crystal boundaries between the sintered crystals of which the structure is made. Each of the individual pieces of the powdered tungsten used in this fabrication may exhibit considerable range in work-function. In order to illustrate this point, the sketch in Fig. 5 has been produced and represents a completely hypothetical situation which may not differ too much from reality to be worth considering. The lines on the diagram represent imaginary crystal boundaries between the solid tungsten crystalites or sintered powder particles and the shading is related to the thermionic emission. Regions that are white represent strong electron emission and regions that are dark represent high work-function areas of weak electron emission. The scale used on this figure implies that each of the individual particles of tungsten that were sintered together to make a porous block had an average linear dimension of approximately 10 microns. The activation material is assumed to have migrated out between the

tungsten particles and activated them locally. The higher the temperature that the cathode is operated, the more likely a situation of this kind exists.

It is important for this discussion to attempt to represent in a two-dimensional diagram a motive function to be associated with this situation. It is quite evident that it is impossible to draw an exact representation but Fig. 6 may help to bring out the ideas that are involved.

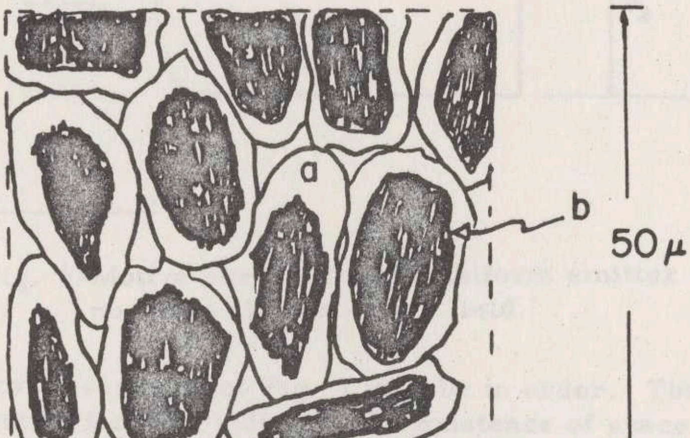

Fig. 5-Hypothetical electron emission distribution over the surface of a very hot dispenser cathode.

In Fig. 5 one of the strongly emitting regions is identified as the "a" region, while a neighboring dark region is identified as the "b" region. The work-function of the "a" region is taken to be $\phi_a$ and the "b" region $\phi_b$.
These work-function steps are shown in Fig. 6. If the a and b regions are of comparable size, then the motive function becomes something close to the average at a distance comparable to the average dimension of the patches of high work-function and low work-function area. This is illustrated by Fig. 6 by the fact that one line starts at $\phi_a$ and the other line starts at $\phi_b$ and they come together at a distance just outside the emitter comparable with the size of the individual patches involved. If the required potential is applied, shown as $V_O$, and the work-function of the collector is $\phi_2$, the motive function the rest of the way across the diode has no slope and therefore corresponds to the zero field condition. If the emitter is heated sufficiently to cause a strong electron emission, then the motive function will take on a form similar to that shown in Fig. 7.

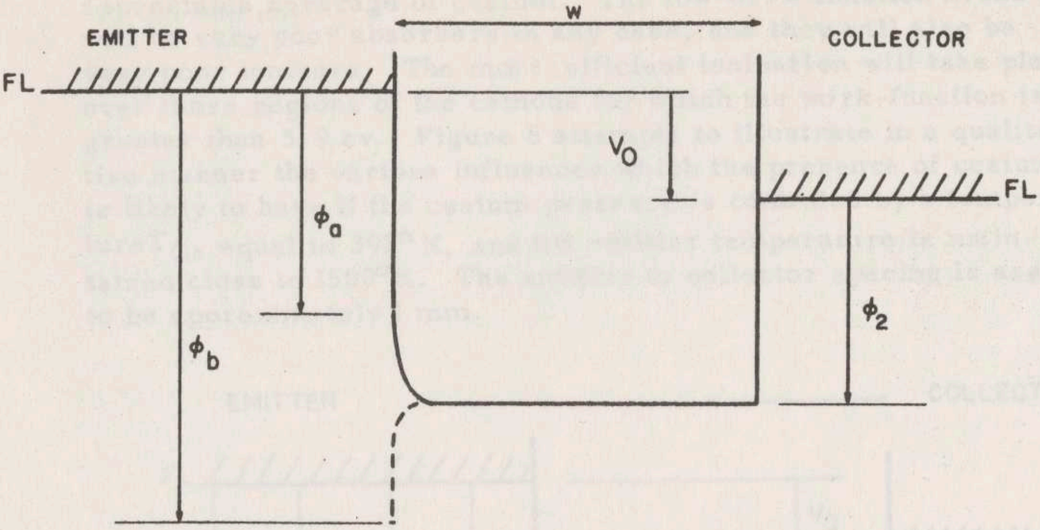

Fig. 6-Motive curves for non-uniform emitter - no space charge - zero field.

A word description of Fig. 7 may be in order. The cruvature of the motive function indicates the existence of space charge. The minimum in the motive function found at B established the minimum electron energy within the interior of the emitter for which electrons can escape from the emitter to the collector. The value of this energy is shown as $\phi'$. Thus any electron emitted from the high work-function area $\overline{\phi}_b$ is accelerated into the space and moves over to the collector. It is to be expected that there will be a gradation in work-function from $\phi_a$ to $\phi_b$ close to the boundaries of the activated area. It is impossible to show in this diagram that additional detail, and therefore only the extremes of the motive function are illustrated. Note that in this figure there is an energy level indicated at $\phi'_+$ which represents a peak in the motive function. Any positive ions generated at the surface will have to traverse this peak in order to find themselves in the space-charge region near B. This quantity $\phi'_+$ is very important because of the fact that if it is less than 3.9 ev then the difference (3.9 - $\phi'_+$) $V_T$ is related exponentially to the probability that an ion produced at the surface of the emitter will be able to pass into a region between the emitter and the collector. It is only here that it can be at all effective in reducing space charge.

The next problem to be considered in this qualitative manner is the influence of cesium vapor introduced at a moderately low density. Equation 19 indicates that at a cesium temperature of 390°K, the approximate mean-free path for cesium ions will be 1 centimeter. Furthermore, the relation given as Eq. 11 indicates that the minimum temperature for a pure tungsten surface to remain, on the average, in a state for which good ionization can take place will be 1410°K. It follows from this that a dispenser cathode operating at 1500°K will not be expected to adsorb an

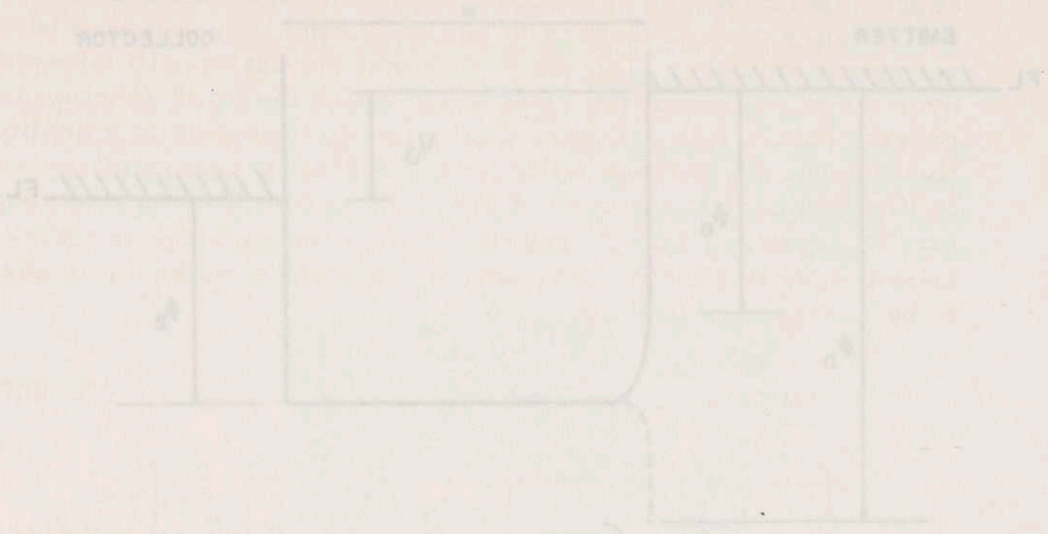

Fig. 6-Motive curves for non-uniform emitter - no space charge - zero field.

A word description of Fig. 7 may be in order. The curvature of the motive function indicates the existence of space charge. The minimum in the motive function found at B established the minimum electron energy within the interior of the emitter for which electrons can escape from the emitter to the collector. The value of this energy is shown as $\phi'$. Thus any electron emitted from the high work-function area $\phi_b$ is accelerated into the space and moves over to the collector. It is to be expected that there will be a gradation in work-function from $\phi_a$ to $\phi_b$ close to the boundaries of the activated area. It is impossible to show in this diagram that additional detail, and therefore only the extremes of the motive function are illustrated. Note that in this figure there is an energy level indicated at $d_1$ which represents a peak in the motive function. Any positive ions generated at the surface will have to traverse this peak in order to find themselves in the space-charge region near B. This quantity $d_1$ is very important because of the fact that if it is less than 3.9 ev then the difference $(3.9 - d_1) V_T$ is related exponentially to the probability that an ion produced at the surface of the emitter will be able to pass into a region between the emitter and the collector. It is only here that it can be at all effective in reducing space charge.

The next problem to be considered in this qualitative manner is the influence of cesium vapor introduced at a moderately low density. Equation 10 indicates that at a cesium temperature of 350°K the approximate mean-free path for cesium ions will be 1 centimeter. Furthermore, the relation given as Eq. 11 indicates that the maximum temperature for a pure tungsten surface to remain, on the average, in a state for which good ionization can take place will be 1410°K. It follows from this that a dispenser cathode operating at 1500°K will not be expected to absorb ca appreciable coverage of cesium. The low work-function areas will be very poor absorbers in any case, and they will also be very poor ionizers. The most efficient ionization will take place over those regions of the cathode for which the work-function is greater than 3.9 ev. Figure 8 attempts to illustrate in a qualitative manner the various influences which the presence of cesium is likely to have if the cesium pressure is controlled by a temperature $T_{Cs}$ equal to 392° K, and the emitter temperature is maintained close to 1500°K. The emitter to collector spacing is assumed to be approximately 1 mm.

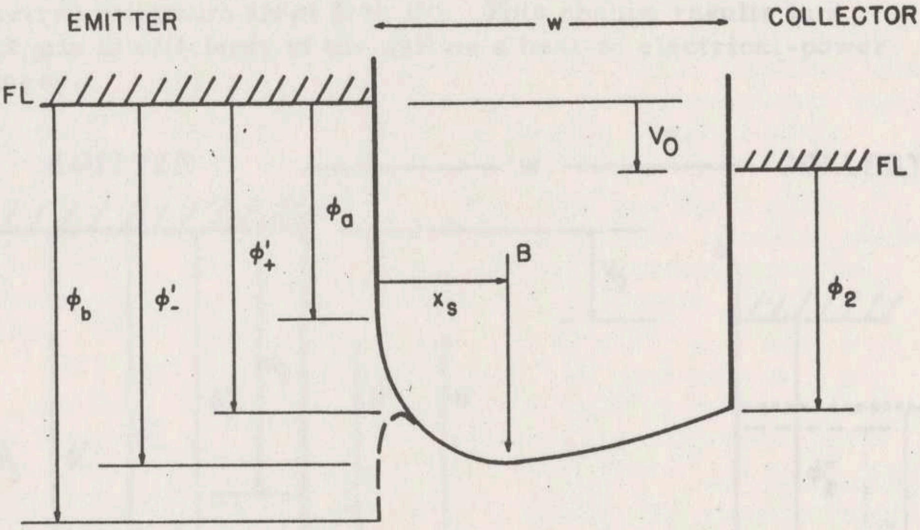

Fig. 7-Motive curve for a nonuniform emitter with electron space charge.

In Fig. 8 the dotted lines are a superposition of Fig. 7 on Fig. 8 so that it will be easier to see a comparison of the two. Note that $\phi_a$ and $\phi_b$ are the same whereas $\phi_2$ shows a measurable decrease to $\phi''_2$. This decrease is the result of the adsorption on the collector of a film of cesium which in all probability will reduce the work-function on the collector by 0.4 ev. or more. If no other change took place as a result of the cesium, this would be distinct advantage. The new work-function is designated by $\phi''_2$. In the drawing of Fig. 8, it is assumed that some of the ions produced at the hot emitter surface will find their way into the neighborhood of the potential minimum at B which in turn acts as an ion trap. In other words once that an ion is caught at an energy level lower in the diagram of Fig. 7 and $\phi'_+$, it will remain in that area more or less indefinitely since recombination between ions and free electrons is such an improbable event that the ion will remain trapped there, the potential minimum for a very long time. As additional ions are trapped there, the potential minimum will rise. This is illustrated schematically in

Fig. 8 in that the potential minimum changes from B to B''. This change in the potential minimum results in a corresponding change in the value of the limiting electron energy needed to transfer current from the emitter to the collectro. The new level is at $\phi''_-$. An objectionable change, from the point of view of being able to deliver <u>ions</u> to the space-charge minimum, is that indicated by the rise in the level diagram associated with $\phi''_+$. Thus the rise to "B'' because of the trapped ions results in a change of the motive function which inhibits the delivery of additional ions. Finally a balance takes place at which the delivery rate of ions is exactly equal to the loss rate. This condition more or less inevitably gives a rise in the electron minimum from B to B''. This change results in a distinct gain in efficienty of the unit as a heat-to electrical-power transducer.

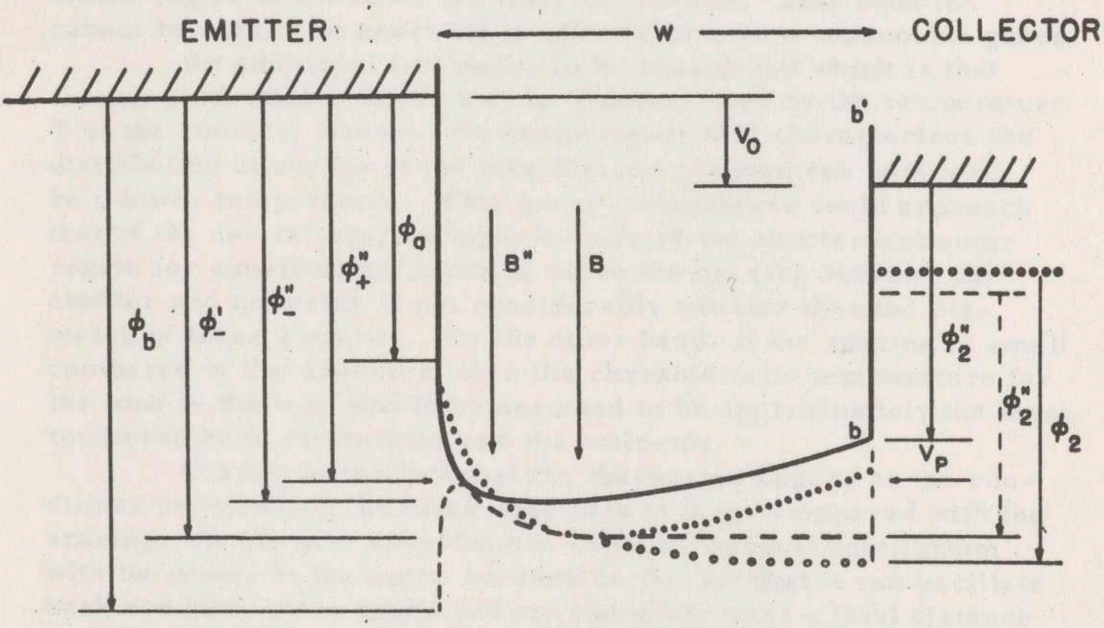

Fig. 8-Motive functions plasma diode compared with vacuum diode.

It will be assumed for the present that if the transducer is working under the most favorable conditions the voltage output $V_O$ added to the receiver work-function $\phi''_2$ will actually be less than the energy difference from the Fermi level of the emitter to the "hump" in the potential function shown as $\phi''_+$ in Fig. 8. In those circumstances the loss of ions from the space-charge region will be back to the hot emitter and not to the collector. If the problem were as simple as this then the density of ions at B'' could be related to the ionization potential $V_i$ by:

$$\frac{n_{+B''}}{n_s} = \frac{1}{4} e^{-\frac{V_i - \phi''}{V_T}} \qquad (21)$$

In this equation $V_i$ is the ionization potential of cesium of 3.9 volts; $n_s$ is the density of atoms in immediate neighborhood of the ionization surface, and the factor (1/4) is a guess made up of a combination of two facts. In the use of an equation of the form of Eq. 21 a factor of 1/2 is usually introduced as a so-called statistical weight and an additional factor of 1/2 is being introduced because of the fact that only part of the cathode is capable of producing ions. Once they are produced and caught in the potential minimum, the entire region is available for their occupation. This equation cannot be said to be exact but is offered here as a reasonable guess.

An additional fact needs to be considered which is that the ion production process will be characterized by the temperature T of the emitter, whereas the temperature that characterizes the distribution in energy of the ions that are trapped can very well be a lower temperature. This lower temperature could approach that of the neutral cesium vapor outside of the emitter-collector region for experimental tubes in which the spacing between the emitter and collector is not considerably smaller than the diameter of these surfaces. On the other hand, if the spacing is small compared to the diameter, then the characteristic temperature for the ions in the trap should be assumed to be approximately the mean temperature of the emitter and the collector.

In spite of the fact that the discussion applies to the conditions under which the mean-free path is large compared with the spacing, the ion may nevertheless come to thermal equilibrium with the atoms in the space because of the fact that it can oscillate back and forth many times and can therefore have a total distance of travel during its lifetime in the potential minimum much longer than the mean-free path. This detail can be considered to be a refinement which acts in the favorable direction from the point of view of the making of a practical and effective transducer.

## Influence of Cesium Pressure in Association with a Nonuniform Emitter.

Under the circumstances illustrated in Fig. 8 an increase in cesium pressure will increase the rate of arrival of cesium atoms at the emitter. This increase in arrival rate will result in more adsorption of cesium on the high work-function areas. Once the cesium is adsorbed there, these areas will no longer be

very effective at producing ions. The effect to be anticipated then will be a raising of the limiting level $\phi''_+$. This change will work to disadvantage in that it makes it more difficult for the ions that are produced to find their way to the region marked B''. The lack of a suitable ion density at B'' will result in a lowering of this level or an increase in the value of $\phi''_-$. Thus it is to be anticipated that other things being equal, the increase in cesium pressure will first result in a raising of the potential minimum for electrons from B toward B'' with an increase in cesium pressure toward its optimum but additional cesium pressure will result in a decrease in the available current for the transducer. The important point is to try to establish the optimum pressure and this may be possible only by experiment. The theory as so far carried out indicates that an optimum is likely.

## Loss of Positive Ions to the Collector

As the output voltage $V_O$ is increased at the expense of some decrease in electron current, the difference in potential between the potential minimum at B'' and the surface of the collector at b will decrease. At the critical condition expressed by:

$$V_O + \phi''_2 = \phi''_+ \tag{22}$$

ions will begin to escape to the collector to the same extent that they escape to the emitter. If $V_O$ is made still larger, then the principal loss of ions will be at the collector surface and this rate of loss must be supplied by the ions which are able to pass over the ion barrier at $\phi''_+$. As $V_O$ is made larger, it will be slightly easier for ions to escape from the emitter. This increase in yield may not be sufficient to maintain the desired trapped ion density.

Under the conditions shown in Fig. 8 there is a difference in potential between the space-charge minimum at B'' and the surface of the collector which is numerically equal to

$$\phi''_- - (V_o + \phi''_2) = V_p. \tag{22a}$$

This potential difference does not represent a loss of energy by the electrons because of the presence of the neutral cesium atoms, but is simply a difference needed to satisfy the space-charge relations. The reader must be reminded that the situation in this part of the plasma discharge is an entirely different one from that found in self-sustained low voltage arcs, because there the difference in potential along the plasma column is one that is very effective at producing ions. The effect to be anticipated then will be a raising of the limiting level $\phi_2''$. This change will work to disadvantage in that it makes it more difficult for the ions that are produced to find their way to the region marked B''. The lack of a suitable ion density at B'' will result in a lowering of this level or an increase in the value of $\phi_2''$. Thus it is to be anticipated that other things being equal, the increase in cesium pressure will first result in a raising of the potential minimum for electrons from B toward B'' with an increase in cesium pressure toward its optimum but additional cesium pressure will result in a decrease in the available current for the transducer. The important point is to try to establish the optimum pressure and this may be possible only by experiment. The theory as so far carried out indicates that an optimum is likely.

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## An Interpretation of Data Taken on An Experimental Diode

A diode was constructed for study by Mr. Thomas Robinson of the Thermo-Electron Engineering Corporation which had two dispenser-type cathodes, each 3 mm in diameter separated from each other a distance of 0. 68 mm. One of these dispensers was heated to 1520°K and before cesium was admitted to the tube, a current voltage characteristic was obtained over the range of applied voltage from zero to -2. 2 volts with the receiving electrode negative with respect to the emitter. These data were plotted as shown in Fig. 9 and compared with the "master curve"(2). The solid line of this figure is the master curve and the index of that curve establishes the value of $(V_R/V_T)$ of 10. 4 and a corresponding value of $V_R$ of 1. 37. These numbers combine in the formulae given here as Eqs. 23 and 24 permit the calculation of the maximum power available from this diode and the voltage output at which maximum power occurs.

$$P_{max} = 3.7 \times 10^{-6} \, V_T^{1/2} \frac{V_R^2}{w^2} \text{ watt/cm}^2 \qquad (23)$$

$$V_O = \frac{0.383 \left( \frac{V_R}{V_T} \right) V_R}{1 + 0.31 \left( \frac{V_R}{V_T} \right)^{4/3}} \qquad (24)$$

In Eq. 23 the distance w is expressed in centimeters for the calculation of the maximum power available in $\text{watts/cm}^2$. The insertion of the figures mentioned gives a predicted $P_{max} = 5.5 \times 10^{-4}$ $\text{watts/cm}_2$ and the voltage output at the maximum power is 0. 683. The current at maximum power is given by;

$$I_{max} = I_m \left[ 1 + 0.31 \left( \frac{V_R}{V_T} \right)^{4/3} \right] \qquad (25)$$

For the present experiment $I_{max} = 7.96\, I_m$.

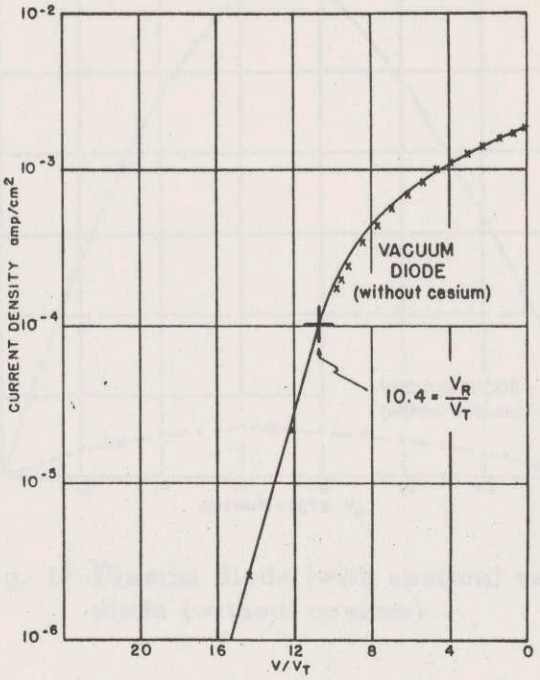


Fig. 9-Experimental results and comparison with space-charge theory for vacuum diode. Data from Robinson of TEE.

Robinson measured the power delivered to the load in his experiment as a function of the output voltage and obtained the curve designated "without cesium" of Fig. 10. It is clear that the maximum power was correctly calculated by theory and the voltage output at maximum power was correct as shown by the circle on that graph. Later cesium was admitted to the tube and the available power again measured as a function of the voltage. This result is also shown in Fig. 10 and it is very evident that the favorable action of the cesium increased the power a factor of 10.

Before the details concerning this second experiment are discussed, a few additional computations should be made relative to the high vacuum experiment. A calculation chart, shown here as Fig. 11 shows that with this operating temperature and spacing, a current density of 100 microamperes per square centimeter should have been expected in terms of the temperature-spacing relation for the critical current density $I_m$ which flows to the collector with a space-charge minimum coinciding with it. This situation is illustrated by the dotted lines of Fig. 12.

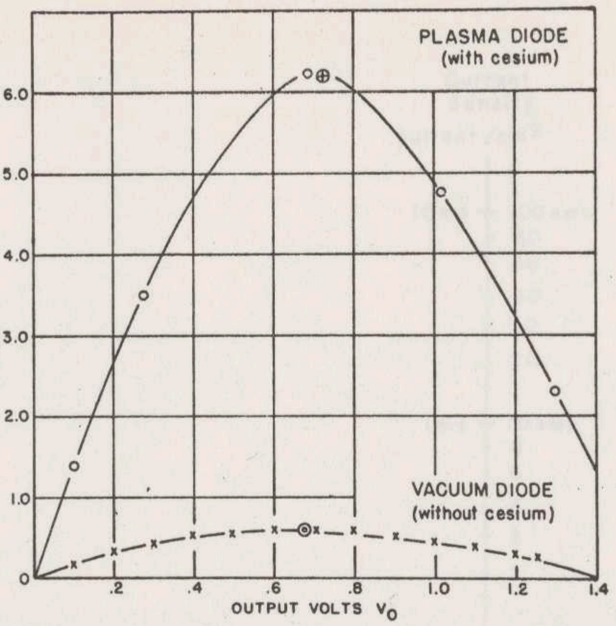
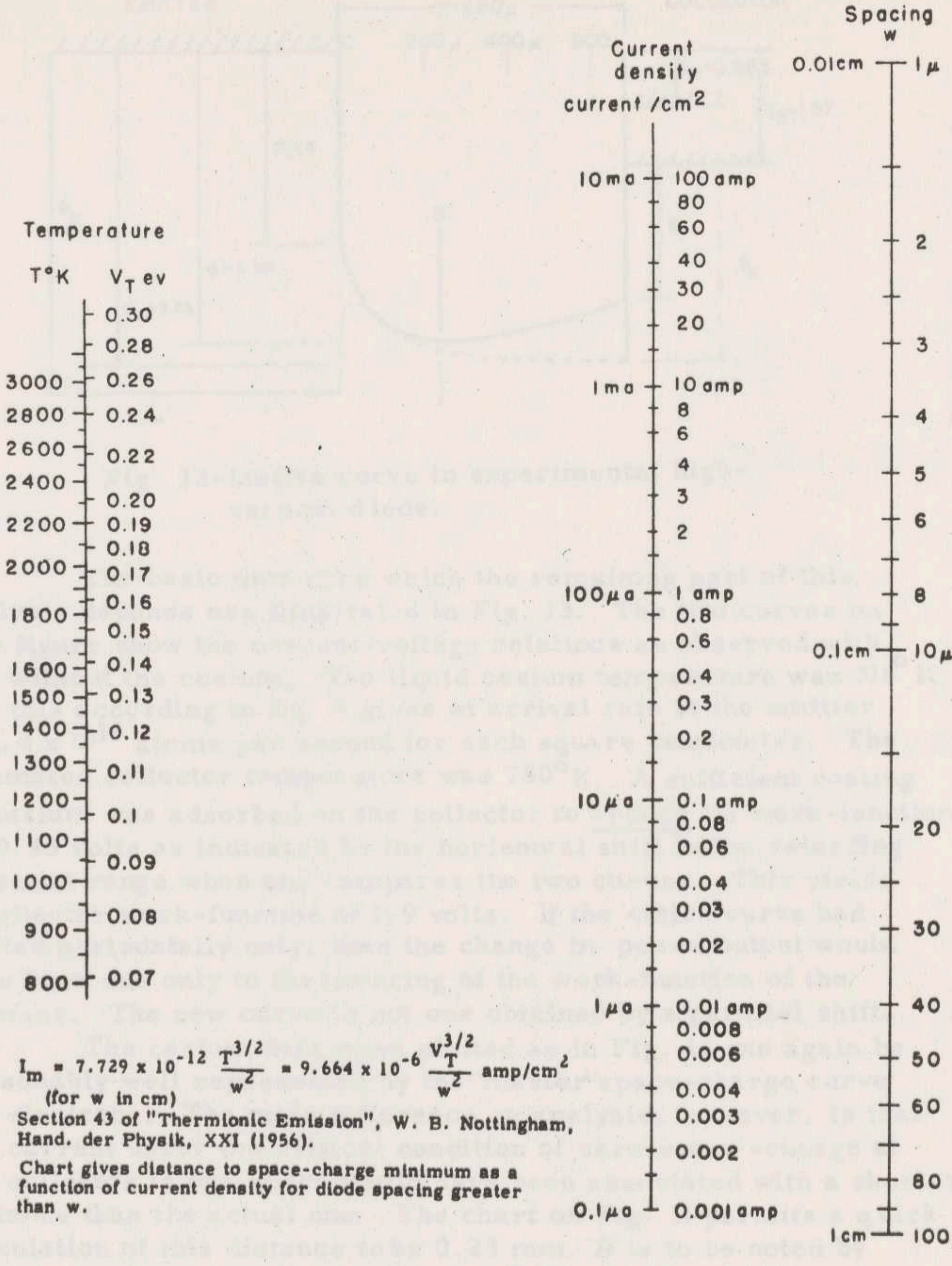


Fig. 10-Plasma diode (with cesium) vacuum diode (without cesium).

The actual value of $\phi_R$ applicable to this experiment may be calculated by:

$$\phi_R = 2.3 \ V_T \left[ 2.08+2 \log_{10} T - \log_{10} I_m \right] \qquad (26)$$

The value of $\phi_R$ is 3.73 ev. The fact that the observed current voltage characteristic follows the "master curve" so well, as is illustrated by Fig. 9, indicates that the inhomogeneity of the collector is relatively small but does seem to be present, and furthermore, such inhomogeneity as may exist at the emitter probably represents a small part of the area. It will become evident as the calculation proceeds that it will be desirable to invoke the concept that some inhomegeneity does exist and that potential functions not unlike those shown in Fig. 7 and 8 will be involved as a step in the analysis to explain the increased power shown by the curve in Fig. 10 in the presence of cesium. The numerical data mentioned are summarized in Fig. 12



$$I_m = 7.729 \times 10^{-12} \frac{T^{3/2}}{w^2} = 9.664 \times 10^{-6} \frac{V_T^{3/2}}{w^2} \text{ amp/cm}^2$$

**(for w in cm)**

**Section 43 of "Thermionic Emission", W. B. Nottingham, Hand. der Physik, XXI (1956).**

**Chart gives distance to space-charge minimum as a function of current density for diode spacing greater than w.**

Fig. 11-Current density-Temperature-Spacing relation in ideal high vacuum diode for onset of space-charge limitation (zero field at surface of collector).

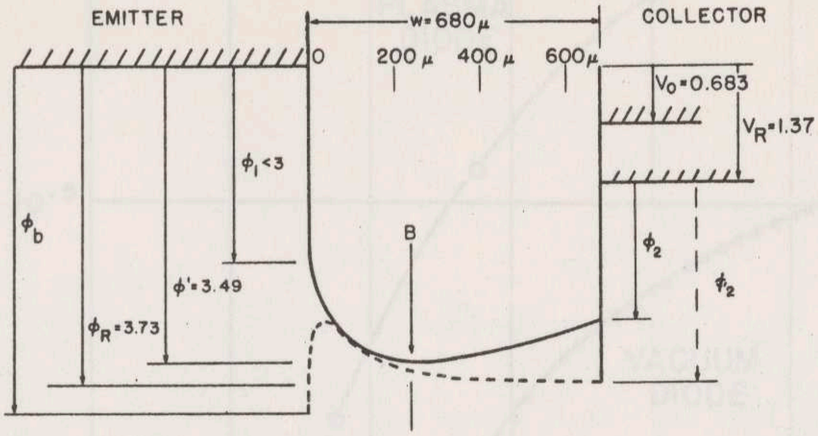


Fig. 12-Motive curve in experimental high-vacuum diode.

The basic data upon which the remaining part of this analysis depends are illustrated in Fig. 13. The two curves on this figure show the current-voltage relations as observed with and without the cesium. The liquid cesium temperature was 316° K and this according to Eq. 9 gives an arrival rate at the emitter of $1.4 \times 10^{15}$ atoms per second for each square centimeter. The estimated collector temperature was 780°K. A sufficient coating of cesium was adsorbed on the collector to reduce its work-function by 0.45 volts as indicated by the horizontal shift in the retarding potential range when one compares the two curves. This yields a collector work-function of 1.9 volts. If the entire curve had shifted horizontally only, then the change in power output would have been due only to the lowering of the work-function of the receiver. The new curve is not one obtained by a parallel shift.

The cesium data when plotted as in Fig. 14 can again be reasonably well represented by the "master" space-charge curve for electrons. The main difference on analysis, however, is that the current under the critical condition of zero space-charge at the collector is one which would have been associated with a shorter distance than the actual one. The chart on Fig. 11 permits a quick calculation of this distance to be 0.23 mm. It is to be noted by reference to Fig. 12 that the new condition, in the presence of some space-charge neutralization from cesium atoms, is practically as though a diode had been created with the collector located precisely at the potential minimum illustrated in Fig. 12 for the condition of maximum power. This result suggests the drawing of the potential distribution as shown in Fig. 15. The number of electrons that cross the boundary at B per second for a unit area is $5.6 \times 10^{15}$. This current would correspond to a density of

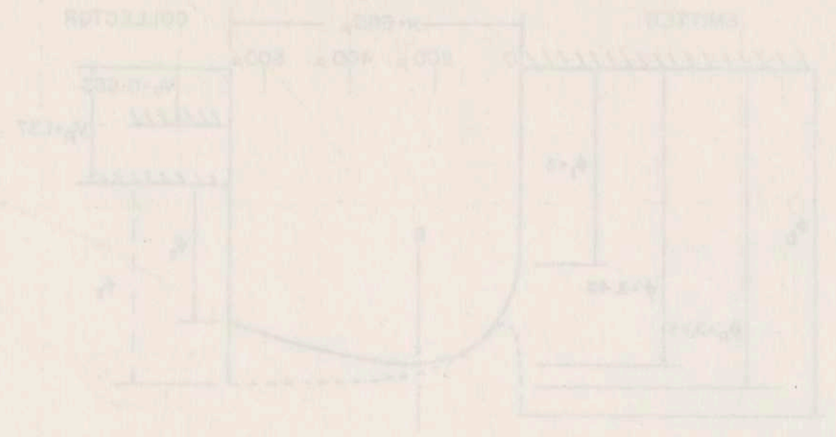


Fig. 12-Motive curve in experimental high-vacuum diode.

The basic data upon which the remaining part of this analysis depends are illustrated in Fig. 13. The two curves on this figure show the current-voltage relations as observed with and without the cesium. The liquid cesium temperature was 310° K and this according to Eq. 9 gives an arrival rate at the emitter of $1.4 \times 10^{17}$ atoms per second for each square centimeter. The estimated collector temperature was 780° K. A sufficient coating of cesium was adsorbed on the collector to reduce its work-function by 0.45 volts as indicated by the horizontal shift in the retarding potential range when one compares the two curves. This yields a collector work-function of 1.9 volts. If the entire curve had shifted horizontally only, then the change in power output would have been due only to the lowering of the work-function of the receiver. The new curve is not one obtained by a parallel shift.

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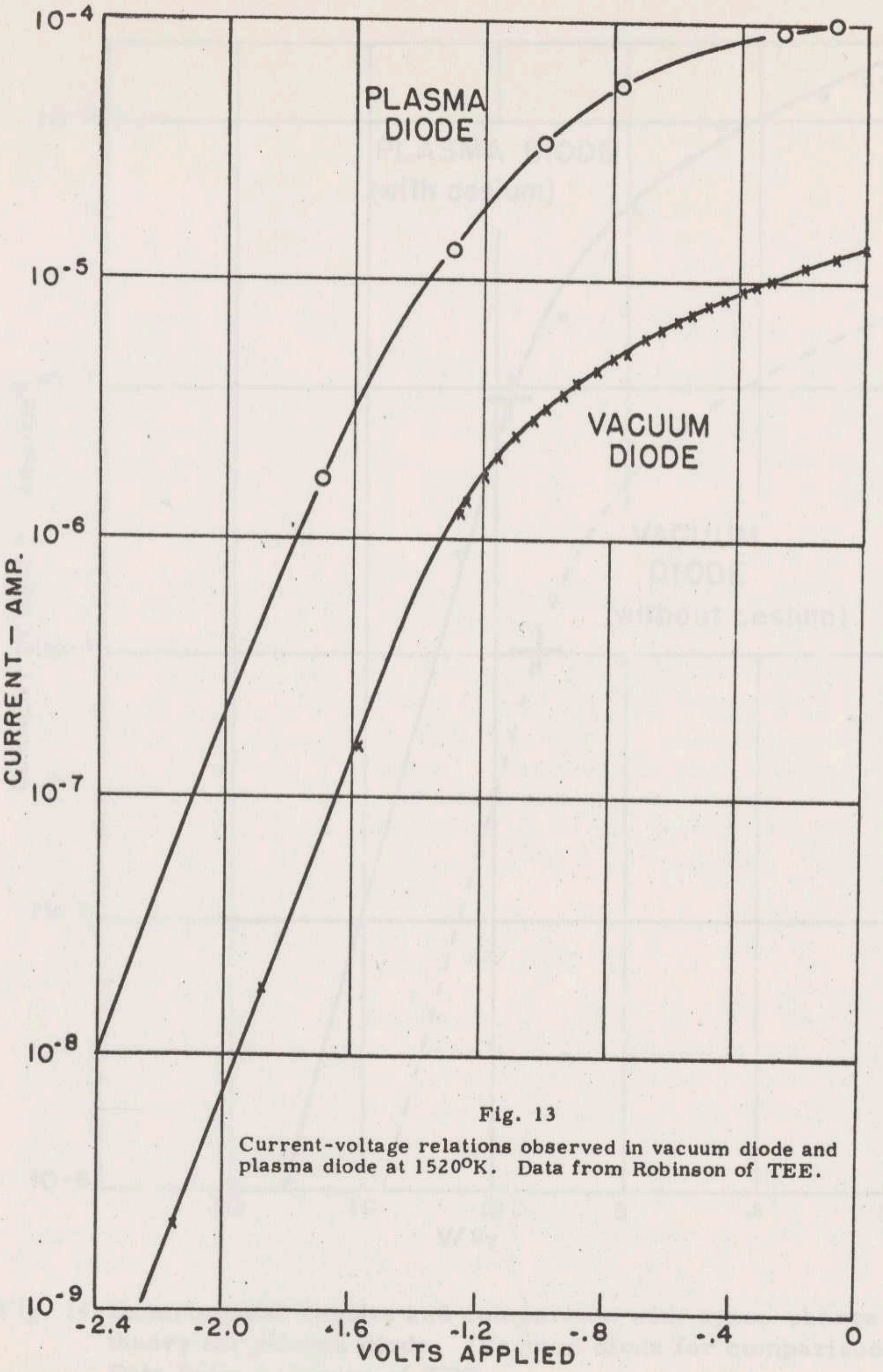

Fig. 13

Current-voltage relations observed in vacuum diode and plasma diode at 1520°K. Data from Robinson of TEE.

Fig. 13-Current-voltage relations observed in vacuum diode and plasma diode at 1520°K. Data from Robinson of TEE.

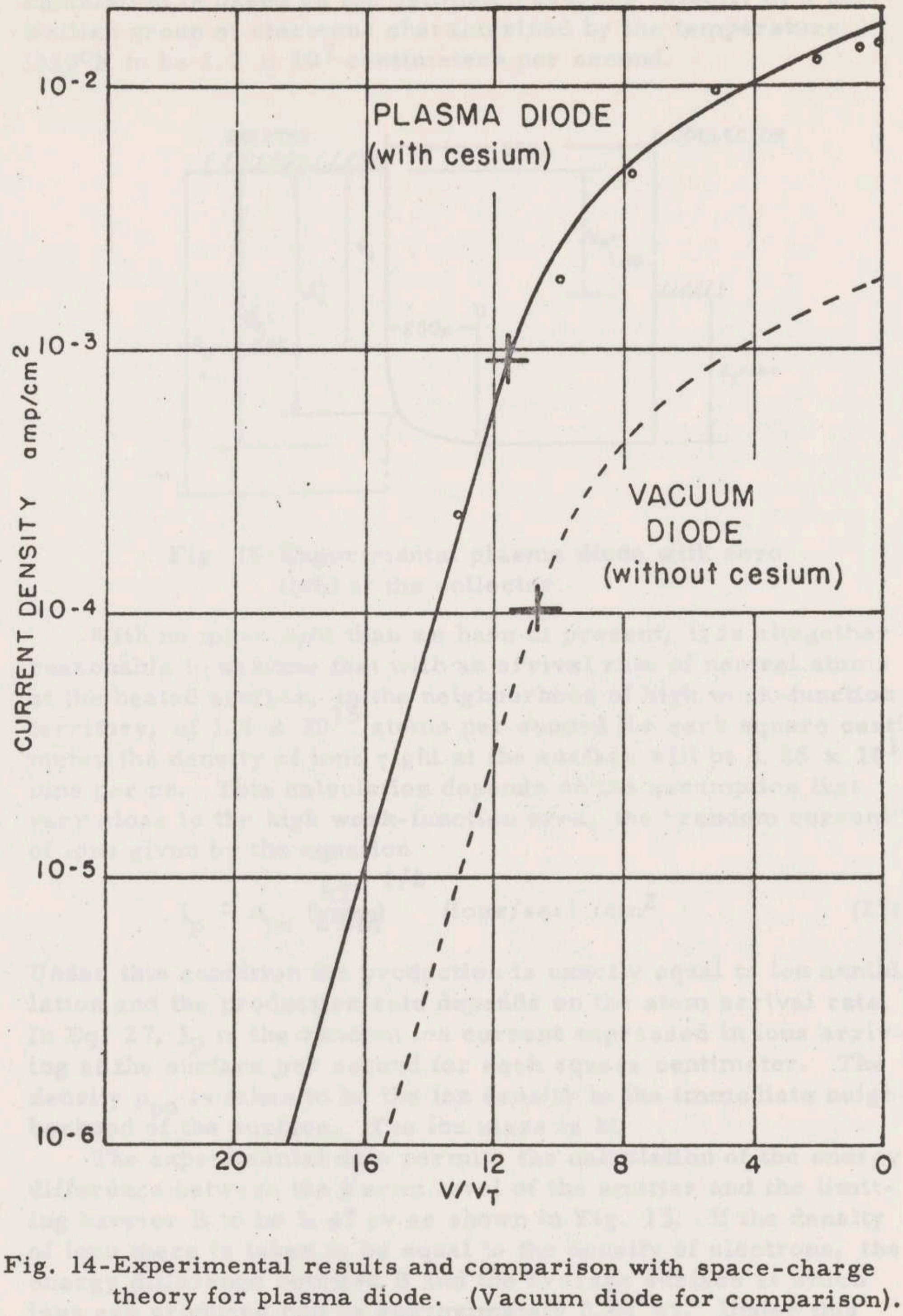

Fig. 14-Experimental results and comparison with space-charge theory for plasma diode. (Vacuum diode for comparison). Data from Robinson of TEE.

electrons of $4.6 \times 10^8$ electrons per cubic centimeter. This calculation is based on the estimated average velocity of a Maxwellian group of electrons characterized by the temperature 1520°K to be $1.2 \times 10^7$ centimeters per second.

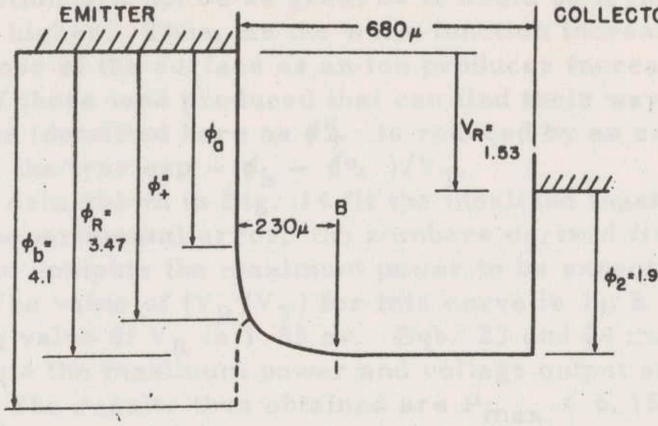

Fig. 15-Experimental plasma diode with zero field at the collector.

With no more data than we have at present, it is altogether reasonable to assume that with an arrival rate of neutral atoms at the heated surface, in the neighborhood of high work-function territory, of $1.4 \times 10^{15}$ atoms per second for each square centimeter the density of ions right at the surface will be $1.25 \times 10^{11}$ ions per cc. This calculation depends on the assumption that very close to the high work-function area, the "random current" of ions given by the equation

$$I_p = n_{po} \left(\frac{kT}{2\pi M}\right)^{1/2} \quad \text{(ions/sec)} /\text{cm}^2 \qquad (27)$$

Under this condition ion production is exactly equal to ion annihilation and the production rate depends on the atom arrival rate. In Eq. 27, $I_p$ is the random ion current expressed in ions arriving at the surface per second for each square centimeter. The density $n_{po}$ is taken to be the ion density in the immediate neighborhood of the surface. The ion mass is M.

The experimental data permits the calculation of the energy difference between the Fermi level of the emitter and the limiting barrier B to be 3.47 ev as shown in Fig. 15. If the density of ions there is taken to be equal to the density of electrons, the energy difference between B and the average surface at which ions are produced can be approximately 0.61 ev. Under this condition, the effective surface potential will be close to 4.1 ev. Fig. 15 has been prepared on the assumption that a surface for ionization does, in fact, exist at 4.1 ev away from the Fermi level of the emitter. On the basis of present knowledge with regard to these surfaces, any ionization surface with a work-func-

tion greater than 3. 5 ev could deliver ions to the region at B with approximately the same efficiency. When the work-function is less than 3. 9, then many of the neutral atoms that arrive at the surface are likely to leave as neutral atoms and therefore the ion production will not be as great as it would be if the work-function were higher. Thus, as the work-function increases, the effectiveness of the surface as an ion producer increases but the fraction of those ions produced that can find their way over the ion barrier identified here as $\phi''_+$ is reduced by an exponential function of the type exp $-(\phi_b - \phi''_+)/V_T$.

Since the data shown in Fig. 14 fit the idealized master curve within experimental error, the numbers derived from it may be used to compute the maximum power to be expected from the device. The value of $(V_R/V_T)$ for this curve is 11. 8. The corresponding value of $V_R$ is 1. 53 ev. Eqs. 23 and 24 may be used to compute the maximum power and voltage output at maximum power. The results thus obtained are $P_{max} = 6.15 \times 10^{-3}$ watts per $cm^2$ and the voltage output is 0. 73 v. These calculated results are represented by the cross with the circle in Fig. 10 and agree well with experiment. The electron current density at maximum power is $8.4 \times 10^{-3}$ amp/$cm^2$.

## Determination of the Optimum Cesium Temperature

It is impossible to predict with confidence the optimum cesium temperature because of the lack of needed experimentally determined data applicable to this problem. This section will, therefore, have to depend on the making of a few simple assumptions and if these hypotheses are proven wrong, then the predictions based on them will certainly be in error.

The first assumption is that the zero field condition illustrated by Fig. 15 is determined by the equality of the positive ion and the electron densities. For equality, the arrival rate of ions and electrons must be related by:

$$\left(\frac{\nu_-}{\nu_+}\right)_{\text{at B}} = \left(\frac{M}{m}\right)^{1/2} \qquad (28)$$

In this equation M is the mass of an ion and m is the mass of an electron. The relations represented here imply that both ions and electrons leave the region by going to the collector and since the velocities are inversely proportional to the square root of the mass ratio the ion current will be correspondingly lower for the zero space-charge condition. In order to use this assumption in a numerical calculation, the "effective" average work-function of the ionization region must be know. The experiment described in the previous section gave a value as $\phi_b = 4.1$. The equation which will relate the important quantities is the following:

$$\phi_R = \frac{\phi_b}{2} - 10.68\ V_T + 2.3\ V_T \log_{10} T + \frac{4490}{T_{Cs}}\ V_T \qquad (29)$$

The numerical validity of this equation may be tested by inserting into it the values associated with the experiment described in the previous section. When this is done the calculated value of $\phi_R$ is 3.47 ev on the assumption that $\phi_b$ is 4.1.

In order to determine the optimum value of $T_{Cs}$, use is first made of the relation shown as Eq. 11. The implication here is that unless some other factor interferes, the best choice of the cesium temperature will be the highest that will result in effective ionization on a tungsten surface. Certain interfering factors will be discussed later in this section.

According to Eq. 11, the maximum cesium temperature is 425°K for an emitter temperature of 1520°K. These two figures are supplemented by the assumption that the previously determined value of $\phi_b$ of 4.1 is suitable. The predicted value of $\phi_R$ at this higher condensation temperature of cesium is then 3.0 ev. This new value represents a reduction in $\phi_R$ of 0.47 and a corresponding increase in electron current delivered to the collector by a factor of 36. The current density expected is $3.2 \times 10^{-2}$ amp/cm$^2$. The chart of Fig. 11 may now be used to determine that the effective spacing of the diode in the presence of this increased cesium pressure will be 39 microns.

At the lower cesium pressure the average work-function of the collector was found to be $\phi_2 = 1.94$ ev. There is no way to know for sure that an increase in cesium pressure will decrease this average work-function still more but for the purpose of illustration at least, it will be assumed that the increase in pressure will reduce the work-function to 1.70. The predicted value of $V_R$ is then:

$$V_R = \phi_R - \phi_2 = 3.00 - 1.70 = 1.30 \qquad (30)$$

The corresponding value of $(V_R/V_T)$ is 10.

The next assumption is that having established the equivalent spacing of a vacuum diode which would have the same current density as the cesium diode the current voltage characteristics will follow the vacuum diode curve associated with this reduced spacing. The new spacing for this calculation is the one given as 39 microns. Eqs. 23 and 24 may be used to determine the predicted maximum power output and corresponding voltage output. The values obtained are $P_{max} = 0.15$ w/cm$^2$ and $V_O$ is 0.65. The current density is 0.23 amp/cm$^2$. The corresponding value for $\phi''_-$ is 2.74 ev. It is implied in these calculations that the true work-function of the emitter $\phi_a$ is less than 2.6 ev. The chief advantage in having a smaller value of $\phi_a$ for the electron emitting portion of the emitter is its influence on the value or $\phi_g = \frac{\phi_i}{2} - 10.65\, V_T + 2.3\, V_T \log_{10} T + \frac{4498}{T_{Cs}}\, V_T$ (29)

The numerical validity of this equation may be tested by inserting into it the values associated with the experiment described in the previous section. When this is done the calculated value of $\phi_g$ is 3.47 ev on the assumption that $\phi_i$ is 4.1.

In order to determine the optimum value of $T_{Cs}$, use is first made of the relation shown as Eq. 11. The implication here is that unless some other factor interferes, the best choice of the cesium temperature will be the highest that will result in effective ionization on a tungsten surface. Certain interfering factors will be discussed later in this section.

According to Eq. 11, the maximum cesium temperature is 480°K for an emitter temperature of 1520°K. These two figures are supplemented by the assumption that the previously determined value of $\phi_i$ of 4.1 is suitable. The predicted value of $\phi_R$ at this higher condensation temperature of cesium is then 3.0 ev. This new value represents a reduction in $\phi_R$ of 0.47 and a corresponding increase in electron current delivered to the collector by a factor of 36. The current density expected is $3.2 \times 10^{-2}$ amp/cm². The chart of Fig. 11 may now be used to determine that the effective spacing of the diode in the presence of this increased cesium pressure will be 39 microns.

At the lower cesium pressure the average work-function of the collector was found to be $\phi_c$ = 1.94 ev. There is no way to know for sure that an increase in cesium pressure will decrease this average work-function still more but for the purpose of illustration at least, it will be assumed that the increase in pressure will reduce the work-function to 1.70. The predicted value of $V_R$ is then:

$$V_R = \phi_R - \phi_c = 3.00 - 1.70 = 1.30 \qquad (30)$$

The corresponding value of $(V_R/V_T)$ is 10.

The next assumption is that having established the equivalent spacing of a vacuum diode which would have the same current density as the cesium diode the current voltage characteristics will follow the vacuum diode curve associated with this reduced spacing. The new spacing for this calculation is the one given as 39 microns. Eqs. 23 and 24 may be used to determine the predicted maximum power output and corresponding voltage output. The values obtained are $P_{max}$ = 0.15 w/cm² and $V_O$ is 0.85. The current density is 0.23 amp/cm². The corresponding value for $\phi_R^*$ is 2.74 ev. It is implied in these calculations that the true work-function of the emitter $\phi_a$ is less than 2.4 ev. The chief advantage to having a smaller value of $\phi_a$ for the electron emitting portion of the emitter is its influence on the value of $\phi''_+$ which is the hump in the diagram of Figs. 15 and 16 that inhibits the loss of ions to the emitter and thus helps maintain the space-charge reduction.

This prediction that an additional 24-fold increase in power is available in the cesium diode over that actually obtained as a result of increasing the cesium pressure is probably optimistic. The calculation has been made in this manner in order to indicate the potential advantages in choosing the most favorable value of the cesium condensation temperature. At this higher temperature Eq. 19 permits the estimation of the cesium ion mean-free path to be 1. 8 mm. Since the electron mean-free path is undoubtedly longer than this, the implication is that a diode spacing of 1 mm would be just as effective in the presence of cesium as a diode spacing of 100 microns. Associated with the current density of 0. 23 amp/cm$^2$, the chart on Fig. 11 permits the location of the space-charge minimum to be calculated as 14. 4 microns.

In the calculations of this paper, electron energy steps are expressed in terms of their true values at the temperature involved. Therefore the fundamental equation obtained directly from the statistical analysis expresses the current density in the following form:

$$I = 120\, T^2\, e^{-\frac{\phi}{V_T}} \text{ amp/cm}^2 \qquad (31)$$

This is the Richardson type of equation and carries with it the number 120 instead of some empirically-determined constant that could be designated as $A_R$ to indicate that it is the Richardson constant found empirically to make the data fit for some set of observations for which the Richardson work-function would be appropriate. Since "true" values of $\phi$ are used in the analysis given here, the coefficient 120 is the only correct one to use.

For calculations in which it is the purpose to determine the temperature at which a certain emission density will be obtained, Eq. 31 is not the most convenient one to use. The theory behind the transformation to a simple equation is given in Section 50 of "Thermionic Emission" and two useful equations may be obtained from Eq. 31 to be applied over the emitter temperature ranges associated with each of these equations:

**Range** $1150^\circ K < T < 2500^\circ K$

$$I = 3 \times 10^{9-(5.04\phi + 1.6)\frac{1000}{T}} \text{ amp/cm}^2 \qquad (32)$$

$$\log_{10} I = 9.48 - (5.04\phi + 1.6)\frac{1000}{T} \qquad (32a)$$

Range $600^{o}K < T < 1150^{o}K$

$$I = 7 \times 10^{8-(5.04\phi + 0.76)\frac{1000}{T}} \text{ amp/cm}^2 \qquad (33)$$

$$\log_{10} I = 8.85 - (5.04\phi + 0.76)\frac{1000}{T} \qquad (33a)$$

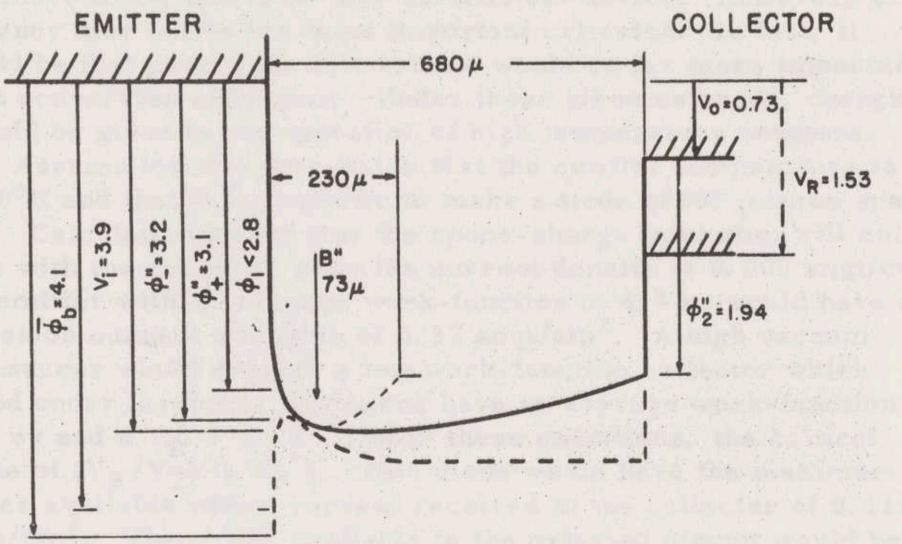

Fig. 16-Motive function for plasma diode.

**The principal purpose of the above equations is to show the steps toward the writing of a simple equation by which the maximum temperature of the collector can be evaluated readily in terms of the available output volts $V_O$ and the energy step $\phi''_-$ illustrated in Fig. 16 and the temperature of the electron emitter of the diode $T_1$. The way this equation is written incorporates also the limitation that the electron current streaming from the collector toward the emitter shall be no more than 5 per cent of the electron current that flows in the opposite direction. This equation subject to these conditions and definitions permits the calculation of the maximum receiver temperature.**

$$_{\max}T_2 = 1000 \frac{5.04\,(\phi''_- - V_O) + 0.76}{0.65 + (5.04\phi' + 1.6)\frac{1000}{T_1}} \qquad (34)$$

**In the analysis just completed it was assumed that $\phi''_-$ is 2.74 ev, $V_O$ is 0.65, and $T_1$ is $1520^{o}K$. With these values substituted into Eq. 34, the maximum temperature of the collector is found to be $1040^{o}K$.**

## Operation Considerations Related to Diodes Having High Work-Function Emitters

If the source of heat is one which lends itself to the use of a very high work-function emitter, then other considerations may be important. Under ordinary circumstances, it should be anticipated that the lower temperature at which the emitter can work, the more likely one is to have an efficient device. However, efficiency may not be the most important criterion. In fact, it could be that power per unit volume would be far more important than conversion effieiency. Under these circumstances, thought should be given to the operation of high temperature emitters.

Assume for this discussion that the emitter temperature is 2400°K and that it is realistic to make a diode of 100 micron spacing. Calculation shows that the space-charge minimum will coincide with the collector when the current density is 0. 009 amp/cm$^2$. An emitter with an average work-function of 4. 4 ev would have an emission current available of 0. 37 amp/cm$^2$. A high vacuum transducer would require a low work-function collector which could under favorable conditions have an average work-function of 2 ev and a $V_R$ = 3. 17. Under these conditions, the critical value of $(V_R/V_T)$ is 15. 3. This diode would have the maximum power available with a current received at the collector of 0. 115 amp/cm$^2$. The power available to the external circuit would be 0. 168 watts/cm$^2$. The question that may now be asked is: what improvement should be expected in this diode by the introduction of cesium, and what would be the most suitable cesium pressure as determined by the temperature of the liquid cesium?

The first effect of cesium if the collector temperature could be maintained at a sufficiently low value would be to reduce its work-function. Assume that the absorbed cesium would reduce the work-function to 1. 4 ev. Such a change would have a very favorable influence on the power output. Secondly, all of the cesium atoms which would in their normal motion come in contact with the emitter would leave as ions. Very few of these ions would be lost to the collector since its surface would be 0. 9 volts more positive than the emitter surface if the external surface conditions were maintained to hold the current at 0. 115 amp/cm$^2$ in the absence of space-charge neutralization. Actually, as the cesium pressure increases, the ions will tend to neutralize space charge and the electron current achieve its full capability of 0. 37 amp/cm$^2$. This current should remain constant as the output voltage is increased to 3 volts. The combination of these two effects would increase the available power a factor of 6. 6, since the maximum power we would expect is:

$$0.37\,(4.4 - 1.4) = 1.1 \text{ watt/cm}^2 \qquad (35)$$

Achievement of this power would depend on there being a negligible number of ions that are generated at the emitter and recombine at the collector. We may assume that the average energy associated with the moving electrons and also the moving ions will be 2 $V_T$ (that is, 2 kT/q) and that if their energies are equal, their average velocities will be inversely proportional to the square root of their masses. This factor for cesium is close to 490. This figure will then be used to establish the order of magnitude of the desired cesium arrival rate if on the average every ion which leaves the cathode travels more or less unimpeded to the collector. The electron current of 0.37 amp/cm$^2$ corresponds to an emission rate of 2.3 x $10^{18}$ electrons per second for each square centimeter. If we assume that just the right number of ions are present to neutralize the space charge of the electrons, there will be no electric field and the average velocity of electrons in transit across the space may be computed to be 1.52 x $10^7$ cm/sec. Since the electrons will travel unimpeded across this space, the density will be 1.51 x $10^{11}$ electrons per cm$^3$. An exactly equal ion density corresponds to an ion emission rate of 4.7 x $10^{15}$ ions per second for each square centimeter. Under these conditions the ion emission rate will be exactly equal to the atom arrival rate and the next question to be answered is the determination of the most suitable cesium condensation temperature.

Were it not for the fact that consideration must be given to the reduced density of atoms between the emitter and the collector, a condensation temperature of 330°K would give the required ion emission. The average temperature in the interelectrode space may be taken as about 1600°K. With this approximate value of the condensation temperature, Eq. 17 shows that the atom evaporation rate at the cesium surface should be increased 2.2 over that calculated by Eq. 8. With this correction factor the theoretical value for the cesium condensation temperature becomes 340°K. At this temperature the mean-free path will be long compared with the spacing and therefore both the electrons and the ions should flow in an unimpeded manner from the emitter to the collector.

If the cesium condensation temperature is raised above the value 340°K then the arrival rate will be greater than that needed and the excess ion production will result in an ion space-charge sheath forming near the surface of the emitter. Such a sheath will inhibit the delivery of the ions to the plasma. It will also create an accelerating field for the electrons. This will increase the electron emission if the output voltage is reduced slightly to take advantage of the lowering of the electron work-function because of the accelerating field produced at the emitter by the ions. The fact that many of the ions that are produced are re-

turned to the emitter surface may also result in the establishing there of an average dipole moment favorable to a still further reduction of the work-function in spite of the high temperature. These two effects increase the emission capability of the cathode over the previously calculated value of 0.37 amp/cm$^2$. Without experimental experience it is difficult to determine the best cesium condensation temperature except to state that for temperatures less than 340°K, an insufficient number of ions will be available to neutralize the space-charge and as temperatures exceed this value, favorable results may come because of the reduction in the emitter work-function and the corresponding exponential increase in available current.

The cesium pressure called for by this discussion is much lower than that used by some experimenters. For example, V. C. Wilson[4] experimented with high temperature emitters at relatively high cesium pressure. The two cesium temperatures he used were 534°K and 564°K. The use of the higher cesium pressure would result in the creation of an ion space charge sheath. The fact that an electron emission of 4 amp/cm$^2$ was observed sets the "effective" electron barrier at B" of Fig. 8 at 3 ev. Eq. 29 shows that any area of the hot surface with a work-function in the range 3 ev to 4.3 ev will deliver ions to help neutralize the electron space charge. The Wilson experiment would seem to indicate that work-function lowering actually takes place.

These thoughts are presented by way of illustration of the use of the equations presented here and the general ideas related to the behavior of a plasma diode as it might be used in conjunction with a high temperature, high work-function emitter. It is clear that many experiments should be performed before too much reliance is placed in these deductions since they depend on an extrapolation based on data not specifically applicable to present needs. Additional experimentally determined facts might come into prominence beyond those considered.

## Concluding Remarks

Three principle objectives have guided the preparation of this report. The first was to discuss in some detail the physical principles that seem to be involved in the better understanding of the design and properties of a plasma diode heat-to-electrical-power transducer. The second objective was to make available an intricate set of empirical equations by which many answers to pertinent questions can be obtained numerically when desired. These equations are consistent with the experimental facts as they are know today. They depend very largely upon the studies of Taylor and Langmuir and some very recent measurements made by Robinson. The final objective has been to apply the ideas here to the experiments of Robinson and indicate by quantitative caused to the emitter surface may also result in the establishment of an average dipole moment favorable to a still further reduction of the work-function in spite of the high temperature. These two effects increase the emission capability of the cathode over the previously calculated value of 0.37 amp/cm$^2$. Without experimental experience it is difficult to determine the best cesium condensation temperature except to state that for temperatures less than 340°K, an insufficient number of ions will be available to neutralize the space-charge and as temperatures exceed this value, favorable results may come because of the reduction in the emitter work-function and the corresponding exponential increase in available current.

The cesium pressure called for by this discussion is much lower than that used by some experimenters. For example, V. C. Wilson[8] experimented with high temperature emitters at relatively high cesium pressures. The two cesium temperatures he used were 518°K and 564°K. The use of the higher cesium pressure would result in the creation of an ion space charge sheath. The fact that an electron emission of 4 amp/cm$^2$ was observed sets the "effective" electron barrier at E' of Fig. 8 at 3 ev. Eq. 29 shows that any area of the hot surface with a work-function in the range 3 ev to 4.3 ev will deliver ions to help neutralize the electron space charge. The Wilson experiment would seem to indicate that work-function lowering actually takes place.

These thoughts are presented by way of illustration of the use of the equations presented here and the general ideas related to the behavior of a plasma diode as it might be used in conjunction with a high temperature, high work-function emitter. It is clear that many experiments should be performed before too much reliance is placed in these deductions since they depend on an extrapolation based on data not specifically applicable to present needs. Additional experimentally determined facts might come into prominence beyond those considered.

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In the present report the detailed derivations involved in the development of the empirical equations have not been given. If the results found here are ultimately of real practical importance, the more interesting derivations can be prepared and made available as a supplement to this report. Twenty of the thirty-six equations used in this report may find direct application to other problems and are therefore summarized for quick reference in an appendix.

## Appendix

Selected equations from the text. Minimum temperature for surface ionization as a function of the atom evaporation rate at the cesium condensation surface:

$$T_{min} = \frac{14,100}{27.56 - \log_{10}\mu_a} \tag{7}$$

Equation for evaporation rate of cesium:

$$\log_{10}\mu_a = 27.48 - \frac{3900}{T_{Cs}} \tag{8}$$

$$\mu_a = 3 \times 10^{27} \times 10^{-\frac{3900}{T_{Cs}}} \text{ atoms/cm}^2 \tag{9}$$

$$\mu_a = 3 \times 10^{27}\, e^{-\frac{8980}{T_{Cs}}} \text{ atoms/cm}^2 \tag{10}$$

Minimum temperature for surface ionization related to cesium temperature:

$$T_m = 3.6\, T_{Cs} \tag{11}$$

Ion evaporation rate for tungsten surface temperatures less than the minimum temperature calculated by Eq. 11:

$$\log_{10}\nu_p = 54.16 - 1.113 \log_{10}\mu_a - \frac{26,000}{T} \tag{12}$$

$$\nu_p = \frac{1.45 \times 10^{54}}{\mu_a{}^{1.113}}\, 10^{-\frac{26,000}{T}} \tag{13}$$

$$\nu_p = \frac{1.45 \times 10^{54}}{\mu_a{}^{1.113}}\, e^{-\frac{5.28}{V_T}} \tag{14}$$

The vapor concentration of cesium atoms in equilibrium with liquid cesium:

$$n_{Cs} = 2.7 \times 10^{23}\, e^{-\frac{8750}{T_{Cs}}} \text{ atoms/cm}^3 \tag{15}$$

$$\log_{10}n_{Cs} = 23.44 - \frac{3800}{T_{Cs}} . \tag{16}$$

Mean-free path of cesium ions in cesium vapor:

$$\log_{10} \lambda_+ = \frac{3800}{T_{Cs}} - 9.7 \tag{19}$$

$$T_{Cs} = \frac{3800}{9.7 + \log_{10} \lambda_+} \tag{20}$$

Maximum power in a vacuum diode used for a plasma diode after the "effective" spacing is determined:

$$P_{max} = 3.7 \times 10^{-6} V_T^{1/2} \frac{V_R^2}{w^2} \text{ watt/cm}^2 \tag{23}$$

Voltage output:

$$V_O = \frac{0.383 \left(\frac{V_R}{V_T}\right) V_R}{1 + 0.31 \left(\frac{V_R}{V_T}\right)^{4/3}} \tag{24}$$

Energy step $\phi_R$ related to current density and temperature:

$$\phi_R = 2.3 V_T \left[ 2.08 + 2 \log_{10} T - \log_{10} I_m \right] \tag{26}$$

Energy step $\phi_R$ related to ion surface work-function $\phi_b$; surface temperature; and cesium temperature:

$$\phi_R = \frac{\phi_b}{2} - 10.68 V_T + 2.3 V_T \log_{10} T + \frac{4490}{T_{Cs}} V_T \tag{29}$$

Simplified emission equation for the range $1150^\circ K < T < 2500^\circ K$:

$$\log_{10} I = 9.48 - (5.04 \phi + 1.6) \frac{1000}{T} \tag{32a}$$

Range $600^\circ K < T < 1150^\circ K$:

$$\log_{10} I = 8.85 - (5.04 \phi + 0.76) \frac{1000}{T} \tag{33a}$$

Maximum temperature of the collector for 5 per cent return emission:

$$_{max}T_2 = 1000 \frac{5.04 (\phi'' - V_O) + 0.76}{0.65 + (5.04 \phi' + 1.6) \frac{1000}{T_1}} \tag{34}$$

## Acknowledgments

This work has been supported in part by the U. S. Army (Signal Corps), the U. S. Air Force (Office of Scientific Research, Air Research and Development Command), and the U. S. Navy (Office of Naval Research).

Also the Thermo-Electron Engineering Corporation has contributed toward the support of this research and supplied significant experimental data.

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Glossary of Symbols

| | |
|---|---|
| "a" | Low work-function region as in Fig. 5 |
| a - a' | Designations of emitter surface in Fig. 1 |
| "b" | High work-function region as in Fig. 5 |
| b - b' | Designations of collector surface as in Fig. 1 |
| B | Designation of electron space-charge minimum or barrier |
| B'' | Designation of electron space-charge minimum or barrier with cesium present |
| FL | Fermi level within emitter or collector |
| $I_m$ | Current density when space-charge minimum is at collector. Energy step is $\phi_R$. Used in Fig. 11 |
| $I_{max}$ | Current density obtained at maximum power as in Eq. 25 |
| $I_p$ | Random current of ions at ionization surface. Same as arrival rate of ions as in Eq. 27 |
| $I_1$ | Emission current density as used in Eq. 1 |
| k | Boltzmann's constant used in Eq. 2. Value is $1.38 \times 10^{-23}$ joule/deg or $1.38 \times 10^{-16}$ erg/deg. |
| m | Mass of an electron $9.108 \times 10^{-28}$ gm or $9.108 \times 10^{-31}$ kg |
| M | Mass of an atom or ion. Atomic weight of cesium is 132.9 gms/gm mole. Atoms/gm mole are $6.045 \times 10^{23}$. $M = 2.20 \times 10^{-22}$ gm or $2.20 \times 10^{-25}$ kg. |
| $n_{Cs}$ | Concentration of cesium atom in atoms/cm$^2$ |
| $n_{po}$ | Concentration of ions at ionization surface as in Eq. 27 |
| $n_s$ | Concentration of cesium atoms in the space between the surfaces of the diode |
| $n_{+B''}$ | Concentration of ions at space-charge minimum B'' as in Fig. 8 |
| P | Power per unit area delivered to the resistance as in Eq. 4 |
| $P_{max}$ | Maximum power as in Eq. 6 |
| q | Electron charge as in Eq. 2. Value is $1.602 \times 10^{-19}$ coulomb or $4.803 \times 10^{-10}$ statcoulomb |
| $R_{10}$ | Load resistance defined by Eq. 3 |
| T | Emitter temperature where there is no need to specify further |
| $T_1$ | Emitter temperature as used in Eq. 1 |
| $_{max}T_2$ | Maximum collector temperature which permits a "return" electron current of 5 per cent |
| $T_C$ | Temperature of collector as in Eq. 17 |
| $T_{Cs}$ | Cesium condensation temperature |
| $T_E$ | Temperature of the emitter as in Eq. 17 |
| $T_m$ | Minimum temperature of a hot tungsten surface which is a function of cesium arrival rate as in Eqs. 7 and 11 |
| $T_{min}$ | Minimum temperature of a hot tungsten surface which is a function of cesium arrival rate as in Eqs. 7 and 11 |
| v | Incremental voltage in "negative" direction as in Eq. 4 |

| | |
|---|---|
| $v_m$ | Incremental voltage associated with maximum power as in Eq. 5 |
| $V_i$ | Ionization potential - value for cesium is 3. 893 ev |
| $V_o$ | A voltage defined as $\phi_1 - \phi_2$ as in Fig. 1 |
| $V_O$ | Output voltage generally |
| $V_p$ | Potential difference between the space-charge minimum and the potential of an electron at the surface of the collector |
| $V_R$ | Potential applied to collector with respect to emitter when space-charge minimum coincides with collector surface as in Fig. 15 |
| $V_T$ | Electron volt equivalent of temperature as defined in Eq. 2 |
| $x_s$ | Distance to potential minimum at B as in Fig. 7 |
| x, y, z | Direction in space between diode surface with x perpendicular to the surface |
| w | Diode spacing. Also used as in Fig. 11 for distance to potential minimum when current density I and temperature T are related |
| $\phi$ | True work-function as in Eq. 31 |
| $\phi_a$ | Work-function of low work-function region of Fig. 5 |
| $\phi_b$ | Work-function of high work-function region of Fig. 5 |
| $\bar{\phi}_b$ | Average of high work-function area as shown in Fig. 16 |
| $\phi_1$ | Emitter true work-function as in Fig. 1 |
| $\phi_2$ | Collector true work-function as in Fig. 1 |
| $\phi_R$ | Energy step from the Fermi level (FL) of the emitter to the surface potential of an electron at the collector when space-charge minimum is there as in Figs. 12 and 15 |
| $\phi'$ | Electron potential at space-charge minimum relative to the Fermi level as in Fig. 4 |
| $\phi'_+$ | Energy at "hump" of motive diagram as in Fig. 7 |
| $\phi'_-$ | Energy at electron potential minimum at B as in Fig. 7 |
| $\phi''_+$ | Energy at "hump" of motive diagram as in Fig. 15. Ions present |
| $\phi''_-$ | Electron potential at the space-charge minimum related to the emitter Fermi level when cesium is present as in Fig. 8 |
| $\phi''_2$ | True work-function of the collector with cesium present as in Fig. 7 |
| $\lambda_+$ | Mean-free path of an ion in cesium vapor |
| $\mu_a$ | Atom arrival rate in atoms per sec for each $cm^2$ as in Eq. 8 |
| $\nu_p$ | Ion emission rate in ions per sec for each $cm^2$ |
| $\nu_+$ | Arrival rate of ions in number per second for a unit area |
| $\nu_-$ | Arrival rate of electrons in number per second for a unit area |
| $\sigma_+$ | Collision cross section for an ion in cesium vapor assumed to be $2 \times 10^{-14} cm^2$ as in Eq. 18 |

# CESIUM PLASMA DIODE AS A HEAT-TO-ELECTRICAL-POWER TRANSDUCER

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# CESIUM PLASMA DIODE AS A HEAT-TO-ELECTRICAL-POWER TRANSDUCER

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## ABSTRACT

The new interest in the direct conversion of heat-to-electrical power has stimulated research in both the application of the high vacuum diode and the plasma diode to accomplish this purpose. The theory of the high vacuum diode is relatively simple and the experimental verification of the theory has been satisfactory. The plasma diode which depends on the ionization of cesium at a hot surface cannot be worked out in all of its detail at present because of the lack of certain fundamental experimental data. It is possible to make use of published results of Taylor and Langmuir and a detailed analysis of recent thermionic studies to carry the underatanding of the plasma diode far enough to make a direct comparison with experiment. This analysis first involves an understanding of the phenomenon of surface ionization. General properties of a plasma and space-charge considerations control the delivery of ions to neutralize electron space charge. When applied to the experimental data available, an interesting result comes as an important simplification. Essential to the theory of the high vacuum diode is the knowledge of the emitter temperature and the diode spacing. The electrical characteristics of the plasma diode have been found to be very closely duplicated by those of a high vacuum diode characterized by an *effective* distance that is reduced from the actual diode spacing. This fact supports the opinion that the efficiency of the plasma diode may be tremendously improved over that of vacuum diodes of practical design.

## Table of Contents

# Glossary of Symbols

| | |
|---|---|
| "a" | Low work-function region as in Fig. 5 |
| a - a' | Designations of emitter surface in Fig. 1 |
| "b" | High work-function region as in Fig. 5 |
| b - b' | Designations of collector surface as in Fig. 1 |
| B | Designation of electron space-charge minimum or barrier |
| B'' | Designation of electron space-charge minimum or barrier with cesium present |
| FL | Fermi level within emitter or collector |
| $I_m$ | Current density when space-charge minimum is at collector. Energy step is $\phi_R$. Used in Fig. 11 |
| $I_{max}$ | Current density obtained at maximum power as in Eq. 25 |
| $I_p$ | Random current of ions at ionization surface. Same as arrival rate of ions as in Eq. 27. |
| $I_1$ | Emission current density as used in Eq. (1) |
| k | Boltzmann's constant used in Eq. (2) Value is $1.38 \times 10^{-23}$ joule/deg or $1.38 \times 10^{-16}$ erg/deg. |
| m | Mass of an electron $9.108 \times 10^{-28}$ gm or $9.108 \times 10^{-31}$ kg |
| M | Mass of an atom or ion. Atomic weight of cesium is 132.9 gms/gm mole. Atoms/gm mole are $6.045 \times 10^{23}$. $M = 2.20 \times 10^{-22}$ gm or $2.20 \times 10^{-25}$ kg. |
| $n_{Cs}$ | Concentration of cesium atom in atoms/cm$^2$ |
| $n_{po}$ | Concentration of ions at ionization surface as in Eq. (27) |
| $n_s$ | Concentration of cesium atoms in the space between the surfaces of the diode |
| $n_{+B''}$ | Concentration of ions at space-charge minimum B'' as in Fig. 8 |
| P | Power per unit area delivered to the resistance as in Eq. (4) |
| $P_{max.}$ | Maximum power as in Eq. 6 |
| q | Electron charge as in Eq. (2). Value is $1.602 \times 10^{-19}$ coulomb or $4.803 \times 10^{-10}$ statcoulomb |
| $R_{10}$ | Load resistance defined by Eq. 3 |
| T | Emitter temperature where there is no need to specify further |
| $T_1$ | Emitter temperature as used in Eq. (1) |
| $\text{max}T_2$ | Maximum collector temperature which permits a "return" electron current of 5 per cent |
| $T_C$ | Temperature of collector as in Eq. (17) |
| $T_{Cs}$ | Cesium condensation temperature |
| $T_E$ | Temperature of the emitter as in Eq. 17 |
| $T_m$ | Minimum temperature of a hot tungsten surface which is a function of cesium arrival rate as in Eqs. (7) and (11) |

Glossary of Symbols, contd.

| | |
|---|---|
| $T_{min}$ | Minimum temperature of a hot tungsten surface which is a function of cesium arrival rate as in Eqs. (7) and (11) |
| $v$ | Incremental voltage in "negative" direction as in Eq. 4 |
| $v_m$ | Incremental voltage associated with maximum power as in Eq. 5 |
| $V_i$ | Ionization potential - value for cesium is 3.893 ev |
| $V_o$ | A voltage defined as $\phi_1 - \phi_2$ as in Fig. 1 |
| $V_O$ | Output voltage generally |
| $V_p$ | Potential difference between the space-charge minimum and the potential of an electron at the surface of the collector |
| $V_R$ | Potential applied to collector with respect to emitter when space-charge minimum coincides with collector surface as in Fig. 15 |
| $V_T$ | Electron volt equivalent of temperature as defined in Eq. (2) |
| $x_s$ | Distance to potential minimum at B as in Fig. 7. |
| x, y, z | Direction in space between diode surface with x perpendicular to the surface |
| w | Diode spacing. Also used as in Fig. 11 for distance to potential minimum when current density I and temperature T are related |
| $\phi$ | True work-function as in Eq. (31) |
| $\phi_a$ | Work-function of low work-function region of Fig. 5 |
| $\phi_b$ | Work-function of high work-function region of Fig. 5 |
| $\overline{\phi}_b$ | Average of high work-function area as shown in Fig. 16 |
| $\phi_1$ | Emitter true work-function as in Fig. 1 |
| $\phi_2$ | Collector true work-function as in Fig. 1 |
| $\phi_R$ | Energy step from the Fermi level (FL) of the emitter to the surface potential of an electron at the collector when space-charge minimum is there as in Figs. 12 and 15 |
| $\phi'$ | Electron potential at space-charge minimum relative to the Fermi level as in Fig. 4 |
| $\phi'_+$ | Energy at "hump" of motive diagram as in Fig. 7 |
| $\phi'_-$ | Energy at electron potential minimum at B as in Fig. 7 |
| $\phi''_+$ | Energy at "hump" of motive diagram as in Fig. 15. Ions present |
| $\phi''_-$ | Electron potential at the space-charge minimum related to the emitter Fermi level when cesium is present as in Fig. 8 |
| $\phi''_2$ | True work-function of the collector with cesium present as in Fig. 7 |
| $\lambda_+$ | Mean-free path of an ion in cesium vapor |
| $\nu_a$ | Atom arrival rate in atoms per sec for each $cm^2$ as in Eq. (8) |
| $\nu_p$ | Ion emission rate in ions per sec for each $cm^2$. |
| $\nu_+$ | Arrival rate of ions in number per second for a unit area |
| $\nu_-$ | Arrival rate of electrons in number per second for a unit area |
| $\sigma_+$ | Collision cross section for an ion in cesium vapor assumed to be $2 \times 10^{-14}$ $cm^2$ as in Eq. (18) |

# Cesium Plasma Diode As a Heat-to-Electrical-Power Transducer

## Introduction

An analysis of the *high vacuum* diode as a heat-to-electrical-power transducer has been worked out in detail(1) based on theories presented in "Thermionic Emission" published some years ago(2). In the high vacuum diode it is of the utmost importance to have the lowest possible work-function on the collector and to have the smallest practical spacing. It is anticipated that under these conditions the work-function of the emitter will be practically without influence so long as the space-charge minimum in front of the emitter is ~~of the order of~~ $V_T$ or more (the electron-volt equivalent of the temperature). Any further decrease in the emitter work-function will do neither good nor harm in controlling the usefulness of the device. In the plasma diode it is not so easy to generalize and since much of the most significant fundamental data relevant to the problem will not be available until additional research studies have been made. These remarks relevant to the plasma diode represent some of my own views on the subject in that they are being developed without the benefit of the experimental evidence needed.

In the high vacuum diode the understanding of the space charge is very important. It controls the properties of diodes that operate within the temperature limits set by available materials and the electron emission properties associated with them. The function of the positive ions in the plasma diode is to alter the effect of electron space charge. Even in the plasma diode there will be space charges near the boundaries of either the emitter or the collector or both.

Assume for the purpose of illustration that the emitter work-function is somewhat higher than the collector work-function and by some mechanism not specified for the moment exactly the right number of ions are available to reduce the space charge to zero. Assume furthermore that the spacing is sufficiently small in relation to the gas pressure so that the drop in potential over the plasma between the surface of the emitter and the collector can be neglected. This condition is represented diagramatically by Fig. 1. Here the *true* work-functions of the emitter and the collector are $\phi_1$ and $\phi_2$. Note particularly that these are *not* the "Richardson" work-functions. The current emitted over the barrier $\phi_1$ is given by the equation:

$$I_1 = 120 \times T_1^2 \, e^{-\frac{\phi_1}{V_T}} \text{ amp/cm}^2 \qquad (1)$$

In this equation T is the temperature of the emitter and $V_T$ is its electron-volt equivalent defined by:

$$V_T = \frac{kT}{q} = \frac{T}{11,600} \qquad (2)$$

The available power is the product $I_1 V_o$. The total resistance in the circuit from the receiving surface on the collector through the external load and to the emitting surface of the emitter may be expressed for a unit area as

$$R_{10} = \frac{V_o}{I_1} \qquad (3)$$

If the total resistance is made less than $R_{10}$, then the current around the circuit will remain constant and the output voltage decrease, thus giving a smaller available electrical power. If the load resistance is made greater than $R_{10}$, then the output voltage increases while at the same time the current decreases. Identify this increase in voltage by the symbol v. The following equation serves to relate these quantities for the calculation of maximum power.

$$P = I_1 \, e^{-\frac{v}{V_T}} (V_o + v) \qquad (4)$$

Equation 4 may be differentiated to obtain an expression for the deviation in the power as a function of v. A maximum of power output will be found when the differential (dP/dv) is zero and the value of v so determined is identified as $v_m$. The result is given as

$$v_m = V_T - V_o = V_T - (\phi_1 - \phi_2) \qquad (5)$$

The physical meaning of this equation may be considered in its relation to Fig. 1. If the work-function difference $(\phi_1 - \phi_2)$ is greater than $V_T$ then $v_m$ is zero or stated in other words maximum power is delivered to the external load when the resistance is chosen to satisfy Eq. 3 and the overall output voltage is exactly the difference between the work-functions. Under the circumstances in which the receiver work-function is actually greater than the emitter work-function, the power output is a maximum when the voltage available is $V_T$. This situation is illustrated by the diagram of Fig. 2 and the total available power is expressed by

$$P_{max} = 120 \times T^2 \, e^{-\frac{\phi_2 + V_T}{V_T}} \, V_T \ \text{w/cm}^2 \qquad (6)$$

In the range of work-function difference for which $(\phi_1 - \phi_2)$ is equal to or less than $V_T$ even though $\phi_1$ is greater than $\phi_2$, maximum power will occur with an available voltage of $V_T$ and Eq. 6 applies.

These statements may be summarized as follows. For work-function differences greater than $V_T$ in which the emitter work-function is larger than the collector work-function, maximum power is available at an overall voltage level equal to the difference in the work-functions, whereas in all other cases the maximum power occurs with an available voltage of $V_T$.

It will be the purpose of the following sections to discuss the problem of ion production as well as the delivery to the plasma of the ions and the current flow expected in the presence of atoms and ions.

## Ionization of Cesium at Heated Surfaces

The classic work on the ionization of cesium at heated tungsten surfaces was largely due to Langmuir and his collaborators. One of the latest in that series of researches was reported by Taylor and Langmuir(3). This work serves as the basis of many of deductions made here. If the arrival rate of cesium atoms for each square centimeter of a heated tungsten surface is less than $10^{13}$ atoms per second (cesium condensation temperature 0°C), a tungsten filament at 970°K will have a neligible surface coverage of cesium atoms and every cesium atom that arrives will leave as a cesium ion. As the arrival rate increases to $10^{18}$ atoms/sec-cm$^2$, the minimum temperature for complete conversion of atoms to ions is approximately 1475°K.

This statement should not be attributed to Taylor and Langmuir but does depend on an analysis of their data which was made available in their Fig. 18. Since no other data are available, these are used to yield a relation between the atom arrival rate $\mu_a$ expressed in atoms per second for each square centimeter and the minimum temperature for which a negligible surface film will form on tungsten. At this temperature practically every atom that arrives will leave the surface as an ion. An empirical equation has been derived to relate this minimum temperature to the atom arrival rate and is

$$T_{min} = \frac{14,100}{27.56 - \log_{10} \mu_a} \tag{7}$$

At any given arrival rate, the production of ions is discontinuously reduced approximately a factor of 10 if the tungsten surface is at a temperature slightly below the minimum temperature given by Eq. 7.

The ideal gas laws are used to convert cesium vapor pressure data to atom arrival data. The following formula relates atom arrival to the temperature of the liquid cesium in equilibrium with its vapor.

$$\log_{10} \mu_a = 27.48 - \frac{3900}{T_{Cs}} \tag{8}$$

The corresponding formulae in the exponential form are given in Eqs. 9 and 10.

$$\mu_a = 3 \times 10^{27} \times 10^{-\frac{3900}{T_{Cs}}} \text{ atoms/cm}^2 \tag{9}$$

$$\mu_a = 3 \times 10^{27} e^{-\frac{8980}{T_{Cs}}} \text{ atoms/cm}^2 \tag{10}$$

It is an interesting fact that Eqs. 7 and 8 may be combined to yield a very simple relation between the temperature of the liquid cesium and the minimum temperature of the heated tungsten surface at which complete ionization takes place. This relation is:

$$T_m = 3.6 \, T_{Cs} \tag{11}$$

The fact that the minimum temperature at which all cesium atoms are converted to ions is linearly related to the temperature of the liquid cesium may not be a pure accident of numbers. In the broader sense of the word both the evaporation of atoms from the liquid and the evaporation from a tungsten surface may possibly involve atomic attractive forces that are not so different from each other. This would imply that their vibrational frequency before evaporation might be characterized by a force function not unlike that of a vibrating molecule. In the molecular case there is a dissociation energy whereas in the case of atomic evaporation there is the latent heat of evaporation which is dominated by a quantum state of vibrational activity which is the maximum that can be attained by the atom as it is on the border line between continued adherence to the surface and evaporation. If the equations of motion are not too dissimilar then the factor that relates these two critical temperatures could well be the ratio of the "activation" energies. Equation 8 implies an activation energy for evaporation from the liquid state that is not too far from 0.773 ev. Taylor and Langmuir deduce from their studies of cesium evaporation from tungsten that the heat of evaporation is 2.83 ev if the fraction of the surface coated is small. Note that the ratio (2.83/0.773) is 3.65 which is consistent with the empirical results presented in the form of Eq. 11.

Results of the Taylor-Langmuir experiments as they relate to the yield of ions from surfaces that have an appreciable fraction covered by adsorbed cesium atoms show an extremely small yield of ions. A detailed analysis of the experimental data available yields some interesting results. The equations to be presented fit quite well over the range of cesium pressures and tungsten surface temperatures studied by Taylor and Langmuir. The purpose in deriving empirical equations to represent their data is to permit computations to be made under conditions that involve considerable extrapolation into unexplored areas. The philosophy here is that estimates based on an extrapolation from existing data are better than pure guesses without a systematic relationto established facts. The ion production rate $\nu_p$ expressed as the number of ions produced per second for each square centimeter area is expressed by the set of equations that follow:

$$\log_{10}\nu_p = 54.16 - 1.113 \log_{10}\mu_a - \frac{26,600}{T} \tag{12}$$

$$\nu_p = \frac{1.45 \times 10^{54}}{\mu_a^{1.113}} \, 10^{-\frac{26,600}{T}} \tag{13}$$

$$\nu_p = \frac{1.45 \times 10^{54}}{\mu_a^{1.113}} \, e^{-\frac{5.28}{V_T}} \tag{14}$$

Attention may be directed specifically to the results shown as Eq. 13. The ion yield decreases rapidly with a decrease in the temperature of the surface for the range below the temperature $T_m$ introduced in Eq. 7 and related to the liquid cesium temperature by Eq. 11. Equation 13 has no significance at temperatures higher than $T_m$ since then the ion yield is precisely equal to the atom arrival rate $\mu_a$. It is evident from this equation that at a fixed surface temperature T, the ion yield decreases almost linearly (1.113 power) with the atom arrival rate. Stated in another way this equation shows that the probability of ionization at a given surface temperature decreases with the 2.113 power of the atom arrival rate. The physical explanation for this result depends on the fact that at a given surface temperature the fraction of the surface covered by adsorbed cesium atoms increases as the rate of arrival of neutral atoms increases. This increase in surface coverage brings about a decrease in the average work-function of the surface. The probability that a cesium atom will evaporate as an ion instead of neutral atom is strongly influenced by the work-function of the surface. Since the ion yield decreases as the average work-function decreases, it is to be anticipated that a relation not unlike that of Eq. 13 might hold. The insertion of numbers into these equations and a comparison with the published data of Taylor and Langmuir may convince the reader that over the range of temperature and cesium arrival rate for which good data exist, the equations represent the observations. It is thought that the failure to represent the true facts over the ranges of the variables likely to be encountered in association with the plasma transducer problem will be less than an order of magnitude, that is a factor of ten. It is hoped that the error will be much less than this in many examples that are of practical interest.

## Kinetics of Ions Formed at Hot Surfaces

The reader should be reminded that when drawings are made which show energy relations of electrons in diode structures, these drawings really relate to the "motive function" of an electron. That is, lines and energy levels apply to the potential of an electron in the space of interest. Figure 1 may be used as an example. We start at the Fermi level in the emitter and the line a' - a represents the potential function of an electron in the neighborhood of the surface. The energy difference between the Fermi level and the surface potential at a is the true work-function $\phi_1$ of the emitter. In this energy diagram the surface of the emitter is joined to the surface of the collector by the line a-b and since that line is drawn horizontally it represents the potential energy of an electron in the field-free space between the emitter and the collector. The work-function of the collector is shown by $\phi_2$ as the energy difference between the Fermi level in the collector and the potential energy of an electron at its surface at the point b. The misalignment of the Fermi level of the emitter with respect to that of the collector is indicated by $V_o$. This separation of Fermi levels can be maintained only by having some auxiliary and external source of power and may be measured directly by a voltmeter connected between the emitter and the collector electrodes. The kinetic energy of an electron in this diagram is represented by the fact that the electron is in an energy level which would exist in the diagram *below* the horizontal line a-b. The separation between the electronic energy level and the line a-b is a direct indication of the actual kinetic energy of the electron associated with the motion across the space from a to b. This direction is taken as the x direction whereas the y and z directions are taken perpendicular to this one and therefore parallel to the planes of the emitter and the collector. We can show in this diagram *only* the kinetic energy associated with the motion in the x direction. It is not suitable to try to show the kinetic energy associated with the other directions in this one-dimensional diagram. Energy levels do not exist for electrons in the space between the electrodes that would be represented by electron levels above the a-b line. Such levels do exist within the conductors but not outside of them.

This same potential diagram applies equally well to ions that might be in the space between a and b. Specifically an ion at rest would be represented as being on an energy level coincident with the line a-b, whereas one which is moving in the x direction with some kinetic energy would be occupying an energy level in this daigram *above* the line a-b. No positive-ion energy levels exist that would be represented by states drawn below the line a-b. These remarks are made to anticipate the discussion with reference to the energy distribution of ions created at a hot surface. One of the few sets of measurements on the ion energy distribution that exists in the literature is that of Fig. 16 of Taylor and Langmuir. Their data are reproduced as Fig. 3. The plot shows the ion current received at a coaxial collector as a function of the applied voltage with the emitter temperature held constant and the rate of arrival of neutral atoms established by a constant bath temperature of $275^{\circ}$K. The critical temperature for this rate of arrival of atoms is $990^{\circ}$K and therefore since the measurements were made at $980^{\circ}$K it was probable that the ion yield was less than 10 per cent of the atom arrival rate. Under this condition the average work-function of the emitter was very close to 3.4 volts. Other evidence would indicate that the work-function of the collector was very close to 2 volts. Therefore under these circumstances, the contact difference in potential was close to 1.4 volts. This fact is exhibited reasonably well by the data of Fig. 3. Under the conditions of this experiment, the heated tungsten surface undoubtedly had considerable nonuniformity in work-function, and also space-charge effects reduce the ion current at zero field. It may be assumed that these two factors account for the relatively poor saturation of the ion current. The evidence seems to be satisfactorily clear that the application of an ion retarding voltage of a fraction of a volt is sufficient to inhibit the flow of ions from the hot tungsten surface over to the collecting electrode. The slope of the semi-log plot is in agreement with the temperature of $980^{\circ}$K.

## A Generalized Discussion of the Simultaneous Emission of Electrons and the Creation of Ions at a Hot Tungsten Surface

In Fig. 4 the emitter work-function $\phi_1$ is assumed to be uniform. The collector work-function is $\phi_2$; the Fermi level of the collector is $V_O$ more negative than that of the emitter. Electrons are being emitted from the surface a-a' and those that impinge on the surface from inside the metal at an energy level $\phi'$ with respect to the Fermi level have a chance of escaping across the space-charge barrier at B. If the electron has an excess energy $V_T$ as it passes the barrier, then its energy state is represented by the dot-dash line of Fig. 4. Cesium ions will evaporate from the emitter surface in an appreciable number if the work-function $\phi_1$ is high enough and the temperature suitable. An ion which originates at the emitter surface with practically no kinetic energy associated with its motion in the x direction would find itself ~~ex~~accelerated by the electrostatic field created by the electron space charge and acquire kinetic energy so that as it passes through the region B it will have acquired a maximum of kinetic energy represented by the vertical distance between the solid potential line and the dotted line of the diagram. If the ion evaporates from the surface with an initial kinetic energy of $V_T$, its energy level will be that of the dash-plus line. In either case the ion will be accelerated across the space, impinge on the collector with considerable energy, and probably absorb an electron from it and evaporate off as a neutral atom. It is presumed that the collector will be at a higher temperature than the liquid cesium surface somewhere within the tube envelope and therefore maintain a surface coverage of less than a monolayer. As the pressure of the cesium is increased, the production rate of ions may increase or decrease depending on the temperature of the heated surface and the rate of arrival of cesium atoms.

This discussion indicates that if the cathode has a uniform work-function structure, ions may be produced at the energy level a or above and will not be as effective in neutralizing space charge as they would be if they could become trapped in the electronic space-charge potential minimum at B. As the cesium pressure is increased, there will be a greater and greater probability that the cesium ion will encounter a neutral cesium atom in such a way as to deliver energy to the atom and thus find itself in an energy level below the surface potential of the collector shown in the diagram at b.

The interaction between the neutral cesium and the ionized cesium may follow either one of two mechanisms. The cesium ion may collide with the cesium atom with an energy-sharing and momentum-conserving collision. The subsequent products then would be a slower-moving ion which might then become trapped, and a faster moving neutral atom. An alternate mechanism which also could have a good probability of occurring would be that the fast-moving ion would come into the neighborhood of a slow-moving cesium atom, absorb the electron from the atom, and continue on its way as a <u>neutral atom</u> with a relatively high kinetic energy. The ion thus created would be a slow-moving ion and therefore lie close to the electronic motive function and be trapped. Either of these processes would accomplish the desired result, namely, that of altering the energy level of the ion to bring it into a state that is below the energy level of the most negative surface. In the example shown in Fig. 4 this is the energy level at b.

As the cesium pressure is increased, the probability of such energy-losing collisions increases in direct proportion to the concentration of cesium atoms in the space between the two electrodes. It may therefore be of some interest to have equations by which this concentration may be estimated with reasonable accuracy. These equations in turn can be combined with an estimated "cross-section" associated with the energy-losing transition to try to determine the fraction of the ions produced that find their way into the space-charge minimum. Finally in the steady state situation as the space-charge minimum is reduced, the rate of arrival of ions into the plasma will equal their rate of loss. If the ion concentration builds up to be practically equal to the electron concentration then the main space-charge field will be wiped out and the only fields that will remain will be space-charge fields near one or the other of the two electrodes or both and the "drift" field needed to carry the electrons across the space from the neighborhood of the emitter space-charge sheath to the surface of the collector.

## Atom Concentration in the Space Between the Emitter and the Collector

In order to calculate an ion mean-free path it is first necessary to have an approximate formula for determining the atom density in the immediate neighborhood of liquid cesium in equilibrium with its vapor at a temperature $T_{Cs}$. This concentration is given by:

$$n_{Cs} = 2.7 \times 10^{23} \, e^{-\frac{8750}{T_{Cs}}} \text{ atoms/cm}^3 \tag{15}$$

A useful form of this equation for calculation purposes is:

$$\log_{10} n_{Cs} = 23.44 - \frac{3800}{T_{Cs}} \tag{16}$$

The fact that the exponents of Eq. 10 and Eq. 15 are different is not an error but results from the inclusion of a temperature coefficient term into the exponent. Although the above two equations serve to give the density of cesium atoms in equilibrium with the liquid, a correction should be used to relate the estimated temperature in the space between the emitter and the collector to the temperature at the cesium liquid surface.

A suitable form of this correction term is given as:

$$n_s = n_{Cs} \left(\frac{2\,T_{Cs}}{T_E + T_C}\right)^{1/2} \tag{17}$$

This equation takes this form on the assumption that the effective temperature in the diode space may be taken as the average of the emitter temperature $T_E$ and the collector temperature $T_C$. To give some idea of the magnitude of this correction, assume for example that the emitter temperatube is 1600°K and the collector temperature is 900°K when the cesium temperature is 500°K. The correction factor is then 0.63.

In order to estimate the mean-free path of an ion it is necessary to know the effective cross-section for an energy-losing collision. This quantity is undoubtedly energy-dependent and therefore any statement concerning its value should preferably be backed by experimental data not known accurately at present. For the purposes at hand, the assumption will be made that the cross-section for energy-losing collisions is $2 \times 10^{-14}$ $\text{cm}^2$. This assumption is the equivalent of stating that if a fast-moving ion comes close enough to a slow-moving atom for the center-to-center distance to be $8 \times 10^{-8}$ cm, there will be a high probability that an exchange of some sort will take place between the two in a manner to leave as a product a slower-moving ion. If the cross-section is represented by $\sigma_+$, and the density of atoms by $n_s$, then the best estimate for the mean-free path for the ions is given by:

$$\bar{\lambda}_+ = \frac{1}{n_s \sigma_+} = \frac{5 \times 10^{13}}{n_s} \text{ cm} \tag{18}$$

Equations 16, 17, and 18 may be combined for numerical calculations. Since the correction term in Eq. 17 is hardly great enough to be compared with the uncertainty in the collision cross-section, an equation can be written which neglects this factor and serves as a quick means of estimating the ion mean-free path in terms of the cesium-condensation temperature. The equation is:

$$\text{Log}_{10} \lambda_+ = \frac{3800}{T_{Cs}} - 9.7 \tag{19}$$

A second useful form of this equation serves to determine the condensation temperature needed to approach a desired mean-free path. This equation is:

$$T_{Cs} = \frac{3800}{9.7 + \log_{10} \lambda_+} \qquad (20)$$

This equation yields the result that approximately 1 mm mean-free path will be associated with the condensation temperature of 437°K and one-tenth of a millimeter will be the approximate mean-free path at 493°K.

If these calculations and estimations are valid, they may be helpful to the designer of a practical heat-to-electrical-power transducer that operates according to the plasma principle. It has been pointed out(1) that the high vacuum transducer must have a very small spacing in order to operate with satisfactory efficiency. The conclusion that one would draw from this discussion is that such a small spacing could work as a definite disadvantage in the plasma transducer.

For illustration purposes, assume a spacing of 0.5 cm for a diode and a cesium pressure adjusted so that the ion-free path would be approximately half this distance or 2.5 mm. The cesium temperature would be 418°K. Equation 11 is used to estimate the minimum temperature of the cathode for moderately efficient ion production. This temperature is $T \approx 1500°K$ and is a reasonable one for a dispenser-type cathode. If its average work-function is approximately 2.5 ev, then the available electron emission would be about 1 amp /cm$^2$. Assume that a space-charge sheath near the surface of the emitter added an additional 0.4 volt step to make the effective value of $\phi'$ shown in Fig. 4 equal to 2.9 ev. This would reduce the available electron emission a factor of 20 and bring it to 0.05 amp/cm$^2$. This current density would be 25,000 times the current density that would have been available from the same cathode in a high vacuum diode of this spacing when the space-charge minimum coincided with the collector. Again with this cathode operating at this temperature, the current density of 0.05 amp.cm$^2$ implies that the location of the space-charge minimum, would be approximately 30 microns from the emitter. The point in mentioning these figures is to lead the way toward an evaluation of the production rate of ions needed to hold the space-charge minimum at this point and give a very small potential difference between the minimum point at B and the electron collector surface at b. The next section will attempt to evaluate these factors.

## Ion and Electron Evaporation from a Nonuniform Cathode

The motive diagram shown in Fig. 4 is a one-dimensional diagram suitable for an emitter and a collector, each of which has a uniform surface work-function. In many practical examples of emitters this situation does not represent the facts. Specifically for pure tungsten the variation of work-function even without the adsorption of films may range from 4.3 to 5.3 ev. Adsorbed films may result in still wider differences. Details concerning dispenser cathodes are very uncertain. Specifically one may picture a work-function structure of a dispenser cathode as one in which there are many islands or wells of strong electron emission to be found at crystal boundaries between the sintered crystals of which the structure is made. Each of the individual pieces of the powdered tungsten used in this fabrication may exhibit considerable range in work-function. In order to illustrate this point, the sketch in Fig. 5 has been produced and represents a completely hypothetical situation which may not differ too much from reality to be worth considering. The lines on the diagram represent imaginary crystal boundaries between the solid tungsten crystalites or sintered powder particles and the shading is related to the thermionic emission. Regions that are white represent strong electron emission and regions that are dark represent high work-function areas of weak electron emission. The scale used on this figure implies that each of the individual particles of tungsten that were sintered together to make a porous block had an average linear dimension of approximately 10 microns. The activation material is assumed to have migrated out between the tungsten particles and activated them locally. The higher the temperature that the cathode is operated, the more likely a situation of this kind exists.

It is important for this discussion to attempt to represent in a two-dimensional diagram a motive function to be associated with this situation. It is quite evident that it is impossible to draw an exact representation but Fig. 6 may help to bring out the ideas that are involved.

In Fig. 5 one of the strongly emitting regions is identified as the "a" region, while a neighboring dark region is identified as the "b" region. The work-function of the "a" region is taken to be $\phi_a$ and the "b" region $\phi_b$.

These work-function steps are shown in Fig. 6. If the a and b regions are of comparable size, then the motive function becomes something close to the average at a distance comparable to the average dimension of the patches of high work-function and low work-function area. This is illustrated by Fig. 6 by the fact that one line starts at $\phi_a$ and the other line starts at $\phi_b$ and they come together at a distance just outside the emitter comparable with the size of the individual patches involved. If the required potential is applied, shown as $V_O$, and the work-function of the collector is $\phi_2$, the motive function the rest of the way across the diode has no slope and therefore corresponds to the zero field condition. If the emitter is heated sufficiently to cause a strong electron emission, then the motive function will take on a form similar to that shown in Fig. 7.

A word description of Fig. 7 may be in order. The curvature of the motive function indicates the existence of space charge. The minimum in the motive function found at B establishes the minimum electron energy within the interior of the emitter for which electrons can escape from the emitter to the collector. The value of this energy is shown as $\phi'_-$. Thus any electron emitted from the high work-function area $\phi_b$ is accelerated into the space and moves over to the collector. It is to be expected that there will be a gradation in work-function from $\phi_a$ to $\phi_b$ close to the boundaries of the activated area. It is impossible to show in this diagram that additional detail, and therefore only the extremes of the motive function are illustrated. Note that in this figure there is an energy level indicated at $\phi'_+$ which represents a peak in the motive function. Any positive ions generated at the surface will have to traverse this peak in order to find themselves in the space-charge region near B. This quantity $\phi'_+$ is very important because of the fact that if it is less than 3.9 ev then the difference $(3.9 - \phi'_+)/V_T$ is related exponentially to the probability that an ion produced at the surface of the emitter will be able to pass into a region between the emitter and the collector. It is only here that it can be at all effective in reducing space charge.

The next problem to be considered in this qualitative manner is the influence of cesium vapor introduced at a moderately low density. Equation 13 indicates that at a cesium temperature of $390^{\circ}$K, the approximate mean-free path for cesium ions will be 1 centimeter. Furthermore, the relation given as Eq. 9 indicates that the minimum temperature for a pure tungsten surface to remain, on the average, in a state for which good ionization can take place will be $1410^{\circ}K$. It follows from this that a dispenser cathode operating at $1500^{\circ}K$ will not be expected to adsorb an appreciable coverage of cesium. The low work-function areas will be very poor absorbers in any case, and they will also be very poor ionizers. The most efficient ionization will take place over those regions of the cathode for which the work-function is greater than 3.9 ev. Figure 8 attempts to illustrate in a qualitative manner the various influences which the presence of cesium is likely to have if the cesium pressure is controlled by a temperature $T_{Cs}$ equal to $392^{\circ}K$, and the emitter temperature is maintained close to $1500^{\circ}K$. The emitter to collector spacing is assumed to be approximately 1 mm.

In Fig. 8 the dotted lines are a superposition of Fig. 7 on Fig. 8 so that it will be easier to see a comparison of the two. Note that $\phi_a$ and $\phi_b$ are the same whereas $\phi_2$ shows a measurable decrease to $\phi''_2$. This decrease is the result of the adsorption on the collector of a film of cesium which in all probability will reduce the work-function of the collector by 0.4 ev or more. If no other change took place as a result of the cesium, this would be a distinct advantage. The new work-function is designated by $\phi''_2$. In the drawing of Fig. 8, it is assumed that some of the ions produced at the hot emitter surface will find their way into the neighborhood of the potential minimum at B which in turn acts as an ion trap. In other words once that an ion is caught at an energy level lower in the diagram of Fig. 7 than $\phi'_+$, it will remain in that area more or less indefinintely since recombination between ions and free electrons is such an improbable event that the ion will remain trapped in the potential minimum for a very long time. As additional ions are trapped there, the potential minimum will rise. This is illustrated schematically in Fig. 8 in that the potential minimum changes from B to B''. This change in the potential minimum results in a corresponding change in the value of the limiting electron energy needed to transfer current from the emitter to the collector. The new level is at $\phi''_-$. An objectionable change, from the point of view of being able to deliver <u>ions</u> to the space-charge minimum, is that indicated by the rise in the level diagram associated with $\phi''_+$. Thus the rise to B'' because of the trapped ions results in a change of the motive function which inhibits the delivery of additional ions. Finally a balance takes place at which the delivery rate of ions is exactly equal to the loss rate. This condition more or less inevitably gives a rise in the electron minimum from B to B''. This change results in a distinct gain in efficiency of the unit as a heat-to-electrical-power transducer.

It will be assumed for the present that if the transducer is working under the most favorable conditions the voltage output $V_O$ added to the receiver work-function $\phi''_2$ will actually be less than the energy difference from the Fermi level of the emitter to the "hump" in the potential function shown as $\phi''_+$ in Fig. 8. In those circumstances the loss of ions from the space-charge region will be back to the hot emitter and not to the collector. If the problem were as simple as this then the density of ions at B'' could be related to the ionization potential $V_i$ by:

$$\frac{n_{+B''}}{n_s} = \frac{1}{4} e^{-\frac{V_i - \phi''_-}{V_T}} \tag{21}$$

In this equation $V_i$ is the ionization potential of cesium of 3.9 volts; $n_s$ is the density of atoms in immediate neighborhood of the ionization surface, and the factor (1/4) is a guess made up of a combination of two facts. In the use of an equation of the form of Eq. 21 a factor of 1/2 is usually introduced as a so-called statistical weight and an additional factor of 1/2 is being introduced because of the fact that only part of the cathode is capable of producing ions. Once they are produced and caught in the potential minimum, the entire region is available for their occupation. This equation cannot be said to be exact but is offered here as a reasonable guess.

An additional fact needs to be considered which is that the ion production process will be characterized by the temperature T of the emitter, whereas the temperature that characterizes the distribution in energy of the ions that are trapped can very well be a lower temperature. This lower temperature could approach that of the neutral cesium vapor outside of the emitter-collector region for experimental tubes in which the spacing between the emitter and collector is not considerably smaller than the diameter of these surfaces. On the other hand, if the spacing is small compared to the diameter, then the characteristic temperature for the ions in the trap should be assumed to be approximately the mean temperature of the emitter and the collector.

In spite of the fact that the discussion applies to the conditions under which the mean-free path is large compared with the spacing, the ion may nevertheless come to thermal equilibrium with the atoms in the space because of the fact that it can oscillate back and forth many times and can therefore have a total distance of travel during its lifetime in the potential minimum much longer than the mean-free path. This detail can be considered to be a refinement which acts in the favorable direction from the point of view of the making of a practical and effective transducer.

## Influence of Cesium Pressure in Association with a Nonuniform Emitter

Under the circumstances illustrated in Fig. 8 an increase in cesium pressure will increase the rate of arrival of cesium atoms at the emitter. This increase in arrival rate will result in more adsorption of cesium on the high work-function areas. Once the cesium is adsorbed there, these areas will no longer be very effective at producing ions. The effect to be anticipated then will be a raising of the limiting level $\phi''_+$. This change will work to disadvantage in that it makes it more difficult for the ions that are produced to find their way to the region marked B''. The lack of a suitable ion density atB'' will result in a lowering of this level or an increase in the value of $\phi''_-$. Thus it is to be anticipated that other things being equal, the increase in cesium pressure will first result in a raising of the potential minimum for electrons from B toward B'' with an increase in cesium pressure toward its optimum but additional cesium pressure will result in a decrease in the available current for the transducer. The important point is to try to establish the optimum pressure and this may be possible only by experiment. The theory as so far carried out indicates that an optimum is likely.

## Loss of Positive Ions to the Collector

As the output voltage $V_O$ is increased at the expense of some decrease in electron current, the difference in potential between the potential minimum at B'' and the surface of the collector at b will decrease. At the critical condition expressed by:

$$V_O + \phi''_2 = \phi''_+ \qquad (22)$$

ions will begin to escape to the collector to the same extent that they escape to the emitter. If $V_O$ is made still larger, then the principal loss of ions will be at the collector surface and this rate of loss must be supplied by the ions which are able to pass over the ion barrier at $\phi''_+$. As $V_O$ is made larger, it will be slightly easier for ions to escape from the emitter. This increase in yield may not be sufficient to maintain the desired trapped ion density.

Under the conditions shown in Fig. 8 there is a difference in potential between the space-charge minimum at B'' and the surface of the collector which is numerically equal to

$$\phi''_- - (V_O + \phi''_2) = V_p. \qquad (22a)$$

This potential difference does not represent a loss of energy by the electrons because of the presence of the neutral cesium atoms, but is simply a difference needed to satisfy the space-charge relations. The reader must be reminded that the situation in this part of the plasma discharge is an entirely different one from that found in self-sustained low voltage arcs, because there the difference in potential along the plasma column is one that is determined by the rate at which energy must be put into the system in order to maintain the excitation and ionization of the plasma. In the transducer, no ionization whatsoever takes place in the space since all of the ions are produced at the heated emitter surface unless some auxiliary electrode is introduced. For the present discussion it is assumed that the only two electrodes in this diode are the nonuniform emitter and the uniform collector.

## An Interpretation of Data Taken on An Experimental Diode

A diode was constructed for study by Mr. Thomas Robinson of the Thermo-Electron Engineering Corporation which had two dispenser-type cathodes, each 3 mm in diameter separated from each other a distance of 0.68 mm. One of these dispensers was heated to 1520°K and before cesium was admitted to the tube, a current voltage characteristic was obtained over the range of applied voltage from zero to - 2.2 volts with the receiving electrode negative with respect to the emitter. These data were plotted as shown in Fig. 9 and compared with the "master curve"(2). The solid line of this figure is the master curve and the index of that curve establishes the value of $(V_R/V_T)$ of 10.4 and a corresponding value of $V_R$ of 1.37. These numbers combine in the formulae given here as Eqs. 16 and 17 permit the calculation of the maximum power available from this diode and the voltage output at which maximum power occurs.

$$P_{max} = 3.7 \times 10^{-6} \; V_T^{1/2} \; \frac{V_R^2}{w^2} \; \text{watt/cm}^2 \tag{23}$$

$$V_O = \frac{0.383 \left(\frac{V_R}{V_T}\right) \; V_R}{1 + 0.31 \left(\frac{V_R}{V_T}\right)^{4/3}} \tag{24}$$

In Eq. 16 the distance $w$ is expressed in centimeters for the calculation of the maximum power available in watts/cm$^2$. The insertion of the figures mentioned gives a predicted $P_{max} = 5.5 \times 10^{-4}$ watts/cm$^2$ and the voltage output at the maximum power is 0.683. The current at maximum power is given by:

$$I_{max} = I_m \left[ 1 + 0.31 \left(\frac{V_R}{V_T}\right)^{4/3} \right] \tag{25}$$

For the present experiment $I_{max} = 7.96 \; I_m$.

Robinson measured the power delivered to the load in his experiment as a function of the output voltage and obtained the curve designated "without cesium" of Fig. 10. It is clear that the maximum power was correctly calculated by theory and the voltage output at maximum power was correct as shown by the circle on that graph. Later cesium was admitted to the tube and the available power again measured as a function of the voltage. This result is also shown in Fig. 10 and it is very evident that the favorable action of the cesium increased the power a factor of 10.

Before the details concerning this second experiment are discussed, a few additional computations should be made relative to the high vacuum experiment. A calculation chart, shown here as Fig. 11, shows that with this operating temperature and spacing, a current density of 100 microamperes per square centimeter should have been expected in terms of the temperature-spacing relation for the critical current density $I_m$ which flows to the collector with a space-charge minimum coinciding with it. This situation is illustrated by the dotted lines of Fig. 12.

The actual value of $\phi_R$ applicable to this experiment may be calculated by:

$$\phi_R = 2.3\, V_T \left[ 2.08 + 2 \log_{10} T - \log_{10} I_m \right] \qquad (26)$$

The value of $\phi_R$ is 3.73 ev. The fact that the observed current voltage characteristic follows the "master curve" so well, as is illustrated by Fig. 9, indicates that the inhomogeneity of the collector is relatively small but does seem to be present, and furthermore, such inhomogeneity as may exist at the emitter probably represents a small part of the area. It will become evident as the calculation proceeds that it will be desriable to invoke the concept that some inhomogeneity does exist and that potential functions not unlike those shown in Fig. 7 and 8 will be involved as a step in the analysis to explain the increased power shown by the curve in Fig. 10 in the presence of cesium. The numerical data mentioned are summarized in Fig. 12.

The basic data upon which the remaining part of this analysis depends are illustrated in Fig. 13. The two curves on this figure show the current-voltage relations as observed with and without the cesium. The liquid cesium temperature was 316°K and this according to Eq. 9 gives an arrival rate at the emitter of $1.4 \times 10^{15}$ atoms per second for each square centimeter. The estimated collector temperature was 780°K. A sufficient coating of cesium was adsorbed on the collector to <u>reduce</u> its work-function by 0.45 volts as indicated by the horizontal shift in the retarding potential range when one compares the two curves. This yields a collector work-function of 1.9 volts. If the entire curve had shifted horizontally only, then the change in power output would have been due only to the lowering of the work-function of the receiver. The new curve is not one obtained by a parallel shift.

The cesium data when plotted as in Fig. 14 can again be reasonably well represented by the "master" space-charge curve for electrons. The main difference on analysis, however, is that the current under the critical condition of zero space-charge at the collector is one which would have been associated with a shorter distance than the actual one. The chart on Fig. 11 permits a quick calculation of this distance to be 0.23 mm. It is to be noted by reference to Fig. 12 that the new condition, in the presence of some space-charge neutralization from cesium atoms, is practically as though a diode had been created with the collector located precisely at the potential minimum illustrated in Fig. 12 for the condition of maximum power. This result suggests the drawing of the potential distribution as shown in Fig. 15. The number of electrons that cross the boundary at B per second for a unit area is $5.6 \times 10^{15}$. This current would correspond to a density of electrons of $4.6 \times 10^{8}$ electrons per cubic centimeter. This calculation is based on the estimated average velocity of a Maxwellian group of electrons characterized by the temperature $1520^{\circ}K$ to be $1.2 \times 10^{7}$ centimeters per second.

With no more data than we have at present, it is altogether reasonable to assume that with an arrival rate of neutral atoms at the heated surface, in the neighborhood of high work-function territory, of $1.4 \times 10^{15}$ atoms per second for each square centimeter the density of ions right at the surface will be $1.25 \times 10^{11}$ ions per cc. This calculation depends on the assumption that very close to the high work-function area, the "random current" of ions given by the equation

$$I_p = n_{po} \left(\frac{kT}{2\pi M}\right)^{1/2} \text{(ions/sec)/cm}^2 \qquad (27)$$

Under this condition ion production is exactly equal to ion annihilation and the production rate depends on the atom arrival rate. In Eq. 20, $I_p$ is the random ion current expressed in ions arriving at the surface per second for each square centimeter. The density $n_{po}$ is taken to be the ion density in the immediate neighborhood of the surface. The ion mass is M.

The experimental data permits the calculation of the energy difference between the Fermi level of the emitter and the limiting barrier B to be 3.47 ev as shown in Fig. 15. If the density of ions there is taken to be equal to the density of electrons, the energy difference between B and the average surface at which ions are produced can be approximately 0.61 ev. Under this condition, the effective surface potential will be close to 4.1 ev. Figure 15 has been prepared on the assumption that a surface for ionization does, in fact, exist at 4.1 ev away from the Fermi level of the emitter. On the basis of present knowledge with regard to these surfaces, any ionization surface with a work-function greater than 3.5 ev could deliver ions to the region at B with approximately the same efficiency. When the work-function is less than 3.9, then many of the neutral atoms that arrive at the surface are likely to leave as neutral atoms and therefore the ion production will not be as great as it would be if the work-function were higher. Thus, as the work-function increases, the effectiveness of the surface as an ion producer increases but the fraction of those ions produced that can find their way over the ion barrier identified here as $\phi''$ is reduced by an exponential function of the type $\exp - (\bar{\phi}_b - \phi''_+)/V_T$.

Since the data shown in Fig. 14 fit the idealized master curve within experimental error, the numbers derived from it may be used to compute the maximum power to be expected from the device. The value of $(V_R/V_T)$ for this curve is 11.8. The corresponding value of $V_R$ is 1.53 ev. Equations 16 and 17 may be used to compute the maximum power and the voltage output at maximum power. The results thus obtained are $P_{max} = 6.15 \times 10^{-3}$ watts per $cm^2$ and the voltage output is 0.73 v. These calculated results are represented by the cross with the circle in Fig. 10 and agree well with experiment. The electron current density at maximum power is $8.4 \times 10^{-3}$ $amp/cm^2$.

Determination of the Optimum Cesium Temperature

It is impossible to predict with confidence the optimum cesium temperature because of the lack of needed experimentally determined data applicable to this problem. This section will, therefore, have to depend on the making of a few simple assumptions and if these hypotheses are proven wrong, then the predictions based on them will certainly be in error.

The first assumption is that the zero field condition illustrated by Fig. 15 is determined by the equality of the positive ion and the electron densities. For equality, the arrival rate of ions and electrons must be related by:

$$\left(\frac{\nu_-}{\nu_+}\right)_{\text{at B}} = \left(\frac{M}{m}\right)^{1/2} \qquad (28)$$

In this equation M is the mass of an ion and m is the mass of an electron. The relations represented here imply that both ions and electrons leave the region by going to the collector and since the velocities are inversely proportional to the square root of the mass ratio the ion current will be correspondingly lower for the zero space-charge condition. In order to use this assumption in a numerical calculation, the "effective" average work-function of the ionization region must be known. The experiment described in the previous section gave a value as $\bar{\phi}_b = 4.1$. The equation which will relate the important quantities is the following

$$\phi_R = \frac{\bar{\phi}_b}{2} - 10.68\, V_T + 2.3\, V_T \log_{10} T + \frac{4490}{T_{Cs}}\, V_T \qquad (29)$$

The numerical validity of this equation may be tested by inserting into it the values associated with the experiment described in the previous section. When this is done the calculated value of $\phi_R$ is 3.47 ev on the assumption that $\bar{\phi}_b$ is 4.1.

In order to determine the optimum value of $T_{Cs}$, use is first made of the relation shown as Eq. 9. The implication here is that unless some other factor interferes, the best choice of the cesium temperature will be the highest that will result in effective ionization on a tungsten surface. Certain interfering factors will be discussed later in this section.

According to Eq. 9, the maximum cesium temperature is 425°K for an emitter temperature of 1520°K. These two figures are supplemented by the assumption that the previously determined value of $\bar{\phi}_b$ of 4.1 is suitable. The predicted value of $\phi_R$ at this higher condensation temperature of cesium is then 3.0 ev. This new value represents a reduction in $\phi_R$ of 0.47 and a corresponding increase in electron current delivered to the collector by a factor of 36. The current density expected is $3.2 \times 10^{-2}$ amp/cm$^2$. The chart on Fig. 11 may now be used to determine that the effective spacing of the diode in the presence of this increased cesium pressure will be 39 microns.

At the lower cesium pressure the average work-function of the collector was found to be $\phi_2 = 1.94$ ev. There is no way to know for sure than an increase in cesium pressure will decrease this average work-function still more but for the purpose of illustration at least, it will be assumed that the increase in pressure will reduce the work-function to 1.70. The predicted value of $V_R$ is then:

$$V_R = \phi_R - \phi_2 = 3.00 - 1.70 = 1.30 \qquad (30)$$

The corresponding value of $(V_R/V_T)$ is 10.

The next assumption is that having established the equivalent spacing of a vacuum diode which would have the same current density as the cesium diode the current voltage characteristics will follow the vacuum diode curve associated with this reduced spacing. The new spacing for this calculation is the one given as 39 microns. Equations 16 and 17 may be used to determine the predicted maximum power output and corresponding voltage output. The values obtained are $P_{max} = 0.15$ w/cm$^2$ and $V_O$ is 0.65. The current dansity is 0.23 amp/cm$^2$. The corresponding value for $\phi''_-$ is 2.74 ev. It is implied in these calculations that the true work-function of the emitter $\phi_a$ is less than 2.6 ev. The chief advantage in having a smaller value of $\phi_a$ for the electron emitting portion of the emitter is its influence on the value of $\phi''_+$ which is the hump in the diagram of Figs. 15 and 16 that inhibits the loss of ions to the emitter and thus helps maintain the space-charge reduction.

This prediction that an additional 24-fold increase in power is available in the cesium diode over that actually obtained as a result of increasing the cesium pressure is probably optimistic. The calculation has been made in this manner in order to indicate the potential advantages in chosing the most favorable value of the cesium condensation temperature. At this higher temperature Eq. 13 permits the estimation of the cesium ion mean-free path to be 1.8 mm. Since the electron mean-free path is undoubtedly longer than this, the implication is that a diode spacing of 1 mm would be just as effective in the presence of cesium as a diode spacing of 100 microns. Associated with the current density of 0.23 amp/cm$^2$, the chart on Fig. 11 permits the location of the space-charge minimum to be calculated as 14.4 microns.

In the calculations of this paper, electron energy steps are expressed in terms of their true values at the temperature involved. Therefore the fundamental equation obtained directly from the statistical analysis expresses the current density in the following form:

$$I = 120\, T^2 e^{-\frac{\phi}{V_T}} \text{ amp/cm}^2 \qquad (31)$$

This is the Richardson type of equation and carries with it the number 120 instead of some empirically-determined constant that could be designated as $A_R$ to indicate that it is the Richardson constant found empirically to make the data fit for some set of observations for which the Richardson work-function would be appropriate. Since "true" values of $\phi$ are used in the analysis given here, the coefficient 120 is the only correct one to use.

For calculations in which it is the purpose to determine the temperature at which a certain emission density will be obtained, Eq. 31 is not the most convenient one to use. The theory behind the transformation to a simpler equation is given in Section 50 of "Thermionic Emission" and two useful equations may be obtained from Eq. 31 to be applied over the emitter temperature ranges associated with each of these equations:

<u>Range</u> $1150^\circ K < T < 2500^\circ K$

$$I = 3 \times 10^{9-(5.04\phi + 1.6)\frac{1000}{T}} \text{ amp/cm}^2 \qquad (32)$$

$$\log_{10} I = 9.48 - (5.04\phi + 1.6)\frac{1000}{T} \qquad (32a)$$

<u>Range</u> $600^\circ K < T < 1150^\circ K$

$$I = 7 \times 10^{8 - (5.04\phi + 0.76)\frac{1000}{T}} \text{ amp/cm}^2 \qquad (33)$$

$$\log_{10} I = 8.85 - (5.04\phi + 0.76)\frac{1000}{T} \qquad (33a)$$

The principal purpose of the above equations is to show the steps toward the writing of a simple equation by which the maximum temperature of the collector can be evaluated readily in terms of the available output volts $V_O$ and the energy step $\phi''_-$ illustrated in Fig. 16 and the temperature of the electron emitter of the diode $T_1$. The way this equation is written incorporates also the limitation that the electron current streaming from the collector toward the emitter shall be no more than 5 per cent of the electron current that flows in the opposite direction. This equation subject to these conditions and definitions permits the calculation of the maximum receiver temperature.

$$_{\max}T_2 = 1000 \; \frac{5.04\,(\phi''_- - V_O) + 0.76}{0.65 + (5.04\,\phi' + 1.6)\,\frac{1000}{T_1}} \qquad (34)$$

In the analysis just completed it was assumed that $\phi''_-$ is 2.74 ev, $V_O$ is 0.65, and $T_1$ is 1520°K. With these values substituted into Eq. 34, the maximum temperature of the collector is found to be 1040°K.

## Operation Considerations Related to Diodes Having High Work-Function Emitters

If the source of heat is one which lends itself to the use of a very high work-function emitter, then other considerations may be important. Under ordinary circumstances, it should be anticipated that the lower temperature at which the emitter can work, the more likely one is to have an efficient device. However, efficiency may not be the most important criterion. In fact, it could be that power per unit volume would be far more important than conversion efficiency. Under these circumstances, thought should be given to the operation of high temperature emitters.

Assume for this discussion that the emitter temperature is 2400°K and that it is realistic to make a diode of 100 micron spacing. Calculation shows that the space-charge minimum will coincide with the collector when the current density is 0.009 amp/cm$^2$. An emitter with an average work-function of 4.4 ev would have an emission current available of 0.37 amp/cm$^2$. A high vacuum transducer would require a low work-function collector which could under favorable conditions have an average work-function of 2 ev and a $V_R = 3.17$. Under these conditions, the critical value of $(V_R/V_T)$ is 15.3. This diode would have the maximum power available with a current received at the collector of 0.115 amp/cm$^2$. The power available to the external circuit would be 0.168 watts/cm$^2$. The question that may now be asked is: what improvement should be expected in this diode by the introduction of cesium, and what would be the most suitable cesium pressure as determined by the temperature of the liquid cesium?

The first effect of cesium if the collector temperature could be maintained at a sufficiently low value would be to deduce its work-function. Assume that the absorbed cesium would reduce the work-function to 1.4 ev. Such a change would have a very favorable influence on the power output. Secondly, all of the cesium atoms which would in their normal motion come in contact with the emitter would leave as ions. Very few of these ions would be lost to the collector since its surface would be 0.9 volts more positive than the emitter surface if the external surface conditions were maintained to hold the current at 0.115 amp /cm$^2$ in the absence of space-charge neutralization. Actually, as the cesium pressure increases, the ions will tend to neutralize space charge and the electron current achieve its full capability of 0.37 amp/cm$^2$. This current should remain constant as the output voltage is increased to 3volts. The combination of these two effects would increase the available power a factor of 6.6, since the maximum power we would expect is:

$$0.37\ (4.4 - 1.4) = 1.1\ \text{watt/cm}^2 \tag{35}$$

Achievement of this power would depend on there being a negligible number of ions that are generated at the emitter and recombine at the collector. We may assume that the average energy associated with the moving electrons and also the moving ions will be 2 $V_T$ (that is, 2 $kT/q$) and that if their energies are equal, their average velocities will be inversely propoertional to the square root of their masses. This factor for cesium is close to 490. This figure will then be used to establish the order of magnitude of the desired cesium arrival rate if on the average every ion which leaves the cathode travels more or less unimpeded to the collector. The electron current of 0.37 amp.cm$^2$ corresponds to an emission rate of 2.3 x 10$^{18}$ electrons per second for each square centimeter. If we assume that just the right number of ions are present to neutralize the space charge of the electrons, there will be no electric field and the average velocity of electrons in transit across the space may be oomputed to be 1.52 x 10$^7$ cm/sec. Since the electrons will travel unimpeded across this space, the density will be 1.51 x $10^{11}$ electrons per $cm^3$. An exactly equal ion density corresponds to an ion emission rate of 4.7 x $10^{15}$ ions per second for each square centimeter. Under these conditions the ion emission rate will be exactly equal to the atom arrival rate and the next question to be answered is the determination of the most suitable cesium condensation temperature.

Were it not for the fact that consideration must be given to the reduced density of atoms between the emitter and the collector, a condensation temperature of 330°K would give the required ion emission. The average temperature in the interelectrode space may be taken as about 1600°K. With this approximate value of the condensation temperature, Eq. 17 shows that the atom evaporation rate at the cesium surface should be increased 2.2 over that calculated by Eq. 8. With this correction factor the theoretical value for the cesium condensation temperature becomes 340°K. At this temperature the mean-free path will be long compared with the spacing and therefore both the electrons and the ions should flow in an unimpeded manner from the emitter to the collector.

If the cesium condensation temperature is raised above the value 340°K then the arrival rate will be greater than that needed and the excess ion production will result in an ion space-charge sheath forming near the surface of the emitter. Such a sheath will inhibit the delivery of the ions to the plasma. It will also create an accelerating field for the electrons. This will increase the electron emission if the output voltage is reduced slightly to take advantage of the lowering of the electron work-function because of the accelerating field produced at the emitter by the ions. The fact that many of the ions that are produced are returned to the emitter surface may also result in the establishing there of an average dipole moment favorable to a still further reduction of the work-function in spite of the high temperature. These two effects increase the emission capability of the cathode over the previously calculated value of 0.37 amp/$cm^2$. Without experimental experience it is difficult to determine the best cesium condensation temperature except to state that for temperatures less than 340°, an insufficient number of ions will be available to neutralize the space-charge and as temperatures exceed this value, favorable results may come because of the reduction in the emitter work-function and the corresponding exponential increase in available current.

The cesium pressure called for by this discussion is much lower than that used by some experimenters. For example, V. C. Wilson(4) experimented with high temperature emitters at relatively high cesium pressure. The two cesium temperatures he used were 534°K and 564°K. The use of the higher cesium pressure would result in the creation of an ion space charge sheath. The fact that an electron emission of 4 amp/cm$^2$ was observed sets the "effective" electron barrier at B'' of Fig. 8 at 3 ev. Equation 29 shows that any area of the hot surface with a work-function in the range 3 ev to 4.3 ev will deliver ions to help neutralize the electron space charge. The Wilson experiment would seem to indicate that work-function lowering actually takes place.

These thoughts are presented by way of illustration of the use of the equations presented here and the general ideas related to the behavior of a plasma diode as it might be used in conjunction with a high temperature, high work-function emitter. It is clear that many experiments should be performed before too much reliance is placed in these deductions since they depend on an extrapolation based on data not specifically applicable to present needs. Additional experimentally determined facts might come into prominence beyond those considered.

## Concluding Remarks

Three principle objectives have guided the preparation of this report. The first was to discuss in some detail the physical principles that seem to be involved in the better understanding of the design and properties of a plasma diode heat-to-electrical-power transducer. The second objective was to make available an intricate set of empirical equations by which many answers to pertinent questions can be obtained numerically when desired. These equations are consistent with the experimental facts as they are known today. They depend very largely upon the studies of Taylor and Langmuir and some very recent measurements made by Robinson. The final objective has been to apply the ideas here to the experiments of Robinson and indicate by quantitative calculation the results that one might hope to obtain by the choice of a more favorable cesium condensation temperature than was actually used.

In the present report the detailed derivations involved in the development of the empirical equations have not been given. If the results found here are ultimately of real practical importance, the more interesting derivations can be prepared and made available as a supplement to this report. Twenty of the thirty-six equations used in this report may find direct application to other problems and are therefore summarized for quick reference in an appendix.

## Appendix

### Selected Equations From the Text

Minimum temperature for surface ionization as a function of the atom evaporation rate at the cesium condensation surface:

$$T_{min} = \frac{14,100}{27.56 - \log_{10} \mu_a} \qquad (7)$$

Equation for evaporation rate of cesium:

$$\log_{10} \mu_a = 27.48 - \frac{3900}{T_{Cs}} \qquad (8)$$

$$\mu_a = 3 \times 10^{27} \times 10^{-\frac{3900}{T_{Cs}}} \text{ atoms/cm}^2 \qquad (9)$$

$$\mu_a = 3 \times 10^{27} e^{-\frac{8980}{T_{Cs}}} \text{ atoms/cm}^2 \qquad (10)$$

Minimum temperature for surface ionization related to cesium temperature

$$T_m = 3.6\, T_{Cs} \qquad (11)$$

Ion evaporation rate for tungsten surface temperatures less than the minimum temperature calculated by Eq. 11

$$\log_{10} \nu_p = 54.16 - 1.113 \log_{10} \mu_a - \frac{26,600}{T} \qquad (12)$$

$$\nu_p = \frac{1.45 \times 10^{54}}{\mu_a^{1.113}} \cdot 10^{-\frac{26,600}{T}} \qquad (13)$$

$$\nu_p = \frac{1.45 \times 10^{54}}{\mu_a^{1.113}} e^{-\frac{5.28}{V_T}} \qquad (14)$$

The vapor concentration of cesium atoms in equilibrium with liquid cesium

$$n_{Cs} = 2.7 \times 10^{23} e^{-\frac{8750}{T_{Cs}}} \text{ atoms/cm}^3 \qquad (15)$$

$$\log_{10} n_{Cs} = 23.44 - \frac{3800}{T_{Cs}} \qquad (16)$$

Mean-free path of cesium ions in cesium vapor

$$\mathrm{Log}_{10} \lambda_+ = \frac{3800}{T_{Cs}} - 9.7 \qquad (19)$$

$$T_{Cs} = \frac{3800}{9.7 + \log_{10} \lambda_+} \qquad (20)$$

Maximum power in a vacuum diode used for a plasma diode after the "effective" spacing is determined.

$$P_{max} = 3.7 \times 10^{-6} V_T^{1/2} \frac{V_R^2}{w^2} \text{ watt/cm}^2 \qquad (23)$$

Voltage output

$$V_O = \frac{0.383 \left(\frac{V_R}{V_T}\right) V_R}{1 + 0.31 \left(\frac{V_R}{V_T}\right)^{4/3}} \qquad (24)$$

Energy step $\phi_R$ related to current density and temperature

$$\phi_R = 2.3\, V_T \left[ 2.08 + 2 \log_{10} T - \log_{10} I_m \right] \qquad (26)$$

Energy step $\phi_R$ related to ion surface work-function $\phi_b$; surface temperature; and cesium temperature

$$\phi_R = \frac{\phi_b}{2} - 10.68\, V_T + 2.3\, V_T \log_{10} T + \frac{4490}{T_{Cs}} V_T \qquad (29)$$

Simplified emission equation for the range $1150^\circ K < T < 2500^\circ K$

$$\log_{10} I = 9.48 - (5.04\, \phi + 1.6) \frac{1000}{T} \qquad (32a)$$

Range $600^\circ K < T < 1150^\circ K$

$$\log_{10} I = 8.85 - (5.04\, \phi + 0.76) \frac{1000}{T} \qquad (33a)$$

Maximum temperature of the collector for 5 per cent return emission

$$_{\max}T_2 = 1000 \, \frac{5.04\,(\phi''_{-} - V_O) + 0.76}{0.65 + (5.04\,\phi' + 1.6)\,\frac{1000}{T_1}} \qquad (34)$$

## References

1. W. B. Nottingham, J. Appl. Phys. 30, 413 (1959); W. B. Nottingham, G. N. Hatsopoulos, and J. Kaye, J. Appl. Phys. 30, 440 (1959); W. B. Nottingham, Report on the 19th Annual Conference on Physical Electronics, M.I.T. 1959.

2. W. B. Nottingham, "Thermionic Emission, "Handbuch der Physik, Springer-Verlag, Germany, Vol. 21, 1956.

3. J. B. Taylor and I. Langmuir, Phys. Rev. 44, 423 (1933).

4. V. C. Wilson, J. Appl. Phys. 30, 475 (1959).

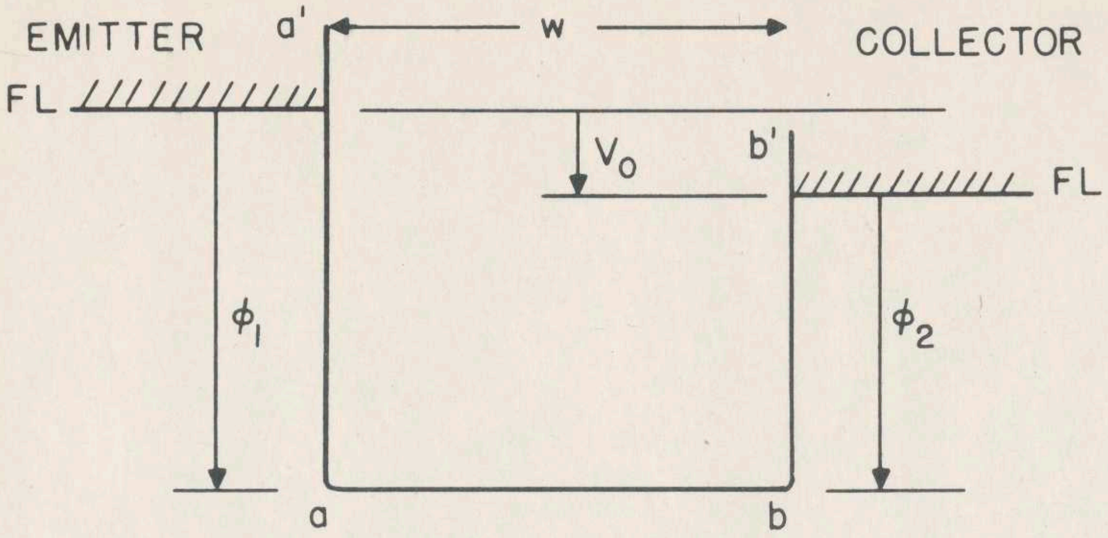


FIG. 1

MOTIVE OF AN ELECTRON WITH ZERO FIELD BETWEEN a and b

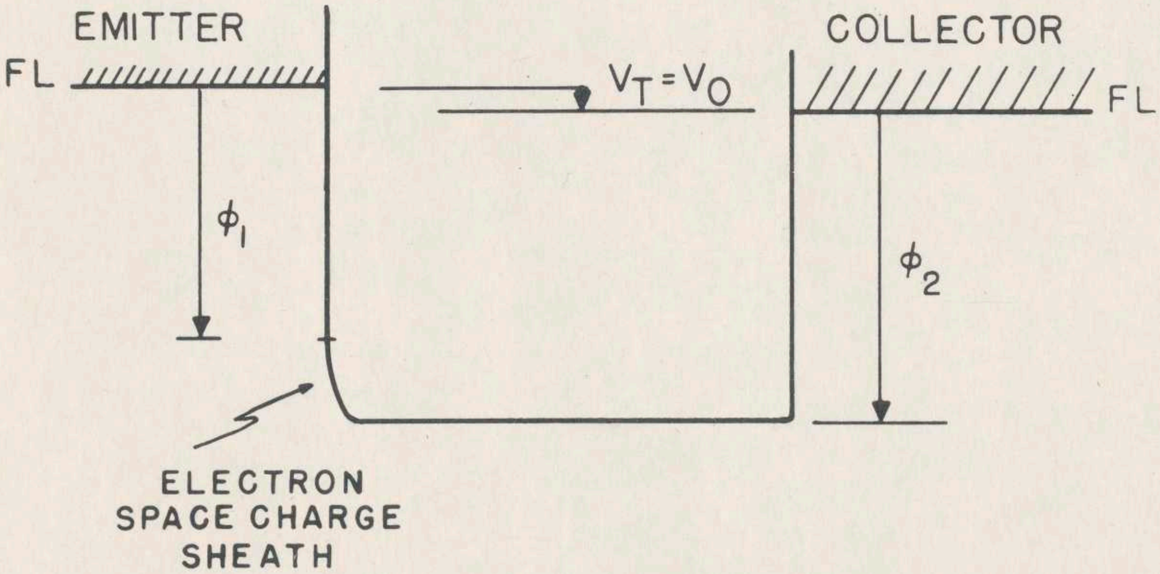


FIG. 2

PLASMA DIODE WITH HIGH COLLECTOR WORK-FUNCTION

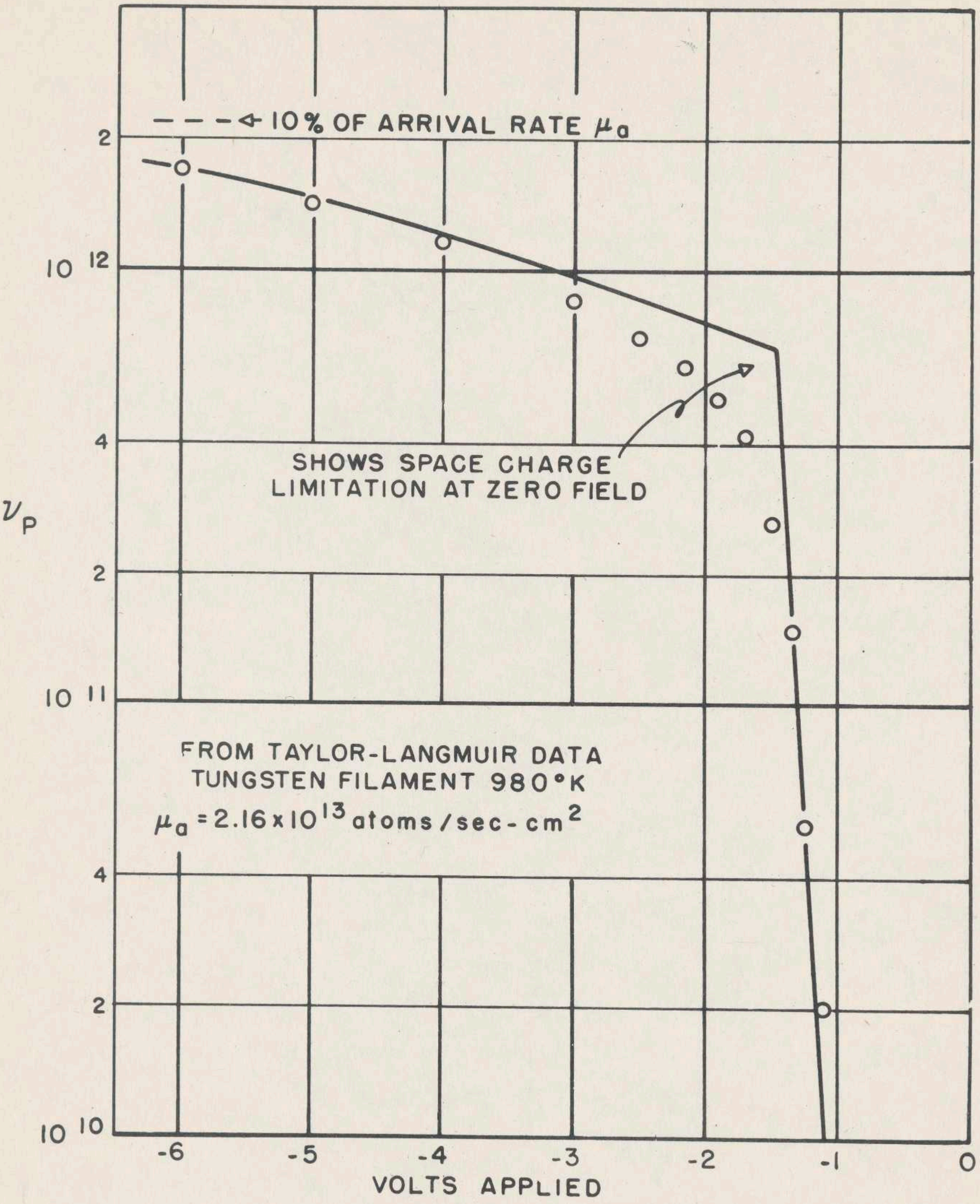


Fig. 3

Ionization rate as a function of applied volts observed by Taylor and Langmuir.

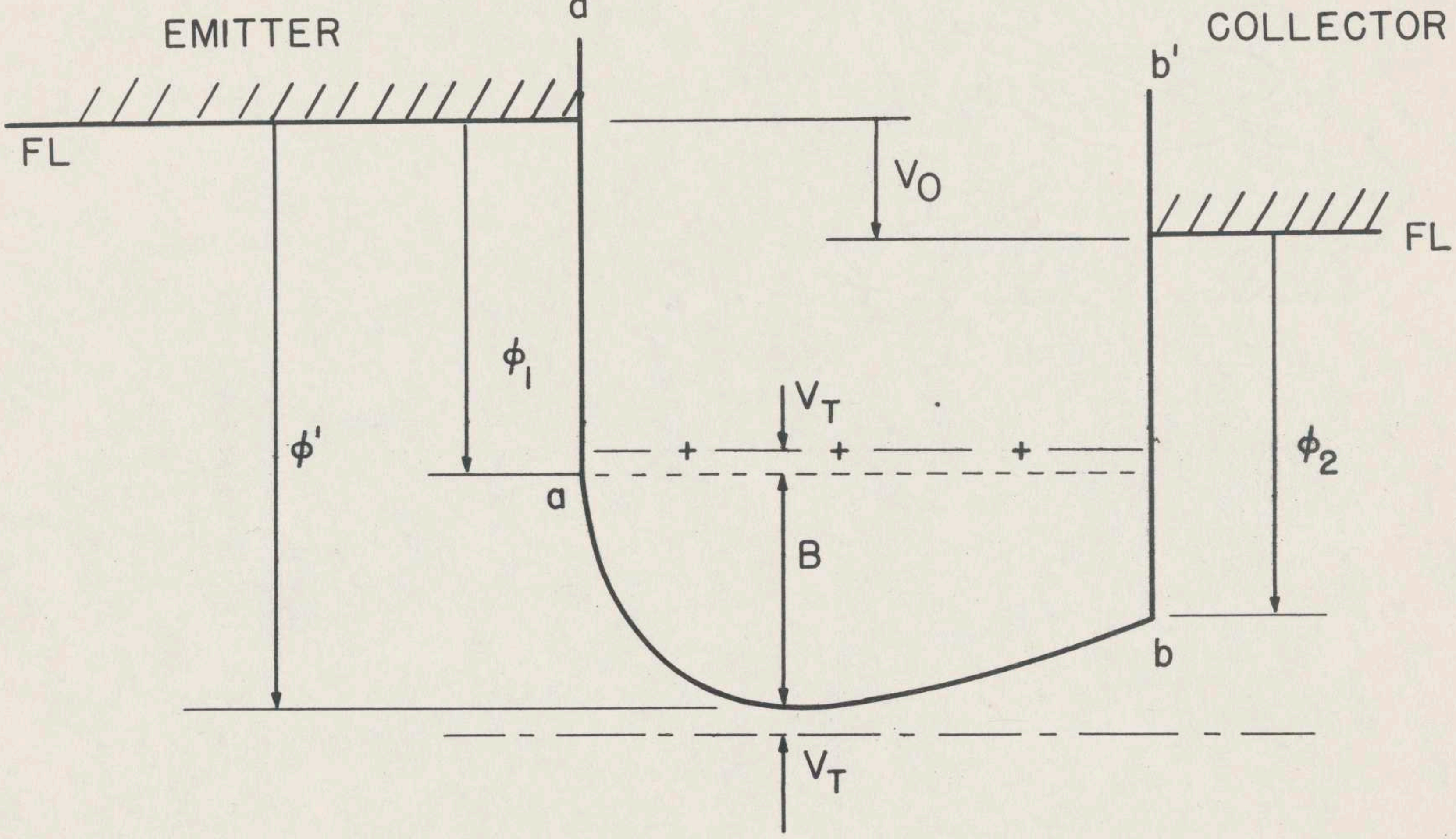


Fig. 4
Motive curve with electron space charge.

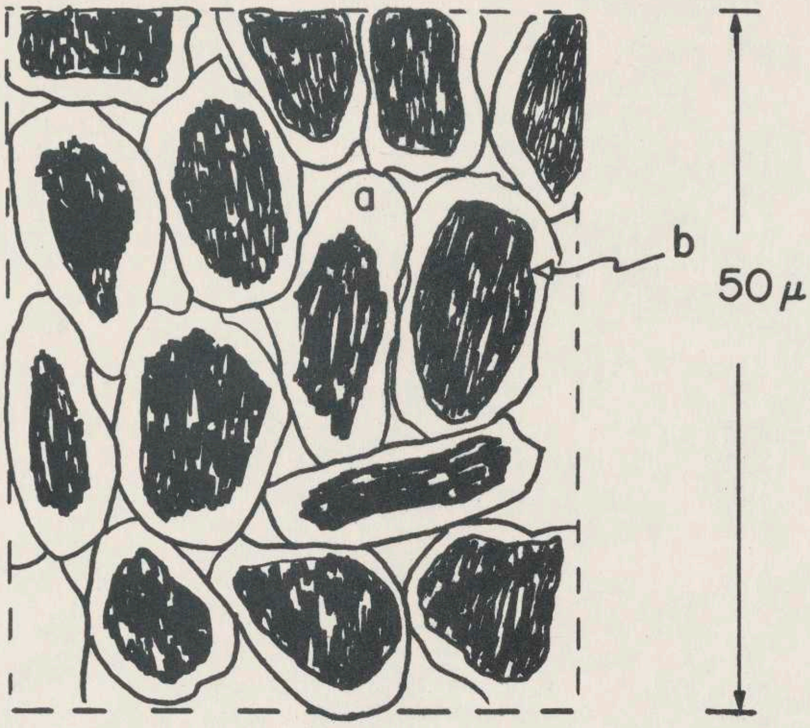


Fig. 5

Hypothetical electron emission distribution over the surface of a very hot dispenser cathode.

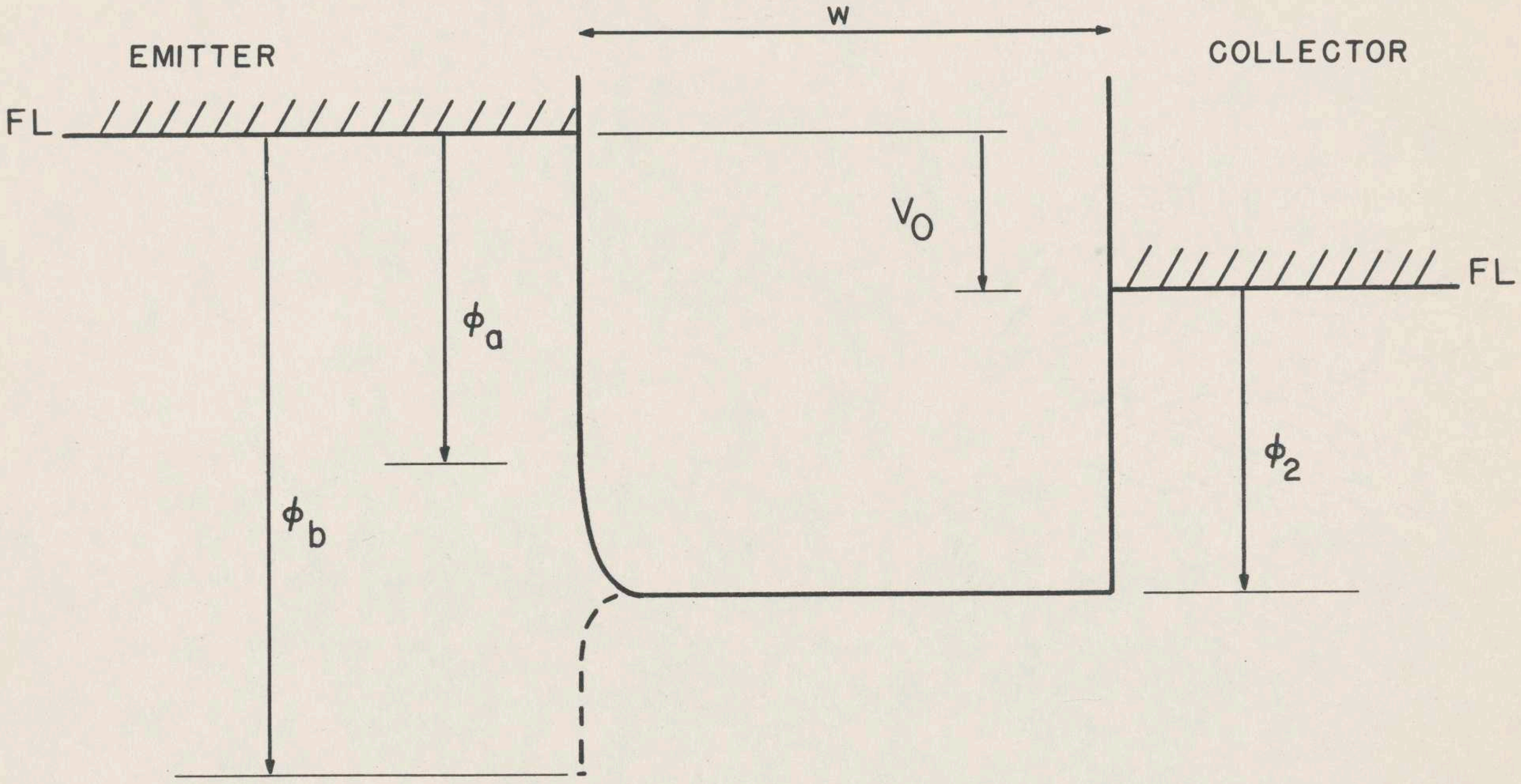


FIG. 6

MOTIVE CURVES FOR NON-UNIFORM EMITTER — NO SPACE CHARGE — ZERO FIELD

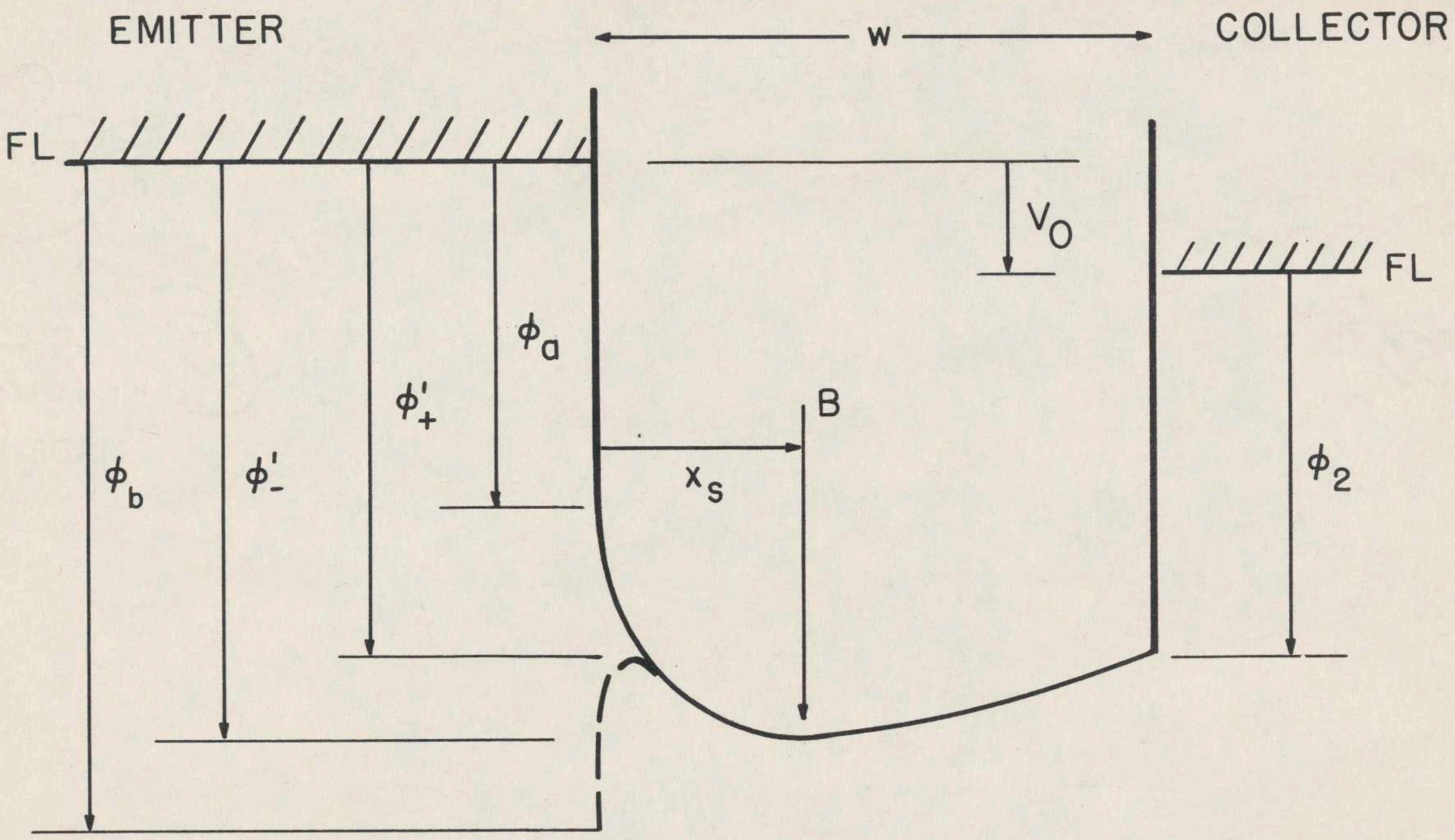


FIG. 7

MOTIVE CURVE FOR A NON-UNIFORM EMITTER
WITH ELECTRON SPACE CHARGE

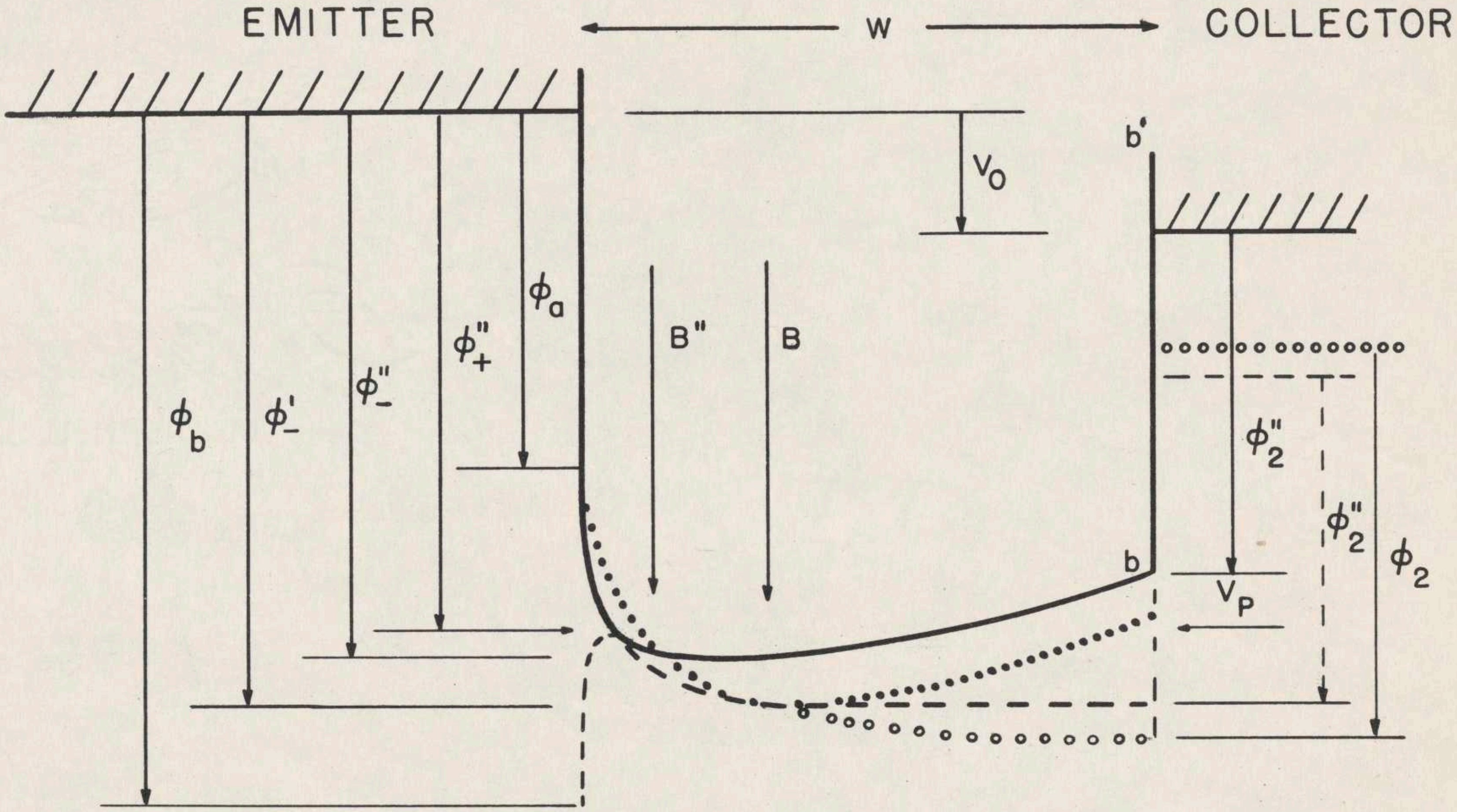


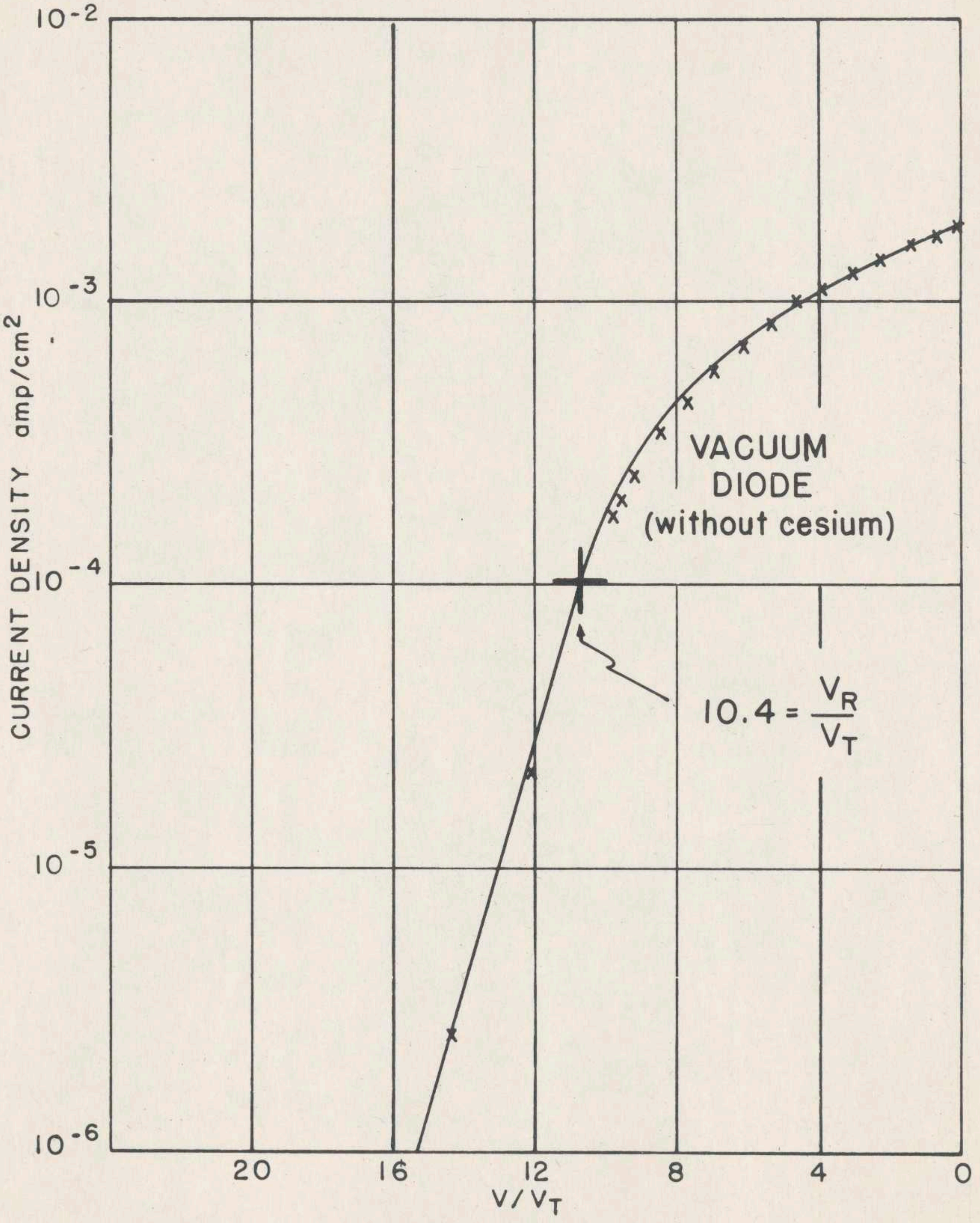
Fig. 8

Motive functions plasma diode compared with vacuum diode.

Zero-field – – – (plasma) ○ ○ ○ ○ ○ (vacuum)
Max power —— (plasma) ● ● ● ● ● (vacuum)

Fig. 9

Experimental results and comparison with space-charge theory for vacuum diode. Data from Robinson of TEE.



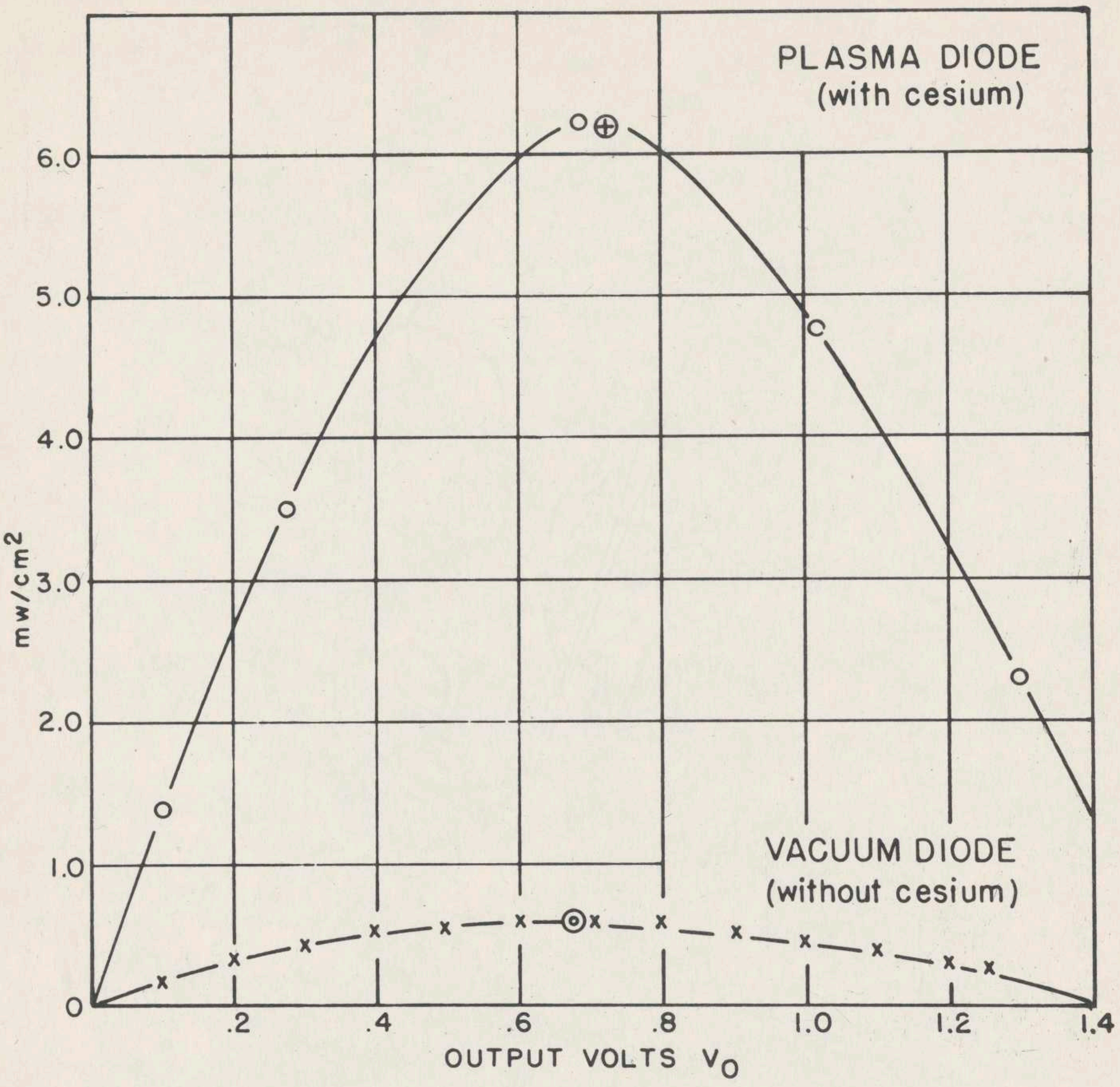
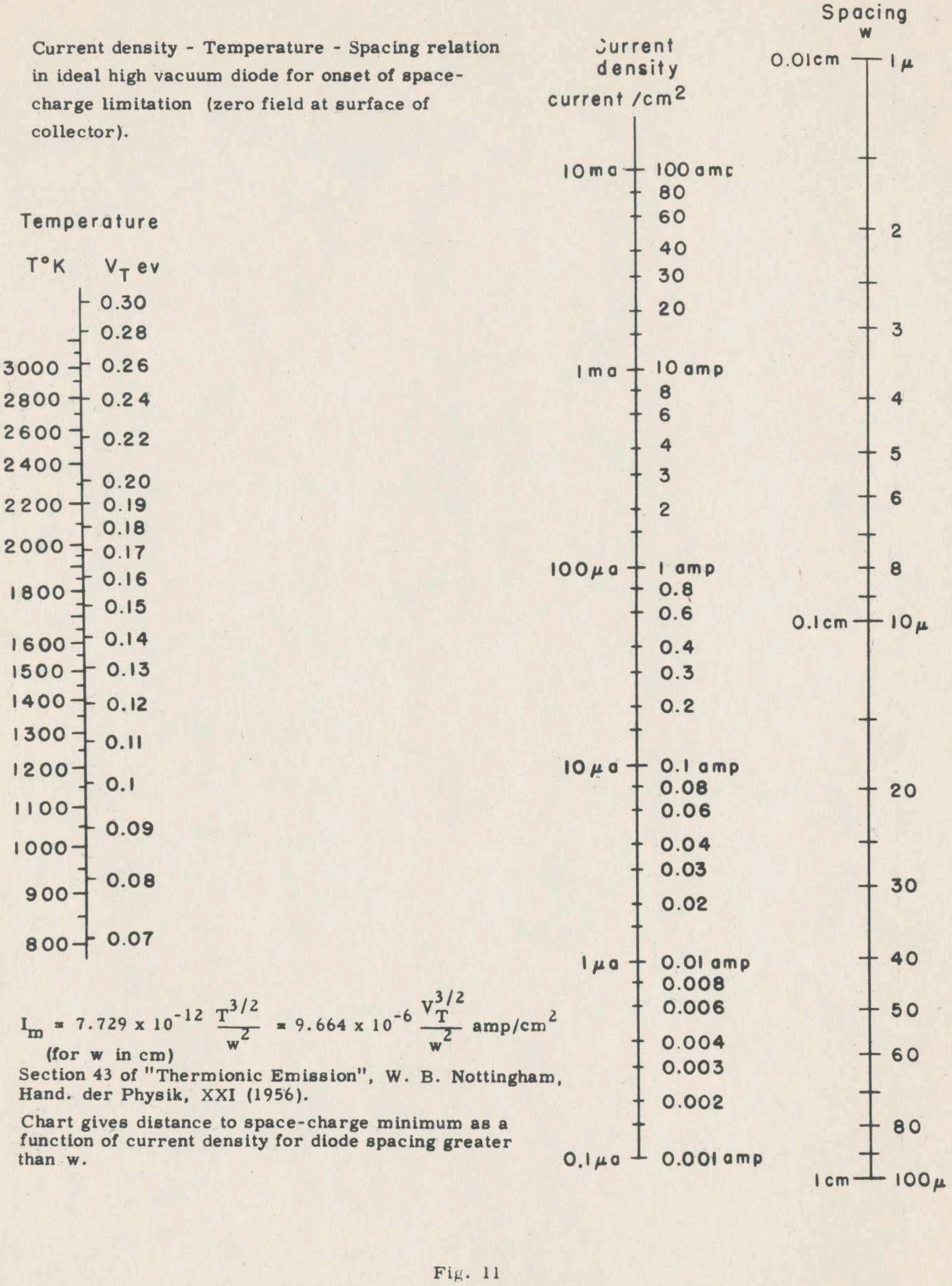


Fig. 10

Observed and calculated power density; x vacuum diode,
○ plasma diode; ◎ , ⊕ theory. Data from Robinson of TEE.

Current density - Temperature - Spacing relation in ideal high vacuum diode for onset of space-charge limitation (zero field at surface of collector).



$$I_m = 7.729 \times 10^{-12} \frac{T^{3/2}}{w^2} = 9.664 \times 10^{-6} \frac{V_T^{3/2}}{w^2} \text{ amp/cm}^2$$

(for w in cm)

Section 43 of "Thermionic Emission", W. B. Nottingham, Hand. der Physik, XXI (1956).

Chart gives distance to space-charge minimum as a function of current density for diode spacing greater than w.

Fig. 11

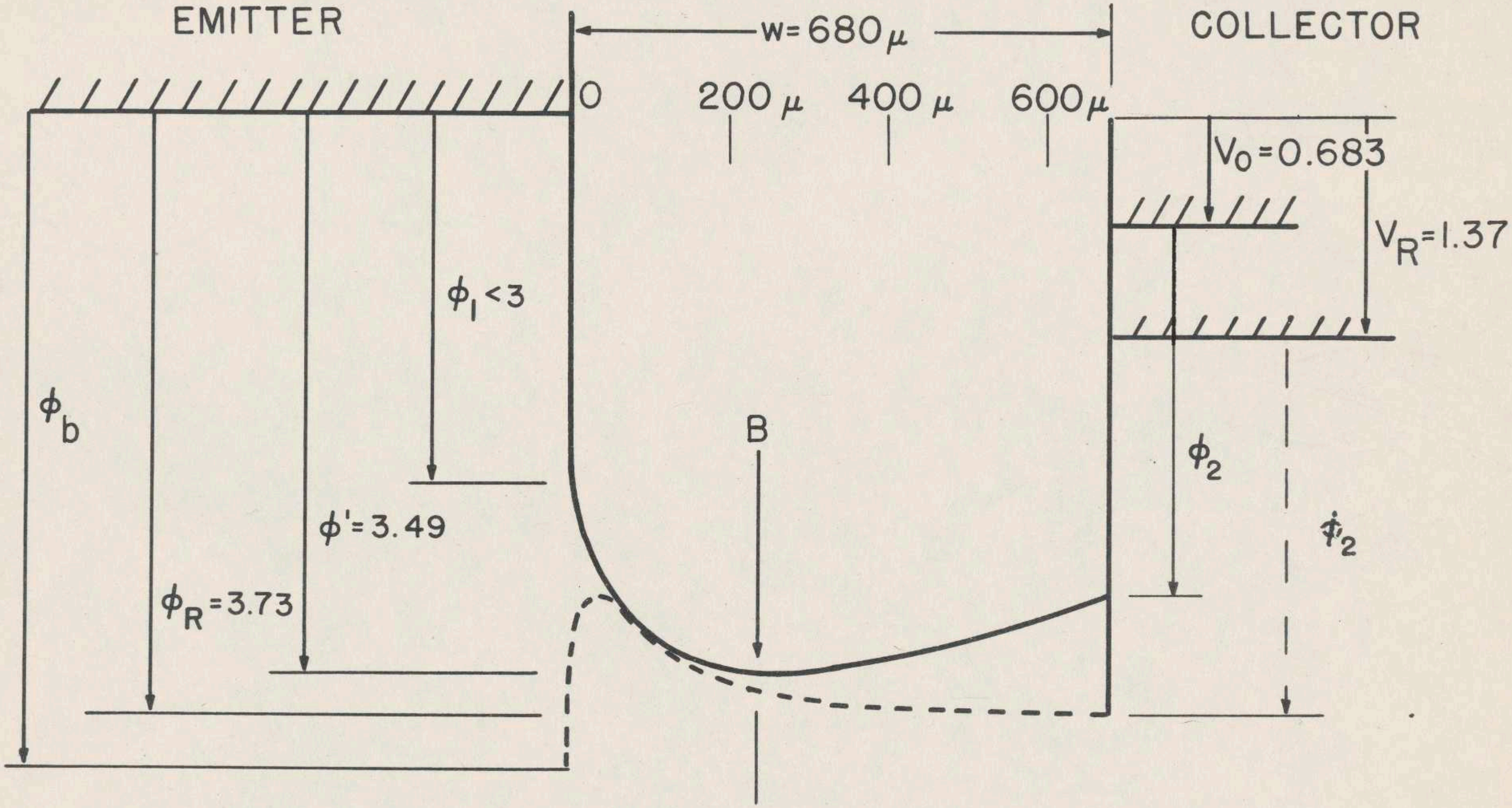


Fig. 12

Motive curve in experimental high-vacuum diode.

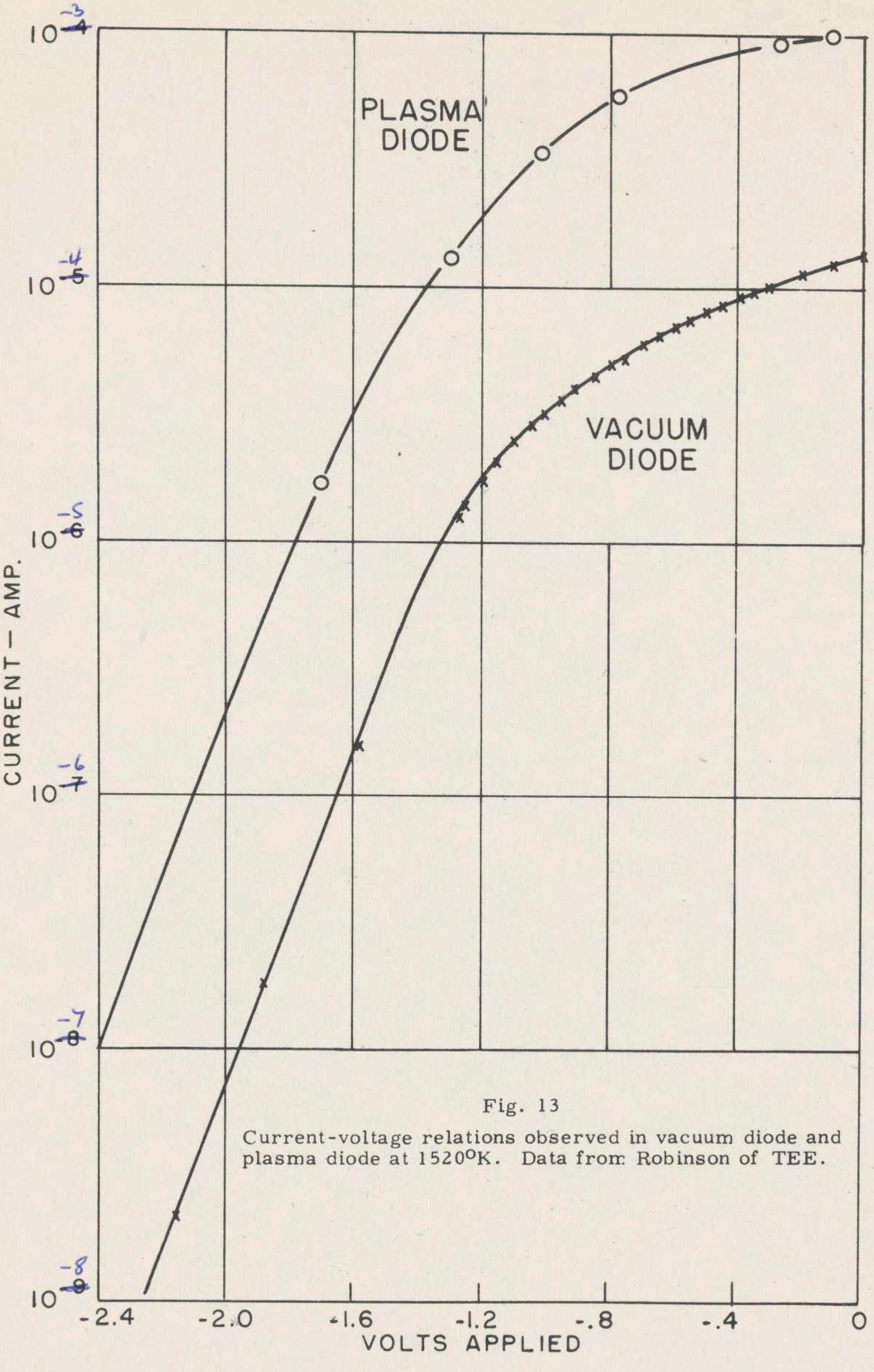
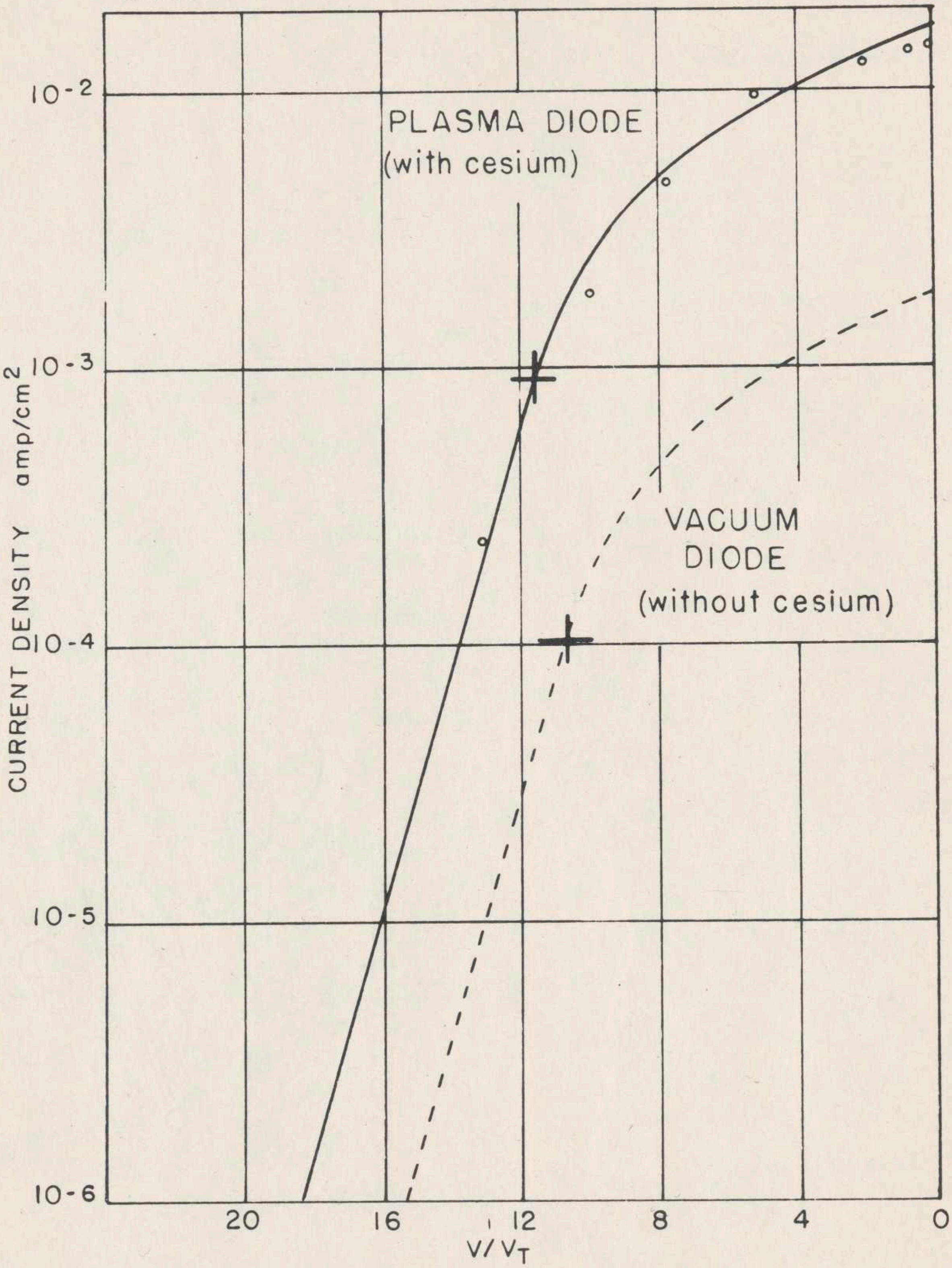


Fig. 13

Current-voltage relations observed in vacuum diode and plasma diode at 1520°K. Data from Robinson of TEE.

Fig. 14

Experimental results and comparison with space-charge theory for plasma diode. (Vacuum diode for comparison). Data from Robinson of TEE.



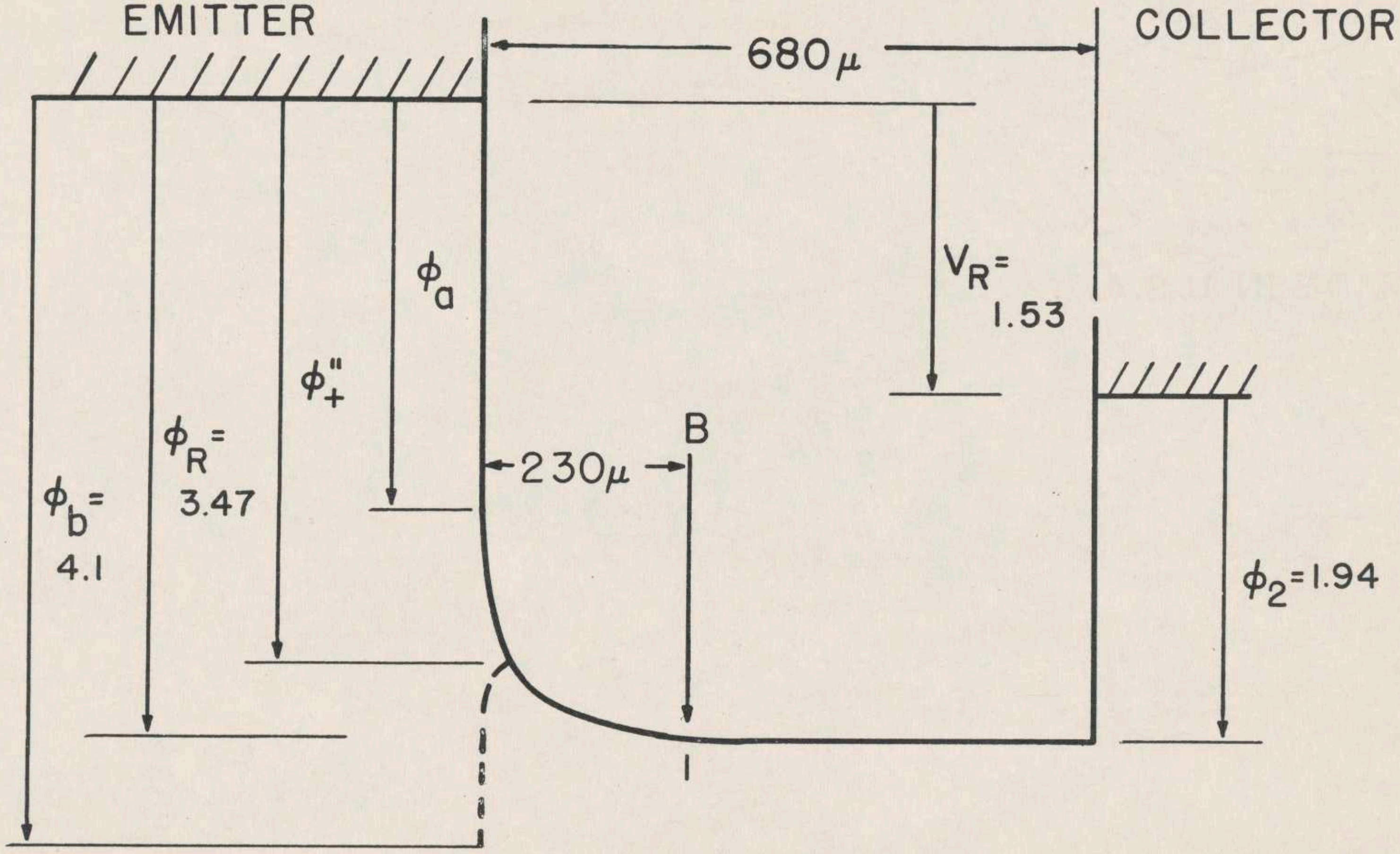


Fig. 15

Experimental plasma diode with zero field at the collector.