MC 241
Box 5 Folder 3

Screen Bombardment with Pulses While a Beam is Sweeping, 1942-43

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(1)

Some points concerning screen bombardment with pulses while beam is sweeping.

δ = diameter of spot is δ expressed in cm.

v_s = sweep velocity in cm/μsec

L = length of pulse in μs.

$T_R = 10.7\,R \times \cancel{10}$ time required for transmission and return of echo (1)

R = range to target in miles.

R_m = max range

T_m = max time for transmission and return

$v_s T_m = r_m$ = max radius of sweep on PPI in cm (2)

$v_s T_R = r$ = radial location for intermediate distance of R. (3)

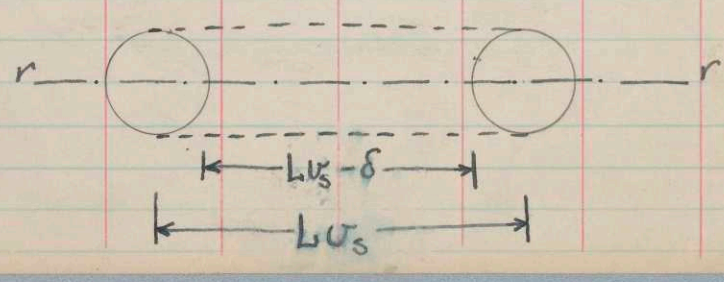
$$r = \left(\frac{r_m}{R_m}\right) R \qquad (3a)$$

In the time L the spot move a distance

$$dr = L v_s \quad \text{cm} \qquad (4)$$

Case I

$$dr > \delta$$



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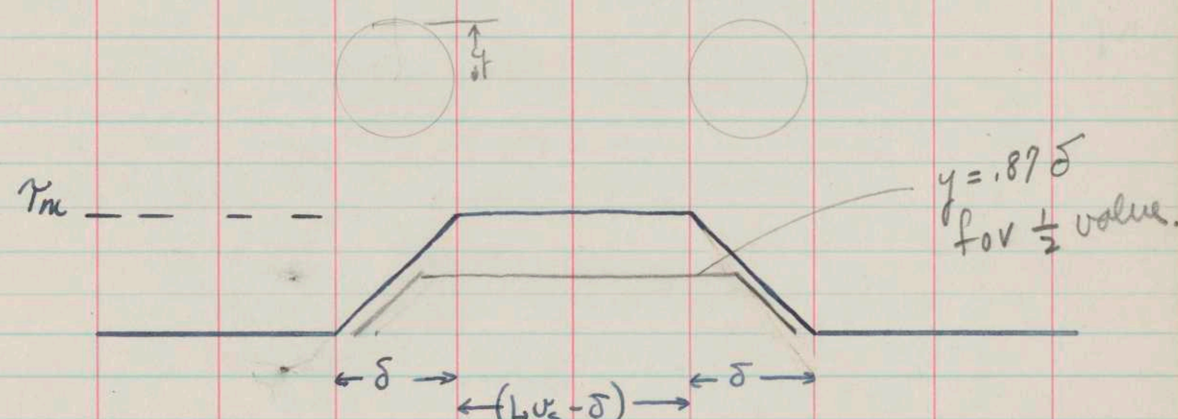
Along radius r-r

distance $(Lv_s - \delta)$ of screen is bombarded max length of time as determined by spot dia. and velocity v_s. This time is

$$\frac{\delta}{v_s} = \tau_m \quad \text{where } \tau_m \text{ is in } \mu s. \qquad (5)$$

The exposure pattern is as follows:

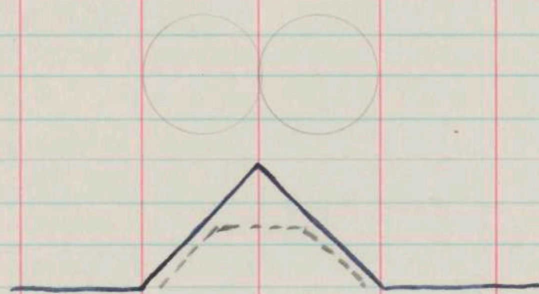


Area of spot which has received ~ 50% of full bombardment or more

$$(Lv_s) \times .87\delta = \text{area.} \qquad (6)$$

Case II $\quad Lv_s = \delta$

The exposure pattern

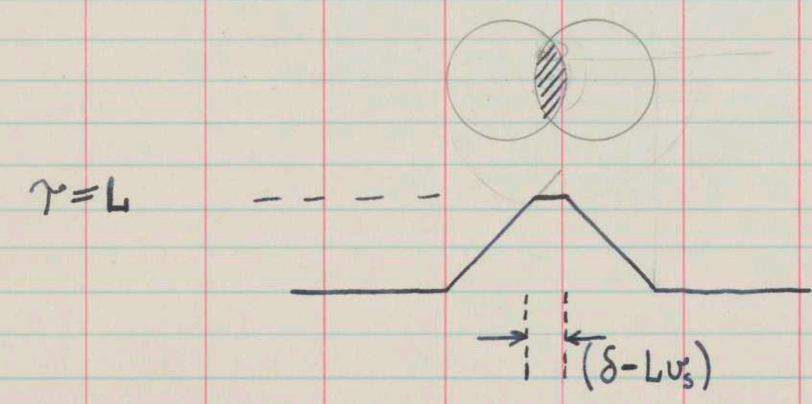


Area for 50% or more exp.

$$.87\delta^2 = \text{area}_{II} \qquad (7)$$

Case III $\quad Lv_s < \delta$

Exposure pattern



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For linear sweep

Since $v_s = \frac{r_m}{10.7 R_m}$

v_s (cm./μs) for various tube radii and ranges

Table I

$r_m \rightarrow$ / R_m (miles) $\downarrow$	10 cm	7.5 cm	5 cm	3.5 cm
10	$.0935 \times 10^{-3}$ (corrected to 93.5)	70	46.7	32.7
20	46.7	35	23.3	16.4
50	18.7	14.0	9.35	6.5
100	9.35	7.0	4.67	3.27×10^{-3}

In order to satisfy Case II for $r_m = 10$ cm and $R_m = 10$ mi with $\delta = .1$ cm

$$L_{II} = \frac{.1}{.0935} = 1.07 \mu s.$$

for $R_m = 100$

$$L_{II} = 10.7 \mu s.$$

$\delta = .2$

$L_{II} = 2.14 \mu s$ $r_m = 10$ $R_m = 10$

$L_{II} = 21.4$ " $r_m = 10$ $R_m = 100$

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This shows that for $L = 1\,\mu s$ the entire range of application to PPI is dominated by <u>Case III</u>

Table II

This is a table for the <u>time</u> required for the beam to move one spot dia.

$\delta = 0.1$ cm.

	Time for one sweep μs.	(9") 10 cm μs	(7") 7.5 cm μs	(5") 5 cm μs	(3") 3.5 cm μs.
10	107	1.07	1.43	2.14	3.06
20	214	2.14	2.86	4.28	6.12
50	535	5.35	7.15	10.70	15.3
100	1070	10.7	14.3	21.4	30.6

T. Soller information

PPI 8 cm 8 cm. 5.5 cm

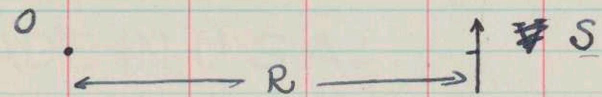
7.8 × 7.8 cm B scan

10 × 4 " C "

At a radius of 10 cm circumference is 63 cm

This is 630 spot dia. per rev. with $\delta = 0.1$ cm.

With 1 R.P.S., time per spot dia. is 1590 μs.



consider target R from observer moving at S mi per hr. at rt angles to line to observer.

$$\frac{d\theta}{dt} = \dot{\theta} = \frac{S}{R} \quad \text{radians per hr.}$$

or

$$\frac{S}{R} \times \frac{1}{3600} \times \frac{1}{2\pi} \times 360 \quad \text{degrees per second}$$

$$= \frac{S}{20\pi R} \quad \text{deg per sec.}$$

$$\frac{\delta}{r} = \text{radians per spot dia}$$

$\frac{S}{3600\,R}$ = radians per sec.

$\frac{S \cdot r}{3600\,R \cdot \delta}$ = spot diameters per second.

$\frac{3600\,R\,\delta \times 10^6}{S\,r}$ = micro sec / spot dia.

Take max value of $S = 500$ ~~±~~ mi/hr.
" " " " $R = 10$ mi.
"

$\frac{3.6 \times 10^9}{.5 \times 10^2} = 7.2 \times 10^7$

$\frac{\delta}{r} \times 7.2 \times 10^7$ = micro sec / spot dia.

take $\begin{cases} \delta = .1 \\ r = 10 \end{cases}$

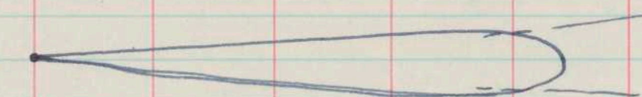
this gives 7.2×10^5 μs per spot dia.

Consider recurrence at 2000 per sec.
then

$2000 \times 7.2 \times 10^{-1} = \underline{1440}$

pulses would come for a change in location of plane by one spot dia.

For search light use or very slow sweep the build up at the ~~recur~~ rec. rate (recurrance rate) is important.



assume the effective "angular opening" of the transmitted and received beam (together) is $\Delta\phi$ radians

Time for lobe to sweep the target will be

$$\frac{\Delta\phi}{\dot{\phi}}$$

Define ~~p_r~~ $p_r \equiv$ recurrance period.

$$N_r = \frac{\Delta\phi}{p_r \dot{\phi}}$$

$\dot{\phi}$ is determined by "frame" recurrence or time of revolution of PPI.

Consider ~~ϕ~~ PPI first.

$$\frac{\dot{\phi}}{2\pi} = R.P.S. = \cancel{F_f}\frac{1}{F_r} = (\text{"frame" recurrance})^{-1}$$

$$\text{or } \dot{\phi} = \frac{2\pi}{F_r}\cancel{F_f}$$

$$N_r = \frac{\Delta\phi F_r}{2\pi \rho_r \cancel{F_f}}$$

This is the max number of echos which can return from a target per sweep across depending on the antenna system.

If

$$\frac{\delta}{r} \doteq \frac{1}{2}\Delta\phi$$

there will be very little loss in resolving power on the screen.

This serves to determine approx the max value of δ as

$$\delta_{max} \doteq \cancel{r\phi\phi}\ \frac{r\Delta\phi}{2}$$

where r = radial location

on a PPI tube (i.e. range) and $\Delta\phi$ is effective lobe width in radians.

$$\delta_{max} \doteq \frac{R}{2}\left(\frac{r_m}{R_m}\right)\Delta\phi$$

If $\Delta\phi$ is expressed in degrees.

$$\delta_{max} = \frac{R}{2}\left(\frac{r_m}{R_m}\right)\frac{\Delta\phi^\circ}{360}2\pi$$

For $\Delta\phi^\circ = 1$

$R = R_m$

$r_m = 10$

$$\delta_{max} = \frac{1\cancel{0}\,\pi}{36\cancel{0}} = .087 \text{ cm.}$$

This indicates that the max dia of the spot at the edge of the tube on a PPI is

(cm) $\delta_{max} = .0087 \times$ (radius of tube) $\times$ (lobe width in deg.)

PP1 — Ā

7" P7. 5" P7

36° per sec. 24 μ

800 — $\left.\begin{matrix}775\\825\end{matrix}\right\}$ pulses per sec.

dish 6 feet in dia.

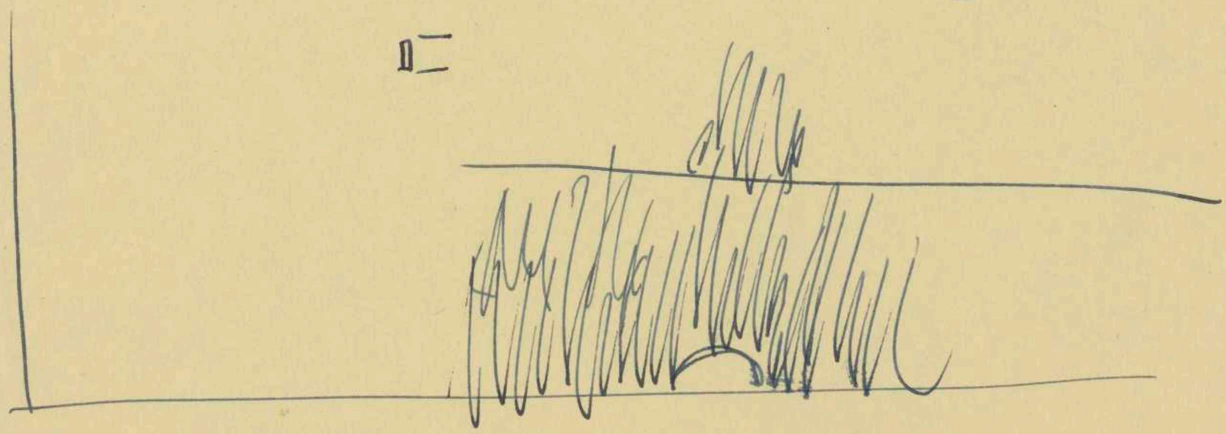
10.7

$\frac{1}{2}$ power ~~point~~ 2.5°
from formula page (11).

$$6^{\circ} \times \frac{\cancel{30}}{\cancel{72}\ 12}\ \frac{10}{10} = \frac{18}{72} \quad \underline{\underline{2.5}}^{\circ}$$

$$\frac{\cancel{18} \times \cancel{10}}{\cancel{72}}$$

1.5 m



According to Van Atta "effective" lobe width in degrees ~~is appox~~ may be calculated using relation.

$$.7(\text{Max voltage round trip}) = 6° \frac{30''}{(\text{Dish inches dia})} \frac{(\lambda)}{10}$$

10 cm
3.2 cm

λ = wave length in cm
D = Dish dia in inches.

$$= 18 \frac{\lambda}{D}$$

Because of response of receiver which is not linear but appox

$$(.7)^{1.5} = .58$$

This about "~~half~~ voltage" at the receiver out put.

For 30" dish and a 9" tube ($\frac{V}{m}=10$)

$$\delta_{max} = 0087 \times 10 \times 6 = .522$$

This means that if the spot were "elongated" at the edge of the tube to .5 cm there would be very little loss in "resolving power."

With a spot dia of 0.1 cm the resolution begins to be set by the CTR tube at r = 2 cm.

For type ~~x~~ "B" scan (asm. - range)

the asm. angular resolution is independent of range.

Assume that 180° is presented in a distance B. across the tube

then max spot ~~size~~ "length" in "B" direction would be

$$\delta_{max} \doteq \cancel{180°} \frac{B \quad 18\lambda}{\cancel{180°}_{10} \quad D} = \frac{B}{10} \times \frac{\lambda}{D}$$

For 9" tube assume $B = 10$ cm

$\lambda = 10$ cm.
$D = 30$ inches.

$$\delta_{max} = .33 \text{ cm}$$

See p(5) "B" scan on 5" tube has "B" = 7.8 cm ∴

$$\delta_{max} = \frac{.78}{3} = .26 \text{ cm.}$$

For type "B" scan the spot width is determined by the resolution in range demanded.

Type "C" scan —

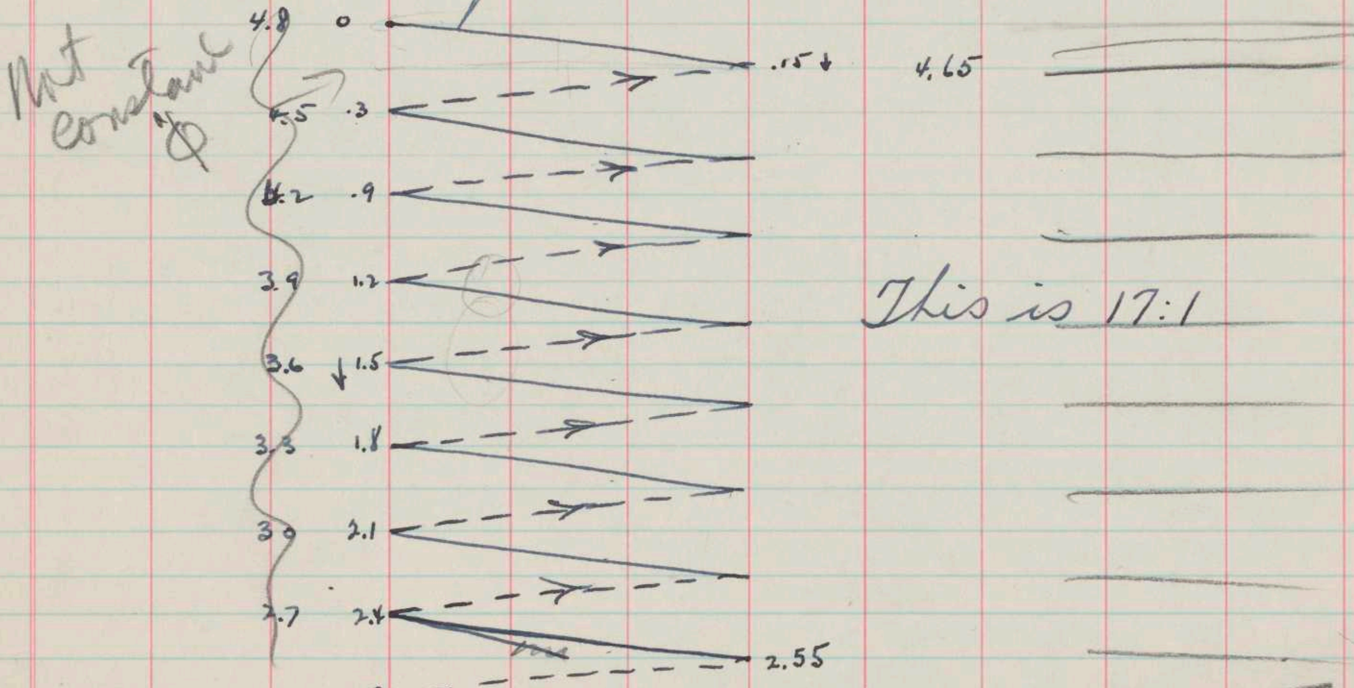
Sperry —

approx 200 R.P.M.

or $\frac{200}{60}$ = RPS = 3.3

time per rev. = .3 sec.

time for 180° = .15 sec.



This is 17:1

vert 12.5 ~~R.P.M.~~

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Although above picture represents actual scanning the presentation on tube showed horizontal lines since the sweeps were put on using contacts.

G.E. Scanner

operates 360 RPM or
6 R.P.S. giving

B

6 lines per second each lasting $\frac{1}{12}$ of a sec. $= 83.3$ ms.

with $\delta_{max} = .26$ cm

there are $\frac{7.8}{.26} = 30$ of δ_{max} to each line.

$\frac{83.3}{30} = 2.8$ ms per δ_{max}

$\frac{2.8}{.5} \doteq 5$ this is max number of recurrences which could be used in overlaps with 2000 ~ pulse rate.

83.3 ms after the first rep. of say a max of 5 pulses, about 5 more will come in due to the finite size of the lobe in the vertical direction.

With a vert scanning vel. of 30° per sec approx $\frac{5}{30} = 167$ ms will be required to pass over a target

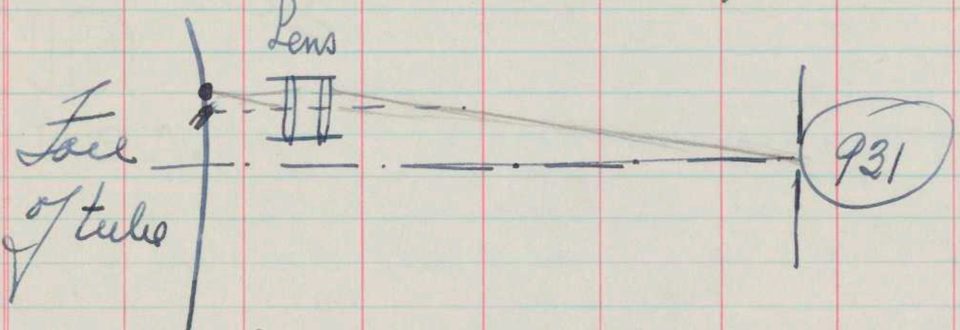
$$\frac{167}{83.3} = 2$$

That is with a "5°" lobe two sets of 5 pulses might be used for build up.

~~The~~ With operation with such biases that a considerable amount of noise comes up on the tube it might be possible to get three or four "trips" to be of some effect and raise the total number of effective pulses to nearly 20.

For searching purposes an adjustable speed vert scan. would be very useful – A low range of 7° per second might be about right. At a higher vert speed perhaps "frame" repetition will be useful.

Consider the possibility of photo-tube scanning.



Lens scans tube about ½ sec. out of phase with the scanning "disk". If the brightness or (light intensity) viewed same as above some adjustable value an "alert" alarm could be sounded.

"Ground" signals could be painted out with aquadag.

This idea should perhaps be expanded.

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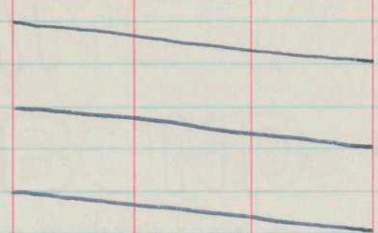
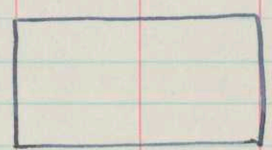
For "C" scan

Vert scan is at rate of 30° per sec. and range of scan is $-10°$ to $+60°$ or 70°.

Complete scan is

$$\frac{70}{30} \times 2 = 4.67 \text{ sec.}$$





70° to 4cm

or

17.5°/cm

← 180° to → 10 cm.

18°/cm

Cross speed

$$\dot{\theta} = \frac{360° \times 6}{\ } = \quad 2160$$

$$= 2160 \text{ deg/per sec.}$$

$$\text{or } \frac{10 \times 2 \times \cancel{6}}{\cancel{6}} = 120 \text{ cm/per sec.}$$

for a vert of 30°/sec.

$$\frac{4 \times 30}{70} = 1.715 \text{ cm/sec.}$$

$$\cancel{\frac{120}{1.715} = 1.70 \text{ slo}}$$

$$\frac{1.715}{120} = \frac{1}{70} = \text{Slope of scan lines.}$$

$$\frac{H}{2(\text{Slope})\,W} = \text{lines ``down''}$$

$$\frac{\cancel{4}^{\,2} \times 7\cancel{0}}{\cancel{2} \times 1\cancel{0}} = 14 \text{ lines.}$$

dist between lines $= \dfrac{H}{14} = .286$ cm.

$$2(\text{Slope})\,W = \frac{2 \times 1\cancel{0}}{7\cancel{0}} = .286 \text{ cm.}$$

Spot dia should be at least .3 cm or else vert and hor. sweeps should be synchronized inorder to trace exactly the same lines. This could be done best by "step" pot. on vertical and make make back "run" follow <u>same</u> lines.

Consider change in range with time where target is moving with respect to the observer.

Max ~~diff.~~ relative vel. when observer and target are moving in opposite direction.

Take this as 800 mi./hr. or

$\frac{800}{3600} = .22$ mi/sec.

~~Zoom of sweep is~~

The miles per spot diameter depends on the range and the length of the throw for that range - this would be r_{max} on PPI and h_{max} on "B".

~~Rec. period~~

~~Vel. of~~ mi/spot dia $= \frac{R_{max}\delta}{r_{max}}$

Time for signal to move one spot dia. is $\dfrac{\frac{R_{max}\delta}{r_{max}}}{\left(\frac{v_T - v_O}{3600}\right)} = \dfrac{R_{max}\,3600\,\delta}{r_{max}(v_T - v_O)}$

These velocities are "radial" components.

consider ang. motion ang. foles.

Take case of

$v_T - V_0 = 800$ mi/hr.

$R_{max} = 100$ mi. $= 100 \times 5280 \times 2.54 = 134 \times 10^4$ cm

$r_{max} = 8$ cm.

$\delta = .1$ cm.

$$\frac{\cancel{100} \times \cancel{1.34 \times 10^4} \times 3.6 \times 10^3}{8 \times 8\cancel{00}} \times .1 = \frac{360}{64} = 5.63$$

This shows that to get build up with frame rep. under these condition the period would need to be less than 5 sec and the spot dia greater than .1

With the range reduced to 10 miles the time would be .56 sec ~~and~~ frame rep. would not be useful.

Consider formula to show relation speed for frame rep with vari spot dia.

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This formula should be expressed in :-

Range in miles per cm $= \left(\frac{R_{max}}{r_{max}}\right)$

Miles per hr relative vel. $= (v_T - v_0)$

seconds per ~~cm~~ spot dia $= \frac{T}{\delta}$

$$\frac{T}{\delta} = \left(\frac{R_{max}}{r_{max}}\right)\frac{3600}{(v_T - v_0)}$$

Plot $\frac{T}{\delta}$ vs $\frac{R_m}{r_m}$ for vari~ $(v_T - v_0)$

$v_T - v_0$	3600/vrel.	R/r	sec/cm. T/δ	sec/mm
10.~~00~~	360	1.0		36
20	180	1		18
50	72	1		7.2
100	36	1		3.6
200	18	10		18
500	7.2	10	72	7.2
1000	3.6	10	36	3.6

$$\frac{I}{\delta} = \left(\frac{R}{cm}\right) \frac{\rho}{V}$$

Use Twin the

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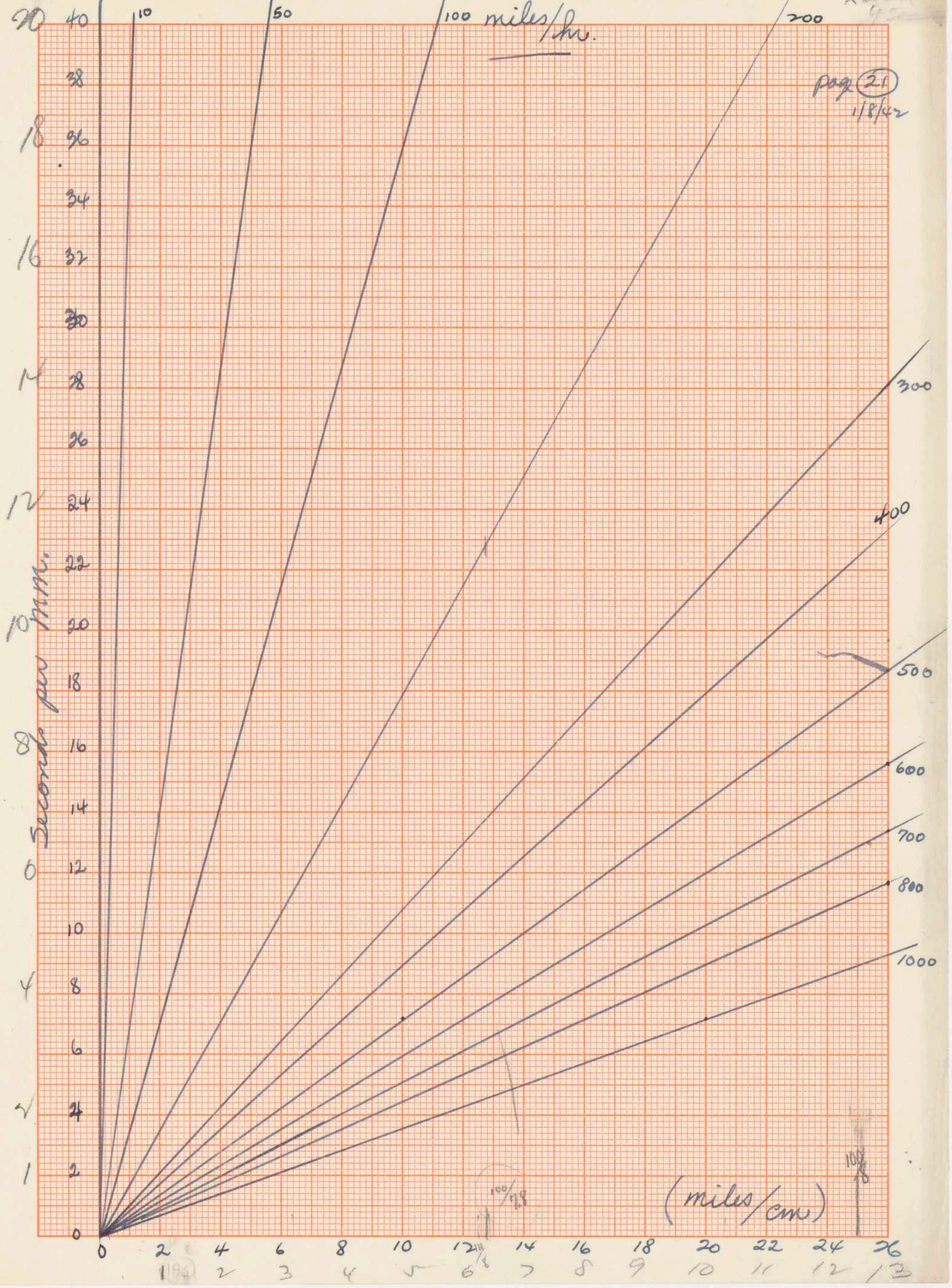

If the uncertainty in reading the spot location is some-what less than the diameter why not take it to be the radius?

Then the fractional uncertainty in location at 1 cm is the radius

$$\frac{\text{(uncertainty)}}{\text{Range}} = \frac{\text{spot radius}}{\text{(deflection)}}$$

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The effect of small angle variations of the observer's base would be to cause a random variation in the location of the return signal and its indication on the CTR tube. If the el. beam is fairly large and advantage is taken of build up then the effect is to bring out the average direction of the target relative to the observer because the build up will be stronger where over lap is more frequent.

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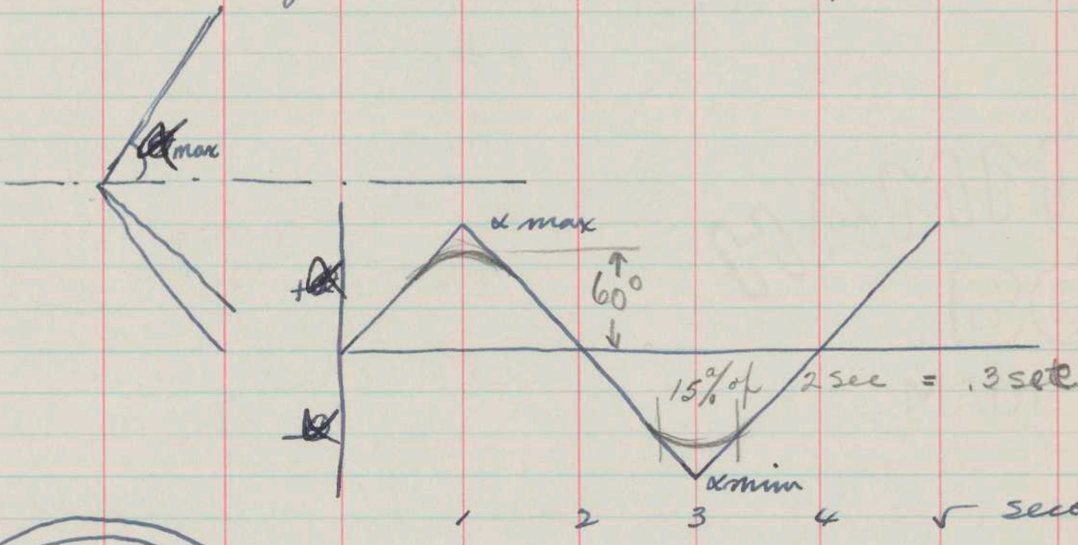
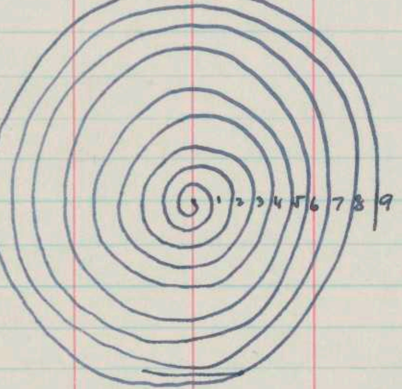
Before this is done consider spiral scanning.

Fast rotation 20 RPS.

Slow sweep. 60° per sec

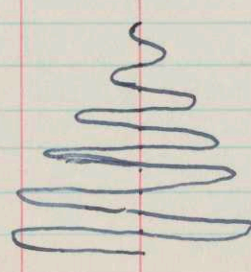
This is 3 degree "pitch"





$\dot{\Phi}$ = constant

$\dot{\alpha} = +$ or $- \dot{\alpha}_{const.}$



pitch $\doteq \frac{60}{54} \times \frac{60}{20} = 3.3°$

Tangential velocity of sweep and radio beam is proportional to α

Consider tube of radius r_m

$vel = r\dot{\theta}$ = tang vel. on CRT

$r = r_m \dfrac{\alpha}{60}$

$\dot{\theta} = 20$ RPS $= 40\pi$ radians per sec.

$vel_{max} = 40\pi r_m$

To cover sky at least the following number of pulses are needed.

$(\text{No}/\text{rev.}) = \dfrac{360 \times D}{18\,\lambda} = \dfrac{20\,D}{\lambda}$ number per rev.

take $\dfrac{D}{\lambda} = 3$

Min number of pulses per rev $= 60$

$\therefore\ 20 \times 60 = 1200$ = min number of pulses per second.

General formula

$$\min f_p = 20\left(\frac{D}{\lambda}\right)(RPS)$$

At what spot ~~diameter~~ radius will there be no over-lap of spots and ∴ $N = 0$ on out side part of scan.

$$\cancel{2} r_b \times f_p = \cancel{2}\pi r_m (RPS)$$

$$r_b = \frac{\pi r_m (RPS)}{f_p}$$

Example $RPS = 20$

$f_p = 2000$

$r_m = 8$

$$r_b = \frac{8\pi}{100} = .25 \text{ cm}$$

This is a dia of 5 mm.

Take $r_m = 5.5$ cm

$r_b = .17$ cm

Assume $r_p = .05$
Inside of what radius will spots begin to overlap.

$$r = \frac{r_p f_p}{\pi(RPS)}$$

$$= \frac{05 \times 2000}{\pi \times 20} = \underline{1.6 \text{ cm.}}$$

Assume that the "pitch" is approx $3°$

$$\left(\frac{deg}{sec}\right) \times \left(\frac{sec.}{Rev}\right) = \frac{\dot{\alpha}}{(RPS)}$$

with an overall of $\pm \alpha_{max} \doteq 1.15 \alpha_{wanted}$

$\cancel{\dot{\alpha} = \alpha_{max}}$

time to go from $\alpha = 0$ to $\alpha = \alpha_{wanted} = p_\alpha$

Max velocity is $\dot{\alpha}_{max} = \frac{1.15 \alpha_{wanted}}{p_\alpha}$

"pitch" is $= \frac{1.15 \alpha_{want}}{p_\alpha (RPS)} = \left(\underline{\text{deg per revolution}}\right)_{max}$

Ex:

$\alpha_{want} = 60$

$P_\alpha = 1$ sec.

$RPS = 20$

$$\text{pitch} = \frac{1.15 \times 60}{20} = \underline{3.4^\circ \text{ per Rev.}}$$

~~Min useful radius r_b~~

$$\cancel{2r_b = r_m \times \beta}$$

What is the "pitch" on the tube ?

$\dot{r}$ (Seconds per rev.) = cm per rev.

$$\frac{\dot{r}}{RPS} = \text{cm per rev}$$

$\dot{r} = \frac{1.15\, r_m}{P_\alpha}$ for tube of rad. r_m and period from center to beginning of return ~~to~~ from edge.

$$\text{cm per rev} = \frac{1.15\, r_m}{P_\alpha (RPS)}$$

Example: $\frac{1.15 \times 8}{1 \times 20} = .46$ cm.

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Any spot dia less than .46 cm would not increase the radial resolution

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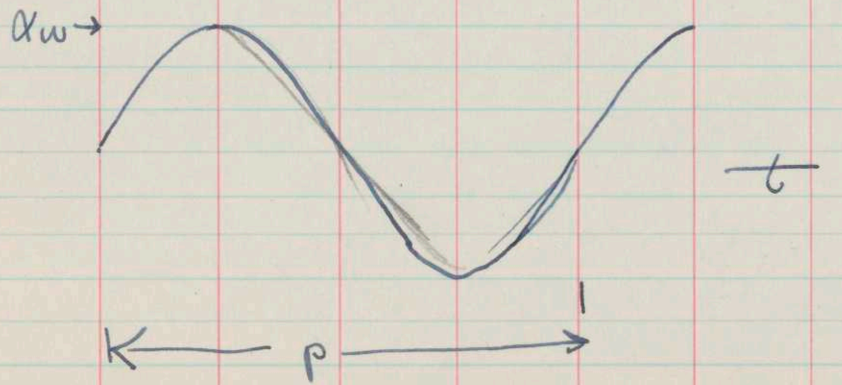
This type of scan is not used but a "B" scan is used.

Here the x coordinate is

$$x = \alpha_{wanted} \sin\frac{2\pi t}{p} \cos 2\pi(RPS)\, t$$

$$\dot{x} = \frac{2\pi\alpha_w}{p} \cos\frac{t}{p} \cos 2\pi(RPS)t - \alpha_w 2\pi(RPS) \sin\frac{t}{p} \sin 2\pi(RPS)t$$

These two eq. depend on the assumption that the tilt α is approx:



$$\alpha \doteq \alpha_w \sin\frac{2\pi t}{p}$$

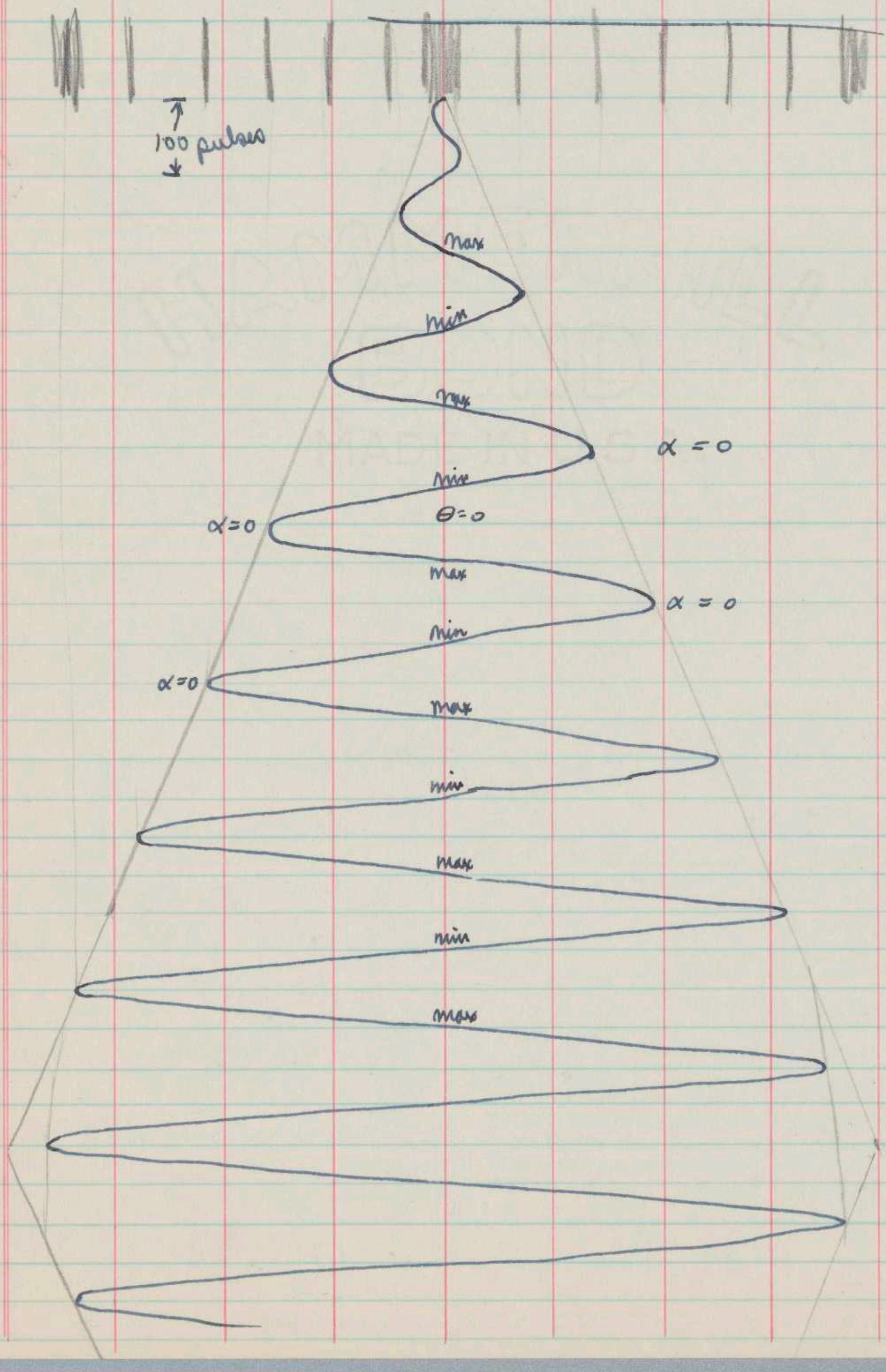
$$\dot{\alpha} \doteq \frac{2\pi\alpha_w}{p} \cos\frac{2\pi t}{p}$$

$$\dot{\alpha}_{max} = \frac{2\pi 60}{4} = 94°/sec.$$

or 4.1°/rev.

Estimate on p 27 gives 70°/sec or 3.5°/rev.

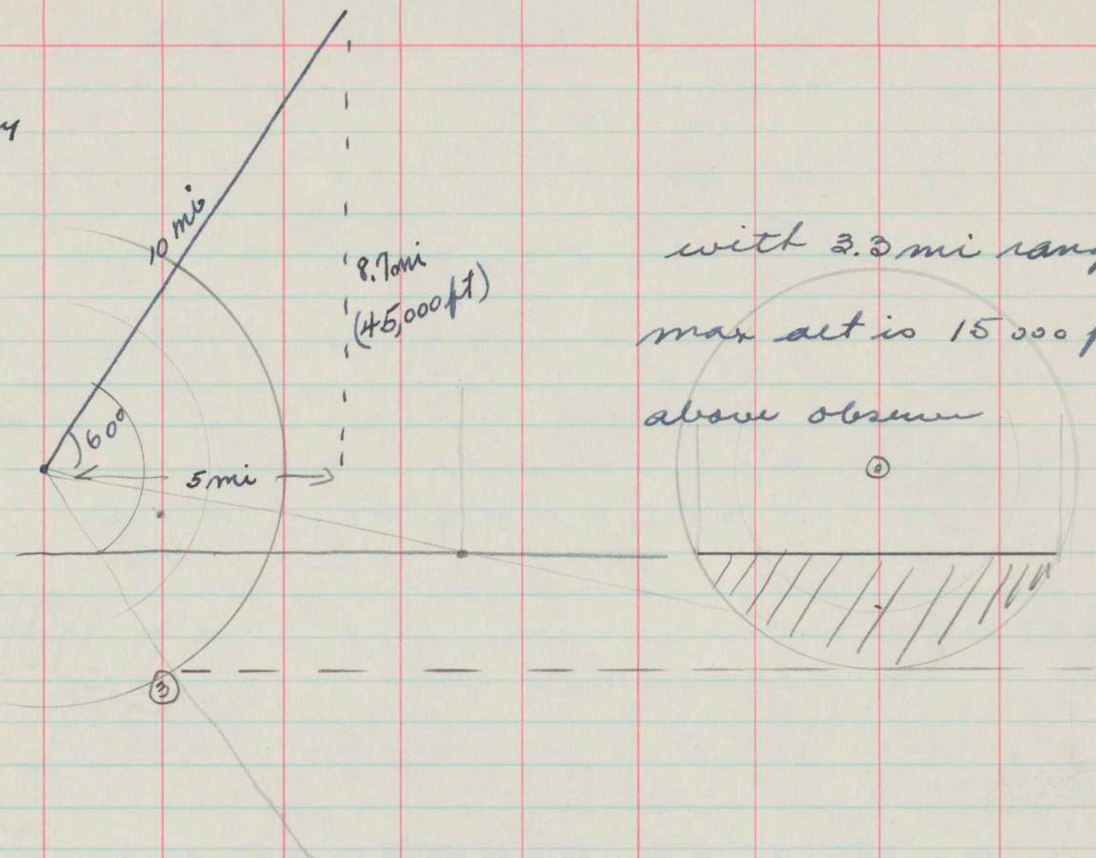
This appox gives a max value of $\dot{\alpha}$ which is too great but it is worth while to still consider it.



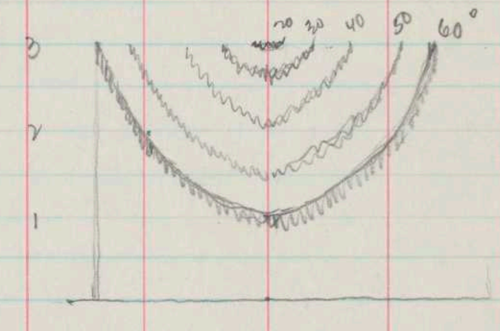
might expect a distribution of noise peaks to look like this.

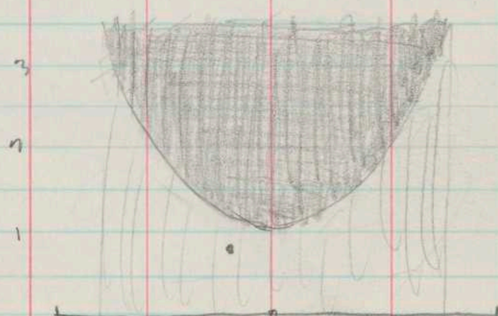
with $p = 4$ sec whole of sky is scanned in 1 second.

Case I
observer very low



with 3.3 mi range
max alt is 15 000 ft
above observer





1/27/42

1/8/42
(32)

I find that the plan of operation involves a "blinder" which kills sensitivity when beam is down ∴ target must be above observer if leo is to be seen.

System proposed by Ramsey in Nov. Before further analysis is done complete "case history" must be looked up.

It seems as though "case history" files should be maintained for development of all systems.

1/13/42
①

Reorganization of equations etc on sheets dated 1/8/42 ① to 23

Description of ~~electrons~~ beam as it hits CTR-Screen.

r_b = radius of el. beam if spot is round.

h_b = heigth of beam ~~pattern~~ spot retangular or formed in an arc.

w_b = widtht of beam spot if rec. or arc.

i_b = el. current in beam spot.

j_b = " " density in " "

(round) $$j_b = \frac{i_b}{\pi r_b^2} \qquad ①$$

(rec.) $$j_b = \frac{i_b}{h_b w_b} \qquad ②$$

Location of beam for a retangular presentation.

x = Horizontal displacement

y = vert displacement.

$\dot{x}$ = horizontal velocity of el. beam spot.
$\dot{y}$ = vert " " " " ".

Location of beam with polar presentation

r = radial distance from origin
θ = ang. displacement from fixed line

$\dot{r}$ = radial velocity
$r\dot{\theta}$ = tang. velocity.

For many equations
r is equivalent to y
and $r\theta$ " " " x
$\dot{r}$ " " " $\dot{y}$
$r\dot{\theta}$ " " " $\dot{x}$

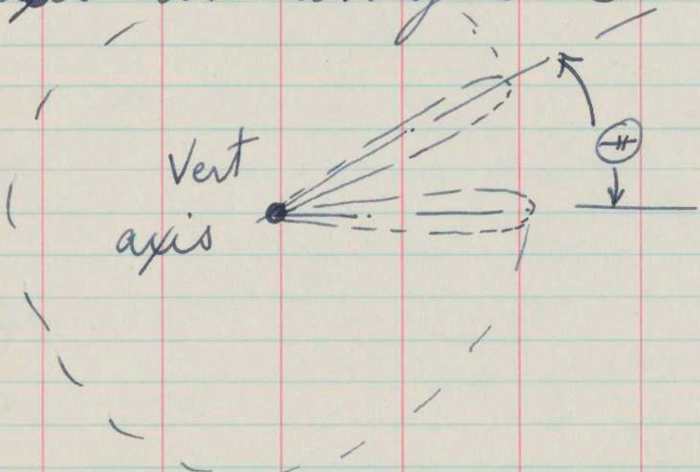
4/3/42

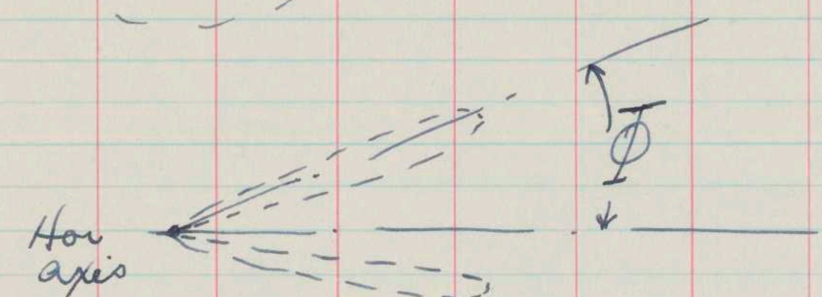
(3)

Helical scan

Description of transmitted and received signal.

Center line of lobe sweeps about a vertical axis an angle Θ

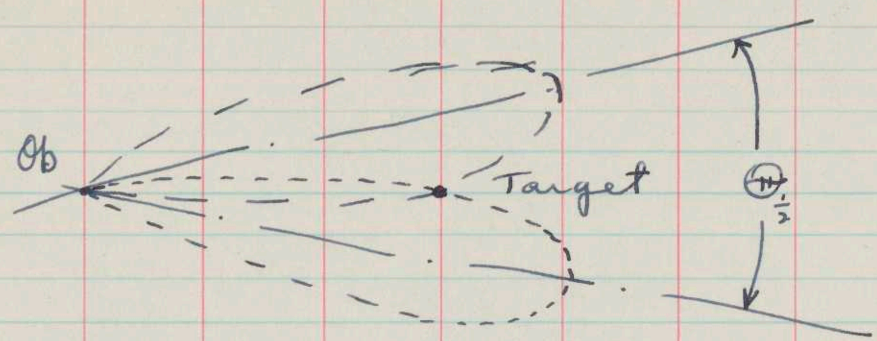




And up and down about a horizontal axis Φ

angular velocities $\dot{\Theta}$ and $\dot{\Phi}$

R = Range to target in miles



For a certain angular displacement $\Theta_{\frac{1}{2}}$ the ideal transmitted and received pattern would move from half-power on one side of the lobe to half power on the other side. Since the response of the receiver is said to be non-linear this approximates the extreme variation of Θ for $\frac{1}{2}$ voltage to max and down to half voltage.

If the lobe is symmetrical about its axis then

$\Theta_{\frac{1}{2}} = \Phi_{\frac{1}{2}}$ where $\Phi_{\frac{1}{2}}$ is defined the same way.

Relative velocities of observer and target.

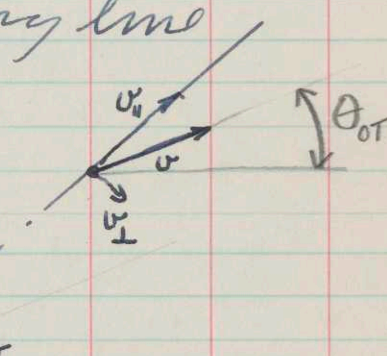
V = vector velocity of observer.

v = " " " target

$V_\parallel$ = component along joining line (line of centers) at angle Θ_T

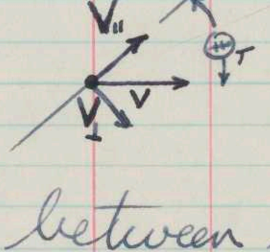
$V_\parallel = V \cos \Theta_T$
and

$V_\perp = V \sin \Theta_T$ = component $\perp$ to joining line



$v_\parallel = v \cos(\Theta_{OT} - \Theta_T)$

$v_\perp = v \sin(\Theta_{OT} - \Theta_T)$



where the angle between the vector velocities V and v is Θ_{OT}
("OT" stands for observer-target)

1/13/42
5a

Notes on 1/13/42 ⑤ may not be sufficiently general. — Consider this again.

Take observer velocity vector to be V and create frame of reference with x in direction of V take y as perp. to x and in plane of wings. Take z as ⊥ to x and y.

The line which joins the observer and the target is now defined in direction by the cosines l_1, m_1, and n_1. The distance $\overline{\overline{\text{observer to target}}} = R$.

Target velocity $= v$

This has components along the x, y, z frame

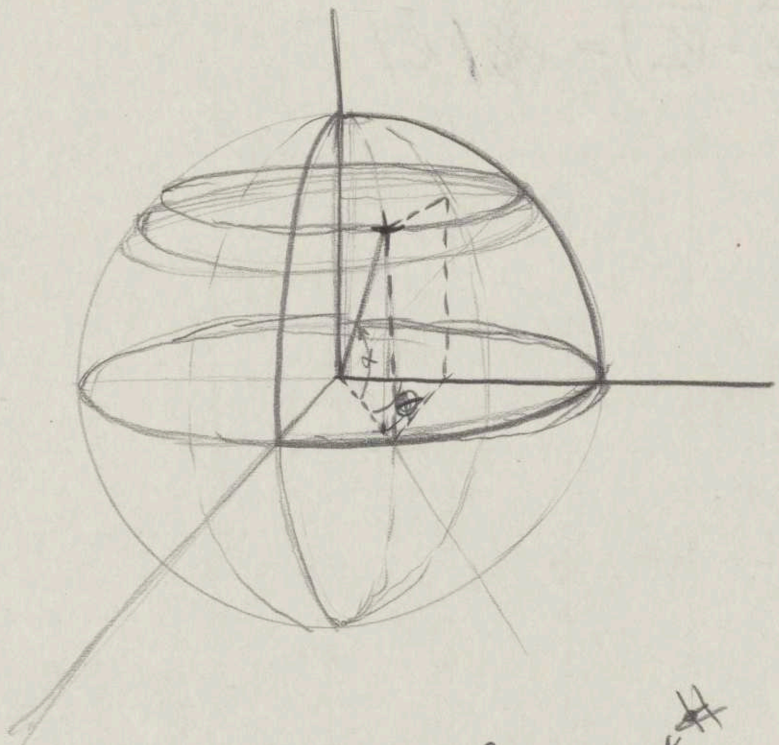
$$\vec{v} = \vec{v}_x + \vec{v}_y + \vec{v}_z$$

The relative velocity vector is

$$\vec{v} - \vec{V} = \vec{V}_{OT} = (v_x - V) + v_y + v_z$$

$\dot{R}$ is the component of $\vec{V}_{OT}$ along direction l_1, m_1, n_1, or direction of unit vector a_1

$$(\vec{a}_1 \cdot \vec{V}_{OT}) = |\dot{R}|$$



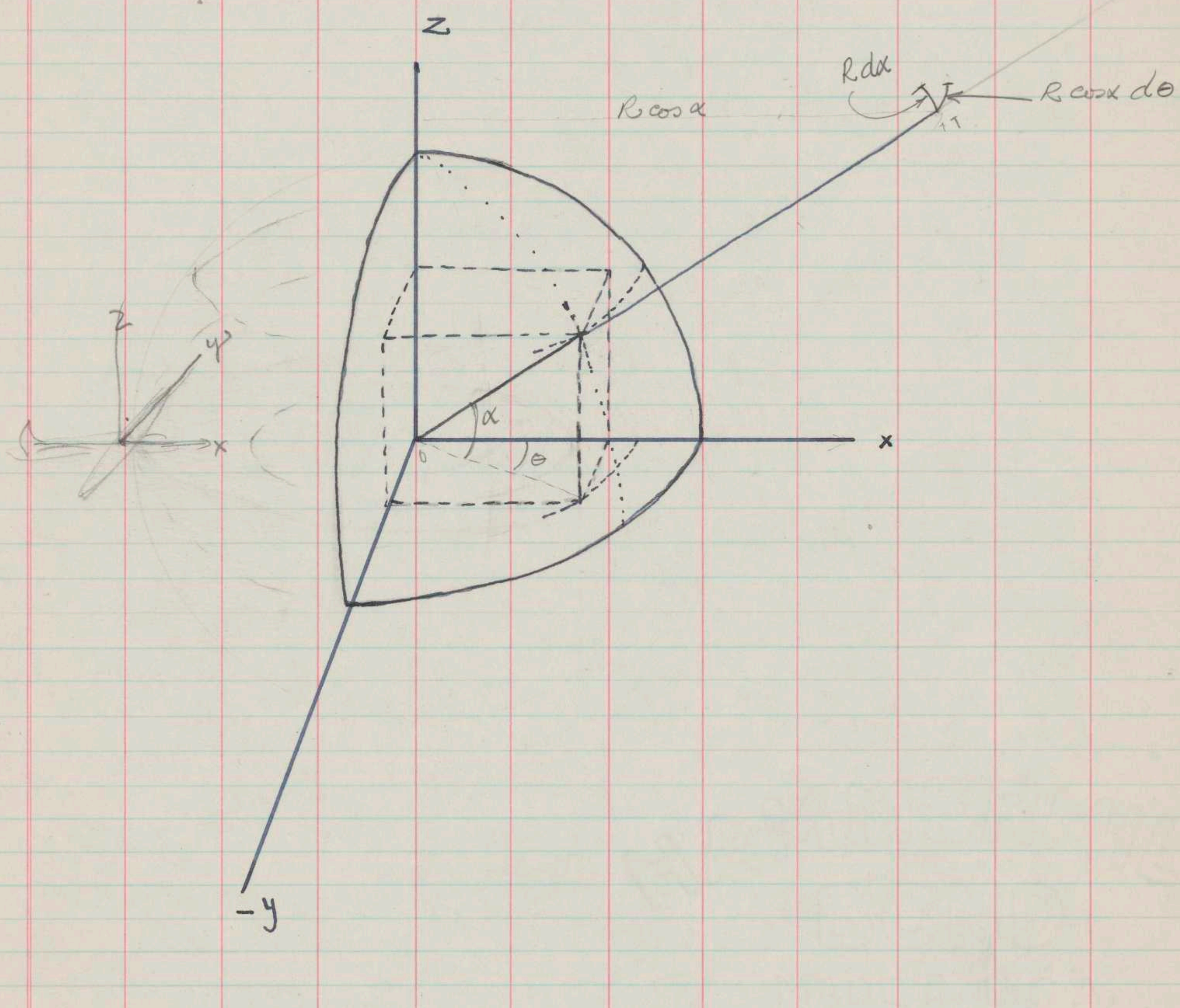
$R \cos\alpha \, d\theta$

$R \, d\dot{\alpha}$

R

1/13/42

5b



Relative velocity

$$\vec{v} - \vec{V} = \dot{\vec{R}} = \vec{a}_1 \dot{R} + R \frac{d\vec{a}_1}{dt}$$

$$\frac{d\vec{a}_1}{dt} = \vec{\alpha}_1 \frac{d\alpha}{dt} + \vec{\theta}_1 \cos\alpha \frac{d\theta}{dt}$$

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5C

$$\vec{v} - \vec{V} = \vec{a}_1 \dot{R} + \vec{\alpha}_1 R \dot{\alpha} + \vec{\theta}_1 R \cos\alpha\, \dot{\theta}$$

$\dot{\alpha}$ is made up of two components which are angular velocity of the observer $\dot{\alpha}_0$ and the component of target velocity along the direction of the unit vector $\vec{\alpha}_1$ this is

$$\vec{\alpha}_1 \cdot \vec{v}.$$

Total is $\vec{\alpha}_1 \cdot \vec{v} + \dot{\alpha}_0 = \dot{\alpha}$

In the same way there are two components of "azimuth angle velocity" which are

$$\vec{\theta}_1 \cdot \vec{v} + \dot{\theta}_0 = \dot{\theta}$$

The max value of $\vec{\alpha}_1 \cdot \vec{v}$ comes when the target is climbing ~~or~~ diving at max velocity.

The max value of $\vec{\theta}_1 \cdot \vec{v}$ comes when target is flying ~~parallel~~ to the observer plane and at rt angle to $\vec{a}_1$!

$$\frac{d(\bar{R})}{dt} = \vec{a}_1 \frac{dR}{dt} + R \frac{d\vec{a}_1}{dt}$$

$$\vec{a}_1 = li + mj + nk$$

$$\frac{d\vec{a}_1}{dt} = \frac{dl}{dt} i + \frac{dm}{dt} j + \frac{dn}{dt} \vec{k}$$

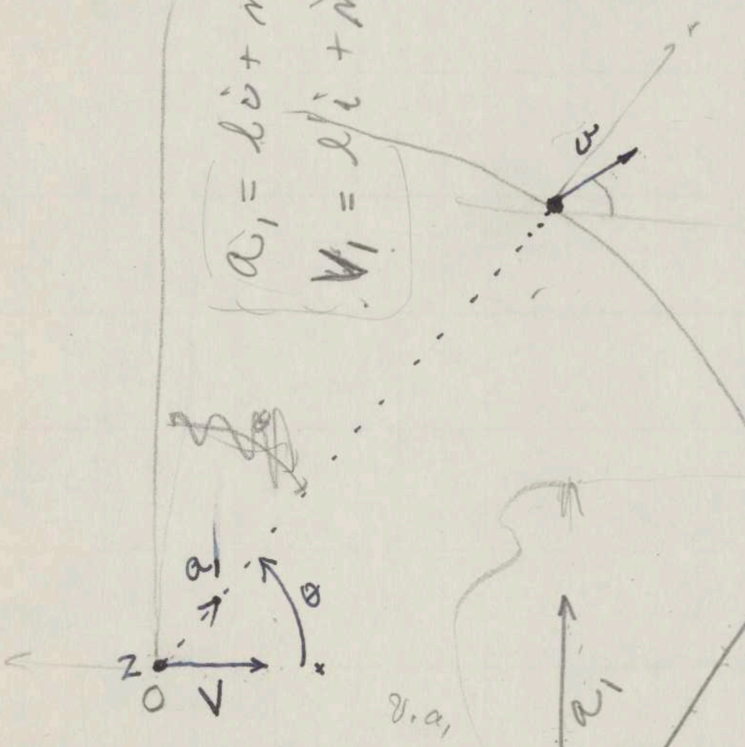
$$\begin{cases} i\frac{dl}{dt} = -i\cos\theta \sin\alpha \frac{d\alpha}{dt} - i\cos\alpha \sin\theta \frac{d\theta}{dt} \\ j\frac{dm}{dt} = -j\sin\theta \sin\alpha \frac{d\alpha}{dt} + j\cos\alpha \cos\theta \frac{d\theta}{dt} \\ k\frac{dn}{dt} = k\cos\alpha \frac{d\alpha}{dt} \end{cases}$$

$$\frac{d\vec{a}_1}{dt} = \left\{k\cos\alpha - \sin\alpha(j\sin\theta + i\cos\theta)\right\}\frac{d\alpha}{dt} + \left[\cos\alpha(i\sin\theta\, j\cos\theta)\right]\frac{d\theta}{dt}$$

$$(\vec{v} - \vec{V}) = \vec{a}_1\dot{R} + R\left\{\quad\right\}\frac{d\alpha}{dt} + R\left[\quad\right]\frac{d\theta}{dt}$$

$a_1 = l\,i + m\,j + n\,k$

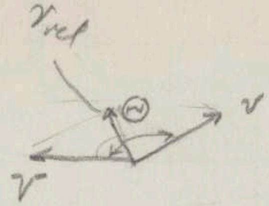
$V_1 = l'\,i + m'\,j + n'\,k$



v_r

v_θ

v_φ



$$v_{rel}^2 = V^2 + v^2 - 2vV\cos\Theta$$

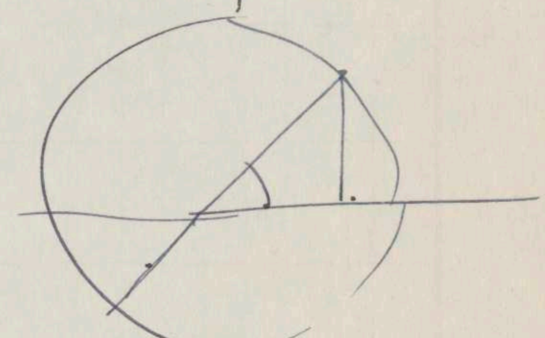
$$\cos\Theta = \cos\varphi\cos\varphi' + \sin\varphi\sin\varphi'\cos(\theta-\theta')$$

$\theta' = \pi$

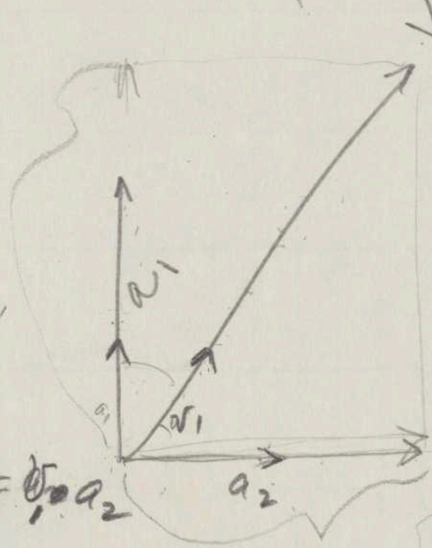
$\varphi' = \pi/2$

$$\cos\Theta = \sin\varphi\cos(\theta-\pi)$$

$$= -\sin\varphi\cos\theta$$



$v \cdot a_1$



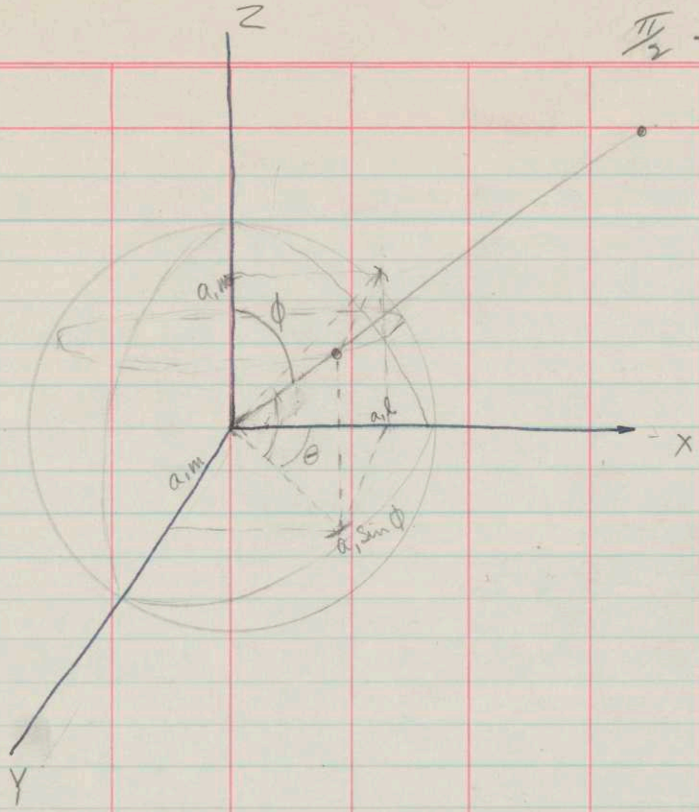
$\boxed{(a_1 \times v_1) \times a_1} = v_1 a_2$

$v \cdot a_2$

a_1

$(a_1 \times V_1) \times a_1 V =$

$\frac{\pi}{2} - \phi = \alpha$ = elevation angle.



Line OP has
dir. cos. l, m, n.

$\cos\phi = n$

li

$(a_1 \sin\phi)\cos\theta = a_1 l$

$(a_1 \sin\phi)\sin\theta = a_1 m$

$a_1 \cos\phi = a_1 n$

$\sin\phi = \sin(\frac{\pi}{2} - \alpha)$

$\sin\phi = \cos\alpha =$
$\cos\phi = \sin\alpha = n$

$$\begin{cases} a_1 l = a_1 \cos\alpha\cos\theta \\ a_1 m = a_1 \cos\alpha\sin\theta \\ a_1 n = a_1 \sin\alpha \end{cases}$$

$\cos^2\alpha\cos^2\theta + \cos^2\alpha\sin^2\theta + \sin^2\alpha = 1$ ok.

$l = \cos\alpha\cos\theta$
$m = \cos\alpha\sin\theta$
$n = \sin\alpha$

$\vec{R} = R\vec{a}_1$

$\cancel{\vec{a}_1 l + \vec{a}_1 m + \vec{a}_1 n = i + j + k}$

$$\begin{cases} \vec{a}_1 l = i \\ a_1 m = j \\ a_1 n = k \end{cases}$$

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(6)

For stationary observer or a "frame" moving with the observer.

$$\cancel{\dot{R} = v_{\parallel} \cos(\Theta_{0T} - \Theta)} -$$

$$\dot{R} = v_{\parallel} - V_{\parallel} = v \cos(\Theta_{0T} - \Theta_T) - V \cos \Theta_T$$

$$R \dot{\Theta}_T = v_{\perp} - V_{\perp} = v \sin(\Theta_{0T} - \Theta_T) - V \sin \Theta_T$$

Other symbols:

p = recurrence period

$\frac{1}{p} = f_p$ = " frequency - or freq. of pulsing.

P = "frame" period.

τ = time ~~length~~ of duration of a pulse at the output of the receiver.

L = time of duration of excitation of a specified area of screen.

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r_m = tube radius

R_m = maximum range for given sweep system

y_m = height of "B" scan for max. range

λ = wave length (cm)

$\nu = \frac{3 \times 10^{10}}{\lambda}$ = freq. c.p.s.

$= \frac{3 \times 10^{4}}{\lambda}$ = freq. meg.c.p.s.

D = dish diameter usually given in inches

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8

Questions:—

1.) How many pulses strike the target and return as the lobe sweeps by?

(a) direct shot.

$$N = \frac{3\,\lambda\, f_p}{D\,(RPM)_\Theta} = f_p \frac{\Theta^\circ_{\frac{1}{2}}}{\dot{\Theta}^\circ} = f_p \frac{18\,\lambda}{D} \cdot \frac{60}{360\,(RPM)} \qquad (1)$$

Example:

$\lambda = 10$ cm
$D = 30$ inches
$f = 2000$
$(RPM)_\Theta = 360$

$\dot{\Theta}^\circ = 6 \cdot (RPM)$

$N = 5.5$ this gives max number of pulses in a pulse group.

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8

2.) By How large an angle can the lobe be directed above or below the target and still get one signal back?

$$\dot{\Theta}^\circ = 6\ (RPM)$$

Angle per pulse $\Theta_1^\circ = \dfrac{\dot{\Theta}^\circ}{f_p} = \dfrac{6\,(RPM)}{f_p}$ (2)

Example:

$$\Theta_1^\circ = \frac{6 \times 360}{2000} = 1.08 \text{ degrees.}$$

$$\Phi = \frac{1}{2}\left(\Theta_{\frac{1}{2}}^{\circ 2} - \Theta_1^{\circ 2}\right)^{\frac{1}{2}}$$

Example

$$= \frac{1}{2}\left(36^\circ - 1.17\right)^{\frac{1}{2}}$$

$$= 2.95^\circ$$

$$\Delta\Phi = \frac{1}{2}\left(\left(\frac{18\,\lambda}{D}\right)^2 - \frac{36\,(RPM)^2}{f_p^2}\right)^{\frac{1}{2}}$$

For one Signal $\pm\Delta\Phi_1^\circ = 3\left[\dfrac{\lambda^2}{4D^2} - \dfrac{(RPM)^2}{f_p^2}\right]^{\frac{1}{2}}$

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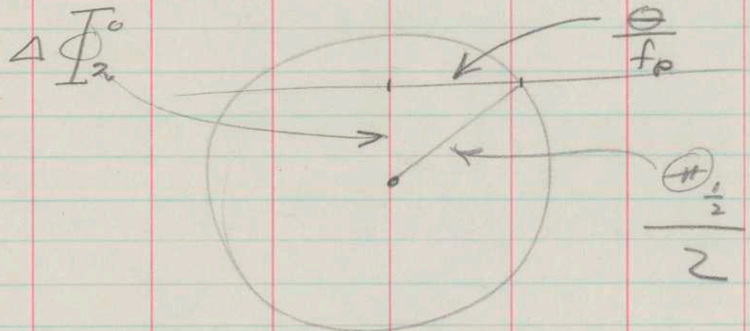
(9)

3) By how large an angle can the lobe be directed above or below the target and still get $\underline{2}$ signals back.



$$\Delta \Phi^\circ_2 = \left[\left(\frac{9\lambda}{.D}\right)^2 - \frac{6^2(RPM)^2}{f_p^2}\right]^{1/2}$$

$$= 3\left[\left(\frac{3\lambda}{D}\right)^2 - \left(\frac{2(RPM)}{f_p}\right)^2\right]^{1/2}$$

Example.

$$\Delta \Phi^\circ_2 = \cancel{\left[9 - \frac{\overset{9}{\cancel{36}} \times (36)^2 \times \cancel{100}}{\underset{10,000}{\cancel{1000000}}}\right]^{1/2}}$$

$$\pm \Delta \Phi^\circ_2 = \cancel{3\left[1 - \frac{36 \times 36}{10,000}\right]^{1/2}} = 3\left[1 - \frac{2 \times (360)^2}{2 \times 10^6}\right]^{1/2}$$

$1 - .13$

$(.87)^{1/2}$

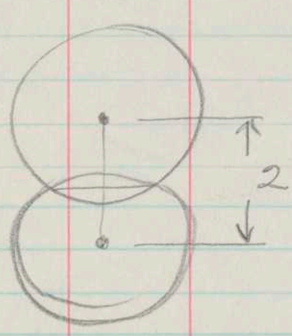
$= 2.8^\circ$

4. What is the maximum speed of the up and down sweep of dish for complete coverage?

$$\text{Seconds per rev.} = \frac{60}{(RPM)}$$

$$\Delta \Phi^{\circ}_{(REV)} = \dot{\Phi}^{\circ} \frac{60}{(RPM)}$$

$$\dot{\Phi}^{\circ} = \frac{\Delta \Phi^{\circ}_{rev} (RPM)}{60}$$



$$\dot{\Phi}^{\circ} = 6\left[\left(\frac{3\lambda}{D}\right)^2 - \left\{\frac{2\,(RPM)}{f_p}\right\}^2\right]^{1/2} \cdot \frac{(RPM)}{60}$$

Example:

$$\dot{\Phi}^{\circ} = 6\overset{.933}{\left[1 - .13\right]}^{1/2} \cdot \frac{\overset{6}{\cancel{360}}}{\cancel{60}} = 33.5^{\circ} \text{ per second.}$$

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Modify eqn above.

$$\dot{\Phi}_0 = \frac{(RPM)}{10}\left[\left(\frac{3\lambda}{D}\right)^2 - \left\{\frac{2\,RPM}{f_p}\right\}^2\right]^{1/2}$$

Same example as above.

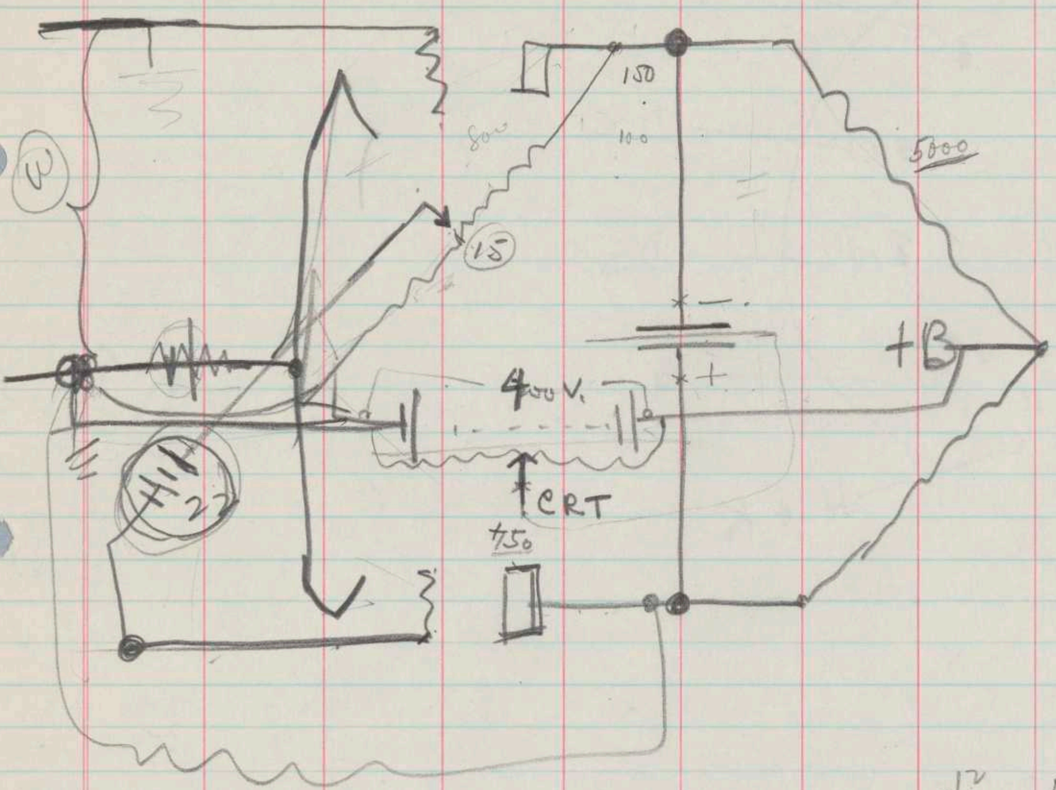
$$\dot{\Phi}_0 = \frac{36\cancel{0}}{1\cancel{0}}\left[1 - \left\{\frac{2\cdot 360}{2000}\right\}^2\right]^{\frac{1}{2}} = 33.5^\circ \text{ per second.}$$

.18

4

6A3

6



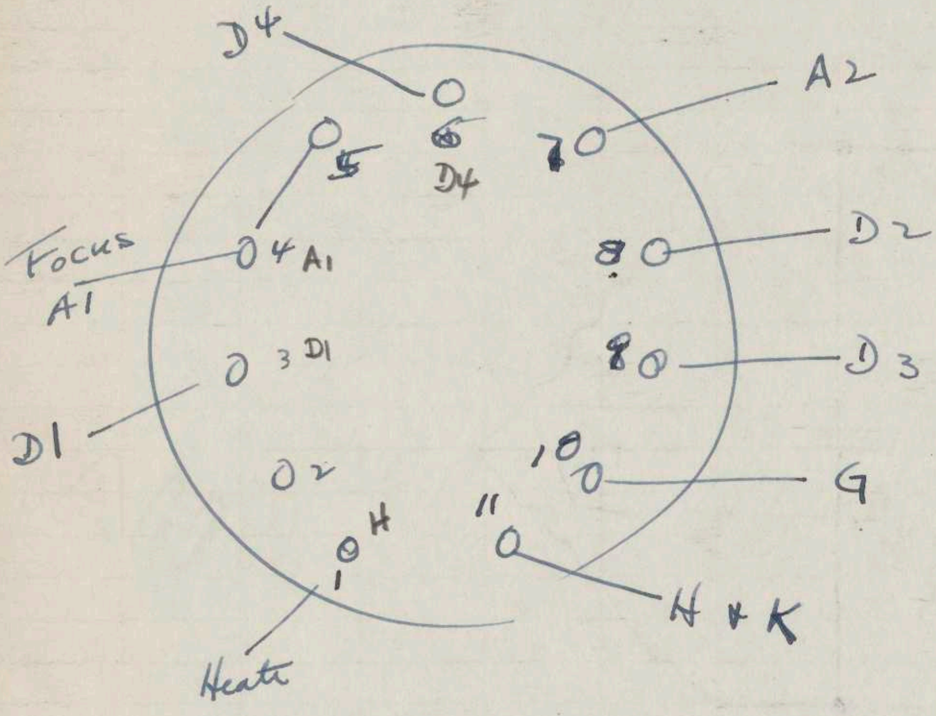
$$20 \times 10^{-12} \times 5 \times 10^{3}$$

$$100 \times 10^{-11}$$

$$1 \times 10^{-7}$$

Du Mont

Bottom View -

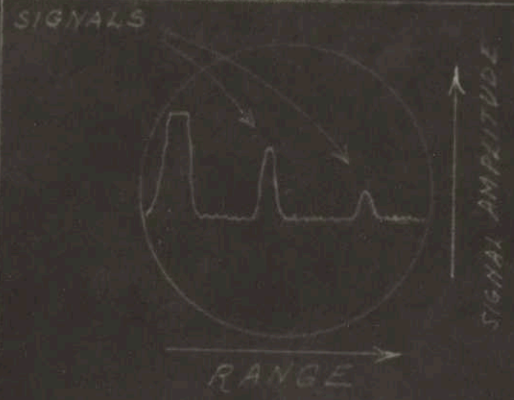
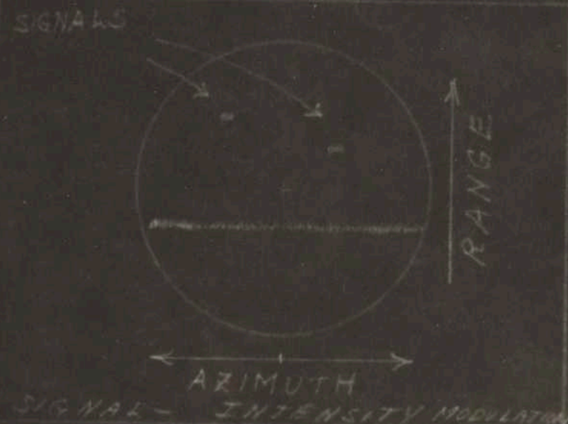
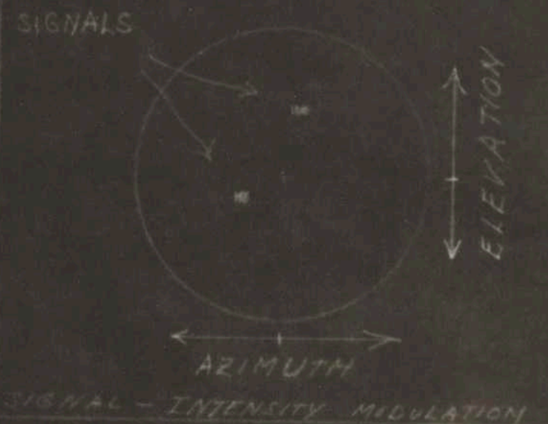
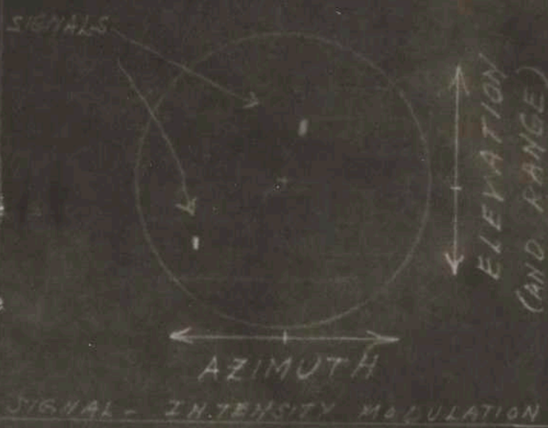


SECRET

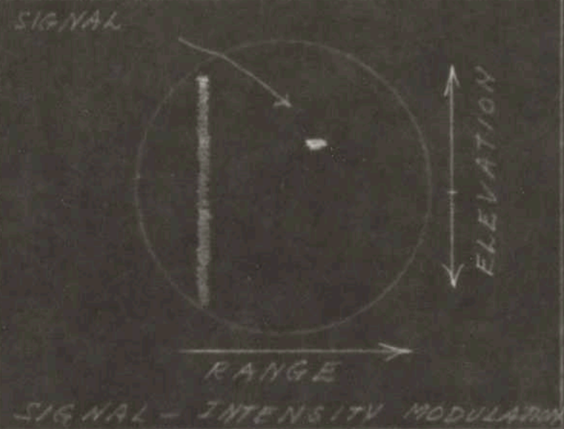
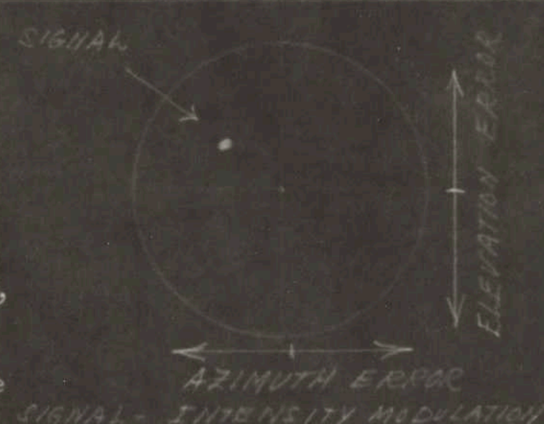
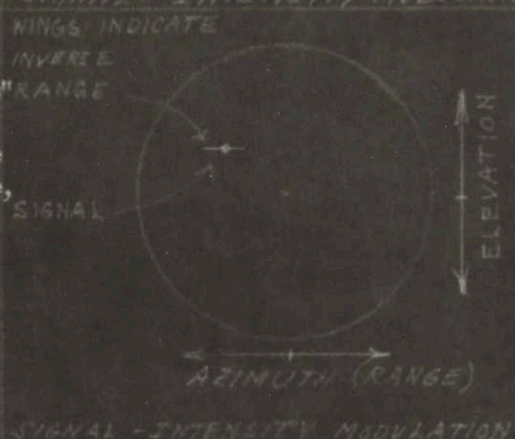
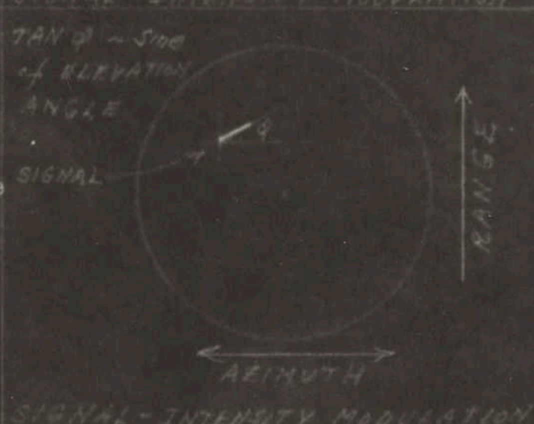
RECOMMENDED DESIGNATION OF RADAR TYPES

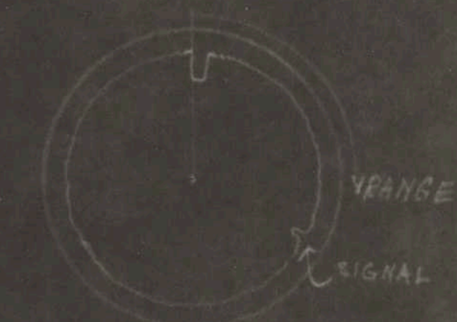
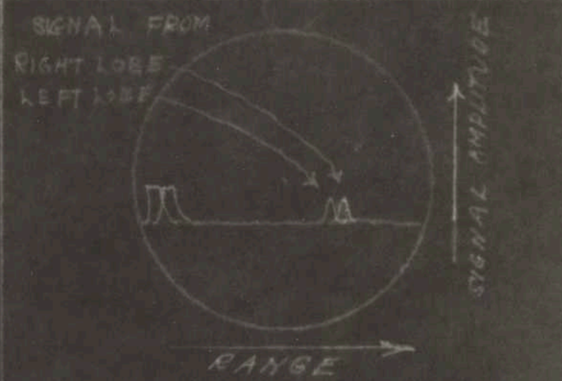
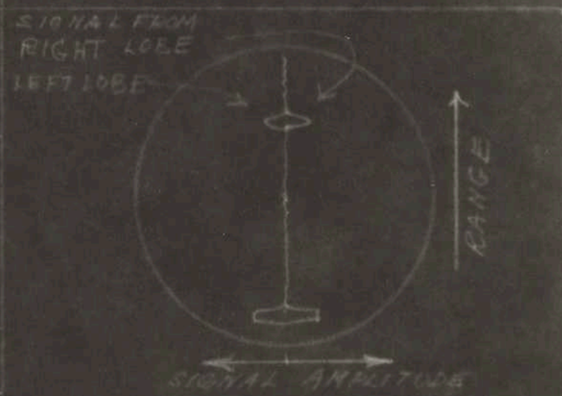
The Massachusetts Institute of Technology - Radiation Laboratory

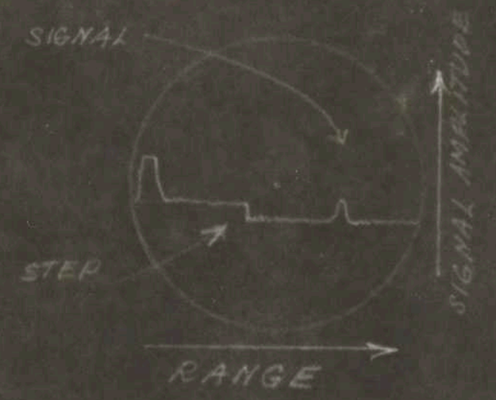
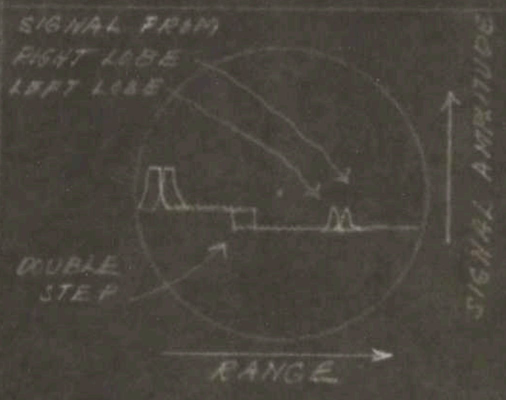
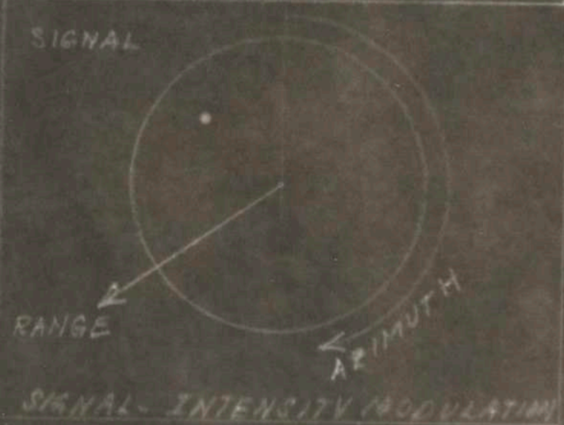
Copy #1 WBN

<table>
<tr><th>TYPE</th><th>DESCRIPTION</th><th>APPEARANCE</th></tr>
<tr><td>A</td><td>A time sweep produces a horizontal range scale. Signal appears as a vertical deflection of the time trace.</td><td>
</td></tr>
<tr><td>B</td><td>Signal appears as bright spot with azimuth angle as the horizontal coordinate and range as the vertical coordinate.</td><td>
</td></tr>
<tr><td>C</td><td>Signal appears as bright spot with azimuth angle as the horizontal coordinate and elevation angle as the vertical coordinate.</td><td>
</td></tr>
<tr><td>D</td><td>A combination of types B and C. Signal appears as bright spot with azimuth angle as the horizontal coordinate and elevation angle as the vertical coordinate. Each horizontal trace is expanded slightly vertically by a compressed time sweep to facilitate separation of signal from noise and to give a rough range indication.</td><td>
</td></tr>
</table>

SECRET

<table>
<tr><th>TYPE</th><th>DESCRIPTION</th><th>APPEARANCE</th></tr>
<tr><td>E</td><td>~~A single signal, only, appears as a bright spot subtended by "wings" which grow as the distance to the target is diminished. Azimuth angle appears as the horizontal coordinate, elevation angle as the vertical coordinate. This has been referred to as "Mark VI" indication.~~</td><td>
</td></tr>
<tr><td>F</td><td>A modification of Type B. Signal appears as a bright spot with range as the horizontal coordinate and elevation as the vertical coordinate.
A single signal, only, appears as a bright spot. Azimuth error angle appears as the horizontal coordinate, elevation error angle as the vertical coordinate. Cross-hairs on the indicator face permit of bringing the system to bear on the target.</td><td>
</td></tr>
<tr><td>G</td><td>A single signal, only, appears as a bright spot subtended by "wings" which grow as the distance to the target is diminished. Azimuth angle appears as the horizontal coordinate, elevation angle as the vertical coordinate. This has been referred to as "Mark VI" indication.</td><td>
</td></tr>
<tr><td>H</td><td>A modification of type B. Signal appears as a bright line whose tangent is proportional to the sine of the angle of elevation. Azimuth appears as the horizontal coordinate and range as the vertical coordinate.</td><td>
</td></tr>
</table>

TYPE	DESCRIPTION	APPEARANCE
I	Used to depict range and direction for a system with a conically scanning antenna. Signal appears as bright circular segment with radius proportional to range. The circular length of the segment is inversely proportional to the error of aiming the system and its position indicates the bearing of the target. True aim results in a complete circle. This has been referred to as "Mark IV" indication.	
J	A modification of Type A in which the time sweep produces a circular range scale near the circumference of the CR tube face. The signal appears as a radial deflection of the time trace. No bearing indication is given.	
K	A modification of Type A for aiming a double lobe system in azimuth (elevation). A horizontal (vertical) time sweep is displaced slightly in the direction of the antenna lobe in use. The signal appears as a double vertical (horizontal) deflection of the time sweep with the ratio of amplitudes indicative of the error in aiming.	
L	A modification of Type A for aiming a double lobe system in azimuth (elevation). A vertical (horizontal) time sweep indicates range. The signal from the left (lower) lobe appears as a horizontal (vertical) deflection to the left (downwards); the signal from the right (upper) lobe as a horizontal (vertical) deflection to the right (upwards). The ratio of signal amplitudes is indicative of the error in aiming.	

TYPE	DESCRIPTION	APPEARANCE
M	A modification of Type A for accurate range finding. A horizontal time sweep is displaced slightly vertically stepwise. The signal appears as a vertical deflection of the time sweep. An auxiliary device for controlling the phase of the signal or the step is used to bring them into coincidence at which point the device registers range.	
N	A combination of Types K and M	
O		
P	For plan position indication presenting range and bearing in polar coordinates. The signal appears as a bright spot at a radial distance from the center of the indicator proportional to range. Its angular position corresponds to the azimuth angle of the target.	

R. F. Bacher
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Some considerations on DT Screens.

Nautical mile 6080 ft. ratio = 1.152
Statute " 5280 .

Time for transmission and return

$T_R = 10.7R$ (μs) for Stat. miles.

$T_R = 12.3\ R$ (μs) for Naut. Miles.

40 mile sweep gives

$12.3 \times 40 = 492$ (μs) take this as 500 μs.

and take spoke radius as 4 cm.
beam velocity is

$$v_s = \frac{4}{500 \times 10^{-6}} = 8 \times 10^3 \text{ cm/second.}$$

δ = diameter of spot in cm $\sim .04$ cm

time for one spot diameter $\tau_\delta = \dfrac{.04}{8 \times 10^3}$

$\tau_\delta = 5 \times 10^{-6}$ or 5 μsec.

If the pulse length is longer than 5 μs then the area hit will be 2δ or more but the time of excitation will be 5 μs or less for any particular part of the tube.

Outside circumfrence of 4 cm sweep is $8\pi = 25$ cm

Measured in spot diameters this is

$$\frac{25}{4\times10^{-2}} = 625$$

Assume 10 seconds for each revolution and 60 ~ rep. we have $10\times60 = 600$

This shows that at every part of the tube inside of 3.8 cm there is some over lapping of spots which occurs at a given range.

At approx (20 mi.) each area receives two pulses; at 10 mi there are three to four pulses ~~the~~ in the pulse group (separation in time is (1/60) sec.)

continue
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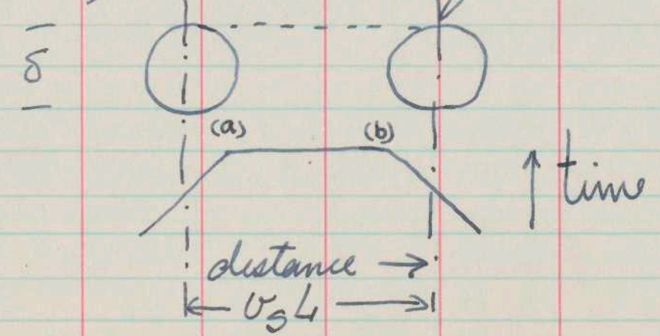
For P-7 guns typical values for constants are $E_{co} = 40$ volts.

$A = 12 \times 10^{-3} (\mu a / v^3)$

10 volts	$i = 12 \times 10^{-3} (10)^3$ for 10 volt drive.		
	$= 12$ μamp.		Max current density
12.5	$i = 23.4$ μamp.	.4	50 ma/cm² to 74
25.0	$= 187.$ μamp.	.8	300 ma/cm² 300

Derivation for general formula for "Q" when the pulse length is longer ~~ com~~ than the time for sweeping one diameter.

Excitation pattern is



$(v_s \delta) = \tau_\delta$

Points between (a) and (b) are bombarded with constant current density j and time τ_δ $\therefore$ $Q_{max} = j\,\tau_\delta$

If τ_n ~~L~~ $< \tau_\delta$ which may be the case for noise (τ_n = duration of noise pulse) and L = duration of signal pulse, then

$$Q^*_{n\,m} = j\,\tau_n \text{ for noise}$$

$$Q_{sm} = j\,L$$

Conversion for j

$$j = \frac{4i}{\pi\delta^2} = \frac{4AV^n}{\pi\delta^2}$$

where A is grid drive factor and V is grid drive and (n) is exponent. (2 or 3)

$$\tau_\delta = \frac{\delta}{v_s} = \frac{\delta}{\left[\dfrac{r_{max}}{T_{max}}\right]} = \frac{\delta}{\left[\dfrac{r_{max}}{12.3\times10^{-6}\,R}\right]}$$

where R = range in nautical miles

r_{max} = max radius of sweep.

Return to

$$Q_{max} = j\,T_\delta = \frac{4i}{\pi\delta^2}\cdot\frac{12.3\times10^{-6}R\,\delta}{r_{max}}$$

$$= 15.6\times10^{-6}\,\frac{iR}{\delta\,r_{max}}$$

or

$$Q_{max} = 15.6\times10^{-6}\,\frac{R}{\delta\,r_{max}}\cdot AV^n$$

Example for $V = 12.5$ volts.

$R = 40$ miles

$r_{max} = 4$ cm

(Graph 24) $\delta = .025$ (this is a function of V)

$n = 3$

$A = 12\times10^{-9}$ amp/(volt)^3

$V = 12.5$

$$Q_{max} = \frac{15.6\times10^{-6}\times\cancel{40}^{10}\times\cancel{12}^{6}\times10^{-3}\times12.5^3\times10^{-6}}{\cancel{25}_{2}\times10^{-3}\times\cancel{4}}$$

$$= 0.146\times10^{-6}\ \text{coulombs/cm}^2$$

With $V = 25$ volts

$$\delta = 26 \times 10^{-3} \text{ cm.}$$

$$Q_{max} = 1.12 \text{ micro-cou./cm}^2$$

With limiting ap. gun. PN-44

$$A = .28 \ \mu\text{amp/volt}^2$$

$$\delta_{12.5} = 12 \times 10^{-3} \qquad \delta_{25} = 13 \times 10^{-3}$$

$\frac{AV^2}{\delta} 156 \times 10^{-6}$

4360 $\quad 10^{-5} \quad \times 13.02$

$$_{12.5}Q_{max} = \frac{156 \times 10^{-6} \times 28 \times 10^{-8} \times 12.5^2}{12 \times 10^{-3}}$$

$$= 0.57 \text{ micro coul./cm}^2 \qquad (12.5V)$$

$$_{(25)}Q_{max} = 2.1 \ \mu c/\text{cm}^2$$

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PN-44 Beam currents for two cases above are:-

$i_{12.5} = 28\times10^{-8} \times 12.5^2 = \underline{\underline{44}}$ μamp.

$i_{25} = 176$ μamp.

For P-7 gun.

$i_{12.5} = 23.4$ μamp.

$i_{25} = 187$ μamp.

A 2000~ vert sweep ~~of~~ 4 cm would give a writing speed of about

$$v_s = \frac{4\times2000}{} = 8000 \text{ cm/sec}$$

which is about the same as a 40 mile PPI sweep.

If lines are to be near or slightly over lapping then horizontal velocity must be

$$v_H = f_v \delta \doteq 2000 \times .02 = 40 \text{ cm/sec.}$$

On this basis one ~~cm~~ of tube would be covered in $\frac{1}{40}$ sec.

In $\frac{1}{30}$ sec the width covered would be 1.33 cm.

Arrange to have ~~deflection~~ 30~ synchronized sweep on horizontal and 1500 or 2400~ sweep on vertical (tie in with P-7) ~~or else 2100~ sweep sum from P-7 puls~~

Synchronize so that beam is on outside of tube for specified time and then jump to test position for 1/30 sec. This is same as present equipment except that deflection modulation is used instead of amplitude modulation.

$$Q = \frac{i}{30 \times a}$$

Assume $a = 4 \times \frac{2400}{30} \times 2 \times 10^{-2} = 6.4\ cm^2$ about 13% of surface

about 200 cm² per second.

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Cont 10/7/43 1

For $Q = .52 \times 10^{-6}$

$i = 15 \times 6.4 = 100 \mu amp.$

~~the~~ For $Q = 2$

$i = 400 \mu amp.$

Summary of formulae.

$$v_s = \frac{r_{max}}{12.3 \times 10^{-6} R} \qquad R = \text{nautical miles.}$$

$$\tau_\delta = \frac{\delta}{v_s} = \frac{12.3 \times 10^{-6} R \delta}{r_{max}}$$

$$j = \frac{4i}{\pi \delta^2} = \frac{4AV^n}{\pi \delta^2}$$

$$Q_{max} = j \tau_\delta = 15.6 \times 10^{-6} \left(\frac{R}{r_{max}}\right) \cdot \left(\frac{AV^n}{\delta}\right)$$

P-7 gun. ($R = 40$ mi. $r_{max} = 4$ cm)

$n = 3$ $A = 12 \times 10^{-9}$ amp/volt^3 | $n = 2$ $A = 28 \times 10^{-8}$

V	δ	i	Q	V	δ	i	Q
12.5	25×10^{-3}	23.4	0.15	12.5	12×10^{-3}	44	.57
25	~~187~~ "	187	1.12	25	$13 \times$ "	176	2.1