What Constitutes a Mathematical System?

Connected with every scientific discipline, there is an ancillary science whose purpose it is to determine, define, and arrange the fundamental conceptions of the discipline. This ancillary science is called the phenomenology of the branch of learning to which it pertains. As to the value of phenomenology in general, very different opinions have been put forward. There are some, of whom Husserl is perhaps the most distinguished representative, who would transfer to the domain of phenomenology the determination of many of the most fundamental questions of philosophy; while there are others, with whom the more speculative interests are but feebly developed, who plunge into the experimental or technical portions of their studies without so much as a glance at the intrinsic nature of the conceptions which they use in their analyses. The proper path, however, lies between these two extremes. There is perhaps no performance of the human intellect that is at once so futile and so useless as a catalogue which is not at the same time something more than a catalogue. Nevertheless, when we stop to take stock of the notions pertaining to an investigation, we often find some which, on analysis, show *lines of* a *sort of* grain, as it were, in such a manner that an investigation progresses in a manner at once more simple and more fruitful if it

goes with this grain than if it goes against it. What I propose to do in this paper is to exhibit this grain in the case of those recent investigations into the constitution of mathematical systems that go by the name of postulate-theory.

This discipline concerns itself with the deduction of the properties of a mathematical system from a few simple properties of certain terms and relations belonging to the system, and the consequent specification of the system as a whole. It has developed an elaborate classification of sets of postulates, according as the constituent postulates are consistent or inconsistent, according as they specify uniquely the form of the system they determine, or merely class it among a certain set of systems, and according as the several postulates are logically independent or not. This last notion has been developed to a high degree of completeness by Professor E. H. Moore, who takes account, not merely of the logical relations of dependence between all the postulates, taken positively, but between all the postulates of a system or their negations. All these investigations, of course, are perfectly legitimate, but what I wish to question in this paper is whether the distinguished mathematicians who have carried out these studies have really been pursuing a method germane to their subject-matter, and have touched on the points of the most fundamental importance for the determination of a mathematical system.