

are no omniscient beings. Now, the author gives no grounds whatsoever for belief in the proposition that there are omniscient beings, and indeed, it is extremely improbable that there are beings which are omniscient in the sense that they possess every possible experience, or even every possible kind of experience, which are the only senses of "omniscient" that are relevant in this context. In addition to this inadequate argument on the basis of logic against the view that religion depends upon ignorance, several more satisfactory arguments to the same effect are urged on the basis of common sense, and it is shown that neither the rise and fall of creeds with the growth of knowledge nor the more intimate changes induced by the same cause in the religious emotions suffice to prove the statement that "ignorance is the fons et origo of the religious emotions", which Professor Keyser categorically states to be false.

Now comes the crucial point of the book: the statement, namely, that "the Rational implies and reveals the Superrational", and that the universe is not exhausted by the domains of sense and of reason, but contains "a realm into which logic cannot go." To support this,

the author introduces an argument from a mathematical analogy - that of limit. He points out the fact that a sequence of terms, such as the sequence of inscribed regular polygons in some circle, taken in order of magnitude, may indicate as its limit a term - the circle - <sup>to</sup> which the members of the sequence approach as close as you please, though they never attain it. Another example he gives is that of the square root of two - this is not a rational number, but may be made the limit of a sequence of rational numbers. Professor Keyser claims that a domain of "objects or spiritual entities" may give us a clue to the nature of <sup>those</sup> entities outside it <sup>which</sup> are limits of sequences of entities contained in the domain, and he points out that the collection of these latter entities may be infinitely richer than the original domain in which the sequences of which they form the limits. In this manner he claims, objects of reason, such as geometrical shapes, are limits of sequences of objects of sense, such as the forms that we see, and further, entities belonging to the superrational form the limits of sequences of objects belonging to the realm of the rational, while the realm of the superrational may still be richer than that of the rational. As examples of objects thus defined,

the universe, considered as a class including, among other things, all classes as its members, is mentioned, and, on the other hand, such notions as that of omniscience.

In this whole discussion, the author's arguments are by no means adequate. One of the chief services of the modern mathematical logicians is to have pointed out that no entirely new realm can be discovered as the limiting realm of regions already given. For example, it has been shown by Cantor, Dedekind, and Weierstrass that one cannot say that a sequence of rational numbers has an irrational limit until, by a method not involving the use of the notion, "limit", one has already obtained access to a domain including both the rational and the irrational numbers. All that the method of limits can do is to lead us from the more explored to the less explored parts of a region which is previously given as a whole; nothing on earth could justify us in saying that the sequence  $1, 1.4, 1.41, 1.414, \dots$  has the limit  $\sqrt{2}$ , or, indeed, any limit whatsoever, did we not already have the knowledge of a system including both the members of the sequence and  $\sqrt{2}$ . The method of limits can lead us from one part to another

21

of a single rational domain, but it certainly cannot lead us from one rational region to another - much less from the rational to the superrational. The introduction of the notion of limit in the discussion of the relation of the rational to the superrational, though it may be a suggestive metaphor, should be nothing more than a metaphor.

Furthermore, the particular examples which the author chooses as illustrations of things superrational which are limits of sequences in the realm of the rational are unfortunate. He tries to show the superrationality of the universe in ~~the following manner~~ in accordance with an argument given <sup>(for a different purpose)</sup> by Mr. Bertrand Russell, whose name is again accidentally omitted in this context. ~~The universe, he says, as it includes everything, must include every class: that is, it is a class which contains as members all classes. No class, however, as he claims, can be a member of itself. Therefore, the universe cannot be a member of itself. On the other hand Mr. Russell's form of the argument deals with all classes that are not members of themselves - he shows that the class of all classes that are not members of themselves, if it is a member of itself, cannot be a member of itself, while if it is not a member of itself, it must be~~

22

a member of itself. From this Mr. Russell draws the conclusion that it is illegitimate to speak of a class either as a member of itself, or as not a member of itself, and that any "universe" to which one attributes classes which either are or are not members of themselves, does not merely fail to belong to the realm of the rational, but cannot belong to any domain whatsoever - not even to that of the superrational. However, on the assumption that no class is a member of itself, in some perfectly legitimate sense, and that the universe is the class of all classes, after stating what is obviously intended to be Mr. Russell's argument in a form in which it is <sup>unnecessarily</sup> deprived of all its cogency, Professor Keyser comes to the conclusion that the universe is superrational, on the basis of this argument. He claims that the universe is the limit of a sequence of rational entities in that it is the limit of a sequence of classes that are amenable to the processes of logic, though he leaves us utterly in the dark as to just what this sequence is. He argues on the basis of common sense against any claim which might be raised that the universe does not exist as a whole. This last point is the Achilles' heel of his argument. The tautologous truth that every existent thing exists is no justification whatever for the assertion of the proposition that there is some single aggregate which contains all existent

things. Professor Keyser must know how little the first verdict of common sense is to be trusted in mathematics: why should he put more faith in it at this point where the common sense of the greatest living mathematicians has led them to diametrically opposite conclusions? Furthermore, there is no reason whatsoever that will bear the slightest examination which can furnish an adequate justification of our belief in the existence of a class of all classes.

After considering a number of supposed entities of the same general character as the class of all classes, which we have <sup>even</sup> less reason to consider superrational and existent than the latter, Professor Keyser comes to the notion of omniscience. He claims that not only is the actual possession of infinite knowledge beyond the reach of man, but even that the human intellect cannot grasp the mere idea of a being who knows all, and therefore has neither the need nor the ability to acquire knowledge, to think. The author gives no argument whatever to show the existence of an omniscient being, ~~so that his claim that omniscience is~~ and nowhere gives a precise definition of omniscience, so that his claim that omniscience is an example of a superrational entity belonging to the boundary of the rational is hardly established. Furthermore, there is nothing beyond the grasp of the human reason in the idea

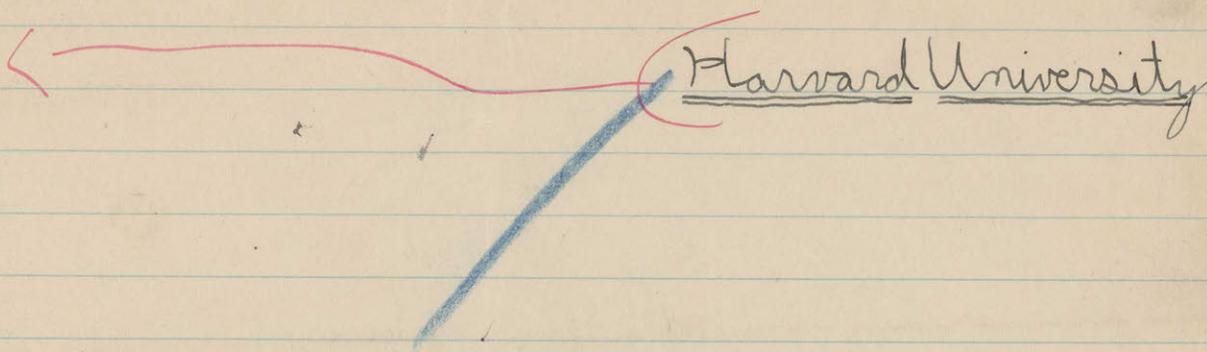
24

of a being who knows all, and therefore does not think nor need to think, or if there is, our author has not pointed it out.

After finally including absolute beauty and absolute love in the category of those things which are superrational, yet form limits to the world of the rational, Professor Keyser brings "Science and Religion" to a close. The book, notwithstanding its occasional technical inaccuracies, bears witness to an unusual breadth of interest and comprehensiveness of view of the author, and gives the non-mathematical reader a real insight into the relation between the scope of human knowledge and that wonderful realm of infinities which has been opened up by the modern mathematicians.

Norbert Wiener,

Harvard University.



## SCIENCE AND RELIGION.\*

Ever since the time when the Handmaiden of Theology went into business on her own account, she has had rather strained relations with her former mistress. The ways of philosophy and religion have lain apart from the beginning of the seventeenth century, when the Aristotelian tradition in physics gave way before the experiments of Galileo and the Thomist tradition in metaphysics gave way before the philosophic doubt of Descartes. In a large measure, the divergence between philosophy and the more orthodox forms of religion has been due to the criticism of the religious concepts of omnipotence, superrationality, omniscience, and so on, in accordance with the new standards of criticism established with regard to the concepts of natural science and mathematics. The old idea of creation has made way for some type of an evolutionary theory; the old belief in miracles is replaced by some sort of a compromise with the law of the uniformity of nature; the old omnipotent, omniscient, personal God is transformed into the Nature of the pantheist, or else is shorn of some measure of his wisdom or his might.

Now, these criticisms of religion date back a fairly considerable time. Within this period science and its standards have not remained immutable. As a matter of fact, many notions- those of infinity and continuity, for example- which presented to the older school of philosopher and mathematician what appeared to be insuperable difficulties, have received a thorough and consistent discussion within the last half-century, and largely within the last decade. These notions are among those which have played the most important part in the criticism of religious concepts, so that it is only natural to expect to find them useful in throwing a new light on the relation of science and religion. This task has been attempted by Professor C. J. Keyser of Columbia, in an essay published as a book,

---

\* The greater part of this essay appeared in the Journal of Philosophy, etc. as a review of Professor Keyser's book on Science and Religion.

69

by the Yale University Press, and entitled  
Science and Religion: the Rational and the Superrational

In this little book, which represents an address delivered on May 4, 1914, before the Phi Beta Kappa alumni in New York, Professor Keyser gives us a remarkably interesting and stimulating account of the relations between science, religion, and the extent of human knowledge, as seen from the point of view of an expert in the theory and technique of modern mathematics. He starts out by distinguishing scientific from religious knowledge in that the former deals with "a certain group of concepts, together with the relations that bind them into a logically organic whole," while, at bottom, religion is "a complex of emotions felt in their integrity," and "does not belong to the rational domain, does not pertain to the field of logic." To put the same idea into other words, scientific knowledge is connotative, religious knowledge is denotative, and therefore logic, which is the science of the interconnections of connotations, is irrelevant to religion. As the author shows, the scientific knowledge of religion is a far different matter from the knowledge of religion which the religious man possesses, for, though the subject-matter of the knowledge may be the same in both cases, the former is knowledge *about* its subject-matter, while the latter is acquaintance with its subject-matter. Professor Keyser gives a very interesting and very thorough discussion of this point, though, with perfect justice, he lays considerable emphasis on the fact that in our actual life these two elements—the direct awareness we have of our own feelings, and the scientific, descriptive, connotative knowledge which we may have even of religious experience—are closely intermingled.

The next thesis considered in the book is "that human ignorance is a necessary condition for the existence of religion." Professors Gilbert Murray and Shotwell are mentioned as advocates of this view, which is discussed in connection with the other familiar opinion that human knowledge is destined to develop beyond all bounds. Professor Keyser deems this latter tenet characteristic of the present age, but is not convinced of its soundness. Leaving all questions as to the validity of this view aside, however, he holds that in some more or less metaphorical sense, the amount of the unexplored in the universe—which, according to a view just mentioned, forms a necessary factor for the existence of religion—is infinite. He points out, in accordance with the Tristram Shandy paradox of Mr. Bertrand Russell (~~whose name is accidentally not mentioned in this connection~~), that the infinitude of the unknown in the universe and the fact that at no time will the human intellect have

exhausted it, may be perfectly compatible with the hypothesis that every single fact in the universe will be learned sooner or later. He also indicates that "it is far from evident that, for the intellect of man, every specific knowable is convertible into a known," while he puts forward the further claim: if it be granted that the sum total of human knowledge is not capable of being extended so as to include any part of the universe you please, though the possibility still might remain that the sphere of the humanly knowable is infinite, though not all-inclusive, even this reduced assumption concerning the unbounded extent of the humanly knowable is extremely questionable. He next presents the alternative view that though the sum total of the humanly knowable might be finite, human knowledge might go on forever approaching it asymptotically. How this alternative is to be distinguished from the last is somewhat difficult to make out, since, by the author's own confession, we are using terms of measurement such as "finite" and "infinite" in an extremely metaphorical and loose sense.

Professor Keyser now returns to the discussion of the hypothesis that the humanly knowable is infinite and includes the whole universe, combined with that assumed throughout up to this point: that religion depends upon human ignorance. He claims that it is possible for the *quantity of the unknown*, which has been assumed to form the basis of religion, to remain absolutely unchanged, on account of the infinitude of the unknown, though every item of fact in the universe becomes known at some time or other. He brings out the difficulties which would ensue were the future duration of the human race on this planet finite, or were time cyclical in its course, and not, so to put it, rectilinear. Herewith he closes the first and most satisfactory part of his argument, which, even if it is too metaphorical to have settled anything finally, is a most interesting discussion of the relations of the known to the unknown. The conclusion is that under none of the hypotheses already considered does any conflict arise between the "limitless progressibility of human knowledge" and the view that religion "essentially depends for sustenance upon human ignorance."

This latter view is now denied. Professor Keyser's chief reason for denying it is by no means satisfactory. It is this: an omniscient being, on the theory that religion depends on ignorance, having no ignorance, can have no religion. But, says the author, religion furnishes one with certain experiences that the non-religious being could not have, so that if a being is omniscient, and consequently possesses all possible experiences, it must possess these experiences, and must consequently have a religion. Professor Keyser sees a contradiction in these two conclusions—that an omniscient being, on the one hand, must not have a religion, and that, on the other hand, an omniscient being, must have a religion—and argues on the basis of this contradiction that the theory that religion depends on ignorance is false. But this argument is fallacious, for the two conclusions in question are perfectly compatible, *provided that there are no omniscient beings*. Now, the author gives no grounds whatsoever for belief in the proposition that there are omniscient beings, and indeed,

it is extremely improbable that there are beings which are omniscient *in the sense that they possess every possible experience, or even every possible kind of experience*, which are the only senses of "omniscient" that are relevant in this context. In addition to this inadequate argument on the basis of logic against the view that religion depends upon ignorance, several more satisfactory arguments to the same effect are urged on the basis of common sense, and it is shown that neither the rise and fall of creeds with the growth of knowledge nor the more intimate changes induced by the same cause in the religious emotions suffice to prove the statement that "ignorance is the *fons et origo* of the religious emotions," which Professor Keyser categorically states to be false.

Now comes the crucial point of the book: the statement, namely, that "the rational implies and reveals the superrational," and that the universe is not exhausted by the domains of sense or reason, but contains "a realm into which logic can not go." To support this, the author introduces an argument from a mathematical analogy—that of limit. He points out the fact that a sequence of terms, such as the sequence of inscribed regular polygons in some circle, taken in order of magnitude, may indicate as its limit a term—the circle—to which the members of the sequence approach as close as you please, though they never attain it. Another example he gives is that of the square root of two—this is not a rational number, but may be made the limit of a sequence of rational numbers. Professor Keyser claims that a domain of "objects or spiritual entities" may give us a clue to the nature of those entities outside it which are limits of sequences of entities contained in the domain, and he points out that the collection of these latter entities may be infinitely richer than the original domain in which lie the sequences of which they form the limits. In this manner, he claims, objects of reason, such as geometrical shapes, are limits of sequences of objects of sense, such as the forms that we see, and further, entities belonging to the superrational form the limits of sequences of objects belonging to the realm of the rational, while the realm of the superrational may still be richer than that of the rational. As examples of objects thus defined, the universe, considered as a class including, among other things, all classes as its members, is mentioned, and, on the other hand, such notions as that of omniscience.

In this whole discussion, the author's arguments are by no means adequate. One of the chief services of the modern mathematical logicians is to have pointed out that no entirely new realm can be discovered as the limiting realm of regions already given. For example, it has been shown by Cantor, Dedekind, and Weierstrass that one can not say that a sequence of rational numbers has an irrational limit until, by a method not involving the use of the notion, "limit," one has already obtained access to a domain including both the rational and the irrational numbers. All that the method of limits can do is to lead us from the more explored to the less explored parts of a region which is previously given as a whole; nothing on earth could justify us in saying that the sequence 1, 1.4, 1.41, 1.414, . . . has the limit  $\sqrt{2}$ , or, indeed, any limit whatsoever,

72

did we not already have the knowledge of a system including both the members of the sequence and  $\sqrt{2}$ . The method of limits can lead us from one part to another of a single rational domain, but it certainly can not lead us from one rational region to another—much less from the rational to the superrational. The introduction of the notion of limit in the discussion of the relation of the rational to the superrational, though it may be a suggestive metaphor, should be nothing more than a metaphor.

Furthermore, the particular examples which the author chooses as illustrations of things superrational which are limits of sequences in the realm of the rational are unfortunate. He tries to show the superrationality of the universe in accordance with an argument given for a different purpose by Mr. Bertrand Russell, whose name is again accidentally omitted in this context. Mr. Russell's form of the argument deals with all classes that are not members of themselves—he shows that the class of all classes that are not members of themselves, if it is a member of itself, can not be a member of itself, while if it is not a member of itself, it must be a member of itself. From this, Mr. Russell draws the conclusion that it is illegitimate to speak of a class either as a member of itself, or as not a member of itself, and that any "universe" to which one attributes classes which either are or are not members of themselves, does not merely fail to belong to the realm of the rational, but can not belong to any domain whatsoever—not even to that of the superrational. However, on the assumption that no class is a member of itself, in some perfectly legitimate sense, and that the universe is the class of all classes, after stating what is obviously intended to be Mr. Russell's argument in a form in which it is unnecessarily deprived of all its cogency, Professor Keyser comes to the conclusion that the universe is superrational, on the basis of this argument. He claims that the universe is the limit of a sequence of rational entities in that it is the limit of a sequence of classes that are amenable to the processes of logic, though he leaves us utterly in the dark as to just what his sequence is. He argues on the basis of common sense against any claim which might be raised that the universe does not exist as a whole. This last point is the Achilles's heel of his argument. The tautologous truth that every existent thing exists is no justification whatever for the assertion of the proposition that there is some single aggregate which contains all existent things. Professor Keyser must know how little the first verdict of common sense is to be trusted in mathematics: why should he put more faith in it at this point where the common sense of the greatest living mathematicians has led them to diametrically opposite conclusions? Furthermore, there is no reason whatsoever that will bear the slightest examination which can furnish an adequate justification of our belief in the existence of a class of all classes.

After considering a number of supposed entities of the same general character as the class of all classes, which we have even less reason to consider superrational and existent than the latter, Professor Keyser comes to the notion of omniscience. He claims that not only is the

actual possession of infinite knowledge beyond the reach of man, but even that the human intellect can not grasp the mere *idea* of a being who knows all, and therefore has neither the need nor the ability to acquire knowledge, to think. The author gives no argument whatever to show the existence of an omniscient being, and nowhere gives a precise definition of omniscience, so that his claim that omniscience is an example of a superrational entity, belonging to the boundary of the rational, is hardly established. Furthermore, there is nothing beyond the grasp of the human reason in the *idea* of a being who knows all, and therefore does not think nor need to think, or if there is, our author has not pointed it out.

After finally including absolute beauty and absolute love in the category of those things which are superrational, yet form limits to the world of the rational, Professor Keyser brings "Science and Religion" to a close. The book, notwithstanding its occasional technical inaccuracies, bears witness to an unusual breadth of interest and comprehensiveness of view of the author, and gives the non-mathematical reader a real insight into the relation between the scope of human knowledge and that wonderful realm of infinities which has been opened up by the modern mathematicians.

For all that, and notwithstanding the unquestionable need of considering modern mathematico-logical research in the discussion of many theological problems, not one single positive new valid conclusion is added to the existing stock of knowledge. While of course <sup>u</sup> further investigation along the indicated lines might bear ~~positive~~ <sup>tangible</sup> fruit, the comprehensiveness of Professor Keyser's attempt justifies us in saying that ~~as far as~~ <sup>without a more precise re-definition of</sup> the existing notions of religious matters, the status of the dispute between science and religion is not very fundamentally affected by the new mathematics.