On the Nature of sensation. Intensities and gravities
Ir his Manual of Psychology, strut, following stump, gives the following argument to prove the existence of 'mere sensations', or sensations which exist without being known: "... within limits we can vary a stimuli without prosmeing any perceptible crifference in the ofyest cognized. If this variation in the stimulus is accompssived by variation in the sesse-experience, then we have a variation in the sense-esperiense which maker no difference to cognition. There is a difference in mere sensation, but not in perception. J hat, as a matter of fact, this is so may be demonstrated as follows. We may vary the physical conditions on which the pitch of a musical note depends, so as to produce a graduated scale of routes increasing or decreasing in fish. Sypulolire the series by $P_{1}, P_{2}, P_{3}, P_{4}, P_{5}, \ldots P_{n}$. Now, if the variation of the Physical conditions is sufficiently gradual, $P_{1}$ may be quite indisturuishabe from $P_{2}$ and similarly $P_{2}$ maybe quite indistinguishable from $P_{3}$ and $P_{3}$ fran $P_{4}$. Mire the less, $P_{4}$ will le perceived as distinctly different from $P_{1}$. But this would be impossible unless the change in the physical conditions were accompanied by a change in the sensation, even when the change is imperceptible. If the fitch -sensation? is regarded as identical with the fitch-sintation $P_{2}$, merely because the one note is indistinguishable from the otter, and if in a like manner? 2 is regarsled as identical with $P_{3}$, and $P_{3}$ with $P_{4}$, and soon, then $P_{1}$ must be identical with $P_{n}$, and it would be impossible that any perceptible difference should ever arise".

It is obvious that in this passage, the word sensation is not used to refer to any given sense datum, like this tone 9 am hearing now', but to a property common to a number of senseuses mon the same argument as stout, uses (Isabsted'e translation) Poincare'

data, or to a certain collection of sense-chatr, for were this not so, it would be nonsense even to entertain the hyprothesis that $P_{1}$ and $P_{2}$, erothen dy different punsical unditions, and rewiring under different craunstanas, could he edentionel, in the only justifiavele sente of this much-abused term. Now, sensations, stout says a few pages furthoron, are fischicol states, and be defines a psychical fact asclifferent from apycholarisal font, which is merely a fact of which pay chology most tube account, in that it is a frost of consciousness. so it would seam that ito ut means to maintain that the differonce velween $P_{1}$ end $P_{2}$ is that they are fundamentally clifferent sorta of objects of consuavoness. Stent obviously wishes to assert also that in the sense of 'imssiousnoss' in which they are different sorts of objects of cansciononess, we do nat have the consciousness of their difference: otherwise, his argument in favor of the existence of mere sensations loses its point. Now, as his own argument shows, we know by imperense what the difference between $P_{1}$ and $P_{2}$ is : it is fainlyclear, then, that stout wishes to assert is tat wehnaw what $P_{1}$ and $P_{2}$ are by some other way than by inference: he doer not merely want to show that the sort of inference by which we Alow $P_{1}$ and $P_{2}$ is not the same as that by which we kern their difference. Now he give absolutely no argument whit ever which is inconsistent with the supposition that we tawoit $P_{1}$ and $P_{2}$ only byimferende. So, in order to discover whether Stouts argument is valid we are led to the important questions corder char sensation -intensity or quality? and what is pow a sensation - intensity or quality? I hose are the questions I propose to discuss in the present paper. a little introspection will convince one that it
is only through the comparison of one sense-datum with another that we are able to arrange our sensentions in stories of intensity and quality. 9 on never able to say that a given note is of a certain pitch without comparing it with other notes, and even in the case of those who have what is called a sense of absolute fitch, their powers of naming immediatsy the fitch of a rote which they hear rests probably in the extreme tenacity of their auditory memory which renders them able to compare a note which therg has now with one which theypheard a considerable time ago. What 9 man to say when $I$ assert that a sense-catum $\underline{x}$ is of a given intensity is that it bears some relation with, a given standard sense -datum of the same ort which I shall call óeguipotentingewt ant a da similes statement holds goal in the case of sersary qualities of a given bind, suchasifth. I $t$ is merely the prejudice of common suse and the aristotelian logic in favor of propositions involving predicates, and against relational propositions, which makes me use in every-day life the phrase of the same intensity as, fin s 'aequipotent with', and af makes moxppeur to derive this relation from the intensities themselves.

But stump ls arguments prove if they prove anything at all, that we do not recognize discatly that two sense dato are of th es
 Oof a thossitive relation: that is, if, when $a$ is asquiphtent with $b$, and If is aequifotent with $\leq$, a must be aequifatent with $\leq$. Mourver, the argument stump t gives shows us exactly how we are able to find out undentur sense.data are tonally aequipebtent, and when they are not I he fact that a sense-datum of fitch $P_{1}$ and another of fitch $P_{2}$ are not aequippotent is discovered when we find a tone-daturn of fitch, say, $P_{k s}$
which is noticed to be of higher pitch than a tone datum of pitch $P_{1}$, but which is not of noticeably different fitch than a cation of fitch $P_{2}$. Now, since the only criterion which we are ever given for the subliminal difference in intension org- two sensecduta, other then tests which bring in such extraneous mutters as the physical intensity of the stimulus producing a given sumps. datum, is that these is same datum distinguishable in intensity or quality fin from one of the saiginal sense-data, and indistinguishable from the other, and since the sole way we pave of discriminating the genuine requpptence of duro data from the relation whish holds between them when they are not noticeably different as to intensity ollas to quality is that in the case of true cequifotence there can the tow subliminal difference between them in respect to intensity or quality, we may define the aequifotence of a and 15 in a given respect of that relation whish hoods between them when all the dates which can be discriminated frame in that respect can also he discriminated fran b, and vice versa. Moreover, if Weber's law hols ls good, or any tach how which makes it impossible for the upper ar lower limina of diserimingianfirm two suse-data procured by stimuli of different intensity to coincide, then we con easily show that this definition of aequipotence has the desirable property of rendering two sense-data dequifpotent when and only when they are produced by stimuli of the sure physical intensity.

We have, then, shown how the notion of is sens ation.interssity
 date of the appinipistic sort when one is noticeably different from the other Chis selatio in regard to its intensity or quality. I his relation is manifestly dovived forest from that which holds betaine ouse datum and another when the first is notiviglly of greater intensity, or higher fitch, ban more vivid lues, the. than the second. I am not
meaning to say that wermay not have same soot of a direst experience that a is of different intensity or fitch, etc. from $b$, apart from any experience that a is of greater or of less intensity, higher or lower pitch than $k$, butut seems to me fairly ahiaus that we usually havesome experience of the direction of the difference between a and 1 when we experience them as different, and that often what we mean to assent in saying that they are felt to be cifferent is hat one is experienced as noticeably more intense or of roviseably higher pitch, etc., than the other. I hese relations of noticeably greater and less in intensity, noticeably higher and lower in fitch, etc. whether they are given as such in our experience one are certainly for more primitive than any experience we er have of a sensation intensity. of we take the latter as a property: namely, the property of being alequipotent with a given sensedatum in a certain respect, then whatever wen whereabout at is simply a paraphrase for certain things we know about she revelation of 'noticeably more intense than', since we know the relation of aequifotence only as a function of the latter relation. If a sensation intensity be taken as a class of sense-data, determined by the relation of aequipatense with a given sense-datum, then, since, as Mar. Russell has pointed out, a proposition about a class does not really emsern the class at all butonly the property by whish the class is defined, our knowledge of sensation - intensities is even more a Arawledge by inference, and the sens rationintensities themselves are even further from beng presented tom thywerepowities seams to me, moresver, that the sort of inference through which we know that a given serse-intensity is different from another is not of a fundmentally different character than that through whish we lanow anything at all about a sensation-intensity. Since sensation-intensities, and in the the only sense in which anything which strut or stump f says demands that they exist, are not ever presented to us, it follows that stands argument in favor of 'mere' sensations is utterly without any punt


IJ his article is based on arne mathematicol－logieal work of mine，whim was published，under the title， Studies in synthetic Aggie，in the Proceeding of 达 Cambridge philowophicond society，vol xVII，taut 1.
 2）Bor kI，chapter ，，$\xi_{3}$ ．，Ed l．
argument to prove the existence of．＇mere＇sensations，MA semeations which exist without
 Inference in the obyest uspusid．If this variation in the stimulus is a complanied by ovinition in the sense．Scpurieme，then we have a variation in the senses sheree whilimbes no difference to cognition．I tors is a difference in mere sensation，fut not in jeraphtion，Tut， asa matter of foot，this is so may be demonstrated as follows．We may vary the Physical conditions on which the fitch of a musical minted depponde，sons Ir produce a grmiunted scale of notes，increaing or decreasing in piton．Syymblize this series by $P_{1}, P_{2}, P_{3}, P_{4}, P_{5}, \ldots . P_{n}$ ．Now，id，the variation in the qumsical anditions is sufficiently gradual，$P_{1}$ may be quite indistinguishable from $P_{2}$ ，and similarly $P_{2}$ may the quite indistingrishatle from $P_{3}$ and $P_{3}$ pan $P_{4}$ ．Tone the less，$P_{4}$ will he perceived as distinctly different from $P_{1}$ ．But this would be impressible unless the change in the physical conditions were accompanied by a change in the sensation，ven when the change is imperseftible，of the pitch－sersation $P$ ，is regarded as intention l with the fit sensation $P_{2}$ ，merely because the its one note is indistinguishable from theothes，and if in a
like manner $P_{2}$ is regarded as identical with $P_{3}$, and $P_{3}$ with $P_{4}$, and so on, then $P_{1}$ must be identical with $P_{x}$, and it would be impossible that any perceptible difference should ever arise".

I he meaning of this passage is a little hard to follow, owing to the ambiguity of the terms, 'sensation', 'note', and identity' If 'islentity' be understood as numerical identity, then $P_{1}, P_{2}$, etc. must be regarded as species of sense-datin, and not as individual sense-clats - as notes in the sense in whish the chromatic scale is a collection of notes, and not in the sense in which this particular trene-datum presented to me when? now press this piano. key is a note-foir it is manifest that stout regards was worthy of refutation the view that $P_{1}$ and $P_{r}$ are identical, and means to maintain that two instances of the same species of note are 'identical' in some sense in whish two sistonces of different notes are not, whereas if $P_{1}$ and $P_{2}$ are individual tones, evoked, as stout says, by different physical conditions, whether $P_{1}$ and $P_{2}$ are of the some fitch or not, $P_{1}$ must be numerically differsint from $P_{2}$, and the hypothesis that $P_{1}$ and $P_{2}$ are not numerically different ode is obviously force as to be unworthy of a refutation. If, on the other hand, $B_{1} P_{1}$ and $P_{2}$ be regarded as nit is individual sense-clata, the only of identity which any sane man would ever assert to hold between them would he some kind of govalitative identity, and the difference between them in which stout is interested is a qualitative difference

If Stout means by 'difference' qualitative difference, it is hard to avoid the that he intones to assert in the passage quoted above that the difference whit $P_{1}$ is inferred to have from $P_{2}$ is the same whicence wee perceives $P_{1}$ to hove from $P_{4}$, and not merely a
relation which happens to hold between $P$ and $Q$ whenever $P$ and $Q$ are perceived to becifferent. He wishes to say that what we mean by the relation of difference which we notice to hold between $P_{1}$ and $P_{4}$ is a relation which abs hols , although we only know this by inference) between $P_{1}$ and $P_{2}$. His argument in favor of this conclusion consists in substance solely in pointing out that unless this is the case, we can have two sensations, each qualitatively identical with another sensation, yest qualitatively different from one another. Be denies that this batter circumstance is possible, without, however, giving the grounds for his dermal. of it be said that the impossibility that $P_{1}$ be qualitatively identical with $P_{2}$, and $P_{2}$ with $P_{3}$, while $P_{1}$ and $P_{3}$ are qualitatively distinct, follows from the fort that ' $P$ is qualitatively distinct from $Q$ ' means, ' $P$ possesses a quality which $Q$ does not possess, then stouts argumont implies that what we notice chen we see e that $P$ and $P$ are distinct is that $P_{1}$ possesses some quality absent in Pomp 4 g git, on the other hand we do not derive the relation of qualitative clisferencein same such manner from the relation of logical difference, in order to justify stouts arguments one would have to have recourse to the alleged obviousness of the transitivity of the relation of qualitative identity. Now, to say that it is obvious that the relation of qualitative identity is transitive would, in this connection, be equivalent to asserting
that it needs no proof to show that the relation we notice when we notice $P_{1}$ to be different from $P_{4}$ must be one whose negation is transitive. That is, it must be prions that the a certain relation we notice must have properties dependent on \#t its behaviour where we do not notice it. I his seems to me to be palpably false, and its whole plausibility appears to depend upon un equivocal use of
the terms 'identity' and 'difference'.
Ix order to understand what stout's position
meansif we are to understand $P_{1}, P_{2}$, etc. as species of sense-data, we must see what he means by the term, 'sensation'. Now, sensations, start says?", BQ马R , Chat. 1, §4.
are psychical states, and he defines a psychical fart as different from a puchobaunal which is merely a fact of which psychology must ta be account, in that it is a fact of consciousness? So
$\Rightarrow$ Ibid, Introduction, Chapter 1, §3.
it would seem that stout means tor maintain that the difference between $P_{1}$ and $P_{2}$ is that they are essentially different sorts of objects of consciousness Stout obviously also wishes to assert that in the sense of 'consciousness' in which they are different sorts of obyists of consciousness, we do not have the consciorsones of their difference: otherwise, his argument in favor of the existence of 'mere' sensations loses' its point. Now, as his own argument shows, we know by inference that these is a difference between $P_{1}$ and $P_{2}$ : it is fairly clear, then, that stout wishes to assert that we know $P_{1}$ and $P_{2}$ in some other way than ky inference - he does not merely care to show that the cletails of the process whereby we detain an inferential knowledge of $P_{1}$ and $P_{2}$ ra not the same as tho bl of the process through which we lean their difference. Whether $P_{1}, p_{2}$, etc. be mental events or kinds of mental events, therefore, stouts argument in favor of the existence of 'mere' sensations assumes that we know -sensation - qualities by means of a process involving no inference. Now, stout gives absolutely no argument to show that we pave other than inferential knowledge of sensation-qualities. Therefore, in order to discover whether start's arguments in favor of the existence of 'mere' sensations are valid, we are

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- led to the important questions, how do we know a sensation quality or intensity's and, what is the nature of a sensation-qualitya As the theories of ranges of sensation-qualities and ranges of sensation-intensities are precisely parallel, 9 shall now use the ore term in my discussions, and now the other, and shall draw my examples from whichever field may be most convenient. Everything I say about the one will, mutatis mutandis, be true of the other. For the purposes of this paper, such circular ranges of qualities as the periphery of the base of the color-pyramid must be regarded as if consisting of several non-circular portionsfor example, we madivide the base of the color-pyramid, as is indeed natural to do, into the region between red and yellow, the region between yellow and green, the region between green and be, and the region between the and red. We shall also regard it as possible for a given sense-datum to belong to several ranges of totality land
intensity? These are the first questions ? propose to cliscuss in the present paper.

A little introspection will convina one that it is only through the comparison of sense-clata that wee are able to arrange them in stooges of intensity and quality.? am never able torsay that a given note is of a certain pitch without compraing it with other notes, and even in the case of chase who have what is called a sense of absolute fitch, their power of naming immediately the pitch of a note which they hear rests probably in the extreme tenacity of their auditory memory, which renders them able to compare a note which they hear now with one which they heard a considerable time ago. What I mean to say when

I assert that a sense-datum $x$ is of a given intensity is that it bears some relation, which 9 shall call isocracy of a given sort, ts a certain standard sense-daturn of the same kind. 95 is merely the prejudice of common sense and the Aristotelian logic in favor of propositions involving predicates, and against props= sitions involving relations, which makes me use in every-dory life the phrase, 'of the same intensity as', instead of some such expression as 'socratic with', and causes me to derive this relation from some such hypothetical entities as intensities.

But the arguments of stump and of stout prove, if they prove anything at all, that we do not recognize directly that two sense-clata are isocratic, if we mene by isocracy a transitive relation: that is, if a must be isocratic with $c$, when a is socratic with 15, and f is isccratic with $\varepsilon$. Moreover, if we mean dy isocracy a transitive relation, the expample stout discusses shows us how we can derive it from the relation n wo sur ronde betupeny a given kind, when we say they as noticed to-bedifferent. I Sis relation, which? shall term hetersphany. of a given sort, or some other relation formally equivalent to it, are the only relations we have any right lo say we notice when we observe that two sense-data ofacertain range are of different intensities, for we know of no process of inference which will entitle us to conclude that the identical relation which we notice to hold between a and f, holds although we do mort notice it, between $C$ and $d$. Now, the fact that the tonal sense-data $P_{1}$ and $P_{2}$ are not isocratic inferred by stout from the fact
is that there is some tonal sense-datum $P_{k}$, which is heterophanice
to $P_{1}$, but not to $P_{2}$. Moreover, apart from criteria solving such extraneous considerations as that of the physical intensity or nature of the stimuli which produce them, the only real test we have of whether any two sense data, whose absence of isocracy (or) as 9 shall call it, heterscracy) is not demonstrated by this heterophony, are isocratic, is to see whether or not there is same sense-datum heterophonic to one and not to the other. I hat is, our sole made of demonstrating that two sense-clata of a given sort (i.e. two members of the field of a given relation of heterophony) are isceratic with one another, is by showing that all the sense-data wt the one are heterophanic to the other" We may, then, define I gt is possible, it is true, to show that $\underline{x}$ is socratic with $y$ by showing that all the data heterapanic to $x$ are hitesphanic to $y$, without discussing whither all the sense-data heterophanie to $y$ are
heterophasic to $x$. T his condition is, as an empirical fact, equivalent to the above, for, were it root, then we should have arne such condition as the following: $P$, would be a tone datum indistinguishable in fitch from $P_{2}$, whish in its turn, would be indistingushable from $P_{3}$, while there would be a datum, $P_{4}$, noticeably higher in fitch than $P_{1}$ and noticeably bowers than $P_{3}$, while as a matter of fact, no such condilinin everoccurs. Moreover, did such a condition occur, we should certainly not term $P_{2}$ and $P_{4}$ socratic: i. 1. of the same actual pitch. I herepre, it seems obvious that the fact that all the data
heterophanic to $x$ are also heterophwinic to $y$ only proves the isocracy of $x$ and $y$ when combined with the empirical premise that if all the data heterplanis to $x$ are heterophamic to $y$, all the data heterophanie to y are heterophanir to $x$, and hence that two in this case the ultimasiate criterion of the isocracy of $x$ and $y$ is that all the senso-data heteroplanic is either are heterophaxic to the other.
the isoperacy of and and in a given respect as the relation whin h hols between them whin they both belong to the freed of the appropriate sort of heterophony, and when all the sense-data heterophonic to the one are also heterophanie to the other. Moreover, if Webers law holds good, or any similar law which makes it impossible for the upper or lower limina of discrimination from two sense-clata, produced by stimuli of different Musical intensity, or wave-length, te. to coincide, we can easily show that this definition of isocracy has the desirable property of rendering two sense-clata socratic when and only when they arefundued by stimuli of the same physical intensity

We have, hen, shown how the notion of a sersation-intensity or quality of agon range can be analysed in terms of the appropriate sort of heterophony. I he relation of heterophenany is manifestly not itself lanown quite directly by us, but en se a function of the relation whish holds between a and tv when $\underline{a}$ is of noticeably greater intensity thant-, or of noticeably higher pitch than b, on of noticeably more vivid hue than be etc. I hese latter relations I shall term various sorts of hyperfhany. I am not meaning to - "deny that we may have same sort of a direct experience that a
is of different intensity or fitch, etc from $1 \mathbf{V}$, aphent from any experience that a is of greater or of less intensity, higher ar ewer pitas, Doe thank, but it seems tor me fairly obvious that we usually have some experience of the direction of the difference between $\underline{a}$ and $t$ when we experience them as different, that of ter what we mean to assent in saying that they are felt to be culferent is that one is experienced as noticeably more intense, or oof noticeably higher fitch, cts, than the other, and that this sense of noticeably different' is precisely the one which is relevant in our discussion of the analysis of Sensationintensities and qualities. These relations of hyperphany, whether they aregiven as swot in our experience or not, are certainly far mare primitive than any experience we deer have of a sensation-intensity. We have seen thirst such a proposition as, ' $\underline{\underline{c}}$ is of the sensation - intensity $Q$ ', is really a paraphrase for such a proposition as,' $a$ is socratic with the standard sense-daturn, $v^{-\prime}$ if we take the sesation-intersity as being the furperty of is ocracy with bo, then whatever we seem to prow about it is really b noon about 15 and the appropriate sort of isocracy, ar, if we carry our analysis further back, about b- and the athopribate soot of hyherphang. If, as is monetise convenient, we regard a sensaction-intensiti as the class", of all the sense-clata
 sties is aver more thoroughly inferential than in the preceding case, and the sensation-intensities are even furcter from hing presented to us, since, as mr. Russell has pointed out, a proposition about a chase doe not really conses the class at all, but only the forperiny try which the class is defined. Te only sensation- intensities and qualities, then, whose existence is in any way demanded by any thing stent are not presented to us, but are known by inference, whether

- there are any directly known entities which mary be cabled sen sation-intensities and qualities or not. Msvemer, it seems tor me that the sars of inference through which we bor that a givensense-intensity is different from another, is not of a fundamentally distinct character from that through which we know anything at all about a sensation-intensity. Therefore,? can see no cogency whatever in Stouts argument in favor of the existence of 'mere' sensations".

1) It should be noted that even if the whole of stouts argument which we have criticised above were valid, it wolff entitle him to say that there may be variations in sensoryqualities without their correlates in cognition, but not that there are uncognized sense-datar or sensory qualities, which is what he sots out to prove.
If the sensation-intensities and qualities whose structure we have explained above are to have the properties which we usually associate with intensities, and, with must in some way form a series, in which a greater intensity always fallows a smaller one. Interpreting this statement in logical terms, it means that there must be a relation', such that (1) its field is a class 2). Q. Whitehead s Qusselle, Prinsifia Mathematicn, *200-*204 of all the sensation-intensities or qualities of a given kind (i.e., derived from a given relation of hyperphany), (2) if it relates a sensation intensity or quality $\alpha$ to another say $\beta$, we should naturally say that $\beta$ is a greater intensity, ''homing, $\alpha,(3)$ if it relates $\alpha$ to $\beta, \alpha$ cannot be identical with $\beta$, (4) if it relates $\alpha$ to $\beta$ and $\beta$ to $\gamma$, then it relates - $\alpha$ to $\gamma$, and (5) if $\alpha$ and $\beta$ are distinct sensatim. intensities ar
qualities of the appropriate kind, then it either relates $\alpha$ to $\beta$ or $\beta$ to $\alpha$. Let it be noticed that relations of hypheph any, although they satisfy conditions (3) and (4), failto satisfy conditions (1), (2), and (5). 以 ow, then, are we find a relation satisfying all five of these conditions? If we turn back to the argument of stout and stumpl, we shall find a simple answer to this question. $P_{1}$ and $P_{2}$, it will be remembered, were taken not to be directly distinguishable from one anther in fitch, and same $P_{k}$ was supposed indistinguishable from $P_{1}$, but of noticeably higher fitch than $P_{2}$. since the difference between $P_{n}$ and $P_{2}$ is subliminal, while $P_{k}$ is suprabiminally higher and fitch than $P_{1}$, start says that the fitch of $P_{2}$ is really cliffeent from that of $P_{1}$, and he obviously infers also that it is really higher than that of $P_{1}$. The relation which one sensory quality or intensity of a given range bears to another quality or intensity of the same range is, then, regarded by stout as a' lowerthan vI less than 'relation when some sense -datum of the second intensity or quality is not heteraphaine to some datum hyperphanie tr a datum of the first intensity. Since no sense-dature is heterafhanic to itself, our criterion of 'less than' or 'lower than' will apply tor suprationincl intervals as well as to subliminal intervals. It will not, however apply to intervals near the upper threshold of a given quality or intensity of sensations: if $x$ be the highest note of a galton whistle, and $y$ bee subliminally lower nate, there will be no note subliminally higher than $x$ and supratiminally sigher than $y$. However, there will be a nate suprabiminally lower than $x$ and subliminally lowecthan $y$.
Since, then, when $\alpha$ is an intensity subliminally or supraliminally less than $\beta$, either some member of $\beta$ will bear the relation of not being hetersphanic to something hyperphania to a member of $\alpha$, our will be hyper Manic to something not heterophianis to some member of $\alpha$, and since whenever this relation holds hetiveen $\alpha$ and $\beta$, when $\alpha$ and $\beta$ are intensities,
we would always call a less than $\beta$, we may define this latter relation, when it holds between sensation-intensities of the proper kind, as the relation of being less thane, which generates a given series of sense-intensities.

But this brings usto the question, when will this relation actually generate a series of sensation -intensities - when, that is well it have properties (1) - (5). What we have just said proves that it will hove property ( 2 ), and shows abs that it will have property (1), unless there is tame sense-clatum of a given range nat notice ably different in intensity rom in quality from anything - a rempasition which is palpably false. Furthermore, (3) and (5) are invariably satisfied by it. This follows from the fact that if $x$ and $y$ are sensations of the appropriate sort, one of the pint following five conditions, which are all manually inconsistent must be satisfied: (a). Everything heterophanic to $x$ in wy be heteraphanic to $y_{1}$ an wee something hyperfhamic to $x$ may not be heterophainc to $y$.
(c) Something to which $x$ is hyperphanic may not he heterophanictoy
(d) Something hyperphanic to $y$ nay not beheterophanic to $x$ (e) something to which $y$ is hyprepphanic may not be heterophanic to $x$. Ix case (a), $x$ and $y$ are socratic, and, owing to the transitivity of isocracy, if $x$ belongs to an intensity $\alpha, y$ does also, and $\alpha$ is the only intensity to which y belongs. In cases $(b)$ and (e), if $x l_{2}$ of intensity $\alpha$, and $y$ of intensity $\beta, \alpha$ is less than $\beta$. I $x$ cases $(\xi)$ and $(\alpha)$, if $x$ is of intensity $\alpha$ and $y$ of inter site $\beta, \beta$ is less than $\alpha$. Since these two later hypotheses are inconsistent with the first, (3) is satisfied; since either d' is identical with $\beta \sigma$ is less or greater than it, $(5)$ is satrefied
(4) must practically be assumed as it stands,
and it is an assumption which we constantly make without thinking of it. We may, it is tine, assume instead that the relation of either being hyperppanic to something not heteroPhonic to something else or of nothang heteraphanic to something hyperphaine to it is transelve I he assumpt ion in this form is equivalent io supposing that the relation of being either supraliminally or subliminally mare intense than is transitive.


We have seen, then, that under turo as three hypotheses of avery general nature, which are almost certainly satisfied by oh relations of hyperphany, the relations between sensationintensities and qualities which we have defined as being 'less than', 'lower than', etc. will generate serves fogey intensities and ananges of qualities. Let it be noted, however, that sensation-intensities and qualities are not thereby treated as quantities. although we have shaver how the pressure-data, brugtenus dots, eta. thin we experience can be arranged in what we may call their natural order, not only are we rot thereby entitled to speak of one intensity as the sun of other intensities, but wee have not even sampuriched given whereby we may correlate the series of sensation - intensities with the series of the renlor rational numbers, and treat one interval between two sensalon intensities is equal to another. It is true that we are able to state one form of Weber's law in terns of relations of hyperphany. We are able to state that is the hypothesis that if $x$ is a given sense-clatum, eroded by a stimulus of intensity $\xi$, and if $y$ is a sensechitum hyperphanic to $x$, evoked by a stimulus of intensity $\eta$, then, whatever $x$ may be, the minimum possible value, ar lower limit of the passible values, of $\frac{\eta}{C \text { is constant". }}$ $1>$ his is the really correct statements of Weberis low as aphiid to 'just noticeable differences' of sensations it involves no reference to just noticeable
differmess at all, for it cloves not refer tr any thypothetionl first datum whose intensity is noticeably greater than that of 2. I his is an important advantage, as no experiment that ever has beendevised, or could enseivally be durisel, could the adequate to prove the existence of such ada tum. Ibis statement of Weber's law also involves, let it be noted, xor reference $l o$ the constructions which we have exiled sensationintensities.
But it is by no means evident that we should regard the liming of discrimination fran all sense-data of a given sort as equal. If we should define all 'just noticeable differences' as equal, we have not yet found an unambiguous way of assigning a real number tor each sensation-intersitg, which is one of the things we must do to he able to regard sensation intensities as quantities, for we are $x_{0}$ t thereby given any unambiguous method of subdividing the interval between two sensation intensities whose menders are only subliminally chpperent into two equal parts. If, on the other hand, we the same relation which hoods between pairs of sense. data of a giver sort when Ins interval between the members of the first pair is not noticeably different in magnitude from the intensity -interval between the members of the second pair, as primitive in our experience, and derive from this some 'numeration of our sensation intensities of a given kind, than it is a proposition which is not a priori certain and which stands in need of experimental justification in
exactly the some sense in which Webers low, in either of its interpetntions, needs experimental verification, that all y just noticeable difference', tophet it crudely, are equal. Webers two may y well be true in one of these senses and false in the other.

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some method of measurement starting from sen-sation-intervals as vrumbive, and deriv from this a numeration of our sensation - intensities, then the proposition that all just-noticeable differences are equal needs experimental verification in the same sense in which Webers law in either its quantitative or its non-quantitative sense neecls experimental verification, and Weber's low may well be tare in one of these senses yet false in the other. So we see that the prefer interpretation of the notion of sensation - intensity clears up many /voblems connected with Webers low.

In conclusion, 9 wish to contrast my method of handling the frablem of sense timon intensity with that of \& tout and ind stump f. Stout and Strmff see acontradiction in the iftransitivity of the relation, an 'sin distinguishable from', and bay that to risalve this contradiction, we must postulate differences of sexsation-intensity where none are seen. They assume unosrscioushy, that is, that the differences of intensity between sensation2 are $A$ in the first instance differences between sensation-intensitios. We tale the differences of intensity as primitive, and derive the intensities as function e of them. I key have to postulate that the

In his lanual of Psychology, st, following
Stumpf, gives the following argument to prove the existence of "mere sensations", or sensations which exist without being known: "........ within the limits we can vary a stimulus without producing any perceptable difference in the object cognized. If this variation in the stimulus is accompanied by variation in the sense-experience, then we have a variation in the sense-experience which makes no difference to cognition. There is a difference in mere sensation, but not in perception. That, as a matter of fact, this is so may be demonstrated as follows. We may vary the physical conditions on which the pitch of a musical note depends, so as to produce a graduated scale of notes increasing or decreasing in pitch. Symbolize the series by $P_{1}, P_{2}, P_{3}, P_{4}, \ldots . P_{n}$. Now, if the variation of the physical conditions is sufficiently gradual, $P$ may be quite indistinguishable from $P$, and similarly $P_{2}$ may be quite indistiguishable from $P_{3}$, and $P_{3}$, from $P$. None the less, $P_{4}$ will be perceived as distinctly different from $P_{1}$. But this would be impossible unless the change in the physical conditions were accompanied by a change in the sensation, even where the change is imperceptible. If the pitch-sensation $P_{1}$ is regarded as identical with the pitch -sensation $P_{2}$, merely because one note is indistinguishable from the other, and if in a like manner $P_{2}$ is regarded as identical with $P_{3}$, and $P_{3}$ with $P_{4}$, and so on, then $P_{l}$ must be identical with $P_{n}$, and it would be impossible that any perceptible difference should ever arise".

It is obvious that in this passage the word "gen-
sation" is not used to refer to any given sense datum, like "this cone I amhearingnow, but toaproperty common to a number of sease -
data, or to a certain collection of sense-data, for were this not so, it would be nonsense even to entertain the hypothesis that $P_{1}$ and $P_{2}$ evoked by different physical conditions, and occurring under different circumstances, could be identical, in the only justifiable sense of this much-abused term. Now, sensations, Stout says a few pages further on, are paychical states, and he defines a pmpical fact as different from psychological fact, which is of merely a fact, which psychology must take account, in that it is a fact of consciousness. So it would seem that Stout means to maintain that the difference between $P$ and $P$ is that they are fundamentally different font of objects of consciousness. Stout obviousDy wishes to assert also that in the sense of "consciousness" in undrenot hour the unsciruness which they are different sorts of objects of consciousness, $n$, difference: otherwise, his argument in favor of the existence of "mere" sensations loses its point. Now, as his argument shows, we know by inference what the difference between $P_{1}$ amd $P_{2}$ is: it is fairly clear, then, that Stout wishes to assert that we know what $P_{1}$ and $P_{2}$ are by some other way than by inference: he doe not merely want to show that the sort of inference by which we know $P$ and $P$ is not the same as that by which we know their difference. Now, he gives absolutely no argument whatever which is inconsistent with the supposition that we know $P_{1}$ and $P_{2}$ only by inference. So in crier to discover whether Stout's argument is valid we are led to the important questions, how do we know a sensation-intensity or quality? and, what is a sensation-intensity or quality? These are the questions I propose to discuss in the present paper.

A little introspection will convince one that it is only through the comparison of one sense-datum with another that we are able to arrange our sensations in stages of intensity and quality.

I am never able to say that a given note is of a certain pitch without comparing it with other notes, and even in the case of those who have what is called a sense of absolute pitch, their power of naming immediately the pitch a note which they hear rests probably an the extreme tenacity of their auditory memory which renders them able to compare a note which they hear now with one which they heard a considerable time a. What I mean to say when I assert that a sense-datum $\underline{x}$ is of a given intensity is that it bears some relation to a given standard sense-datum of the same sort which I shall call aequipotence in a given respect and a similar statement holds good in the cas of sensory qualities of a given kind, such as pitch". It is merely the prejudice of common sense and the Aristotelian logic, in favor of propositions involving predicates, and a ainst"relational propositions, which makes me use in every-day life the phrase "of the same intensity as" instead of some parase such as "aequipotent with", and makes me appear to derive this relation from the intensities themselves.

But Stumpf's arguments prove, if they prove anything at all, that we do not recognize directly that two sense-data are aequipotent, if we take the relation of being aequipotent as o transitive relation: that is, if when a is aequipotent with $b_{e}$ and $\underline{b}$ is aequipotent with $\underline{c}$, $\underline{a}$ must be aequipotent with $\underline{c}$. Moreover, the argument Stumpf gives shows us exactly how we are able to find out when two sense-data are tonally aequipotent, and when they are not. The fact that a sense-datum of pitchp, and another of pitch $P_{\lambda}$ are not aequipotent is discovered when we find a tone-datum of pitch, say, $P_{k}$,

which is noticed to be of higher pitch than a tone datum of pitch $P$, but which is not of noticeably different pitch than a datum of pitch $P$. Now, since the only criterion which we are ever given for the subliminal difference in intensity or quality of sense-data, other than tests which bring in such extraneous matters as the physical intensity of the stimulus produoing a given sense-datum, is thet there is some datum distinguishabie in intensity of quality, respectively, from one of the original sense-data, and indistinguishable from the other, and since the sole way we have of discriminating the genuine aequipotence of two data from the relation Which holds between them when they are not noticeably different as to intensity or as to quality is that in the case of true aequipotence of there can be no subliminal difference between them in respect to intensity or quality, we may define the aequipotence of $\underline{a}$ and $\underline{b}$ in a given respect as that relation which holds between them when all the data which can be discriminated from $a$ in that respect can also be discriminated from $\underline{b}$ and vice versa. Horeover, if Weber's law holds good, or any similar law which makes it impossible for the uper or lower limina of discrimination from two sense-data produced by stimuli of different intensity to coincide, then we can easily show that this definitaon of aequipotence has the desirable property of rendering two sense-data aequipotent when and only when they are produced by stimuli of the same physical intensity.

We have, then, shown how the notion of a sensation-intensity or quality of a given range can be analysed in terms of the relation which holds between two sense-data of the appropriate sort when one is noticeably different from the other in regard to its intensity or quality. This relation is manifestly dexived from that which holds between one datum and another when the first is noticeably of greater intensity, or higher pitch, more vivid hue, $\phi t c$, than the second. I am not meaning to say that we may not have some sort of a direct experience that a is of different intensity or piton, etc., from b apart from any experience

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that $\mathfrak{a}$ is of greater or of less intensity, higher or lower pitch than $\underline{b}$, but it seems to me fairly obvious that we usually have some experience of the direction of the difference between $\underline{a}$ and $\underline{b}$ then we experience them as different, and often what we mean to assert in saying that they are felt to be different is that one is experienced as noticeably more intense or of noticeably higher pitch, etc., than the other. These relations of noticeably greater and less in intensity, noticeably higher and lower in pitch, etc. whether they ere given as such in our experience or not, are certainly far more primitive than any experience we ever have of a sensation intensity. If we take the latter as a property: namely, the property of being aequipotent with a given sense-datum in al certain respect, then whatever we seem to know about it is simply a paraphrase for certain things we know about certain sense-data and the relation of Inoticeably more intense than', since We know the ablation of aequipotence only as a function of the latter relation. If a sensation intensity be taken as a class of serise-data, determined by the relation of aequipotence with a given sense-datum, then since, as Mr. Russel 11 has pointed out り a proposition about o class does not really concern the class at all, but only the property by which the class is defined, ont knowledge of sensation-intensities is even more a knowledge by inference, and the sensation-intensities themselves are even fur the from being presertedoto us than if they wore properties. It seems to me, moreover, that the sort of inference through which we know that a given sense-intensity is different from another is not of a. fundamentally different character from that through which we know anything at all about a sensation-intensity. Since sensation-intensities and qualities, in the only sense in which anything who stout or stumps says demands that they exist, are not ever presented to us, it follows that Stout's argument in favor of 'mere' sensations is utterly without any point

In order that sense-intensities and qualities may have those properer usually associate with their names, it is necessary that in some sense they should form series, in which a greater intensity always follows passer one. Interpreting this statement in logical terms, it means that there must be a relation, such that:
(1) Its field is a class of sengetion-intensitios qu a given kind,
(2) if it relates a sensation -intensity $\alpha$ to af entities whin 'enter intr' the relation say $\beta$, we should naturally say that $\beta$ is a greater intensity than $\alpha$
(3) if it relates $\alpha$ to $\beta, \alpha$ cannot be the same as $\beta \ldots$,
(4) if it relates $\alpha$ to $\beta$ and $\beta$ to $\gamma$, then it relates $\alpha$ to $\gamma$, and,
(5) if $\alpha$ and $\beta$ belong to its field and are distinct, then it either relates
$\beta$ or $\beta$ to $\alpha$.
Let it be remarked to start with that the relation, 'noticeably less than? between sense-data, although it satisfies (3) and (4), fails to satisfy (1) and (3) and (5). Mow, how are we to find such a relation? If we turn back to the argument of Stout and Stumps, we shall find the answer to this question very readily. $P$ and $P$, yourremember, were taken to be 'tones' indistinguishable directly from one another, and some $P^{\prime}$ iras taker van indistinguishable from $P$, but noticeably higher in k 1 pitch than $P_{I}$ Stout says that $P$ is really higher than $P_{1}$. The relation which one sense-intensity of a given sort has to another intensity of the same sort is, then, taken by Stout to be a 'greater-than' relation then some datum of the first intensity is not noticeably differcent in intensity from some datum which is of noticeably greater intensity than some datum of the second intensity. It will, if Weber's law be true, itwill sufficient condition for one datum the more intense than another if this relation relate the second datum to the first, but it need not bo a necessary condition. Our ranges of sensation-intensities do ot on in infinitum: sooner or later, in ascending such a range,
we find an experience such that there is no other experience noticeably more intense than it. It will be impossible to arrange one such experience before another by the criterion we have just discussed, for our criterion consists in discovering whether there is some dum noticeably more intense than one of these, and indistinguishable in intengity from the other. Nevertheless, it seems natural to oall one atum really merefintense tyan nother if there is some datum only sublininally less intense than the second and noticeably less intense than the first. This criterion is irreducible to that already given, and, may hold between two experiences of such intensity that there is nothing noticeably more intense than either. Let us, therefore, define the celation, iless intense than', attong sense-data of a given kind, as the relation between ondatum and another when The is either indistinguishable in the appropriate respeot from some datum noticeably legs intense than the other, or is noticeably less intense than some datum indistinguishable from the uther, and one intensity less than another if some member of it is less intense than some member of the other.

This relation 1 less than' among sensation-intensities evidently has the properties which we numbored (1) and (2) above, whatever properties the relation 'noticeably less intense than' may have (3) is satisfied on account of the definition of the relation: for suppose that $\alpha$ stands in this relation to $\alpha$ - then there is a term belonging to $\alpha$ which is noticeably more intense, or noticeably less intense, than some sonse-datum indistinguishable from another member of $\alpha$. It follows th that there are two terms of $\alpha$ which are not aequipotent, for $X$ and $y$ are defined as aequipotent when and only when all the terms/notiodably different from - are noticeably different from $y$, and vice versa. Now, a sensation-intensity is defined as the class of all sense-data possessing a certain sort of aequipotence with a. given se se-datum, and
it follows from the definition of aequipotence that if $X$ is aequipotent with $\underline{y}$ and $\underline{y}$ is aequipotent with $\underline{z}, \underline{x}$ is aequipotent with z. We thus show that it is contradiotory, according to our definitdon, for $\alpha$ to be a greater intensity than

In a similar manner, we show that it is 20 gically demonstrable that (5) is satisfied by the relation 'greater thon' among sensation-intensities. Por, if two sensation-intensities are dipferent, es the relation aequipotence is transitive, some member of the one mat not be aequipotent with some member of the other. That is, some member of one must not be indistinguishable fromi some datum indistirygujshable from some member of the other: in other words, some member of one must be either noticeably more intense or noticeably less intense than something indistinguishable from some member tof the ather. Therefore, by the definition (2) 'less than', one the is intesarties or greater than the other, as some member of it is less intense or more intense than some member of the other.

In a similar manner, it can be shown that a sufficient condition for (4) to satisified is that the relation between $x$ and $y$ when $x$ is more intense than $y$ yhoud transitive: that is, that if $X$ beaxs this relation to $z$, and $\underline{z}$ bears it to $\underset{\sim}{ }, \underline{x}$ bears it to $y$. This relationwe assune in our everyday life to be transitive .

We have seen, then, that under a single hypothesis of, very general nature almost certainly satilsfich by the relations in iis a noticeably brighter than', etc., the relation between the sensation-qualities and intensitfies in question which we have defined as 'is a less degree of pressure than', 'is a lower pitoh than', 'is a smaller degree of brightmess than', will generate the series of pressures or tones or brightnesses: , as the case may be.

Let it be noted, however, that our theory up to this point, has been purely ordinal, and that we have not said anything which will entitle us to treat these pressures, brightness and tones, as quant ities. Such a theory can be developed, but it is far more complicated. than that here developed, and it starts with, tetradic relation, "the interval between $\underline{x}$ and $y$ seems greater than that between $\underline{u}$ and $\mathscr{V}^{\prime \prime}, r$ rather than the dyadic relation, "noticeably more intense than". In our ordinal theory of sensation- intensities, not only are we not entitled to speak of sums or products of sensationtintensities, but we have not even any indication of any way to correlate the series of sensation-intensities with the series of real numbers, and to treat one interval between two sensation-intensities as equal to another. It is true that we are able to state one form of Weber's law in terms of the relation of noticeable difference between the sensations. We are able to say, for example, that if $\underline{z}$ is a given sound-datum, evoked by a stimulus of vibration per second, and $\mathcal{F}$ a sound-datum evoked by a stimulus of $\nmid$ vibrations per second, and noticeably higher in pitch than $\mathcal{X}$ then, when $\underline{X}$ is fixed, the minimum or lower limit of $\eta$ is constant*.


But it is by no means evident that we should take all "just noticeable differences" as equal. If we define all "just noticeable differences" as equal, then we have not yet found an unambiguous way of assigning a ragl number to each sensation-intensity, which is one of the things we must be able to do in order to be able to regardjt asaquantitys, for we are not thereby given any definite method of subdividing the interval between two sensation-intensities
whose members are only subliminally different into equal parts. If, on the other hand, we take some method of measurement starting from sensation-intervals as primitive, and derive from, the proposition that all "just noticeable differences"are equal needs experimental verification in he same sense in which Weber's law in either its quantifive or its non-quantitive sense needs experimental verification, and Weber's law may well befrue in one of these senses yet false in the other. So we see that the proper interpretation of the notion of sensation-intensity clears up many problems connected with Weber's law.

In conclusion I wish to contrast my method of handing the problem of sensation-intensity with that $f$ Stout and Stumpf. Stout and Stump see a contradiction in the intransitivity of the relation "is indistinguishable from", and say that to resolve this contradicion, we must postulate differences of sensation-intensity where none are seen. They assume unconsciously, that is, that the differences of intensity between sensations are in the first instance differences between sensation-intensities. We take the differences of intensity as primitive, and derive the intensities as functions of them. They have to postulate that the indistinguishability of subliminally different sensations is due to our inability to perceive relations of difference which really subsist between them: to us the relation of indistinguishability is not merely an ill-perceived qualitative identity, , but qualitative identity is defined as a perfectly determinate logical function of indisting ishability. We postulate nothing, and need be bothered by no qualms of conscidpee as to the existence of anything postulated: we construct all the relations and classes we need from asingle relation, "noticeably greater than". This is the chief methodological point to notice in this paper, and is due to Mr. Russell.

