[Ca 1915]

A Further Contribution to the Theory of Relative Position

By Norbert Wiener, Ph. D.

In a recent article contributed to the Proceedings of this society" I showed how from the relation of complete temporal succession between events we can derive the series of the instants of time. In this article I propose to show how we may, in a somewhat similar manner, derive the various series of sensory intensities — e.g. brightness-intensities, loudness-intensities, etc. — from such relations between sense-data as 'noticeably lighter than', 'noticeably louder than', etc. The method of my previous article does, in fact, enable us to derive a series from such relations as I have just been mentioning, but this series will not possess all the properties which we would naturally expect of a series which we could properly call a series of sensory intensities: e.g. its terms will be, as we would naturally anticipate, classes of sense data, but the sense-data belonging to a given member of the series will bear to one another no transitive, symmetrical, reflexive relation, independent of the particular member of the series chosen, which could properly be called 'being of the same sensory intensity as'.

We shall first define this latter relation, and then by means of it obtain the series of sensory

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intensities. Let P be e.g. the relation between a sound x and a sound y when x is noticeably louder than y. Then P_{se}, which I defined in my last article as $(\dot{-}P\dot{-}\breve{P})\text{`}C\text{`}P$,[1] will be the relation of two sounds between which no difference in loudness is noticable — i.e. between two sounds whose difference in loudness is subliminal. In general, P_{se} is not reflexive, though it is when $P \subset \dot{J}$, as in the case in question. Even in the case in question, however, P_{se} is not transitive, for it is a familiar psychological fact that though the differences in loudness between x and y and between y and z may be subliminal, yet that between x and z may be clearly noticeable. It is clear, then, that P_{se} is not the relation of 'being of the same sensory intensity' for which we are looking. But the relation between x and y when ~~every~~ x and y are both sounds, and every sound whose difference in loudness from the one is subliminal is only subliminally different in loudness from the other, and vice versa, is transitive, reflexive, and symmetrical, and is just such a relation as we would naturally call '[redacted] of the same sensory intensity as'. This relation, considered as a function of P, I shall call P_{si}, and I shall give a symbolical definition of it as follows:

$$P_{si} = (\overrightarrow{\overrightarrow{P_{se}}} \mid \overrightarrow{P_{se}})\text{`}C\text{`}P \qquad Df.$$

The next step, obviously, is to define a sensory intensity — e.g. a loudness — as the class of sense-

[1] Except where I expressly define the meaning of a symbol, the symbolism used in this article is that of the Principia Mathematica of Whitehead and Russell.