The Relations between Certain series Observed in the White Mountains.

A group of mountains such as the White Mountains furnishes many interesting examples of all sorts of series standing in every kind of relation to one another, together with the levels correlated with them. The first series to strike our eye is that of altitude. Here the asymmetric transitive relation is that of 'higher than'; the levels are those of points of equal altitude. The series is dense, because between every two heights there is another one; it is continuous, because ~~at~~ every two segments of the series which together make up the whole series can be separated by a term itself on the series. The series has a last term, 6280 feet, the height of Mount Washington; but no first term. why not?

Correlated with this we find the vegetation-series whose levels are marked by broad zones of vegetation spread across the mountain. Beginning with the fields and meadows at the bottom, one ascends first through a mixed forest of maple, birch, pine, and other trees. This continues for some distance up the slope of the mountain (usually not more than to 700 feet above sea-level) when its place is taken through a fairly abrupt transition by a forest composed entirely of birches, both white and yellow. At 1000-1500 feet this in turn gives place by a very abrupt change to a forest composed of spruce and some balsam fir. This forest is much more free from underbrush than the first ~~two~~, and is carpeted more richly with moss and wood sorrel. At about 3000-3700 feet the trees begin to be dwarfed, and continue so to the tree-line, at 3500-4500 feet. From here to about 5000 feet the flora is sub-alpine, consisting chiefly of mountain cranberry and plants of similar habit, and much contorted spruce and alpine azalea, rising scarcely three inches above the ground. Lastly, from 5000 feet up, we have the true alpine region, bearing only a few small, very hardy herbs. Thus we have a series of plants in order of hardiness, approximately corresponding to certain regions in the series of altitudes. The plant series is discrete, whether we take as our unit plant or region, for each plant or region except the first and the last has both a next predecessor and a next successor. The plant series is correlated not only with the altitude series, but with the temperature series, which we shall discuss later.

A series which is correlated with the altitude series in a different way is the series of barometric pressures. The altitude of a place varies about as the logarithm of the difference between average sea-level pressure and the average pressure at the place. This correlation is fairly perfect. Both the series participating in it, the 'higher than' series and the 'more average barometric pressure than' series are continuous, and so, by the rule given on page 20 of Huntington's paper, can be put in one-one correspondence in an infinitude of ways. A little consideration will show that they tend to be found in one-one correspondence in nature (cf. §3 of Huntington's paper). Perfect as their correlation is, however, it is not absolutely perfect, and so the correspondence becomes rather of the type which I call regional; - a certain average pressure is found within a certain range of height and vice versa. The more limited these regions become, the more does the correspondence approach one-one correlation.

Another example of regional correspondence is that between the mean ~~winter~~ annual temperature and the altitude. The higher one ascends, the colder does the climate become, owing partly to the free radiation through the dry air, which prevents the earth from storing up the bright sunshine it receives, and partly to the free sweep given to the winds on the bare backs of the ridges. The temperature-altitude correlation is even less perfect than the temperature-pressure correlation, for in the deep ravines which abound in the White Mountains, owing to the scanty sunshine, the temperature is lower than on the top of Mt. Washington. Snow stays there sometimes into August. Thus the temperature series is correlated both with the altitude-series and the average-amount-of-sunshine series, forming a triadic relation.

A very interesting type of series is that formed by the peaks in a range. Of course the large, named peaks form a discrete series, but every two consecutive peaks are united by a saddle, and this saddle is by no means even. On the contrary, it is usually covered with a great number of boulders and rocks lying in confusion, so that the waterparting has many more crests than the range has named peaks. These boulders and rocks in turn are not smooth. Their surfaces are covered with miniature summits and crests, cols and saddles, and we can continue this series of peaks between peaks, as far as we can see, until we reach molecules or atoms. We have then a series between any two of whose terms there is a third, as far as we know, and which is ideally, at least, denumerable. That is, we have here a denumerable dense series, whose serial relation is 'further peak than'. Now, the White Mountains consist of parallel ranges, to which in turn the same analysis can be applied with the same results. That is, a mountain system furnishes us with an excellent example of a dense denumerable two-dimensional series. This is correlated with a two-dimensional continuum, the surface of the ground, in such a way that to each and every member of the dense

It would not be denumerable in the order obtained by this relation!

denumerable series one member and one only of the continuum corresponds. The reverse relation, however, does not hold. If we read § 59 of Huntingtons' paper, we shall see that the peaks form a subclass of the type R of the two-dimensional linear continuous series formed by the surface of the ground. It is interesting to note that the minima of the ground form another such subclass, absolutely distinct from the first. Perhaps these two sub-classes might be compared with such a class as that of all the proper fractions with an odd denominator, arranged in order of magnitude, or that of all those with an even denominator. Together they form a dense denumerable series, while each by itself does the same. Since all dense denumerable series are ordinally similar, since between every two 'hills' there is a 'valley', and since between every two terms ~~those~~ of a continuum there is another term, and since the 'hills' form a 'framework' of the continuum, every continuous series has an infinitude of 'frameworks', an infinitude of subclasses of 'rational elements'. Thus, the ~~rational~~ products of π and a rational number and the series of rational numbers themselves both form frameworks of the real number series. We have, ~~thus~~, in the case of the peak and valley series a framework correspondence with the area series. A two-dimensional series bears, as we can see, the same relation to its two-dimensional frameworks as a one-dimensional series to its one-dimensional frameworks.

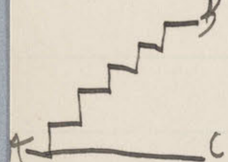
We have observed relations between discrete and continuous series, between dense denumerable and continuous series; now let us observe the relations between a discrete and a dense denumerable series. Let us take as our discrete series that of the named peaks in the White Mountain region[1]; as our dense series, that of all the ground maxima in the White Mountain region. Here every term of the discrete series is a member of the dense series, but not vice versa. If an integer be assigned to every named peak, the unnamed summits can be assigned to the proper fractions. This latter assignment can be made in an infinitude of ways. Let us call some ~~sate~~ particular summit between summit 0 and summit 1 summit $\frac{1}{2}$. The fractions be

[1] I am taking the peaks up as if they formed a one-dimensional complex, for simplicity's sake. To deal with a two dimensional complex of peaks, treat it as a one-dimensional complex of one-dimensional complexes of peaks.

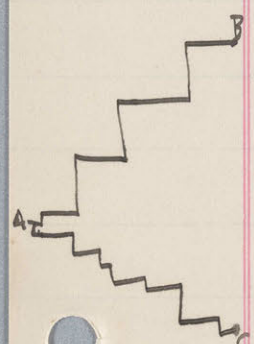
tween 0 and $\frac{1}{2}$ can be put in one-one correspondence with those between 0 and one by making every fraction correspond to its product with two so that $\frac{1}{4}$ corresponds to $\frac{1}{2}$. The peaks between the peak zero and the peak $\frac{1}{2}$ can be put in one-one correspondence with those between peak zero and peak 1 by dividing the two intervals into an equal number of smaller intervals, and putting the peaks nearest the extremities of corresponding intervals into correspondence. Let the peak in the 0 - $\frac{1}{2}$ interval corresponding to peak $\frac{1}{2}$ in the 0-1 interval be called $\frac{1}{4}$. By continuing this method ad indefinitum we can give a fraction to any peak we please and vice versa. We might, then ~~[illegible]~~ write our result as " Named peaks: all maxima :: real integers : real rational numbers." We have, then, in an indefinitely complex mountain range ~~system~~ a working model, as it were, of a two-dimensional number system.

Two dense denumerable series can be put into one-one correspondence, as we have just seen, in an infinitude of ways, for any part of each series, since it can be put into one-one correspondence with the whole of the series it belongs to, can be put into one-one correspondence with any part of the other. The same thing is true of a continuum, and for the same reason. With a discrete series ~~having a first and a last term~~, however, this is not the case if there is any definite order of succession of the terms. Therefore, if the named peaks be numbered as to their order of successions, no single number will apply exclusively to any of the unnamed peaks or to any point on the ground not a named peak. These may be numbered in an infinity of different ways.

Two discrete series may stand in such relations to one another as one-two correspondence. Thus, if one took as one series the saddles of all the two-peaked mountains in New Hampshire, arranged as to the mean height of their peaks, and as the other all the peaks of all the two-peaked mountains in New Hampshire arranged in such an order that of two peaks from different mountains the one from the mountain ~~of~~ with the greater mean height of its peaks will precede, and of two peaks from the same mountain the taller one precedes, it is plain that to every member of the first series, two consecutive members and two only of the second correspond, while to every two adjacent members in the second series, one or two members, as the case may be, in the first series correspond. With a dense series of any sort, however, such relations cannot hold, because there are no adjacent terms. Regional correspondence is the closest approach to such conditions found there. In this type, which we have mentioned already, any region between two given boundaries in one series corresponds to a region between two given boundaries in the other, and vice versa. There are several types of such correspondence. Let us ~~first~~ take up some of those, where both ~~of these~~ series are dense. In the first place, a region in each series may change smoothly when the corresponding region in the other series changes smoothly. Such is the relation between the series of average barometric pressure and the altitude series. The annual average of barometric pressure has a given range for every range of altitude, and a slight change in the one corresponds to a slight change in the other. In such a case the means of either of the two ranges form a one-one correspondence with the other; for every altitude ~~there~~ is one ~~and one~~ mean annual average of barometric pressure and only one, and vice versa.



Another type of regional correspondence is that shown between a broken line and its projection. Suppose, for example, we ~~have~~ for one series ~~the~~ ~~dis~~ ~~distances~~ points along a trail which goes partly over vertical cliffs and partly over level plains, and for the other series, the horizontal projections of these ~~distances~~ points. Then if the law of correspondance be that a point on the trail corresponds to its projection, it is easy to see that while to each member of the first series one member of the second series and one only corresponds, to one member of the second series there may correspond a member of the first series or a region.



By projecting broken lines on broken lines, we can make two series correspond so that to each member or region of one there corresponds a member or region of the other. It can be observed readily that if the correspondence be made not between points but regions, we shall obtain two series such that every region of one will correspond with a unique region of the other, but not vice versa, or, if both lines be broken, such that the regional correspondence on neither side is unique. Relations of unilateral regional character are found between y and $D_x y$ ~~at the~~ in any curve.

These regional correspondences can be extended to any degree of complexity. Complex regional relations may be found between discrete and dense series, but these are not worth ~~while~~ a while going into. One might say that the regional type of correspondence is second in importance only to the one-one type.

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