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THE INFLUENCE OF THE BOUNDARY LAYER ON THE WAVE RESISTANCE OF A SHIP

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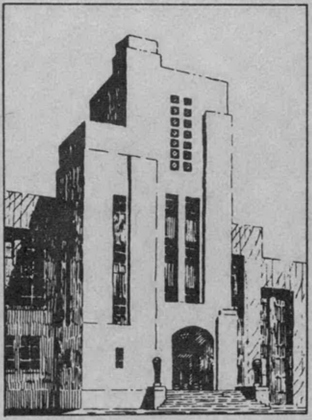
by

V.M. Lavrentieff

Academy of Sciences of the U.S.S.R.



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April 1952

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THE INFLUENCE OF THE BOUNDARY LAYER ON THE WAVE RESISTANCE OF A SHIP

(Vliianie Pogranichnogo Sloia na Volnovoe Soprotivlenie Korablia)

by

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Central Scientific Research Institute of the Maritime Fleet

Reports of the Academy of Sciences of the U.S.S.R.
1951, Vol. LXXX, No. 6
(Presented by Academician A.I. Nekrasov, 20 August 1951)

Translated by Ralph D. Cooper

April 1952

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THE INFLUENCE OF THE BOUNDARY LAYER ON THE WAVE RESISTANCE OF A SHIP

The experimental investigation of the linear theory of the wave resistance of a ship in an ideal fluid shows that, in general, this theory is satisfactorily confirmed by tests for sufficiently sharp closed bodies with the exception of the range of gravitational numbers (Froude numbers), which correspond to the interfering humps and hollows on the resistance curves. It is known that the viscosity of the fluid considerably smooths out the modulation in wave resistance curves.

The influence of the viscosity of the fluid on the wave resistance is manifested mainly through the influence of the boundary layer. This last can be taken into account approximately, if, in place of flow about the actual ship with wetted surface $y = \phi(x,z)$, the flow of an ideal fluid about a body (Figure 1) with wetted surface $y = f(x,z) = \phi(x,z) + \delta^*(x,z)$ is considered, which represents the hull of the ship surrounded by a displacement layer δ^* together with its wake, whose width may be approximated by a constant equal to twice the thickness δ^* at the stern perpendicular. Such an assumption is equivalent to considering that separation of the boundary layer does not occur, a condition which is satisfied practically by all vessels for which a study of wave resistance has any significance.

The thickness of the displacement layer δ^* can be determined approximately at every point on the surface of a ship by the method of successive approximations; however, it should be stated that for practical purposes it will be sufficient to use the available solution for a flat plate having the same length and draft as the ship.

For determining wave resistance in deep water (the formulas for shallow water, either in depth or in breadth, are obtained in an analogous way) we use the well-known formulas:

$$R_w = \frac{4\gamma g}{\pi v^2} \int_0^{\pi/2} (I^2 + J^2) \sec^3 \theta \, d\theta \qquad [1]$$

$$\left.\begin{matrix} I(\theta) \\ J(\theta) \end{matrix}\right\} = \int_S\int \frac{\partial f}{\partial x} \exp\left(\frac{gz}{v^2} \sec^2\theta\right) \left\{\begin{matrix} \cos\left(\frac{gx}{v^2}\sec\theta\right) \\ \sin\left(\frac{gx}{v^2}\sec\theta\right) \end{matrix}\right\} dx\,dz \qquad [2]$$

where γ is the specific weight of water,

g is the acceleration of the force of gravity,

v is the speed of the ship, and

$y = f(x,z)$ is the equation of the surface of the body.

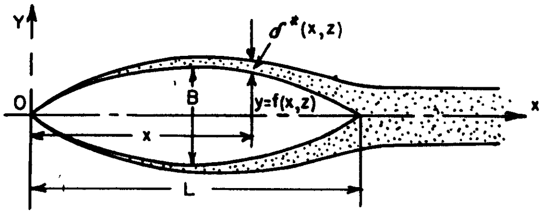

Figure 1

In the integrals [2], the integration extends over the centerline area S of the ship, the axis OZ being directed upward.

Designating the length, beam, and draft of the ship by L, B, and T, we introduce the nondimensional quantities:

$$\xi = \frac{x}{L};\ \eta = \frac{y}{B/2};\ \zeta = \frac{z}{T};\ f(x,z) = \frac{B}{2}f(\xi,\zeta);\ \frac{\partial f}{\partial x} = \frac{1}{4}\frac{B}{L}\frac{\partial f}{\partial \xi} \qquad [3]$$

$F=v/\sqrt{gL}$ is the gravitational number.

Then Formulas [1] and [2] take the form

$$R_w = \frac{\gamma}{\pi}\frac{B^2T^2}{LF^2}\int_0^{\pi/2}(i^2 + j^2)\sec^3\theta d\theta \qquad [1a]$$

$$\left.\begin{matrix} i(\theta) \\ j(\theta) \end{matrix}\right\} = \int_S\int \frac{\partial f}{\partial \xi}\exp\left(\frac{T}{L}\frac{\sec^2\theta}{F^2}\zeta\right)\left\{\begin{matrix} \cos\left(\frac{\sec\theta}{F^2}\xi\right) \\ \sin\left(\frac{\sec\theta}{F^2}\xi\right) \end{matrix}\right\} d\xi\, d\zeta \qquad [2a]$$

The wave resistance per ton of displacement will be

$$\frac{R_w}{\Delta} = \frac{1}{\pi\delta}\frac{B}{L}\frac{T}{L}\frac{1}{F^2}\int_0^{\pi/2}(i^2 + j^2)\sec^3\theta d\theta \qquad [4]$$

where $\Delta = \gamma\delta\ BLT$ is the displacement of the ship and δ is the block coefficient.

Keeping in mind that

$$f(\xi,\zeta) = \phi(\xi,\zeta) + \delta^*(\xi,\zeta) \qquad [5]$$

we find for the resistance per ton of displacement, the formula*

$$\frac{R_w}{\Delta} = \frac{1}{\pi\delta}\frac{B}{L}\frac{T}{L}\frac{1}{F^2}\left\{\int_0^{\pi/2}(i_\infty^2 + j_\infty^2)\sec^3\theta d\theta + \right.$$
$$\left. + 2\int_0^{\pi/2}(i_\infty i_1 + j_\infty j_1)\sec^3\theta d\theta + \int_0^{\pi/2}(i_1^2 + j_1^2)\sec^3\theta d\theta\right\} \qquad [6]$$

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*Translator's Note: In the original paper, because of a typographical error, the second integrand was incorrectly given as $\left\{(i_\infty i_1 + j_\infty i_1)\sec^3\theta d\theta\right\}$

where:

$$\left.\begin{matrix} i_\infty(\theta) \\ j_\infty(\theta) \end{matrix}\right\} = \iint_S \frac{\partial \phi}{\partial \xi} \exp\left(\frac{T}{L}\,\frac{\sec^2\theta}{F^2}\,\zeta\right) \left\{\begin{matrix} \cos\left(\frac{\sec\theta}{F^2}\,\xi\right) \\ \sin\left(\frac{\sec\theta}{F^2}\,\xi\right) \end{matrix}\right\} d\xi\, d\zeta \qquad [7]$$

$$\left.\begin{matrix} i_1(\theta) \\ j_1(\theta) \end{matrix}\right\} = \iint_S \frac{\partial \delta^*}{\partial \xi} \exp\left(\frac{T}{L}\,\frac{\sec^2\theta}{F^2}\,\zeta\right) \left\{\begin{matrix} \cos\left(\frac{\sec\theta}{F^2}\,\xi\right) \\ \sin\left(\frac{\sec\theta}{F^2}\,\xi\right) \end{matrix}\right\} d\xi\, d\zeta \qquad [8]$$

From Equation [6] it follows that the wave resistance of a ship in a viscous fluid can be represented as consisting of three parts:

1. The resistance of the ship in an ideal fluid (at a Reynolds number $Re = vL/\nu = \infty$), corresponding to the first term of [6];

2. the wave resistance of a flat plate caused by the presence of its boundary layer (the third term in [6]), and

3. the wave resistance obtained as a result of the interference of the wave motions created separately by the hull of the ship and by the boundary layer, to which the second term of [6] corresponds.

At higher values of the Reynolds number the third term of [6] proves to be of second order of smallness in comparison to the second and may be neglected.

The most simple way to make computations with the formulas is to start from the exponential expressions for the boundary layer. Let us assume, for instance, "the one-seventh power law," taking into account that at the aft end in the region of positive pressure gradient, the velocity profile in the boundary layer is less full than for a flat plate (as is known, for a flat plate in the region of usual Reynolds numbers, "the one-eleventh power law" gives good agreement with experiments).

Thus, we shall start with the formula

$$\delta^*(\xi) = \frac{\delta^*(x)}{B/2} = 0.0924\,\frac{L}{B}\,\frac{\xi^{4/5}}{\sqrt[5]{Re}} \qquad [9]$$

In this case Formula [8] takes the form:

$$\left.\begin{matrix} i_1(\theta) \\ j_1(\theta) \end{matrix}\right\} = 0.074\,\frac{L}{B}\quad \frac{1}{\sqrt[5]{Re}} \iint_S \frac{\exp\left(\frac{T}{L}\,\frac{\sec^2\theta}{F^2}\,\zeta\right)}{\sqrt[5]{\xi}} \left\{\begin{matrix} \cos\left(\frac{\sec\theta}{F^2}\,\xi\right) \\ \sin\left(\frac{\sec\theta}{F^2}\,\xi\right) \end{matrix}\right\} d\xi\, d\zeta \qquad [8a]$$

Since, in our assumption, the displacement thickness δ^* appears as a function of ξ only and does not depend on ζ, and if, in addition, for the purpose of calculating i_1 and j_1 we consider the centerline area of the ship as a rectangle, the integration over ζ can be carried out beforehand, giving:

$$\left.\begin{matrix} i_1(\theta) \\ j_1(\theta) \end{matrix}\right\} = 0.074 \frac{L}{B} \frac{L}{T} \frac{F^2 \cos^2\theta}{\sqrt[5]{Re}} \left[\exp\left(-\frac{T}{L}\frac{\sec^2\theta}{F^2}\right) - 1\right] \int_0^1 \xi^{-1/5} \left\{\begin{matrix} \cos\left(\frac{\sec\theta}{F^2}\xi\right) \\ \sin\left(\frac{\sec\theta}{F^2}\xi\right) \end{matrix}\right\} d\xi \qquad [8b]$$

From this it is seen that if the wave resistance of the plate is of order $Re^{-2/5}$, then the wave resistance created by the interference (the second term in [6]) will be of order $Re^{-1/5}$.

Analogous formulas for i_1 and j_1 can be obtained by assuming an arbitrary exponential law for the velocity distribution in the boundary layer. In particular, for a laminar layer the integrals for i_1 and j_1 are expressed in final form by Fresnel integrals.

The practical calculation of the integrals for i_1 and j_1 presents no difficulties, and the calculation of wave resistance including the effect of the boundary layer can be made for closed bodies, which are given either analytically or in the form of ordinates of the theoretical plan. In the latter case, it is necessary to use the transformation of G.E. Pavlenko.

Formulas [6] and [8a] show that the wave resistance has a "measurable effect."

Submitted 18 June 1951

NAVY-DPPO PRNC WASH DC