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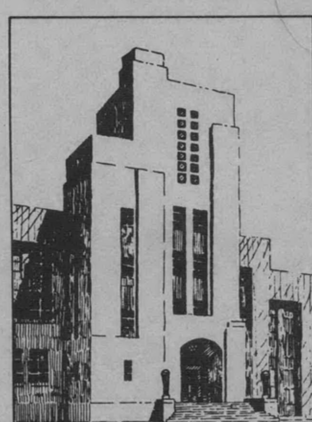
THE PRESENT STATUS OF THEORETICAL RESEARCH ON SHIP PROPELLERS WITH RESPECT TO ITS TECHNICAL APPLICATION

(DER STAND DER FORSCHUNG ÜBER DEN SCHIFFSPROPELLER IM HINBLICK AUF DIE TECHNISCHE BERECHNUNG)

by Dr.-Ing. H. Lerbs
Hamburg Model Basin



Translated by E. N. Labouvie, Ph.D.

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Dr.Ing. H. Lerbs, Hamburg Model Basin

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SYMBOLS

Circulation Γ (Line integral of the velocity; $m^2 s^{-1}$)

Density $\rho = \gamma/g (kg\ s^2 m^{-4})$

Airfoil:

Lift	$A (kg)$
Drag	$W (kg)$
Speed of Advance	$V (m\ s^{-1})$
Area of the Airfoil	$F (m^2)$
Lift coefficient	$c_a = A/\frac{\rho}{2} FV^2$
Drag coefficient	$c_w = W/\frac{\rho}{2} FV^2$
Drag-lift coefficient	$\epsilon = c_w/c_a$

Propeller:

Thrust	$S (kg)$
Torque	$M (m\ kg)$
Power input	WPS
Speed of Advance	$v (m\ s^{-1})$
Angular velocity	$\omega = 2\pi n (s^{-1})$
Blade-tip radius	$R (m)$
Intermediate radius	$r = x \times R\ (m)$
Area of Propeller Disc	$F_p = R^2\pi (m^2)$
Thrust loading Coefficient	$c_s = S/\frac{\rho}{2} F_p V^2$
Power loading Coefficient	$c_L = 75\ WPS/\frac{\rho}{2} F_p V^2$
Efficiency	$\eta = c_s/c_L$

THE PRESENT STATUS OF THEORETICAL RESEARCH ON SHIP PROPELLERS

WITH RESPECT TO ITS TECHNICAL APPLICATION

In reporting on the status of research on ship propellers I wish to survey and correlate for you some of the more recent investigations in this particular field from which a practical method can be developed for the technical calculation of the conventional ship propeller and also of the more complicated propulsion systems such as propellers with guide vanes, counter-rotating propellers and tandem propellers. Naturally, nothing more can be accomplished in the time allotted than to outline the ideas developed in the studies referred to; however, since they are scattered over a wide area of technical and mathematical literature, I assume that even a brief survey presented from a unified point of view will be advantageous. Moreover, it will be necessary to limit this review to propellers of constant initial velocity, the so-called "free running propellers."

The first practical method for the design of a ship propeller based on theoretical research was published in 1926 by Helmbold.[1] He made extensive use of the results which had been obtained previously in the field of airplane propellers under small load. Up to that time the prevailing method of calculating propeller data used in shipbuilding was based on the interpolation of a propeller within methodical series tests. It was soon found, however, that in contrast to this method, the theoretical approach was considerably more elastic in that it is not bound to the constants necessarily occurring in the geometrical structure of a methodical series. Moreover, it yields more detailed data than the method of interpolation, especially when the investigation no longer concerns the propeller as a whole but rather the flow about the individual propeller sections. Such problems arise when investigating cavitation or the construction of suitable guide vanes. It is true that this method used to require more time for calculation than did the customary interpolation but numerous attempts have been made recently to reduce the time outlay to a minimum by using suitable aids such as curve sheets[2] and the like. In this respect also, recent investigations deserve credit for the progress made toward improving the methods to such an extent that by the extensive use of tabulated values of mathematical functions and by a certain arrangement of the method of calculation it is now possible to reduce the computational time to such an extent that it compares favorably with the time required for interpolation from methodical series tests.

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[1]References are listed on page 18.

All modern research pertaining to the flow produced by a propeller and to the force generated by this flow are based on the airfoil theory of Prandtl.[3,4] The conclusions derived from this theory regarding the flow about a single airfoil may be logically applied to the ship propeller if the propeller blade is conceived as being the sum of an infinite number of single blade elements and if, in accordance with this, the action of the propeller is considered as being the sum of the actions of all these individual blade elements. It is well known that the motion of an airfoil produces a force with the two components of lift and drag; the first component is perpendicular to, and the second is parallel to the direction of the relative velocity. The problem now is to clarify the origin of these two force components and to observe their effects upon the flow. It will be sufficient for our purpose to emphasize the following results obtained by this theory. The lift necessarily depends on the existence of a vortex which is conceived to be located in the wing and which is carried along by the latter, whence it is called a "bound vortex." This vortex produces additional velocities in the flow which increase the pure speed of advance of the wing above it while decreasing the speed of advance below. According to Bernoulli's principle, differences in pressure correspond to these differences in speed and result in the action of a force on the wing which we call lift. The relation of the intensity of the vortex which is measured by its circulation, the speed of advance, and the generated lift is determined by the theorem of momentum. According to this theorem, the lift is perpendicular to the resulting speed and, for each unit of length of the span-width, it is equal to the product of the density of the fluid, the circulation and the speed of advance (principle of Kutta-Joukowski). Primarily, this principle holds good for the two-dimensional flow only, i.e., for a wing of infinite span over which the lift has a constant value per unit of length. For a wing of finite span the lift decreases from a maximum value at the middle to zero at the wing tips. This is due to the fact that the pressure difference between the overpressure on the lower side and the underpressure on the upper side of the wing becomes zero at the wing tips. This equalization of pressure produces a transverse flow of the fluid across the wing[4] which is directed from the middle toward the wing tip on the lower side of the wing while its direction is reversed on the upper side. Hence, the particles of the fluid which pass over the wing are diverted somewhat laterally toward the middle while those which pass under the wing are similarly diverted toward the wing tips. Such a movement of the fluid, however, can be conceived to result from a vortex sheet located in the wake of the wing. As already pointed out, the transverse velocity of the fluid and thus the circulation in the vortex sheet are directly related to the variation of the lift and circulation about the wing. According to the principle of Biot-Savart, this vortex sheet, like any vortex, produces additional velocities in the entire surrounding fluid. Thus, in addition to causing transverse velocities behind the wing, it generates downward velocities about the wing itself (by analogy with electro-dynamics, the term "induced" is used in reference to this kind of distance-effect). This result is of great significance. A resultant velocity about the bound vortex is now obtained which is composed of two components, viz., the speed of advance and the downward velocity induced by the trailing vortex sheets, hence this resultant velocity has a downward direction. Since, according to the principle of Kutta-Joukowski, the force exerted on the bound vortex is perpendicular to the resulting velocity, the latter is inclined rearward, i.e., opposite to the direction of the motion of the wing. This explains the generation of a drag in a fluid assumed to be frictionless. This component of the drag is called the "induced drag" since it results from the downward velocity induced by the trailing of the vortex sheet from the wing of finite length. The work performed in moving the wing against this induced drag equals the kinetic energy which the trailing vortex system imparts to the fluid.

Let us now apply these ideas to the propeller whose individual blades are considered as the sum of blade elements. First, consider each blade on which forces are exerted as the center of a bound vortex which, as in the case of a single wing of finite length, generates a trailing vortex sheet since the force exerted by the blade is variable in radial direction. This force has zero values at the propeller hub and the wing tips and between these two points it follows a hitherto unknown law. The direction of this trailing vortex sheet is determined by the direction of the stream-lines, hence it assumes a spiral form. Again there is the problem, as in the case of a single wing, of computing at the place of the bound vortices, the induced velocities of this system of bound vortices and vortex sheets and then of calculating from these the speed of advance, the peripheral velocity, the resulting velocity, and, finally, the resultant force according to the principle of Kutta-Joukowski. This resultant force can be resolved into a thrust component and a tangential component whose integration over the radius finally yields the exerted thrust and torque. The solution still presents considerable difficulties in this general form since it can only be said that the additional velocities produced by the bound vortices cancel each other in the case of symmetrical screws. The field of the induced velocities of a helical vortex sheet is still not known initially and no details can be stated concerning the shape of the vortex sheet which, as we know, depends on induced velocities still to be computed. Moreover, the variation of the circulation within the vortex is not known since it is determined by the distribution of the circulation over the blade, i.e., over the bound vortex which as yet is entirely unknown. In order to make any progress we shall first make an assumption and then apply two principles of Betz which will lead to a solution of the problem.

The trailing vortex sheets of the blades induce a field of velocity which appears externally as the propeller slip stream. In general, the induced velocity will have an axial, a tangential, and a radial component at any fixed point of this slip stream. The axial component indicates an increase in the velocity toward the rear, the tangential component indicates a rotation of the fluid contained within the slip stream, and, finally, the radial component brings about a contraction of the slip stream whereby the stream lines tend to move toward the axis. These components are not mutually independent, however.[4]

First, it may be stated that the axially induced velocity component of any fluid particle increases continuously from a zero value infinitely far ahead of the propeller to a definite finite value infinitely far behind the propeller. According to the theorem of momentum, this variation is directly related to the generation of thrust and it is familiar as one of the results of the simple slip stream theory. This continuous increase of the axial component results in a corresponding contraction of the slip stream and thus of the vortex sheets whereby the radial distance of a fluid particle from the axis is reduced. Since the moment of momentum of a particle which arises from the tangential component remains constant during the contraction of the slip stream, the tangential component has to increase correspondingly. This means that the kinetic energy of its movement is also increased. This can only happen at the expense of the kinetic energy of the axial movement so that it is decreased accordingly. On the other hand, due to the action of a centrifugal force, the tangential component produces a pressure gradient which is directed across the flow toward the interior of the slip stream. This pressure gradient tends to increase the axial component. Thus, the contraction of the slip stream, i.e., the induced radial component, and the centrifugal pressure gradient have mutually opposing effects on the axial velocity component. It probably will not matter a great deal, therefore, if these two effects are neglected in the subsequent analysis. The extent to which they may safely be disregarded will be explained briefly later in the paper.

A principle of Betz[3] applies for the case of a slip stream free from contraction; it states that for symmetrical propellers the additional velocities resulting from the trailing vortex sheets at the bound vortex are equal to one-half the velocity which is induced at a corresponding point infinitely far behind the propeller. Attention has already been directed to the continuous increase of the axial component. The tangential component, on the other hand, behaves differently. According to a general principle of Stokes on the geometry of vortex fields, a tangential component along any closed line can be present in a flow only if the vortices are surrounded by this closed line. From this it follows that the rotational motion of the fluid forward of the propeller is zero and attains a finite value in a discontinuity where the free vortex sheet begins. The Stokes principle states further that the tangential component no longer varies along the slip stream behind the propeller. This is explained by the fact that the circulation of the trailing vortex sheets at a certain distance from the axis is determined exclusively by the variation of circulation at the corresponding point of the propeller blade. Thus the circulation remains constant downstream. In order to harmonize these conclusions with the Betz principle, a core of small yet finite thickness must be attributed to the bound vortex over which the tangential component increases from a zero value in front of the core through its mean value at the mid-point to its maximum value at the rear of the core where the vortex sheet is shed.

Clarification of the radial distribution of the additional velocities and their distribution along the circumference remain to be accomplished before the forces on the propeller blade can be calculated. In the case of the distribution along the circumference, it may be stated qualitatively that the additional velocities vary from a maximum value within the vortex sheets to one generally smaller between them. In order to avoid this variation, the assumption is made that the vortex sheets are very close to each other, i.e., that there are an infinite number of blades in the propeller. Although in that case the calculations involve an excessively large mean value for the induced velocity, it is later reduced to the mean value corresponding to the finite number of blades. First, however, let us continue with the consideration of the propeller with an infinite number of blades for which there is no variation of the induced velocity components along the circumference so that the only remaining question concerns the radial distribution. We know that this question is related to the distribution of the circulation, i.e,, the distribution of the forces over the blade, and that it can only be answered either by assuming a certain distribution of circulation or by establishing certain definite conditions by which this distribution would be determined.

A question of practical importance concerns a way to so distribute the thrust and thus the circulation for a given total thrust over the blade that the loss of energy which is unavoidable with the generation of thrust and which arises from the appearance of the induced velocities may be kept at a minimum. This question constitutes a variation problem and seems to complicate our problem; it will be seen, however, that the answer yields a very simple and easy result which was first formulated by Betz.[4] It is obtained by increasing the circulation at any point of the propeller blade by a small amount and analyzing the increase in thrust and torque thus produced. If the thrust distribution over the blade is such that it corresponds to the maximum obtainable efficiency, then any increase in thrust must be produced at every point with the same degree of efficiency otherwise it could be transferred from those places where additional thrust would entail a decrease in efficiency to other areas where it would result in an increase in efficiency and enhance the total efficiency. Inasmuch as optimum efficiency is assumed already, the efficiency of a variation of thrust must have the same value at every point of the blade for the case of free running propellers.

The calculation based on this way of reasoning yields the result that at optimum thrust distribution $tg\beta = k\ tg\beta_i$ (Cf. Figure 1), where k is independent of the radius. This means that in this case the direction of the resultant velocity at the blade section results from the direction which is formed with v and $r\omega$ by multiplication with a factor which has the same value for each radius. Thus, the resultant velocities in their entirety form a true helicoidal surface also. The calculation shows, moreover, that the factor referred to is identical with the efficiency of a blade section which should now be designated as induced efficiency η_i since the losses resulting from the induced velocities are indeed the only ones now under consideration. At the optimum condition, therefore, each blade section has the same induced efficiency; consequently, the induced efficiency of any blade section is identical to that of the entire propeller. Finally, the optimum condition can be expressed in still a third form. In fact, where k is independent of r, the expression $tg\beta = k\ tg\beta_i$ is equivalent to the statement that v' is independent of the radius so that the quantities w and $\vartheta = \frac{w}{v}$ likewise become independent of the radius. Hence, for the case where the loss of energy is at a minimum, the following equivalent relations exist:

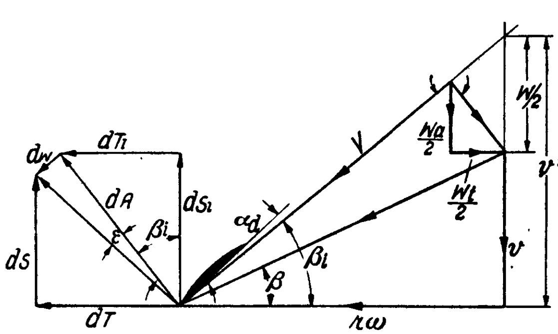

Figure 1 - Velocity- and Force-Diagram on a Blade Section of r = x × R

$$\eta_i = \frac{tg\beta}{tg\beta_i} = \frac{\frac{v}{r\omega}}{\frac{v'}{r\omega}} = \frac{v}{v'} = \frac{v}{v + \frac{w}{2}} = \frac{1}{1 + \frac{\vartheta}{2}}$$

where η_i and the other quantities, especially the so-called "displacement coefficient" ϑ, are independent of the radius so that they are applicable to the analysis of the propeller as a whole. Consequently, these relations hold good for the blade tip as well, so that the optimum condition can also be expressed:

$$\eta_i = \frac{\frac{v}{R\omega}}{\frac{v'}{R\omega}} = \frac{\lambda}{\lambda_i}$$

where λ designates the advance coefficient.

It should be pointed out that the components of the induced velocity are not completely determined by the optimum condition since this requires only that the end point of the resultant induced velocity lie along the direction of β_i. When the vortex sheets are sufficiently close to each other, it can be shown that the resultant induced velocity is perpendicular to the direction β_i, i.e., perpendicular to the resultant velocity. This is considered to be sufficiently satisfied in the following discussion.* From the geometrical relations represented in Figure 1 the following expressions can then be given for the induced components at the radius $x = \frac{r}{R}$ of the blade:[5]

$$\frac{1}{2}\frac{w_a}{v} = \frac{\vartheta}{2}\frac{x^2}{x^2 + \lambda_i^2}, \qquad \frac{1}{2}\frac{w_t}{v} = \frac{\vartheta}{2}\frac{x\lambda_i}{x^2 + \lambda_i^2}$$

$$x = \frac{r}{R}, \qquad \lambda_i = \frac{v'}{R\omega} = \frac{\lambda}{\eta_i}, \qquad \vartheta = \frac{w}{v}$$

The next task is to determine ϑ and λ_i or else η_i which, according to the optimum condition, is the same thing. In accordance with the principle of Kutta-Joukowski, therefore, we express the forces acting on the blade section from which the thrust and torque developed by the propeller are obtained, by integration over the entire propeller blade. The following expressions are obtained for the element of thrust and the tangential component of force:

$$\left.\begin{aligned} dS_i^\infty &= \Gamma\infty\rho\left(\omega r - \frac{w_t}{2}\right)dr = \Gamma\infty\rho vR\left(\frac{x}{\lambda} - \frac{1}{2}\frac{w_t}{v}\right)dx \\ dT_i^\infty &= \Gamma\infty\rho\left(v + \frac{w_a}{2}\right)dr = \Gamma\infty\rho vR\left(1 + \frac{1}{2}\frac{w_a}{v}\right)dx \end{aligned}\right\} \Gamma - 2r\pi w_t = 2R\pi x\, w_t$$

*Investigations conducted since this paper was originally written indicate that the "normal condition" is satisfied for vortex sheets of true helical shape without any restriction to the mutual distance of the sheets.

The indices ∞ and i must be used for the elements of force since for the present we are considering a propeller with an axially symmetrical slip stream, i.e., a propeller of infinitely many blades operating in an ideal, i.e., frictionless, fluid. If the above expressions are substituted for the induced velocities, it will be found that the integration over the radius x is possible. Thus the desired relation is obtained between the thrust loading factor of the power loading factor of the propeller and the quantities ϑ and λ_i or their equivalents η_i and λ respectively. In order to facilitate the later application of these results to a propeller with a finite number of blades, it is desirable to introduce first the dimensionless circulation

$$G_\infty = \frac{x^2}{x^2 + \lambda_i^2}$$

which is related to Γ_∞ as follows:

$$\Gamma_\infty = 2r\pi w_t = 2r\pi w \frac{x\lambda\left(1 + \frac{\vartheta}{2}\right)}{x^2 + \lambda_i^2} = 2\pi \frac{wv}{\omega}\left(1 + \frac{\vartheta}{2}\right) \frac{x^2}{x^2 + \lambda_i^2} = G_\infty \times 2 \frac{wv}{\omega}\left(1 + \frac{\vartheta}{2}\right)$$

The dependence of the circulation on the radius is thus expressed by G_∞, while the products of the other quantities constitute a constant factor for each particular optimum propeller. The integration of the elements of force over the radius yields integrals of the form

$$\int_0^1 \frac{x^m}{(x^2 + \lambda_i^2)^n}\, dx$$

which we shall designate by K; these integrals depend only on m and n and they are related to G_∞ as follows:

$$K^{\infty}_{m,n} = \int_0^1 \frac{x^m}{(x^2 + \lambda_i^2)^n}\, dx = \int_0^1 G_\infty \frac{x^{m-2}}{(x^2 + \lambda_i^2)^{n-1}}\, dx$$

This is essential to the application of the results to a propeller with a finite number of blades. Expressed in this form, the integration over the element of thrust now yields the following formula which represents the dependence of the thrust loading coefficient upon ϑ and λ_i:

$$c_{si}^{\infty} = 4\vartheta \times K_{3,1}^{\infty} + 2\vartheta^2 \times K_{5,2}^{\infty}$$

$$K_{3,1}^{\infty} = \frac{1}{2}\left[1 - \lambda_i^2 \times \ln\left(1 + \frac{1}{\lambda_i^2}\right)\right]$$

$$K_{5,2}^{\infty} = \frac{1}{2}\left[1 + \frac{\lambda_i^2}{1 + \lambda_i^2} - 2\lambda_i^2 \times \ln\left(1 + \frac{1}{\lambda_i^2}\right)\right]$$

The next step toward a complete solution of the problem is to apply this expression for the thrust load factor of a propeller with an infinite number of blades to a propeller with a finite number of blades; in doing so the assumption of a frictionless fluid is retained for the present. The first consideration will be to transform c_{si}^{∞} to $c_{si}^{\mathfrak{z}}$.[6] This transition to a propeller with a finite number of blades can be accomplished if it is recalled that, other conditions being equal, the mean values of the induced velocities for a fixed radius differ from the corresponding values for a propeller with an infinite number of blades. The smaller the number of blades, the greater is this difference, i.e., the greater the distance between the trailing vortex sheets. The value of the induced velocities remains constant on the blade itself but between the blades it varies according to the distance between the vortex sheets. Since the circulation depends only on the mean value of the induced tangential velocity component, the following expression is justified: $\Gamma_{\mathfrak{z}} = X \times \Gamma_{\infty}$. According to the method of calculation just outlined, the corresponding values for a propeller with a finite number of blades are obtained by substituting $X \times \Gamma_{\infty}$ for Γ_{∞}; thus

$$K_{m,n} = \int_0^1 X \times G_{\infty} \times \frac{x^{m-2}}{(x^2 + \lambda_i^2)} dx = \frac{x^m}{(x^2 + \lambda_i^2)^n} dx$$

and, finally, according to Kramer's analysis:[6]

$$c_{si}^{\infty} = 4\vartheta \times K_{3,1}^{\infty} + 2\vartheta^2 \times K_{5,2}^{\infty}$$

The "mean value factor" was obtained by Goldstein by solving the potential theory problem involved in the determination of the velocity field of the helicoidal vortex sheets.[7] It will be found that X depends only on x and λ_i, in addition to the blade number $\mathfrak{z}$.[8] Consequently, the integrals $K_{3,1}^{\mathfrak{z}}$

and $K^{\gamma}_{5,2}$ can be expressed as numerical functions of γ and λ_i. Thus the thrust load factor c^{γ}_{si} appears as a function of γ, λ_i, and ϑ, or, in terms of the optimum condition, of γ, λ and η_i. On the basis of this relation, represented in Figure 2, it is possible to read off the efficiency with which a propeller with γ blades generates a certain thrust loading factor in a frictionless fluid at a given advance coefficient.

Before discussing this result further, we shall eliminate the sole remaining assumption, viz., that of a frictionless fluid, and turn to a propeller with a finite number of blades operating in a viscous fluid to obtain a complete approximate solution of our problem. The friction which occurs produces a component of force dW on the blade element in the direction of the resultant relative velocity (Cf. Figure 1). This causes a change in the direction of the lift so that the thrust decreases while the torque increases. The resultant drag is measured in terms of the drag-lift coefficient $\epsilon = \frac{c_w}{c_a}$ in percentages of the lift. From Figure 1 the relation between the element of thrust dS_i in the ideal fluid and dS in the viscous fluid can be readily expressed as:

$$\frac{dS_i}{dS} = \frac{\cos\beta_i}{\cos(\beta_i + \epsilon)}$$

This relation varies from one radius to another, but fortunately the variation is only slight. Hence, it can be replaced by its mean value or even by its value at the blade tip which would be sufficient for practical calculations. This avoids the tedious integration over the radius which involves an additional assumption with respect to the radial dependence of ϵ. Therefore, the same relation between the thrust loading coefficients can be written which at first was only applicable for the elements of thrust, viz.,

$$\frac{c_{si}}{c_s} = \frac{\cos\beta_i}{\cos(\beta_i + \epsilon)}$$

where β_i now designates the value at the blade tip ($\operatorname{tg}\beta_i = \lambda_i$) or a mean value over the blade. This relation is important for practical calculations since it enables the thrust loading coefficient of the propeller to be converted into the greater value c_{si} which it would generate in the ideal fluid under otherwise equal conditions.

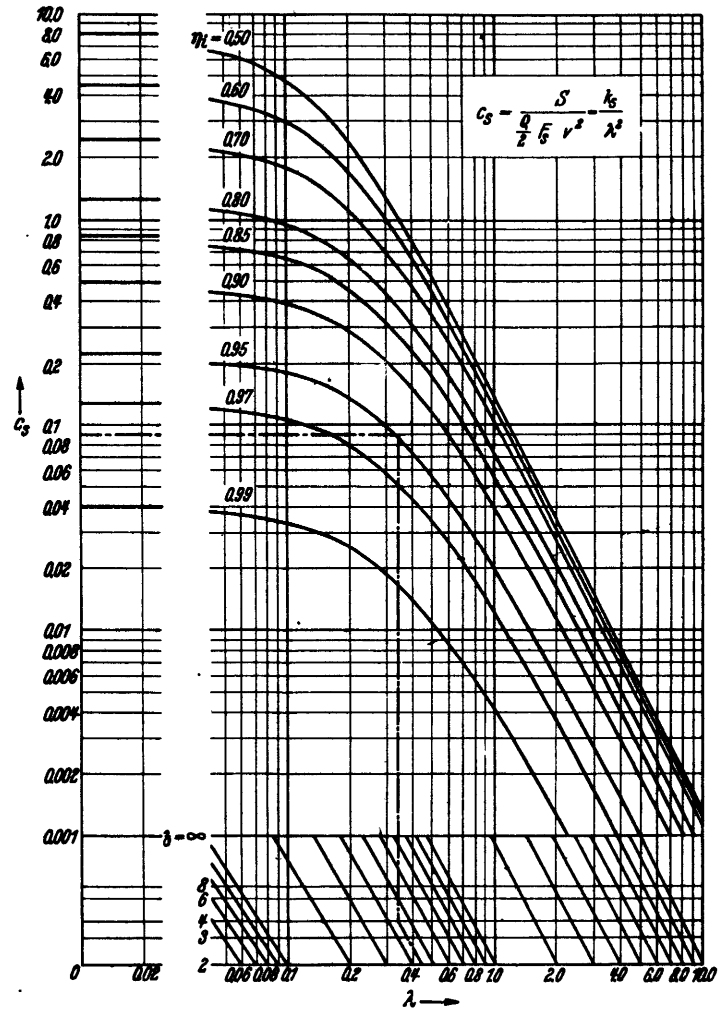

Figure 2 - Induced Efficiency η_i As a Function of λ, c_{si} and γ
(According to Kramer, Luftforschung, Vol. 15, No. 7, 1938, pp. 326-333)

The effect of the viscous fluid on the propeller results in a special efficiency η_ϵ which expresses the change in the thrust and torque brought about by the appearance of dW (Cf. Figure 1). Assuming that the drag-lift coefficient is constant along the radius, a more detailed calculation shows,[1,9] that the formula

$$\eta_\epsilon = \frac{1 - 2\epsilon\lambda_i}{1 + \frac{2}{3}\frac{\epsilon}{\lambda_i}}$$

expresses these losses with sufficient accuracy.

Now the total efficiency η of a propeller may be separated into $\eta = \eta_i \times \eta_\epsilon$, where η_i expresses the kinetic losses which accompany the generation of thrust in an ideal fluid and which result from the induced velocities, and where η_ϵ represents the loss in power input through friction. In a broader sense, η_ϵ represents all the losses produced by factors which affect the drag-lift coefficient, for instance, cavitation. Unfortunately, it is not possible within the scope of this paper to discuss the phenomena of cavitation in more detail. Nevertheless, we have seen that by this method of approach, it becomes possible to determine velocity and pressure of the propeller flow at any station of the blade. This permits the design of a propeller under any condition of cavitation provided the profile properties of each blade element are known for the corresponding pressure and velocity of the propeller flow.

At this point, a remark should be made regarding the optimum condition which is essential to our method of analysis in so far as it determines the thrust distribution or the distribution of the induced velocities, respectively. This condition was formulated so that the induced losses are at a minimum and the losses caused by the drag-lift coefficient, especially the frictional losses, are not taken into consideration. That a practically useful result is obtained by limiting the minimum condition to only one part of all the losses is explained by the fact that the induced component of the loss is by far the major portion of the power loss. This is seen from the small value of the drag-lift coefficient of a wing section which, in the normal range of propeller operation, amounts to about 3 percent of the lift. Hence, there is justification for retaining the simple and obvious result of the optimum condition just developed, or, in other words, for regarding the induced velocities as independent of friction and for designating a propeller designed in accordance with this as the propeller of minimum energy loss. Of course, as pointed out previously, it is possible to set up this minimum condition in a similar manner for the total loss; this has been done by Bienen[9] and more recently by Flügel.[10] It may be noted here that according to Flügel's investigations, the total losses for propellers under small loads are at a minimum when the total efficiency is constant along the radius.

It is possible to solve the practical problems of propeller design on the basis of the results of the theory as presented thus far. The calculation will be set up so that a certain thrust loading coefficient is required for a given advance coefficient, i.e., so that thrust, number of revolutions, diameter and speed are given while the required power input is to be determined. It is also possible to set up the calculation so that power input and number of revolutions are given while the attainable velocity is to be found.

The method of solution begins by converting the required thrust loading factor c_s of the propeller to the somewhat greater value c_{si} which the propeller would generate with the same speed coefficient in an ideal fluid; then, with the aid of the diagram of Figure 2, the efficiency η_i of this thrust in an ideal fluid is determined. Hence, by means of the optimum condition, all quantities required for the calculation of the induced velocities on a blade section and thus for the calculation of pitch as well are already determined. The required chord length of the blade is obtained by first expressing the element of life by means of the square of the resultant relative velocity V, in accordance with the definition of the lift coefficient, and then by expressing it according to the principle of Kutta-Joukowski:

$$dA = \frac{\rho}{2} V^2 \times c_a \times \gamma \times t \times dr$$

$$dA = \gamma \times \Gamma_\infty \times \rho \times V \times dr = \gamma \times 2r\pi \times X w_t V \, dr$$

By equating, we obtain the expression

$$c_a \gamma t = 4r\pi X \frac{w_t}{V}$$

From the geometrical relations of Figure 1 we obtain for w_t/V the relation

$$\frac{w_t}{V} = 2 \times \sin\beta_i \times tg(\beta_i - \beta)$$

and, finally,

$$c_a \times \gamma \times t = 8x\pi R X \sin \beta_i \times tg(\beta_i - \beta)$$

Thus, all the quantities required for the calculation of the geometrical data of a propeller are determined.

As an example, use the numerical values indicated in Reference 2 and, designing the propeller according to the method of calculation developed above, compare our result with the result obtained in Reference 2 and with the model test described therein.

Given: $c_s = 1.273$; $\lambda = 0.187$; $\epsilon = 0.025 = 1.43°$

Required: η and the shape of the propeller.

1. $c_{si} = c_s \times \cos\beta_i / \cos(\beta_i + \epsilon)$

λ_i	β_i^0	$\cos\beta_i$	$(\beta_i + \epsilon)°$	$\cos(\beta_i + \epsilon)$	c_{si}	η_i
0.187	10.58	0.983	12.02	0.978	1.280	0.72
0.260	14.55	0.968	15.98	0.961	1.283	0.715
0.262	14.67	0.967	16.10	0.961	1.282	0.715
0.262	-	-	-	-	-	-
$\vartheta = \frac{2(1 - \eta_i)}{\eta_i} = 0.797$						

2. $\frac{1}{2}\frac{w_a}{v} = \frac{\vartheta}{2}\frac{x^2}{x^2 + \lambda_i^2}$; $\frac{1}{2}\frac{w_t}{v} = \frac{\vartheta}{2}\frac{x\lambda_i}{x^2 + \lambda_i^2}$; $tg\,\beta_i = \frac{1 + \frac{1}{2}\frac{w_a}{v}}{\frac{x}{\lambda} - \frac{1}{2}\frac{w_t}{v}}$; $tg\,\beta = \frac{\lambda}{x}$

x	x^2	$x^2+\lambda_i^2$	$\frac{1}{2}\frac{w_a}{v}$	$x\lambda_i$	$\frac{1}{2}\frac{w_t}{v}$	$tg\,\beta_i$	β_i^0	$\frac{\lambda}{x}$	β^0	$(\beta_i-\beta)^0$
0.353	0.125	0.194	0.257	0.093	0.191	0.741	36.53	0.530	27.92	8.61
0.471	0.222	0.291	0.305	0.123	0.169	0.555	29.03	0.397	21.67	7.36
0.588	0.346	0.415	0.333	0.154	0.148	0.445	24.00	0.318	17.62	6.38
0.706	0.498	0.567	0.351	0.185	0.130	0.371	20.33	0.265	14.83	5.50
0.823	0.677	0.746	0.362	0.216	0.116	0.318	17.62	0.227	12.78	4.84
0.941	0.885	0.954	0.370	0.247	0.103	0.278	15.53	0.199	11.28	4.25

3. $c_a \times t = \frac{8\pi R}{\gamma} \times x \times X \sin\beta_i \times tg(\beta_i - \beta) = 17.80 \times x \times X \times \sin\beta_i \times tg(\beta_i - \beta)$

x	X^1	$\sin\beta_i$	$tg(\beta_i-\beta)$	c·t (m)	t (m)	c_a	$\frac{d}{t}$	α^0	ϵ
0.353	0.99	0.595	0.151	0.558	0.921	0.606	0.0923	~7	>0.035
0.471	0.98	0.485	0.130	0.516	1.029	0.502	0.0719	1.8	0.020
0.588	0.97	0.407	0.112	0.463	1.082	0.428	0.0582	1.2	0.020
0.706	0.92	0.348	0.096	0.387	1.065	0.363	0.0498	1.2	0.020
0.823	0.79	0.303	0.085	0.298	0.946	0.315	0.0433	1:2	0.021
0:941	0.49	0.268	0.075	0.166	0.634	0.262	0.0457	0.5	0.023

4. $\frac{H}{2\pi} = R \times x \times tg(\alpha + \beta_i) = 2.125 \times x \times tg(\alpha + \beta_i)$

x	$(\alpha + \beta_i)^0$	$tg(\alpha + \beta_i)$	$\frac{H}{2\pi}$(m)
0.353	43.53	0.950	0.712
0.471	30.83	0.597	0.598
0.588	25.20	0.471	0.589
0.706	21.53	0.395	0.592
0.823	18.82	0.341	0.595
0.941	16.03	0.288	0.578

Mean value $\frac{H}{2} = 0.590$

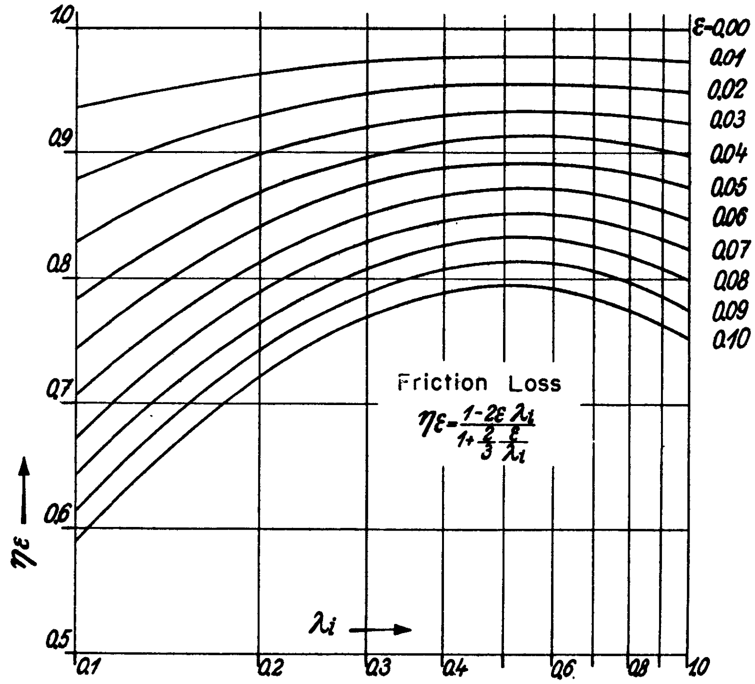

Figure 3

A comparison of our results with those of Reference 2 indicates that the same data for the shape of the propeller are obtained as before. The efficiency now becomes $\eta = \eta_i \times \eta_\epsilon = 0.715 \times 0.928 = 0.664$ where η_ϵ can be taken from Figure 3. The efficiency now obtained is about 4 percent greater than the previously calculated value $\eta = 0.636$. According to the model test previously carried out, this means that the velocity of the ship as now predicted is too large by about 0.5 percent whereas its previous value was too small by the same amount. The same can be said here as in Reference 2 with regard to the correction of the value of the pitch.

An indication of the limitations of the approximate theory which assumes a slip stream free from contraction and neglects the pressure gradient resulting from the centrifugal force may be obtained by comparing its results with those of an exact theory of Betz-Helmbold[11] which takes the factors of contraction and pressure gradient into consideration for an infinite number of blades. It will be found that for efficiencies of η_i equals about 0.5 and lower, i.e., in the entire region of practical importance, the agreement is surprisingly good.[5] Thus, in practice, it is entirely sufficient and justifiable to take advantage of the extremely simple method of numerical calculation of the approximate theory, an advantage which is unfortunately missing in the exact theory.

Finally, we shall discuss the two component efficiencies η_i and η_ϵ. It will be seen in Figure 2 that the kinetic losses at a constant advance coefficient increase with the thrust loading coefficient while at a constant thrust loading coefficient the losses increase with the advance coefficient. This behavior of the propeller results in the first case from an increase of the kinetic energy in the rotation of the slip stream. This second case is particularly interesting since it is possible to convert a part of the energy loss due to the stream rotation into an additional thrust by means of guide vanes, and thus to convert it back again into useful energy. The questions of practical importance concerning the increase that can be expected from a guide vane and its dependence on the advance coefficient and the thrust loading coefficient may be answered by referring to the diagram for η_i. The induced efficiency for a propeller with an infinite number of blades must be compared with the efficiency which, with an equal thrust loading coefficient, would result for a purely axial motion of the propeller slip stream. This can be read at the extreme left of the ordinate of Figure 2. The difference between these two efficiencies is due mainly to the rotation in the slip stream and it gives the approximate increase in energy made possible through the use of ideal guide vanes which operate without any energy loss. This increase in energy is then to be multiplied by the efficiency of the guide vanes. As shown by one of Betz's investigations,[12] this in turn depends upon the distribution of thrust over the blades of the guide vanes and on the length and number of the blades, just as is the case for a propeller.

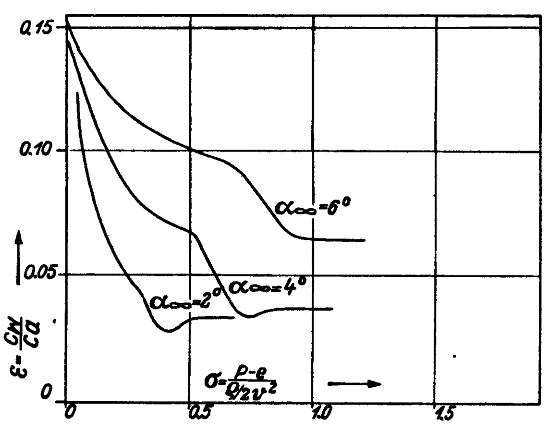

Figure 4 - Drag-Lift Coefficient of a Circular Back Profile (d/l = 0.0385) as Functions of σ and α_∞ (According to O. Walchner, Hydromechanische Probleme des Schiffsantriebs, Vol. I, pp. 256 - 267)

The component efficiency η_ϵ decreases with increasing ϵ and reaches a maximum for $\lambda_i \sim 0.5$ (Cf. Figure 3). At the normal operation of a propeller, the value of ϵ amounts to about 0.03 - 0.04 so that the corresponding losses at the most favorable advance coefficient are about 8 percent. With the inception of cavitation, the drag-lift ratio and thus the energy loss can increase considerably. Values of the magnitude of 0.1 may result for ϵ under certain conditions (Cf. Figure 4). Thus, losses of as much as 20 percent as compared with η_i can be accounted for; according to what has been said previously, this means a loss of about 12 percent as compared with the propeller that is unaffected by cavitation.

The two component efficiencies differ in their dependence on the advance coefficient; η_i decreases with increasing λ while η_ϵ increases with the advance coefficient within the range of practical importance. Consequently, for a constant thrust loading coefficient, there is an advance coefficient at which the given problem is solved with a maximum total efficiency of $\eta = \eta_i \times \eta_\epsilon$. One is justified, therefore, in speaking of an optimum diameter of a propeller or of an optimum number of revolutions respectively. However, this holds good only for that value of the induced advance coefficient λ_i which corresponds to the maximum of η_ϵ; outside of this limit the maximum total efficiency is related to the smallest possible advance coefficient.

These remarks about the two component efficiencies of a propeller conclude my lecture. I should like to emphasize that while these considerations were confined to free running propellers, the side propellers of multi-propeller ships correspond with sufficient accuracy. The application of the theory to the case where the mean initial velocity taken in the direction of the circumference depends to a large extent upon the radius (as in single-propeller ships) presents no methodological difficulties. Here also the optimum condition which involves the constancy of the variation of efficiency is used as a basis. Then the corresponding thrust distribution[14] is obtained which, when compared with the free running propeller, is displaced toward the hub corresponding to the distribution of the wake.

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