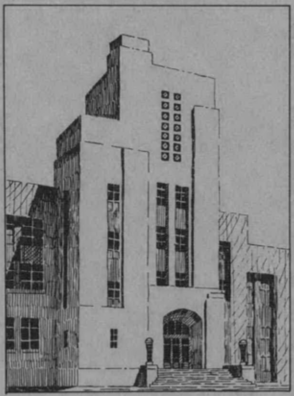
THE DAVID W. TAYLOR MODEL BASIN

UNITED STATES NAVY

CONTRIBUTION TO STRENGTH TESTING ON SHIPS UNDERWAY

BY DR. W. DAHLMANN AND ING. K. REMMERS



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RESTRICTED

CONTRIBUTION TO STRENGTH TESTING ON SHIPS UNDERWAY

(BEITRAG ZUR FESTIGKEITSMESSUNG AM FAHRENDEN SCHIFF)

by

Dr. W. Dahlmann and Ing. K. Remmers

(Schiffbau, Schiffahrt und Hafenbau, 1 January 1940)

Translated by M. C. Roemer

The David W. Taylor Model Basin
Bureau of Ships
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CONTRIBUTION TO STRENGTH TESTING ON SHIPS UNDERWAY

(Report of the Hamburg Strength Testing Laboratory on Tests Aboard the MS DUISBURG
of the Hapag, from 3 March 1939 to 8 April 1939)

The purposes of the trip were:

1. To measure the variation in static load in the longitudinal structure during loading,
2. To check the effect of temperature changes,
3. To measure variations in plane stress condition in a seaway.

ARRANGEMENT OF TEST STATIONS

In addition to strain and temperature tests, a study of the possibility of the buckling of plates under changing load conditions was made. Deflections of the longitudinal structure of the ship were also measured. As shown in Figure 1, the strain gages were distributed over seven stations on the port side. At Stations I to V the gages were arranged tri-axially, at Station VI in longitudinal and transverse direction, and at Station VII only longitudinally. The tri-axial strain gage consisted of Orthotest instruments made in quantity production by the firm of Carl Zeiss, Jena, which were built into mounting frames made especially for this purpose. The base length in each case was 200 millimeters (7.87 inches). Views of individual stations are shown in Figure 2. The test range of the instruments exceeds 200μ, and for this range the guaranteed accuracy of the Orthotest itself is ±1μ. Since the greater portion of the measuring rods of the mounting frame consists of Indilatan steel, while

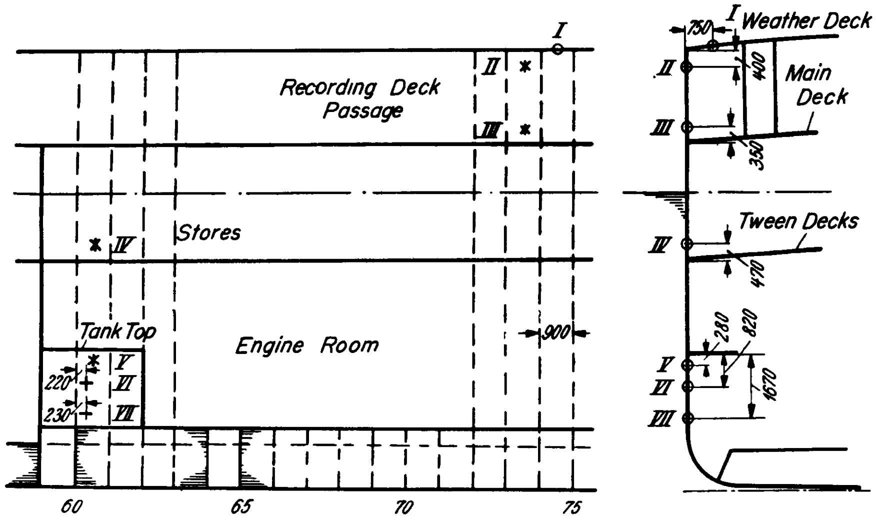
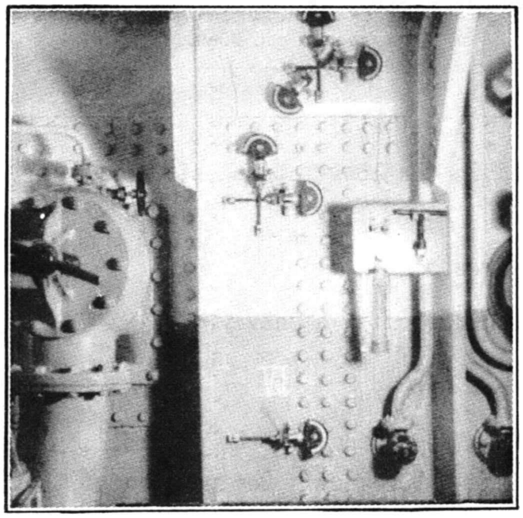


Figure 1 - Arrangement of Strain Gages

All the instruments are inboard on the port side, and are, with the exception of VI and VII, placed midway between frames



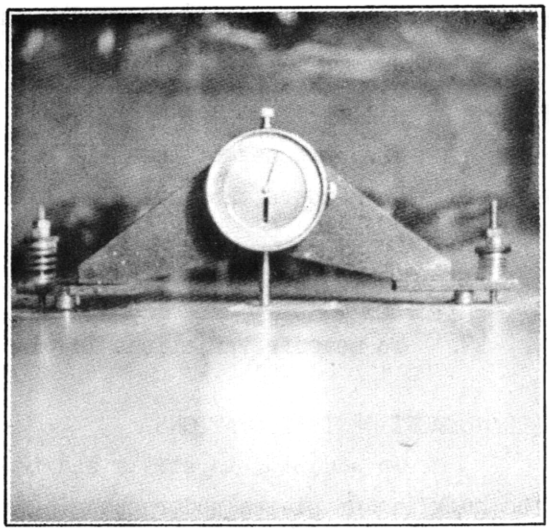


Figure 3 - Dial Gage for Measuring Buckling

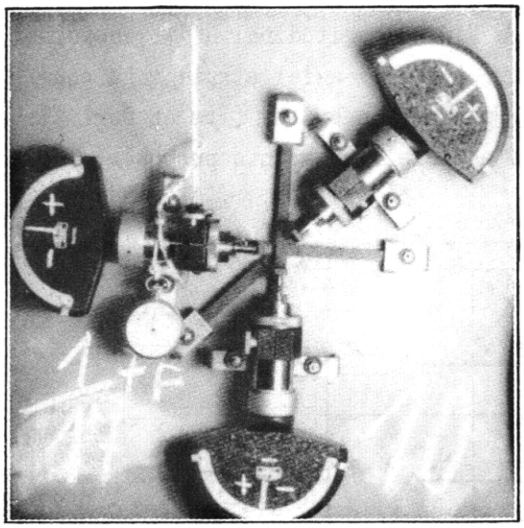


Figure 2 - Arrangement of Strain Gages

the expansion coefficient of that part of the Orthotest proper which is involved corresponds quite closely to that of steel used in ship structures, calculation shows that for the 200-millimeter (7.87-inch) base length there will be an expansion of 1.1μ in the instrument for each degree of temperature change.

The method of installing the thermometers is shown in Figure 2.

The instrument used to measure the buckling of plates under changing loads was an ordinary Zeiss dial indicator on a mounting saddle with a base length of 200 millimeters (7.87 inches), as shown in Figure 3. The instrument was attached at various points on the shell plating to determine maximum buckling. During the major portion of the trip this gage was at Station III.

Longitudinal deflections of the ship were observed in the usual manner by means of two Zeiss theodolites (see Figure 4).

RESULTS OF TESTS

Measurements of longitudinal deflection yielded very small values owing to the uniformity of changes in loading and temperature. The deflections observed were less than 5 millimeters (0.197 inch) over a base length of 59 meters (194 feet). It was again evident that measurements with a theodolite are highly unreliable, chiefly

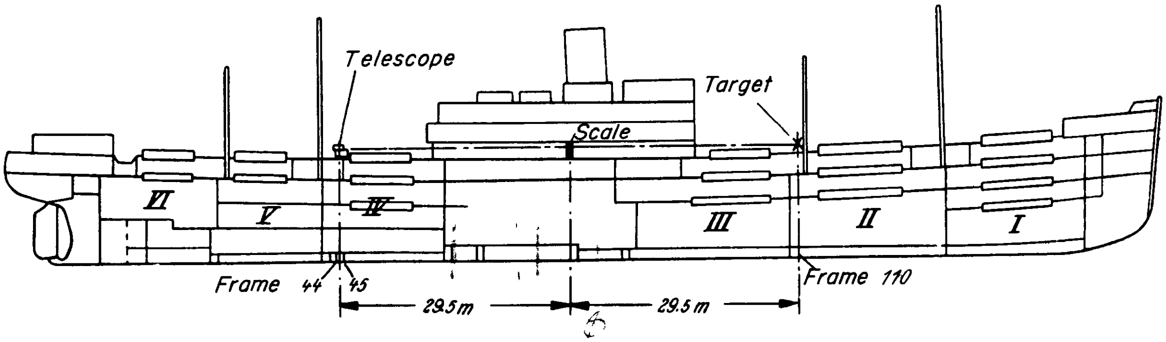


Figure 4 - Deflection Test Gear

There were two sets of identical gear, one on the port and one on the starboard side.

because it is impossible to obtain simultaneously a sharp focus on the scale at a distance of 29.5 meters (96.78 feet) and on the target at 59 meters (194 feet). Unfortunately it was impossible to obtain deflection data in a seaway because of the lack of waves during the trip. Therefore an improved test instrument is being constructed.

Observation of the buckling of plates in a light seaway showed a maximum change in deflection of 15μ at the middle of a base length of 200 millimeters (7.874 inches).

Calculations must be made to determine how much the actual length $2s$ of the arch exceeds that of the chord $2a$ which is 200 millimeters (7.874 inches). If the curve is assumed to be parabolic, the length of arc, according to Figure 5, will be approximately

$$2s = 2a\left[1 + \frac{2}{3}\left(\frac{b}{a}\right)^2 - \frac{2}{5}\left(\frac{b}{a}\right)^4\right]$$

This will give

$$2s - 2a = \sim 3\mu$$

i.e., the effect of buckling is negligible.

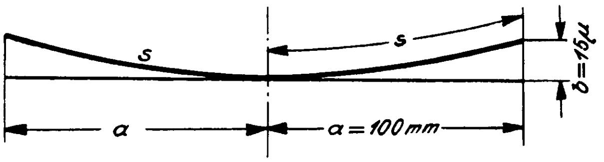


Figure 5

DATA ON CHANGE IN STRESS DUE TO LOADING

The data are compiled in Table 1. For the period from 20 March to 31 March during which the changes in stress due to loading occurred, there were no appreciable strains in the ship plating at Stations V, VI, and VII as a result of loading, which was uniform for all hatches. Consequently there were no appreciable ship deflections either, particularly since the temperature of the plating at Stations V, VI, and VII, which were below the water line, remained constant because the temperature of the sea water was approximately constant. On the other hand the gages at the remaining stations showed considerable movements due to changes in temperature.

TABLE 1

Temperatures and Orthotest Readings at the Seven Recording Stations

Temperatures are in degrees C and Orthotest readings are in μ.
Straight lines at the head of the columns indicate the direction of the strain readings.

Date	Time	Sea Water °C	Outside Air °C	Room Temperatures °C			I (Weather Deck)					II (Sheer Strake)					III				
				Passage	Stores	Below	Ship's Skin °C	Instrument °C	—	|	/	Ship's Skin °C	Instrument °C	—	|	/	Ship's Skin °C	Instrument °C	—	|	\
March 1939																					
20	7:00	The temperature of sea water corresponds quite closely to the temperature of the ship's side at Stations V and VI	8.0				Temperatures of ship's side and instruments not recorded	During sunlight the deck remained considerably colder than the ship's side at Station II	+ 2	- 10	+ 6	1.5	4.0	- 1	- 4	- 5	1.5	4.0	+ 6	+ 7	- 10
	16:00								- 2	- 6	+ 1	14.8	14.5	- 5	- 20	- 15	14.8	14.5	+ 8	+ 5	- 9
21	18:00		5.0	11.0	16.0	17.0			- 30	- 11	- 2	8.0	9.8	- 4	- 5	- 12	8.0	9.8	+ 9	+ 16	- 7
22	8:30		5.0	7.0	13.7	15.5			- 31	- 4	+ 10	2.5	5.0	- 6	+ 2	- 7	2.5	5.0	+ 7	+ 24	- 6
23	8:30		6.0	6.8	13.5	13.5			- 32	- 9	+ 9	4.4	5.2	- 5	+ 9	- 8	4.4	6.0	+ 8	+ 25	- 6
25	12:00			13.0	16.8				- 45	- 6	- 6	17.9	13.5	- 6	- 25	- 23	17.9	14.0	+ 10	+ 20	- 15
26	8:00		6.0	7.5	12.8	20.0			- 48	- 18	- 9	6.0	5.5	- 13	- 1	- 20	6.0	6.0	+ 7	+ 37	- 17
27	8:00		5.0	9.0	15.5	15.5			- 48	- 11	- 7	5.3	6.9	- 16	+ 6	- 22	5.3	7.5	+ 6	+ 36	- 10
28	8:00		4.0	10.0	14.7	15.0			- 48	- 10	- 7	6.0	7.8	- 16	+ 6	- 22	6.0	7.8	+ 6	+ 36	- 10
29	7:15		5.5	10.5	16.2	18.2			- 50	- 10	- 7	6.0	7.9	- 16	+ 7	- 22	6.0	8.0	+ 7	+ 36	- 11
30	9:50			16.4	15.7	14.8			- 59	- 9	- 13	22.3	17.7	- 22	- 26	- 28	22.3	15.5	+ 2	+ 19	- 18
	20:45			12.1	16.0	16.0			- 60	- 13	- 10	8.3	10.8	- 15	+ 4	- 26	8.3	11.0	+ 5	+ 31	- 17
31	8:00			12.5	16.0	18.5			- 60	- 6	- 7	12.2	11.0	- 17	- 8	- 24	12.2	11.5	+ 3	+ 35	- 17
April 1939																					
1	7:00		9.0	14.0	16.7	21.7			- 59	- 14	- 7	13.8	11.7	- 16	- 16	- 31	13.8	12.0	+ 4	+ 32	- 19
	8:00		10.0	17.0	16.7	21.8			- 59	- 13	- 9	23.5	17.9	- 20	- 35	- 39	23.5	17.5	+ 2	+ 21	- 21
	9:10		12.0	22.1	17.3	22.0			- 60	- 10	- 11	33.8	25.8	- 24	- 56	- 47	33.8	23.6	+ 1	+ 4	- 28
	10:00		11.3	23.5	17.2	22.2			- 60	- 10	- 11	37.5	28.9	- 24	- 62	- 50	37.5	24.5	+ 1	0	- 30
	11:00		11.5	26.5	17.1	22.0			- 62	- 13	- 15	40.0	33.5	- 24	- 62	- 52	40.0	27.3	+ 1	- 5	- 35
	12:00		11.0	26.6	16.5	21.3			- 62	- 13	- 15	36.3	34.4	- 21	- 55	- 50	36.3	27.1	0	- 5	- 31
	13:15		11.3	23.0	15.5	21.0			- 62	- 15	- 16	26.2	27.2	- 17	- 36	- 41	26.2	23.8	+ 1	- 6	- 24
	14:00		11.2	21.6	15.5	20.9			- 61	- 16	- 16	23.0	24.1	- 18	- 31	- 37	23.0	22.2	+ 2	+ 11	- 23
	15:00		11.2	20.5	15.9	21.5			- 61	- 17	- 15	21.0	21.7	- 16	- 33	- 35	21.0	20.8	+ 2	+ 15	- 22
	16:00		11.5	19.2	16.4	21.5			- 60	- 17	- 15	18.7	19.9	- 16	- 23	- 34	18.7	19.3	+ 3	+ 18	- 21
	17:00		11.8	18.0	16.6	22.0			- 60	- 17	- 15	17.0	20.3	- 16	- 23	- 27	17.0	17.5	- 3	+ 23	- 21
	20:35		9.5	14.5	16.2	21.0			- 59	- 13	- 14	8.9	12.6	- 16	- 5	- 25	8.9	10.5	+ 2	+ 36	- 20
	22:30		9.7	13.6	16.3	21.5			- 59	- 10	- 14	10.1	12.4	- 16	- 7	- 24	10.1	11.2	+ 3	+ 38	- 20
2	0:30		9.3	14.0	17.6	21.6			- 59	- 11	- 14	10.5	14.0	- 17	- 7	- 23	10.5	11.0	+ 2	+ 39	- 20

EFFECTS OF CHANGES IN TEMPERATURE

Tests made on April 1 in a calm sea show the extraordinary effect of exposure to the sun. These results are also included in Table 1. While Stations IV, V, VI, and VII, which now were all below the water line, showed no appreciable strains owing to the uniform temperature of the water, the instruments at the other stations indicated temperature changes proportionate to the temperature rise of the plates. It should be noted that extensions due to temperature varied in degree in the various directions because of the fixation of the plates in the ship structure.

In Figure 6 the temperature strains of Stations II and III are plotted as functions of time. The extensometer reading is corrected with the temperature coefficient and the temperature of the gage which was also measured (see Table 2). A curve has also been plotted for strains corresponding to unrestricted extensions. It is particularly noticeable that longitudinal extension is greatly restricted. This fact is explained by the uniform temperature of the submerged part of the hull.

DATA ON ALTERATION OF PLANE STRESS CONDITION OF THE PLATING IN A SEAWAY

The ship traverses a wave in such a short time, that, even with three

TABLE 1 (Continued)

IV (Store)					V					VI				VII			Deflection Gage	Remarks
Ship's Skin °C	Instrument °C	—	|	\	Ship's Skin °C	Instrument °C	—	|	/	Ship's Skin °C	Instrument °C	—	|	Ship's Skin °C	Instrument °C	—		
	9.0	- 20	- 6	- 9		Temperature not closely enough recorded, corresponds to that of instrument VI	- 1				10.5	- 6	+ 2	Temperature not recorded corresponds closely enough to the skin temperature at Stations V and VI	10.5	- 8		Before loading
14.1	14.5	- 23	- 14	- 15	5.8		0			5.8	11.3	- 6	+ 2		11.0	- 8	5.0	In Bremen
10.0		- 21	- 5	- 14	6.0		- 1			6.0	13.0	- 7	+ 1		13.0	- 11	5.6	Out of Bremen
5.3	9.5	- 31	- 1	- 12	3.5		0			3.5	11.5	- 8	+ 1		11.5	- 11	5.9	Off Hamburg
6.6	10.0	- 32	- 2	- 13	3.5		0			3.5	9.7	- 8	+ 1			- 11	5.8	In Hamburg
11.0	11.7	- 30	- 4	- 12	5.0		- 1	- 10	- 29	5.0	11.5	- 8	0		11.0	- 12	4.8	In Hamburg
5.0	9.3	- 34	+ 4	- 11	5.0		- 3	- 10	- 31	5.0	12.5	- 5	0		12.5	- 13	5.7	At sea
7.5	11.0	- 34	- 2	- 10	6.0		- 3	- 10	- 37	6.0	12.2	- 5	+ 1		11.6	- 13	5.9	Off Antwerp
6.0	10.5	- 35	+ 2	- 10	5.9		- 3	- 10	- 37	5.9	11.6	- 5	+ 2		11.2	- 13	5.8	In Antwerp
7.8	10.5	- 34	+ 2	- 10	6.5		- 3	- 11	- 39	6.5	13.8	- 5	+ 1		13.2	- 13	6.2	Off Rotterdam
6.0	11.0	- 32	+ 1	- 8	6.2		- 2	- 11	- 39	6.2	11.5	- 4	+ 2		11.0	- 13	5.6	In Rotterdam
6.1	10.9	- 34	+ 1	- 10	7.0		- 3	- 12	- 39	7.0	11.3	- 5	+ 2		10.7	- 13	6.3	In Rotterdam
6.0	11.5	- 34	+ 1	- 7	6.8		- 3	- 12	- 40	6.8	14.4	- 5	+ 1		13.8	- 13	5.7	Out of Rotterdam
8.5	12.5	- 40	0	- 7	9.1		- 4	- 12	- 42	9.1	16.6	- 4	+ 1		16.7	- 14	6.0	At sea
8.5	12.5	- 39	- 1	- 6	9.0		- 3	- 12	- 42	9.0	16.6	- 7	0		16.7	- 14	4.9	Sun, astern, port
9.0	12.9	- 38	- 1	- 6	9.2		- 3	- 12	- 42	9.2	17.1	- 7	0		17.2	- 14	4.0	Sun to port
9.0	12.1	- 38	0	- 6	9.2		- 2	- 12	- 42	9.2	17.2	- 7	0		17.1	- 13	3.7	Sun to port
9.2	13.2	- 39	0	- 6	9.5		- 2	- 12	- 42	9.5	17.3	- 6	0		17.1	- 13	3.7	Sun to port
9.1	13.1	- 39	0	- 6	9.5		- 2	- 12	- 42	9.5	17.1	- 7	0		17.0	- 13	4.2	Fog
9.4	12.6	- 40	0	- 6	9.6		- 2	- 12	- 42	9.6	16.9	- 7	0		16.8	- 14	4.9	Fog
9.7	12.7	- 40	- 1	- 6	10.0		- 3	- 12	- 42	10.0	17.1	- 7	+ 1		17.0	- 14	5.0	Fog
10.1	12.9	- 41	- 2	- 7	10.3		- 4	- 12	- 43	10.3	17.4	- 7	+ 1		17.3	- 14	5.2	Fog lifts
10.3	13.1	- 42	- 2	- 7	10.5		- 4	- 12	- 43	10.5	17.4	- 8	+ 1		17.3	- 13	5.1	
10.1	13.6	- 42	- 4	- 7	10.4		- 4	- 12	- 44	10.4	17.3	- 8	+ 1		17.2	- 14	5.1	
10.1	13.0	- 41	- 6	- 7	10.6		- 3	- 11	- 44	10.6	17.5	- 7	+ 1		17.5	- 13	5.8	
10.3	13.0	- 40	- 5	- 7	10.8		- 3	- 11	- 45	10.8	17.8	- 8	+ 1		17.8	- 14	5.8	
10.7	13.0	- 38	- 7	- 7	11.0		- 4	- 12	- 47	11.0	18.0	- 8	+ 1		17.6	- 14	5.9	

TABLE 2 (To Figure 6)

Influence of Temperature Changes April 1, 1939 - At Sea - Very Calm Weather

Time	Temperature		Related to Readings at 7 o'clock*		Instrument Readings			Elongation in μ Related to Readings at 7 o'clock			Values Corrected with 1.1μ when Δt_2 = 1°C			Values for Unimpeded Elongation $\Delta t_1 \cdot 2.4\mu$	
	Ship's Plating t_1 °C	Instruments t_2 °C	Δt_1 °C	Δt_2 °C	—	|	/	Δ—	Δ|	Δ/	Δ—	Δ|	Δ/		
7:00	13.8	11.7	-	-	- 16	- 16	- 31	-	-	-	-	-	-	-	Station II
8:00	23.5	17.9	+ 9.7	+ 6.2	- 20	- 35	- 39	4	19	8	11	26	15	23	
9:10	33.8	25.8	+ 20.0	+ 14.1	- 24	- 56	- 47	8	40	16	24	59	33	48	
10:00	37.5	28.9	+ 23.7	+ 17.2	- 24	- 62	- 50	8	46	19	27	69	39	56	
11:00	40.0	33.5	+ 26.2	+ 21.8	- 24	- 62	- 52	8	46	21	31	73	46	62	
12:00	36.3	34.4	+ 22.5	+ 22.7	- 21	- 55	- 50	5	39	19	29	66	45	54	
13:15	26.2	27.2	+ 12.4	+ 15.5	- 17	- 36	- 41	1	20	10	17	38	27	29	
14:00	23.0	24.1	+ 9.2	+ 12.4	- 18	- 31	- 37	2	15	6	15	29	19	22	
15:00	21.0	21.7	+ 7.2	+ 10.0	- 16	- 33	- 35	0	17	4	11	29	15	17	
16:00	18.7	19.9	+ 4.9	+ 8.2	- 16	- 23	- 34	0	7	3	9	16	12	11	
17:00	17.0	20.3	+ 3.2	+ 8.6	- 16	- 23	- 27	0	7	- 4	9	16	5	7	
20:35	8.9	12.6	- 4.9	+ 0.9	- 16	- 5	- 25	0	- 11	- 6	1	- 10	- 5	- 11	
22:30	10.1	12.4	- 3.7	+ 0.7	- 16	- 7	- 24	0	- 9	- 7	0.7	- 8	- 6	- 8	
0:30	10.5	14.0	- 3.3	+ 2.3	- 17	- 7	- 23	1	- 9	- 8	3	- 7	- 6	- 7	
7:00	13.8	12.0	-	-	+ 4	+ 32	- 19	-	-	-	-	-	-	-	Station III
8:00	23.5	17.5	+ 9.7	+ 5.5	+ 2	+ 21	- 21	2	11	2	8	17	8	23	
9:10	33.8	23.6	+ 20.0	+ 11.6	+ 1	+ 4	- 28	3	28	9	15	42	22	48	
10:00	37.5	24.5	+ 23.7	+ 12.5	+ 1	0	- 30	3	32	11	16	48	25	56	
11:00	40.0	27.3	+ 26.2	+ 15.3	+ 1	- 5	- 35	3	27	16	19	46	34	62	
12:00	36.3	27.1	+ 22.5	+ 15.1	0	- 5	- 31	4	27	12	20	46	29	54	
13:15	26.2	23.8	+ 12.4	+ 11.8	+ 1	- 6	- 24	3	26	5	15	40	17	29	
14:00	23.0	22.2	+ 9.2	+ 10.2	+ 2	+ 11	- 23	2	21	4	13	34	15	22	
15:00	21.0	20.8	+ 7.2	+ 8.8	+ 2	+ 15	- 22	2	17	3	11	27	12	17	
16:00	18.7	19.3	+ 4.9	+ 7.3	+ 3	+ 18	- 21	1	14	2	9	23	10	11	
17:00	17.0	17.5	+ 3.2	+ 5.5	+ 3	+ 23	- 21	1	9	2	7	15	8	7	
20:35	8.4	10.5	- 4.9	- 1.5	+ 2	+ 36	- 20	2	- 4	1	0	- 6	- 1	- 11	
22:30	10.1	11.2	- 3.7	- 0.8	+ 3	+ 38	- 20	1	- 6	1	0	- .7	0	- 8	
0:30	10.5	11.0	- 3.3	- 1.0	+ 2	+ 39	- 20	2	- 7	1	1	- 8	0	- 7	
* 24 Hour Time.					—	|	/	Δ—	Δ|	Δ\	Δ—	Δ|	Δ\		

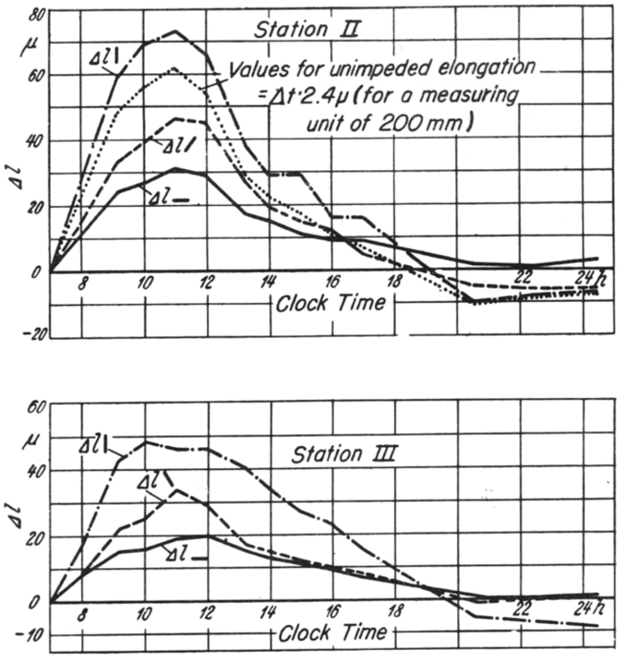


Figure 6 - Extension of Plating due to Rising Temperature

l = 200 millimeters (see Tables 1 and 2)

extensometers at one station, readings are impossible. Hence, an automatic recorder is necessary. To record at all stations simultaneously, an automatic Robot camera with an electric shutter was first experimentally used. With this camera several series of 8 to 20 exposures were made in the course of a few seconds in a seaway (see Figure 7). The negatives were enlarged to about 60 x 60 centimeters (23.6 x 23.6 inches) and were analyzed directly. The gage readings and the data of Series 9, 10, and 13 are given in Table 3.

The size and direction of the preliminary strains and stresses in the plating at the stations must be known to convert the strain data to stresses in the material. As proposed by Dr. Ing. P. Maack, the method of determining pre-stresses in ship plating, using "virgin" base lengths established on the plates as a control, is to be put into practice as soon as possible.*

Since, as usual, the preliminary stress was unknown in the present case, it was assumed, solely to obtain an idea of its effect, that at Station II the plate had just reached a point of zero strain immediately preceding the reading. This was by no means actually true. It was further assumed that in the region of the test station the strain condition of the plate was homogeneous.

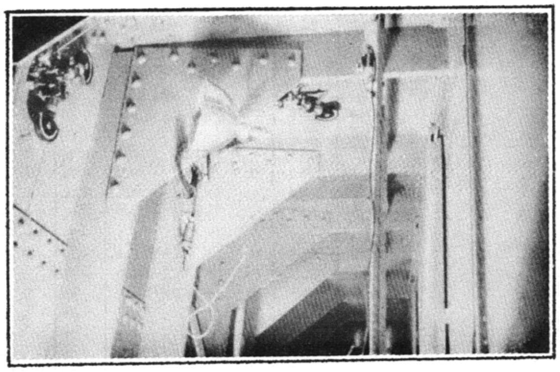


Figure 7 - Robot Camera in Place at a Station

To analyze the tri-axial strain measurements, i.e., to determine

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* See P. Maack, "Kritische Betrachtungen und Vorschläge zu Festigkeitsmessungen am Schiffskörper" (Critical Studies and Suggestions on Strength Measurements of Ship Forms), Boyson and Maasch, Hamburg, 1935.

TABLE 3

Strain Variations at Station II

In the Bay of Biscay, in a head sea.

	Orthotest Readings					
	Clock Time			Longitudinal	Transverse	Forward ∠ 45° Aft
	Hrs.	Min.	Sec.			
Film 4 Series 9 April 4, 1939	23	42	30	- 20.5	- 8.5	- 28
			30 1/2	- 22	- 10	- 29
			32	- 20	- 11	- 27
			32 1/2	- 13	- 17	- 23
			33 1/2	- 10	- 20	- 23
			34 1/2	- 10	- 20.5	- 22
			35	- 8	- 18	- 21.5
			36 1/2	- 13	- 13	- 23
			37	- 20	- 9	- 26
			38	- 21	- 9	- 27
			39	- 22	- 10	- 27
			40	- 17	- 14	- 26
			41 1/2	- 15	- 17	- 25
Mean				- 16.3	- 13.6	- 25.2
Film 4 Series 10 April 4, 1939	23	43	47 1/2	- 23	- 7	- 27
			48	- 27	- 5	- 28
			49	- 28.5	- 8	- 29
			50	- 18	- 13	- 26
			51	- 9	- 20	- 22
			52	- 6.5	- 22.5	- 20
			53	- 5	- 22	- 20
			54	- 3	- 18	- 20
Mean				- 15	- 14.4	- 24
Film 4 Series 13 April 5, 1939	0	03	18	- 16	- 10	- 19
			28	- 19	- 5	- 20
			32	- 17	- 10	- 19.5
			34	- 12	- 13	- 18
			37	- 18	- 5	- 18
			39	- 10	- 14	- 19
			40	- 8	- 17	- 19
			41	- 5	- 12	- 17
Mean				- 13.2	- 10.8	- 18.7

Note: It is striking, that the mean value of the readings from 23:42 or 23:43 to 0:03 o'clock has appreciably changed. The increase of temperature of the instruments resulted from the radiation of a 500-W "Nitraphotolamp," placed close to the instruments. This naturally does not effect evaluation of the data of each series, since for example, Series 10 only covers a time of 6 1/2 seconds.

the strain ellipse, the theory of elasticity gives the following ratios regarding strains in the directions of the principal axes of the strain ellipse; see Figure 8.*

$$\epsilon_\alpha = \epsilon_o \cdot \cos^2\alpha + \epsilon_{\frac{\pi}{2}} \cdot \sin^2\alpha \qquad [1]$$

$$\epsilon_{\alpha+\frac{\pi}{2}} = \epsilon_o \cdot \sin^2\alpha + \epsilon_{\frac{\pi}{2}} \cdot \cos^2\alpha$$

$$\epsilon_\alpha + \epsilon_{\alpha+\frac{\pi}{2}} = \epsilon_o + \epsilon_{\frac{\pi}{2}} = \epsilon_{\alpha+\frac{\pi}{4}} + \epsilon_{\alpha+\frac{3}{4}\pi} = \text{constant} \qquad [2]$$

$$\epsilon_\alpha - \epsilon_{\alpha+\frac{\pi}{2}} = \cos 2\alpha \left(\epsilon_o - \epsilon_{\frac{\pi}{2}}\right) \qquad [3]$$

Since ϵ_o, $\epsilon_{\frac{\pi}{2}}$, and α determine the ellipse, three strain readings, Δl_α, $\Delta l_{\alpha+\frac{\pi}{2}}$ and $\epsilon_{\frac{\pi}{2}}$ will be sufficient. Equation [3] gives a relation to a fourth direction, so that an additional reading in this direction supplies a possible check.

Development of Equations [1] to [3] gives the directions of the principal axes as

$$\tan 2\alpha = -\frac{D_{\alpha+\frac{\pi}{2}}}{D_\alpha} \qquad [4]$$

and the principal strains as

$$\epsilon_o = \frac{1}{2} \cdot \left(\epsilon_\alpha + \epsilon_{\frac{\pi}{2}} + \sqrt{D_\alpha^2 + D_{\alpha+\frac{\pi}{4}}^2}\right) \qquad [5]$$

$$\epsilon_{\frac{\pi}{2}} = \frac{1}{2} \cdot \left(\epsilon_\alpha + \epsilon_{\frac{\pi}{2}} - \sqrt{D_\alpha^2 + D_{\alpha+\frac{\pi}{4}}^2}\right) \qquad [6]$$

Here

$$D_\alpha = \epsilon_\alpha - \epsilon_{\alpha+\frac{\pi}{2}} \text{ and}$$

$$D_{\alpha+\frac{\pi}{4}} = \epsilon_{\alpha+\frac{\pi}{4}} - \epsilon_{\alpha+\frac{3}{4}\pi}$$

Equations [4], [5] and [6] give the ellipse, and with it the strain condition about the point of intersection of the three base lengths used.

It should be noted that the basis used to derive Equations [4] to [6] was a strain condition equal to zero.

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* Dr.-Ing. T. Wyss, "Die Kraftfelder in festen elastischen Körpern" (The Fields of Force in Solid Elastic Bodies), Julius Springer, Berlin.

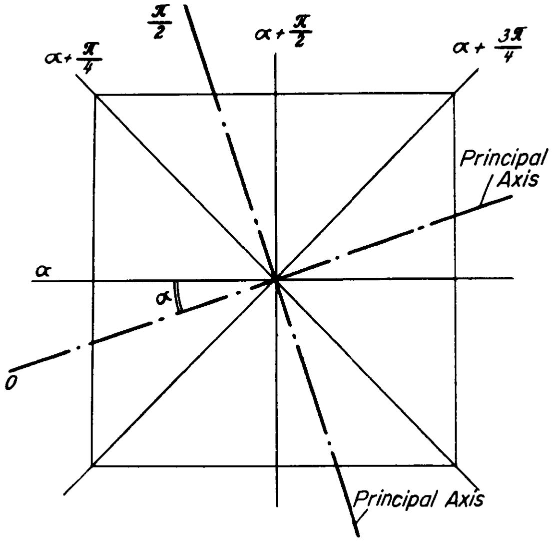


Figure 8

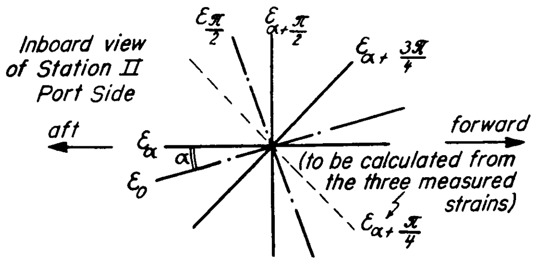


Figure 9 - Strain Ellipse, Station II, Port Side

For the test data of Series 10 the foregoing equations will be used to carry out several calculations for the plane strain condition.

It is first assumed that at the moment when the instruments at Station II recorded

-3, -18, -20 [illegible]

in accordance with Table 3, Series 10, the plate was free of strain. Since the longitudinal gage in this series recorded the greatest contraction, the ship was in a wave trough at the time. We shall now predetermine the direction and the magnitude of the principal stresses when the instruments record -27, -5, and -28.

Compared with

	- 3	- 18	- 20, the extension
	- 27	- 5	- 28
will be	$\Delta l_\alpha = +24$	$\Delta l_{\alpha+\frac{\pi}{2}} = -13$	$\Delta l_{\alpha+\frac{3}{4}\pi} = +8$

in μ per 200 millimeters of base length. (The change of the Orthotest indicator from + to - signifies extension.)

Using the nomenclature of Figure 9 for the plate strains in the various directions, the following strains result from the measured differences:

$$\epsilon_\alpha = \frac{\Delta l}{l} = \frac{24}{200 \times 1000} = +12 \times 10^{-5}; \quad \epsilon_{\alpha+\frac{\pi}{2}} = -6.5 \times 10^{-5}$$

First let it be assumed that before the recording of

$$\epsilon_{\alpha+\frac{3}{4}\pi} = +4 \times 10^{-5} \text{ and } \epsilon_{\alpha+\frac{\pi}{4}} = \epsilon_\alpha + \epsilon_{\alpha+\frac{\pi}{2}} - \epsilon_{\alpha+\frac{3}{4}\pi} = 1.5 \cdot 10^{-5}$$

From this it follows that

$$D_\alpha = \epsilon_\alpha - \epsilon_{\alpha+\frac{\pi}{2}} = 18.5 \cdot 10^{-5}; \quad D_{\alpha+\frac{\pi}{4}} = \epsilon_{\alpha+\frac{\pi}{4}} - \epsilon_{\alpha+\frac{3}{4}\pi} = -2.5 \cdot 10^{-5}$$

Thus we get

$$\tan 2\alpha = -\frac{D_{\alpha+\frac{\pi}{4}}}{D_\alpha} = \frac{2.5}{18.5} = 0.135$$

in accordance with

$$\alpha = 3^\circ\,51'$$

The strains in the directions of the principal axes given by α are

$$\epsilon_0 = \frac{1}{2} \cdot (5.5 + 18.7) \cdot 10^{-5} = 12.1 \cdot 10^{-5} \text{ and}$$

$$\epsilon_{\frac{\pi}{2}} = \frac{1}{2} \cdot (5.5 - 18.7) \cdot 10^{-5} = -6.6 \cdot 10^{-5}$$

when

$$\sqrt{D_\alpha^2 + D_{\alpha+\frac{\pi}{4}}^2} = 18.7 \cdot 10^{-5}$$

To convert these strains into stresses the familiar equations for the condition of plane stress are used.

$$\sigma_0 = \frac{mE}{m^2-1} \cdot (m \cdot \epsilon_0 + \epsilon_{\frac{\pi}{2}}) \text{ and}$$

$$\sigma_{\frac{\pi}{2}} = \frac{mE}{m^2-1} \cdot (m \cdot \epsilon_{\frac{\pi}{2}} + \epsilon_0)$$

With $mE/m^2 - 1 = 692{,}000$ kg/cm² the following stresses result

$$\sigma_0 = 692{,}000 \cdot \left(\frac{121}{3} - 6.6\right) \cdot 10^{-5} = 233 \text{ kg/cm}^2$$

$$\sigma_{\frac{\pi}{2}} = 692{,}000 \cdot \left(-\frac{66}{3} + 12.1\right) \cdot 10^{-5} = -69 \text{ kg/cm}^2$$

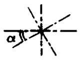
This result indicates that under the assumptions made, the direction of principal stress at the test station forms an angle of $\alpha = 3^\circ 51'$ with the longitudinal direction, that in this direction there occurs a tensile stress of 233 kg/cm² (3,314 pounds per square inch), and in the direction of principal stress normal to this a compressive stress of 69 kg/cm² (981 pounds per square inch). Table 4 gives a summary of the α, ϵ_0, and $\epsilon_{\frac{\pi}{2}}$ values obtained for the strain alterations in Series 10. In this case it was arbitrarily assumed that in the middle record of the instruments at Station II the plate was in unstrained condition. Then according to Table 4 the direction of princi pal stress will turn from the horizontal by 1°41' to 13°17', while the stresses will vary between a tensile stress of 124 kg/cm² (1,764 pounds per square inch) and a compressive stress 136 kg/cm² (1,934 pounds per square inch). Perpendicular to these there will be stress changes from 74 kg/cm² (1,052 pounds per square inch) tension, to 67 kg/cm² (953 pounds per square inch) compression. [illegible]

Actually, however, conditions will be such that before the test the plate has a rather high pre-stress and a corresponding strain, over which the strain and stress alterations due to the alternating stressing of loading and seaway are superposed. To distinguish the effect of such a pre-stress on the stress value derived from data on stress changes let it be assumed that the plate, before the readings of -16, -14, -25 were obtained from the instruments (mean value Series 9), had a prestrain corresponding to a stress of 500 kg/cm² (7,111 pounds per square inch) tension precisely in longitudinal direction, and that it was unstressed in the direction normal

TABLE 4

Evaluation of the Recordings of Photographic Series 10, Related to the Plating in Unstrained State at the Mean Recording of the Instruments at 23:43 o'clock (-16, -14, -25)

Note: All strain values are to be multiplied by 10^{-5}. $\Sigma\alpha = \epsilon_0 + \epsilon_{\frac{\pi}{2}}$

$\sigma_{\frac{\pi}{2}} = 6.92\left(\frac{10}{3}\epsilon_{\frac{\pi}{2}} + \epsilon_0\right)$		kg/cm²	- 59	- 67	- 26	- 5	+ 49	+ 74	+ 59	+ 1
$\sigma_0 = 6.92\left(\frac{10}{2}\epsilon_0 + \epsilon_{\frac{\pi}{2}}\right)$		kg/cm²	+ 59	+ 97	+ 124	+ 20	- 64	- 89	- 104	- 136
$\epsilon_{\frac{\pi}{2}} = \frac{1}{2}(\Sigma\alpha - x)$			- 3.64	- 4.6	- 3.01	- 0.54	+ 3.23	+ 4.8	+ 4.3	+ 2.01
$\epsilon_0 = \frac{1}{2}(\Sigma\alpha + x)$			+ 3.64	+ 5.6	+ 6.26	+ 1.04	- 3.74	- 5.3	- 5.8	- 6.51
$x = \sqrt{D_\alpha^2 + D_{\alpha+\frac{\pi}{4}}^2}$			+ 7.28	+ 10.2	+ 9.28	+ 1.58	- 6.97	- 10.1	- 10.1	- 8.52
	α		7° 59'	5° 39'	2° 19'	9° 13'	10° 32'	13° 17'	10° 38'	1° 41'
2α			15° 58'	11° 18'	4° 38'	18° 20'	21° 4'	26° 34'	21° 16'	3° 22'
$\tan 2\alpha = -\frac{D_{\alpha+\frac{\pi}{4}}}{D_\alpha}$			+ 0.286	+ 0.200	+ 0.081	+ 0.333	+ 0.385	+ 0.500	+ 0.489	+ 0.059
$D_{\alpha+\frac{\pi}{4}} = \epsilon_{\alpha+\frac{\pi}{4}} - \epsilon_{\alpha+\frac{3\pi}{4}}$			- 2.0	- 2.0	- 0.75	- 0.5	- 2.5	- 4.5	- 3.5	- 0.5
$D_\alpha = \epsilon_\alpha - \epsilon_{\alpha+\frac{\pi}{2}}$			+ 7.0	+ 10.0	+ 9.25	+ 1.5	- 6.5	- 9.0	- 9.5	- 8.5
$\Sigma_\alpha = \epsilon_\alpha + \epsilon_{\alpha+\frac{\pi}{2}}$			± 0	+ 1.0	+ 3.25	+ 0.5	- 0.5	- 0.5	- 1.5	- 4.5
$\epsilon_{\alpha+\frac{\pi}{4}} = \epsilon_\alpha + \epsilon_{\alpha+\frac{\pi}{2}} - \epsilon_{\alpha+\frac{3\pi}{4}}$			- 1.0	- 0.5	+ 1.25	± 0	+ 1.0	+ 2.0	+ 1.0	- 2.0
Strains $\epsilon = \frac{\Delta l}{l}$	$\epsilon_{\alpha+\frac{3\pi}{4}}$		+ 1.0	+ 1.5	+ 2.0	+ 0.5	- 1.5	- 2.5	- 2.5	- 2.5
	$\epsilon_{\alpha+\frac{\pi}{2}}$		- 3.5	- 4.5	- 3.0	- 0.5	+ 3.0	+ 4.25	+ 4.0	+ 2.0
	ϵ_α		+ 3.5	+ 5.5	+ 6.25	+ 1.0	- 3.5	- 4.75	- 5.5	- 6.5
Actual Δl, Related to Mean Value	/ $\Delta l_{\alpha+\frac{3\pi}{4}}$	μ	+ 2	+ 3	+ 4	+ 1	- 3	- 5	- 5	- 5
	| $\Delta l_{\alpha+\frac{\pi}{2}}$	μ	- 7	- 9	- 6	- 1	+ 6	+ 8.5	+ 8	+ 4
	— Δl_α	μ	+ 7	+ 11	+ 12.5	+ 2	- 7	- 9.5	- 11	- 13
Orthotest Reading Station II	\	- 25	- 27	- 28	- 29	- 26	- 22	- 20	- 20	- 20
	|	- 14	- 7	- 5	- 8	- 13	- 20	- 22.5	- 22	- 18
	—	- 16	- 23	- 27	- 28.5	- 18	- 9	- 6.5	- 5	- 3

to this. Now let the effect that this pre-strain will have on the strain alteration, which would result from changing the readings to -6.5, -22.5, -20 (Series 10), be investigated.

If a longitudinal pre-stress of 500 kg/cm² (7,111 pounds per square inch) is assumed, the corresponding strains already present when the instruments read -16, -14, -25, will be

$$\epsilon_{\alpha_\gamma} = \frac{\sigma}{E} = \frac{500}{2.1\cdot 10^{6}} = 23.8\cdot 10^{-5}$$

Transverse contraction gives the strain

$$\epsilon_{(\alpha+\frac{\pi}{2})_\gamma} = -\,23.8\cdot 10^{-5}\cdot \frac{10}{3} = -\,7.93\cdot 10^{-5}$$

Since the strains at 45 degrees and 135 degrees must be equal, write

$$\epsilon_{(\alpha+\frac{\pi}{4})_\gamma} = \epsilon_{(\alpha+\frac{3}{4}\pi)_\gamma} = \frac{\epsilon_{\alpha_\gamma} + \epsilon_{(\alpha+\frac{\pi}{2})_\gamma}}{2} = 7.95\cdot 10^{-5}$$

Superposed upon these pre-strains will be the measured strains corresponding to -16, -14 and -25 on -6.5, -22.5, -20.

$$\epsilon_{\alpha} = -\,4.75\cdot 10^{-5},\ \epsilon_{\alpha+\frac{\pi}{2}} = 4.25\cdot 10^{-5},$$

$$\epsilon_{\alpha+\frac{\pi}{4}} = 2\cdot 10^{-5},\ \epsilon_{\alpha+\frac{3}{4}\pi} = -\,2.5\cdot 10^{-5}$$

As a result of pre-strain and measured strain in a seaway, the total strain of the plate at the test station will then be

$$\epsilon_{\alpha} = (23.8 - 4.75)\cdot 10^{-5} = 19.05\cdot 10^{-5}$$

$$\epsilon_{\alpha+\frac{\pi}{2}} = (-7.93 + 4.25)\cdot 10^{-5} = -\,3.68\cdot 10^{-5}$$

$$\epsilon_{\alpha+\frac{\pi}{4}} = (7.94 + 2.0)\cdot 10^{-5} = 9.94\cdot 10^{-5}$$

$$\epsilon_{\alpha+\frac{3}{4}\pi} = (7.94 - 2.5)\cdot 10^{-5} = 5.44\cdot 10^{-5}$$

Further,

$$D_{\alpha} = \epsilon_{\alpha} - \epsilon_{\alpha+\frac{\pi}{2}} = 22.73\cdot 10^{-5}$$

$$D_{\alpha+\frac{\pi}{4}} = \epsilon_{\alpha+\frac{3}{4}\pi} = 4.50\cdot 10^{-5} \text{ from which}$$

$$\tan 2\alpha = \frac{D_{\alpha+\frac{\pi}{4}}}{D_{\alpha}} = -\,0.198 \quad \text{and therefore} \quad \alpha = -\,5^{\circ}36'$$

The minus sign indicates that the axial rotation is opposite to the previous one. As the result of the assumed pre-stress, therefore, the principal axes have experienced a rotation of 13°17' + 5°36' = 18°53'. In addition, write

$$\epsilon_{0} = \frac{1}{2}(15.37 + \sqrt{22.73^{2} + 4.5^{2}})\cdot 10^{-5} = 19.27\cdot 10^{-5}$$

$$\epsilon_{\frac{\pi}{2}} = \frac{1}{2}(15.37 - \sqrt{22.73^{2} + 4.5^{2}})\cdot 10^{-5} = -\,3.9\cdot 10^{-5}$$

and as resultant stresses get

$$\sigma_o = 6.92\left(\frac{192.7}{3} - 3.9\right) = +417\ \mathrm{kg/cm^2}$$

$$\sigma_u = 6.92\left(-\frac{39}{3} + 19.27\right) = +43\ \mathrm{kg/cm^2}$$

The foregoing considerations show the importance of determining pre-stresses and the effects of temperature. No published reports treating either of these influences, from the viewpoints of shipbuilding theory or of technique, exist at present. An attempt to determine the preliminary stresses by measuring a virgin base length applied to the annealed and unprocessed plate has been planned by the Strength Testing Laboratory and will be carried out at the proper time. Moreover it is planned to repeat and supplement tri-axial tests in conjunction with the appropriate deflection tests in the longitudinal structure. This is to be done as soon as possible by an improved method for wider variations in loading. During the test trip with the Motorship DUISBURG reported herein, stress variations due to loading were small, and the sea was relatively calm. Therefore no comparison can be made between test data and those of longitudinal strength calculation.

Thanks are due the Hapag for rendering this test possible and for assistance in carrying it out.