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SHIP RESISTANCE IN WATER OF LIMITED DEPTH
RESISTANCE OF SEAGOING VESSELS IN SHALLOW WATER

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SHIP RESISTANCE IN WATER OF LIMITED DEPTH

Resistance of Seagoing Vessels in Shallow Water

(Schiffswiderstand auf beschränkter Wassertiefe

Widerstand von Seeschiffen auf flachem Wasser)

by

Otto Schlichting, Berlin.

Computations by Dr. Eng. Strohbusch.

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Translated by M. C. Roemer

U.S. Experimental Model Basin
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SHIP RESISTANCE IN WATER OF LIMITED DEPTH

Resistance of Seagoing Vessels in Shallow Water

Summary

Within the range of normal velocities, the velocity loss Δv of a ship in water of limited depth f, as compared to the deep water velocity v^{∞}, is calculated for constant wave resistance, i.e., for velocities causing waves of equal length, as the sum of the theoretical loss Δc of the velocity of the propagated wave and an increase Δw_b in the displacement flow determined by experiment. The change in ship resistance is then set equal to the change in frictional resistance ΔW_r associated with the first-named loss of velocity. The quantities $\frac{\sqrt{\otimes}}{f}$ and $\frac{(v^{\infty})^2}{g f}$ serve as suitable parameters for these changes. When the ship velocity exceeds the critical wave velocity – $\frac{(v^f)^2}{g f} > 1$ – the changes in the wave resistance from the deep water resistance for equal velocities are determined empirically as ratios depending primarily, as functions, upon the variables $\sqrt{\frac{D}{L}} \cdot \frac{1}{f}$ and $\frac{v^2}{g f}$.

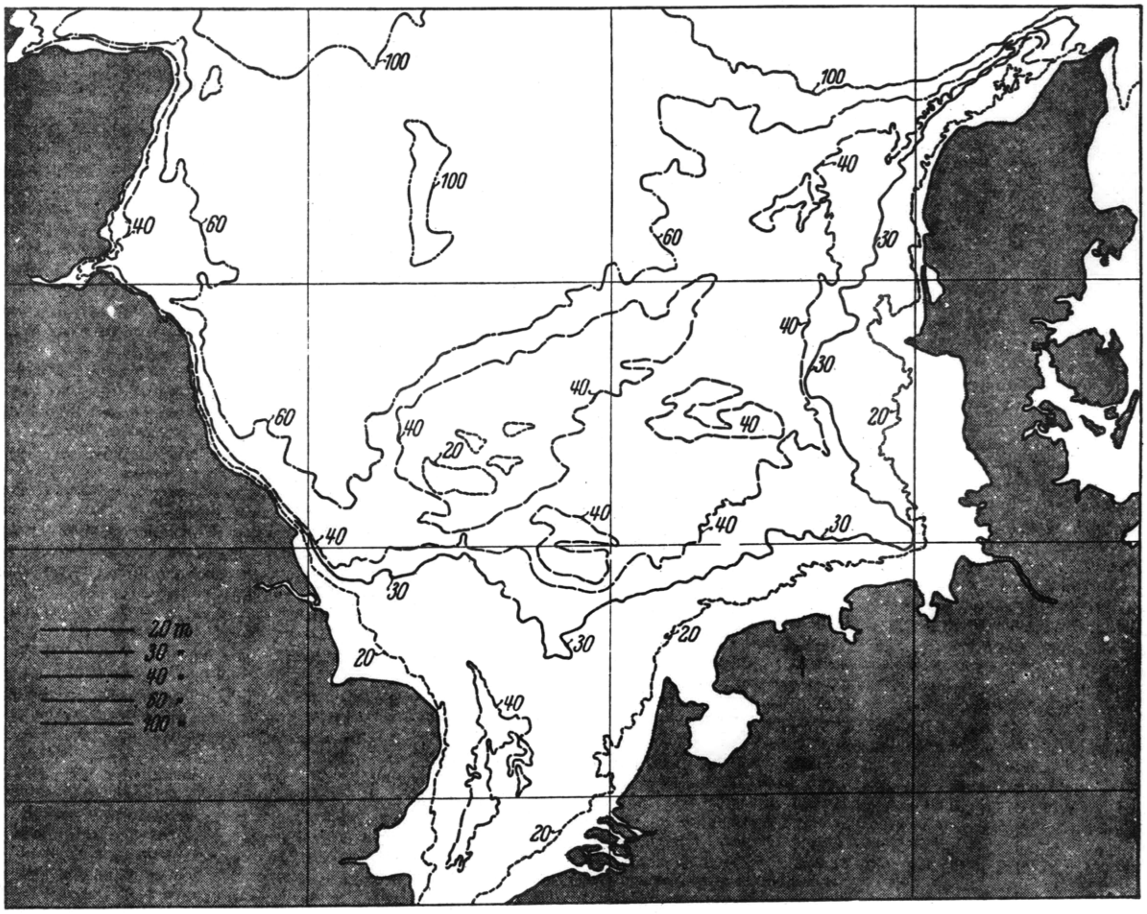
1. Purpose of the Investigation.

Ship resistance in shallow water was discussed before this society by Dr. Weitbrecht in 1920, but also, as appears in his paper, a number of times elsewhere in shipping circles. Special interest in this problem is due to the phenomenon that in very shallow water ship resistance undergoes an extraordinary increase at high speeds, and then drops from the peak of resistance to a value markedly lower than that for deep water when the velocity is further increased or the depth decreased. This phenomenon is in agreement with the theory of wave motion. It was discussed very clearly by Oberregierungsrat Krey, among others, in the periodical Schiffbau, 1913, and as early as 1903 it had been systematically studied, by thorough trial runs with a destroyer, by Chief Naval Constructor Paulus.

The motive for the tests which are the primary subject of discussion here, however, was not supplied by the decrease in resistance occurring under extraordinary conditions, but by the general increase in resistance, especially for larger vessels, generally caused by shallow water, or respectively, the loss in velocity resulting from it at a given engine output. These tests were carried out, for the Reichsmarineamt, in the Hamburg and Vienna Model Basins during the war, with models of one heavy and two light cruisers, for the particular purpose of investigating the effect of shallow water, such as is found in the field of operations in the North and Baltic Seas, in comparison with the depth of water of about 60 meters (196.85 ft.) of the trial course at Neukrug (See maps, Fig. 1).

2. Test Data Obtained from Different Scale Models.

The test data are distinctive in that they were obtained for all depths on various scales, whose range was systematically adapted to the depth of water and



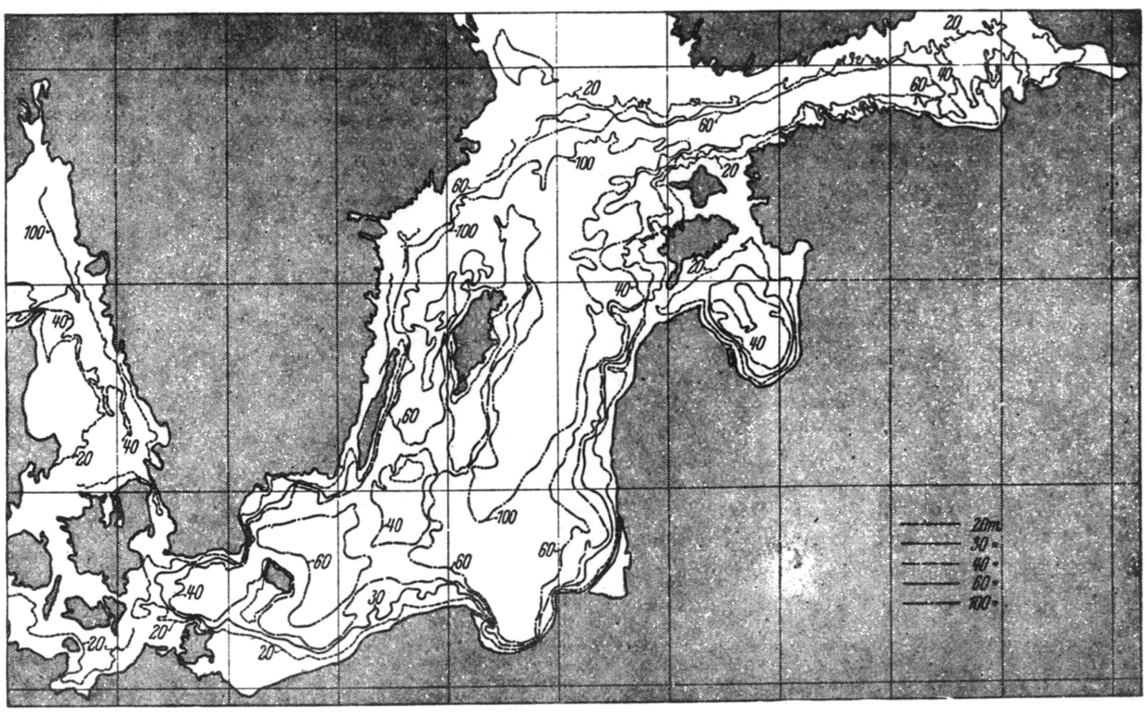


Fig. 1. Depth charts of North and Baltic Seas.

ship velocity according to breadth of the basin. This method, like all those described by Zeyss in his discussion of Weitbrecht's paper, was prescribed by the Navy, because it had been found that the standard model scales first used by the Hamburg Basin for shallow water tests had given too large resistance values and thus also excessively large differences in resistance as compared to deep water values. This idea was confirmed by tests repeated at the order of the Navy with considerably smaller models, since these yielded materially smaller differences from deep water resistances. This discovery is of fundamental importance in the analysis of towing tests in shallow water. It renders all such investigations unreliable for which the admissibility of the scale applied has not been specifically established.

The experimental results were then applied in the usual manner to the full-scale ship, each for its particular purpose, by direct conversion according to Froude's method. The scope of the test, however, renders it desirable, beyond this, to make clear the systematic relationships between the test data, in order to permit general conclusions to be drawn from them. The result of this work is discussed in the following.

3. Comparison of Results of Tests at Ship Velocities Producing Equal Wave Lengths.

Analysis of the experimental results begins with the fact that, for a constant wave length λ, corresponding to the deep water velocity, the wave speed c in shallow water of depth f, undergoes a decrease Δc to be computed by wave theory according to the formula

$$c^2 = \frac{e^{\frac{4\pi f}{\lambda}} - 1}{e^{\frac{4\pi f}{\lambda}} + 1} \cdot \frac{g\lambda}{2\pi}$$

Figure 2 gives the ratio between the shallow water and deep water wave velocity as a function of $\frac{c_\infty}{\sqrt{gf}}$, and Figure 3 shows the absolute velocities and wave lengths for various depths. According to this the deep water velocity is a relative value, such that $\frac{c_\infty}{\sqrt{gf}} \leqq \sim 0.4$.

In accordance with this theory, we investigate the changes in velocity and resistance which a ship undergoes in shallow water at constant wave lengths, on the assumption that in this case, i.e., with constant wave distribution along the ship, constant wave resistances occur.

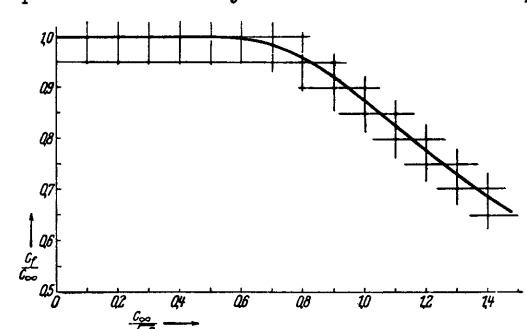


Fig. 2. Ratio $\frac{c^f}{c^\infty}$ of the shallow water and deep water velocities of waves of equal length as function of deep water velocity and depth f.

Strictly speaking, when the wave lengths remain constant, the only basis for the theoretical influence on velocity

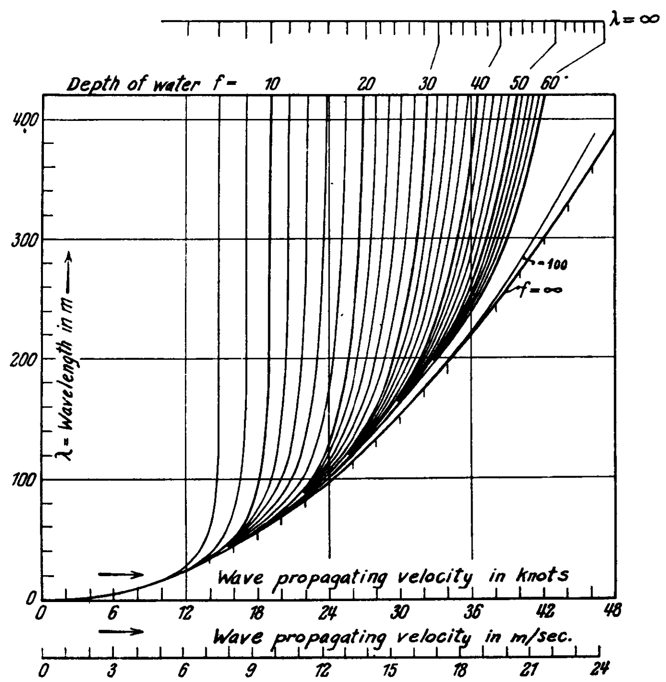


Fig. 3. Wave length λ and speeds of propagation c for water depths f.

will be Lord Kelvin's ideal wave system shown in Fig. 4, to which the ship wave system can be traced back. It occurs on the free surface of the water through the progression of two points of pressure, one of which is to be thought of as acting on the bow and the other on the stern, and is formed of similar divergent and transverse wave trains such as are actually propagated by a ship. In practice, it may be produced by a pair of rods, immersed like stem and sternpost and arranged at a distance equal to the ship's length.

If we imagine the space between the points of pressure as occupied by a ship body, a displacement flow is set up, and the formation of waves is altered simultaneously. However, no material reciprocal action between these phenomena and no essential change in the velocity of wave propagation need result, generally speaking. Under these assumptions, in running in shallow water with the same wave lengths as in deep water, a loss in speed made up of two

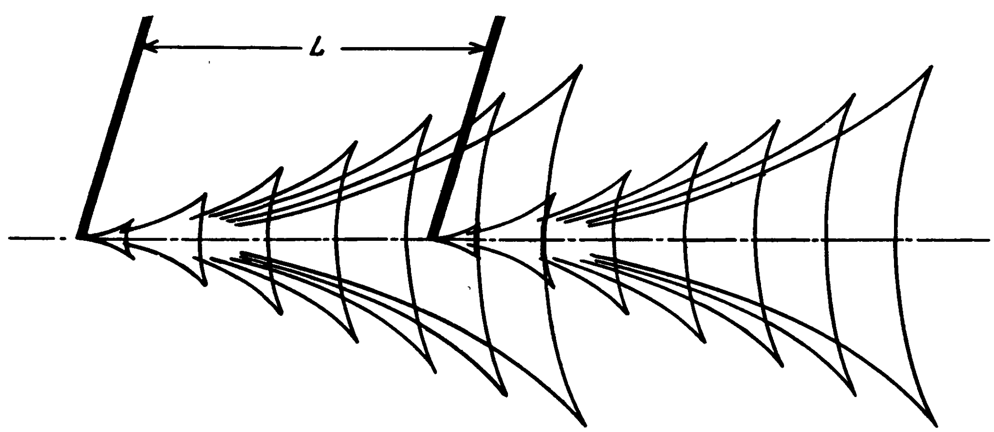
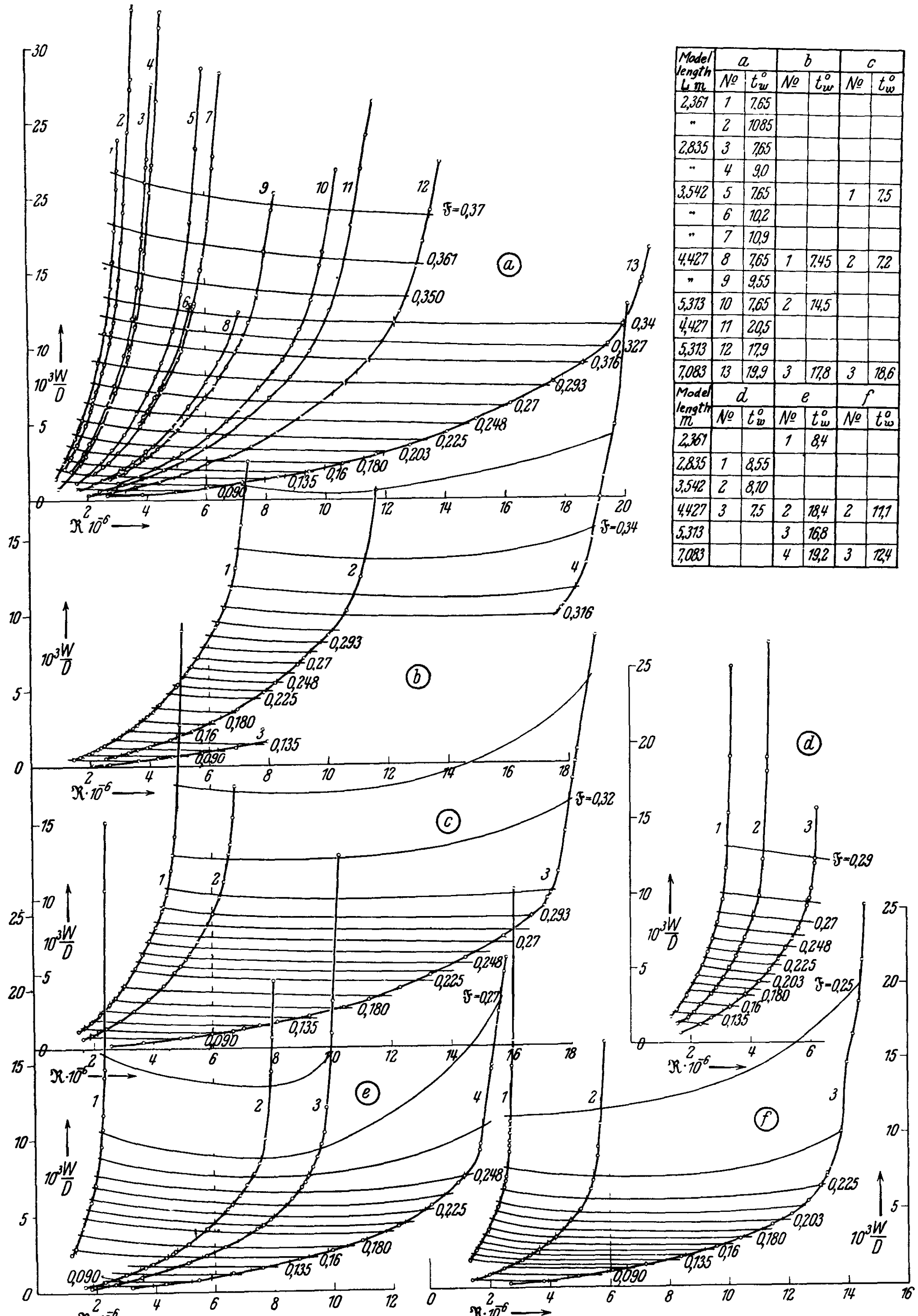


Fig. 4. Wave system produced by advancing points of flow according to Lord Kelvin.

components will result. The one will correspond to the theoretical decrease in the wave propagating velocity Δc, the other to the average increase in velocity of displacement flow Δw_b resulting from the limitation of the section of the channel through which the water flows. The former decrease in velocity determines the



Model length L m	a №	a t_w^o	b №	b t_w^o	c №	c t_w^o
2,361	1	7,65				
"	2	10,85				
2,835	3	7,65				
"	4	9,0				
3,542	5	7,65			1	7,5
"	6	10,2				
"	7	10,9				
4,427	8	7,65	1	7,45	2	7,2
"	9	9,55				
5,313	10	7,65	2	14,5		
4,427	11	20,5				
5,313	12	17,9				
7,083	13	19,9	3	17,8	3	18,6

Model length m	d №	d t_w^o	e №	e t_w^o	f №	f t_w^o
2,361			1	8,4		
2,835	1	8,55				
3,542	2	8,10				
4,427	3	7,5	2	18,4	2	11,1
5,313			3	16,8		
7,083			4	19,2	3	12,4

Fig. 5. Resistances W/D for equal Froude numbers $\mathfrak{F}$, plotted against $\mathfrak{R}$ for water depths f/L, from table.

Heavy Cruiser Model	Figure →	a	b	c	d	e	f
$\frac{\sqrt{\boxtimes}}{L} = 0{,}0767$, $\frac{T}{L} = 0{,}045$	$\frac{f}{L}$	$\sim\infty$	0,216	0,164	0,129	0,115	0,094
$\frac{B}{L} = 0{,}136$	$\frac{\sqrt{\boxtimes}}{f}$	~ 0	0,35	0,46	0,56	0,67	0,815

"ideal" ship velocity w_i relative to the flow, and therewith the shallow water resistance W^f. Since it is assumed that the wave resistance in running with the same wave lengths will remain constant, the shallow water resistance can be lower only by the difference ΔW_r between the frictional resistances attributed to the deep water and shallow water relative velocities. For the conditions of comparison resulting from the foregoing, the velocity and resistance differentials for running in deep and in shallow water have been found in towing tests and checked as to regularity.

4. Method of Analyzing the Data.

The resistances have been evaluated for every depth investigated, as the ratio between model resistance and displacement W/D for equal Froude's numbers $\mathcal{F} = \frac{v}{\sqrt{gL}}$, and are plotted in Fig. 5a-f as functions of the Reynolds number $\mathcal{R} = \frac{vL}{\nu}$ successively according to the ratio of depth of water to length of model f/L (and according to $\frac{\sqrt{\boxtimes}}{f}$). This chart, through the regular fall of the W/D curves according to $\mathcal{R}$, permits detection of viscosity effect in turbulent flow, and, through deviations from this fall in the zone of lower Reynolds numbers, the influence of laminar flow and in the upper zone the influence in the tank walls on resistance resulting from excessively large model scales. It permits thus not only the detection of erroneous data, but also the application of the reliable data to a common Reynolds number, and thus the elimination simply of resistance differences arising in the range of the model from changes in the relative viscosity. For the large cruiser the W/D values for $\mathcal{R} = 8 \times 10^6$ were compared, and for the small cruisers for $\mathcal{R} = 4 \times 10^6$ and 6×10^6 (curve b, Fig. 8) were compared.

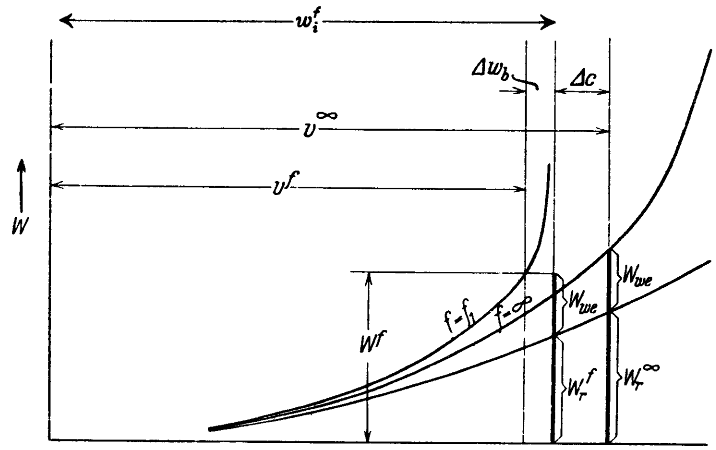


Fig. 6. Relationship of resistance and velocity values for deep and shallow water (derivation from Δw_b).

For these Reynolds numbers the resistance values $\frac{W}{D}$ listed according to depths $\frac{f}{L}$ (or $\frac{\sqrt{\boxtimes}}{f}$) were plotted as curves one over the other together with the frictional resistance value W_r/D according to Froude's number (see Figs. 6-8). The frictional resistance values ascend quadratically, since they relate to the same Reynolds number. The resistance curves for deep water have the following relationship with those for shallow water, according to the foregoing (see Fig. 6): If the loss Δc in the wave velocity shown in Figs. 2 and 3, at a water depth f, is deducted from the deep water velocity, wave lengths being equal, then, according to our assumption, the "ideal" shallow water relative velocity w_i^f, producing the same wave resistance W_{we}, and in the corresponding

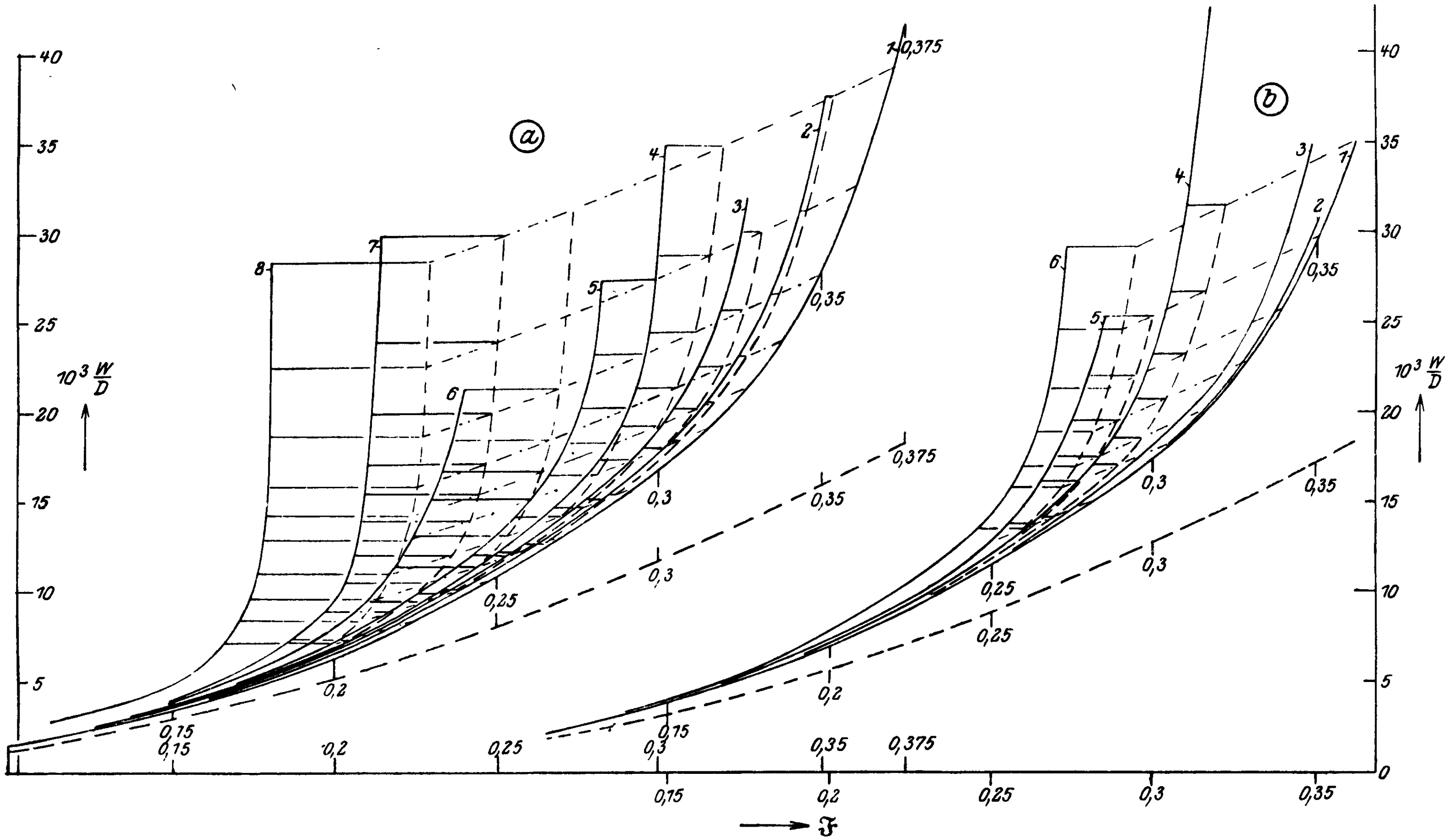


Fig. 7. Resistance values W/D and W_r/D derived as functions of the Froude number with constant Reynolds number for depths f/L.

	$\frac{D}{L^3} \cdot 10^3$	$\frac{\sqrt{\otimes}}{f}$	Nr.	1	2	3	4	5	6	7	8
a	3,26	0,0765	$\frac{f}{L}$	$\sim\infty$	0,22	0,17	0,132	0,115	0,085	0,0705	0,0565
			$\frac{\sqrt{\otimes}}{f}$	~ 0	0,351	0,453	0,582	0,670	0,905	10,88	13,6
b	2,68	0,071	$\frac{f}{L}$	$\sim\infty$	0,31	0,216	0,138	0,115	0,104	—	—
			$\frac{\sqrt{\otimes}}{f}$	~ 0	0,23	0,30	0,517	0,621	0,69	—	—

frictional resistance the shallow water frictional resistance W_i^f will be obtained. The ship speed with a water depth f and resistance $W_{we} + W_r^f$ thus becomes the shallow water velocity v^f appropriate to v^∞ and the difference $w_i^f - v^f$ will be the velocity increase appropriate to the displacement flow Δw_b.

The relationship between v^∞ and v^f was also sufficient for extrapolation of the compared shallow water resistances. This was requisite inasmuch as the scale tests could not be applied or did not extend to the Reynolds number used as a basis with the influence of the basin walls in mind. According to our statements to this association last year, the effect of a change in viscosity on model resistance is dependent on wave formation, because this affects separation. The high degree of accuracy of comparison required for the present tests presented an opportunity to determine in detail the systematic variation of the influence of viscosity with Froude's number. It made evident the unconditional necessity of deriving the viscosity influence for shallow water tests from deep water tests at the velocities v^f corresponding to v^∞ .

The systematic determination of Δw_b is carried out by the following simple method: As in Figs. 7 and 8, curves of the wave resistances W_{we} ascribed to the deep water velocities v^∞ are plotted over the frictional resistance curve W_r as equidistant lines, and on these are laid out the losses Δc from the wave velocity to the deep water velocity for various depths of water. By connecting the points of intersection thus formed we obtain the shallow water resistance curve at the relative velocity w_i^f , and the velocity sections intersected between the shallow water resistance curve and the resistance curve at the ship speed v^f will indicate the increase in velocity Δw_b of the displacement flow.

Since Δw_b is a value which depends wholly upon the process of deduction, the measure of its regularity appears to be of decisive importance as to its validity, and should therefore be checked.

5. Regularity of the Empirical Velocity Increase due to the Displacement Flow, Δw_b, and Presentation of the Total Results of Calculation.

According to the streamline theory, the percentage increase in velocity of the flow of displacement caused by decreasing the depth of water is not dependent upon the velocity of this flow, but only upon the relative decrease in the depth of water. We will assume that the decisive factor for this is the ratio of the midship section to the depth of water, and thus derive Δw_b as a function of $\frac{\sqrt{\boxtimes}}{f}$. By carrying out the steps given under Section 4, we find that Δw_b actually is not especially dependent upon the velocity and that furthermore, for the vessels and draughts investigated, it is to a great extent an identical function of $\frac{\sqrt{\boxtimes}}{f}$. The circumstance that the surface

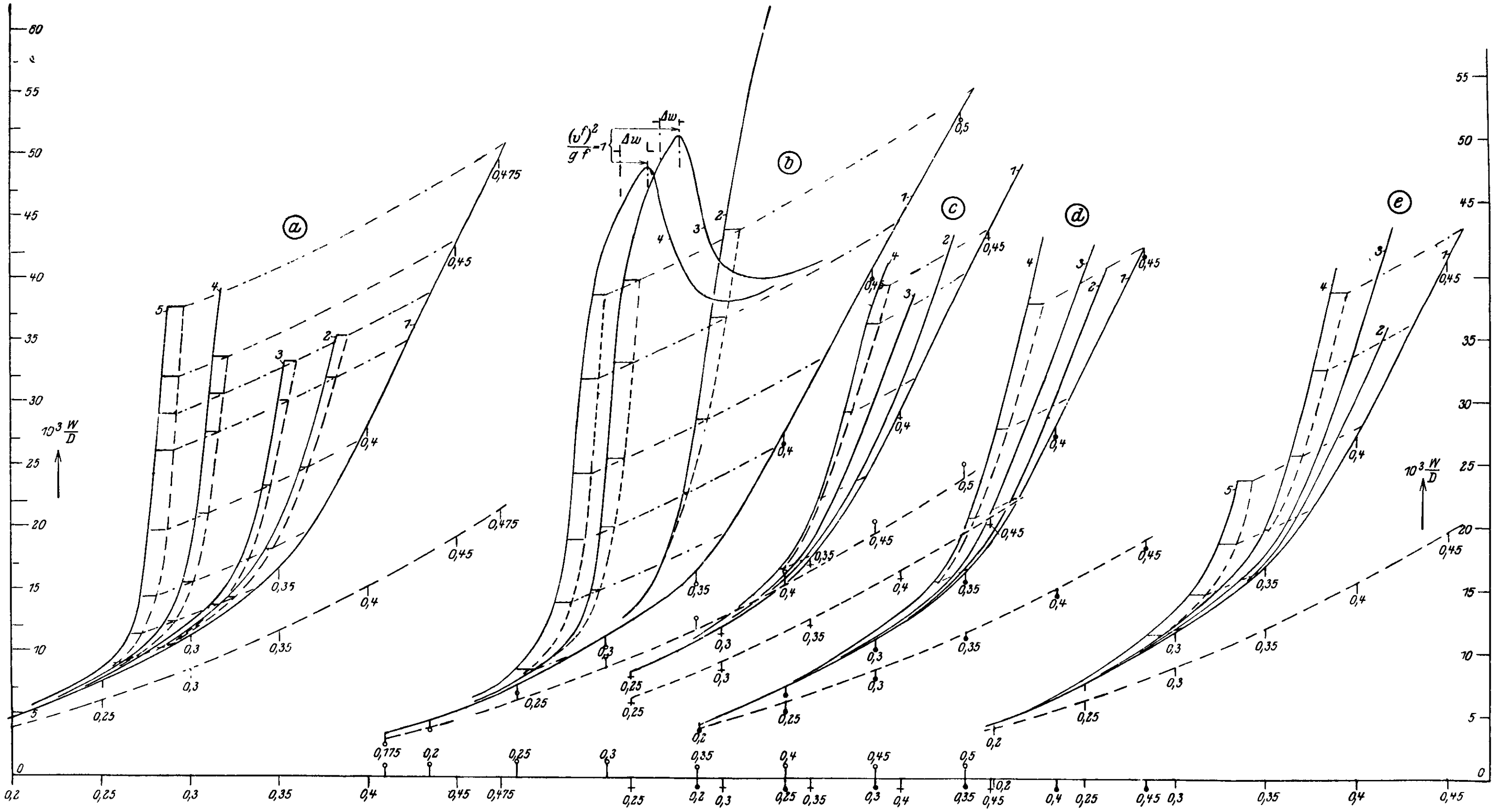


Fig. 8. Resistance values W/D and W_r/D derived as functions of the Froude number with constant Reynolds number for depths f/L.

	$\frac{D}{L^3} \cdot 10$	$\frac{\sqrt{\boxtimes}}{L}$	Nr	1	2	3	4	5
a	2,06	0,059	$\frac{f}{L}$	$\sim\infty$	0,207	0,163	0,1182	0,0965
			$\frac{\sqrt{\boxtimes}}{f}$	~ 0	0 285	0,364	0,50	0,615
b	1,7	0,055	$\frac{f}{L}$	$\sim\infty$	0,163	0,111	0,0965	
			$\frac{\sqrt{\boxtimes}}{f}$	~ 0	0 337	0,495	0,57	
c	1,7	0,055	$\frac{f}{L}$	$\sim\infty$	0,326	0,26	0,207	
			$\frac{\sqrt{\boxtimes}}{f}$	~ 0	0,169	0,212	0,265	
d	2,06	0,059	$\frac{f}{L}$	$\sim\infty$	0,326	0,26	0,207	
			$\frac{\sqrt{\boxtimes}}{f}$	~ 0	0,187	0 228	0,285	
e	1,78	0,055	$\frac{f}{L}$	$\sim\infty$	0,310	0,247	0,198	0,148
			$\frac{\sqrt{\boxtimes}}{f}$	~ 0	0,177	0,222	0,278	0,370

of the water actually does not posse~s the constancy assumed in theory, but is variable with the formation of waves, i.e., Froude's number, as well as with the drop caused by displacement flow dependent upon the value $\frac{(v^\infty)^2}{gf}$, becomes noticeable only at very high values of $\frac{(v^\infty)^2}{gf}$ and $\frac{\sqrt{\boxtimes}}{f}$, which are hardly attainable in practice. The deviations of the individual values from the mean values for Δw_b, generally do not exceed from 1 to 2 per cent. The mean values of Δw_b are given in Fig. 9 as functions of $\frac{\sqrt{\boxtimes}}{f}$.

This regularity in the computed values of increases in velocity of displacement flow justifies the determination of the total loss per cent in ship velocity Δv, and of the resistance differential $\Delta W = \Delta W_r$ on the same principle. The additional loss in velocity Δc, based on wave propagation, is determined by $\frac{(v^\infty)^2}{g\,f}$, and so also the difference of ΔW from deep water resistance, if the latter is expressed in per cent of the frictional resistance, and if the frictional resistance is regarded as constant within the difference between deep and shallow water velocity. This would appear permissible outside the range of the model.

In accordance with this, there are further presented in Fig. 9, the total velocity loss of the ship $\Delta v = \Delta w + \Delta c = \frac{v^\infty - vf}{v^\infty}$, with respect to $\frac{\sqrt{\boxtimes}}{f}$ and $\frac{(v^\infty)^2}{gf}$, corresponding to equal wave distribution, and the corresponding resistance differential ΔW with respect to $\frac{(v^\infty)^2}{gf}$ only, both for the investigated ships and draughts. Here Δw has been derived from Δw_b in per cent of v^∞. The method of resolving the alterations in the operating conditions of the ship in shallow water here applied, is applicable directly in that range of speed in shallow water, in which the wave length does not exceed that proper to the highest attained deep water speed. In any other case, however, a fairly reliable extrapolation of the W/D values is generally possible, since according to Section 7, these are only slightly dependent upon the individual shipform for high Froude's numbers.

6. Results of Flow Measurements.

Resistance measurements were supplemented by flow measurements in order to determine the scale similarity of the displacement flow of the various sizes of models, and the alteration of flow with Froude's number and the depth of water. The scale similarity is affected by the circumstance that viscosity as well as the extent of the displacement flow with respect to the width of the basin varies with the size of the model. The velocities of flow w_m were measured relative to the model of the large and the small cruisers at half draught (0.5T) in the plane of the main frame (0.5L), in this case at stations on both sides at distances of 1.5 of the beam of the ship B as well as 0.5 and 0.9 of the breadth b of the basin, and in addition at the centers of the propeller circles (M). From these are formed the relationships between the local velocities of displacement flow to the speed of the ship $\frac{w_m - v}{v}$, which are presented on Figs. 10 and 11 as functions of the Froude numbers, with positive sign wherever they represent an increase in relative speed in the plane of the main frame, i.e. back flow, and an increase in propeller thrust in the propeller circle, i.e. forward flow.

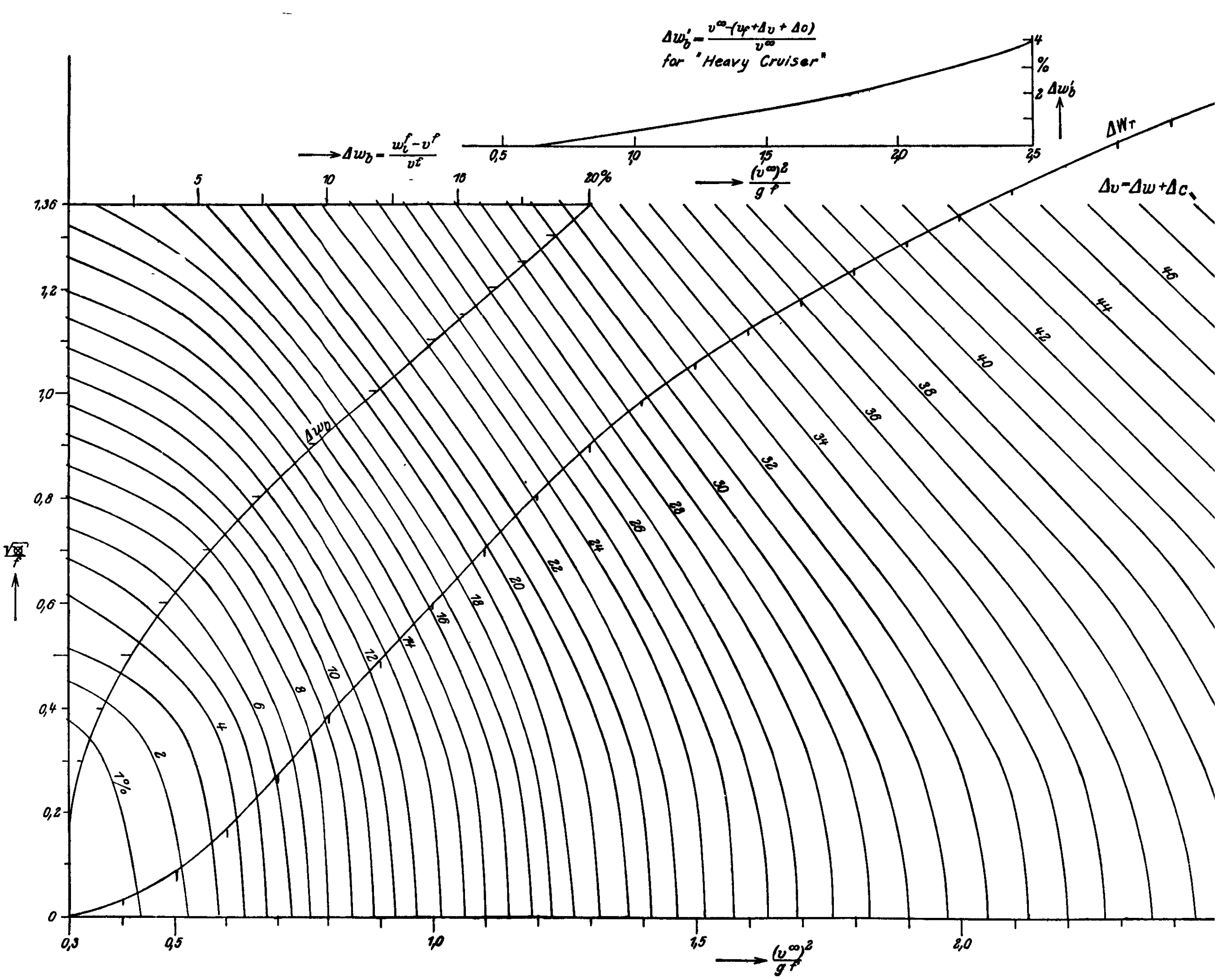


Fig. 9. Chart of the calculated values in per cent for determining the effect of depth of water on shi] ance and speed:

Velocity increase of displacement flow Δw_b against $\frac{\sqrt{\boxtimes}}{f}$, total velocity loss $\Delta v = \Delta w + \Delta c = \frac{v^{\infty} - v^f}{v^{\infty}}$ in contou against $\frac{(v^{\infty})^2}{g\,f}$ and $\frac{\sqrt{\boxtimes}}{f}$, resistance decrease ΔW_r against $\frac{(v^{\infty})^2}{g\,f}$, additional velocity loss $\Delta w_b'$ for "] cruiser" against $\frac{(v^{\infty})^2}{g\,f}$.

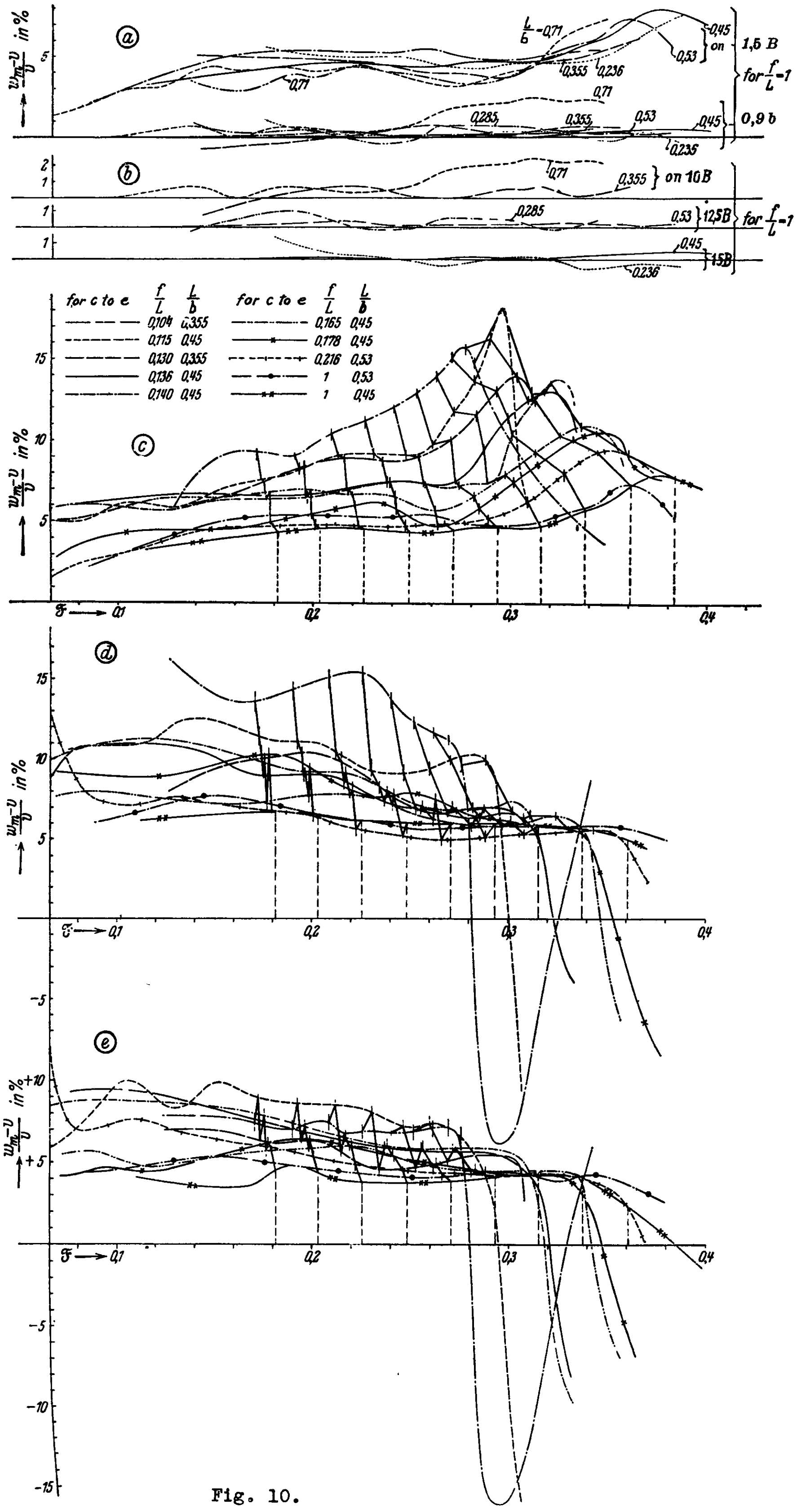

a
b
c
d
e
for c to e
f/L
L/b
0,104 0,355
0,115 0,45
0,130 0,355
0,136 0,45
0,140 0,45
0,165 0,45
0,178 0,45
0,216 0,53
1 0,53
1 0,45
1,5 B
0,9 b
on 10B
12,5B
15B
for f/L=1

Fig. 10.

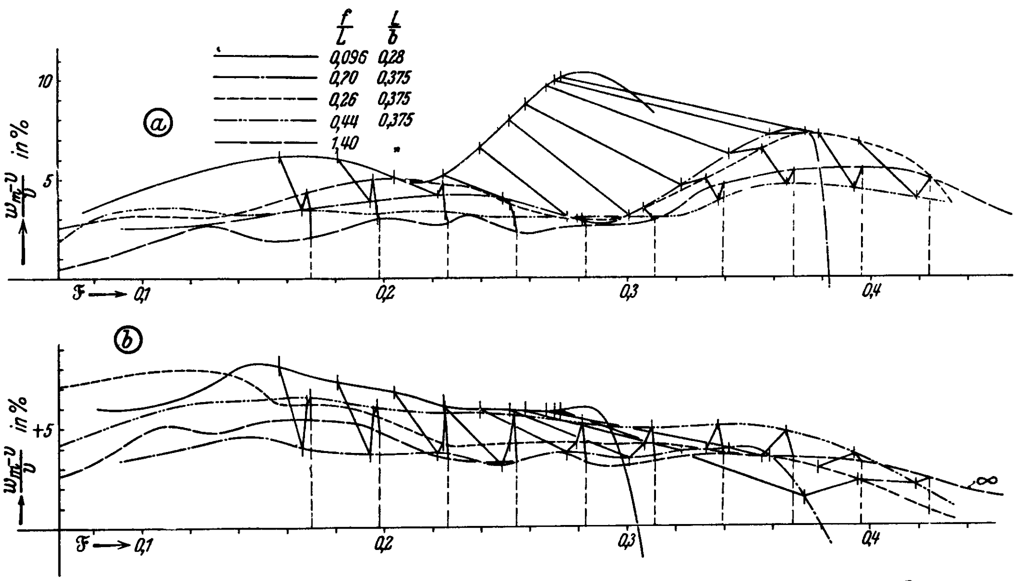


Fig. 10 and 11. Measured displacement flow velocity $\frac{w_m - v}{v}$ as function of the Froude number.

Fig. 10, a and b. Comparison according to $\frac{\text{model length}}{\text{basin width}} = \frac{L}{b}$.

Fig. 10, c to d and 11, a and b. Comparison according to $\frac{\text{water depth}}{\text{model length}} = \frac{f}{L}$.

Fig. 10. Heavy cruiser. Fig. 11. Light cruiser.

Test Stations	a	b	c	d	e	a	b
	1.5 B 0.9 b	10 B 12.5 B 15 B	1.5 B	inner	outer	1.5 breadths from center line in plane of main frame	Propeller disc (center)
	Distance to each side of center line in plane of midship section.			Propeller disc (center)			

Figures 10a and 10b show the scale effect of the local displacement flow velocities in the plane of the main frame for various relative sizes of models $\frac{L}{b}$ at a depth of water $\frac{f}{L}$ = 1, (i.e., f $\sim \infty$), Fig. 10a for the station close to the model (1.5B) and near the wall of the basin (0.9b). The displacement flow velocities at both stations, according to this, show satisfactory agreement, at least for model sizes 0.53 and 0.45 $\frac{L}{b}$, while those for the models size 0.71 $\frac{L}{b}$ are characterized by wide fluctuations. The difference at the basin wall is especially great (Station 0.9b). Here the velocity of displacement flow above f = 0.25 rises to a considerable value, while for the models of other sizes it remains about zero, so that for these the displacement flow is not affected by the breadth of the basin. To learn whether this phenomenon might be due solely to turbulent flow, the displacement flow velocities of two models, as shown in Fig. 10b, at geometrically similar stations, i.e., stations occupying the same positions with relation to the beam of the models, were compared. The excessively high velocity values found here also for the model size 0.71 $\frac{L}{b}$ confirms the existence in this case of a displacement flow along the walls of the basin, resulting in an increase of the velocity of flow due to limitation of the breadth of the basin.

In Fig. 12 the lateral extent of the displacement flow in comparison to the beam of the model and length of the model for various Froude numbers is given. For a model of the length $\cong 0.7\,b$ this would indicate admissible model speeds up to $\mathfrak{F} \cong 0.24$ $\left(\text{or } \frac{v^2}{g\,b} < 0.03\right)$, and for model lengths of $\cong 0.52\,b$, speeds up to $\mathfrak{F} \cong 0.36$ $\left(\text{or } \frac{v^2}{g\,b} < 0.065\right)$. Adherence to these conditions simultaneously renders it possible to carry the model tests in both cases up to equally high Reynolds numbers, temperatures being equal. In order further to prevent the depth t of the basin from affecting the wave velocity, $\frac{v^2}{g\,t} < 0.4$, must be even smaller theoretically.

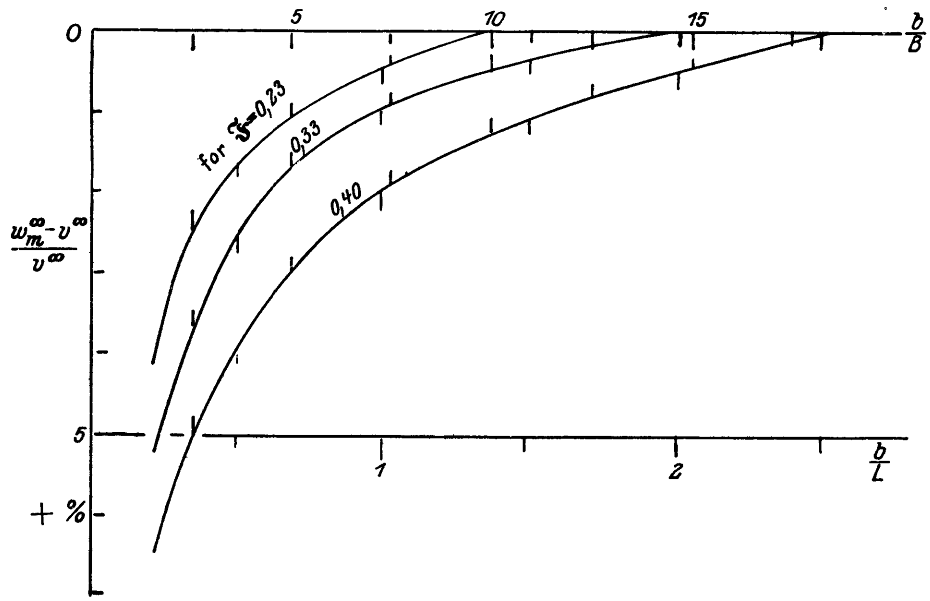


Fig. 12. Displacement flow velocities $\frac{w_m^\infty - v^\infty}{v^\infty}$ from measurement in the plane of the midship section of the heavy cruiser as functions of the ratio of twice the distance b of the test station from the center line, to the model dimensions L and B.

In addition, Figs. 10 and 11 show the effect on local displacement flow near the shipform (at 1.5B and in the propeller circles) of the Froude numbers as well as the depth of water. It changes comparatively slightly with the Froude number in the lower velocity brackets in deep water, so that wave formation here produces no appreciable flow. The range in which there is still almost pure eddy and frictional resistance, that is, where outside the boundary layer there is approximately perfect potential flow, may be recognized, according to our remarks before this Society last year on the resistance of geometrically similar vessels, by the coincidence of the resistance coefficients with respect to the ship's wetted surface, $\frac{W}{\frac{\varrho}{2}\,v^2 \cdot O}$, plotted against the Reynolds number for equal and for variable Froude's numbers and is determined approximately by the Froude number 0.18 for the vessels investigated in deep water. This range of resistances for the smaller models, in view of the low Reynolds numbers, will be found partly in the laminar zone of boundary layer separation, which appears to manifest itself in the form of a pressure loss by the diminution of the plotted forward flow values in the zone of the propeller circles. When, on the other

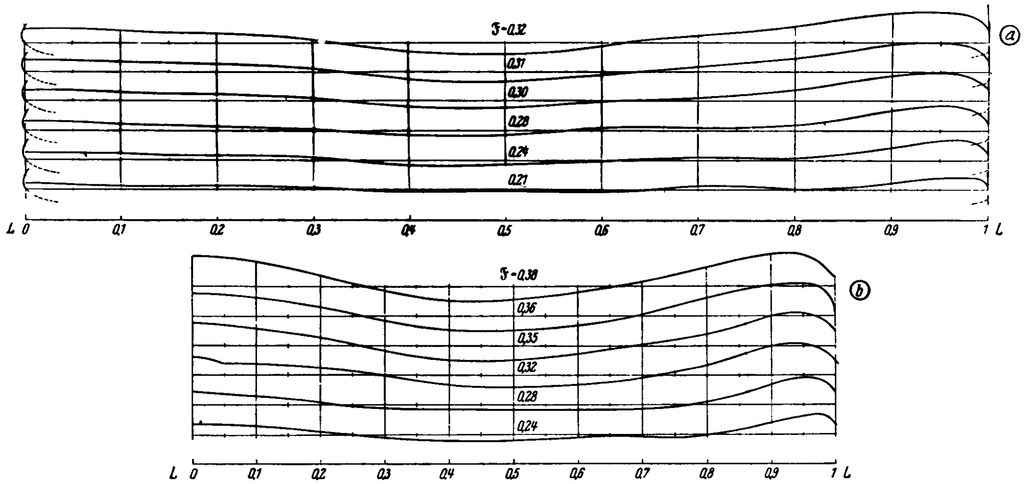


Fig. 13, a and b. Wave contours along the ship.
a) Heavy cruiser. b) Light cruiser.

hand, a velocity value of $\mathfrak{F} \sim 0.32$ is exceeded in deep water, a pronounced increase in the percentage velocities of flow takes place. As may be seen in the wave photographs shown in Figs. 13a and 13b, this velocity limit is characterized by a wave distribution in which the wave length is approximately equal to the length of the ship and in which the hollow amidships is superimposed over the wave trough, and at the same time the stern wave crest is superposed over the echo of the bow wave.

To investigate the effect of depth of water on the local displacement flow it is advisable, as in Sections 4 and 5, to compare the velocity coefficients for velocities producing equal wave lengths. By the use of suitable Froude numbers, we therefore establish a relation between the deep water velocities (v^{∞}) and those shallow water velocities (v^{f}) which are lower by the velocity differentials $\Delta c + \Delta w_b$ determined in Section 5 between the wave making velocity and the velocity of displacement flow (vertical curves plotted in Figs. 10, 11). The differentials (Δw_m) found for various depths of water (L/f or $\sqrt{F}/f$), of the local displacement flow velocities corresponding to these shallow water velocities according to measurement are compared in Fig. 14 with the calculated increase in velocity (Δw_b) of the ideal relative velocities (w_i) for various Froude numbers corresponding to the equivalent deep water velocity. Here, it is true, Δw_b is derived as an increase in the mean relative velocity, while Δw_m represents the absolute changes in the velocities of a displacement or an orbital flow, which have contrary directions in the midship section and the stern section of the ship. However, with respect to their effects on resistance, both the velocity values seem to be comparable. It would be possible, and within certain limits perhaps more justifiable, to think of Δw_b as an additional oscillating velocity causing increased resistance, and to take the resistance difference corresponding to Δw_b as the basis of comparison. However it seemed simpler, to say the least, to base the shallow-water velocity on changes in velocity rather than changes in resistance.

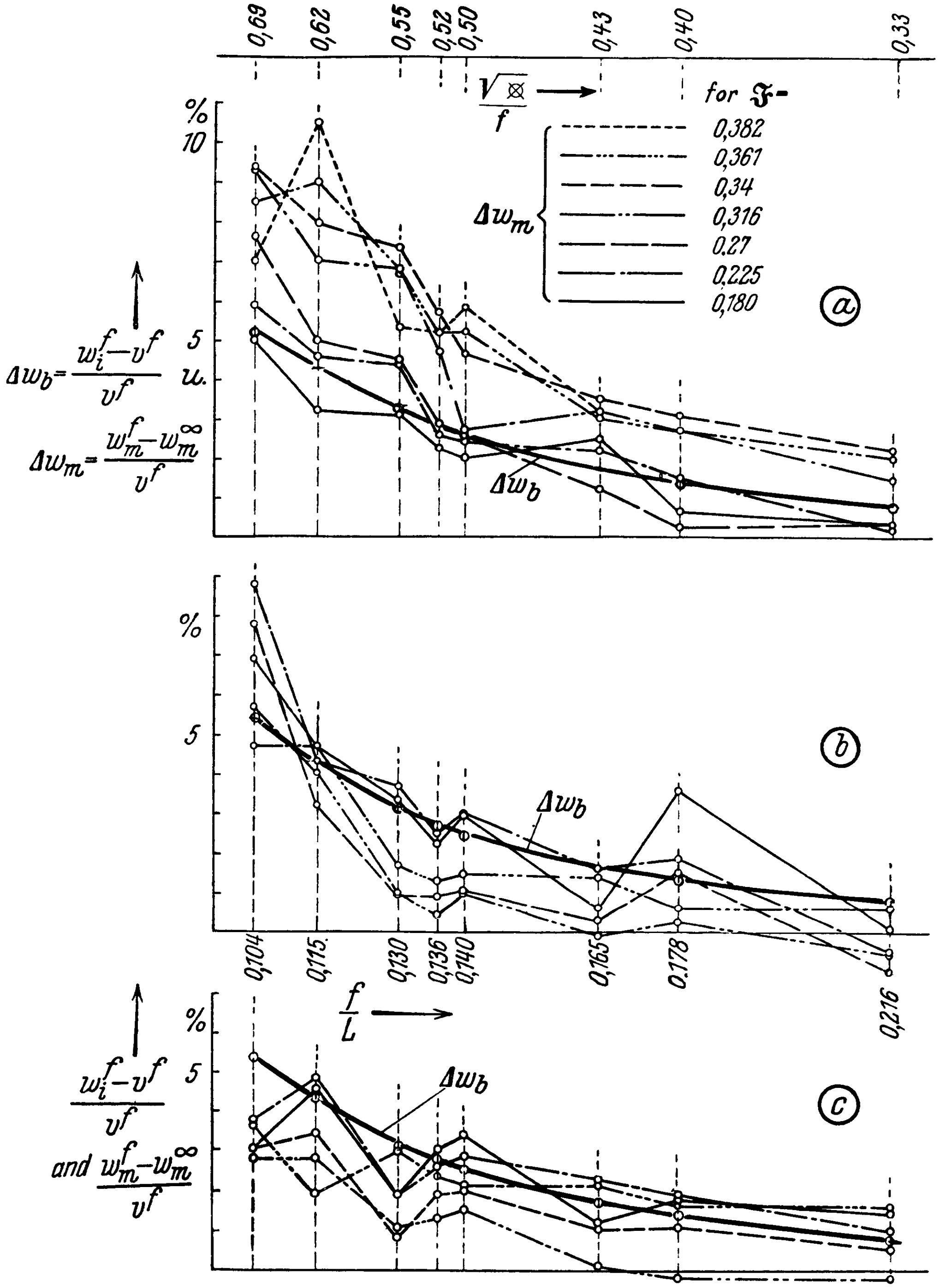


Fig. 14

Test station	a and d	b	c	e
	1.5 B double the distance from center line	inner	outer	propeller circle
		propeller circle		

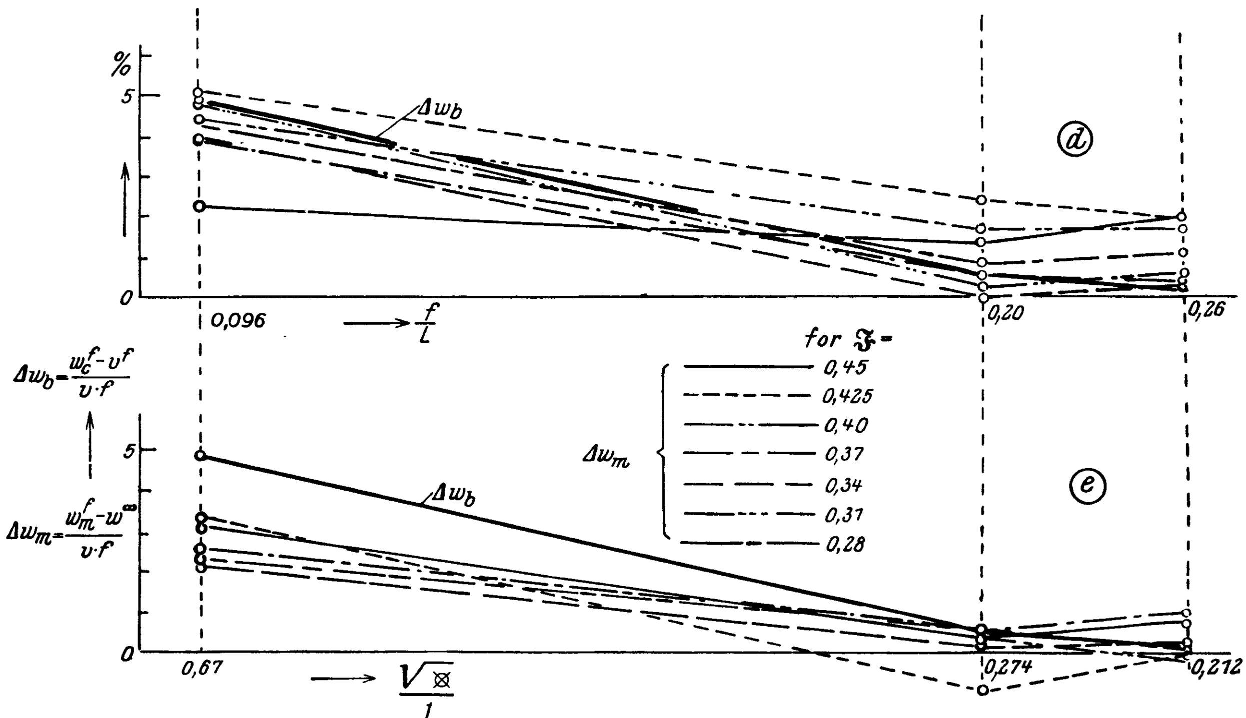


Fig. 14. Increase in velocity of displacement flow by calculation Δw_b, and by measurement Δw_m, for Froude numbers of the deep water velocity for constant wave lengths in decreasing depth of water.

In view of the various conditions of derivation and signification of Δw_b and Δw_m, the agreement between the measured and computed values as to their magnitudes and mean values seems remarkable. Especially at Station 1.5 B in the midship section, the differences between them are within the limits of accuracy attained in the repetitive tests and in the scale-comparing tests. In the forward flow values the deviations are greater, probably due to the fact that here the velocities of flow of the entire ship wave train are superimposed over the displacement flow, and the influence of viscosity takes on greater weight, as a disturbing factor in comparing for corresponding velocities.

Since, according to this, systematic deviations from the values computed for shallowing water depths, of the increase in displacement flow are hardly noticeable, it likewise seems justified by the flow measurements, to use the computed Δw_b values as a basis for determining the absolute variations in velocities which produce the same wave lengths in water of limited depth.

For comparison with these results the displacement flow velocities for the midship section plane of a streamlined body of rotation, the axial section of which corresponded to the plane of flotation of the investigated large cruiser, and the effect of the depth of water upon it were computed according to Taylor's method*. This calculation showed a similar increase in velocity of flow due to depth of water, which, however, was much too low numerically (see Fig. 15).

* T.I.N.A. 1895 (On Solid Stream Forms, etc.).

7. Effect of Shallow Water at Velocities Greater than Wave Velocities.

Up to this point, these investigations have been confined to running conditions in which shallow water causes a decrease in velocity or increase in resistance. According to the wave theory the opposite may occur with respect to wave resistance when the ratio between ship velocity and depth of water is such that the value $\frac{(v^f)^2}{g\ f}$ becomes greater than unity, i.e. in running at a velocity greater than the wave velocity. An idea of the physical conditions under which this reversal will occur is supplied by consideration of the possibilities of satisfying continuity conditions, that in all cross sections of a body of water the sums of the products of the elements of the cross sections and of the velocities of flow prevailing in them must be equal. This condition, as a rule, is satisfied, so that when the cross section of a body of water is diminished by shallow water and a ship's hull, velocity increases, which simultaneously offsets that diminution of cross section brought about by the fall in water level connected with increased velocity. But since the per cent loss in cross section due to fall in the water level increases in the same proportion as the depth decreases, i.e. it increases the more rapidly as the latter drops, a limiting depth must finally be reached when the depth continuously decreases, in which the increase in velocity required to offset the diminution of cross section becomes so high, that it is no longer possible to affect it by the reduction of pressure due to falling water level. The differential equation of the continuity condition shows that this critical point occurs when $\frac{(v^f)^2}{g\ f}$ is equal to unity. On the other side it may be seen that when the water depth decreases below this, fulfillment of the continuity condition is rendered easily possible by raising the water level and simultaneously decreasing the velocity of the ship, because any increase in water level increases the cross section relatively the more as the depth of water decreases, and therefore requires a decrease in velocity which is relatively easier to bring about.

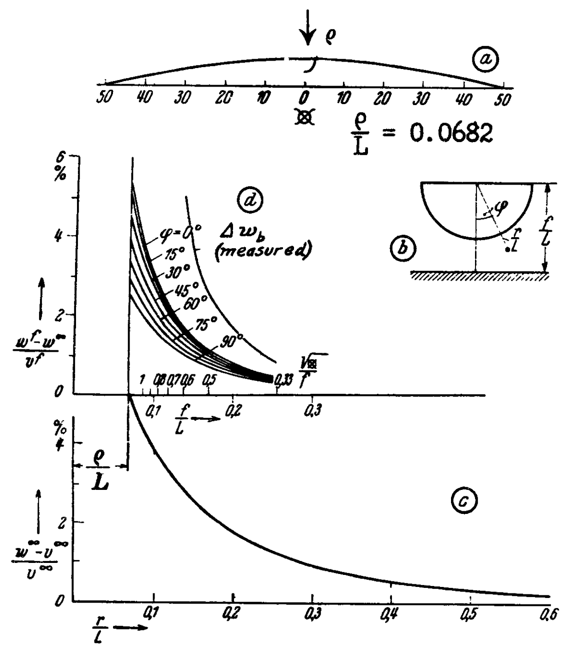


Fig. 15. Velocities calculated by Taylor's method for the plane of the main frame of half a solid of rotation; $\frac{w^\infty - v^\infty}{v^\infty}$ of the displacement flow and of the increase in velocity $\frac{w^f - w^\infty}{v^\infty}$ in decreasing depth of water, as compared with Δw_b.

The familiar phenomenon initially mentioned, of a very slight increase or even decrease in ship resistance in very shallow water and at high velocity is based, according to the foregoing, on the fact that under the conditions explained previously a very considerable diminution in the height of the thwartship waves occurs – which, according to the theory of wave systems propagated by progressive points of disturbance, may even fall to zero – so that the decrease in wave resistance offsets wholly or in part the increase in frictional resistance. It was impossible, however, to establish a basis for immediate calculation of the change in velocity and resistance resulting in shallow water under these conditions. Therefore it was attempted to obtain an idea of these changes by non-dimensional plotting of shallow and deep water test data.

Since there will be no hydrodynamic connection between equal wave resistances when velocities change due to variations in depth of water above the range of wave velocity, it seems advisable to compare the changes in resistance occurring at equal ship velocities. These velocity changes will have to be determined for hydrodynamically similar vessels and conditions of flow. For vessels capable of attaining velocities higher than wave velocities it may be assumed that for high deep water velocities they are composed of a comparatively closely limited δ/β value of about 0.64. For such vessels, furthermore, differences in lines have small effect on the resistance. Their resistance values, i.e. particularly their wave resistance values, will be rather closely limited in dependence upon the Froude number by the value D/L^3 in deep water, so that when the Froude numbers are equal there will be approximate hydrodynamic similarity between vessels with equal D/L^3. Since they govern the reduction of cross section and wave propagation, the values $\frac{\sqrt{\boxtimes}}{f}$ and $\frac{v^2}{gf}$ are also decisive of resistance in shallow water even in the range of super-wave velocities. Here, due to the slightness of the changes in δ/β, $\sqrt{\frac{D}{L}}\,\frac{1}{f}$ may be substituted for $\frac{\sqrt{\boxtimes}}{f}$. For this condition of comparison equal values of D/L^3 will also give the agreement in Froude numbers required for hydrodynamic similarity.

These details show that the value $\frac{v^2}{gf}$ is not alone decisive of the development of resistance at velocities greater than wave velocity, since it takes into account only the wave movement itself and not the reduction of cross section due to the ship. Therefore the "critical" condition will occur at a ship velocity v^f, as has already been found by Weitbrecht, for which $\frac{(v^f)^2}{g\ f}$ is less than unity; for since $w_i^f - v^f$ will be the increase Δw in displacement velocity determined by $\frac{\sqrt{\boxtimes}}{g\ f}$, $(w_i^f)^2/g \cdot f$ will be equal to unity. It was hardly necessary to prove by the present scale tests that Froude's law of similitude is valid for shallow water also, since its validity is unquestionable especially in a case, such as the present, in which we are dealing with pronounced wave phenomena.

8. Checking Results by Comparison with Trial Run Data, and Conclusions.

Applicability of the method explained for finding the effect of depth of water on operating conditions has been checked with trial run data of the same ship in various depths of water. Among the trial run data, those of BREMSE, a wartime cruiser, are noteworthy because the corresponding model resistances are contained in sheets 8a to 8d. For the range in which resistance increases in shallow water, the engine

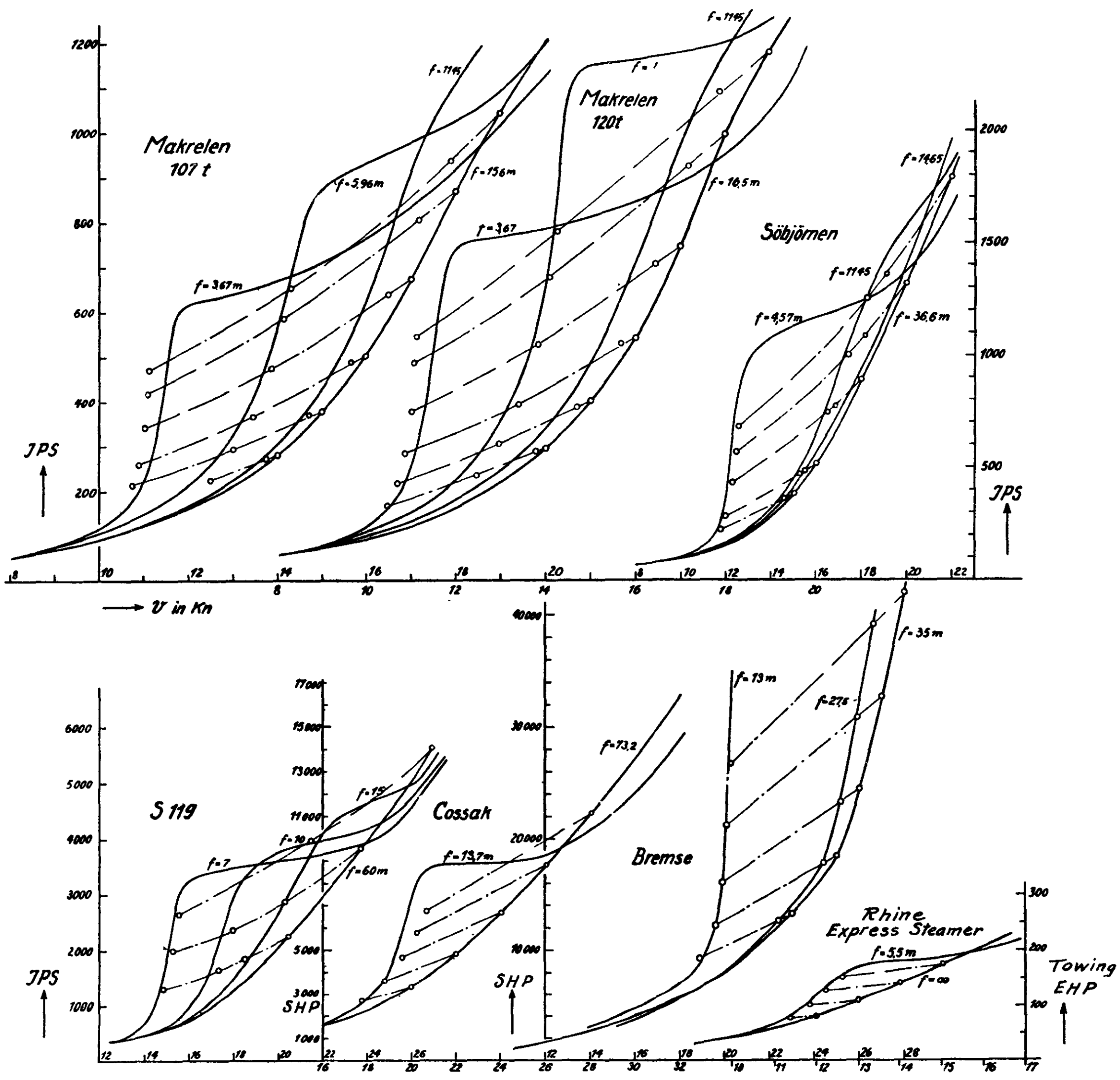


Fig. 16. Comparison of engine outputs (single points) calculated for shallow depths (with $\frac{(v^f)^2}{g\,f} < 1$) with those measured over the measured mile and in model tests (curves).

outputs which are to be expected from this process are determined from the deep-water values obtained on the measured mile as in Fig. 9, and they are shown in Fig. 16 opposite those measured on the mile runs.

In carrying out the required conversion of the engine output into resistances and from these back into engine outputs, the propulsive efficiencies for various depths of water were not specially determined, as a rule, but assumed to be equal. Furthermore, according to Fig. 9, only mean values of the increase in velocity of displacement flow and the theoretical losses of wave-making velocity were taken as a basis. In so far, Fig. 16 shows generally satisfactory agreement between the computed and the measured efficiency values; the deviations appear to be due rather to scattering of the trial run values or inaccurate depth readings than to errors in calculation.

The generally good agreement permits us to conclude that the test values of the smaller models likewise have not been materially impaired by the viscosity effect of the bottom of the basin on the displacement flow.

In the range of decreasing resistance in shallow water, i.e. the zone of super-wave velocity, the ship resistances were again first derived from the mile-run values, and from the former the wave resistances W_w^∞ and W_w^f for deep and shallow water by means of an assumed frictional resistance curve of the ship. The ratio $\frac{W_{we}^f}{W_{we}^\infty}$ is given in Fig. 17 as function of $\frac{v^2}{gf}$ and $\frac{\sqrt{D/L}}{f}$ in the form of contour curves for vessels with equal D/L^3. In the same manner the comparative values of wave resistances of the model of the light cruiser BREMSE have also been plotted, for which elimination of the influence of the basin walls seemed to be assured even at velocities higher than wave velocities.

Figure 17 shows that for the vessels tested the changes in wave resistance are included comparatively satisfactorily in the relative values given.

From the comparative values obtained it is likewise possible by taking the slip values corresponding to the propeller thrust for any time to determine the effect of depth of water on propeller RPM, which is of great importance in operating a ship. Likewise, by reversing the process, it is possible to use the trial run values for various depths of water to determine the increase in velocity of the displacement flow for similar vessels. In addition it is possible from these to judge the effect of depth of water on speed indicators.

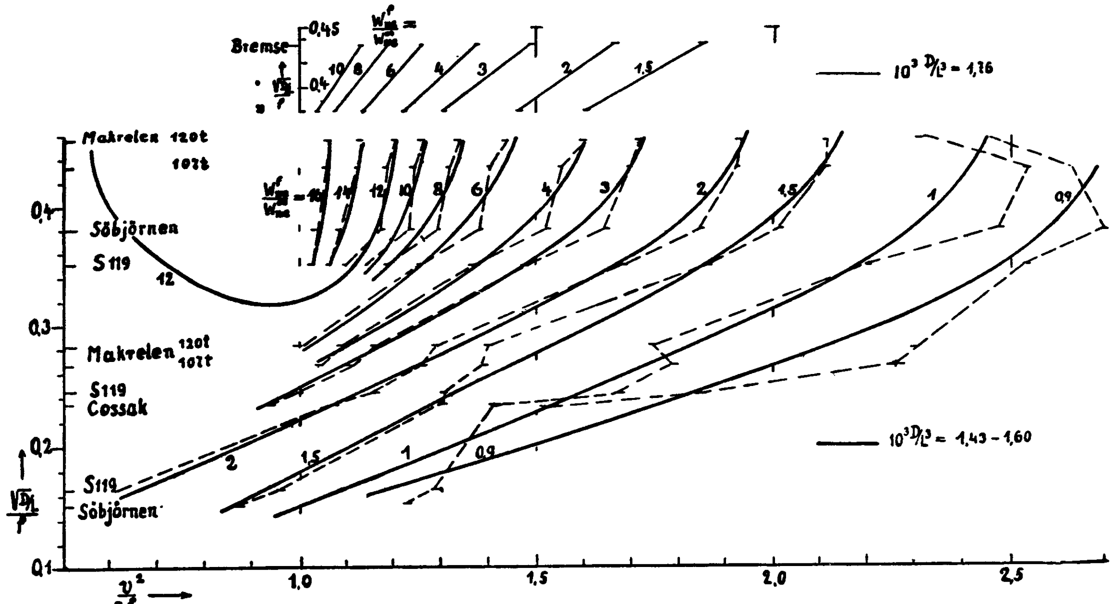


Fig. 17. Ratio $\frac{\text{shallow water wave resistance}}{\text{deep water wave resistance}} = \frac{W_{we}^f}{W_{we}^\infty}$ against $\frac{v^2}{gf}$, $\frac{\sqrt{D/L}}{f}$ and $\frac{D}{L^3}$ for $\frac{v^2}{gf} > 1$ based on model tests and runs over the measured mile.

The results of the investigation may be summarized by stating that the two comparative methods given permit determination of the effect of depth of water on ship resistance, at least in principle and generally also with sufficient practical accuracy. The effect of shallow water is determined primarily by decreasing the wave propagating velocity. For moderate limitations of depth ($\frac{\sqrt{\boxtimes}}{f} < 1$) it is the sole critical factor. It may be easily estimated by changing relative wave velocities as shown in Fig. 2. This alteration begins only when $v^2/g \cdot f$ is greater than 0.4.

If the depth of water decreases greatly with respect to the ship dimensions, supplementary losses in velocity $\Delta w'$ will occur in the "uncritical" zone for high corresponding deep water velocities ($\frac{(v^{\infty})^2}{gf} > 1$) due to falling of the water level. These may be taken into account as a function of $\frac{(v^{\infty})^2}{gf}$ and of the Froude number. For example, Fig. 9 shows an improvement for the heavy cruiser as an additional loss in velocity with reference to the deep water velocity v^{∞} and as a function of $\frac{(v^{\infty})^2}{gf}$. Only for very high values of $\frac{(v^{\infty})^2}{gf}$ does it become valid, and attains importance numerically only for extraordinarily high values of $\frac{\sqrt{\boxtimes}}{f}$.

9. Concluding Remarks.

The regularity of the data obtained from the present investigations will serve to justify the simplicity of the assumptions used as basis of comparison. It may be considered as clear confirmation of the two bases from which it is itself derived:- The concept explained by Froude of the working together of frictional, eddy, and wave resistance, and the teachings of the wave theory. It extends the previous theory, perhaps, in so far as it recognizes an important general factor in displacement flow.

In general, to attain such knowledge, those experiments are especially suitable in which, as here, only one of the primary unique, external experimental conditions – size of vessel, velocity, temperature of water, depth of water – is altered. After the frictional resistance has been investigated rather thoroughly by altering the first three factors, the chief nautical requirement seems to me to be the clearing up of additional eddy and wave formation conditions. The problem as to how the train of waves formed by the ship is superimposed over the hollows formed by displacement flow, appears to be satisfactorily solved quite recently in normal, simple conditions by the investigations of Wigley. Nevertheless, our general knowledge of what conditions prevail when the two surface effects are freely superimposed is still slight. It would be of great value to determine to what extent complete transformation of wave formation such as is noted in the case of speedboats, is based upon similar conditions as exist in shallow water, i.e. transformation of sink areas into source areas (Absenkungs = in Stauflächen). Wherever knowledge of hydrodynamics used in shipbuilding lags behind the knowledge of aerodynamics in this and other respects, a contributing factor is the fact that the possibility of detailed observation from a stationary position is lacking. Therefore it would be advantageous to the science of naval architecture to devise such a device for use in a water tunnel.

DISCUSSION

Dr. Ing. G. Kempf, Hamburg:

I desire to be brief and will therefore touch only upon the first part of the paper. The author describes a new process for deriving from ship resistance in water of infinite depth, the resistance curves for shallow water.

The basic diagram shows the curves of frictional resistance and total resistance in deep water as functions of the velocity. The author arrives at the total resistance curve for shallow water in two steps. The first step is as follows: He computes the retardation which the wave, corresponding to the original ship velocity in deep water, experiences in shallow water, and thus obtains a new lower velocity at which the vessel propagates the same wave train in shallow water, and assumes that at this velocity the wave resistance will be the same as previously in deep water at the original velocity. From the point thus found, the author makes his next step horizontally by assuming a further decrease in velocity due to lessening of the cross section in shallow water, to the new resistance curve. This second step is debatable, at the least. The author himself expresses doubt whether the wave resistance which he has assumed to remain the same when passing from deep to shallow water actually does so, since the wave becomes higher in shallow water.

I am of the opinion that it would be clearer to proceed from the point obtained by the first step in assuming constant wave resistance, not horizontally to the shallow water curve, but vertically upward, by saying that on the one hand the wave resistance will increase by a definite amount in shallow water, and that secondly, frictional resistance will also increase as a result of increased potential flow in the decreased cross section.

It is to the credit of the author that he had the happy thought of being able to proceed as far as the first point by computing the retardation of the wave system in shallow water and assuming the wave resistance to be constant. Further investigation and computation will have to determine how best to proceed empirically from this point, whether horizontally or vertically.

Ministerialrat Schlichting (Conclusion):

The objection just now expressed by Dr. Kempf has already been considered in my paper. On Page 15 I stated: "However, it seemed simpler to base the shallow-water velocity on changes in velocity rather than changes in resistance." Thus I left no doubt that the described method should not be considered as absolutely precise and correct from the hydrodynamical standpoint, but that it is a question of an economical process of comparison. If, as Dr. Kempf proposes, comparison is made according to differences in resistance, these must be referred to wave resistance. As to the general applicability of the method, even the introduction of frictional resistance was not very desirable. I wished to avoid introduction of wave resistance likewise. For this reason I intentionally refrained from making comparison according to differences in resistance. There is also another adverse argument: I refer to Fig. 7. According to this there are velocities at which it is impossible to grasp the difference in resistance between the resistance curve computed from the loss of progress of the wave and the actually observed resistance curve, because these curves lie too far apart. Furthermore, I consider it extremely difficult to report a correct method. The method seemed to me to be adequate since the agreement between the resistance values computed by its means and those measured is absolutely acceptable. For the rest, I thank you for the interest shown in the paper.

Geheimrat Schütte:

Expresses thanks for contributions to resistance problem.

Prof. Dr. Horn, Berlin:

(Sent in): I had at first intended to participate in the oral discussion, but subsequently, first because of the lateness of the hour and second because Dr. Kempf had covered the points I had intended to touch upon, of his own initiative, I had refrained. Now, however, having meanwhile considered the paper more carefully, I regard it as necessary to express a brief opinion after all.

I agree with Dr. Kempf's criticism in so far as the velocity differential introduced by the author between the wave-making velocity in deep water and that in shallow water is based upon physical laws, but not with the assumption that equality of wave lengths also leads to equality of wave resistances. In my opinion, it is more probable that at equal wave lengths the wave resistance in shallow water is greater than in deep water for the reason that the three-dimensional flow which exists in deep water tends to approximate two-dimensional flow more and more closely in going over to shallow water, which is known to have as a result corresponding increases in differences in pressure or level, and this must lead to increased height of the waves and consequently to greater wave resistance in the wave system even when the wave length remains the same. However, the author wishes to have the assumption made by him considered obviously as a working hypothesis, and as such it fulfills its purpose if it permits placing the resistances prevailing at velocities separated by Δc when wave lengths are equal in a certain relationship to each other. The fact that this relationship is not valid, inasmuch as it is based on an assumed equality of wave resistances, is not of great importance, since even so a correction empirically derived from model test data with respect to the velocity differential is introduced, which probably is adapted at the same time to correct the error existing in the first assumption.

This correction, then, which is embodied in the additional velocity differential Δw_b, and which is refused by Dr. Kempf in his discussion, I was inclined to regard, at first, only with respect to its effect on the influence of viscosity but not with respect to the influence of waves, as based on physical laws. I adopted this view because of my opinion that a body proceeding on the surface of the water at a velocity v, will propagate a train of waves which, since it is stationary with respect to the body, must likewise progress at velocity v and therefore possess a wave length corresponding to the latter. Another process of reasoning in which we apply the principle of inversion, i.e., regard the fixed body as in a flow having a velocity equal to v but in opposite direction, shows, however, that in the zone in which a greater velocity exists due to the disturbance of displacement – and this is by far the greater portion of the ship's length – in fact, a greater wave length is set up than would correspond to the speed of advance v, and since for the reasons previously mentioned, the displacement flow in shallow water is more pronounced than in deep water, the wave length corresponding to that for deep water must also be produced at the shallow water velocity which is less than the deep water velocity by a greater amount than given by Δc.

I am therefore inclined to regard this additional velocity correction Δw_b as actually justified. More than this, as already stated, because of its empirical nature, it renders it possible to correct an error in the resistance calculation which was connected with the introduction of the first velocity differential.

Thus I have become convinced that the method developed by the author embodies a highly skillful and fortunate combination of theoretical and empirical methods, and supplies a valuable basis for solution of the highly complicated problem of shallow water resistance, which will at least be satisfactory to the engineer. Among other things, it is highly satisfactory that the much disputed question how it happens that the so-called critical velocity $v = \sqrt{g \cdot f}$ does not bring about the maximum resistance value, but that this occurs, rather, at a lower velocity – a problem which in some cases has even awakened doubts as to the validity of Froude's law of similitude – has for the first time found a practically satisfactory answer.

Dr. Ing. Weinblum, Berlin:

The problem treated by Ministerialrat Schlichting has also attracted great attention in hydrodynamics. Unfortunately, the familiar solutions can not yet be quantitatively applied to practical problems. Essentially, we have to consider two papers: that of Michell (Phil. Trans. 1898) and that of Havelock (Proc. Royal Soc. 1922).

Michell investigates a vessel the draught of which is equal to the depth of the water (it would be better to speak of a sled gliding over the floor without friction). He assumes that only the so-called long waves affect wave resistance.

This (arbitrary) assumption leads to the surprising result that at a ship velocity $v < c$ (c = velocity of free waves) the resistance becomes zero, when $v = c$ resistance will be = infinity, and when $v > c$ (supersonic velocity) resistance increases linearly with v. For the present, therefore, not much can be accomplished with this solution.

Much more enligntening is the limit case treated by Havelock. This is the case of a simple pressure system moving on the surface of the water. As a matter of fact, one of the values of interest to us, the relative limitation of the depth of water, $\frac{\sqrt{\boxtimes}}{f}$, is eliminated here too; nevertheless, the resistance curves given by Havelock are not without practical interest (they have been reproduced in Baker's new book "Ship Design"). The curves, which reproduce non-dimensional coefficients of wave resistance as functions of a Froude number for various relative depths of water, all indicate a maximum which corresponds exactly to a velocity $v = \sqrt{gf}$ only in very shallow depths of water, but which otherwise is reached somewhat earlier. Weitbrecht has already called attention to the fact that the maximum resistance does not occur exactly when $v = \sqrt{gf}$ (Jahrbuch, STG. 1921) as shown by tests. Outside the maximum, humps are evident which correspond to the humps in the deep water resistance curve, but which also occur with a lower Froude number than in deep water. At very high velocities the shallow water resistances in some instances are considerably less than in deep water.

Havelock concludes from his results that the most important effect of shallow water consists in a decrease in wave-making velocity. Theory therefore supports the hypothesis formed by Schlichting; the good agreement between calculation and test in itself indicates that the author has succeeded by means of an astonishingly simple assumption - wave resistance for the time being only a function of wave length - in reaching the nub of the physical problem. Regarding the second part, displacement flow, Havelock's theory is unable to give any information. Only, based on theory, it is to be assumed that when $v \sim \sqrt{gf}$, accentuated wave effects may occur, which yields a possible explanation of deviations from the fundamental assumption in this zone.

The wave diagram by Kelvin, reproduced in Fig. 4, has been improved by Hogner (a phase displacement occurs between the echo- and cross-waves). The assumption of the author that the introduction of a shipform between the points of pressure does not materially alter the velocity of wave propagation, is confirmed to a large extent by theory. As long as the height of the waves in comparison to their length is not very great, the elementary formula $c = \sqrt{\frac{g\lambda}{2\pi}}$ applies with surprising accuracy for various wave forms.

The reported data on the effect of the dimensions (width) of the basin on models of various sizes at various Froude numbers are of great value to experimental technique (so far as I know, they are the first of their kind); here we are at present wholly dependent upon experiment, since theory presents no solution as yet.