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THE STRESS DISTRIBUTION IN THE FLANGES OF THIN-WALLED BOX-GIRDERS

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THE STRESS DISTRIBUTION IN THE FLANGES OF THIN-WALLED BOX-GIRDERS

I. The Opinions of the Leading Experts on the Effective Width.

Experiments and Reports of Read, Bruhn, Biles, Hovgaard, Pietzker, Lienau, Foster, King, Robb, Abell, Schute, Bortsch, Eggenschwyler, and von Karman.

The statics of structural work can boast of great progress during the past decade in Germany. Notably the statically indeterminate space lattices have been made amenable to calculation even in the most difficult cases. Great obstacles were encountered in the statics of full-walled girders. Altho Kirchoff had already created a plate theory, which later was considerably extended by other investigators, these difficult problems presuppose such a wide knowledge of mathematics that the practical engineer remained a stranger to their application. It was only in recent times that the statics of plates was again taken up among technicians, and led to new views especially in the field of reinforced concrete construction. A short time ago, the approximate formulas for reinforced concrete plates were subjected to revision on the basis of this theory which led to a considerably improved economy of material and to an increase in strength over the structural methods previously used (see Dr. H. Marcus, The Simplified Calculation of Flexible Plates, Bauing, 1924, p 600).

Since the difficulty of the statics of plates kept the classical students from this field, it is not to be wondered at that in shipbuilding no use was made of these theories for a long time. That was the more regrettable since the basic problems of the statics of ship structures must be built up on the theory of thin plates. The methods of the ordinary theory of strength, on that account very frequently fail in shipbuilding. Sometimes they lead to loads which already lie above the breaking limit, sometimes they would yield dimensions which have been long outmoded, altho experience has shown that they are sufficiently large for ordinary loads. For these reasons, it is not astonishing that the technic of Naval Architecture followed an entirely different line of development than civil engineering. Merchant ships even now are built to a large extent according to tables given out by the great classification associations and in which the dimensions of the chief members for a ship of a given size, and the properties of the materials used are specified. Recently, the British Freeboard Committee even fixed the standard strength of ships for a long time in advance, acting from the standpoint that the required strength of merchant ships is sufficiently well known from experience. The formulas for standard strength are purely empirical and hardly take consideration of strength theories. Bruhn, director of Norske Veritas, in a paper before the I.N.A., 1920 sharply criticised this method. He also pointed out that the effective width and the stress distribution in broad flanges of girders is still entirely unexplained, altho numerous attempts were being made to render this difficult problem amenable to calculation. The problem of the stress distribution in the working shell of the ship must be considered in connection with longitudinal strength just as in connection with the transverse strength. It is among the factors which determine the choice of frame spacing and bulkhead stiffener spacing, the scantlings of the floors, frames, deck beams, deck girders, and bulkhead stiffeners.

Before taking up the theoretical treatment of the problems we shall briefly present the observations of the experts in the field and examine them critically.

The question as to what part of a ship's hull can be considered as effective first arose as a result of the experiments of Read and Stanbury in 1894. These engineers measured the deflection of two merchant ships under different loads and found that the measured deflection was greater than the calculated. They accounted for this by introducing the "reduced modulus of elasticity" theory.

This explanation was not accepted for long and a succession of engineers occupied themselves with the problem. The first to present his ideas to the public was the Director of Norske Veritas, Bruhn (see Bruhn: Beanspruchung von Schiffsverbanden, Schiffbau I p 561). Particularly relevant is the viewpoint of Bruhn on buckling strength. He assumes that a ship's plating is stiffened so as to be proof against buckling. With this assumption, he bases his theory on experiments he had carried out with plates of rubber. These plates had been made sufficiently thick to resist formation of lobes and subjected to loads similar to those of stiffened ship plates. His views are briefly as follows: Not all the parts of a bridge erected amidships are effective. The portions of the side plating of the bridge which are cut out of the plating by a straight line with the gradient of 1:4 from the main deck are not effective. (See Fig 1). Moreover, the ends of the bridge which are marked out of the bridge deck by the extension of this straight line 1:4 (folded back on the deck) are not effective. (See Fig. 1).

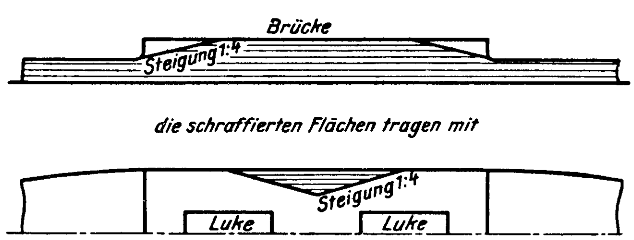

Abb. 1. Tragende Teile nach Bruhn.

Obviously Bruhn's experiments can have only a qualitative value, since the modulus of elasticity of rubber differs altogether too widely from that of iron.

In order to determine the actual decrease of stress in the deck towards the middle, tests and stress measurements must be carried out with iron bodies. However, in any event, it may be said even now that without the occurrence of buckling phenomena there will take place a decrease in stress towards the middle of the deck, although it is doubtful whether this stress decrease is as great as reported by Bruhn.

(Bruhn already points out the special significance of shear stresses for this problem). He assumes that under normal conditions the stress curve under compression does not differ materially from the stress curve under tension. He undertakes the theoretical proof of his views by means of stress trajectories. Airy's stress function proves that actually tensile stresses are not distinguishable from compressive stresses in a thin plate. Later Bruhn, at the instance of Lloyds' Register, undertook a long series of tests with heavy riveted girders, which he reported before the INA in 1905 (see INA 1905, p 126). The girders were patterned after ship members and were stressed to failure. Although no stress measurements were made on the flanges, it may nevertheless be concluded from the breaking loads that the plating was effective almost without exception at the dangerous points, since the girders only failed under the calculated breaking loads, when the riveting yielded prematurely because of excessive shear stress, or when local stresses led to failure due to concentrated individual loads.

At the same time the classical tests with the destroyer WOLF were being carried out by Biles, on which later views regarding the effective width were based. The essential result of these tests is the confirmation of previous tests by Read, according to which the actual deflection of the ship is materially greater than the calculated deflection. In this connection it must be borne in mind that here too the influence of shear stress on the deflection had been neglected. As a second material result, the tests with extensometers by Strohmeyer show that the stress decrease from the stringer towards the middle of the ship in the deck is only immaterial in the neighborhood of the maximum moment.

Using the results of Biles tests, Pietzker gave an entirely new explanation of the increased deflection. He too completely neglects the influence of the shear forces, although it is precisely in this field that bridge building statics had already supplied the means of calculation. He attributes the increase in deflection of the shipform to a decrease in the moment of inertia, regarding the exceeding of the buckling strength in the unstiffened plate areas as the chief reason. He regards only a strip forty times as wide as the plate thickness at the longitudinal stiffeners as resistant to buckling. As the reason for his theory he cites American tests said to have been published in "Stahl und Eisen", 1908. However, I have been unable to find this place in Stahl und Eisen which he cites. Peculiarly Pietzker regards only 40 times the plate thickness next to the longitudinals as effective on the tension side, also. It is unknown how far Pietzker was able to bear out his views by tests with war vessels. At all events, Pietzker's views can not apply to merchant ships. In his paper on "Material Stresses in the Longitudinal Members of Steel Merchant Ships", Prof. Lienau (STG 1913) pointed out that Pietzker's calculation contradicts experience, at least in the case of merchant vessels, and proposed improvements on this method of calculation.

The effective flange width of carlings, frames, and deck beams and other ship members is also taken to be 40 times the plate thickness by Pietzker. It is doubtful whether this theory, which is based only on deflection measurements, can stand up under close scrutiny. In particular there results a very wide contradiction of the theory of the behavior of a ship as a girder. According to Pietzker's theory, only a strip near the deck edge could be taken into account as flange. Pietzker seeks to explain this contradiction by stating that his theory is not valid for stiffened plate walls, although he fails to set the limits of validity.

Very recently, Th. Hoffmann attempted to reconcile the investigations on the WOLF with Pietzker's theory. He found that calculation by Pietzker's method would in the case of this vessel yield only about 1/3 of the moment actually taken up by the WOLF. The remaining 2/3 must therefore be taken up by the unstiffened plating, and furthermore Hoffmann shows that this is possible [1].

Paths similar to Pietzker's were trodden by Hovgaard. He, however, regards 80 plate thicknesses as the effective width even for war ships, since otherwise the calculations can not be made to agree with test results on actual ships. For bulkhead stiffeners, however, he introduces another method of calculation (see Strength of Watertight Bulkheads, Schiffbau 1908/1910). Here Hovgaard determines the deflection, or as the case may be, the elastic curve by experiment, and then estimates the approximate fixation above and below. Then he calculates backward to find the effective width. In view of the extraordinarily great influence exerted on deflection by fixation, compared to which the influence of the effective width is relatively small, this method can lay no claim whatever to producing useful results. The effective width which Hovgaard computes in this manner fluctuates between 12 and 20 cm.

In England students have been unable to decide upon a clear stand regarding effective width, in spite of numerous tests. Rather, the course was adopted of determining for all times the strength of given members by tests of full scale ships. To be noted in this connection are the costly tests of the English Bulkhead Committee undertaken under the direction of Foster King, and in which, unfortunately, no stress measurements were made. Foster King gave as his reason before the INA that conditions were too complicated and that in consequence even accurate stress measurements would have yielded no satisfactory results. As the sole result of a stress measurement in a girder he was able to report that apparently the neutral axis of the built-up girder moves toward the plating when water pressure increases.

1) See INA, 1925, p. 41.

Later Robb, in a report to the INA (see INA 1920, p. 210) attempted to solve the problem from the scientific standpoint. He took the same course as Buchsbaum in his paper: Loading of Bulkhead Stiffeners, Schiffbau 1907/08, p 756, postulating the fixation of the bulkhead stiffeners as equal above and below, and computing deflections for a series of different degrees of fixation. Then he attempted to determine the effective width for each specific instance.

The increase in the deflection of the shipform he attributed to the shear stresses, comparing the ship to a wide flanged T-girder.

Neither is Robb's method capable of withstanding criticism, particularly since the fixation of bulkhead stiffeners above and below will differ greatly, the rigidity of adjacent members evidencing wide variations. He was unable to give an exact method of calculating the deflection of the shipform by shear.

In general we may say that the leading English shipbuilders and the two English Classification Societies, Lloyds' Register and the British Corporation, do not agree with Pietzker's assumptions. The development of building regulations after 1920, particularly, indicates an opposite tendency. Probably under the influence of Prof. Abell (see INA 1920, p. 261) Lloyds' Registry reduced the number of side girders even for ships with open floors, although, according to Pietzker, the strength of ships would thereby be materially reduced. However, it has become evident that the ships built according to this method satisfied strength requirements completely.

It is self-evident that the stress distribution in wide girder flanges is of considerable importance to structural engineers also. One of the first experiments to clear up this problem was carried out by Schüle (see Mitteilungen der eidgenössischen Materialprüfungsanstalt in Zürich 1909, No. 13). Schüle compressed concrete plates of 0.5 and 1 m length and width respectively and about 12 cm thickness by opposite concentrated forces and measured the strain decrease. From these tests conclusions were then drawn as to the effective width of wide flanges, assumed to be 28% of the length of span.

On a similar basis Bortsch attempted to express the problem mathematically for wide plates, and to obtain the effective width by superposition of various loads (see Bauingenieur 1921, p. 662). He also estimates up to 28% of the length of span as the proper flange width for calculation.

In his dissertation, 1920, Eggenschwyler set up an approximate equation which he derived from the conditions of equilibrium for infinitely small parts of the flange. Since he here made assumptions not strictly true, his formula applies only to very wide flanges.

Very recently von Karman carried out an investigation for the special case of infinitely wide flanges by means of the least work of deformation (see Beiträge z. technischen Mechanik, 1924). In this case he finds that for a continuous girder with a cosine form moment, approximately 1/11 of the span length may be considered effective.

If we glance over the numerous papers on stress distribution, we will find that hitherto opinions have differed widely. A deck beam with a span of about 10 m, according to the various theories, has an effective width of from 12 cm to 280 cm. Some experts regard buckling as determinative of the effective width, and others stress diminution without buckling.

Since stress diminution without buckling is the real basis, only such cases will be treated in our further investigations in which buckling is fundamentally excluded. Only when this case has been entirely cleared up, will it be possible in a subsequent paper to treat the influence of buckling.

II. The Theory of Stress Distribution in the Broad Flange of a Beam.

1. The fundamental principles for the solution of the stress problem.

The Airy stress function.

The principal component of a beam is the web. Normal and shearing stresses are set up by the bending of the web, by the deflection and stiffness of the web. If we consider any bent beam and cut it up horizontally in a number of parallel strips, we see that under the influence of the deflection a strip at the nth part of the heighth of the web, would by itself experience at its edges only the nth part of the change in length experienced by the uppermost or undermost fiber of the web. In order to set up further deformation, we must apply shear forces between the individual fibers. If we now consider the flange by itself, it experiences first a bending stress, due to the flexure, which is in the ratio of its height to the height of the entire beam. Besides, it experiences a change in length of the same magnitude as that of the uppermost fiber of the web due to the shear forces that the web exerts on it at its upper end.

If we apply this view to a symmetrical box beam, we can consider the flange as a plate loaded at the places where the webs are attached. (See Fig. 2). If we now give these shear forces a distribution as they actually appear in a girder, the stress condition of the flange is completely determined. Inversely, we can calculate also the necessary shear forces from the distribution of the normal stresses on the boundary.

Even without computation we can make several important statements for a symmetrical body as represented by a ship as usually constructed. Because of symmetry, the shear stresses must be zero in the middle of the flanges, while in the middle of the web they attain a maximum value. For the calculation of the web, we can, for the present, with sufficient accuracy use as a basis the laws of ordinary bending. In doing this it will no longer be permissible to neglect the influence of the shearing stresses upon the deflection, since the shear stresses attain

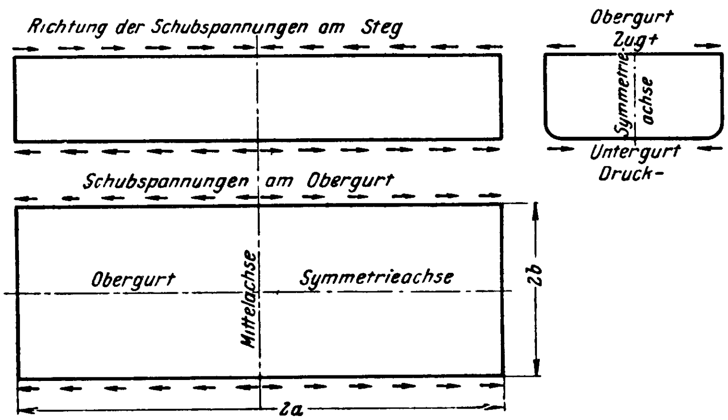

Abb. 2.

values approaching those of the normal stresses. It is to be noted here that even the influence of the flange causes very large shear stresses on the upper end of the web, increasing somewhat up to the neutral axis, but that may be assumed with sufficient accuracy to be uniformly distributed over the entire web. At the points of application of the loads, however, an alteration occurs inasmuch as the maximum shear stresses occur at the points of application. Concerning this point, I refer to the treatise of Karl Huber in the Föppl's Anniversary Number (contributed to Technischen Mechanik 1924, p. 25). In this work there is also shown by means of experiments the rise of shearing stresses in the flange of a T-beam from the free edge to the web.

The stress distribution in plates is calculated by means of Airy's stress function. The Airy stress function presupposes that we are dealing with a condition of plane stress. The condition of plane stress in a plate can only be assumed, however, if the plate is so thin that the stresses perpendicular to its plane are very small. Since in the case of stiffened girders in ship building, very thin-walled girders are usually involved, this condition is fulfilled with sufficient accuracy.

Assuming a plane stress condition, we can set up the Airy's differential equations for the stresses directly. Since calculation with this function is comparatively new to engineers the differential equation is derived in the appendix.

The differential equation is

$$\frac{\partial^4 F(xy)}{\partial y^4} + 2\frac{\partial^4 F(xy)}{\partial x^2 \partial y^2} + \frac{\partial^4 F(xy)}{\partial x^4} = 0 \tag{1}$$

or abbreviated

$$\Delta \Delta F(xy) = 0. \tag{1a}$$

herein

$$\frac{\partial^2 F(xy)}{\partial y^2} = \sigma_x, \quad (2) \qquad \frac{\partial^2 F(xy)}{\partial x^2} = \sigma_y, \quad (3) \qquad \frac{\partial^2 F(xy)}{\partial x \partial y} = -\tau, \quad (4)$$

where,

σ_x = normal stress in the X-direction,

σ_y = normal stress in the Y-direction

and, $\tau = \tau_x = \tau_y$ = shear stress.

Since we are confronted with a partial differential equation, we can choose any expression we like for the stresses in one direction. Then the partial differential equation must fall in a series of total differential equations which of necessity give us the stresses in the other direction. The solutions of the differential equation of the fourth order are known for a series of expressions, which follow.

$$F(xy) = a_1 x^2 + b_1 y^2 + c_1 xy + a_2 x^3 + b_2 y^3 + c_2 x^2 y + c_3 x y^2 + d_1 (x^4 + y^4) + d_2 (x^4 - 6x^2 y^2 + y^4). \tag{5}$$

The following general expression may also be written

$$F(xy) = f_1(x + iy) + f_2(x - iy) + [f_3(x + iy) + f_4(x - iy)](x^2 + y^2). \tag{6}$$

Here both the real and the imaginary part must satisfy the differential equation independently.

Functions which progress according to powers of x and y are in general not suited to the solution of difficult problems. The expression

$$F(xy) = \sum F(y) \cos kx \qquad \text{oder} \qquad F(xy) = \sum \sin kx\, F(y). \tag{7}$$

is more applicable. The total differential equation that follows from this expression becomes then for a term of cos k x

$$\frac{d^4 F(y)}{d y^4} - 2k^2 \frac{d^2 F(y)}{d y^2} + k^4 F(y) = 0. \tag{8}$$

The general solution of this differential equation is familiar. It is

$$F(y) = A_1 \mathfrak{Cos}\, ky + B_1 \mathfrak{Sin}\, ky + C_1 y \mathfrak{Cos}\, ky + D_1 y \mathfrak{Sin}\, ky, \tag{9}$$

where $\mathfrak{Sin}$ and $\mathfrak{Cos}$ are understood to be the hyperbolic functions. Under the assumption that for the present we are dealing with a symmetrical problem, the terms containing B_1 and C_1 disappear and the solution becomes

$$F(y) = \sum A\,\mathrm{Cos}\,ky + By\,\mathrm{Sin}\,ky \tag{10}$$

Obviously no difficulty as to principle would result in finding a solution to the unsymmetrical beam likewise. However, because of the simplified computation, we shall deal first with symmetry. From the previously given solution, the equations for the stresses for a term cos k x follow

$$\begin{aligned}
&\text{a)} \quad \sigma_x = \frac{\partial^2 F}{\partial y^2} = \sigma_m \cos(kx)\,\frac{d^2 F(y)}{dy^2}\,;\\
&\text{b)} \quad \sigma_y = \frac{\partial^2 F}{\partial x^2} = -\,\sigma_m \cdot k^2 \cos(kx)\cdot F(y)\,; \qquad (11)\\
&\text{c)} \quad \tau = -\,\frac{\partial^2 F}{\partial x\,\partial y} = \sigma_m \cdot k \sin(kx)\,\frac{dF(\cancel{x}\,y)}{dy}\,;
\end{aligned}$$

oder:

$$\begin{aligned}
&\text{a)} \quad \sigma_x = \sigma_m \cos kx\,[k^2(A\,\mathrm{Cos}\,ky + By\,\mathrm{Sin}\,ky) + 2Bk\,\mathrm{Cos}\,ky]\,;\\
&\text{b)} \quad \sigma_y = -\sigma_m k^2 \cos kx\,(A\,\mathrm{Cos}\,ky + B y\,\mathrm{Sin}\,ky)\,; \qquad (12)\\
&\text{c)} \quad \tau = k\cdot\sigma_m \sin kx\,(A\,k\,\mathrm{Sin}\,ky + Bky\,\mathrm{Cos}\,ky + B\,\mathrm{Sin}\,ky)\,.
\end{aligned}$$

From these equations, we see that for constant values of y, the stresses in the X-direction depend only upon the trigonometric functions cosine and sine. Now, however, the theory of series shows us that it is possible to express any desired function in a given interval by a Fourier's series, i.e., to replace a function by a sum of sine and cosine terms of which each individual term contains a constant. To each of these sine or cosine terms there will then correspond a completely definite stress distribution which is expressed by equation (12). Through superposition of stresses we can then reckon the final stress condition with any desired accuracy.

When we have determined the stress condition with sufficient accuracy, and fulfilled the boundary conditions, we can consider the flange having the unequal stress distribution as replaced by a flange over which the stresses perpendicular to the x-axis are uniformly distributed. (See Fig. 3). The width of this imaginary flange we designate as the "effective width" (Mittragende Breite).

For a thickness of the above flange of d cm the total force that the flange can carry is equal to

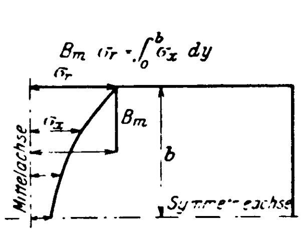

Abb. 3. Die mittragende Breite.

$$P = d\int_0^b \sigma_x\,dy\,.$$

The imaginary flange with uniform stress distribution has then the width

$$B_m = \frac{d \int \sigma_x \cdot dy}{d\,\sigma_r}, \tag{14}$$

where d shall designate the thickness of the flanges and σ_r the stress on the web; therefore, effective width

$$B_m = \frac{\int \sigma_x \, dy}{\sigma_r}, \qquad \text{wo} \qquad \sigma_r = \sigma_m \cdot \sum A_n \cos kx\,. \tag{15}$$

Having computed the effective width at various points by means of this formula we can calculate point by point the exact normal stress at the edge, i.e. at the junction point of the web and the flange by means of the formula

$$W_0 = \frac{J_1}{e_1}\left(1 + \frac{F_2}{F_1}\right) + F_2 e_1 \tag{16}$$

A repetition of the computation with the more exact stress is not necessary, since B_m varies only slightly.

The moment of inertia is found from

$$J = J_1 + \frac{F_1 F_2}{F_2 + F_1} \cdot e_1^2, \tag{17}$$

where F_1 = area of the web
F_2 = effective area of the flange
J_1 = moment of inertia of the web
J = moment of inertia of the entire beam
e_1 = the distance of the center of gravity of the web from the flange.

2. Calculation of Stresses in the Flange.

In calculating a particular case, we first make the assumption that the stress distribution is similar in individual cross-sections of the flange. This assumption is valid however only in the first approximation or for very particular boundary conditions. The calculation indicates that differences appear on the ends of the beam. Since, however, it is known that the ordinary beam theory does not consider this deviation on the end, we may also neglect it in our calculation for the time being. Then, however, by addition of a second solution and by superposition, we will be able to eliminate this error and to determine the magnitude of the deviation with sufficient accuracy.

We will now take as a basis of calculation a box girder with two webs as shown in Fig. 2. Let the webs be so loaded thru a cosine form moment that the moment on the girder end is zero. If the stress distributions in the web and in the flange are similar in the individual cross-sections, the longitudinal stress must also be distributed according to a cosine function. For the individual stresses, the following boundary conditions will then apply.

(a) For y = b, $\sigma_y = 0$; this condition will be fulfilled if

$$A\,\mathrm{Cof}\,k\,b + B\,b\,\mathrm{Sin}\,k\,b = 0 = F(y)_b$$

$$\frac{A}{B} = -b\,\frac{\mathrm{Sin}\,k\,b}{\mathrm{Cof}\,k\,b}. \tag{18}$$

(b) The stress in the flange for y = b must be equal to the stress on the upper edge of the web; therefore,

$$\sigma_m \cdot \cos k\,x = \sigma_r \cos k\,x \cdot [k^2\,F(y) + 2\,B\,k\,\mathrm{Cof}\,k\,y]_{y=b}. \tag{19}$$

but for y = b, $k^2F(y) = o$, [Translator's Note: This follows from Eq. (11b) since $\sigma_y = 0$ for y = b] Therefore, according to (19) we still have

$$2\,B\,k \cdot \mathrm{Cof}\,k\,b = 1 \tag{20}$$

$$B = \frac{1}{2\,k} \cdot \frac{1}{\mathrm{Cof}\,k\,b} \tag{20}$$

$$B = \frac{1}{2\,k} \cdot \frac{\mathrm{Tg}\,k\,b}{\mathrm{Sin}\,k\,b} \tag{20}$$

$$A = -\frac{b}{2\,k} \cdot \frac{\mathrm{Tg}\,k\,b}{\mathrm{Cof}\,k\,b}. \tag{21}$$

With this our equations will read: For the Stresses

$$\sigma_x = -\sigma_m \cos k\,x \cdot \left[k^2\,\frac{b}{2\,k} \cdot \mathrm{Tg}\,k\,b \cdot \left(\frac{\mathrm{Cof}\,k\,y}{\mathrm{Cof}\,k\,b} - \frac{y}{b}\,\frac{\mathrm{Sin}\,k\,y}{\mathrm{Sin}\,k\,b}\right) - \frac{2\,k\,\mathrm{Tg}\,k\,b}{2\,k\,\mathrm{Sin}\,k\,b}\,\mathrm{Cof}\,k\,y\right] \tag{22}$$

or longitudinal stress

$$\sigma_x = -\sigma_m \cos k\,x \cdot b \cdot \frac{k}{2}\,\mathrm{Tg}\,k\,b\left[+\left(\frac{\mathrm{Cof}\,k\,y}{\mathrm{Cof}\,k\,b} - \frac{y}{b}\,\frac{\mathrm{Sin}\,k\,y}{\mathrm{Sin}\,k\,b}\right) - \frac{2\,\mathrm{Cof}\,k\,y}{b\,k\,\mathrm{Sin}\,k\,b}\right]; \tag{22}$$

Transverse stress

$$\sigma_y = \sigma_m \cdot \cos k\,x \cdot \frac{b\,k}{2}\,\mathrm{Tg}\,k\,b\left(\frac{\mathrm{Cof}\,k\,y}{\mathrm{Cof}\,k\,b} - \frac{y}{b}\,\frac{\mathrm{Sin}\,k\,y}{\mathrm{Sin}\,k\,b}\right) \tag{23}$$

and shear stress

$$\tau = \sigma_m \sin k\,x \cdot \frac{b\,k}{2}\,\mathrm{Tg}\,k\,b\left(-\frac{\mathrm{Sin}\,ky}{\mathrm{Cof}\,k\,b} + \frac{y}{b}\,\frac{\mathrm{Cof}\,ky}{\mathrm{Sin}\,k\,b} + \frac{\mathrm{Sin}\,k\,y}{b\,k\,\mathrm{Sin}\,k\,b}\right) \tag{24}$$

and the stress function

$$F(x\,y) = -\sigma_m \cos k\,x\,\frac{1}{k^2}\,\frac{b\,k}{2}\,\mathrm{Tg}\,k\,b\left(\frac{\mathrm{Cof}\,k\,y}{\mathrm{Cof}\,k\,b} - \frac{y}{b}\,\frac{\mathrm{Sin}\,k\,y}{\mathrm{Sin}\,k\,b}\right). \tag{25}$$

Then the effective width becomes

$$B_m = \frac{d\int_0^b \sigma_x dy}{d\sigma_m \cos kx} = \frac{\int_0^b \sigma_x \cdot dy}{\sigma_m \cos kx} = -\frac{\sigma_m \cos kx\, bk}{\sigma_m \cos kx\, 2k} \mathfrak{Tg}\, kb \left(\mathfrak{Tg}\, kb - \frac{1}{\mathfrak{Tg}\, kb} - \frac{1}{bk}\right). \quad (26)$$

$$\boxed{B_m = -\frac{b}{2} \mathfrak{Tg}\, kb \left(\mathfrak{Tg}\, kb - \frac{1}{\mathfrak{Tg}\, kb} - \frac{1}{bk}\right).} \quad (27)$$

We see that for a cosine form distribution of the stresses the effective width is actually constant. If we now set the length of the girder as 2a and the width 2b, and take the center of coordinates in the middle of the rectangle, then if

$$x = a,$$
$$\cos kx = 0$$

it follows that

$$k_1 = \frac{\pi}{2a}, \quad (28)$$

If we choose another distribution of the stresses then we must represent the desired stress condition by superposition of higher cosine terms. Then we will have

$$k = m\frac{\pi}{2a}, \quad (29)$$

where m is any integral, positive number.

If we substitute this value for k in Eq (9) and designate the functions as follows

$$F(y) = \frac{bm\pi}{4a} \mathfrak{Tg} \frac{m\pi}{2a} b \left(\frac{\mathfrak{Cof} \frac{m\pi}{2a} y}{\mathfrak{Cof} \frac{m\pi}{2a} b} - \frac{y}{b} \frac{\mathfrak{Sin} \frac{m\pi}{2a} y}{\mathfrak{Sin} \frac{m\pi}{2a} b} \right) \quad (30)$$

$$F'(y) = \left(\frac{m\pi}{2a}\right) \frac{bm\pi}{4a} \mathfrak{Tg} \frac{m\pi b}{2a} \left(\frac{\mathfrak{Sin} \frac{m\pi}{2a} y}{\mathfrak{Cof} \frac{m\pi}{2a} b} - \frac{y}{b} \frac{\mathfrak{Cof} \frac{m\pi}{2a} y}{\mathfrak{Sin} \frac{m\pi b}{2a}} - \frac{2a}{bm\pi} \frac{\mathfrak{Sin} \frac{m\pi}{2a} y}{\mathfrak{Sin} \frac{m\pi}{2a} b} \right) \quad (31)$$

$$F''(y) = \left(\frac{m\pi}{2a}\right)^2 \frac{bm\pi}{4a} \mathfrak{Tg} \frac{m\pi b}{2a} \left(\frac{\mathfrak{Cof} \frac{m\pi}{2a} y}{\mathfrak{Cof} \frac{m\pi}{2a} b} - \frac{y}{b} \frac{\mathfrak{Sin} \frac{m\pi}{2a} y}{\mathfrak{Sin} \frac{m\pi}{2a} b} - \frac{2\mathfrak{Cof} \frac{m\pi}{2a} y}{\frac{bm\pi}{2a} \mathfrak{Sin} \frac{m\pi}{2a} b} \right). \quad (32)$$

Then the stress function becomes

$$F(xy) = -\left(\frac{2a}{m\pi}\right)^2 \sigma_m \cdot \cos\frac{m\pi}{2a} x F(y), \tag{33}$$

where the minus sign is arbitrarily chosen so as to get a positive σ_x and

$$\sigma_y = \frac{\partial^2 F}{\partial x^2} = \sigma_m \cos\frac{m\pi}{2a} x(+1) F(y), \tag{34}$$

$$\tau = -\frac{\partial^2 F}{\partial x \partial y} = \sigma_m \sin\frac{m\pi}{2a} x(-1)\frac{2a}{m\pi} F'(y), \tag{35}$$

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} = \sigma_m \cdot \cos\frac{m\pi}{2a} \cdot x(-1)\left(\frac{2a}{m\pi}\right)^2 F''(y). \tag{36}$$

The boundary condition in the X-direction, that for x = a, σ_x = o, is automatically satisfied for all odd values of m if

$$k = \frac{m\pi}{2a} \qquad \text{und} \qquad m = 1,\ 3,\ 5,\ \text{usf.}$$

The shear stresses set up by the web must attack at the edge y = b. Their equation is as follows:

$$\tau_r = \sigma_m \sin\frac{m\pi}{2a} x \cdot \frac{b}{2}\frac{m\pi}{2a}\,\mathfrak{Tg}\frac{m\pi}{2a}\cdot b\left(-\mathfrak{Tg}\frac{m\pi}{2a}\cdot b + \frac{1}{\mathfrak{Tg}\frac{m\pi}{2a}b} + \frac{1}{b\frac{m\pi}{2a}}\right) \tag{37}$$

or

$$\tau_r = \sigma_m \sin\frac{m\pi}{2a}\cdot x\cdot\frac{1}{2}\left(-b\frac{m\pi}{2a}\mathfrak{Tg}^2\frac{m\pi b}{2a} + \frac{m\pi}{2a}\cdot b + \mathfrak{Tg}\, b\frac{m\pi}{2a}\right).$$

On the other hand, there really should be no shearing stresses present at the boundary x = a. If the conditions are to be fulfilled, however, there are shearing stresses present there according to the following equation:

$$\tau_a = \sigma_m \cdot \frac{b}{2}\frac{m\pi}{2a}\cdot\mathfrak{Tg}\frac{m\pi}{2a}b\left(-\frac{\mathfrak{Sin}\frac{m\pi}{2a}y}{\mathfrak{Cof}\frac{m\pi}{2a}b} + \frac{y}{b}\frac{\overset{\cosh}{\cancel{\mathfrak{Sin}}}\frac{m\pi}{2a}y}{\mathfrak{Sin}\frac{m\pi}{2a}b} + \frac{1}{b\frac{m\pi}{2a}}\cdot\frac{\mathfrak{Sin}\frac{m\pi}{2a}y}{\mathfrak{Sin}\frac{m\pi}{2a}b}\right) \tag{38}$$

In any event, we can determine without further calculation that in ordinary girders, i.e., when the length of the girder exceeds at least three times the width of the flange, an influence of this shearing stress in the middle of the girder where the dangerous cross-section is found is no longer possible. In order to obtain exact solutions, however, we must remove these shear forces by superposition. We can accomplish this mathematically either by applying opposite equal shear stresses there, or by substituting a second Fourier series for the equation for the shear stresses at the point where x = a. Because of the rapid convergence, from 2 to 3 terms will then usually be sufficient to cause the shear stresses completely to disappear. These additional shear stresses then unfortunately set up stresses in the vicinity of the ends of the flange, which are superposed on the stresses already present.

3. Simple Examples for Determining the Stresses in Girder Flanges.

As we have already indicated, we shall presuppose a similar distribution of the normal stresses in the flange. Since the distribution of the normal stresses is similar for all sections in the web also, i.e., linear, the stresses in the longitudinal direction of the girder must likewise follow the same distribution as is assumed for the moments. The first cases to be studied will be a few simple ones for a statically determinate girder as follows.

(a) A cosine form distribution of the moments over the girder length.

(b) A triangular form moment produced by a concentrated load in the middle of the girder.

(c) A moment distributed in parabola form such as is produced by uniform distribution of the load over the entire girder length.

(d) A moment distributed in trapezoidal form by the action of two symmetrically located, concentrated loads on the girder.

For simplicity we will let the ratio of the girder length to the flange breadth be divisible by the number π, so that for example $b/a = 1/\pi$ or $b/a = 1/m\pi$, or $b/a = \frac{1}{\pi/n}$.

(a) Girder flange with cosine form moment and $a/b = \pi$

The stress distribution for the cosine form moment is given by the following three equations, for which

$$\frac{\pi b}{2a} = \frac{\pi b}{2\pi b} = 0{,}5 = \frac{1}{2}.$$

Then the normal stress in the X-direction is:

$$\sigma_x = \sigma_m \cdot \cos\frac{\pi}{2a} \cdot x \left(\frac{b\pi}{4a}\right) \mathfrak{Tg}\,0{,}5 \left[-\left(\frac{\mathfrak{Cof}\frac{\pi}{2a}\cdot y}{\mathfrak{Cof}\,0{,}5} + \overset{y/b}{\cancel{\frac{b}{y}}}\, \frac{\mathfrak{Sin}\frac{\pi}{2a}\cdot y}{\mathfrak{Sin}\,0{,}5} \right) + \frac{2\,\mathfrak{Cof}\frac{\pi}{2a}\cdot y}{0{,}5\,\mathfrak{Sin}\,0{,}5} \right]$$

The normal stress in the Y-direction is:

$$\sigma_y = \sigma_m \cdot \cos\frac{\pi}{2a} \cdot x \left(\frac{b\pi}{4a}\right) \mathfrak{Tg}\,0{,}5 \left[+ \frac{\mathfrak{Cof}\frac{\pi}{2a}\cdot y}{\mathfrak{Cof}\,0{,}5} - \frac{y}{b}\,\frac{\mathfrak{Sin}\frac{\pi}{2a}\cdot y}{\mathfrak{Sin}\,0{,}5} \right]$$

The shear stress is:

$$\tau = \sigma_m \cdot \sin\frac{\pi}{2a}\, x \left(\frac{b\pi}{4a}\right) \mathfrak{Tg}\,0{,}5 \left[- \frac{\mathfrak{Sin}\frac{\pi}{2a}\cdot y}{\mathfrak{Cof}\,0{,}5} + \frac{y}{b}\,\frac{\mathfrak{Cof}\frac{\pi}{2a}\cdot y}{\mathfrak{Sin}\,0{,}5} + \frac{\mathfrak{Sin}\frac{\pi}{2a}\cdot y}{\frac{b\pi}{2a}\cdot \mathfrak{Sin}\,0{,}5} \right]$$

The maximum shear stress for y = b is:

$$\tau_{\max} = \sigma_m \cdot \frac{1}{4} \mathfrak{Tg}\, 0{,}5 \left(- \mathfrak{Tg}\, 0{,}5 + \frac{1}{\mathfrak{Tg}\, 0{,}5} + \frac{1}{0{,}5} \right) = \sigma_m \cdot \frac{0{,}462}{4} (-0{,}46 + 4{,}16) = 0{,}43\, \sigma_m .$$

The dangerous cross-section of our girder lies in the middle where x = o. It is sufficient for us to calculate the effective width only at the place x = o, since in this case it is constant. From our previous statements, we may write the effective width directly:

$$B_m = \frac{1}{\sigma_r} \int_0^b \sigma_x \, dy$$

or

$$B_m = \frac{b}{2} \mathfrak{Tg} \left(\frac{\pi b}{2a} \right) \cdot \left(- \mathfrak{Tg} \frac{\pi}{2a} b + \frac{1}{\mathfrak{Tg} \frac{\pi b}{2a}} + \frac{2a}{b \pi} \right);$$

$$B_m = \frac{b}{2} \mathfrak{Tg}\, 0{,}5 \left(- \mathfrak{Tg}\, 0{,}5 + \frac{1}{\mathfrak{Tg}\, 0{,}5} + 2 \right);$$

$$B_m = \frac{b}{2} \cdot 0{,}462 \, (-0{,}46 + 2{,}16 + 2) = 0{,}86\, b .$$

The distribution of the stresses over the cross-section on the other hand are computed in a separate table, namely at the points:

$$y = 0, \qquad y = \frac{b}{4}, \qquad y = \frac{b}{2}, \qquad y = \frac{3b}{4} \quad \text{und} \quad y = b .$$

accurate data can be obtained from Table I. The stress curve is shown graphically in Fig. 4.

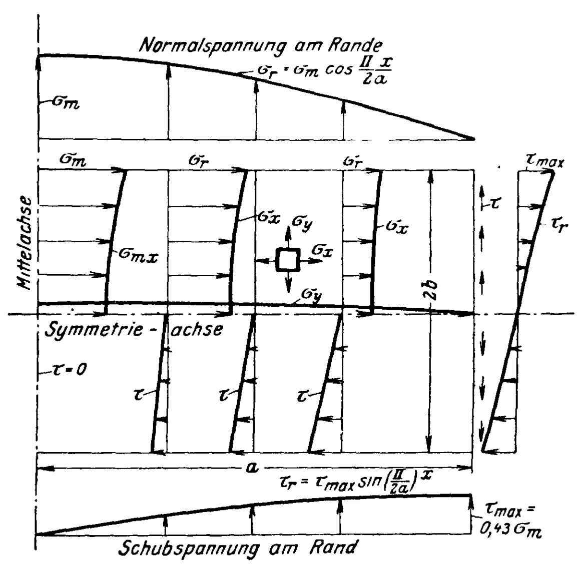

Abb. 4. Spannungsbild für cosinusförmiges Moment. $a : b = \pi$ und $m = 1$.

Let us choose the cosine form moment as so distributed over the length that the stresses take on different signs e.g., m = 3 or 5 and therefore

$$F(xy) = -\sigma_m \cdot \left(\frac{2a}{m\pi}\right)^2 \cos\frac{3\pi}{2a} x\, F(y).$$

Then we again have for the normal and shearing stresses the three equations

$$\sigma_x = \sigma_m \cdot \cos\frac{3\pi}{2a} x \cdot 0{,}75\,\mathfrak{Tg}\,1{,}5 \left[-\frac{\mathfrak{Cof}\,\frac{1{,}5\,y}{b}}{\mathfrak{Cof}\,1{,}5} + \frac{y}{b}\,\frac{\mathfrak{Sin}\,\frac{1{,}5\,y}{b}}{\mathfrak{Sin}\,1{,}5} + \frac{2}{1{,}5}\,\frac{\mathfrak{Cof}\,\frac{1{,}5\,y}{b}}{\mathfrak{Sin}\,1{,}5} \right]$$

$$\sigma_y = \sigma_m \cos\frac{3\pi}{2a} x \cdot 0{,}75\,\mathfrak{Tg}\,1{,}5 \left[\frac{\mathfrak{Cof}\,\frac{1{,}5\,y}{b}}{\mathfrak{Cof}\,1{,}5} - \frac{y}{b}\,\frac{\mathfrak{Sin}\,\frac{1{,}5\,y}{b}}{\mathfrak{Sin}\,1{,}5} \right]$$

$$\tau = \sigma_m \cdot \sin\frac{3\pi}{2a} \cdot x \cdot 0{,}75\,\mathfrak{Tg}\,1{,}5 \left[-\frac{\mathfrak{Sin}\,\frac{1{,}5\,y}{b}}{\mathfrak{Cof}\,1{,}5} + \frac{y}{b}\,\frac{\mathfrak{Cof}\,\frac{1{,}5\,y}{b}}{\mathfrak{Sin}\,1{,}5} + \frac{\mathfrak{Sin}\,\frac{1{,}5\,y}{b}}{1{,}5\,\mathfrak{Sin}\,1{,}5} \right].$$

where

$$\frac{3b\pi}{2a} = \frac{3}{2}\,\frac{b\pi}{b\pi} = 1{,}5.$$

The stress distribution for a girder loaded in this manner is the same as for a girder with ratio of the sides a/b = $\pi/3$ with cosine form moment if for the time being we disregard the shear stress distribution at the end of the girder. The formula for the effective width becomes in this case,

$$B_m = \frac{3b\pi}{4a} \cdot \mathfrak{Tg}\,\frac{3\pi b}{2a} \cdot \left(\frac{2a}{3\pi}\right) \left(-\mathfrak{Tg}\,\frac{3\pi b}{2a} + \frac{1}{\mathfrak{Tg}\,\frac{3\pi b}{2a}} + \frac{2a}{3b\pi} \right)$$

or

$$B_m = \frac{1}{2}\,\mathfrak{Tg}\,1.5 \cdot \left(-\mathfrak{Tg}\,1.5 + \frac{1}{\mathfrak{Tg}\,1{,}5} + \frac{2}{3} \right) b$$

$$B_m = 0.75 \cdot 0.915 \cdot \frac{2b}{3} (-0.92 + 1{,}09 + 0{,}67) = 0{,}458\,b \cdot 0{,}84 = 0{,}386\,b$$

Hence, the effective width for b = $\frac{3a}{\pi}$ is

$$B_m = 0.386 \cdot \frac{3}{\pi} \cdot a = 0.37\,a.$$

that is, 18½ per cent of the length of the span on each side.

The calculation of the stresses σ_x, σ_y, and τ for m = 3 is carried through in Table 2 and is shown graphically in Fig. 5.

Here the stress decrease in the direction of the axis of symmetry is very marked.

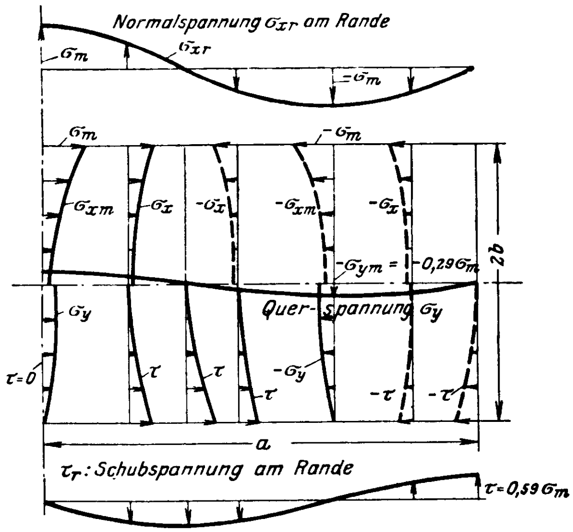

Abb. 5. Spannungsbild für cosinusförmiges Moment a/b = π und m = 3.

The calculation for m = 5 are given in Table 3. The effective width becomes,

$$B_m = \frac{5\pi b\, 2a}{4a\; 5\pi}\,\mathfrak{Tg}\,\frac{5\pi b}{2a}\left(\mathfrak{Tg}\,\frac{5\pi b}{2a} + \frac{1}{\mathfrak{Tg}\,\frac{5\pi b}{2a}} + \frac{2a}{5\pi b}\right) = \frac{1}{2}\, b\cdot 0{,}986\cdot(-0{,}986 + 1{,}015 + 0{,}4)$$

$$B_m = 0{,}5\cdot 0{,}43\cdot 0{,}986\cdot b = 0{,}21\, b.$$

For this order of magnitude of m, the influence of

$$\left(-\,\mathfrak{Tg}\,\frac{m\pi b}{2a} + \frac{1}{\mathfrak{Tg}\,\frac{m\pi b}{2a}}\right),$$

disappears, since this quantity approaches zero, hence we can write

$$B_m = \frac{b}{2}\cdot\frac{2a}{m\pi b} = \frac{a}{m\pi}, \tag{39}$$

that is, B_m, the effective width, henceforth depends only upon the length of the girder.

(b) Calculation of a Girder Flange for a Moment That is Produced by a Concentrated Load in the Middle of the Girder.

The moment curve of such a girder is a triangle of which the vertex lies in the line of application of the force and whose altitude is given by the moment $h = \frac{Pl}{4}$.

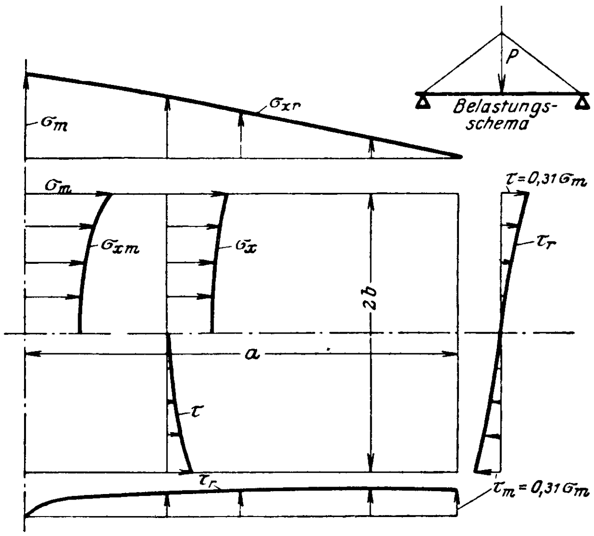

Abb. 6. Spannungsbild bei Einzellast. $a/b = \pi$

A function of this kind can be expressed through a Fourier series. As is shown by the development in the appendix, the Fourier series can be written,

$$F(x) = \frac{8\,\sigma_m}{\pi^2} \cdot \left(\cos\frac{\pi}{2a}\,x + \frac{1}{3^2}\cos\frac{3\pi}{2a}\,x + \frac{1}{5^2}\cos\frac{5\pi}{2a}\,x + \cdots\right). \tag{40}$$

We see from the construction of this function, that we can obtain without difficulty the desired stress condition through superposition of the stress conditions given in Tables 1, 2, and 3. Since the coefficients of the series decrease as squares, the influence of the third term on the stress curve is already very small. Therefore, we can in this series limit ourselves with sufficient accuracy to three terms.

For calculating the shear stresses we must differentiate the series. The differentiated series will then read as follows:

$$F'(x) = -\frac{8\,\sigma_m}{\pi^2}\left(\frac{\pi}{2a}\right)\left(\sin\frac{\pi}{2a}\,x + \frac{3}{3^2}\sin\frac{3\pi}{2a}\,x + \frac{5}{5^2}\sin\frac{5\pi}{2a}\,x + \cdots\right)$$

Since the series is absolutely convergent, the differentiation is permissible so that the superposition can be undertaken without hesitation. Therefore, the stress function is

$$F(xy) = \sum(-1)\,\frac{8}{\pi^2}\,\frac{\sigma_m}{m^2}\cos\frac{m\pi}{2a}\,x\left(\frac{2a}{m\pi}\right)^2\frac{m\pi b}{4a}\,\mathfrak{Tg}\,\frac{m\pi b}{2a}\left[\frac{\mathfrak{Cof}\,\frac{m\pi}{2a}\cdot y}{\mathfrak{Cof}\,\frac{m\pi}{2a}\,b} - \frac{y}{b}\,\frac{\mathfrak{Sin}\,\frac{m\pi}{2a}\cdot y}{\mathfrak{Sin}\,\frac{m\pi}{2a}\cdot b}\right] \tag{41}$$

and

$$\sigma_x = \sum \frac{8}{\pi^2} \frac{\sigma_m}{m^2} \cos\frac{m\pi}{2a}x\, \mathfrak{Tg}\, \frac{m\pi}{2a}b \left(\frac{m\pi b}{\underset{4}{\cancel{\pi}} a}\right) \left[\frac{2\, \mathfrak{Cof} \frac{m\pi}{2a}\cdot y}{\frac{bm\pi}{2a}\, \mathfrak{Sin}\, \frac{m\pi}{2a}\cdot b} - \left(\frac{\mathfrak{Cof}\frac{m\pi}{2a}\cdot y}{\mathfrak{Cof}\frac{m\pi}{2a}\cdot b} - \frac{y}{b} \frac{\mathfrak{Sin}\frac{m\pi}{2a}\cdot y}{\mathfrak{Sin}\frac{m\pi}{2a}\cdot b} \right) \right] \quad (42)$$

and

$$\tau = \sum \frac{8}{\pi^2} \frac{\sigma_m}{m^2} \sin\frac{m\pi}{2a}x \cdot \mathfrak{Tg}\frac{m\pi}{2a}b \left(\frac{m\pi b}{4a}\right) \left[- \frac{\mathfrak{Sin}\frac{m\pi}{2a}\cdot y}{\underset{\text{cosh}}{\cancel{\mathfrak{Sin}}}\frac{m\pi}{2a}\cdot b} + \frac{y}{b} \frac{\mathfrak{Cof}\frac{m\pi}{2a}\cdot y}{\mathfrak{Sin}\frac{m\pi}{2a}\cdot b} + \frac{\mathfrak{Sin}\frac{m\pi}{2a}\cdot y}{\frac{bm\pi}{2a}\mathfrak{Sin}\frac{m\pi}{2a}\cdot b} \right] \quad (43)$$

(See Fig. 6.)

For x = a

$$\sin\frac{m\pi}{2a}x = \sin\frac{m\pi}{2} = +1, \quad -1, \quad +1 \cdots$$

wherein m = 1, 3, 5 .... signs change.

Then for x = o the effective width is

$$B_m = \frac{8}{\pi^2} \cdot \frac{b}{2} \cdot \frac{\sigma_m}{\sigma_r} \sum \frac{1}{m^2} \mathfrak{Tg}\frac{m\pi}{2a}b \left(- \mathfrak{Tg}\frac{m\pi}{2a}b + \frac{1}{\mathfrak{Tg}\frac{m\pi}{2a}b} + \frac{2a}{m\pi b} \right) \quad (44)$$

This gives for x = o and $\frac{a}{b\pi}$ = 1 and $\sigma_r = 0.94\, \sigma_m$

[Translators Note: $\sigma_r = \sigma_x$ for y = b; (See Eq. 19). However here $\sigma_x = \sigma_r = 0.94$ for x = o, y = b. See Table 4. D.W.]

$$B_m = \frac{8}{\pi^2} \cdot \frac{b}{2} \cdot \left(\frac{1}{0{,}94}\right) \left[\mathfrak{Tg}\, 0{,}5 \left(- \mathfrak{Tg}\, 0{,}5 + \frac{1}{\mathfrak{Tg}\, 0{,}5} + 2 \right) + \frac{1}{9} \mathfrak{Tg}\, 1{,}5 \left(- \mathfrak{Tg}\, 1{,}5 + \frac{1}{\mathfrak{Tg}\, 1{,}5} + \frac{2}{3} \right) + \ldots \right]$$

or

$$B_m = \frac{0{,}82}{0{,}94} \cdot \left(B_m + \frac{1}{9} B_{m_1} + \frac{1}{25} B_{m_2} \right),$$

$$B_m = \frac{0{,}82}{0{,}94} \left(0{,}86 + \frac{1}{9} \cdot 0{,}39 + \frac{1}{25} \cdot 0{,}21 + \ldots \right) b$$

$$B_m = 0{,}79\, b.$$

As is evident from the diagram, the effective width varies only slightly with increasing x, so that in general one can calculate with a constant B_m.

(c) Load Uniformly Distributed Over the Length of the Beam.

The uniformly distributed load gives a parabola form moment, which attains the maximum value $M = \frac{pl^2}{8}$ in the middle.

We can express this function also, as shown in the appendix, by means of a Fourier series. It is

$$F(x) = \frac{32\,\sigma_m}{\pi^3}\left(\cos \cancel{\frac{2a}{\pi}}\,\frac{\pi}{2a}\, x - \frac{1}{3^3}\cos\frac{3\pi}{2a}x + \frac{1}{5^3}\cos\frac{5\pi}{2a}x - + \ldots\right). \tag{45}$$

The first derivative of this function gives

$$F'(x) = -\frac{32}{\pi^3}\,\sigma_m\left(\frac{\pi}{2a}\right)\cdot\left[\sin\frac{\pi}{2a}x - \frac{3}{3^3}\sin\frac{3\pi}{2a}x + \frac{5}{5^3}\sin\frac{5\pi}{2a}x - + \ldots\right].$$

The differentiated function is absolutely convergent for all values of x, therefore, we can in this case also undertake the superposition directly. When the distribution is of parabolic form, the influence of the terms of higher order decreases as the cube of m so that two terms are wholly sufficient to describe the

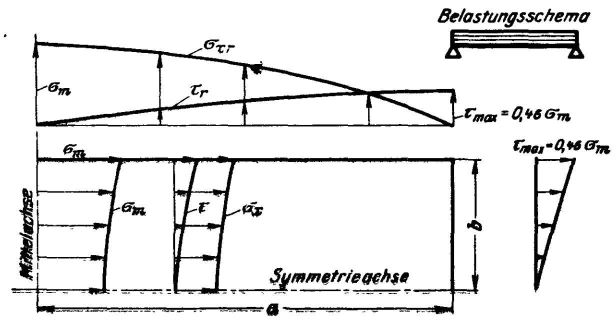

Abb. 7. Spannungsbild bei gleichmäßig verteilter Last. $a : b = \pi$.

stress condition. The functions for the shear and normal stresses can then be expressed through the following equations.

$$F(x\,y) = \frac{32}{\pi^3}\sum \sigma_m \cdot (-1)^{\frac{m+1}{2}}\frac{1}{m^3}\cos\frac{m\pi}{2a}x\left(\frac{2a}{m\pi}\right)^2\cdot F(y). \tag{46}$$

$$\sigma_x = \frac{32\,\sigma_m}{\pi^3}\sum(-1)^{\frac{m+1}{2}}\frac{1}{m^3}\cos\frac{m\pi}{2a}x\left(\frac{2a}{m\pi}\right)^2\cdot F''(y). \tag{47}$$

$$m = 1,\ 3,\ 5,\ \ldots$$

$$\tau = \frac{32}{\pi^3}\,\sigma_m\sum(-1)^{\frac{m+1}{2}}\frac{1}{m^3}\sin\frac{m\pi x}{2a}\left(\frac{2a}{m\pi}\right)[F'(y)], \tag{48}$$

$$\sigma_y = -\frac{32}{\pi^3}\,\sigma_m\sum(-1)^{\frac{m+1}{2}}\cdot\frac{1}{m^3}\cos\frac{m\pi}{2a}x[F(y)]. \tag{49}$$

For x = a, $\sin\frac{m\pi}{2} = \pm 1$, therefore, in calculating the shearing stresses for x = a the signs will all be positive. (See Fig. 7) The effective width at the place x = o is

$$B_m = \pm\frac{32}{\pi^3}\sum\frac{1}{m^3}\,\frac{b}{2}\,\mathfrak{Tg}\frac{m\pi b}{2a}\left(-\mathfrak{Tg}\frac{m\pi b}{2a} + \frac{1}{\mathfrak{Tg}\frac{m\pi}{2a}b} + \frac{2a}{m\pi b}\right),$$

or

$$B_m = \frac{32}{\pi^3}\left(0{,}86\,b - \frac{1}{27}\cdot 0{,}39\,b\right),$$

$$B_m = \frac{32}{31}\,(0{,}86 - 0{,}01)\,b = 0{,}88\,b\,.$$

[Translator's Note: Here $\sigma_r = \sigma_x = \sigma_m$ for x = o, y = b. See Table 5. D.W.]

(d) Loading through two concentrated loads that are arranged symmetrically on the girder.

This gives a Fourier series of the form

$$F(x) = -\frac{4\,h\,2a}{\pi^2\,c}\left(\sin\frac{\pi}{2a}\,c\cos\frac{\pi}{2a}\,x - \frac{1}{3^2}\,\sin\frac{3\pi}{2a}\,c\cos\frac{3\pi}{2a}\,x + \ldots\right). \tag{50}$$

For c = a/2

$$\sin\frac{\pi}{2a}\cdot c = \sin\frac{\pi}{4} = +\sin\frac{3\pi}{4} = -\sin\frac{5\pi}{4}\ \text{usw.}$$

therefore,

$$F(x) = \frac{16\,h}{\pi^2}\left(\sin\frac{\pi}{4}\cos\frac{\pi}{2a}\,x - \frac{1}{3^2}\sin\frac{3\pi}{4}\cos\frac{3\pi}{2a}\,x + \frac{1}{5^2}\sin\frac{5\pi}{4}\cos\frac{5\pi}{2a}\,x - + \ldots\right)$$

or

$$F(x) = \frac{16\,h}{\pi^2}\sin\frac{\pi}{4}\left(\cos\frac{\pi}{2a}\,x - \frac{1}{3^2}\cos\frac{3\pi}{2a}\,x - \frac{1}{5^2}\cos\frac{5\pi}{2a}\,x + \ldots\right) \tag{51}$$

and

$$F'(x) = (-1)\,\frac{16\,h}{\pi^2}\,\frac{\pi}{2a}\,\sin\frac{\pi}{4}\left(\sin\frac{\pi}{2a}\,x - \frac{3}{3^2}\sin\frac{3\pi}{2a}\,x - \frac{5}{5^2}\sin\frac{5\pi}{2a}\,x + \ldots\right).$$

Three terms are sufficient for determining the normal stress. Later an accurate solution will be undertaken for the side ratio a/b = 4 because this case of loading is the basis of the experiments which have been conducted by the author in the structural laboratory of the Technische Hochschule. The distribution of the stresses is evident from Fig. 8. The stresses will be represented by the equations

$$\sigma_x = \frac{16}{\pi^2}\cdot\sigma_m\,(-1)^{\frac{m+1}{2}}\,\sin\frac{m\pi}{4}\cdot\frac{1}{m^2}\cos\frac{m\pi}{2a}\,x\left(\frac{2a}{m\pi}\right)^2 F''(y), \tag{52}$$

$$\tau = \frac{16}{\pi^2}\cdot\sigma_m\,(-1)^{\frac{m+1}{2}}\,\sin\frac{m\pi}{4}\cdot\frac{1}{m^2}\sin\frac{m\pi}{2a}\,x\left[\left(\frac{2a}{m\pi}\right)F'(y)\right] \tag{53}$$

For x = a we will get $\sin\frac{m\pi}{2a}\ x = \sin\frac{m\pi}{2}$

and $(-1)^{\frac{m+1}{2}}\ \cdot\ \sin\frac{m\pi}{2} = (\pm 1) = 1$

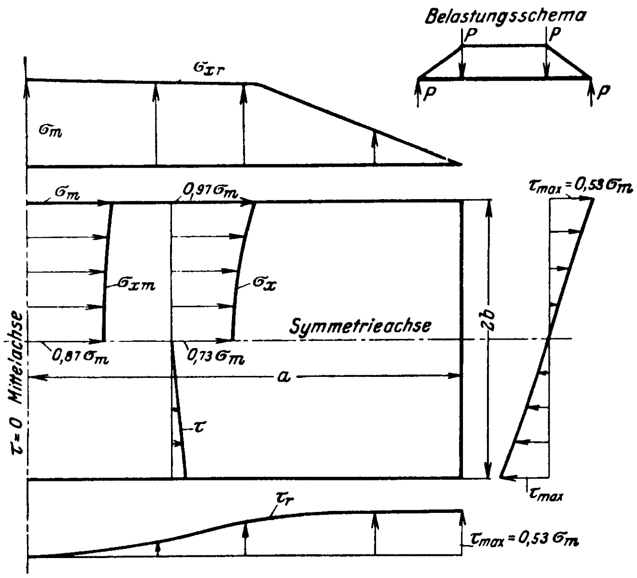

Abb. 8. Spannungsbild bei zwei Lasten. $a:b=\pi$.

Therefore the signs change ++ -- (see Table 6). The effective width at the place x = o is

$$B_m = \frac{16}{\pi^2} \sum \pm \frac{1}{m^2} \cdot \frac{b}{2} \sin\frac{\pi}{4} \mathfrak{Tg}\frac{m\pi}{2a} \cdot b \left(- \mathfrak{Tg}\frac{m\pi}{2a} b + \frac{1}{\mathfrak{Tg}\frac{m\pi}{2a} \cdot b} + \frac{2a}{m\pi b} \right) \tag{54}$$

or

$$B_m = \frac{16}{\pi^2} \cdot \sin\frac{\pi}{4} \left(0{,}86\, b - \frac{1}{9} 0{,}39\, b - \frac{1}{25} 0{,}21\, b \right)$$

or

$$B_m = \frac{16}{\pi^2} \cdot 0{,}71\, b\, (0{,}86 - 0{,}04 - 0{,}01)\,,$$

$$B_m = 0{,}926\, b\,.$$

This shows a marked increase of the effective width because of the accumulation of shear stresses at the end of the girder.

4. Stress Distribution at the Ends of the Flange.

The previous solutions have the fault that at the ends of the girder x = a the boundary conditions are not strictly fulfilled, for although the normal stresses vanish there (σ_x = o) the shear stresses do not. We can cause them to disappear if we solve the equation for the shear stresses at the place x = a in a Fourier series

$$\tau = \sum \sigma_m \left(\sin \frac{m\pi}{2a} x\right) \cdot \frac{b}{2} \cdot \frac{m\pi}{2a} \mathfrak{Tg} \frac{m\pi}{2a} b \left[-\frac{\mathfrak{Sin} \frac{m\pi}{2a} \cdot y}{\mathfrak{Cof} \frac{m\pi}{2a} b} + \frac{y}{b} \frac{\mathfrak{Cof} \frac{m\pi}{2a} \cdot y}{\mathfrak{Sin} \frac{m\pi}{2a} b} + \frac{\mathfrak{Sin} \frac{m\pi}{2a} \cdot y}{\frac{b m\pi}{2a} \mathfrak{Sin} \frac{m\pi}{2a} b} \right] \quad (38)$$

and superpose a second stress condition according to this series upon the first solution. For the accuracy of our solution, it is required that the second Fourier series be so chosen that its integral series disappears at the place y = b. This condition as a matter of fact is at the same time the condition that the stresses σ_y disappear at the boundary y = b. If we designate our first solution for x = a with $-\sum_m A \sigma_m \frac{2a}{m\pi} F_1'(y)$ and the second solution that we use to cause the disappearance of the shear stresses with $\sum_n B \sigma_m \sin ky \cdot F_2(a)$, our condition equation will now be for x = a

(shear stress first solution) = -(shear stress second solution)

$$\tau_1 = -\sigma_m \sum_m A \frac{2a}{m\pi} F'(y) = -\sum_n C \cdot \sigma_m \cdot \sin \frac{n\pi}{2b} \cdot y \cdot F_2'(a) = -\tau_2 \,. \quad (55)$$

This condition gives the equation for the shear stresses:

$$\tau = \sigma_m \sum \sin \frac{m\pi}{2a} x \cdot \frac{b}{2} \cdot \frac{m\pi}{2a} \mathfrak{Tg} \frac{m\pi}{2a} \cdot b \left(-\frac{\mathfrak{Sin} \frac{m\pi}{2a} \cdot y}{\mathfrak{Cof} \frac{m\pi}{2a} \cdot b} + \frac{y}{b} \frac{\mathfrak{Cof} \frac{m\pi}{2a} \cdot y}{\mathfrak{Sin} \frac{m\pi}{2a} \cdot b} + \frac{\mathfrak{Sin} \frac{m\pi}{2a} \cdot y}{b \frac{m\pi}{2a} \mathfrak{Sin} \frac{m\pi b}{2a}} \right) \quad (56)$$

$$+ \sum \frac{4a^3 n^2}{b^3 m^3 \pi} \frac{A \sigma_m}{\left[1 + \left(\frac{an}{bm}\right)^2\right]^2} \sin \frac{n\pi}{2b} y [F_2'(x)] ,$$

This equation now will also satisfy the last boundary condition that for x = a the shear stress vanishes.

The Fourier-coefficients of the series are so chosen that the second solution represents a periodic function, which represents the shear stress of – b – and from there on repeats itself as a mirror image. The derivation of the coefficients is calculated in the appendix. Since the Fourier coefficients of this second series depend upon the factors in the first series, we can express the coefficients of the second series as sums of the Fourier coefficients of the first series. Let us take

$$C_n = \frac{4}{\pi} \cdot \frac{a^3}{b^3} \sum A_m \frac{n^2 m}{\left[m^2 + \left(\frac{a}{b}\right)^2 n^2\right]^2} \,. \quad (57)$$

Then the nth term of our second series for the concentrated load in the middle reads

$$\tau_2 = \sigma_m \cdot \frac{4a^3}{\pi b^3}\left[\frac{1 \cdot n^2}{\left[1+\left(\frac{a}{b}\right)^2 n^2\right]^2} - \frac{1}{3^2}\frac{3n^2}{\left[3^2+\left(\frac{a}{b}\right)^2 n^2\right]^2} + \frac{1}{5^2}\frac{5n^2}{\left[5^2+\left(\frac{a}{b}\right)^2 n^2\right]^2}\right] \sin\frac{n\pi}{2b} y F_2'(x)\,. \quad (58)$$

Since the function corresponds exceptionally well to the first function, 2 or at most 3 terms are sufficient for the calculation. From the expression

$$\tau_2 = \sum C \cdot \sin\frac{n\pi}{2b} y$$

follows an expression for the stress function,

$$F_2(y) = C \cos\frac{n\pi}{2b} y\,.$$

For the second solution of the differential equation

$$\Delta\Delta F_2(xy) = 0$$

this again gives the formula

$$F_2(x) = A \cdot \mathfrak{Cof}\,\frac{n\pi x}{2b} + B \cdot x \cdot \mathfrak{Sin}\,\frac{n\pi x}{2b}\,. \quad (59)$$

The coefficients A and B must now again be so chosen that the boundary conditions will be fulfilled. The boundary conditions for our plate equation are as follows:

1. for y = b it follows that $\sigma_y = \frac{\delta^2 F}{\delta x^2} = 0$,

2. for x = a it follows that $\sigma_x = \frac{\delta^2 F}{\delta y^2} = 0$

3. for x = a it follows that $\tau_1 = -\tau_2$

The first condition is automatically fulfilled by the expression

$$F_2(y) = C \cdot \cos\frac{n\pi}{2b} y$$

We can fulfill the second condition when F_2 (x) assumes the value 0 at the place x = a. The third condition is fulfilled thru the derivation of the coefficients. It is necessary, however, that the function $\frac{\partial F(x)}{\partial x}$ take the value -1 at the place x = a. When Cn are the Fourier coefficients, the function for $\tau_2 = -\frac{\partial^2 F}{\partial y \partial x}$ reads

$$-\frac{\partial^2 F_2}{\partial x \partial y} = \tau_2 = \sigma_m \sum C_n \cdot \sin\frac{n\pi}{2b} \cdot y \left(A \,\mathfrak{Sin}\frac{n\pi}{2b} x + B\left[x\,\mathfrak{Cof}\left(\frac{n\pi}{2b}\right) x + \frac{2b}{n\pi}\mathfrak{Sin}\left(\frac{n\pi}{2b}\right) x\right]\right) \tag{60}$$

and

$$F_2 = \sigma_m C_n \left(\frac{2b}{n\pi}\right) \cos\frac{n\pi}{2b} y \left(\frac{2b}{n\pi} A \,\mathfrak{Cof}\frac{n\pi}{2b} x + B x \frac{2b}{n\pi}\mathfrak{Sin}\frac{n\pi}{2b} x\right). \tag{61}$$

The two equations for determination of the constants A and B are, therefore,

$$A\,\mathfrak{Cof}\frac{n\pi}{2b} \cdot a + B a \,\mathfrak{Sin}\frac{n\pi}{2b} \cdot a = 0\,, \tag{62}$$

$$A\,\mathfrak{Sin}\frac{n\pi}{2b} a + B\left(a\,\mathfrak{Cof}\frac{n\pi}{2b} a + \frac{2b}{n\pi}\mathfrak{Sin}\frac{n\pi}{2b} \cdot a\right) = -1\,. \tag{63}$$

By multiplication with cosh or as the case may be with sinh and by subtraction of the first equation from the second we obtain

$$-Ba\,\mathfrak{Sin}^2\frac{n\pi}{2b} a + Ba\,\mathfrak{Cof}^2\frac{n\pi}{2b} a + B\frac{2b}{n\pi}\mathfrak{Sin}\frac{n\pi}{2b} a\,\mathfrak{Cof}\frac{n\pi}{2b} \cdot a = -\mathfrak{Cof}\frac{n\pi}{2b} \cdot a \tag{64}$$

or

$$Ba + B\frac{2b}{n\pi}\mathfrak{Cof}\frac{n\pi}{2b} a\,\mathfrak{Sin}\frac{n\pi}{2b} a = -\mathfrak{Cof}\frac{n\pi}{2b} \cdot a\,; \tag{64a}$$

whence

$$B = \frac{-\mathfrak{Cof}\frac{n\pi}{2b} \cdot a}{a + \frac{b}{n\pi} \cdot \mathfrak{Sin}\frac{n\pi}{b} \cdot a} \tag{65}$$

and

$$A = -Ba \cdot \mathfrak{Tg}\frac{n\pi}{2b} \cdot a \tag{66}$$

or

$$A = -Ba\frac{\mathfrak{Sin}\frac{n\pi}{2b} \cdot a}{\mathfrak{Cof}\frac{n\pi}{2b} \cdot a} = +\frac{a\,\mathfrak{Sin}\frac{n\pi}{2b} \cdot a}{a + \frac{b}{n\pi}\mathfrak{Sin}\frac{n\pi}{b} \cdot a}\,. \tag{66}$$

Consequently we can write down the second solution for our stress function. It is written as follows:

$$F_2(xy) = -\sigma_m\left(\frac{2b}{n\pi}\right)^2(\pm)\,C\cos\frac{n\pi}{2b}\cdot y\,\frac{-a\,\mathfrak{Sin}\frac{n\pi}{2b}\cdot a\,\mathfrak{Cos}\frac{n\pi}{2b}x + x\,\mathfrak{Cos}\frac{n\pi}{2b}\cdot a\cdot\mathfrak{Sin}\frac{n\pi}{2b}x}{a+\frac{2b}{n\pi}\,\mathfrak{Sin}\frac{n\pi}{2b}a\cdot\mathfrak{Cos}\frac{n\pi}{2b}\cdot a}. \qquad (67)$$

The normal stresses and the shear stresses then follow thru differentiation.

$$\sigma_x = \frac{\delta^2 F_2}{\delta y^2} = \pm\sigma_m(\pm)\,C\cos\frac{n\pi}{2b}y\,\frac{a\,\mathfrak{Sin}\frac{n\pi}{2b}a\cdot\mathfrak{Cos}\frac{n\pi}{2b}x - x\,\mathfrak{Cos}\frac{n\pi}{2b}a\,\mathfrak{Sin}\frac{n\pi}{2b}x}{a+\frac{2b}{n\pi}\cdot\mathfrak{Sin}\frac{n\pi}{ab}a\,\mathfrak{Cos}\frac{n\pi}{2b}a}, \qquad (68)$$

$$\tau = -\frac{\delta^2 F_2}{\delta x\,\delta y} = \sigma_m(\pm)\,C\sin\frac{n\pi}{2b}y\,\frac{a\,\mathfrak{Sin}\frac{n\pi}{2b}a\,\mathfrak{Sin}\frac{n\pi}{2b}x - \mathfrak{Cos}\frac{n\pi}{2b}\cdot a\left(x\,\mathfrak{Cos}\frac{n\pi}{2b}x+\frac{2b}{n\pi}\mathfrak{Sin}\frac{n\pi}{2b}x\right)}{a+\frac{2b}{n\pi}\mathfrak{Sin}\frac{n\pi}{b2}a\,\mathfrak{Cos}\frac{n\pi}{2b}\cdot a}, \qquad (69)$$

$$\sigma_y = \frac{\delta^2 F_2}{\delta x^2} = \pm\sigma_m(\pm)\,C\cos\frac{n\pi}{2b}y\,\frac{a\,\mathfrak{Sin}\frac{n\pi}{2b}\cdot a\,\mathfrak{Cos}\frac{n\pi}{2b}x - \mathfrak{Cos}\frac{n\pi}{2b}\cdot a\left(x\,\mathfrak{Sin}\frac{n\pi}{2b}x+\frac{4b}{n\pi}\mathfrak{Cos}\frac{n\pi}{2b}x\right)}{a+\frac{2b}{n\pi}\mathfrak{Sin}\frac{n\pi}{2b}\cdot a\,\mathfrak{Cos}\frac{n\pi}{2b}\cdot a}. \qquad (70)$$

If we join this second solution to the first by superposition, all shear stresses disappear at the place x = a. Also, the remaining boundary conditions continue to be strictly fulfilled.

A further series of simplifications of this equation may be undertaken. Abbreviating, we write $F(xy) = F_1(xy) + F_2(xy)$, where we shall understand by F_1 and F_2 the first and second solutions. The hyperbolic functions now have the property that for high values $\frac{n\pi}{2b}\cdot a$ or $\frac{m\pi}{2a}\cdot b$ that is for greater lengths or for higher coefficients n or m it will be true approximatelv:

$$\mathfrak{Sin}\frac{n\pi}{2b}\cdot a \approx \mathfrak{Cos}\frac{n\pi}{2b}\cdot a \approx \frac{1}{2}e^{\frac{n\pi}{2b}\cdot a}$$

or

$$\mathfrak{Sin}\frac{m\pi}{2a}\cdot b \approx \mathfrak{Cos}\frac{m\pi}{2a}b \approx \frac{1}{2}e^{\frac{m\pi}{2a}\cdot b}$$

It is sufficiently accurate to apply the abbreviation if $\frac{n\pi}{2b}\cdot a$ is greater than 2.5 or $\frac{a}{b}$ is greater than $\frac{5}{n\pi}$; for $n\cdot\frac{a}{b}$ is greater than 1.7.

We can then cancel out the first term in the denominator at once; since in the expression

$$a\left(1+\frac{2b}{an\pi}\mathfrak{Cos}\frac{n\pi}{2b}a\,\mathfrak{Sin}\frac{n\pi}{2b}a\right)$$

the number 1 is small in comparison to the product

$$\frac{2b}{a n \pi} \mathrm{Cof} \frac{n\pi}{2b} a \,\mathrm{Sin} \frac{n\pi}{2b} a .$$

We have then the following stress function as the second solution

$$F_2(xy) = \sigma_m \left(\frac{2b}{n\pi}\right)^2 (\pm)\, C \cos \frac{n\pi}{2b} \cdot y \,\frac{a n \pi}{2b} \cdot 2\, e^{\left(-\frac{n\pi}{2b} \cdot a\right)} \left(\mathrm{Cof} \frac{n\pi}{2b} x - \frac{x}{a} \mathrm{Sin} \frac{n\pi}{2b} x\right). \quad (71)$$

whence

$$\sigma_x = -\sigma_m (\pm)\, C \cos \frac{n\pi}{2b} \cdot y \,\frac{a n\pi}{2b} \cdot 2\, e^{\left(-\frac{n\pi}{2b} \cdot a\right)} \left(\mathrm{Cof} \frac{n\pi}{2b} x - \frac{x}{a} \mathrm{Sin} \frac{n\pi}{2b} x\right), \quad (72)$$

$$\tau = -\frac{\partial^2 F}{\partial x\, \partial y} = \sigma_m (\pm)\, C \sin \frac{n\pi}{2b} y \,\frac{a n\pi}{2b} \cdot 2 e^{\left(-\frac{n\pi}{2b} \cdot a\right)} \left(\mathrm{Sin} \frac{n\pi}{2b} x - \frac{x}{a} \mathrm{Cof} \frac{n\pi}{2b} x - \frac{2b}{a n\pi} \mathrm{Sin} \frac{n\pi}{2b} x\right), \quad (73)$$

$$\sigma_y = \frac{\partial^2 F}{\partial x^2} = +\sigma_m (\pm)\, C \cos \frac{n\pi}{2b} y \,\frac{a n\pi}{2b} \cdot 2 e^{\left(-\frac{n\pi}{2b} \cdot a\right)} \left(\mathrm{Cof} \frac{n\pi}{2b} x - \frac{x}{a} \mathrm{Sin} \frac{n\pi}{2b} x - \frac{4b}{a n\pi} \mathrm{Cof} \frac{n\pi}{2b} x\right). \quad (74)$$

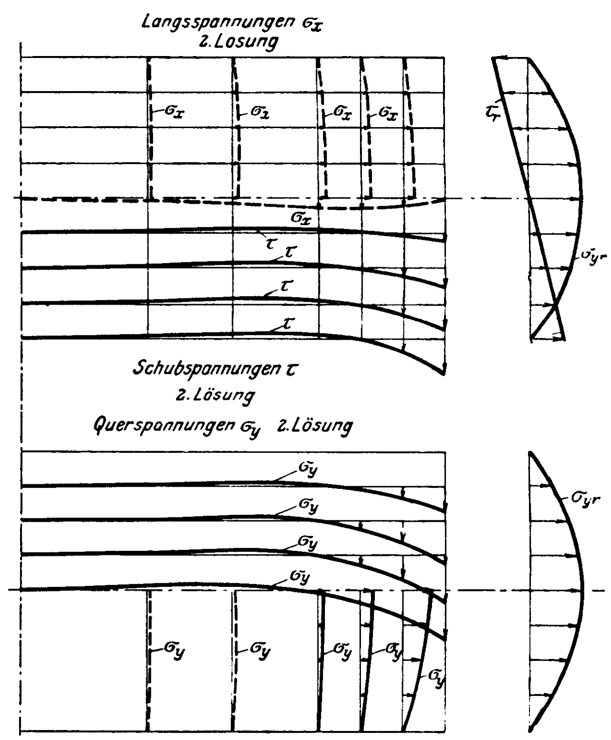

Abb. 9. Spannungsbild der 2. Lösung für cosinusförmiges Moment. $a : b = \pi$.

For the second solution the formula permits even further simplification since for large $\frac{n\pi}{2b}$

$$\mathfrak{Cof}\frac{n\pi}{2b}y = \mathfrak{Sin}\frac{n\pi}{2b}\cdot y = \frac{1}{2}e^{\frac{n\pi y}{2b}}$$

The following equation for the stress function will then apply

$$F_2(xy) = \sigma_m\left(\frac{2b}{n\pi}\right)^2(\pm)\,C\cos\frac{n\pi}{2b}y\,e^{\frac{n\pi}{2b}(x-a)}\cdot\frac{an\pi}{2b}\left(1-\frac{x}{a}\right), \tag{75}$$

$$\sigma_{x_2} = \frac{\partial^2 F}{\partial y^2} = -\sigma_m(\pm)\,C\cos\frac{n\pi}{2b}y\,e^{\frac{n\pi}{2b}(x-a)}\cdot\frac{an\pi}{2b}\left(1-\frac{x}{a}\right), \tag{76}$$

$$\tau_2 = -\frac{\partial^2 F}{\partial x\,\partial y} = \sigma_m(\pm)\,C\sin\frac{n\pi}{2b}y\,e^{\frac{n\pi}{2b}(x-a)}\cdot\frac{an\pi}{2b}\left(1-\frac{x}{a}-\frac{2b}{an\pi}\right). \tag{77}$$

$$\sigma_{y_2} = \frac{\partial^2 F}{\partial x} = +\sigma_m(\pm)\,C\cos\frac{n\pi}{2b}y\cdot e^{\frac{n\pi}{2b}(x-a)}\frac{an\pi}{2b}\left(1-\frac{x}{a}-\frac{4b}{an\pi}\right). \tag{78}$$

If we now take as a basis the simplest case of cosine form load distribution which we treated in our first example, then m = 1 and our formula for the coefficients C of solution 2 reads

$$C_n = \frac{4a^3}{\pi b^3}\cdot\frac{n^2}{\left[1+\left(\frac{a}{b}\right)^2 n^2\right]^2},$$

and therefore

$$C_1 = \tfrac{1}{3};\qquad C_3 = -0{,}045;\qquad C_5 = 0{,}016\,.$$

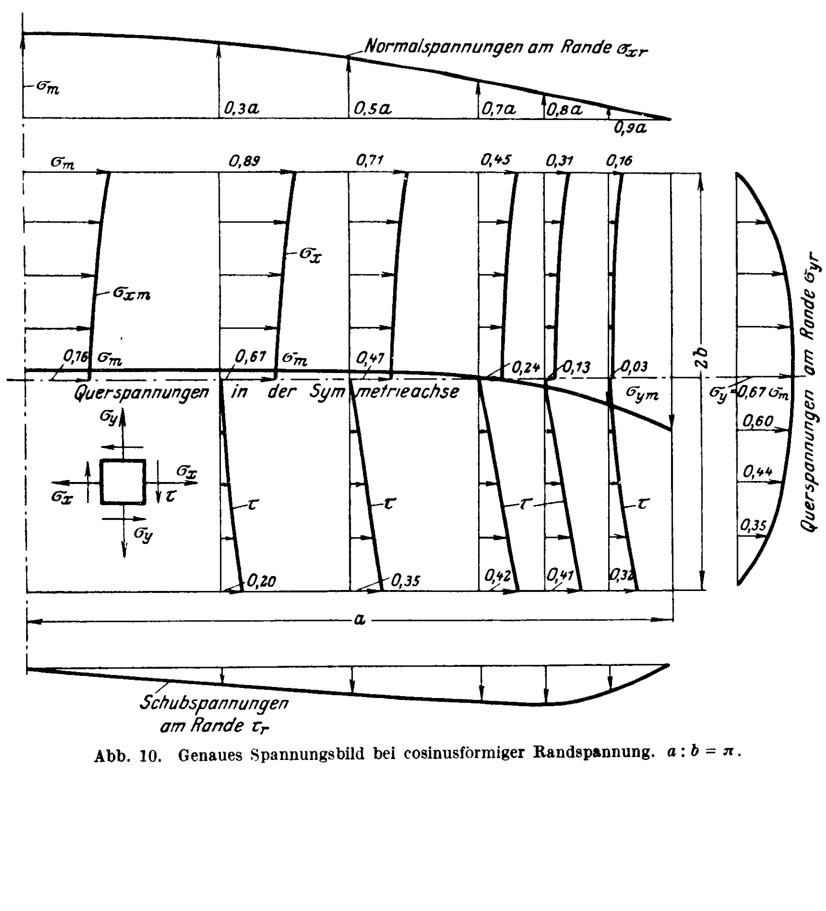

Abb. 10. Genaues Spannungsbild bei cosinusförmiger Randspannung. $a : b = \pi$.

For calculation of the effective width at the ends, we use in addition the integral

$$J_2 = \int \sigma_x \cdot dy \, ;$$

which gives us

$$J_2 = -\sigma_m \cdot C \cdot \frac{2b}{n\pi} \cdot \frac{an\pi}{2b}\left(1 - \frac{x}{a}\right) e^{\frac{n\pi}{2b}(x-a)} \int_0^b \cos\frac{n\pi}{2b} \cdot y\,dy \, ,$$

$$J_2 = -\sigma_m \cdot C \cdot a\left(1 - \frac{x}{a}\right) e^{\frac{n\pi}{2b}(x-a)} \, . \qquad (79)$$

For x = 0.9 a and for two terms of the second Fourier series, this formula yields the value

$$J_2 = -\sigma_m \cdot 0{,}1\, a\left(C_1 e^{\frac{n\pi}{2b}(x-a)} + C_3 e^{\frac{n\pi}{2b}(x-a)}\right)$$
$$= -\sigma_m \cdot 0{,}1\, a \cdot (0{,}333 \cdot 0{,}61 + 0{,}045 \cdot 0{,}22)$$

or

$$J_2 = -\sigma_m \cdot 0{,}022 \cdot a = -\sigma_m \cdot 0{,}022\, \pi b \, ,$$

Therefore

$$J_2 = 0.068\, \sigma_m\, b$$

But according to solution (1)

$$B_{m_1} = 0{,}86\, b \, .$$

Therefore

$$J_1 = \int \sigma_x \cdot dy = 0{,}86\, b \cdot \sigma_r = 0{,}86\, b \cdot 0{,}16\, \sigma_m = 0{,}137\, \sigma_m \cdot b$$

and

$$B_m = \frac{J_1 - J_2}{\sigma_r} = \frac{0{,}137 - 0{,}067}{0{,}16\, \sigma_m}\, \sigma_m \cdot b \, ,$$

$$B_m = \frac{0{,}69}{0{,}16} \cdot b = \underline{0{,}43\, b} \, .$$

We see that the effective width decreases very greatly at the end, in this case by half. However, it is plainly evident from Fig. 9 and it follows from the structure of the equation that the effect of the second solution at a distance b from the short side assumes an extraordinarily low per cent value so that a loss of only about 10 per cent is involved.

The stresses σ_x, σ_y and τ for the case of a cosine form moment distribution and a side ratio a/b = π are calculated in Tables 7, 8, 9, and 10. Herein, solutions 1 and 2 are treated separately in order to show the influence clearly.

The stresses given by solution 2 are plotted by themselves in Fig. 9. It is evident that the stresses σ_{x_2} according to the second solution on the whole leave the boundary stresses unchanged since they become zero there. On the other hand they decrease the longitudinal stresses σ_{x_1} in the middle. However, this is true only in proximity to the ends, where the stresses from the first solution are very small.

Especially noticeable is the occurrence of a very high transverse stress σ_{y_2} at the ends of the girder. This stress acts as a negative stress and therefore a compressive one, and because of the possibility of buckling, must be especially considered. It also gives us a very good idea of the stress distribution in a girder that has very wide flanges.

The shear stress τ_2 follows the course of the transverse stress σ_y. We see that the noticeable effect of these stresses extends to a distance of b/2 from the short edge.

The obvious thought is to use these simplifications for the first solution of various wide girders also. Actually, it can be done when

$$\frac{m\pi b}{2a} > 2$$

Then we will again have

$$\mathfrak{Tg}\,\frac{m\pi b}{2a} \approx 1$$

and

$$\mathfrak{Cof}\,\frac{m\pi b}{2a} = \mathfrak{Sin}\,\frac{m\pi b}{2a} = \frac{1}{2}\,e^{\frac{m\pi b}{2a}},$$

so that the abbreviated solution 1 for the stress function becomes

$$F_1(xy) = -\left(\frac{2a}{m\pi}\right)^2 \cdot \sigma_m \cdot \cos\frac{m\pi x}{2a} \cdot \frac{b\,m\pi}{4a} \cdot 2 \cdot e^{-\frac{m\pi b}{2a}} \left(\mathfrak{Cof}\,\frac{m\pi}{2a}\cdot y - \frac{y}{b}\cdot \mathfrak{Sin}\,\frac{m\pi y}{2a}\right). \qquad (80)$$

The formula for the effective width becomes especially simple because

$$-\mathfrak{Tg}\,\frac{m\pi b}{2a} + \frac{1}{\mathfrak{Tg}\,\frac{m\pi}{2a}\cdot b} \approx 0.$$

that is

$$\boxed{B_m \approx \frac{b}{2}\cdot\frac{2a}{b\,m\pi} = \frac{a}{m\pi}} \qquad (81)$$

when m = 1 and the span is 2a, i.e. 16 per cent of the span on each side of the web.

B. Exact Calculated Example for a beam with two symmetrical loads and a length-width ratio a/b = 4.

The exact calculation for this case is carried out in tables 15–30. The stress function for this case again takes the form of two solutions F_1 (xy) and F_2 (xy) taken together so that when superimposed one upon the other the shear stress at the end where x = a disappears. The first solution is

$$F_1(xy) = -\frac{16}{\pi^2}\cdot 0{,}71\,\sigma_m\left[\left(\frac{2a}{\pi}\right)^2\cos\frac{\pi}{2a}x\,F_1\left(\frac{\pi}{2a}y\right) - \frac{1}{9}\left(\frac{2a}{3\pi}\right)^2\cos\frac{3\pi}{2a}x\,F_1\left(\frac{3\pi}{2a}y\right)\right.$$
$$\left. - \frac{1}{25}\left(\frac{2a}{5\pi}\right)^2\cos\frac{5\pi}{2a}x\,F_1\left(\frac{5\pi}{2a}y\right) + \frac{1}{49}\left(\frac{2a}{7\pi}\right)^2\cos\frac{7\pi}{2a}x\,F_1\left(\frac{7\pi}{2a}y\right) + \dots\right]$$

and the second solution

$$F_2(xy) = -\sigma_m\left(\frac{2b}{n\pi}\right)^2 C\cos\frac{n\pi}{2b}\cdot y\cdot\frac{a\pi n}{2b}\left(1-\frac{x}{a}\right)e^{-\frac{n\pi}{2b}(a-x)}(-1)^{\frac{n+1}{2}},$$

where

$$C = \pm\frac{16}{\pi^2}\sin\frac{m\pi}{4}\sum\frac{4a^3}{\pi b^3}\frac{n^2}{m\left[m^2+\left(\frac{a}{b}\right)^2 n^2\right]^2}.$$

The total solution can be expressed

$$F(xy) = F_1(xy) + F_2(xy),$$

and the stresses follow therefrom by means of differentiation as in the previous example. For the sake of clarity we will write only the stress equations for the second solution. These are

$$\sigma_{x_2} = +\sum\sigma_m C\cos\frac{n\pi}{2b}y\,\frac{a\pi n}{2b}\left(1-\frac{x}{a}\right)e^{-\frac{n\pi}{2b}(a-x)}(-1)^{\frac{n+1}{2}},$$
$$\tau_2 = -\sum\sigma_m C\sin\frac{n\pi}{2b}y\,\frac{a\pi n}{2b}\left(1-\frac{x}{a}-\frac{2b}{a\pi n}\right)e^{-\frac{n\pi}{2b}(a-x)}(-1)^{\frac{n+1}{2}},$$
$$\sigma_y = -\sum\sigma_m C\cos\frac{n\pi}{2b}y\,\frac{a\pi n}{2b}\left(1-\frac{x}{a}-\frac{4b}{a\pi n}\right)e^{-\frac{n\pi}{2b}(a-x)}(-1)^{\frac{n+1}{2}},$$

where n = 1, 3 and 5.

The coefficients C again take the form of a sum of Fourier coefficients with the signs arranged as in the first solution. They become therefore for n = 1

$$C_1 = \frac{16}{\pi^2}\cdot 0{,}71\cdot\frac{256}{\pi}\left(\frac{1}{17^2}+\frac{1}{3\cdot 25^2}-\frac{1}{5\cdot 41^2}-\frac{1}{7\cdot 65^2}\right),$$
$$C_1 = 1{,}14\cdot 0{,}815\,(0{,}345+0{,}053-0{,}12-0{,}003),$$
$$C_1 = 1{,}14\cdot 0{,}815\cdot 0{,}383 = 0{,}356.$$

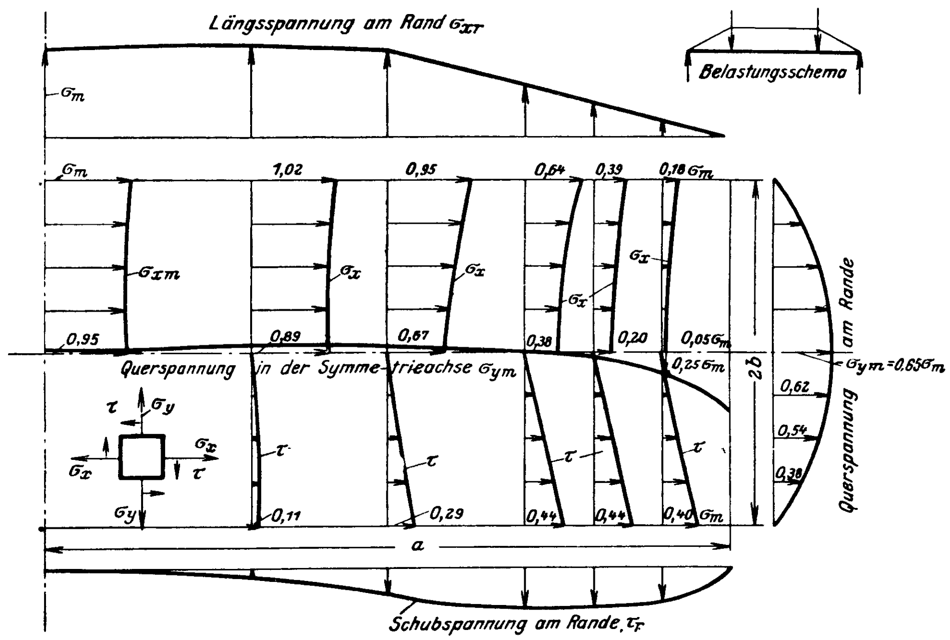

Abb. 11. Genaues Spannungsbild bei trapezförmiger Randspannung. $a : b = 4$.

Further, approximately

$$C_3 = -1{,}14 \cdot \frac{1}{9\pi}\left(1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \cdots\right),$$
$$C_3 = -0{,}045\,,$$
$$C_5 = +0{,}015 \text{ usf.}$$

It is seen that these coefficients are little different from the values found for a different length-width ratio and a different stress distribution.

The calculated stresses are plotted in Fig. 11. It is seen that the shear stress at the edge falls off at the ends, explaining the large effective width.

The transverse stress σ_y again takes on a higher negative value toward the axis of symmetry as we may already observe in the previous section.

The effective width for a/b = 4 becomes: for the term with m = 1

$$B_{m1} = \frac{b}{2}\,\mathfrak{Tg}\,\frac{\pi b}{2a}\left(-\mathfrak{Tg}\,\frac{\pi b}{2a} + \frac{1}{\mathfrak{Tg}\,\frac{\pi b}{2a}} + \frac{2a}{b\pi}\right)$$
$$= \frac{b}{2}\cdot 0{,}373 \cdot (-0{,}37 + 2{,}69 + 2{,}55)$$
$$= 0{,}186\,b \cdot 4{,}87 = 0{,}91\,b\,.$$

For m = 3

$$B_{m3} = \frac{l}{2}\,\mathfrak{Tg}\,\frac{3\pi}{2a}\left(-\mathfrak{Tg}\,\frac{3\pi}{2a} + \frac{1}{\mathfrak{Tg}\,\frac{3\pi}{2a}} + \frac{2a}{3b\pi}\right),$$

that is

$$B_{m3} = 0{,}51\,b\,.$$

similarly

$$B_{m5} = 0{,}29\,b$$

and

$$B_{m7} = 0{,}18\,b\,.$$

From which follows the effective width for x = 0.

$$\begin{aligned} B_m &= \frac{16}{\pi^2}\sin\frac{\pi}{4}\left(0{,}91\,b - \frac{1}{9}\,0{,}51\,b - \frac{1}{25}\,0{,}29\,b + \frac{1}{49}\cdot 0{,}18\right), \\ B_m &= 1{,}14\,(0{,}91\,b - 0{,}06\,b - 0{,}01\,b + \cdots), \\ &= 1{,}14\cdot 0{,}84 = 0{,}96\,b\,. \end{aligned}$$

The effective width for x/a = 0.9 taking into consideration solution 2 follows

For solution 1 and x/a = 0.9

$$\begin{aligned} \int_0^b \sigma_x\,dy = 0{,}91\,b\cdot 0{,}16\,\sigma_m &= 0{,}146\,b\cdot\sigma_m \\ +\,0{,}51\,b\cdot 0{,}05\,\sigma_m &= 0{,}026\,b\cdot\sigma_m \\ -\,0{,}29\,b\cdot 0{,}03\,\sigma_m &= 0{,}009\,b\cdot\sigma_m \\ \hline \text{Summe} &= 0{,}161\,b\cdot\sigma_m\,. \end{aligned}$$

Also for solution 2 and x/a = 0.9,

$$\begin{aligned} \int_0^b \sigma_x\,dy &= \sigma_m\cdot a\left(1-\frac{x}{a}\right)x\,(0{,}356\cdot 0{,}53 - 0{,}045\cdot 0{,}15) \\ &= \sigma_m\cdot 4\,b\cdot 0{,}1\,(0{,}19 - 0{,}01) = 0{,}072\,b\cdot\sigma_m\,. \end{aligned}$$

Therefore

$$\int \sigma_x\,dy = J_1 - J_2 = (0{,}161 - 0{,}072\,b\,\sigma_m) = 0{,}089\,b\,\sigma_m\,.$$

From which we get the effective width for x/a = 0.9

$$B_m = \frac{0{,}089\,b}{0{,}18} = 0{,}49\,b\,.$$

Similarly,

$$\begin{aligned} &\text{für}\quad x = 0{,}8\,a\,,\quad B_m = 0{,}73\,b\,, \\ &\text{für}\quad x = 0{,}7\,a\,,\quad B_m = 0{,}76\,b\,, \\ &\text{für}\quad x = 0{,}5\,a\,,\quad B_m = 0{,}81\,b\,, \\ &\text{für}\quad x = 0{,}3\,a\,,\quad B_m = 0{,}90\,b\,. \end{aligned}$$

We see that the effective width at the end is almost the same as for a beam with a cosine-form moment distribution and a length-width ratio of π. The reason is readily seen. The stress distribution at the ends is similar for nearly all beams.

We can for the first approximation assume that each moment line has a linear ascent at the end and the distribution of stress corresponds to this linear ascent. It follows from the reduction of the effective width at the end that the stresses there are higher, as calculation shows. The shear stresses distort more toward the end and the effective width in the middle becomes several per cent higher.

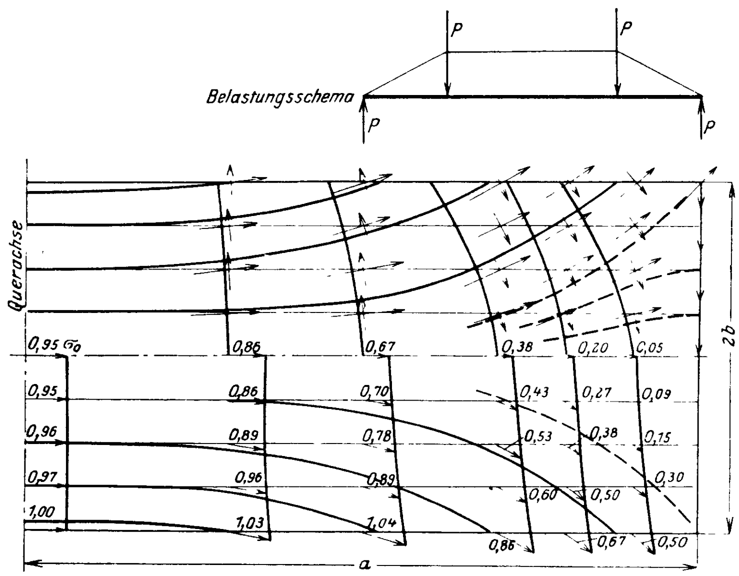

Abb. 12. Spannungstrajektorien.

The stresses are calculated in tables 15 to 30.

For the simplest case, the direction and magnitude of the principal stresses are found with the help of the well known formulae

$$\operatorname{tg} 2\varphi = -\frac{2\tau}{\sigma_x - \sigma_y} \quad \text{und} \quad \sigma_0 = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

From this the course of the principal stress is approximately established. The curves are plotted in Fig. 12. They show us also physically on what the proportionately large effective width is based. The bent lines of stress are crowded closely together in a wedge shape at the middle of the end resulting in a tensile component in the direction of the axis of the support.

6. The Fixed and the Continuous Girder.

It is not within the province of this paper to compute all possible cases of boundary conditions. However, a particularly simple instance with a continuous girder shall be studied briefly.

We choose as our example a girder on an infinite number of supports, which is loaded in the middle by concentrated loads (see Fig. 13).

The equation for the stress function is

$$F(xy) = -\frac{8\sigma_m}{\pi^2}\sum\frac{1}{m^2}\cos\frac{m\pi}{2a}x\left(\frac{2a}{m\pi}\right)^2 F(y)\,.$$

Because of perfect symmetry, solution F_2 does not occur. The effective width according to this example shows good agreement for wide flanges with the solution of Karman mentioned at the beginning.

This equation is the same as that for a freely supported girder with half the span 2a; i.e., in the case of a continuous girder the effective width will be considerably decreased, in the case of very wide flanges to one half. This fact must be considered especially in the case of continuous deck stringers, and likewise of hatch beams, heavy web frames, and similar structures.

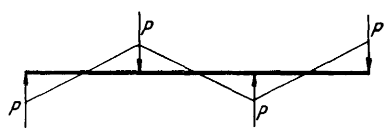

Abb. 13.

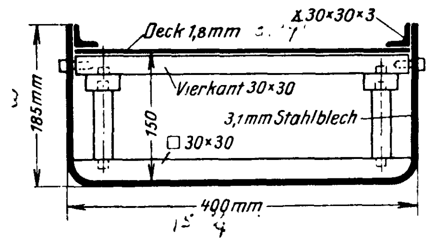

Abb. 14. Versuchskorper.

In case it is not desired to set up special equations for the continuous girders, the effective width for the positive moments can be calculated approximately. On an estimate, it will be as great as in the case of a girder the distance between the supports of which is the length of the positive moment area.

The effective width over the supports is approximately as great as in a girder with overhanging ends, whose overhang extends from the moment zero point to the support. The decrease in effective width will generally be very considerable above the supports.

Up to the present we have considered only cases in which every web is provided with a flange on one side. If we consider a stiffened wall with many web girders spaced equally, the stress condition in longitudinal direction will be altered only slightly. On the other hand, the stresses in transverse direction will travel in the direction of the web, so that a stress condition will result similar to that in T-beams. With these latter, however, extreme caution is required, since the flanges are rarely proof against buckling.

The effective width for such series of girders is to be taken as double that of those derived. In the web, the shear stresses will accordingly attain the double value.

Regarding the stress distribution in the deck and flat bottom of the ship, several statements may likewise be made. Since the ends have a relatively slight influence, the stress distribution will not vary greatly from that in a rectangular flange. The large aspect ratio will in general assure universal effectiveness of the deck. A similar stress condition will prevail in all decks. On the other hand, caution is necessary in such ships, in which heavy loads lie in the midship zone since here a very unfavorable shear stress distribution will cause a decrease in the effective width. These cases must be especially examined.

The effective width of the bridge superstructures is governed by the laws here derived for rectangular girders.

III EXPERIMENTS

The author received his inspiration for the present work from some experiments conducted by him in the structural laboratory at Danzig University. The experiments were based on the exact calculation of the case of a girder with two symmetrical loads. For this purpose a box girder with a cross section as shown in Fig. 14 was built. The bottom and sides were 3 mm (0.12 in.) and the deck 1.8 mm (0.07 in.) thick. The joining of the sides to the deck was by means of 30 x 30 x 3 (1.2 x 1.2 x 0.12 in.) angles.

At the supports transverse stiffeners were built-in which transmitted the compression to the webs. The ends have 10 mm (0.39 in.) thick transverse bulkheads for the absorption of the compressive forces.

The length is 1600 mm (63 in.), the width 400 mm (15.7 in.), and the height (inside) 150 mm (5.9 in.) so that the length-width ratio of the flange a/b = 4 was attained.

The mathematical moment of inertia was found to be 1700 cm^4 (41 $in.^4$). The loading was effected at both ends by loads of 5.1 tons, while the supports were located at 40 cm. (16 in.) each from the end.

The neutral axis theoretically lay 7.95 cm (3.1 in.) above the lower edge of the bottom so that the following section moduli were calculated

(a) for the bottom $W_B = \frac{1700}{7.95} = 214$ cm^3 (13 $in.^3$)

(b) for the deck $W_D = \frac{1700}{7.55} = 225$ cm^3 (13.7 $in.^3$)

(c) for the upper edge of the web $W_S = \frac{1700}{10.55} = 161$ cm^3 (9.8 $in.^3$)

This gives a tensile stress for the deck of

$$k_z = \frac{5{,}1 \cdot 10^3 \cdot 40}{225} = 885\ \mathrm{kg/cm^2}\,.$$

In the bottom a compressive stress

$$k_p = \frac{5.1 \cdot 40 \cdot 10^3}{214} = 950 \text{ kg/cm}^2$$

was to be expected.

The compressive stress lies above the buckling limit for a 3 mm (0.12 in.) thick plate. However, in a plate this limit can undoubtedly be exceeded without permanent deformation according to the results of Hoffman published in the INA. On the other hand we must content ourselves with the fact that the deflections and also the stresses in the deck become higher than shown by calculation. For the rest we will not go further into the stresses in the web and bottom here since this would exceed the scope of this paper.

In all experiments sources of error exist and we must from the beginning take into account their order of magnitude. In the experimental body there doubtless existed a defect in the non-uniform quality of the material. The deck, due to the

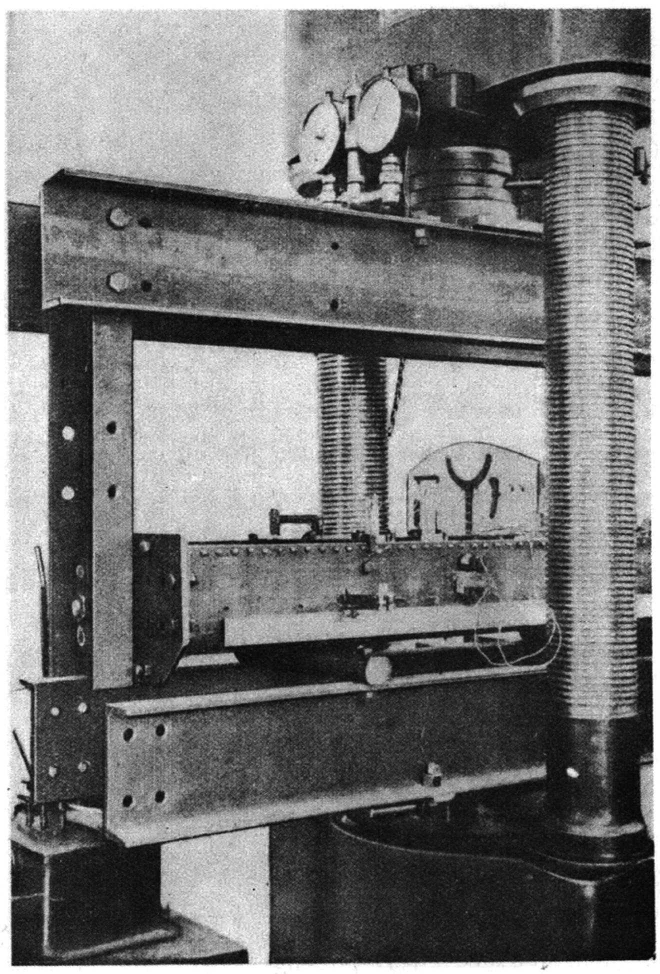

Abb. 15. Versuchsanordnung.

longitudinal stresses, was buckled upward and had certainly received other small bumps while riveting which evidenced themselves in a particularly undesirable manner. It was not to be expected, therefore, that the relatively small transverse strains could be measured. On the other hand, however, the longitudinal strains were not seriously influenced by this defect. A particular source of error lay in the statically indeterminateness of the system whose structure may be seen in Fig. 15. The testing machine used was too powerful for the experiment, so that it did not exert pressure uniformly. Hence, it became evident that the girder was not loaded symmetrically. However, the stress distribution was not materially altered thereby.

MEASURING INSTRUMENTS

For the measurement of the deflections simple lever instruments were constructed as shown in Fig. 16a–16b. They had multiplications of 30 and 20 respectively.

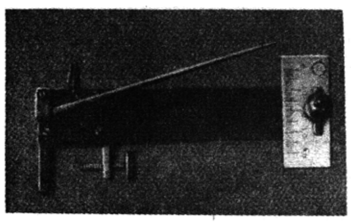

Abb. 16 a.

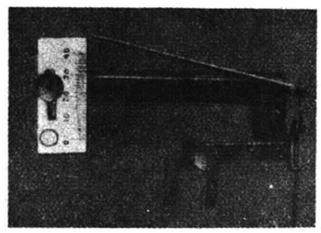

Abb. 16 b.

The Okhuizen (Huggenberger) extensometer was used for the measurement of strains. It was especially well adapted for this because it is extremely simple to handle and can stand comparatively rough treatment. It is based on the principle of the double lever. The lever and pointer each have a multiplication of from 30 to 35 so that a total multiplication of about 1000 is obtained (see Fig. 17). The exact multiplication factor for each instrument is given in the tables.

For determining strains the body was loaded and unloaded twice so that it was possible to get 5 readings, the differences between which gave 4 measurements, (see table 32).

DETERMINATION OF STRESSES

As is evident from previous derivations it is not sufficient in the case of a plane stress condition simply to measure the strains in one direction, since then the essential relationship between σ_x, σ_y and τ will not be taken into consideration. We know, however, that for a uniform state of stress a circle is deformed

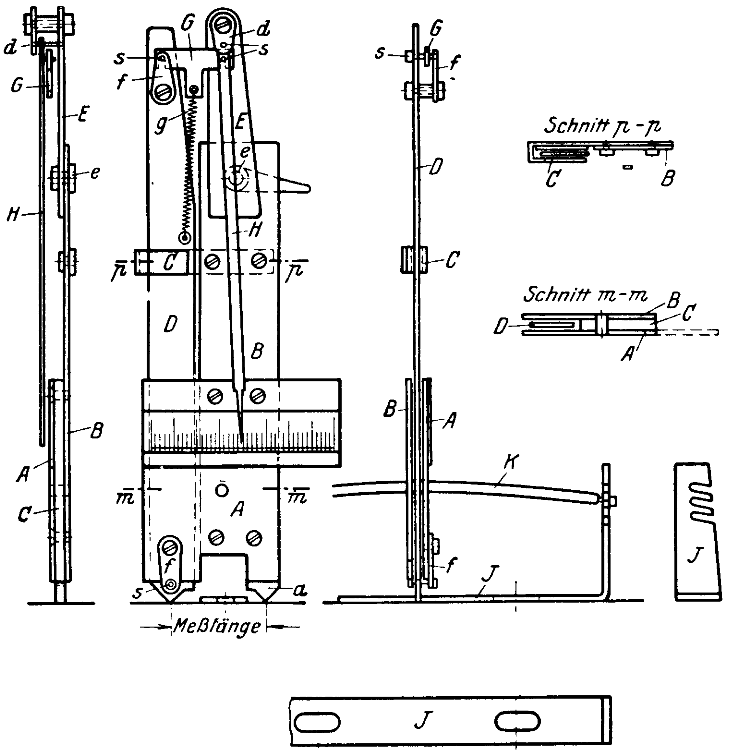

Abb. 17. Okhuizenscher Dehnungsmesser

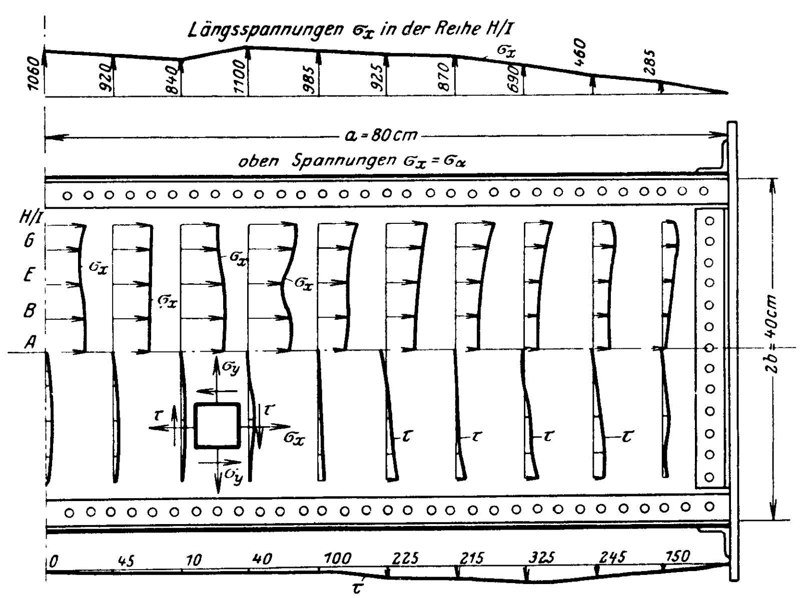

Abb. 18. Normal- und Schubspannungen gemessen.

into an ellipse. When no transverse stress exists the strain ellipse is flattened out due to the transverse contraction.

I am not going to stop here to develop the relation between stress and strain but I will refer the reader to text books on strength of materials, the dissertation by von Wyss, and the brief derivation in the appendix (see bibliography). I should like to mention only that the sum of each pair of strains lying perpendicular on each other yields a constant value in a uniform stress condition. In view of the small base length of the strain gages, 2 cm, an approximately uniform stress condition exists in the range of measurements so that we can use the foregoing fact for the correction of the errors of measurement.

For the rest, all formulas are given in the tables so that the calculation of stresses and the direction of principal stresses is clearly to be seen.

In the tables

Σ_α is the sum of two measured strains lying perpendicularly upon each other.

D_α is the difference of two measured strains lying perpendicularly upon each other.

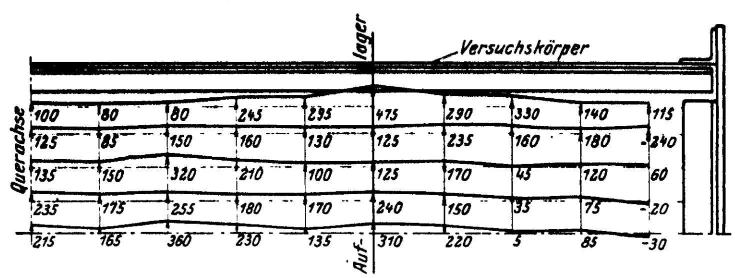

Abb. 19. Querspannungen gemessen.

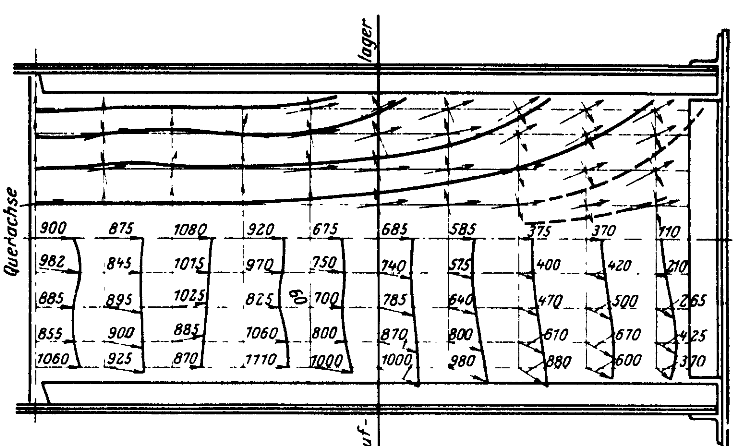

Abb. 20. Spannungstrajektorien nach den Versuchen.

In our girder, four strains at each of 95 points in the flange were measured, namely,

λ_α	parallel to the web
$\lambda_\alpha + \frac{\pi}{2}$	perpendicular to the web
$\lambda_\alpha + \frac{\pi}{4}$	45° to the web
$\lambda_\alpha + \frac{3\pi}{4}$	135° to the web

200 measurements taken at 50 points are evaluated in the tables.

The reduction of the strains and the calculation of the stresses are undertaken in 20 tables, examples of which are shown in tables 32 and 33.

The results are plotted in Figures 18 to 20. It is evident that the value of longitudinal stress σ_x and of shear stress τ are in good agreement with theory.

Also the direction of principal stresses calculated point by point, show the same character, as is to be expected according to theory. Only the transverse stress σ_y does not agree, due to the bulging in the transverse direction as already mentioned.

DEFLECTIONS

In conclusion we will briefly compare the measured deflection with the theoretical. The measuring bar was fixed at two points each 2 cm from the supports at mid height of the body so that the distance between the fixed points was 76 cm. This arrangement gave the deflections at the measuring points as found in Table 31.

The table shows that the deflections in the mid portion exceed those calculated by from 30 to 50%. This is to be attributed to the fact that in this region the buckling limit on the compression side had already been greatly exceeded. At the ends, however, the bottom showed a considerable resisting capacity although stress above the buckling limit had certainly been reached. The maximum shear deflection amounts to about 25% of the calculated maximum bending deflection.

In ships likewise a deflection of the order of magnitude of 15 per cent due to shear is reached so the increase of the deflection can be accounted for in part by the shearing forces, in part by the falling off of stress, according to the present theory, and in part by the buckling of the stiffened walls.

IV. APPENDIX.

Development of Formulas

1. Differential Equation of the Stress Function.

The stress condition must satisfy the following equations in order to fulfill the conditions of equilibrium and the compatibility conditions. Equilibrium is expressed thru the equations

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau}{\partial y} = 0, \qquad (1) \qquad\qquad \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau}{\partial x} = 0. \qquad (2)$$

The compatibility of the deformations is obtained from the equation

$$E\,\varepsilon_x = \sigma_x - \frac{\sigma_y}{m} = E\frac{\partial \xi}{\partial x}, \qquad (3)$$

$$E\,\varepsilon_y = \sigma_y - \frac{\sigma_x}{m} = E\frac{\partial \eta}{\partial y}, \qquad (4)$$

$$\gamma = \frac{\partial \xi}{\partial y} + \frac{\partial \eta}{\partial x} = \frac{\tau}{G}. \qquad (5)$$

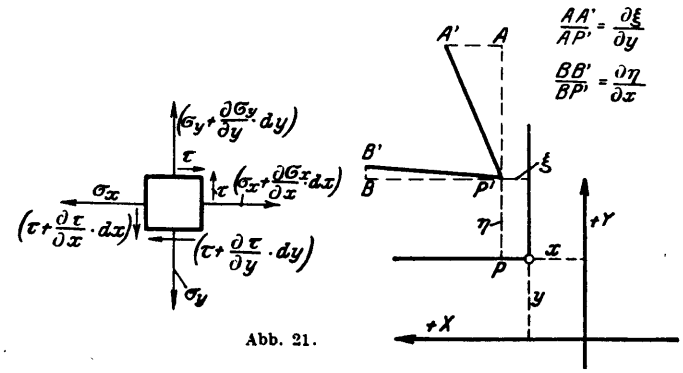

Abb. 21.

From (5) it follows by a double partial differentiation

$$\frac{1}{G}\cdot\frac{\partial^2 \tau}{\partial x\,\partial y} = \frac{\partial^3 \xi}{\partial x\,\partial y^2} + \frac{\partial^3 \eta}{\partial x^2\,\partial y}\,; \qquad (5)$$

In the same way, (3) and (4) yield

$$E\frac{\partial^3 \xi}{\partial x\,\partial y^2} = \frac{\partial^2 \sigma_x}{\partial y^2} - \frac{1}{m}\,\frac{\partial^2 \sigma_y}{\partial y^2}$$

and

$$E\frac{\partial^3 \eta}{\partial x^2\,\partial y} = \frac{\partial^2 \sigma_y}{\partial x^2} - \frac{1}{m}\,\frac{\partial^2 \sigma_x}{\partial y^2}.$$

Substitution in (5) gives

$$\frac{E}{G}\,\frac{\partial^2 \tau}{\partial x\,\partial y} = \frac{\partial^2 \sigma_x}{\partial y^2} + \frac{\partial^2 \sigma_y}{\partial x^2} - \frac{1}{m}\left(\frac{\partial^2 \sigma_x}{\partial x^2} + \frac{\partial^2 \sigma_y}{\partial y^2}\right), \qquad (6)$$

$$\frac{E}{G}\,\frac{\partial^2 \tau}{\partial x\,\partial y} = \frac{2}{m}(m+1)\frac{\partial^2 \tau}{\partial x\,\partial y}.$$

If, according to Airy we set

$$\sigma_x = \frac{\partial^2 F}{\partial y^2}; \quad (7) \qquad \sigma_y = \frac{\partial^2 F}{\partial x^2} \quad (8) \qquad \text{und} \qquad \tau = \frac{-\partial^2 F}{\partial x\;\partial y}, \quad (9)$$

then we will get from (6)

$$\frac{\partial^4 F}{\partial y^4}+\frac{\partial^4 F}{\partial x^4}-\frac{1}{m}\cdot 2\frac{\partial^4 F}{\partial x^2 \partial y^2}=-\frac{2}{m}\frac{(m+1)\,\partial^4 F}{\partial x^2 \partial y^2} \qquad (10)$$

or

$$\frac{\partial^4 F}{\partial y^4}+\frac{\partial^4 F}{\partial x^4}+2\frac{\partial^4 F}{\partial x^2 \partial y^2}=0. \qquad (11)$$

(2) Influence of the Flange and the Effective Width on the Stresses.

According to Fig. 22, let

F_1 = the area of the web

F_2 = the effective area of the flange

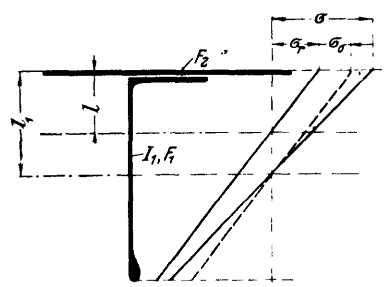

Abb 22

where

$$F_2=\frac{d}{\sigma_r}\int_0^b \sigma_x\,dy$$

when d = the thickness of the flange plate.

Further, let

l_1 = distance of the center of gravity of the web from the flange

l = the distance of the center of gravity of the entire girder from the flange,

J_1 = moment of inertia of the web

J = moment of inertia of the entire girder.

Then, from the assumption of linear stress distribution in the girder,

$$J=F_1(l_1-l)^2+J_1+F_2 l^2. \qquad (1)$$

(Translator's Note: J_2 is omitted, but compensated for by taking l_1 to the outside of the flange. D.W.)

$$\frac{l_1-l}{l_1}=\frac{F_2}{F_2+F_1} \qquad \text{oder} \qquad l=\frac{l_1 F_1}{F_1+F_2}. \qquad (2)$$

(Translator's Note: These two equations come from the conditions for finding center of gravity of the section, first about the flange and then about the gravity axis of the stiffener. D.W.)

$$J=\frac{F_1 F_2^2 l_1^2}{(F_1+F_2)^2}+J_1+F_2\left(\frac{l_1 F_1}{F_1+F_2}\right)^2; \qquad (3)$$

$$J=\frac{F_1+F_2}{(F_2+F_1)^2}\cdot F_1\cdot F_2\cdot l_1^2+J_1,$$

therefore

$$J=\frac{F_1 F_2 l_1^2}{F_2+F_1}+J_1,$$

and

$$W_0 = \frac{J_1}{l} + \frac{l_1^2}{l}\,\frac{F_1 F_2}{F_1 + F_2}, \tag{4}$$

(Translator's Note: W_0 corresponds to section modulus $\frac{I}{h}$. D.W.)

$$W_0 = \frac{J_1}{l_1}\,\frac{F_2 + F_1}{F_1} + \frac{l_1^2 F_1 F_2 (F_1 + F_2)}{l_1 F_1 (F_1 + F_2)},$$

$$W_0 = \frac{J_1}{l_1}\left(1 + \frac{F_2}{F_1}\right) + l_1 F_2 .$$

From this it follows that the moment absorbed by the entire girder is

$$M = W_0 \cdot \sigma_r . \tag{5}$$

$$M = \sigma_r\left(\frac{J_1}{l_1} + \frac{J_1}{l_1}\cdot\frac{F_2}{F_1} + F_2 l_1\right) \tag{6}$$

or

$$M = \frac{J_1}{l_1}\,\sigma_r\left(1 + \frac{F_2}{F_1} + \frac{F_2 l_1^2}{J_1}\right)$$

also

$$\sigma_r = \frac{M}{J_1}\,l_1\,\frac{1}{1 + \frac{F_2}{F_1} + \frac{F_2 l_1^2}{J_1}} . \tag{7}$$

If here $F_2 = 0$, then $\sigma_r = \frac{Ml}{J_1} = \frac{M}{W_1}$ the ordinary formula for the stress at the outermost fiber. We now calculate $F_2 = d\,\frac{\sigma\, x\, dy}{\sigma_r}$ under the assumption that the stress distributes itself over the girder as the bending moment. We then set the prevailing effective width in Eq. (7) and obtain the stress distribution very accurately. Calculation shows that a repetition is not necessary, since for a small stress variation, the effective width barely changes.

Development of the Formulas for Calculation of the Stresses from the Measured Strains.

Let λ_0 be the strain in the principal axis. Then the strain in the direction α is, according to the ellipse construction, (Fig. 23),

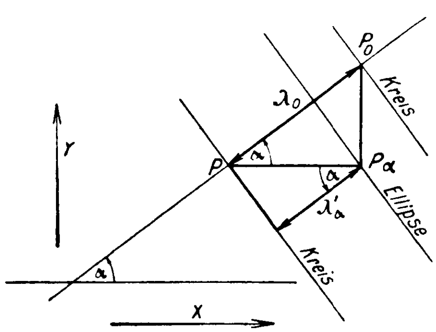

Abb. 23.

$$\lambda'_\alpha = (P P_\alpha)\cos\alpha = (P P_0 \cos\alpha)\cos\alpha .$$

$$\lambda'_\alpha = \lambda_0 \cos^2\alpha$$

for an extension of the principal axis in the y-direction by $\lambda_{\frac{\pi}{2}}$ we get

$$\lambda''_\alpha = \lambda_{\frac{\pi}{2}} \cdot \cos^2\left(\frac{\pi}{2} - \alpha\right) = \lambda_{\frac{\pi}{2}} \cdot \sin^2\alpha .$$

Therefore

$$\lambda_\alpha = \lambda'_\alpha + \lambda''_\alpha = \lambda_0 \cos^2\alpha + \lambda_{\frac{\pi}{2}} \cdot \sin^2\alpha . \tag{1}$$

Similarly we find

$$\lambda_{\alpha+\frac{\pi}{2}} = \lambda_0 \sin^2\alpha + \lambda_{\frac{\pi}{2}} \cos^2\alpha .$$

Thru addition it follows

$$\lambda_\alpha + \lambda_{\alpha+\frac{\pi}{2}} = \lambda_0 + \lambda_{\frac{\pi}{2}} = \text{const.} \tag{3}$$

That is, the sum of two strains, whose directions are perpendicular to each other, is constant if the stress condition is uniform. Thru subtraction it follows

$$\lambda_\alpha - \lambda_{\alpha+\frac{\pi}{2}} = (\lambda_0 - \lambda_{\frac{\pi}{2}}) \cos 2\alpha. \tag{4}$$

From Eq. (4) it follows

$$\lambda_{\alpha+\frac{\pi}{4}} - \lambda_{\alpha+\frac{3\pi}{4}} = (\lambda_0 - \lambda_{\frac{\pi}{2}}) \cos 2\left(\alpha + \frac{\pi}{4}\right) = -(\lambda_0 - \lambda_{\frac{\pi}{2}}) \sin 2\alpha. \tag{5}$$

therefore

$$\operatorname{tg} 2\alpha = -\frac{\lambda_{\alpha+\frac{\pi}{4}} - \lambda_{\alpha+\frac{3\pi}{4}}}{\lambda_\alpha - \lambda_{\alpha+\frac{\pi}{2}}}. \tag{6}$$

If we set

$$\lambda_\alpha + \lambda_{\alpha+\frac{\pi}{2}} = \sum(\alpha),$$

$$\lambda_\alpha - \lambda_{\alpha+\frac{\pi}{2}} = D(\alpha),$$

$$\lambda_{\alpha+\frac{\pi}{4}} - \lambda_{\alpha+\frac{3\pi}{4}} = D\left(\alpha + \frac{\pi}{4}\right),$$

then it follows that

$$\operatorname{tg}(2\alpha) = -\frac{D\left(\alpha+\frac{\pi}{4}\right)}{D(\alpha)} \tag{6}$$

and

$$\alpha = -\frac{1}{2} \operatorname{arc\,tg} \frac{D\left(\alpha+\frac{\pi}{4}\right)}{D(\alpha)}. \tag{6 b}$$

We can find the maximum strains from Eq. (3) and (4) which with the newly introduced designations read:

$$\lambda_0 + \lambda_{\frac{\pi}{2}} = \sum(\alpha)$$

$$\lambda_0 - \lambda_{\frac{\pi}{2}} = \frac{D(\alpha)}{\cos 2\alpha}.$$

However,

$$\frac{1}{\cos 2\alpha} = \sqrt{1+\operatorname{tg}^2 2\alpha} = \frac{1}{D(\alpha)}\sqrt{D^2(\alpha)+D^2\left(\alpha+\frac{\pi}{4}\right)}, \tag{7}$$

$$\lambda_0 = \frac{1}{2}\left(\sum(\alpha) + \sqrt{D^2(\alpha)+D^2\left(\alpha+\frac{\pi}{4}\right)}\right), \tag{8}$$

$$\lambda_{\frac{\pi}{2}} = \frac{1}{2}\left(\sum(\alpha) - \sqrt{D^2(\alpha)+D^2\left(\alpha+\frac{\pi}{2}\right)}\right). \tag{9}$$

Determination of the Stresses

We know that the derived strain is

$$\varepsilon_0 = \frac{1}{E}\left(\sigma_0 - \frac{\sigma_{\frac{\pi}{2}}}{m}\right), \tag{10}$$

$$\varepsilon_{\frac{\pi}{2}} = \frac{1}{E}\left(\sigma_{\frac{\pi}{2}} - \frac{\sigma_0}{m}\right). \tag{11}$$

Therefore

$$\frac{\sigma_0}{E}\left(1-\frac{1}{m^2}\right) = \varepsilon_0 + \frac{1}{m}\varepsilon_{\frac{\pi}{2}},$$

$$\sigma_0 = \frac{E\,m^2}{m^2-1}\cdot\frac{1}{2}\left[\frac{(1+m)\sum(\alpha)}{m} + \frac{\overset{(m-1)}{\cancel{(1-m)}}}{m}\sqrt{D^2(\alpha)+D^2\left(\alpha+\frac{\pi}{4}\right)}\right], \tag{12}$$

$$\sigma_0 = \frac{E\,m}{2}\left[\frac{\sum(\alpha)}{m-1} + \frac{1}{m+1}\sqrt{D^2(\alpha)+D^2\left(\alpha+\frac{\pi}{4}\right)}\right].$$

In the same way,

$$\sigma_{\frac{\pi}{2}} = \frac{m\,E}{2}\cdot\left[\frac{\sum(\alpha)}{m-1} - \frac{1}{m+1}\sqrt{D^2(\alpha)+D^2\left(\alpha+\frac{\pi}{4}\right)}\right], \tag{13}$$

It follows, however, from the conditions of equilibrium

$$\sigma_x = \sigma_0\cos^2\alpha + \sigma_{\frac{\pi}{2}}\sin^2\alpha \tag{14}$$

or from

$$\cos^2\alpha = \tfrac{1}{2}(1+\cos 2\alpha), \qquad \sin^2\alpha = \tfrac{1}{2}(1-\cos 2\alpha),$$

$$\sigma_\alpha = \tfrac{1}{2}\left(\sigma_0 + \sigma_{\frac{\pi}{2}}\right) + \tfrac{1}{2}\left(\sigma_0 - \sigma_{\frac{\pi}{2}}\right)\cos 2\alpha,$$

and

$$\tau_\alpha = -\tfrac{1}{2}\left(\sigma_0 - \sigma_{\frac{\pi}{2}}\right)\sin 2\alpha, \tag{15}$$

where the shear stress is positive if it turns clockwise. In the direction of the principal axis $\tau = 0$. Then

$$\sigma_\alpha = \frac{mE}{2}\left[\frac{\sum(\alpha)}{m-1} + \frac{\sqrt{D^2(\alpha)+D^2\left(\alpha+\frac{\pi}{4}\right)}}{m+.1}\cos 2\alpha\right], \tag{16}$$

$$\tau_\alpha = -\frac{mE}{2}\cdot\frac{\sqrt{D^2(\alpha)+D^2\left(\alpha+\frac{\pi}{4}\right)}}{m+1}\cdot\sin 2\alpha. \tag{17}$$

We have, however,

$$\cos 2\alpha = \frac{D(\alpha)}{\sqrt{D^2(\alpha)+D^2\left(\alpha+\frac{\pi}{4}\right)}},$$

$$\sin 2\alpha = \frac{-D\left(\alpha+\frac{\pi}{4}\right)}{\sqrt{D^2(\alpha)+D^2\left(\alpha+\frac{\pi}{2}\right)}}.$$

Therefore

$$\sigma_\alpha = \frac{mE}{2}\left(\frac{\sum(\alpha)}{m-1} + \frac{D(\alpha)}{m+1}\right), \tag{18}$$

$$\sigma_{\alpha+\frac{\pi}{2}} = \frac{mE}{2}\left[\frac{\sum(\alpha)}{m-1} - \frac{D(\alpha)}{m+1}\right], \tag{19}$$

$$\tau_\alpha = \frac{mE}{2}\cdot\frac{D\left(\alpha+\frac{\pi}{4}\right)}{m+1}, \tag{20}$$

$$\tau_{\alpha+\frac{\pi}{2}} = -\frac{mE}{2}\,\frac{D\left(\alpha+\frac{\pi}{4}\right)}{m+1}. \tag{21}$$

The maximum shear stress follows from Eq. (17) for

$$\sin 2\alpha = \pm 1;$$

$$\tau_{\max} = \pm\frac{mE}{2}\,\frac{\sqrt{D^2(\alpha)+D^2\left(\alpha+\frac{\pi}{4}\right)}}{m+1}. \tag{22}$$

The sign of the radical is the same as the sign of $D(\alpha)$, since when α lies between $\pm 45^\circ$, the cosine of 2α is positive.

$$\frac{D(\alpha)}{\cos 2\alpha} = \sqrt{D^2(\alpha)+D_2\left(\alpha+\frac{\pi}{4}\right)}.$$

Further we have

$$\tau_\alpha = \frac{mE}{2(m+1)}\,D\left(\alpha+\frac{\pi}{4}\right) = G\,D\left(\alpha+\frac{\pi}{4}\right) = G\gamma_\alpha, \tag{23}$$

$$\gamma_\alpha = D\left(\alpha+\frac{\pi}{4}\right) = \varepsilon_{\alpha+\frac{\pi}{4}} - \varepsilon_{\alpha+\frac{3\pi}{4}}.$$

Now with our strain gages, the displacement λ is obtained on a 2 cm base length. Our formulas apply only for an ε over a 1 cm. base length. Therefore, the stresses on the right side must be divided by 2 if we replace ε with α

For m = 10/3 it follows that

$$\left(\frac{mE}{2(m+1)}\right) = \frac{10}{26}\cdot 2150 = 83{,}0 = A\,, \tag{24}$$

$$\left(\frac{mE}{2(m-1)}\right) = \frac{10}{14}\cdot 2150 = 153{,}6 = B. \tag{25}$$

Therefore

$$\sigma_0 = 76{,}8\sum(\alpha) + 41{,}5\sqrt{}\,, \tag{26}$$

$$\sigma_{\frac{\pi}{2}} = 76{,}8\sum(\alpha) - 41{,}5\sqrt{}\,, \tag{27}$$

$$\sigma_\alpha = 76{,}8\sum(\alpha) + 41{,}5\,D(\alpha), \tag{28}$$

$$\sigma_{\left(\alpha+\frac{\pi}{2}\right)} = 76{,}8\sum(\alpha) - 41{,}5\,D(\alpha), \tag{29}$$

$$\tau_\alpha = \frac{83}{2}\,D(\alpha) = \tau_{\alpha+\frac{\pi}{4}}\,, \tag{30}$$

$$\tau_{\max} = \pm\frac{83{,}0}{2}\sqrt{D^2(\alpha) + D^2\left(\alpha+\frac{\pi}{4}\right)} = 41{,}5\sqrt{}\,, \tag{31}$$

(Note: For E = 29 x 10^6 and strain on 1 in. base,

$$\frac{mE}{2(m+1)} = \frac{10}{26}\cdot 29 \times 10^6 = 11.16 \times 10^6$$

$$\frac{mE}{2(m-1)} = \frac{10}{14}\cdot 29 \times 10^6 = 20.7 \times 10^6$$

$$\sigma_0 = 20.7 \times 10^6 \sum(\alpha) + 11.16 \times 10^6\sqrt{}$$

$$\sigma_{\frac{\pi}{2}} = 20.7 \times 10^6 \sum(\alpha) - 11.16 \times 10^6\sqrt{}$$

$$\sigma_\alpha = 20.7 \times 10^6 \sum(\alpha) + 11.16 \times 10^6\, D\alpha$$

$$\sigma_{\left(\alpha+\frac{\pi}{2}\right)} = 20.7 \times 10^6 \sum(\alpha) - 11.16 \times 10^6\, D\alpha$$

$$\tau_\alpha = 11.16 \times 10^6\; D\left(\alpha+\frac{\pi}{4}\right) = \tau_{\alpha+\frac{\pi}{4}}$$

$$\tau_{max} = \pm\, 11.16 \times 10^6\sqrt{D^2(\alpha) + D^2\left(\alpha+\frac{\pi}{4}\right)}$$

$$= 11.16 \times 10^6\sqrt{}$$

where

$$\sqrt{} = \sqrt{D^2(\alpha) + D^2\left(\alpha+\frac{\pi}{4}\right)},$$

$$D(\alpha) = \lambda_\alpha - \lambda_{\left(\alpha+\frac{\pi}{2}\right)},$$

$$D_{\left(\alpha+\frac{\pi}{4}\right)} = \lambda_{\left(\alpha+\frac{\pi}{4}\right)} - \lambda_{\left(\alpha+\frac{3\pi}{4}\right)},$$

$$\sum(\alpha) = \lambda_\alpha + \lambda_{\left(\alpha+\frac{\pi}{2}\right)}.$$

3. Development of Fourier Series

Let f(x) be a function that is to be replaced by means of a Fourier Series. The coefficients of this Fourier series of the form

$$F(x) = A_0 + \sum A_n \cdot \cos kx + \sum B_n \sin kx$$

are calculated with the definite integrals

$$A_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)\,dx,$$
$$A_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos kx\,dx,$$
$$B_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin kx\,dx,$$

where f(x) signifies the function to be expressed and 4a the period (usually 2π). Now let

$$\begin{aligned}
\varphi(x) &= f(x) + f(2\pi - x) = f(x) + f(-x),\\
\psi(x) &= f(x) - f(-x),\\
\varphi_1(x) &= \varphi(x) + \varphi(\pi - x),\\
\varphi_2(x) &= \varphi(x) - \varphi(\pi - x),\\
\psi_1(x) &= \psi(x) + \psi(\pi - x),\\
\psi_2(x) &= \psi(x) - \psi(\pi - x).
\end{aligned}$$

Let us now set in the particular case

A.

$$f(x) = f(\pi - x) \qquad \text{und} \qquad f(x) = -f(2\pi - x),$$

so that

$$\varphi(x) = 0; \qquad \psi(x) = 2f(x); \qquad \varphi_1(x) = 0,$$
$$\varphi_2(x) = 0; \qquad \psi_1(x) = 4f(x); \qquad \psi_2(x) = 0.$$

therefore:

$$A_0 = A_n = 0;$$

further when K is even:

$$B_n = 0,$$

when K is odd:

$$B_n = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} 4f(x)\cdot\sin kx\,dx.$$

B. If we set

$$f(x) = f(2\pi - x) = f(-x),$$

and

$$f(x) = -f(\pi - x),$$

then

$$\varphi(x) = 2f(x), \qquad \psi(x) = 0,$$
$$\varphi_1(x) = 0, \qquad \psi_1(x) = 0,$$
$$\varphi_2(x) = 4f(x), \qquad \psi_2(x) = 0;$$

that is when k is even,

$$A_0 = A_n = B_n = 0.$$

when k is odd then,

$$A_0 = B_n = 0;$$

$$A_n = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} f(x)\cos kx \; dx.$$

4. Development of the Fourier Series for Various Cases of Loading and Development of Formulas for the Theory of Thin Plates.

(a) A Fourier series is to be presented for a function consisting of straight lines, so that the straight lines extend from x = o, y = h, to the point x = 2b, y = -h, and from this point to the point x = 4b, y = +h. On the negative side of the x-axis the function shall be repeated mirror image like. (Fig. 24). We, therefore, have the postulate that

1. $f(x) = f(2\pi - x)$,

2. $f(x) = -f(\pi - x)$.

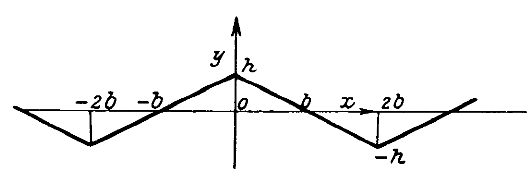

Abb. 24.

From this it follows that

$$f(x) = h - \left(\frac{h}{a}\right)x.$$

the period is $4a$,
therefore

$$\cos kx = \cos\frac{m\pi}{2a}\cdot x.$$

Therefore

$$A_0 = B_n = 0 \text{ und } m = 1,\ 3,\ 5$$

$$A_n = \frac{4}{2a}\int_0^a\left(h - \frac{h}{a}x\right)\cos\frac{m\pi}{2a}x\,dx = \frac{2}{a}\cdot h\left[\frac{\sin\frac{m\pi}{2a}x}{\frac{m\pi}{2a}}\right]_0^a - \frac{2h}{a^2}\left[\frac{x\sin\frac{m\pi}{2a}x}{\frac{m\pi}{2a}}\right]_0^a$$

$$+\frac{2h}{a^2} - \left[\frac{\cos\frac{m\pi}{2a}x}{\left(\frac{m\pi}{2a}\right)^2}\right]_0^a = \frac{2h}{a}\cdot\frac{2a}{m\pi} - \frac{2h}{a}\cdot\frac{2a}{m\pi} - \frac{2h}{a^2}\cdot\frac{4a^2}{m^2\pi^2}(-1),$$

$$A_n = \frac{8h}{\pi^2}\cdot\frac{1}{m^2}.$$

The series is

$$f(x)=\frac{8h}{\pi^2}\sum\frac{1}{m^2}\cdot\cos\frac{m\pi}{2a}\cdot x.$$

This can be differentiated and therefore

$$f'(x)=\frac{8h}{\pi^2}\cdot\sum\left(\frac{m\pi}{2a}\right)\cdot\frac{1}{m^2}\sin\frac{m\pi}{2a}x.$$

(b) A function is to be represented whose straight lines take the same course as in the foregoing problem but whose apexes are cut off by parallels to the x axis (Fig. 25). Once more the assumption is valid that

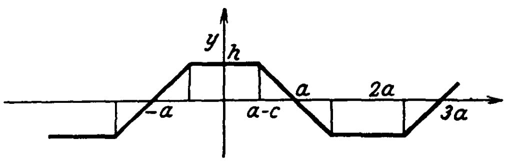

Abb. 25.

$$f(x)=f(2\pi-x)\quad\text{und}\quad f(x)=-f(\pi-x);$$

for

$$A_0=B_n=0$$

and for odd values of n

$$A_n=\frac{4}{2a}\int_0^a f(x)\cdot\cos\frac{m\cdot\pi}{2a}x\,dx$$

$$=\frac{2}{a}\int_0^{a-c} f_1(x)\cos\frac{m\pi}{2a}x\,dx+\frac{2}{a}\int_{a-c}^{a} f_2(x)\cos\frac{m\pi}{2a}x\,dx,$$

$$f_1(x)=h;\qquad f_2(x)=\frac{ah}{c}-\frac{h}{c}x=\frac{h}{c}(a-x).$$

Therefore

$$A_n=\left(\frac{2}{a}\right)h\cdot\left[\frac{\sin\frac{m\pi}{2a}x}{\frac{m\pi}{2a}}\right]_a^{a-c}+\left(\frac{2}{a}\right)\frac{h}{c}\cdot a\left[\frac{\sin\frac{m\pi}{2a}x}{\frac{m\pi}{2a}}\right]_{a-c}^{a}-\left(\frac{2}{a}\right)\frac{h}{c}\cdot\left[x\frac{\sin\frac{m\pi}{2a}x}{\frac{m\pi}{2a}}\right]_{a-c}^{a}$$

$$+\left(\frac{2}{a}\right)\frac{h}{c}\left[\frac{-\cos\frac{m\pi}{2a}x}{\left(\frac{m\pi}{2a}\right)^2}\right]_{a-c}^{a}$$

$$A_n=\frac{2h}{a}\cdot\frac{2a}{m\pi}\sin\left(\frac{m\pi}{2a}(a-c)\right)+\frac{2h}{c}\cdot\frac{2a}{m\pi}\cdot\left(\sin\frac{m\pi}{2}-\sin\frac{m\pi}{2a}(a-c)\right)$$

$$-\frac{2}{a}\cdot\frac{h}{c}\frac{2a}{m\pi}\left[a\sin\frac{m\pi}{2}-(a-c)\sin\frac{m\pi}{2a}(a-c)\right]$$

$$-\frac{2h}{ac}\cdot\frac{4a^2}{m^2\pi^2}\left[\cos\frac{m\pi}{2}-\cos\frac{m\pi}{2a}(a-c)\right]\qquad m=1,\ 3,\ 5.$$

We have

$$\cos\frac{m\pi}{2a}(a-c)=\sin\frac{m\pi}{2}\cdot\sin\frac{m\pi}{2a}c$$

and

$$\sin\frac{m\pi}{2a}(a-c)=\sin\frac{m\pi}{2}\cos\frac{m\pi}{2a}c,$$

$$\cos\frac{m\pi}{2}=0 \qquad \text{und} \qquad \sin\frac{m\pi}{2}=+1; \quad -1; \quad +1.$$

Therefore

$$A_n=\frac{4h}{m\pi}\cdot\cos\frac{m\pi}{2a}\cdot c+\frac{4h}{m\pi}\cdot\frac{a}{c}-\frac{4h}{m\pi}\frac{a}{c}\cos\frac{m\pi}{2a}\cdot c$$
$$-\frac{4h}{m\pi}\cdot\frac{a}{c}+\frac{4h}{m\pi}\left(\frac{a}{c}-1\right)\cos\frac{m\pi}{2a}\cdot c+\frac{8h}{m^2\pi^2}\cdot\frac{a}{c}\sin\frac{m\pi}{2}\cdot\sin\frac{m\pi}{2a}.$$

Therefore

$$f(x)=\frac{8h}{\pi^2}\cdot\frac{a}{c}\sum-\frac{(-1)^{\frac{(m+1)}{2}}\sin\frac{m\pi}{2a}\cdot c}{m^2}\cos\frac{m\pi}{2a}x.$$

This series can also be differentiated.

(c) A function is to be represented as a Fourier series so that a parabolic curve will be formed convex upward from -a to +a and concave upward from a to 2a, etc. (Fig. 26).

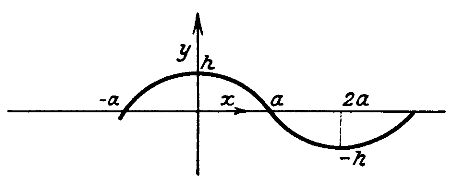

Abb. 26.

Again we have:

$$f(x)=f(-x), \qquad \text{und} \qquad f(x)=-f(\pi-x)$$

and

$$f(x)=h-\frac{h}{a^2}\cdot x^2$$

The function to be represented lies in the interval -b to +b, wherefore

$$A_0=B_n=0,$$

$$A_n = \frac{4}{2a}\int\limits_0^a f(x)\cos\frac{m\pi}{2a}\cdot x\,dx = \frac{2}{a}\cdot h\cdot\frac{2a}{m\cdot\pi}\left[\sin\frac{m\pi}{2a}\cdot x\right]_0^a - \frac{2}{a}\cdot\frac{h}{a^2}\cdot\frac{2a}{m\cdot\pi}\cdot x^2\left[\sin\frac{m\pi}{2a}\cdot x\right]_0^a$$

$$+\frac{2}{a}\cdot\frac{h}{a^2}\cdot\frac{2a}{m\cdot\pi}\int\limits_0^a \sin\frac{m\pi}{2a}x\cdot 2x\,dx = \frac{4h}{m\pi} - \frac{4h}{m\pi} - \frac{4h}{a^2}\cdot\frac{1}{m\cdot\pi}\left[\frac{2x\cos\frac{m\pi}{2a}x}{\frac{m\pi}{2a}}\right]_0^a$$

$$+\frac{4h}{a^2}\cdot\frac{1}{m\cdot\pi}\cdot 2\left[\frac{\sin\frac{m\pi}{2a}x}{\left(\frac{m\pi}{2a}\right)^2}\right]_0^a,$$

$$A_n = \frac{4}{a^2}\cdot\frac{2h}{m\cdot\pi}\cdot\frac{4a^2}{m^2\pi^2} = \frac{32h}{\pi^3 m^3}\cdot\sin\left(\frac{m\pi}{2}\right);$$

Therefore

$$f(x) = \frac{32h}{\pi^3}\pm\sum\frac{1}{m^3}\cdot\cos\frac{m\pi}{2a}\cdot x.$$

Differentiating

$$f'(x) = \frac{32h}{\pi^3}\cdot\sum\pm\frac{m\pi}{2a}\cdot\frac{1}{m^3}\cdot\sin\frac{m\pi}{2}x.$$

(d) Determination of the Fourier Coefficient for a series of the form

$$\tau = -\frac{\partial^2 F}{\partial x\,\partial y} = \sigma_m\cdot\frac{b}{2}\cdot\frac{m\pi}{2a}\mathfrak{Tg}\frac{m\pi}{2a}\cdot b\left[-\frac{\mathfrak{Sin}\frac{m\pi}{2a}\cdot y}{\mathfrak{Cos}\frac{m\pi}{2a}\cdot b}+\frac{y}{b}\,\frac{\mathfrak{Cos}\frac{m\pi}{2a}y}{\mathfrak{Sin}\frac{m\pi}{2a}\cdot b}+\frac{2a}{m\cdot\pi\cdot b}\,\frac{\mathfrak{Sin}\frac{m\pi}{2a}\cdot y}{\mathfrak{Sin}\frac{m\pi}{2a}b}\right].$$

In this case we make the following assumptions:

1. The function is odd which likewise follows from the form of the equation itself.

2. The graph of the function from 0 to -b is repeated mirror image like from b to 2b; the same is true from -b to -2b. (Fig. 27).

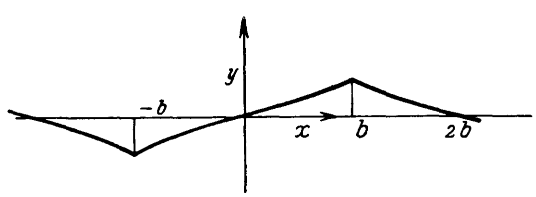

Abb. 27.

From this it follows that the period is 4b and we have the conditional equations

$$1.\ f(x) = -f(-y), \qquad 2.\ f(\pi - y) = f(y).$$

This gives

$$\psi(x) = 2f(y),$$

$$\psi_1(x) = 4f(y).$$

Therefore

$$B_k = \frac{4}{2b}\int_0^b f(y)\sin kx\,dx.$$

The K coefficients are found to be K = $\frac{n \cdot \pi}{2b}$ · y since the value of the function for y = b is to be a maximum value. This imposes the condition at the same time that at the point y = b, cos $\frac{n\pi}{2b}$ · y = 0, so that σ y for y = C is zero.
The stress function is:

$$F(x,y) = -\sigma_m \cdot \frac{a \cdot b}{m \cdot \pi} \cdot \cos\frac{m\pi}{2a} \cdot x \cdot \mathfrak{Tg}\frac{m\pi}{2a}b \cdot \left[\frac{\mathfrak{Cof}\frac{m\pi}{2a} \cdot y}{\mathfrak{Cof}\frac{m\pi}{2a} \cdot b} - \frac{y}{b}\,\frac{\mathfrak{Sin}\frac{m\pi}{2a} \cdot y}{\mathfrak{Sin}\frac{m\pi}{2a} \cdot b}\right]$$

and

$$\tau_a = \left[-\frac{\partial^2 F}{\partial x\,\partial y}\right]_{X=a} = \left[-\frac{\partial F(x)}{\partial x}\right]_{X=a} \cdot \frac{\partial F y}{\partial y} = -\sigma_m \cdot F'(y) \cdot \frac{4a}{m\pi}.$$

Our equation simplified then will read

$$B_n = \frac{4a}{bm\pi}\int_0^b \sigma_m F'(y)\sin\frac{n\pi}{2b} \cdot y\,dy$$

$$= \frac{4a}{bm\pi} \cdot \sigma_m \left([F(y)]^b \sin\frac{n\pi}{2b} \cdot y - \frac{n\pi}{2b}\int_0^b F(y)\cos\frac{n\pi}{2b}y\,dy\right).$$

Now, however, at the point x = a

$$\tau_a = \sigma_m \left\{\frac{b}{2}\,\frac{m\pi}{2a}\left[-\frac{\mathfrak{Sin}\frac{m\pi}{2a}y}{\mathfrak{Cof}\frac{m\pi}{2a}b} + \frac{y}{b}\,\frac{\mathfrak{Cof}\frac{m\pi}{2a}y}{\mathfrak{Sin}\frac{m\pi}{2a}b} + \frac{2a}{bm\pi} \cdot \frac{\mathfrak{Sin}\frac{m\pi}{2a} \cdot y}{\mathfrak{Sin}\frac{m\pi}{2a}b}\right]\mathfrak{Tg}\frac{m\pi}{2a}b\right\},$$

therefore,

$$B_n = \frac{2}{b} \cdot \sigma_m \left\{\frac{b}{2}\left[-\frac{\mathfrak{Cof}\frac{m\pi}{2a}y}{\mathfrak{Cof}\frac{m\pi}{2a}b} + \frac{y}{b}\,\frac{\mathfrak{Sin}\frac{m\pi}{2a} \cdot y}{\mathfrak{Sin}\frac{m\pi}{2a} \cdot b}\right]_0^b \mathfrak{Tg}\frac{m\pi}{2a} \cdot b\left(\sin\frac{n\pi}{2b}y\right)_0^b\right.$$

$$\left. - \sigma_m \frac{2}{b} \cdot \frac{b}{2} \cdot \frac{n\pi}{2b}\int_0^b \left(-\frac{\mathfrak{Cof}\frac{m\pi}{2a}y}{\mathfrak{Cof}\frac{m\pi}{2a}b} + \frac{y}{b}\,\frac{\mathfrak{Sin}\frac{m\pi}{2a} \cdot y}{\mathfrak{Sin}\frac{m\pi}{2a} \cdot b}\right)\mathfrak{Tg}\frac{m\pi}{2a}b\cos\frac{n\pi}{2b}y\,dy.\right.$$

The first summation will be zero, and therefore

$$B_n = -\sigma_m \mathfrak{Tg}\frac{m\pi}{2b} b \cdot \frac{n\pi}{2b} \int_0^b \left(\frac{-\mathfrak{Cos}\frac{m\pi}{2a}y}{\mathfrak{Cos}\frac{m\pi}{2a}b} + \frac{y}{b}\,\frac{\mathfrak{Sin}\frac{m\pi}{2a}\cdot y}{\mathfrak{Sin}\frac{m\pi}{2a}b} \right) \cos\frac{n\pi}{2b} y\, dy$$

$$= -\sigma_m \cdot \mathfrak{Tg}\frac{m\pi}{2a} b \cdot \frac{n\pi}{2b} \Bigg\{ \left(\frac{2a}{m\pi}\right)^2 \cdot \frac{-1}{1+\left(\frac{an}{bm}\right)^2} \left(\frac{\frac{m\pi}{2a}\mathfrak{Sin}\frac{m\pi}{2a}y \cos\frac{n\pi}{2b}y + \frac{m\pi}{2b}\mathfrak{Cos}\frac{m\pi}{2a}y \sin\frac{n\pi}{2b}y}{\mathfrak{Cos}\frac{m\pi}{2a}b} \right)_0^b$$

$$+\frac{y}{b} \cdot \frac{\int \mathfrak{Sin}\frac{m\pi}{2a}y \cos\frac{n\pi}{2b}y\, dy}{\mathfrak{Sin}\frac{m\pi}{2a}b} - \frac{1}{b}\,\frac{\iint \left(\mathfrak{Sin}\frac{m\pi}{2a}y \cos\frac{n\pi}{2b}y\, dy\right) dy}{\mathfrak{Sin}\frac{m\pi}{2a}b} \Bigg\}$$

$$= -\sigma_m \cdot \mathfrak{Tg}\frac{m\pi}{2a} b \cdot \frac{n\pi}{2b} \Bigg\{ \frac{+4a^2 \cdot \frac{n\pi}{2b}\mathfrak{Cos}\frac{m\pi}{2a}b \cdot (-1)^{\frac{1+n}{2}}}{\pi^2\left[m^2+\left(\frac{an}{b}\right)^2\right]\mathfrak{Cos}\frac{m\pi}{2a}b}$$

$$+\frac{y}{b\,\mathfrak{Sin}\frac{m\pi}{2a}b} \cdot \left(\frac{2a}{m\pi}\right)^2 \cdot \frac{1}{1+\left(\frac{an}{bm}\right)^2} \left(\frac{m\pi}{2a}\mathfrak{Cos}\frac{m\pi}{2a}y \cos\frac{n\pi}{2b}y + \frac{n\pi}{2b}\mathfrak{Sin}\frac{m\pi}{2a}y \sin\frac{n\pi}{2b}y\right)_0^b$$

$$-\frac{1}{b}\,\frac{1}{\mathfrak{Sin}\frac{m\pi}{2a}b}\left(\frac{2a}{m\pi}\right)^2 \frac{1}{1+\left(\frac{an}{bm}\right)^2} \Bigg[\frac{m\pi}{2a}\left(\frac{2a}{m\pi}\right)^2 \frac{\left(\frac{m\pi}{2a}\mathfrak{Sin}\frac{m\pi}{2a}y \cos\frac{n\pi}{2b}y + \frac{n\pi}{2b}\mathfrak{Cos}\frac{m\pi}{2a}y \sin\frac{n\pi}{2b}y\right)_0^b}{1+\left(\frac{an}{bm}\right)^2}$$

$$+\frac{n\pi}{2b}\cdot\left(\frac{2a}{m\pi}\right)^2 \frac{1}{1+\left(\frac{an}{bm}\right)^2}\left(\frac{m\pi}{2a}\mathfrak{Cos}\frac{m\pi}{2a}y \sin\frac{n\pi}{2b}y - \frac{n\pi}{2b}\mathfrak{Sin}\frac{m\pi}{2a}y \cos\frac{n\pi}{2b}y\right)\Bigg]_0^b \Bigg\}$$

$$= -\sigma_m \mathfrak{Tg}\frac{m\pi}{2a} b \left(\frac{n\pi}{2b}\right) \Bigg\{ (-1)^{\frac{n+1}{2}} \frac{4a^2}{\pi^2} \frac{\frac{n\pi}{2b}}{m^2+\left(\frac{a}{b}\right)^2 n^2} - \frac{4a^2}{\pi^2}\,\frac{\frac{n\pi}{2b}(-1)^{\frac{n+1}{2}}}{m^2+\left(\frac{a}{b}\right)^2\cdot n^2}$$

$$+\frac{1}{b}\,\frac{1}{\mathfrak{Sin}\frac{m\pi}{2a}b}\,\frac{1}{\left[1+\left(\frac{an}{bm}\right)^2\right]^2}\left(\frac{2a}{m\pi}\right)^3 \left[\underset{(-)}{\pm}\frac{n\pi}{2b}\mathfrak{Cos}\frac{m\pi}{2a}b \underset{(-)}{\pm}\frac{n\pi}{2b}\mathfrak{Cos}\frac{m\pi}{2a}\cdot b\right](-1)^{\frac{n+1}{2}} \Bigg\}$$

$$= -\sigma_m \cdot \mathfrak{Tg}\frac{m\pi}{2a}\cdot b\left(\frac{n\pi}{2b}\right)\frac{1}{b}\cdot\frac{(-1)^{\frac{1+n}{2}}}{\left[1+\left(\frac{an}{bm}\right)^2\right]^2}\left(\frac{2a}{m\pi}\right)^3 \frac{\frac{n\pi}{2b}\cdot 2}{\mathfrak{Tg}\frac{m\pi}{2a}b}$$

$$= -\sigma_m \frac{n^2\pi^2}{4b^2}\cdot\frac{2}{b}\,\frac{m'8a^3}{\pi^3}\,\frac{(-1)^{\frac{1+n}{2}}}{\left[m^2+\left(\frac{a}{b}\right)^2 n^2\right]^2},$$

$$B_n = -\sigma_m \cdot \frac{4a^3}{\pi b^3}\,\frac{(-1)^{\frac{1+n}{2}}\, n^2\cdot m}{\left[m^2+\left(\frac{a}{b}\right)^2\cdot n^2\right]^2},$$

and our Fourier series is

$$\tau_{(a)} = -\sigma_m(-1)^{\left(\frac{1+n}{2}\right)}\frac{4a^3}{\pi b^3}\left[\left(\frac{1A_1}{\left(1+\left(\frac{a}{b}\right)^2\right)^2}+\frac{3A_3}{\left(3^2+\left(\frac{a}{b}\right)^2\right)^2}+\frac{5A_5}{\left(5^2+\left(\frac{a}{b}\right)^2\right)^2}\right)\right]\sin\frac{\pi}{2b}y$$

$$-\left(\frac{1A_1\cdot 3^2}{\left(1+\left(\frac{a}{b}\right)^2 3^2\right)^2}+\frac{3A_3\cdot 3^2}{\left(3^2+\left(\frac{a}{b}\right)^2 3^2\right)^2}+\frac{5A_5\cdot 3^2}{\left(5^2+\left(\frac{a}{b}\right)^2 3^2\right)^2}\right)\sin\frac{3\pi}{2b}y+\dots$$

Since the coefficients A_1, A_3, A_5 diminish at least as the square, the coefficients of the second series will be very small even under ordinary conditions.

Very generally therefore we will have

$$-\tau_a = -\sigma_m(-1)^{\frac{n+1}{2}}\frac{4a^3}{\pi b^3}\sum\frac{m A_m\cdot n^2}{\left(m^2+\left(\frac{a}{b}\right)^2 n^2\right)^2}\sin\frac{n\pi}{2b}y\,.$$

For a large a/b we have

$$\tau_a = \sigma_m\sum\sin\frac{n\pi}{2b}y\frac{4}{\pi}\cdot\pm\frac{An\cdot m}{n^2}\frac{b}{a} = \pm\sum An\frac{4}{\pi}\frac{b}{a}\frac{m}{n^2}\sin\frac{n\pi}{2b}y,$$

for m = 1

$$\tau_a \approx \frac{4}{\pi}\cdot\frac{b}{a}\frac{1}{n^2}\sin\frac{n\pi}{2b}y\,.$$

5. Derivation of Formulas for Hyperbolic Functions

a) $$J = \int_0^b \mathfrak{Cos}\frac{n\pi}{2a}y\cos\frac{m\pi}{2b}y\,dy = \frac{2a}{n\pi}\cdot\mathfrak{Sin}\frac{n\pi}{2a}y\cdot\cos\frac{m\pi}{2b}y$$

$$+\int\frac{2a}{n\pi}\cdot\frac{m\pi}{2b}\cdot\mathfrak{Sin}\frac{n\pi}{2a}y\cdot\sin\frac{m\pi}{2b}y\cdot dy = \frac{2a}{n\pi}\cdot\mathfrak{Sin}\frac{n\pi}{2a}y\cdot\cos\frac{m\pi}{2b}y$$

$$+\left(\frac{2a}{n\pi}\right)^2\frac{m\cdot\pi}{2b}\mathfrak{Cos}\frac{n\pi}{2b}y\sin\frac{n\pi}{2a}y - \int_0^b\left(\frac{2a}{n\pi}\right)^2\cdot\left(\frac{m\pi}{2b}\right)^2\cdot\mathfrak{Cos}\frac{m\pi}{2a}y\cdot\cos\frac{m\pi}{2b}y\,dy,$$

$$J = \left(1+\left[\frac{2a}{n\pi}\right]^2\left[\frac{m\pi}{2b}\right]^2\right) - \frac{2a}{n\pi}\mathfrak{Sin}\frac{n\pi}{2a}y\cos\frac{m\pi}{2b} + \left(\frac{2a}{n\pi}\right)^2\frac{m\pi}{2b}\mathfrak{Cos}\,k_1 y\sin k_2 y,$$

$$J = \left(\frac{2a}{n\pi}\right)^2\frac{1}{1+\left(\frac{am}{bn}\right)^2}\left(\frac{n\pi}{2a}\mathfrak{Sin}\frac{n\pi}{2a}y\cos\frac{m\pi}{2b}y + \frac{m\pi}{2b}\mathfrak{Cos}\frac{n\pi}{2a}y\sin\frac{m\pi}{2b}\cdot y\right) =$$

$$m = 1,\ 3,\ 5;\quad n = 1,\ 3,\ 5\,.$$

b) $$J = \int \mathfrak{Sin}\, k_1 y \sin k_2 y\, dy = \frac{1}{k_1} \mathfrak{Cof}\, k_1 y \sin k_2 y - \int \frac{k_2}{k_1} \mathfrak{Cof}\, k_1 y \cos k_2 y\, dy$$

$$= \frac{1}{k_1} \mathfrak{Cof}\, k_1 x \sin k_2 x - \frac{k_2}{k_1^2} \mathfrak{Sin}\, k_1 x \cos k_2 x - \left(\frac{k_2}{k_1}\right)^2 \int \mathfrak{Sin}\, k_1 x \sin k_2 x\, dx,$$

$$J \cdot \left(1 + \left[\frac{k_2}{k_1}\right]^2\right) = \frac{1}{k_1} \mathfrak{Cof}\, k_1 x \sin k_2 x - \frac{k_2}{k_1^2} \mathfrak{Sin}\, k_1 x \cos k_2 x,$$

$$J = \frac{1}{k_1^2} \cdot \frac{1}{1 + \left(\frac{k_2}{k_1}\right)^2} \left(\frac{n\pi}{2a} \mathfrak{Cof}\, \frac{n\pi}{2a} y \sin \frac{m\pi}{2b} y - \frac{m\pi}{2b} \mathfrak{Sin}\, \frac{n\pi}{2a} y \cos \frac{m\pi}{2b} y\right)$$

$m = 1, 3, 5;\quad n = 1, 3, 5.$

c) $$J = \int_0^b dx\, \mathfrak{Cof}\, k_1 y \sin k_2 y = \frac{1}{k_1} \mathfrak{Sin}\, k_1 y \sin k_2 y - \int \frac{1}{k_1} \cdot k_2 \mathfrak{Sin}\, k_1 y \cos k_2 y,$$

$$J = \frac{1}{k_1} \mathfrak{Sin}\, k_1 y \sin k_2 y - \frac{k_2}{k_1^2} \mathfrak{Cof}\, k_1 y \cos k_2 y - \int \frac{k_2^2}{k_1^2} \mathfrak{Cof}\, k_1 y \sin k_2 y,$$

$$J = \frac{1}{k_1^2} \cdot \frac{1}{1 + \left(\frac{k_2}{k_1}\right)^2} (k_1 \mathfrak{Sin}\, k_1 y \sin k_2 y - k_2 \mathfrak{Cof}\, k_1 y \cos k_2 y)_0^b \qquad m = 1, 3, 5;\quad n = 1, 3, 5.$$

d) $$J = \int \mathfrak{Sin}\, k_1 y \cos k_2 y = \frac{1}{k_1} \mathfrak{Cof}\, k_1 y \cos k_2 y + \int \frac{k_2}{k_1} \mathfrak{Cof}\, k_1 y \sin k_2 y,$$

$$J = \frac{1}{k_1} \mathfrak{Cof}\, k_1 y \cos k_2 y + \frac{k_2}{k_1^2} \mathfrak{Sin}\, k_1 y \sin k_2 y - \frac{k_2^2}{k_1^2} \cdot J$$

$$= \frac{1}{k_1^2} \cdot \frac{1}{C} (k_1 \mathfrak{Cof}\, k_1 y \cos k_2 y + k_2 \mathfrak{Sin}\, k_1 y \sin k_2 y) \qquad m = 1, 3, 5;\quad n = 1, 3, 5.$$

e) $$J = \int \sin^2 k_1 y\, dy = -\frac{1}{k_1} y \sin k_1 y \cos k_1 y + \frac{k}{k_1} \int \cos^2 k_1 y\, dy$$

$$= -\frac{1}{k_1} [\sin k_1 y \cdot \cos k_1 y]_0^b + [y]_0^b - \int_0^b \sin^2 k_1 y\, dy,$$

$$J = \frac{b}{2}.$$

Literatur.

T. B. Abell: A Study of framing of ships. I.N.A. 1921, S. 163.
Bach: Elastizität und Festigkeit. 9. Aufl. 1924.
Bortsch: Die mitwirkende Plattenbreite, Bauing. 1921, S. 662.
Bruhn: Beanspruchung von Schiffsverbänden. Schiffbau I, S. 561.
— Querfestigkeit. Schiffbau III, S. 11.
— Freebord u. Strength of ships. I.N.A. 1920.
Buchsbaum: Beanspruchung von Schottversteifungen. Schiffbau 07/08. S. 756.
Eggenschwyler: Berechnung von Schleusentoren und ähnlichen Bauwerken. Diss. 1922.
Foster King: I.N.A. 1916, S. 155, und Schiffbau 1919.
A. und L. Föppl: Drang und Zwang 1920.
Henky: Näherungsverfahren zur Berechnung des Spannungszustandes. Beiträge zur technischen Mechanik, S. 62.
Hort: Differentialgl. des Ingenieurs.
Hovgard: Festigkeit wasserdichter Schotte. S. 1910.
Huber: Schubspannungen im gebogenen T-Balken, Beiträge zur technischen Mechanik, S. 25.
Isherwood: A new System of ships Construction. I.N.A. 1908.
v. Kàrmàn: Die mittragende Breite. Beiträge zur technischen Mechanik. S. 114.
Lilly: Web stresses in Plate Girders. Eng. Febr. 1907.
Lienau: Materialspannungen in den Längsverbänden stählerner Handelsschiffe. S. T. G. 1913, S. 603. Schwere Decksunterzüge. Schiffbau 1913/14.
Lorenz: Technische Elastizitätslehre 1913.
Henri Marcus: Theorie elastischer Gewebe. Springer 1924.
Müller-Breslau: Neuere Methoden der Festigkeitslehre.
A. Nádai, Dr.: Formänderungen und Spannungen in rechteckigen elastischen Platten. Forsch.-Arb. Ing. 1915.
— Über die Biegung durchlaufender Platten und der rechteckigen Platte mit freien Rändern. Z. ang. Math. Mech. 1922, Heft 1.
— Theorie der Plattenbiegung. Z. ang. Math. Mech. 1922, Heft 5.
Pietzker: Festigkeit der Schiffe. Berlin 1911.
Read and Stanbery: On the relation between stress and strain in the structure of vessels. I.N.A. 1894.
Reihsner: Über die Knicksicherheit ebener Bleche. Zentralbl. Bauverw. 1909, S. 93.
Timoschenko, L.: Sur la stabilité des systemes élastiques. Ann. des ponts et chaussées.
— Biegung von Stäben. Beiträge zur technischen Mechanik, S. 74.
Wyss: Spannungen in den Knotenblechen eiserner Fachwerke. Forsch.-Arb. 262.

V. Tabellen.

Tabelle 1.

Spannungen im Flansch bei cosinusförmigem Moment.

Seitenverhältnis $a : b = \pi$; $(b \cdot \pi) : 2a = 0{,}5$, $m = 1$; $\mathfrak{Tg}\,\frac{mb}{2a} = 0{,}464$.

$y : b$	0	$^1/_4$	$^1/_2$	$^3/_4$	1
$\frac{\pi}{2a} \cdot y$	0	0,125	0,25	0,375	0,50
$\mathfrak{Cof}\,\frac{\pi}{2a} \cdot y$	0	1,01	1,03	1,07	1,128
$\mathfrak{Sin}\,\frac{\pi}{2a} \cdot y$	1	0,125	0,25	0,38	0,52
a) $\mathfrak{Cof}\,\frac{\pi}{2a} \cdot y : \mathfrak{Cof}\,\frac{\pi}{2a} \cdot b$	0,89	0,90	0,92	0,95	1,0
b) $\mathfrak{Cof}\,\frac{\pi}{2a} \cdot y : \mathfrak{Sin}\,\frac{\pi}{2a} \cdot b$	1,92	1,94	1,97	2,06	2,16
c) $\mathfrak{Sin}\,\frac{\pi}{2a} \cdot y : \mathfrak{Cof}\,\frac{\pi}{2a} \cdot b$	0,00	0,110	0,224	0,340	0,46
d) $\mathfrak{Sin}\,\frac{\pi}{2a} \cdot y : \mathfrak{Sin}\,\frac{\pi}{2a} \cdot b$	0,000	0,240	0,486	0,736	1,00
e) $\frac{y}{b} \cdot \left(\mathfrak{Cof}\,\frac{\pi}{2a}\, y : \mathfrak{Sin}\,\frac{\pi}{2a} \cdot b\right)$	0,00	0,490	0,99	1,54	2,16
f) $\left(\mathfrak{Cof}\,\frac{\pi}{2a} \cdot y : \mathfrak{Sin}\,\frac{\pi}{2a} \cdot b\right)$	7,68	7,76	7,88	8,24	8,64
g) $\left(\mathfrak{Sin}\,\frac{\pi}{2a} \cdot y : \mathfrak{Sin}\,\frac{\pi}{2a} \cdot b\right)$	0,00	0,060	0,24	0,55	1,00
h) $2 \cdot \left(\mathfrak{Sin}\,\frac{\pi}{2a}\, y : \mathfrak{Sin}\,\frac{\pi}{2a} \cdot b\right)$	0,00	0,48	0,972	0,472	2,00
$g + f$	7,68	7,82	8,12	8,79	9,64
$-a + g + f$	6,79	6,92	7,20	7,84	8,64
$\sigma_x = (-a + g + f) \cdot \frac{\mathfrak{Tg}\,0{,}5}{4}\, \sigma_m$	0,77	0,805	0,836	0,912	1,00 σ_m
$a - g$	0,789	0,84	0,68	0,40	0
$\sigma_y = (a - g)\; 0{,}116\, \sigma_m$	0,103	0,097	0,79	0,046	0 σ_m
$e + h$	0,00	0,97	1,96	3,01	4,16
$-c + e + h$	0	0,86	1,74	2,67	3,70
$\tau = (-c + e + h) \cdot 0{,}116\sigma_m$	0	0,10	0,202	0,31	0,43 σ_m

Tabelle 2.

Spannungen bei cosinusförmigem Moment

für $\frac{a}{b} = \pi$ und $m = 3$, also $\frac{\pi b}{2a} \cdot m = \frac{3}{2}$

oder $\frac{a}{b} = \frac{\pi}{3}$; $\frac{3}{4} \cdot \mathfrak{Tg}\frac{\pi b}{2a} = \frac{3}{4}\,\mathfrak{Tg}\,1{,}5 = \frac{3}{4} \cdot 0{,}915 = 0{,}680$.

$y:b$	0	1/4	1/2	3/4	1
$\frac{3\pi}{2a} \cdot y$	0	0,375	0,75	1,125	1,50
$\mathfrak{Cos}\frac{3\pi}{2a} \cdot y$	1,00	1,07	1,29	1,70	2,35
$\mathfrak{Sin}\frac{3\pi}{2a} \cdot y$	0,00	0,38	0,82	1,38	2,13
a) $\mathfrak{Cos}\,\frac{3\pi}{2a} \cdot y : \mathfrak{Cos}\,\frac{3\pi}{2a} \cdot b$	0,42	0,46	0,55	0,72	1,00
b) $\mathfrak{Cos}\,\frac{3\pi}{2a} \cdot y : \mathfrak{Sin}\,1{,}5$	0,48	0,50	0,61	0,80	1,10
c) $\mathfrak{Sin}\,\frac{3\pi}{2a} \cdot y = \mathfrak{Cos}\,1{,}5$	0,00	0,16	0,35	0,59	0,90
d) $\mathfrak{Sin}\,\frac{3\pi}{2a} \cdot y : \mathfrak{Sin}\,1{,}5$	0	0,18	0,38	0,65	1,00
e) $\frac{y}{e}\,\mathfrak{Cos}\,\frac{3\pi}{2a} \cdot y\,\mathfrak{Sin}\,1{,}5$	0	0,13	0,31	0,60	1,10
f) $\frac{4}{3}\,\mathfrak{Cos}\,\frac{3\pi}{2a} \cdot y\,\mathfrak{Sin}\,1{,}5$	0,64	0,67	0,81	1,07	1,47
g) $\frac{y}{1}\,\mathfrak{Sin}\,\frac{3\pi}{2a}\,y : \mathfrak{Sin}\,1{,}5$	0	0,05	0,19	0,49	1,00
h) $\frac{2}{3} \cdot \mathfrak{Sin}\,\frac{3\pi}{2a}\,y : \mathfrak{Sin}\,1{,}5$	0	0,12	0,25	0,43	0,67
$(g+f)$	0,64	0,72	1,00	1,56	2,13
$g + f - a =$	0,22	0,26	0,45	0,84	1,47
$\sigma_x = \frac{3}{4}\,\mathfrak{Tg}\,1{,}5\,(g + f - a) =$	0,14	0,178	0,306	0,57	1,00 σ_m
$a - g$	0,42	0,41	0,36	0,23	0,00
$\sigma_y = 0{,}68 \cdot (a - g)$	0,29	0,28	0,25	0,16	0,00 σ_m
$c + h$	0,00	0,25	0,56	1,08	1,77
$c + h - c$	0,00	0,09	0,21	0,49	0,87
$\tau = 0{,}68\,\sigma_m\,(c + h - c)$	0,00	0,06	0,14	0,33	0,59 σ_m

Tabelle 3.

Spannungen bei cosinusförmigem Moment

$\frac{a}{b} = \pi$, $m = 5$; also $\frac{m \cdot \pi b}{2a} = \frac{5}{2} = 2{,}5$ und $\frac{2a}{5b\pi} = \frac{2}{5}$; $\frac{5\pi b}{2a}\,\mathfrak{Tg} \cdot \frac{5\pi b}{2a} = 2{,}5\,\mathfrak{Tg}\,2{,}5 = 0{,}986$.

$y:b$	0	1/4	1/2	3/4	1
$\frac{5 \cdot \pi}{2a} \cdot y = \frac{5}{2}\,\frac{y}{b}$	0	0,625	1,25	1;875	2,5
$\mathfrak{Cos}\frac{5\pi}{2a} \cdot y$	1,00	1,20	1,89	3,34	6,13
$\mathfrak{Sin}\frac{5\pi}{2a} \cdot y$	0,00	0,67	1,60	3,18	6,05
a) $\mathfrak{Cos}\,\frac{5\pi}{2a} \cdot y : \mathfrak{Cos}\,2{,}5$	0,16	0,195	0,31	0,54	1,00
b) $\mathfrak{Cos}\,5\pi a : \mathfrak{Sin}\,2{,}5$	0,16	0,198	0,31	0,55	1,02

Fortsetzung von Tabelle 3.

$y:b$	0	$^1/_4$	$^1/_2$	$^3/_4$	1
c) $\mathfrak{Sin}\,\frac{5\pi y}{2a} : \mathfrak{Cof}\,2{,}5$	0,00	0,11	0,25	0,52	0,98
d) $\mathfrak{Sin}\,\frac{5}{2a}\pi y : \mathfrak{Sin}\,2{,}5$	0,00	0,11	0,26	0,53	1,00
e) $\frac{y}{b}\,\mathfrak{Cof}\,\frac{5}{2a}\pi \cdot y : \mathfrak{Sin}\,2{,}5$	0,00	0,05	0,16	0,41	1,02
f) $\frac{4}{5}\,\mathfrak{Cof}\,\frac{5\pi}{2a} \cdot y : \mathfrak{Sin}\,2{,}5$	0,13	0,16	0,25	0,44	0,82
g) $\frac{y}{b}\,\mathfrak{Sin}\,\frac{5\pi}{2a} \cdot y : \mathfrak{Sin}\,25,$	0,00	0,03	0,13	0,40	1,00
h) $\frac{2}{5}\,\mathfrak{Sin}\,\frac{5\pi}{2a} \cdot y : \mathfrak{Sin}\,2{,}5$	0,00	0,04	0,10	0,21	0,40
$f+g$	0,13	0,19	0,38	0,84	1,82
$f+g-a$	0,03	0,00	0,07	0,30	0,82
$\sigma_x = \frac{5}{2}\,0{,}98\,(f+g-a)$	$-0{,}03$	0,00	0,08	0,37	1,00 σ_m
$(a-g)$	0,16	0,165	0,18	0,14	0,00
$\sigma_y = 1{,}25 \cdot 0{,}98\,(a-g)$	0,20	0,20	0,22	0,18	0,00 σ_m
$e+h$	0,00	0,09	0,26	0,62	1,42
$e+h-c$	0,00	$-0{,}02$	$+0{,}01$	0,10	0,44
$\tau = 1{,}23$	0,00	0,02	0,00	0,12	0,54 σ_m

Tabelle 4.

Moment durch Einzellast in der Mitte. Seitenverhältnis $a : b = \pi$

$$F(x) = \frac{8}{\pi^2} \cdot \sigma_m \cdot \left(\cos\frac{\pi x}{2a} + \frac{1}{9}\cos\frac{3\pi}{2a}\cdot x + \frac{1}{25}\cdot\cos\frac{5\pi}{2a}\cdot x + \ldots\right)$$

Spannungen für $x = 0$

$y:b$		0	$^1/_4$	$^1/_2$	$^3/_4$	1
für $m=1$ σ_{x_1}		0,777	0,805	0,836	0,912	1,00
$\frac{1}{9}$ für $m=3$ σ_{x_3}		0,015	0,020	0,034	0,065	0,11
$\frac{1}{25}$ für $m=5$ σ_{x_5}		—	—	0,003	0,015	0,04
$m=7$		—	—	—	—	0,02
Normalspannungen	Summe	0,792	0,825	0,873	0,992	1,15
$\sigma_x = \frac{8}{\pi^2}\cdot$ Summe		0,64	0,67	0,71	0,80	0,96 σ_m
Schubspannungen für $x = a$						
Schubspannungen für $m=1$ τ_1		0,00	0,10	0,20	0,31	0,43
für $m=3$ τ_3		$-0{,}00$	$-0{,}01$	$-0{,}02$	$-0{,}04$	$-0{,}07$
für $m=5$ τ_5		0,00	0,00	0,00	0,01	0,02
	Summe	0,00	0,09	0,18	0,28	0,38
$\tau = \frac{8}{\pi^2}\sum$		0,00	0,07	0,15	0,23	0,31 σ_m
Spannungen für $x = \frac{a}{3}$; $\sigma_{r_1} = \cos\frac{\pi}{6}\,\sigma_m = 0{,}82\,\sigma_m$; $\sigma_{r_3} = 0$; $\sigma_{r_5} = \cos\frac{\pi}{6}\,\sigma_m$						
$\sigma_{x_1} =$		0,64	0,66	0,69	0,75	0,82
$\sigma_{x_5} =$		0	—	0,00	$-0{,}01$	$-0{,}03$
	Summe	0,64	0,66	0,69	0,74	0,79
$\sigma_{x_5} = \frac{8}{\pi^2}\sum$		0,52	0,54	0,57	0,61	0,66 σ_m

Fortsetzung von Tabelle 4.

Für $x = \frac{a}{3}$ wird $\tau_1 = \sin\frac{\pi}{6} = \frac{1}{2}$; $\tau_3 = \sin\frac{\pi}{2} = 1$; $\tau_5 = \sin\frac{5\pi}{6} = \sin\frac{\pi}{6} = \frac{1}{2}$.

$y:b$		0	$^1/_4$	$^1/_2$	$^3/_4$	1
Also $\tau_1 =$		0,00	0,05	0,10	0,16	0,22
$\tau_3 =$		—	+ 0,01	+ 0,02	+ 0,04	+ 0,07
$\tau_5 =$		—	—	—	0,01	+ 0,02
	Summe	0,00	0,06	0,12	0,21	0,31 σ_m
$\tau = 0{,}82\sum$		0,00	0,05	0,10	0,18	0,25 σ_m
		Normalspannungen am Rande				
$\cos\frac{\pi}{2a}x$		1,0	0,89	0,75	0,31	0,0
$+\frac{1}{9}\cos\frac{3\pi}{2a}x$		0,11	0,02	—0,08	—0,09	0,0
$+\frac{1}{25}\cos\frac{5\pi}{2a}x$		0,04	+ 0,03	—0,03	+ 0,04	0,00
	Summe	1,15	0,94	0,60	0,26	0,00
$\sigma_y = 0{,}82\cdot$	Summe	0,94	0,77	0,49	0,21	0,00 σ_m
		Schubspannungen am Rande				
$\frac{x}{a} =$		0,00	0,3	0,5	0,8	1,0
τ für $m = 1$		0,00	0,20	0,30	0,41	0,43
τ für $m = 3$		0,00	0,07	0,92	—0,04	—0,07
τ für $m = 5$		0,00	0,02	—0,02	—0,00	+ 0,02
	Summe	0,0	0,29	0,30	0,37	0,38
$\tau_y = 0{,}82\cdot$	Summe	0,0	0,24	0,25	0,30	0,31 σ_m

Tabelle 5.

Gleichmäßig verteilte Last. $a:b = \pi$.

$$F(x) = \frac{32}{\pi^3}\sigma_m\left(\cos\frac{\pi}{2a}\cdot x - \frac{1}{3^3}\cos\frac{3\pi}{2a}\cdot x + \frac{1}{5^3}\cos\frac{5\pi}{2a}\cdot x - \right.$$

$y:b$		0	$^1/_4$	$^1/_2$	$^3/_4$	1
		Normalspannungen für $x = 0$				
σ_x für $m = 1$		0,777	0,805	0,836	0,912	1,00
$\frac{1}{27}\cdot\sigma_x$ für $m = 3$		—0,005	—0,007	—0,011	—0,022	—0,037
$\frac{1}{25}\cdot\sigma_x$ für $m = 5$		—	—	0,001	0,003	0,008
	Summe	0,772	0,798	0,826	0,893	0,971
$\sigma_x = \frac{32}{31}\cdot$	Summe	0,800	0,825	0,853	0,924	1,000 σ_m
		Schubspannungen für $x = a$				
τ für $m = 1$		0,00	0,10	0,20	0,31	0,43
$\frac{1}{27}\tau$ für $m = 3$		+ 0,00	+ 0,00	+ 0,01	+0,01	+ 0,02
$\frac{1}{125}\tau$ für $m = 5$		—0,00	—	—	0,01	0,02
	Summe	0,00	0,10	0,21	0,32	0,45
$\tau = \frac{32}{31}\cdot$	Summe	0,00	0,10	0,22	0,33	0,47 σ_m

Fortsetzung von Tabelle 5.

Normal- und Schubspannungen für $x = \frac{a}{3}$.

$$\cos\frac{\pi}{6} = 0{,}82;\quad \cos\frac{3\pi}{6} = 0;\quad \cos\frac{5\pi}{6} = -\cos\frac{\pi}{6} = -0{,}82.$$

$y : b$		0	$^1/_4$	$^1/_2$	$^3/_4$	1
σ_x für $m = 1$		0,64	0,66	0,69	0,75	0,82
σ_x für $m = 5$		0,00	—	0,00	—	—0,01
	Summe	0,64	0,66	0,69	0,75	0,81
$\sigma_x = \frac{32}{31}\cdot$	Summe	0,66	0,68	0,71	0,77	0,83 σ_m

$$\tau\ \sin\frac{\pi}{6} = 0{,}5;\quad \sin\frac{3\pi}{6} = 1;\quad \sin\frac{5\pi}{6} = \sin\frac{\pi}{6} = \frac{1}{2}.$$

τ für $m = 1$		0,0	0,05	0,10	0,16	0,21
τ für $m = 3$		—	—	0,01	0,01	0,02
τ für $m = 5$		—	—	—	—	0,01
	Summe	0,0	0,05	0,11	0,17	0,24
$\tau = \frac{32}{31}\cdot$	Summe	0,0	0,05	0,11	0,18	0,25 σ_m

Normal- und Schubspannungen am Rande $y = b$

$x : a$		0,0	0,3	0,5	0,8	1,0
$\cos\frac{\pi}{2a}\cdot x$		1,0	0,89	0,71	0,31	0,0
$\frac{1}{27}\cos\frac{3\pi}{2a}\cdot x$		—0,04	—0,01	+ 0,03	+ 0,03	0,0
$\frac{1}{125}\cos\frac{5\pi}{2a}x$		0,01	—0,01	—0,01	+ 0,01	0,0
	Summe	0,97	0,87	0,73	0,35	0,0
$\sigma_x = \frac{32}{31}\cdot$	Summe	1,00	0,90	0,75	0,36	0,0 σ_m
$\sin\frac{\pi}{2a}\cdot x$		0	0,20	0,30	0,40	0,43
$\frac{1}{27}\sin\frac{3\pi}{2a}\cdot x$		—	—	—	0,01	0,02
	Summe	0	0,20	0,30	0,41	0,45
$\tau = \frac{32}{31}\cdot$	Summe	0	0,20	0,31	0,42	0,46 σ_m.

Tabelle 6.

Belastung durch zwei Einzellasten $a : b = \pi$.

$$F(x) = \frac{16\,h}{\pi^2}\cdot\sin\frac{\pi}{4}\cdot\left(\cos\frac{\pi}{2\,a}x - \frac{1}{3^2}\cos\frac{3\,\pi}{2\,a}\cdot x - \frac{1}{5^2}\cos\frac{5\,\pi}{2\,a}x + \right)$$

Normalspannungen σ_x. Für $x = 0$.

$y : b$		0	$^1/_4$	$^1/_2$	$^3/_4$	1
σ_x für $m = 1$		0,777	0,805	0,836	0,912	1,000
$-\frac{1}{9}\sigma_x$ für $m = 3$		—0,077	—0,020	—0,034	—0,065	—0,111
$-\frac{1}{25}\sigma_x$ für $m = 5$		—0,001	0,000	0,003	0,015	0,040
	Summe	0,760	0,785	0,805	0,842	0,812
$\frac{16\cdot 0{,}71}{9{,}9} = 1{,}14\cdot\sum$		0,870	0,890	0,910	0,96	1,00 σ_m

Fortsetzung von Tabelle 6.

Schubspannungen τ für $x = a$.

$y:b$		0	$^1/_4$	$^1/_2$	$^3/_4$	1
$m = 1$ $\tau = \tau_1$ für $m = 1$		0,00	0,100	0,200	0,310	0,430
$\tau_3 = +\frac{1}{9}t_3$ für $m = 3$		—	+ 0,007	+ 0,016	+ 0,037	+ 0,066
$\tau_5 = +\frac{1}{25}t_5$ für $m = 5$		—	—	—	— 0,005	— 0,022
τ_7 f. $\frac{1}{49}t_7$ für $m = 7$		—	—	—	—	0,010
	Summe	0,00	0,107	0,216	0,342	0,464
$\tau = 1{,}14\cdot$	Summe	0,00	1,22	0,247	0,391	0,530 σ_m

Normal- und Schubspannungen für $x = \frac{a}{3}$.

σ_x für $m = 1$		0,64	0 66	0,69	0,75	0,82
$-\frac{1}{25}\sigma_x$ für $m = 5$		—	—	—	0,01	0,03
	Summe	0,64	0,66	0,69	0,76	0,85
$1{,}14\cdot$	Summe	0,73	0,75	0,79	0,87	0,97
τ für $m = 1$		0,00	0,05	0,10	0,16	0,22
$-\tau$ für $m = 3$		—	— 0,01	— 0,02	— 0,04	— 0,07
$-\tau$ für $m = 5$		—	—	—	—	— 0,01
	Summe	0,00	0,04	0,08	0,12	0,14
$\tau = 1{,}14\cdot$	Summe	0,00	0,05	0,09	0,14	0,16 σ_m

Normal- und Schubspannungen am Rande $y = b$.

$x = a$		0	0,3	0,5	0,8	1,0
$\cos\frac{\pi}{2a}\cdot x$		1,0	0,89	0,71	0,31	0,00
$-\frac{1}{9}\cos\frac{3\pi}{2a}\cdot x$		— 0,11	— 0,02	+ 0,08	+ 0,09	0,00
$-\frac{1}{25}\cos\frac{5\pi}{2a}\cdot x$		— 0,004	— 0,03	+ 0,03	— 0,04	0,00
	Summe	0,85	0,84	0,82	0,36	0,00
$\sigma_x = 1{,}14\cdot$	Summe	0,97	0,96	0,94	0,41	0,00 σ_m
τ für $m = 1$		0	0,20	0,30	0,41	0,43
τ für $m = 3$		0	— 0,07	0,02	0,04	0,07
τ für $m = 5$		0	— 0,02	+ 0,02	0,0	— 0,02
	Summe	0	0,11	0,34	0,45	0,48
$\tau = 1{,}14\cdot$	Summe	0	0,13	0,39	0,51	0,55 σ_m

Tabelle 7.

Trigonometrische Funktionen zur Berechnung der Spannungen.

$y:b =$	0	$^1/_4$	$^1/_2$	$^3/_4$	1
$\frac{\pi}{2b}y =$	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
$\frac{\pi}{2b}y$ (Bogen)	0,0°	22,5°	45°	67,5°	90°
$\frac{3\pi}{2b}y$ (Grad)	0,0°	67,5°	135°	202,5°	270°
$\frac{5\pi}{2b}y$ (Grad)	0°	112,5°	225°	337,5°	450°

Fortsetzung von Tabelle 7.

$y:b=$	0	$^1/_4$	$^1/_2$	$^3/_4$	1
$\cos\frac{\pi}{2a}y$	1,0	0,92	0,71	0,38	0,00
$\cos\frac{3\pi}{2a}y$	+ 1,0	+ 0,38	— 0,71	— 0,92	— 0,00
$\cos\frac{5\pi}{2a}y$	+ 1,0	— 0,38	— 0,71	+ 0,92	+ 0,00
$\sin\frac{\pi}{2a}y$	0	0,38	0,71	0,92	1,00
$\sin\frac{3\pi}{2b}y$	0	0,92	0,71	— 0,38	— 1,00
$\sin\frac{5\pi}{2a}y$	0	0,92	— 0,71	— 0,38	+ 1,00

Tabelle 8.

Berechnung der Lösung 2 für cosinusförmiges Moment. $a:b=\pi$.

$x:a=$	0,0	0,3	0,5	0,7	0,8	0,9	0,10
$1-\frac{x}{a}$	1,0	0,7	0,5	0,3	0,2	0,1	0,0
$1-\frac{x}{a}-\frac{2b}{a\pi}$	0,8	0,5	0,3	0,1	0,0	— 0,1	— 0,2
$1-\frac{x}{a}-\frac{2b}{3a\pi}$	—	—	—	—	0,13	0,03	— 0,07
$1-\frac{x}{a}-\frac{2b}{5a\pi}$	—	—	—	—	—	—	— 0,04
$1-\frac{x}{a}-\frac{4b}{a\pi}$	0,6	0,3	0,1	— 0,1	— 0,2	— 0,3	— 0,4
$1-\frac{x}{a}-\frac{4b}{3a\pi}$	—	—	—	0,17	0,07	— 0,03	— 0,13
$1-\frac{x}{a}-\frac{4b}{5a\pi}$	—	—	—	—	—	—	0,08
$-\frac{\pi a}{2b}\left(1-\frac{x}{a}\right)$	— 5,0	— 3,5	— 2,5	— 1,5	— 1,0	— 0,5	— 0,0
$-\frac{3\pi a}{2b}\left(1-\frac{x}{a}\right)$	—	—	— 7,5	— 4,5	— 3,0	— 1,5	— 0,0
$-\frac{5\pi a}{2b}\left(1-\frac{x}{a}\right)$	—	—	—	— 7,5	— 5,0	— 2,5	0,0
$e^{-k\left(1-\frac{x}{a}\right)}$	0,01	—	0,09	0,23	0,37	0,61	1,0
$e^{-3k\left(1-\frac{x}{a}\right)}$	—	—	0,00	0,01	0,05	0,22	1,0
$e^{-5k\left(1-\frac{x}{a}\right)}$	—	—	—	—	0,01	0,08	1,0
$\frac{\pi}{2b}e^{-k\left(1-\frac{x}{a}\right)}$	0,05	—	0,45	1,15	1,85	3,05	5,00
$\frac{\pi a}{2b}e^{-k\left(1-\frac{x}{a}\right)}$	—	—	—	0,15	0,75	3,30	15,00
$\frac{\pi a}{2b}e^{-k\left(1-\frac{x}{a}\right)}$	—	—	—	—	0,25	2,00	25,00
$0{,}333\frac{\pi a}{2b}e^{-k\left(1-\frac{x}{a}\right)}$	0,02	—	0,15	0,38	0,62	1,02	1,67
$-0{,}045\frac{3\pi a}{2b}e^{-k\left(1-\frac{x}{a}\right)}$	—	—	—	0,01	0,03	0,15	0,67
$+0{,}016\frac{5\pi a}{2b}e^{-k\left(1-\frac{x}{a}\right)}$	—	—	—	—	—	+ 0,03	+ 0,42

Tabelle 9.

Cosinusförmiges Moment. Genaue Lösung. $a : b = \pi$.
Spannungen σ_x.

1. Lösung. Seitenverhältnis $a : b = \pi$.

$x : a$	0,0	0,3	0,5	0,7	0,8	0,9	1,0 σ_m
$y : b = 1 : 1$ $\sigma_x =$	1,0	0,89	0,71	0,45	0,31	0,16	0,0 σ_m
$y : b = 3 : 4$ $\sigma_x =$	0,91	0,81	0,65	0,41	0.28	0,15	0,0 σ_m
$y : b = 1 : 2$ $\sigma_x =$	0,84	0,75	0,59	0,38	0,26	0,13	0,0 σ_m
$y : b = 1 : 4$ $\sigma_x =$	0,81	0,72	0,57	0,36	0,25	0,13	0,0 σ_m
$y : b = 0$ $\sigma_x =$	0,78	0,69	0,55	0,35	0,24	0,12	0,0 σ_m

2. Lösung. $C_1 = 0{,}333$, $C_3 = 0{,}045$, $C_5 = 0{,}016$, $C_2 = 0{,}008$.

$(1 - \frac{x}{a})$	1,0	0,7	0,5	0,3	0,2	0,1	0,0
$y : b = 1 : 1$ σ_x	0	0	0	0	0	0	0
$y : b = 3 : 4$	0,01	0,01	0,03	0,04	0,05	0,04	0,10
σ_x	—	—	—	0,00	+ 0,01	+ 0,01	—
$\sigma_{x_1} =$	—	0,01	0,03	0,05	0,06	0,05	0,00 σ_m
$y : b = 1 : 2$	0,01	0,01	0,06	0,08	0,08	0,07	0,00
	—	—	—	—	0,01	0.01	—
$\sigma_{x_2} =$	0,01	0,01	0,06	0,08	0,09	0,08	0,00 σ_m
$y : b = 1 : 4$	0,02	0,02	0,07	0,10	0,10	0,08	0,00
	—	—	—	—	—	— 0,00	0,00
$\sigma_{x_2} =$	0,02	0,02	0,07	0,10	0,10	0,08	0,00 σ_m
$y : b = 0$ σ_x	0,02	0,02	0,08	0,11	0,11	0,09	0,00 σ_m

Tabelle 10.

Gesamtspannungen σ_x bei cosinusförmigem Moment aus Lösung 1 und 2.
Seitenverhältnis $a : b = \pi$.

$x : a$		0,0	0,3	0,5	0,7	0,8	0,9	1,0
$y : b = 1$		1,0	0,89	0,71	0,45	0,31	0,16	0,0 σ_m
$y : b = 3 : 4$								
	1. Lösung	0,91	0,81	0,65	0,41	0,28	0,15	0,00
	2. Lösung	0,01	0,01	0,03	0,05	0,06	0,05	0,00
	Summe	0,90	0,80	0,62	0,36	0,22	0,10	0,00 σ_m
$y : b = 1 : 2$								
	1. Lösung	0,84	0,75	0,59	0,38	0,26	0,13	0,00
	2. Lösung	0,01	0,01	0,06	0,08	0,09	0,08	0,00
	Summe	0,83	0,74	0,53	0,30	0,15	0,05	0,00 σ_m
$y : b = 1 : 4$								
	1. Lösung	0,81	0,72	0,57	0,36	0,25	0,13	0,00
	2. Lösung	0,02	0,02	0,07	0,10	0,10	0,08	0,00
	Summe	0,79	0,70	0,50	0,26	0,15	0,05	0,00 σ_m
$y : b = 0$								
	1. Lösung	0,78	0,89	0,55	0,35	0,24	0,12	0,00
	2. Lösung	0,02	0,02	0,08	0,11	0,11	0,09	0,00
	Summe	0,76	0,67	0,47	0,24	0,13	0,03	0,00 σ_m

Tabelle 11.

Cosinusförmiges Moment. Genaue Lösung.
Schubspannungen τ für $a : b = \pi$.

1. Lösung.

$x : a$		0,0	0,3	0,5	0,7	0,8	0,9	1,0
$\sin \frac{\pi}{2} \cdot \frac{x}{a}$		0,0	0,45	0,71	0,89	0,95	0,99	1,00
$y = 0$	τ_1	0,0	0	0	0	0	0	0 σ_m
$y : b = 1 : 4$	τ_1	0,0	0,05	0,07	0,09	0,10	0,10	0,10 σ_m
$y : b = 1 : 2$	τ_1	0,0	0,09	0,14	0,18	0,19	0,20	0,20 σ_m
$y : b = 3 : 4$	τ_1	0,0	0,14	0,22	0,28	0,29	1,31	0,31 σ_m
$y : b = 1 : 1$	τ_1	0,0	0,18	0,30	0,38	0,41	0,43	0,43 σ_m

2. Lösung.

$C_1 = 0{,}333, \quad C_3 = 0{,}045, \quad C_5 = 0{,}016.$

$x : a$	0	0,3	0,5	0,7	0,8	0,9	1,0
$y : b = 0$	0	0	0	0	0	0	0
$y : b = 1 : 4$	0,0	0,00	0,02	0,02	0,0	−0,04	−0,13
	—	—	—	—	—	−0,01	+0,05
$\tau_2 = \frac{2b}{n\pi} \sin n \frac{\pi}{8} F_2'(x)$	—	—	—	—	—	—	−0,02
$\tau =$	0,0	0,00	0,02	+0,02	0,00	−0,05	−0,10 σ_m
$y : b = \frac{1}{2}$	0,0	0,01	0,04	0,03	0,0	−0,07	−0,24
$\tau_2 = \frac{2b}{n\pi} \sin n \frac{\pi}{4} F_2'(x)$	—	—	—	—	—	−0,01	+0,04
	—	—	—	—	—	—	+0,01
$\tau =$	0,0	0,01	0,04	0,03	0,0	−0,08	−0,19 σ_m
$y : b = \frac{3}{4}$	0,0	0,02	0,04	0,04	0,00	−0,09	−0,30
$\tau_2 = \frac{2b}{n\pi} \sin n \frac{3\pi}{8} F_2'(x)$	—	—	—	—	—	—	−0,02
	—	—	—	—	—	—	+0,01
$\tau =$	0,00	0,02	0,04	0,04	0,00	−0,09	−0,31 σ_m
$y : b = 1$	0,00	0,02	0,05	0,04	0,00	−1,10	−0,33
$\tau_2 = \frac{2b}{n\pi} \sin \frac{n\pi}{4} F'(x)$	—	—	—	—	—	+0,01	−0,05
	—	—	—	—	—	—	−0,02
$\tau =$	0,00	0,02	0,05	0,04	0,0	−0,09	−0,41 σ_m

Tabelle 12.

Gesamtspannungen für $a : b = \pi$. τ aus Lösung 1 und 2.

$x : a$		0,0	0,3	0,5	0,7	0,8	0,9	1,0
$y : b = 0$			0	0	0	0	0	0
$y : b = 1 : 4$								
	Lösung 1	0,0	0,05	0,07	0,09	0,10	0,10	0,10
	Lösung 2	0,00	0,00	0,02	0,02	0,00	−0,05	−0,10
	$\tau = \sum$	0,0	0,05	0,09	0,11	0,10	0,05	0,0 σ_m

Fortsetzung von Tabelle 12.

$x:a$		0,0	0,3	0,5	0,7	0,8	0,9	1,0	
$y:b=0$			0	0	0	0	0	0	
$y:b=1:2$									
	Lösung 1	0,0	0,09	0,14	0,18	0,19	0,20	0,20	
	Lösung 2		0,00	0,04	0,03	0,00	—0,08	—0,19	
	$\tau=\sum$	0,00	0,19	0,18	0,21	0,19	0,12	0,01	σ_m
$y:b=3:4$									
	Lösung 1	0,0	0,14	0,22	0,28	0,29	0,31	0,31	
	Lösung 2		0,02	0,04	0,04	0,00	—0,09	—0,31	
	$\tau=\sum$	0,0	0,16	0,26	0,32	0,29	0,22	0,00	σ_m
$y:b=1:1$									
	Lösung 1	0,0	0,18	0,30	0,38	0,41	0,43	0,43	
	Lösung 2		0,02	0,05	0,04	0,00	—0,09	—0,41	
	$\tau=\sum$	0,00	0,20	0,35	0,42	0,41	0,32	0,02	σ_m

Tabelle 13.

Cosinusförmiges Moment. Genaue Lösung.

Spannungen σ_y. $a:b=\pi$.

1. Lösung. $\sigma_y=\sigma_{ym}\cos\frac{\pi}{2a}x$.

$x:a$	0,0	0,3	0,5	0,7	0,8	0,9	1,0	
$\cos\frac{\pi}{2a}\cdot x$	1,0	1,0	0,71	0,45	0,31	0,16	0,0	
$y=0$	0,10	0,09	0,07	0,05	0,03	0,02	0,00	σ_m
$y:b=1:4$	0,10	0,09	0,07	0,04	0,03	0,02	0,00	σ_m
$y:b=1:2$	0,08	0,07	0,06	0,04	0,02	0,01	0,0	σ_m
$y:b=3:4$	0,05	0,04	0,03	0,02	0,01	0,01	0,0	σ_m
$y:b=1$	0,00	0,00	0,0	0,0	0,0	0,0	0,0	σ_m

2. Lösung.

$C_1=0{,}333,\quad C_3=0{,}045,\quad C_5=0{,}016.$

$x:a$		0,0	0,3	0,5	0,7	0,8	0,9	1,0	
$y=0\ \left(\frac{2b}{n\pi}\right)^2 F''(x)\cos 0$		0,01	0,01	0,02	—0,04	—0,12	—0,31	—0,67	
	$n=3$	—	—	—	—	—	+0,01	+0,09	
	$n=5$	—	—	—	—	—	—	—0,03	
	$\sigma_y=\sum$	0,01	0,01	0,02	—0,04	—0,12	—0,30	0,61	σ_m
$y:b=1:4$	$n=1$	0,01	0,01	0,02	—0,04	—0,11	—0,28	—0,62	
$\left(\frac{2b}{n\pi}\right)^2 F_2''(x)\cos n\frac{\pi}{8}$	$n=3$	—	—	—	—	—	—	+0,03	
	$n=5$	—	—	—	—	—	—	—0,01	
	$\sigma_y=\sum$	0,01	0,01	0,02	—0,04	—0,11	—0,28	—0,60	σ_m
$y:b=1:2$	$n=1$	0,01	0,01	0,01	—0,03	—0,08	—0,22	—0,48	
$\left(\frac{2b}{n\pi}\right)^2 F_2''(x)\cos n\frac{\pi}{4}$	$n=3$	—	—	—	—	—	—0,01	—0,06	
	$n=5$	—	—	—	—	—	—	+0,02	
	$\sigma_y=\sum$	0,01	0,01	0,01	—0,03	—0,08	—0,23	—0,52	σ_m
$y:b=3:4$	$n=1$	0,00	0,00	0,01	—0,02	—0,05	—0,12	—0,25	
$\left(\frac{2b}{n\pi}\right)^2 F_2''(x)\cos\frac{3\pi}{8}$	$n=3$	—	—	—	—	—	—0,01	—0,08	
	$n=5$	—	—	—	—	—	—	—0,02	
	$\sigma_y=\sum$	0,0	0,0	0,01	—0,02	—0,05	—0,13	—0,35	σ_m
$y:b=1$	σ_y	0	0	0	0	0	0	0	σ_m

Tabelle 14.

Cosinusförmiges Moment. $a : b = \pi$.

Zusammenstellung der Lösungen 1 und 2 für die Spannungen σ_y.

$\frac{x}{a} =$		0,0	0,3	0,5	0,7	0,8	0,9	1,0
$y : b = 0$								
	1. Lösung	0,10	0,09	0,07	0,05	+ 0,03	+ 0,02	+ 0,0
	2. Lösung	0,01	0,01	0,02	— 0,04	— 0,12	— 0,31	— 0,61
	$\sigma_y = \sum$	0,11	0,10	0,09	0,01	— 0,09	— 0,29	— 0,61 σ_m
$y : b = 1 : 4$								
	1. Lösung	0,10	0,09	0,07	0,04	0,03	0,02	0,00
	2. Lösung	0,01	+ 0,01	+ 0,02	— 0,04	— 0,11	— 0,28	— 0,60
	$\sigma_y = \sum$	0,11	0,10	0,09	0,0	— 0,08	— 0,26	0,60 σ_m
$y : b = 1 : 2$								
	1. Lösung	0,08	0,07	0,06	0,04	0,02	0,01	0,00
	2. Lösung	0,01	0,01	0,01	— 0,03	— 0,08	— 0,33	— 0,44
	$\sigma_y = \sum$	0,09	0,08	0,07	0,01	— 0,06	— 0,32	— 0,44 σ_m
$y : b = 3 : 4$								
	1. Lösung	0,05	0,04	0,03	0,02	0,01	0,01	0,00
	2. Lösung	0,00	0,00	0,01	— 0,02	— 0,05	— 0,13	— 0,35
		0,05	0,04	0,04	± 0,00	— 0,04	— 0,12	— 0,35 σ_m
$y : b = 1 : 1$								
	Lösung	0,00	0	0	0	0	0	0 σ_m

Tabelle 15.

Cosinusförmiges Moment: $a : b = 4$ und $m = 1$.

Also $\frac{m\pi}{2a} \cdot b = \frac{\pi}{8} = 0{,}392$; $\frac{2 \cdot 2a}{m \cdot b \cdot \pi} = \frac{2 \cdot 8}{\pi} = 2{,}55 \cdot 2 = 5{,}1$.

$\frac{m\pi b}{4a} = 0{,}196$; $\mathfrak{Tg}\,\frac{\pi b}{2a} = 0{,}373$; $\frac{\pi b}{4a} \cdot \mathfrak{Tg}\,0{,}392 = 0{,}073$.

$y : b$	0	$^1/_4$	$^1/_2$	$^3/_4$	1
$\frac{m\pi}{2a} \cdot y = \frac{m\pi}{8} \frac{(y)}{b}$	0	0,98	0,196	0,294	0,392
$\mathfrak{Cof}\,\frac{m\pi}{8} \cdot \frac{y}{b} = \mathfrak{Cof}\,\frac{m\pi}{2a}\,y$	1,00	1,005	1,019	1,044	1,078
$\mathfrak{Sin}\,\frac{m\pi}{8} \cdot \frac{y}{b}$	0	0,098	0,20	0,30	0,40
a) $\mathfrak{Cof}\,\frac{m\pi \cdot y}{8 \cdot b} : \mathfrak{Cof}\,\frac{ka}{b}$	0,93	0,94	0,95	0,98	1,00
b) $\mathfrak{Cof}\,\frac{m\pi}{8} \cdot \frac{y}{b} : \mathfrak{Sin}\,\frac{ka}{b}$	2,5	2,53	2,55	2,62	2,69
c) $\mathfrak{Sin}\,\frac{ky}{b} : \mathfrak{Cof}\,\frac{ka}{b}$	0	0,091	0,19	0,28	0,37
d) $\mathfrak{Sin}\,\frac{ky}{b} : \mathfrak{Sin}\,\frac{ka}{b}$	0	0,25	0,50	0,75	1,00
e) $\frac{y}{b}\left(\mathfrak{Cof}\,\frac{ky}{b} : \mathfrak{Sin}\,\frac{ka}{b}\right)$	0	0,63	0,28	1,97	2,69
f) $\frac{4a}{bm\pi} \cdot \left(\mathfrak{Cof}\,\frac{ka}{b} : \mathfrak{Sin}\,\frac{ka}{b}\right)$	12,8	12,9	13,0	13,4	13,7
g) $\frac{y}{b}\left(\mathfrak{Sin}\,\frac{kg}{b} : \mathfrak{Sin}\,\frac{ka}{b}\right)$	0	0,66	0,25	0,56	1,00

Fortsetzung von Tabelle 15.

$y:b$	0	$^1/_4$	$^1/_2$	$^3/_4$	1
h) $\frac{4a}{bm\pi}\left(\mathfrak{Sin}\frac{ky}{b}:\mathfrak{Sin}\frac{ka}{b}\right)$	0	0,64	0,128	1,90	2,55
$a-g$	0,93	0,88	0,70	0,42	0
$\sigma_y=\frac{m\pi b}{4a}\cdot\mathfrak{Tg}\frac{m\pi b}{2a}(a-g)=$	0,072	0,068	0,054	0,032	0 σ_m
$f-(a-g)$	11,7	12,0	12,3	13,0	13,7
$\sigma_x=K(f-a+g)$	0,86	0,88	0,90	0,95	1,00 σ_m
$e+h$	0	1,27	2,56	3,87	5,24
$e+h-c$	0	1,18	2,37	3,59	4,87
$\tau=K\cdot(e+h-c)$	0	0,09	0,17	0,26	0,36 σ_m

Die mittragende Breite $B_1=b\,\mathfrak{Tg}\,\pi b\left(-\mathfrak{Tg}\frac{\pi b}{2a}+\frac{1}{\mathfrak{Tg}\frac{\pi b}{2a}}+\frac{2a}{bm\pi}\right)$,

$$=b\cdot 0{,}186\,(-0{,}373+2{,}69+2{,}55)\,,$$

$$B_1=b\cdot 0{,}186\cdot 4{,}87=0{,}910\,b\,.$$

Tabelle 16. Cosinusförmiges Moment. $a:b=4$. $m=3$

$\frac{m\pi b}{2a}=\frac{3\pi}{8}=1{,}176$; $\frac{2a}{m\pi b}=0{,}85$; $\frac{m\pi b}{4a}\mathfrak{Tg}\,1{,}176=0{,}588\cdot 0{,}826=0{,}485$

$y:b$	0	$^1/_4$	$^1/_2$	$^3/_4$	1
$\frac{m\pi}{2a}y\;\frac{m\pi}{8}\left(\frac{y}{b}\right)$	0	0,294	0,588	0,882	1,176
$\mathfrak{Cof}\frac{m\pi}{8}\frac{y}{b}=\mathfrak{Cof}\frac{m\pi}{2a}y$	1,00	1,044	1,175	1.424	1,78
$\mathfrak{Sin}\frac{m\pi}{8}\frac{y}{b}$	0	0,30	0,62	1,00	1,47
a) $\mathfrak{Cof}\frac{m\pi}{2a}y:\mathfrak{Cof}\frac{m\pi}{2a}b$	0,56	0,59	0,66	0,80	1,00
b) $\mathfrak{Cof}\frac{m\pi}{2a}y:\mathfrak{Sin}\frac{m\pi}{2a}b$	0,68	0,81	0,80	0,96	1,21
c) $\mathfrak{Sin}\frac{m\pi}{2a}y:\mathfrak{Cof}\frac{m\pi}{2a}b$	0	0,17	0,35	0,56	0,82
d) $\mathfrak{Sin}\frac{m\pi}{2a}y:\mathfrak{Sin}\frac{m\pi}{2a}b$	0	0,20	0,42	0,68	1,00
e) $\frac{y}{b}\cdot\left[\mathfrak{Cof}\frac{m\pi}{2a}y:\mathfrak{Sin}\frac{mb\pi}{2a}\right]$	0	0,18	0,40	0,72	1,21
f) $\frac{4a}{bm\pi}\left[\mathfrak{Cof}\frac{m\pi}{2a}y:\mathfrak{Sin}\frac{mb\pi}{2a}\right]$	1,16	1,20	1,36	1,64	2,16
g) $\frac{y}{b}\left(\mathfrak{Sin}\frac{m\pi}{2a}y:\mathfrak{Sin}\frac{m\pi b}{2a}\right)$	0,0	0,05	0,21	0,51	1,00
h) $\frac{2a}{bm\pi}\left(\mathfrak{Sin}\frac{m\pi}{2a}y:\mathfrak{Sin}\frac{m\pi b}{2a}\right)$	0	0,17	0,36	0,58	0,85
$a-g$	0,56	0,54	0,45	0,29	0,00
$\sigma_y=\mathfrak{Tg}\frac{m\pi b}{2a}\frac{m\pi b}{4a}(a-g)$	0,27	0,26	0,27	0,14	0,00 σ_m
$f-(a-g)$	0,60	0,66	0,91	1,35	2,06
$\sigma_x=\frac{m\pi b}{4a}\mathfrak{Tg}\frac{m\pi b}{2a}(f-a+g)$	0,29	0,32	0,44	0,66	1,00 σ_m
$e+h$	0	0,18	0,41	0,74	1,24
$e+h-c$	0	0,09	0,20	0,35	0,59 σ_m
$\tau=\frac{m\pi b}{4a}\mathfrak{Tg}\frac{m\pi b}{2a}(e+h-c)$					

$$B_3=\frac{b}{2}\cdot 0{,}826\,(-0{,}826+1{,}21+0{,}85)=b\cdot 0{,}413\cdot 1{,}23=0{,}51\,b\,.$$

Tabelle 17.

Cosinusförmiges Moment. $a:b=4$.

$\underline{m=5}$; $\frac{5\pi b}{2a}=\frac{5}{8}\pi=1{,}96$; $2\cdot\frac{2a}{5\pi b}=2\cdot 0{,}51=1{,}02$; $\mathfrak{Tg}\,\frac{5\pi b}{2a}=0{,}961$; $\frac{5\pi b}{4a}\cdot\mathfrak{Tg}\,1{,}96=0{,}98\cdot 0{,}96=0{,}94$.

$y:b$	0	$^1/_2$	$^3/_4$	1
$\frac{m\pi}{2a}y=\frac{m\pi}{8}\cdot\left(\frac{y}{b}\right)$	0,0	0.98	1,48	1,96
$\mathfrak{Cof}\,\frac{m\pi}{8}\cdot\frac{y}{b}$	1,00	1,52	2,31	3,62
$\mathfrak{Sin}\,\frac{m\pi}{8}\cdot\frac{y}{b}$	0,00	1,14	2,08	3,48
a) $\mathfrak{Cof}\,\frac{m\pi}{8}\cdot\frac{y}{b}:\mathfrak{Cof}\,k\,\frac{a}{b}$	0,27	0,42	0,64	1,00
b) $\mathfrak{Cof}\,\frac{m\pi}{8}\cdot\frac{y}{b}:\mathfrak{Sin}\,k\cdot\frac{a}{b}$	0,29	0,44	0,67	1,04
c) $\mathfrak{Sin}\,k\,\frac{y}{b}:\mathfrak{Cof}\,k\,\frac{a}{b}$	0,00	0,31	0,57	0,96
d) $\mathfrak{Sin}\,k\,\frac{y}{b}:\mathfrak{Sin}\,k\,\frac{a}{b}$	0,00	0,33	0,60	1,00
e) $\frac{y}{b}\left(\mathfrak{Cof}\,k\,\frac{y}{b}:\mathfrak{Sin}\,k\,\frac{a}{b}\right)$	0	0,22	0,56	1,04
f) $\frac{4a}{b\,m\pi}\left(\mathfrak{Cof}\,k\,\frac{y}{b}:\mathfrak{Sin}\,k\,\frac{a}{b}\right)$	0,29	0,45	0,68	1,06
g) $\frac{y}{b}\left(\mathfrak{Sin}\,k\,\frac{y}{b}:\mathfrak{Sin}\,k\,\frac{a}{b}\right)$	0,0	0,17	0,45	1,00
h) $\frac{2a}{b\,m\pi}\left(\mathfrak{Sin}\,k\,\frac{y}{b}:\mathfrak{Sin}\,k\,\frac{a}{b}\right)$	0,0	0,17	0,31	0,51
$a-g$	0,27	0,25	0,19	0,0
$\sigma_y=\frac{m\pi b}{4a}\cdot\mathfrak{Tg}\,\frac{m\pi b}{2a}(a-g)$	0,25	0,24	0,18	0,0 σ_m
$f-(a-g)$	0,02	0,20	0,49	1,06
$\sigma_x=k\,(f-a+g)$	0,02	0,19	0,46	1,06 σ_m
$e+h$	0,0	0,39	0,81	1,55
$e+h-c$	0,0	0,08	0,24	0,59
$\tau=k\,(e+h-c)$	0,0	0,07	0,23	0,56 σ_m

$$B=\frac{b}{2}\mathfrak{Tg}\left(k\,\frac{a}{b}\right)\cdot\left[-\mathfrak{Tg}\left(k\,\frac{a}{b}\right)+\frac{1}{\mathfrak{Tg}\,k\,\frac{a}{b}}+\frac{2a}{b\,m\cdot\pi}\right].$$

$$B_5=\frac{b}{2}\cdot 0{,}96\left(-0{,}96+\frac{1}{0{,}96}+0{,}51\right).$$

$$B_5=b\cdot 0{,}48\cdot 0{,}59=0{,}285\,b\,.$$

Tabelle 18.

Cosinusförmiges Moment.

$\underline{a:b=4;}$ $\underline{m=7;}$ $\frac{7\pi b}{2a}=2{,}75$; $\frac{4a}{7\pi b}=0{,}73$; $\mathfrak{Tg}\,\frac{7\pi b}{2a}=0{,}99$; $\frac{7\pi b}{4a}\cdot\mathfrak{Tg}\,2{,}75=1{,}38\cdot 0{,}99=1{,}37$.

$y:b$	0	$^1/_2$	$^3/_4$	1
$\frac{m\pi}{2a}\cdot y=\frac{m\pi}{8}\cdot\frac{y}{b}$	0,0	1,38	2,06	2,75
$\mathfrak{Cof}\,\frac{m\pi}{8}\cdot\frac{y}{b}$	1,00	2,11	3,99	7,85
$\mathfrak{Sin}\,\frac{m\pi}{8}\cdot\frac{y}{b}$	0,00	1,86	3,82	7,79

Fortsetzung von Tabelle 18.

$y:b$	0	1/2	3/4	1
a) $\mathfrak{Cof}\frac{m\pi}{8}\cdot\frac{y}{b}:\mathfrak{Cof}\,k\frac{a}{b}$	0,13	0,27	0,51	1,00
b) $\mathfrak{Cof}\frac{m\pi}{8}\cdot\frac{y}{b}:\mathfrak{Sin}\,k\frac{a}{b}$	0,13	0,27	0,51	1,00
c) $\mathfrak{Sin}\,k\frac{y}{b}:\mathfrak{Cof}\,k\frac{a}{b}$	0,00	0,24	0,49	0,99
d) $\mathfrak{Sin}\,k\frac{y}{b}:\mathfrak{Sin}\,k\frac{a}{b}$	0,0	0,24	0,49	1,00
e) $\frac{y}{b}\left(\mathfrak{Cof}\,k\frac{y}{b}:\mathfrak{Sin}\,k\frac{a}{b}\right)$	0	0,14	0,38	1,01
f) $\frac{4a}{b\,m\pi}\left(\mathfrak{Cof}\,k\frac{y}{b}:\mathfrak{Sin}\,k\frac{a}{b}\right)$	0,10	0,20	0,37	0,74
g) $\frac{y}{b}\left(\mathfrak{Sin}\,k\frac{y}{b}:\mathfrak{Sin}\,k\frac{a}{b}\right)$	0	0,13	0,37	1,00
h) $\frac{2a}{b\,m\pi}\left(\mathfrak{Sin}\,k\frac{y}{b}:\mathfrak{Sin}\,k\frac{a}{b}\right)$	0,0	0,09	0,18	0,36
$a-g$	0,13	0,14	0,14	0
$\sigma_y=\frac{m\pi b}{4a}\cdot\mathfrak{Tg}\frac{m\pi\cdot b}{2a}(a-g)$	0,18	0,19	0,19	0,00 σ_m
$f-(a-g)$	−0,03	0,06	0,24	0,74
$\sigma_x=K(f-a+g)$	−0,04	0,08	0,33	1,00 σ_m
$e+h$	0,0	0,23	0,56	0,37
$e+h-c$	0,0	−0,01	0,07	0,38
$\tau=k(e+h-c)$	0,0	−0,11	0,10	0,51 σ_m

$$B=\frac{b}{2}\cdot\frac{2a}{b\,m\pi}=\frac{a}{m\pi}=\frac{4b}{3{,}14\cdot 7}=0{,}32\,a$$

$$B_7=\frac{b}{2}\cdot 0{,}99\,(-0{,}99+1{,}01+0{,}37)$$

$$B_7=0{,}18\,b.$$

Tabelle 19.

Zwei symmetrische Lasten. Seitenverhältnis: $a:b=4$.

$$F_1(x,y)=-1{,}61\cdot 0{,}71\,\sigma_m\left[\left(\frac{\pi}{2a}\right)^2\cdot\cos\frac{\pi}{2a}\cdot x\cdot F\left(\frac{\pi}{2a}\cdot y\right)-\frac{1}{9}\left(\frac{3\pi}{2a}\right)^2\cos\frac{3\pi}{2a}\cdot x\,F\left(\frac{3\pi}{2a}y\right)\right.$$
$$\left.-\frac{1}{25}\cdot\left(\frac{5\pi}{2a}\right)^2\cdot\cos\frac{5\pi}{2a}x\cdot F\left(\frac{5\pi}{2a}y\right)+\frac{1}{49}\left(\frac{7\pi}{2a}\right)^2\cos\frac{7\pi}{2a}x\cdot F\left(\frac{7\pi}{2a}\cdot y\right)\right]$$

$$\sigma_x=1{,}61\cdot 0{,}71\,\sigma_m\left[\frac{b\pi}{4a}\cos\frac{\pi}{2a}x\cdot F_1''\left(\frac{\pi}{2a}y\right)-\frac{1}{9}\,\frac{3\pi b}{4a}\cos\frac{3\pi}{2a}x\cdot F_1''\left(\frac{3\pi}{2a}y\right)-\frac{1}{25}\,\frac{5\pi b}{4a}\cos\frac{5\pi}{2a}x\,F_1''\,\frac{5\pi}{2a}y\right].$$

σ_x für $x=0$.

$y:b$		0	1/4	1/2	3/4	1
σ_x für $m=1$		0,86	0,88	0,90	0,94	1,00
$\frac{1}{9}\sigma_x$ für $m=3$		0,03	−0,04	−0,05	−0,07	−0,11
$\frac{1}{25}\sigma_x$ für $m=5$		—	—	—	−0,02	−0,04
$\frac{1}{49}\sigma_x$ für $m=7$		—	—	—	+0,01	+0,02
	Summe	0,83	0,84	0,84	0,86	0,87
$\sigma_x(x=0)=$		0,93	0,94	0,94	0,98	1,00 σ_m

Fortsetzung von Tabelle 19

τ für $x = a$

$y:b$	0	$^1/_4$	$^1/_2$	$^3/_4$	1
$+$	0	0,09	0,17	0,26	0,36
$\left(\sin\frac{3\pi}{2a}x\right) = -1 +$	0	+ 0,01	+ 0,02	+ 0,04	+ 0,06
$\sin\frac{5\pi}{2a}x = +1 -$	—	—	0,00	— 0,01	— 0,02
$\sin\frac{7\pi}{2a}x = -1 -$	—	—	—	—	— 0,01
Σ	0	0,10	0,19	0,29	0,39
$\tau = 1{,}14\,\sigma_m \cdot \Sigma$	0	0,11	0,22	0,32	0,44 σ_m
	σ_y für $x = 0$				
$m = 1$	0,072	0,068	0,054	0,032	0,00
$m = 3$	— 0,030	— 0,030	— 0,024	— 0,016	— 0,00
$m = 5$	— 0,010	— 0,009	— 0,009	— 0,007	— 0,00
$m = 7$	0,004	0,004	0,004	0,004	0,00
Σ	0,036	0,033	0,023	0,013	0,00
$\sigma_y = 1{,}14 \cdot \Sigma$	0,041	0,038	0,026	0,015	0,00 σ_m

Tabelle 20.

Trigonometrische Funktionen von x.

$\frac{x}{a}$	0	0,3	0,5	0,7	0,8	0,9	1,0
$\frac{\pi}{2}\cdot\frac{x}{a}$ (Bogen)	0	$\frac{3\pi}{20}$	$\frac{\pi}{4}$	$\frac{7\pi}{20}$	$\frac{2\pi}{5}$	$\frac{9\pi}{20}$	$\frac{\pi}{2}$
$\frac{\pi}{2}\cdot\frac{x}{a}$ (Grad)	0°	27°	45°	63°	72°	81°	90°
$\frac{3\pi}{2}\cdot\frac{x}{a}$ (Grad)	0°	81°	135°	189°	216°	243°	270°
$\frac{5\pi}{2}\cdot\frac{x}{a}$ (Grad)	0°	135°	225°	315°	360°	405°	450°
$\frac{7\pi}{2}\cdot\frac{x}{a}$ (Grad)	0°	189°	315°	441°	504°	567°	630°
$\cos\frac{\pi}{2}\cdot\frac{x}{a}$	1,0	0,89	0,71	0,45	0,31	0,16	0,00
$\cos\frac{3\pi}{2}\cdot\frac{x}{a}$	1,0	0,16	— 0,71	— 0,99	— 0,81	— 0,45	0,00
$-\frac{1}{9}\cos\frac{3\pi}{2}\cdot\frac{x}{a}$	— 0,11	— 0,02	+ 0,08	+ 0,11	+ 0,09	+ 0,05	0,00
$\cos\frac{5\pi}{2}\cdot\frac{x}{a}$	1,0	— 0,71	— 0,71	+ 0,71	+ 1,0	+ 0,71	0,00
$-\frac{1}{25}\cos\frac{5\pi}{2}\cdot\frac{x}{a}$	— 0,04	+ 0,03	+ 0,03	— 0,03	— 0,04	— 0,03	0,00
$\cos\frac{7\pi}{2}\cdot\frac{x}{a}$	1,00	— 0,99	+ 0,71	+ 0,16	— 0,81	— 0,89	0,00
$\frac{1}{49}\cos\frac{7\pi}{2}\cdot\frac{x}{a}$	+ 0,02	— 0,02	+ 0,01	+ 0,003	— 0,02	— 0,02	0,00
$\sin\frac{\pi}{2}\cdot\frac{x}{a}$	0,0	0,45	0,71	0,89	0,95	0,99	1,00

Fortsetzung von Tabelle 20.

$\frac{x}{a}$	0	0,3	0,5	0,7	0,8	0,9	1,0
$\sin\frac{3\pi}{2}\cdot\frac{x}{a}$	0,0	0,99	0,71	—0,16	—0,59	—0,89	—1,00
$-\frac{1}{9}\sin\frac{3\pi}{2}\cdot\frac{x}{a}$	—0,0	—0,11	—0,08	+0,02	+0,06	+0,10	+0,11
$\sin\frac{5\pi}{2}\cdot\frac{x}{a}$	0,0	0,71	—0,71	—0,71	0,00	0,71	1,00
$-\frac{1}{25}\sin\frac{5\pi}{2}\cdot\frac{x}{a}$	0,0	—0,03	+0,03	+0,03	+0,00	—0,03	—0,04
$\sin\frac{7\pi}{2}\cdot\frac{x}{a}$	0,0	0,16	—0,71	+0,99	+0,59	—0,45	—1,00
$\frac{1}{49}\sin\frac{7\pi}{2}\cdot\frac{x}{a}$	0,0	0,00	—0,01	+0,02	+0,01	—0,01	—0,02

Tabelle 21.

Zwei symmetrische Lasten. $a:b=4$. Normalspannungen durch Lösung 1.

$$\sigma_x = \frac{16}{\pi^2}\cdot\sin\frac{m\pi}{4}\cdot\frac{(-1)^{\frac{m+1}{2}}}{m^2}, \qquad \cos\frac{m\pi}{2a}\,x\left(\frac{2a}{m\pi}\right)^2 F''(y)\,.$$

$x:a$	0,0	0,3	0,5	0,7	0,8	0,9	1,00
$y:b=0$							
$\cos\frac{\pi}{2a}\cdot x\left(\frac{2a}{\pi}\right)^2 F''(y)$	+0,86	+0,77	0,61	0,39	0,26	0,13	0,00
$-\frac{1}{9}\cos\frac{3\pi}{2a}\,x\left(\frac{2a}{3\pi}\right)F''(y)$	—0,03	—0,01	+0,02	+0,03	+0,03	+0,01	0,00
Summe	0,83	0,76	0,63	0,42	0,29	0,14	0,00
$\sigma_x = 1{,}14\cdot$ Summe	0,95	0,87	0,72	0,48	0,33	0,16	0,00 σ_m
$y:b=\frac{1}{4}$							
$\cos\frac{\pi}{2a}\,x\left(\frac{2a}{\pi}\right)^2 F''(y)$	+0,88	+0,79	0,62	0,40	0,27	0,13	0,0
$-\frac{1}{9}\cos\frac{3\pi}{2a}\,x\left(\frac{2a}{3\pi}\right)^2 F''(y)$	—0,04	—0,02	0,03	0,03	0.03	0,01	0,0
Summe	0,84	0,77	0,65	0,43	0,30	0,14	0,0
$\sigma_x = 1{,}14\cdot$ Summe	0,95	0,88	0,74	0,49	0,34	0,16	0.05 σ_m
$y:b=\frac{1}{2}$							
$\cos\frac{\pi}{2a}\,x\left(\frac{2a}{\pi}\right)^2 F''y$	+0,90	+0,81	0,64	0,41	0,28	0,14	0,0
$-\frac{1}{9}\cos\frac{3\pi}{2a}\,x\left(\frac{2a}{3\pi}\right)^2 F''y$	—0,05	—0,01	+0,04	0,05	0,04	0,02	0,0
$-\frac{1}{25}\cos\frac{5\pi}{2a}\,x\left(\frac{2a}{5\pi}\right)^2 F''y$	—0,01	+0,01	0.01	—0,01	—0,01	—0,01	0,0
Summe	+0,84	+0,81	0,69	0,45	0,31	0,15	0,0
$\sigma_x = 1{,}14\cdot$ Summe	+0,96	+0,90	0,79	0,51	0,35	0,17	0,0 σ_m
$y:b=\frac{3}{4}$	0,95	0,85	0,67	0,43	0,29	0,15	0,0
$\cos\frac{\pi}{2a}\,x\left(\frac{2a}{\pi}\right)F''(y)$	—0,07	—0,01	+0,05	+0,07	+0,06	+0,03	0,0
$-\frac{1}{9}\cos\frac{3\pi}{2a}\,x\left(\frac{2a}{3\pi}\right)F''(y)$	—0,02	+0,01	0,01	—0,01	—0,02	—0,01	0,0
$-\frac{1}{25}\cos\frac{5\pi}{2a}\,x\left(\frac{2a}{5\pi}\right)F''(y)$	+0,01	—0,01	0,00	+0,01	—0,03	—0,01	0,0
Summe	0,87	0,84	0,73	0,50	0,32	0,16	0,0
$\sigma_x = 1{,}14\cdot$ Summe	0,97	0,96	0,83	0,57	0,36	0,18	0,0 σ_m

Fortsetzung von Tabelle 21.

$x : a$	0,0	0,3	0,5	0,7	0,8	0,9	1,00	
$y : b = 1$								
$\cos \frac{\pi}{2a} \cdot x F(y)$	1,0	0,89	0,71	0,45	0,31	0,16	0,00	
$\frac{1}{9} \cos \frac{3\pi}{2a} x F(y)$	$-0{,}11$	$-0{,}02$	$+0{,}08$	$+0{,}11$	$+0{,}09$	$+0{,}05$	0,0	
	$-0{,}04$	$+0{,}03$	$+0{,}03$	$-0{,}03$	$+0{,}04$	$-0{,}03$	0,00	
	$+0{,}02$	$-0{,}02$	$+0{,}01$	—	$-0{,}02$	$-0{,}02$	0,00	
Summe	0,87	0,90	0,83	0,56	0,34	$+0{,}16$	0,00	
$\sigma_x = 1{,}14 \cdot$ Summe	1,00	1,02	0,95	0,64	0,39	0,18	0,00	σ_m
Erwünschtes Resultat	1,00	1,00	1,00	0,60	0,40	0,20	0,00	σ_m

Tabelle 22.

Zwei symmetrische Lasten. $a : b = 4$.

Schubspannungen der ersten Lösung $\tau = \sum \frac{16}{\pi_2} \sin \frac{m\pi}{4} \cdot \sin \frac{m\pi}{2} \sin \frac{n\pi}{2a} \cdot x \left(\frac{m\pi}{2a}\right) F'(y)$.

$x : a$	0,0	0,3	0,5	0,7	0,8	0,9	1,0	
für $y : b = 0$ $\tau =$	0	0	0	0	0	0	0	σ_m
$y : b = 1 : 4$								
$\sin \frac{\pi}{2a} x \frac{2a}{\pi} F'(y)$	0,0	0,04	0,06	0,08	0,09	0,09	0,09	
		$-0{,}01$	$-0{,}01$	$+0{,}01$	$+0{,}01$	$+0{,}01$	$+0{,}01$	
$\sum$	0,0	0,03	0,05	0,08	0,10	0,10	0,10	
$\tau = 1{,}14 \sum$	0,0	0,03	0,06	0,09	0,11	0,11	0,11	σ_m
$y : b = 1 : 2$								
$\sin \frac{\pi}{2a} \cdot x \left(\frac{2a}{\pi}\right) \cdot F'(y)$	0,0	0,08	0,12	0,15	0,16	0,17	0,17	
	0,0	$-0{,}02$	$-0{,}02$	0,00	0,01	0,02	0,02	
$\sum$	0,0	0,06	0,10	0,15	0,17	0,19	0,19	
$\tau = 1{,}14 \sum$	0,0	0,07	0,11	0,17	0,19	0,21	0,22	σ_m
$y : b = 3 : 4$								
$\sin \frac{\pi}{2a} \cdot x \left(\frac{2a}{\pi}\right) F'(y)$	0,0	0,12	0,18	0,23	0,25	0,26	0,26	
		$-0{,}04$	$-0{,}03$	$+0{,}01$	$+0{,}02$	$+0{,}03$	$+0{,}04$	
		$-0{,}01$	$+0{,}01$	$+0{,}01$	$+0{,}00$	$-0{,}01$	$-0{,}01$	
$\sum$	0,0	0,07	0,16	$+0{,}25$	0,27	0,28	0,29	
$\tau = 1{,}14 \cdot \sum$	0,0	0,08	0,18	0,28	0,31	0,32	0,33	σ_m
$y : b = 1 : 1$								
$\sin \frac{\pi}{2a} \cdot x \left(\frac{2a}{\pi}\right) F'y$	0,0	0,16	0,25	0,32	0,34	0,36	0,36	
	0,0	$-0{,}06$	$-0{,}05$	$+0{,}01$	$+0{,}04$	$+0{,}06$	$+0{,}06$	
	0,0	$-0{,}02$	$+0{,}02$	$+0{,}02$	$+0{,}00$	$-0{,}02$	$-0{,}02$	
	0,0	$+0{,}00$	$-0{,}01$	$+0{,}01$	$+0{,}01$	$-0{,}01$	$-0{,}01$	
$\sum$	0,0	$+0{,}08$	0,21	$+0{,}36$	0,39	0,39	0,39	
$\tau = 1{,}14 \cdot \sum$	0,0	0,09	0,26	0,39	0,42	0,46	0,46	σ_m

Tabelle 23.

Zwei symmetrische Lasten. $a : b = 4$.

Querspannung $\sigma_y = \frac{16}{\pi^2} \sin \frac{m\pi}{4} \cdot \sin \frac{m\pi}{2} \cdot \cos \frac{m\pi}{2a} x \cdot F(y)$.

$x : a =$	0,0	0,3	0,5	0,7	0,8	0,9	1,0	
a) $y : b = 0$								
$\cos \frac{\pi}{2a} \cdot x \cdot Fy$	0,07	0,06	0,05	0,03	0,02	0,01	0,0	
$-\frac{1}{9} \cos \frac{3\pi}{2a} \cdot x \cdot F(y)$	—0,03	—0,01	0,02	0,03	0,02	0,01	0,0	
$-\frac{1}{25} \cos \frac{5\pi}{2a} \cdot x \cdot F(y)$	—0,01	0,01	0,01	—0,01	—0,01	—0,01	0,0	
		0,00	0,00	+0,01	0,00	0,0	0,0	
$\sum$	0,03	0,06	0,08	0,06	0,03	0,01	0,0	
$\sigma_y = 1,14 \sum =$	0,034	0,074	0,09	0,07	0,034	0,01	0,0	σ_m
b) $y : b = 1 : 4$; $\cos \frac{\pi}{2a} x F y$	0,07	0,06	0,05	0,03	0,02	0,01	0,00	
$-\frac{1}{9} \cos \frac{3\pi}{2a} x F y$	—0,03	—0,01	+0,02	0,03	0,02	0,01	0,00	
$-\frac{1}{25} \cos \frac{5\pi}{2a} x F y$	—0,01	0,01	0,01	—0,01	—0,01	—0,01	0,00	
$\sum$	0,03	0,06	0,08	0,05	0,03	0,01	0,00	
$\sigma_y = 1,14 \sum =$	0,03	0,07	0,09	0,06	0,03	0,01	0,00	σ_m
c) $y : b = 1 : 2$; $\cos \frac{\pi}{2a} x F y$	0,05	0,05	0,04	0,02	0,02	0,01	0,00	
$-\frac{1}{9} \cos \frac{3\pi}{2a} x F y$	—0,02	—0,00	+0,02	0,02	0,02	0,01	0,00	
$-\frac{1}{25} \cos \frac{5\pi}{2a} \cdot x F y$	—0,01	0,01	0,01	—0,01	—0,01	—0,01	0,00	
$\sum$	0,02	0,06	0,07	0,03	0.03	0,01	0,00	
$\sigma_y = 1,14 \sum$	0,02	0,07	0,08	0,04	0,03	0,01	0,00	σ_m
d) $y : b = 3 : 4$; $\cos \frac{\pi}{2a} x F y$	0,03	0,03	0,02	0,01	0,01	0,005	0,00	
$-\frac{1}{9} \cos \frac{3\pi}{2a} x F y$	—0,02	0,00	0,01	0,02	0,01	0,007	0,00	
$-\frac{1}{25} \cos \frac{5\pi}{2a} x F y$	—0,01	+0,01	0,01	—0,01	—0,01	—0,005	0,00	
$\sum$	0,00	0,04	0,04	0,02	0,01	0,01	0,00	
$\sigma_y = 1,14 \sum$	0,00	0,05	0,05	0,03	0,01	0,01	0,00	σ_m
e) $y : b = 1 : 1$; σ_y	0,00	0,00	0,00	0,00	0,00	0,00	0,00	

Tabelle 24.

Berechnung der Lösung 2 für Moment durch zwei Lasten.
Seitenverhältnis $a : b = 4$.

$1 - \frac{x}{a}$	1,0	0,7	0,5	0,3	0,2	0,1	0,0
$1 - \frac{x}{a} - \frac{2b}{a\pi}$	0,84	0,54	0,34	0,14	0,04	—0,06	—0,16
$1 - \frac{x}{a} - \frac{2b}{3a\pi}$	0,95	0,65	0,45	0,25	0,15	+0,05	—0,05

Fortsetzung von Tabelle 24.

$1-\frac{a}{x}$	1,0	0,7	0,5	0,3	0,2	0,1	0,0
$1-\frac{x}{a}-\frac{2b}{a\pi}$	—	—	—	—	0,17	0,07	—0,03
$1-\frac{x}{a}-\frac{4b}{a\pi}$	0,68	0,38	0,18	—0,02	—0,12	—0,22	—0,32
$1-\frac{x}{a}-\frac{4b}{3a\pi}$	—	—	—	0,19	0,09	—0,01	—0,11
$1-\frac{x}{a}-\frac{4b}{3a\pi}$	—	—	—	—	—	0,04	—0,06
$-\frac{\pi a}{2b}\left(1-\frac{x}{a}\right)$	—6,28	—4,40	—3,14	—1,88	—1,25	—0,63	—0,00
$-\frac{3\pi a}{2b}\left(1-\frac{x}{a}\right)$	—	—	—	—	—5,64	—3,75	1,88
$-\frac{5\pi a}{2b}\left(1-\frac{x}{a}\right)$	—	—	—	—	—6,28	—3,14	0,00
$e^{-\left(\frac{\pi a}{2b}\left[1-\frac{x}{a}\right]\right)}$	0,00	0,013	0,04	0,15	0,29	0,53	1,00
$e^{-\left(\frac{3\pi a}{2b}\left[1-\frac{x}{a}\right]\right)}$	—	—	—	0,004	0,02	0,15	1,00
$e^{-\left(\frac{5\pi a}{2b}\left[1-\frac{x}{a}\right]\right)}$	—	—	—	—	00,0	0,04	1,00
(a) $\frac{\pi a}{2b}\cdot e^{-\left(\frac{\pi a}{2b}\left[1-\frac{x}{a}\right]\right)}$	0,00	0,08	0,27	0,96	1,80	3,35	6,28
(b) $\frac{3\pi a}{2b}\cdot e^{-\left(\frac{3\pi a}{2b}\left[1-\frac{x}{a}\right]\right)}$	—	—	—	0,07	0,43	2,90	18,84
(c) $\frac{5\pi a}{2b}\cdot e^{-\left(\frac{5\pi a}{2b}\left[1-\frac{x}{a}\right]\right)}$	—	—	—	—	0,0	1,26	31,40
0,356 · (a)	0,00	0,03	0,10	0,34	0,65	1,20	2,25
— 0,045 (b)	—	—	—	—	—0,02	—0,13	—0,85
+ 0,015 (c)	—	—	·	—	—	+ 0,02	+ 0,47

Tabelle 25.

Normalspannungen σ_{x2} (2. Lösung).
Seitenverhältnis $a:b=4$; Moment durch 2 Lasten.

$$\sigma_{x2}=\sum\sigma_m\cdot C_n\cos\frac{n\pi}{2b}\cdot y\,\frac{a\pi n}{2b}\cdot\left(1-\frac{x}{a}\right)e^{-\frac{a\pi n}{2b}\left(1-\frac{x}{a}\right)}(-1)^{\frac{n+1}{2}}$$

$C_1=0{,}356$; $C_3=0{,}045$; $C_5=0{,}015$; $C_7=0{,}008$.

$1-\frac{x}{a}$	1,0	0,7	0,5	0,3	0,2	0,1	1,0	
	σ_{x2} für $y=0$; $\cos\frac{n\pi}{2b}y=0$.							
σ_{x2} f. $n=1$	0,0	0,02	0,05	0,10	0,13	0,12	0,0	
$n=3$	—	—	—	—	—	—0,01	0,0	
$n=5$	—	—	—	—	—	0,00	0,0	
σ_{x2} f. $y=0=\sum$	0,0	0,02	0,05	0,10	0,13	0,11	0,0	σ_m
	σ_{x2} für $y=\frac{b}{4}$; $\cos\frac{n\pi}{2b}\cdot y=\cos\frac{\pi}{8}$; $\cos\frac{3\pi}{8}$							
σ_{x2} f. $n=1$	0,0	0,02	0,05	0,09	0,12	0,10	0,00	
$n=3$	—	—	—	—	—	—0,01	0,00	
$\sigma_{x2}\left(\text{f. } y=\frac{b}{4}\right)=\sum$	0,0	0,02	0,05	0,09	0,12	0,09	0,00	σ_m

Fortsetzung von Tabelle 25.

$1-\frac{x}{a}$	1,0	0,7	0,5	0,3	0,2	0,1	1,0
σ_{x2} für $y=\frac{b}{2}$; $\cos\frac{n\pi}{2b}\cdot y=\cos\frac{\pi}{4}$; $\cos\frac{3\pi}{4}$							
σ_{x2} f. $n=1$	0,0	0,01	0,03	0,07	0,09	0,08	0,00
$n=3$	—	—	—	—	—	+0,01	0,00
$\sigma_{x2}\left(\text{f. } y=\frac{b}{2}\right)=\sum$	0,0	0,01	0,03	0,07	0,09	0,09	0,00 σ_m
σ_{x2} für $y=\frac{3b}{4}$; $\cos\frac{n\pi}{2b}\cdot y=\cos\frac{3\pi}{8}$; $\cos\frac{9\pi}{8}$.							
σ_{x2} f. $n=1$	0,00	0,01	0,02	0,04	0,05	0,05	0,00
$n=3$	—	—	—	—	—	+0,01	0,00
$\sigma_{x2}\left(\text{f. } y=\frac{3b}{4}\right)=\sum$	0,00	0,01	0,02	0,04	0,05	0,06	0,00 σ_m
σ_{x2} (f. $y=b$) $=$	0	0	0	0	0	0	0 σ_m

Tabelle 26.

Schubspannungen τ_2 (2. Lösung).

Seitenverhältnis $a:b=4$ und Moment durch 2 Lasten.

$$\tau_2=\sum C_n \sin\frac{n\pi}{2b}\cdot y\left(1-\frac{x}{a}-\frac{2b}{a n\pi}\right)\frac{a n\pi}{2b}e^{-\frac{n\pi}{2b}(a-x)}.$$

$\frac{x}{a}$	0	0,3	0,5	0,7	0,8	0,9	0,0
$\left(1-\frac{x}{a}-\frac{2b}{a\pi}\right)\cdot e^{-\frac{\pi}{2b}(a-x)}$.	0,0	0,04	0,09	0,13	0,07	—0,20	—1,00
$\left(1-\frac{x}{a}-\frac{2b}{3a\pi}\right)e^{-\frac{3\pi}{2b}(a-x)}$	—	—	—	0,02	0,06	+0,15	—1,00
$\left(1-\frac{x}{a}-\frac{2b}{5a\pi}\right)e^{-\frac{5\pi}{2b}(a-x)}$	—	—	—	—	—	+0,09	—1,00
τ_2 für $y=b$; $\sin\frac{n\pi}{2b}\cdot y=+1$; -1; $+1$.							
τ_2 für $n=1$	0,0	0,02	0,03	0,05	0,02	—0,07	—0,356
$n=3$	—	—	—	—	0,00	0,01	—0,045
$n=5$	—	—	—	—	—	0,00	—0,015
	—	—	—	—	—	—	—0,008
τ_2 (für $y=b$) $=\sum$	0,0	0,02	0,03	0,05	0,02	—0,07	—0,424 σ^m
τ_2 für $\cdot y=\frac{3b}{4}$; $\sin\frac{n\pi}{2b}\cdot y=\sin\frac{3\pi}{8}$; $\sin\frac{9\pi}{8}$; $\sin\frac{15\pi}{8}$.							
τ_2 für $n=1$	0,0	0,02	0,03	0,04	0,02	—0,06	— 0,33
$n=3$	—	—	—	—	—	—	— 0,01
$n=5$	—	—	—	—	—	—	+ 0,01
$\tau_2\left(\text{für } y=\frac{3b}{4}\right)=\sum$	0,0	0,02	0,03	0,04	0,02	—0,06	0,33 σ^m

Fortsetzung von Tabelle 26.

$\frac{x}{a}$	0	0,3	0,5	0,7	0,8	0,9	0,0
τ_2 für $\cdot\, y = \frac{b}{2}$; $\sin\frac{n\pi}{2b}\cdot y = \sin\frac{\pi}{4}$; $\sin\frac{3\pi}{4}$; $\sin\frac{5}{4}$.							
τ_2 für $n = 1$	0,0	0,01	0,02	0,04	0,02	—0,05	—0,25
$n = 3$	—	—	—	—	—	—0,02	+0,03
$n = 5$	—	—	—	—	—	—	+0,01
$\tau_2\left(\text{für } y = \frac{b}{2}\right) = \sum$	0,0	0,01	0,02	0,04	0,02	—0,07	—0,21 σ_m
τ_2 für $y = \frac{b}{4}$; $\sin\frac{n\pi}{2b}\cdot y = \sin\frac{\pi}{8}$; $\sin\frac{3\pi}{8}$; $\sin\frac{5\pi}{8}$.							
$\tau_2 =$ für $n = 1$	0,0	0,01	0,01	0,02	0,01	—0,03	—0,14
	—	—	—	—	—	—0,01	+0,04
	—	—	—	—	—	—	—0,01
$\tau_2\left(\text{für } y = \frac{b}{4}\right) = \sum$	0,0	0,01	0,01	0,02	0,01	—0,04	0,11 σ_m
τ_2 (für $y = 0$) $=$	0	0	0	0	0	0	0 σ_m

Tabelle 27.

Querspannungen σ_{y2} für ein Moment aus 2 Lasten.
Seitenverhältnis $a : b = 4$.

$$\sigma_{y2} = \sum C_n\,\sigma_m\cdot\cos\frac{n\pi}{2b}\cdot y\left(1 - \frac{x}{a} - \frac{4b}{an\pi}\right)\frac{an\pi}{2b}\,e^{-\frac{a\pi}{2b}(a-x)}$$

$1 - \frac{x}{a}$	1,0	0,7	0,5	0,3	0,2	0,1	0,0
$\left(1 - \frac{x}{a} - \frac{4b}{a\pi}\right)e^{-\frac{\pi}{2b}(a-a)}$	0,0	0,03	0,05	—0,02	—0,22	—0,73	—2,00
$\left(1 - \frac{x}{a} - \frac{4b}{3a\pi}\right)e^{-\frac{3\pi}{2b}(a-x)}$	—	—	—	+0,01	+0,04	0,03	—2,00
$\left(1 - \frac{x}{a} - \frac{4b}{5a\pi}\right)e^{-\frac{5\pi}{2b}(a-x)}$	—	—	—	—	—	—	—2,00
σ_{y2} für $y = 0$.							
$\sigma_{y2} = 0{,}356\cdot\cos 0\, F(x)$	0,0	0,01	0,02	—0,01	—0,08	—0,26	0,71
$= -0{,}045\cos 0\, F(x)$	—	—	—	—	—0,002	+0,00	+0,09
$= 0{,}015\cos 0\, F(x)$	—	—	—	—	—	—	—0,03
σ_{y2} (für $y = 0$) $= \sum$	0,0	0,01	0,02	—0,01	—0,00	—0,26	—0,65 σ_m
σ_{y2} für $y = \frac{b}{4}$; $\cos\frac{\pi}{8}$; $\cos\frac{3\pi}{8}$; $\cos\frac{5\pi}{8}$.							
σ_{y2} f. $n = 1$ —	0,0	0,01	0,02	—0,01	—0,07	—0,24	—0,66
$n = 3$ +	—	—	—	—	—	—	+0,03
$n = 5$ —	—	—	—	—	—	—	+0,01
$\sigma_{y2}\left(\text{für } y = \frac{b}{4}\right) = \sum$	0,00	0,01	0,02	—0,01	—0,07	—0,24	—0,62 σ_m
σ_{y2} für $y = \frac{b}{2}\cdot\cos\frac{n\pi}{2b}\cdot y = \cos\frac{\pi}{4}$; $\cos\frac{3\pi}{4}$; $\cos\frac{5\pi}{4}$.							
σ_{y2} $n = 1$ —	0,0	0,01	0,01	0,00	—0,05	—0,18	—0,50
$n = 3$ +	—	—	—	—	—	—	—0,06
$n = 5$ —	—	—	—	—	—	—	+0,02
$\sigma_{y2}\left(\text{für } y = \frac{b}{2}\right) = \sum$	0,0	0,01	0,01	0,00	—0,05	—0,18	—0,54 σ_m

Fortsetzung von Tabelle 27.

σ_{y2} für $y = \frac{3b}{4}$. $\cos \frac{n\pi}{2b} \cdot y = \cos \frac{3\pi}{8}$; $\cos \frac{9\pi}{8}$; $\cos \frac{15\pi}{8}$.

$1 - \frac{x}{a}$	1,0	0,7	0,5	0,3	0,2	0,1	0,0	
σ_{y2} $n = 1 +$	0,00	0,004	0,01	0,00	—0,03	—0,10	—0,27	
$n = 3 -$	—	—	—	—	—	—	—0,08	
$n = 5 +$	—	—	—	—	—	—	—0,03	
$\sigma_{y2}\left(\text{f. } y = \frac{3b}{4}\right) = \sum$	0,00	0,004	0,01	0,00	—0,03	—0,10	—0,38	σ_m
σ_{y2} f. $y = b =$	0	0	0	0	0	0	0	

Tabelle 28.

Zusammenstellung der Spannungen für 2 symmetrische Lasten. $a : b = 4$.

$F(xy) = F_1(xy) + F_2(xy)$.

Normalspannungen $\sigma_x = \sigma_{x1} + \sigma_{x2}$.

$x : a$		0,0	0,3	0,5	0,7	0,8	0,9	1,0	
$y : b = 0$	1. Lösung	0,95	0,87	0,75	0,48	0,33	0,16	0,0	
	2. Lösung	0,00	0,02	0,05	0,10	0,13	0,11	0,0	
	$\sigma_x = \sum$	0,95	0,89	0,67	0,38	0,20	0,05	0,00	σ_m
$y : b = \frac{1}{4}$	1. Lösung	0,95	0,88	0,74	0,49	0,34	0,16	0,0	
	2. Lösung	0,00	0,02	0,05	0,09	0,12	0,09	0,0	
	$\sigma_x = \sum$	0,95	0,90	0,69	0,40	0,22	0,07	0,0	σ_m
$y : b = \frac{1}{2}$	1. Lösung	0,96	0,90	0,79	0,51	0,35	0,17	0,0	
	2. Lösung	0,00	0,01	0,03	0,07	0,09	0,09	0,0	
	$\sigma_x = \sum$	0,96	0,91	0,76	0,44	0,26	0,08	0,0	σ_m
$y : b = \frac{3}{4}$	1. Lösung	0,97	0,96	0,83	0,57	0,36	0,18	0,00	
	2. Lösung	0,00	0,01	0,02	0,04	0,05	0,06	0,00	
	$\sigma_x = \sum$	0,97	0,97	0,81	0,53	0,31	0,12	0,00	σ_m
$y : b = 1$ σ_x		1,00	1,02	0,95	0,64	0,39	0,18	0,00	σ_m

Tabelle 29.

2 symmetrische Lasten. $a : b = 4$.

Schubspannungen τ. Lösung 1 und Lösung 2.

$\tau = \tau_1 + \tau_2$.

$x : a$		0,0	0,3	0,5	0,7	0,8	0,9	1,0	
a) $y : b = 0$	$\tau =$	0	0	0	0	0	0	0	σ_m
b) $y : b = 1 : 4$.	1. Lösung	0	0,03	0,06	0,09	0,11	0,11	0,11	
	2. Lösung	0	0,01	0,01	0,02	0,01	—0,04	—0,11	
	$\tau = \sum$	0	0,04	0,07	0,11	0,12	0,07	0,00	σ_m
c) $y : b = 1 : 2$	1. Lösung	0	0,07	0,11	0,17	0,19	0,21	0,22	
	2. Lösung	0	0,01	0,02	0,04	0,02	—0,06	—0,21	
	$\tau = \sum$	0	0,08	0,13	0,21	0,21	0,15	0,01	σ_m
d) $y : b = 3 : 4$.	1. Lösung	0	0,08	0,18	0,28	0,31	0,32	0,33	
	2. Lösung	0	0,01	0,03	0,04	0,02	—0,05	—0,34	
	$\tau = \sum$	0	0,09	0,21	0,32	0,33	0,27	—0,01	σ_m
e) $y : b = 1$	1. Lösung	0	0,09	0,26	0,39	0,42	+0,46	+0,46	
	2. Lösung	0	0,02	0,03	0,05	0,02	—0,06	—0,43	
	$\tau = \sum$	0	0,11	0,29	0,44	0,44	0,40	0,03	σ_m

Tabelle 30.

Zusammenstellung.

2 symmetrische Lasten. $a : b = 4$.
Normalspannungen σ_y aus Lösung 1 und 2. $\sigma_y = \sigma_{y1} + \sigma_{y2}$.

$x : a$		0	0,3	0,5	0,7	0,8	0,9	1,0
$y : b = 0$	1. Lösung	0,03	0,06	0,08	0,06	0,03	0,01	0,0
	2. Lösung	0,0	0,01	0,02	—0,07	—0,08	—0,26	—0,65
	$\sigma_y = \sum$	0,03	0,07	0,10	—0,01	—0,05	—0,25	—0,65 σ_m
$y : b = 1 : 4$	1. Lösung	0,03	0,07	0,09	0,07	0,03	0,01	0,0
	2. Lösung	0,00	0,01	0,02	—0,01	—0,07	—0,24	—0,62
	$\sigma_y = \sum$	0,03	0,08	0,11	+0,06	—0,04	—0,24	—0,62 σ_m
$y : b = 1 : 2$	1. Lösung	0,02	0,07	0,08	0,05	0,03	0,01	0,00
	2. Lösung	0,00	0,01	0,01	0,00	—0,05	—0,18	—0,54
	$\sigma_y = \sum$	0,02	0,08	0,09	0,05	—0,02	—0,17	—0,54 σ_m
$y : b = 3 : 4$	1. Lösung	0,00	0,05	0,05	0,03	0,01	0,01	0,00
	2. Lösung	0,00	0,0	0,01	0,00	—0,03	—0,10	—0,38
	$\sigma_y = \sum$	0,00	0,05	0,06	0,03	—0,02	—0,09	—0,38 σ_m
$\sigma_y = y : b = 1 : 0$		0,0	0,0	0,0	0,0	0,0	0,0	0,0 σ_m

Tabelle 31.

Durchbiegungen.

Abstand des Meßpunktes von der Mitte in cm	10	17,5	25	32,5	47,5	55	62,5	70
Durchbiegung durch Momente errechnet in mm	0,38	0,32	0,23	0,11	—0,25	—0,52	0,75	—0,99
Durchbiegung durch Schub errechnet in mm	0	0	0	0	—0,06	—0,12	—0,17	—0,23
Summe	0,38	0,32	0,23	0,11	—0,31	—0,64	—0,92	—1,22
Durchbiegung, gemessen im Mittel der beiden Seiten in mm	0,58	0,43	0,30	0,15	—0,34	—0,73	—1,15	—1,57

Tabelle 32. Messungen. Querschnitt 0,0 α = 0 cm.

Allgemeines			Meßdose (Belastung)					Ausmittelung					Dehnung			
	Ort	Apparat	0 0 t	150 10,2 t	0 0 t	150 10,2 t	0 0 t	1	2	3	4	Mittel	n	n-Mittel	Korr.	λ
A_0	α	17	17,0	24,5	17,1	25,0	17,0	7,5	7,4	7,9	8,0	7,7	1,04	8,0	— 0,2	7,8
	$\alpha + \frac{\pi}{2}$		28,2	27,5	28,2	27,5	28,2	— 0,7	— 0,7	— 0,7	— 0,7	— 0,7		— 0,7	— 0,1	—0,6
	$\alpha + \frac{\pi}{4}$		13,9	16,1	13,9	16,1	13,9	3,2	3,2	3,2	3,2	3,2		3,4	+ 0,1	3,5
	$\alpha + \frac{3\pi}{4}$		18,9	22 2	18,9	22,2	19,0	3,3	3,3	3,3	3,2	3,3		3,4	+ 0,1	3,5
C_0	α	17	21,6	29,2	21.5	29,2	21,5	7,6	7,7	7,7	7,7	7,7	1,04	8,0	+ 0,4	8,4
	$\alpha + \frac{\pi}{2}$		13,9	13,0	13,8	13,0	13,6	— 0,9	— 0,8	— 0,8	— 0,6	— 0,8		— 0,8	+ 0,3	— 0,5
	$\alpha + \frac{\pi}{4}$		8,2	11,5	8,1	11,5	8,2	3,3	3,3	3,4	3,4	3 4		3,5	— 0,3	3,2
	$\alpha + \frac{3\pi}{4}$		19,1	24,0	19,1	24,0	19,2	4,9	4,9	4,9	4,8	4,9		5,1	— 0,4	4,7
E_0	α	2	27,9	35,9	27,8	35,8	27,8	8,0	8,1	8,0	8,0	8,0	0,94	7,5	+ 0,2	7,7
	$\alpha + \frac{\pi}{2}$		20,5	19,5	20,5	19,5	20,3	— 1,0	— 1,0	— 1,0	— 0,8	— 1,0		— 0,9	+ 0,1	— 0,8
	$\alpha + \frac{\pi}{4}$		13,0	17,0	13,0	17,1	13,0	4,0	4,0	4,1	4,1	4,0		3,8	— 0,1	3,7
	$\alpha + \frac{3\pi}{4}$		21,2	24,8	21,3	24,8	21,3	3,6	3,5	3,5	3,5	3,5		3,3	— 0,1	3,2
G_0	α	17	18,0	26,0	18,6	25,8	18,6	8,0	7,4	7,2	7,2	7,5	1,04	7,8	— 0,2	7,6
	$\alpha + \frac{\pi}{2}$		19,1	18,0	19,1	18,0	19,1	— 1,1	— 1,1	— 1,1	— 1,1	— 1,1		— 1,1	— 0,1	— 1,2
	$\alpha + \frac{\pi}{4}$		16,0	19,1	16,0	19,1	16,0	3,1	3,1	3,1	3,1	3,1		3,2	+ 0,1	3,3
	$\alpha + \frac{3\pi}{4}$		17,0	19,9	17,0	19,9	17,0	2,9	2,9	2,9	2,9	2,9		3,0	+ 0,1	3,1
$\frac{H}{J_0}$	α	17	22,4	31,6	22,6	31,4	22,5	9,2	9,0	8,8	8,9	9,0	1,04	9,4	+ 0,2	9,6
	$\alpha + \frac{\pi}{2}$	11	23,9	21,7	23,6	21,8	23,5	— 2,2	— 1,9	— 1,8	— 1,7	— 1,9	1,12	— 2,1	+ 0,1	— 2,0
	$\alpha + \frac{\pi}{4}$	11	9,5	13,2	9,7	13,2	9,6	3,7	3,5	3,5	3,6	3,6	1,12	4,0	— 0,1	0,8
	$\alpha + \frac{3\pi}{4}$	11	19,4	23,1	19,8	23,2	19,3	3,7	3,3	3,4	3,9	3,6	1,12	4,0	— 0,2	3,8

Tabelle 33. Errechnung der Spannungen. Querschnitt $0{,}0\,\alpha = 0$ cm.

Feld Nr.	Feld Richtung	Dehnung λ	Σ_α / $\Sigma_{\alpha+\frac{\pi}{4}}$	D_α / $D_{\left(\alpha+\frac{\pi}{4}\right)}$	D_α^2 / $D^2_{\left(\alpha+\frac{\pi}{4}\right)}$	$D_\alpha^2 + D^2_{\left(\alpha+\frac{\pi}{4}\right)}$	$\sqrt{D_\alpha^2 + D^2_{\left(\alpha+\frac{\pi}{4}\right)}}$	$\operatorname{tg} 2\alpha : \frac{D_{\left(\alpha+\frac{\pi}{4}\right)}}{D_\alpha}$	α	A $76{,}8\,\Sigma_\alpha$	$B = 41{,}5\,D_\alpha$ / $B' = 41{,}5\,D_{\left(\alpha+\frac{\pi}{4}\right)}$	C $41{,}5\sqrt{\dots}$	$\sigma_0 = A + C$ / $\sigma_{\frac{\pi}{2}} = A - C$	$\sigma_\alpha = A + B$ / $\sigma_{\alpha=\frac{\pi}{2}} = A - B$	$\tau_\alpha = B'$ / $\tau_{\left(\alpha+\frac{\pi}{2}\right)} = -B'$	$\tau_{\max} = \pm C$
A_0	α	7,8														
	$\alpha+\frac{\pi}{2}$	—0,8	7,0	8,6	73,8	73,8	8,6	0,02	0,5°	540	360	360	900	900	—10	$\mp$ 360
	$\alpha+\frac{\pi}{4}$	3,5														
	$\alpha+\frac{3\pi}{4}$	3,5	7,0	0,0	0,0						—10		180	180	+10	
C_0	α	8,4														
	$\alpha+\frac{\pi}{2}$	—0,5	7,9	8,9	79	81,3	9,05	0,17	5°	607	370	375	982	977	—60	$\mp$ 375
	$\alpha+\frac{\pi}{4}$	3,2														
	$\alpha+\frac{3\pi}{4}$	4,7	7,9	—1,5	2,3						—60		232	237	+60	
E_0	α	7,7														
	$\alpha+\frac{\pi}{2}$	—0,8	6,9	8,5	72	72,3	8,52	—0,06	—1,5°	530	355	355	885	885	20	$\mp$ 355
	$\alpha+\frac{\pi}{4}$	3,7														
	$\alpha+\frac{3\pi}{4}$	3,2	6,9	+0,5	0,3						20		175	175	—20	
G_0	α	7,6														
	$\alpha+\frac{\pi}{2}$	—1,2	6,4	8,8	77,5	77,5	8,8	—0,02	—0,5°	490	365	365	855	855	10	$\mp$ 365
	$\alpha+\frac{\pi}{4}$	3,3														
	$\alpha+\frac{3\pi}{4}$	3,1	6,4	0,2	0,04						10		125	125	—10	
$\frac{H}{J_0}$	α	9,6														
	$\alpha+\frac{\pi}{2}$	—2,0	7,6	11,6	135	135	11,6	0,0	0°	580	480	480	1000	1000	0	$\mp$ 480
	$\alpha+\frac{\pi}{4}$	3,8														
	$\alpha+\frac{3\pi}{4}$	3,8	7,6	0,0	0,0						0		100	100	0	

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