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HYDRODYNAMIC FORCES ON AN ANCHOR CABLE

by

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ABSTRACT

The holding power required of an anchor for a ship anchored in shoal water with a length of anchor line at least five times the water depth, i.e., with a scope of five, can normally be assumed to be equal to the estimated drag of the ship. Anchoring in deep water necessitates a relatively shorter anchor line, which results in a considerable hydrodynamic force on the anchor cable and a tension in the cable much greater than the drag of the ship.

Curves have been computed from which the magnitude and direction of the tensions in the anchor cable can be determined when the drag of the ship, the velocity of the current, the depth of the water, and the type and length of the anchor cable are known. Formulas are given for ship drag, current parameter, breaking strength of wire-rope and chain cables, safe working loads on cables, and holding power of an anchor. An illustrative example applies these calculations to the determination of diameter and length of a wire-rope anchor cable and of size of anchor required in a given problem.

INTRODUCTION

In March 1945 the Bureau of Ships (1)* requested the David Taylor Model Basin to calculate the forces acting on the anchor cable of a ship. An anchor selected for a ship must have sufficient holding power to take the greatest loads normally encountered in service. Ordinarily, however, it is not practicable to carry anchors which will hold a ship under any storm conditions and on any types of bottom. Likewise the length of anchor cable carried is limited by practical considerations. The holding power of models of several types of anchors in different types of sea bottom has been investigated experimentally (2) (3). Also, tests of the holding power of full-scale anchors of new designs have been conducted by the Norfolk Naval Shipyard from time to time, as requested by the Bureau of Ships.

The holding power required of an anchor for a given ship is usually computed for the standard conditions of a 70-knot wind and a 4-knot tide in the same direction. It is desirable where practicable to anchor with a length of anchor line at least five times the water depth, i.e., with a scope of five or greater. With such a scope, under the standard conditions of wind and current, an anchor cable would make a small angle with the horizontal at both the anchor and the ship, resulting in a tension in the cable at the anchor very

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* Numbers in parentheses indicate references on page 12 of this report.

nearly equal to the drag of the ship. Consequently the required holding power of an anchor in depths of water where a proper scope can be used may be assumed to be equal to the estimated drag of the ship.

When anchoring in deeper water, however, it is not practicable to pay out a length of line which is five times the depth of the water. Under this condition, the hydrodynamic force on the anchor cable cannot be neglected, and the tension in the cable may be much greater than the drag of the ship. An anchor with a holding power greater than the estimated drag on the ship would then be necessary to prevent dragging of the anchor. Since this condition would also impose greater tension in the cable, the cable size should be selected accordingly.

To meet the various conditions of anchoring, curves have been computed from which the magnitude and direction of the tensions in the anchor cable at the anchor and at the ship can be determined when the drag of the ship, the velocity of the current, the depth of the water, and the type and length of the anchor line are known. The application of these curves is illustrated by a numerical example.

CHARTS OF FORCES ON AN ANCHOR LINE

Figure 1 shows diagrammatically and serves to define the geometrical quantities and the forces in the anchor line which are employed in the subsequent figures and discussion.

The laws of force on a cable in a stream are well known (4). On each element of the cable the hydrodynamic forces consist of a component normal to the cable, whose magnitude diminishes as the square of the sine of the inclination of the element with the horizontal, and a tangential component whose magnitude is small compared with the normal component, except when the inclination of the cable is very small.

A general solution for the shape and tension of a cable in a stream, when the weight of the cable or the tangential force component or both are taken into account, cannot be expressed in simple analytical terms. It is usually necessary to express these solutions in terms of new functions, defined as integrals, for which tables must be computed. However, the use of these tables is as simple as the use of tables of the trigonometric functions, and by their use numerical solutions of cable problems can be obtained readily. The application of such tables to the solution of cable problems in which the weight of the cable can be neglected has been illustrated by numerous examples in a recent Taylor Model Basin report (5).

New charts have now been constructed in which the weight of the cable is not neglected; see Figures 2, 3, 4, and 5. The tangential component

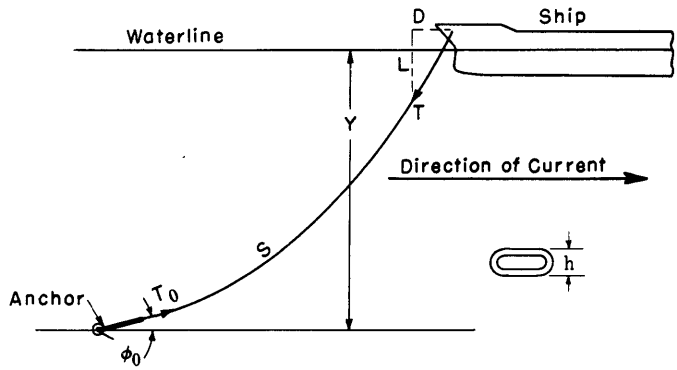

Figure 1 - Dimensions and Forces on an Anchor Cable

Here D is the horizontal component of tension in the anchor line at the ship (drag of the ship), in pounds,
L is the vertical component of tension in the anchor line at the ship, in pounds,
$T = \sqrt{L^2 + D^2}$, the tension in the anchor cable at the ship, in pounds,
$T_0 = T - WY$, the tension in the anchor cable at the anchor, in pounds,
W is the weight per unit length of the anchor cable in water, in pounds per foot,
Y is the depth of the water, in feet,
ϕ_0 is the angle of the anchor line with the horizontal at the anchor, in degrees,
$\mu = R/W$, the current parameter,
R is the drag per unit length of the anchor cable, $0.34\ V^2d$ for wire rope and $0.20\ V^2h$ for chain,
V is the current speed, in knots,
d is the diameter of the wire rope, in inches,
h is the outside width of a link of chain, in inches, and
S is the length of the anchor line, in feet.

of the hydrodynamic force on the line is neglected in these computations, but the resulting error is partly offset by the assumption that the velocity of the current is the same at all depths; actually the current velocity is known to approach zero rapidly near the bottom.

Figures 2, 3, 4, and 5 are polar diagrams of the vertical and horizontal components of the cable tension at the waterline, expressed as dimensionless quantities in terms of the weight WY in water of a length of anchor line equal to the depth. Plots of this type are called polar diagrams because the polar coordinates, i.e., the radius from the origin and the angle of the radius with the D/WY axis, give the magnitude of the tension and the inclination of the anchor cable with the horizontal at the ship.

In anchoring a ship it is considered desirable to pay out enough line to permit the cable to be horizontal at the anchor. The reason for this is that the holding power of an anchor falls off linearly with the angle of inclination of the cable at the anchor; the holding power is reduced by one-half for an angle of 30 degrees with the horizontal, as shown in Figure 13 of Reference (3). Consequently, it is desirable to have a method for estimating the length of line needed to ensure that the cable will be horizontal at the anchor. Figure 2 was devised for this purpose.

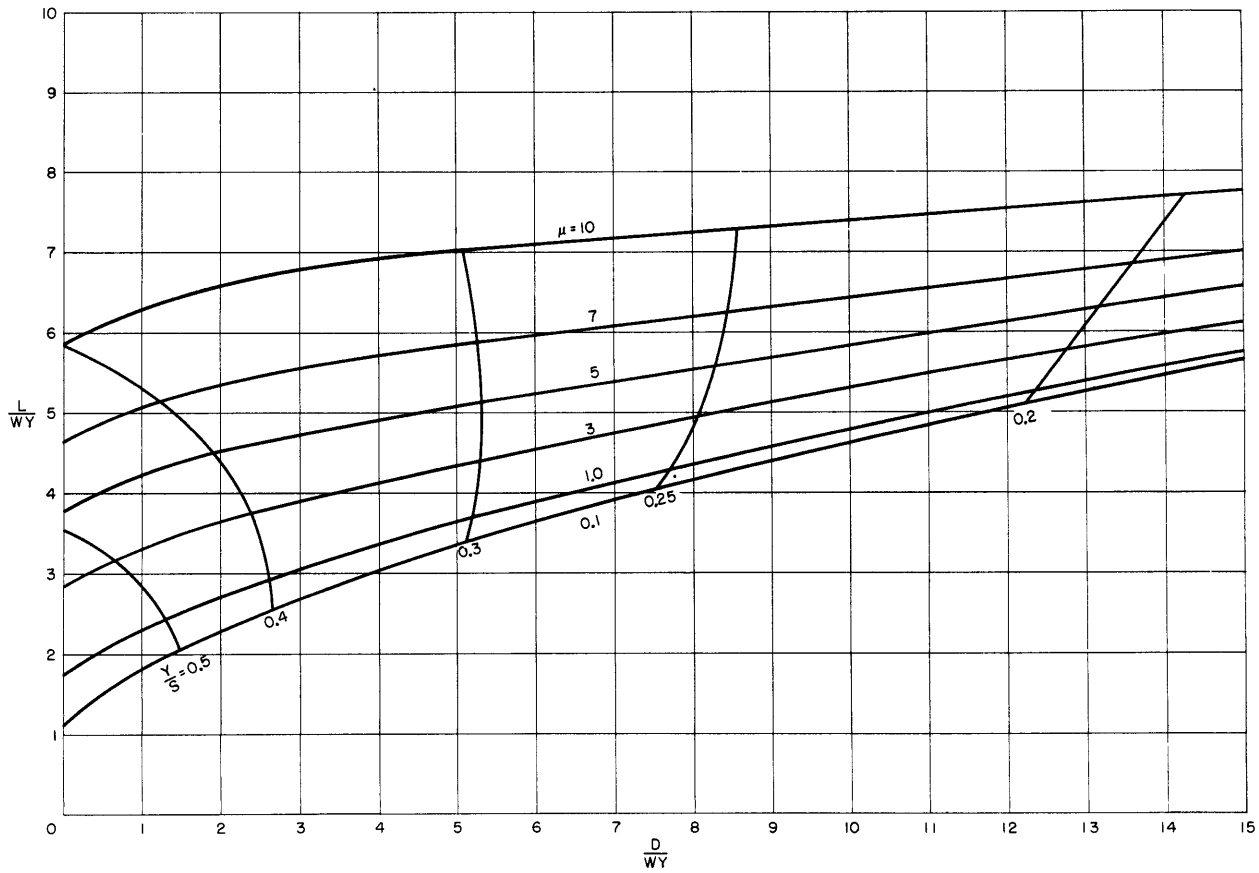

Figure 2 - Forces in an Anchor Cable with Anchor Line Horizontal at Anchor (ϕ_0 = 0)

These curves also apply when part of the anchor line is lying on the bottom. In this case the value for S in Y/S is the length of cable above the bottom.

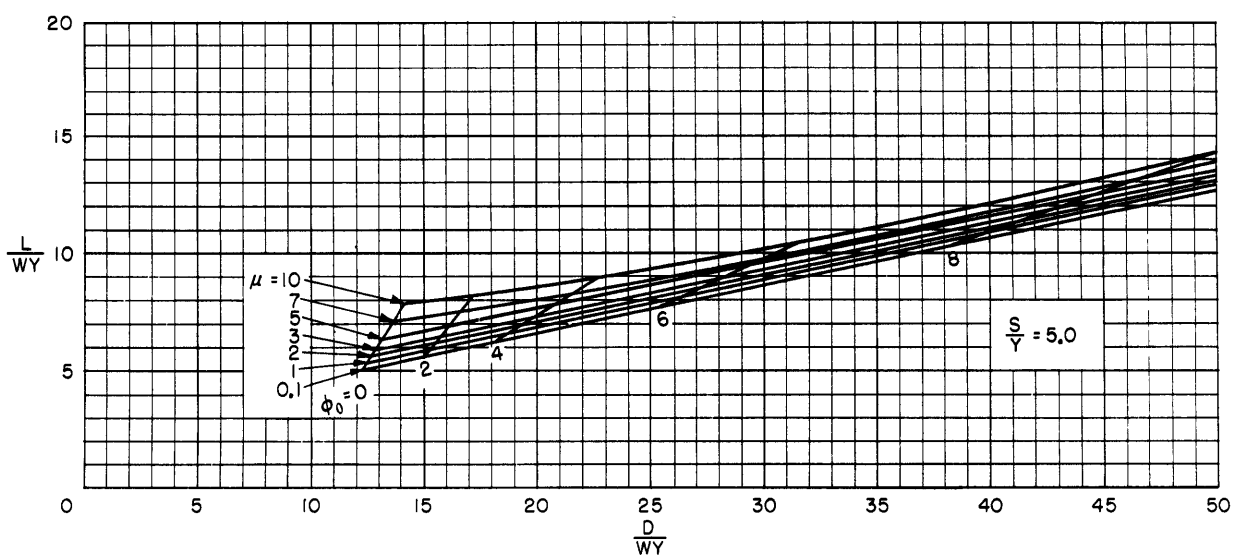

Figure 3 - Forces in an Anchor Cable for a Scope of 5.0

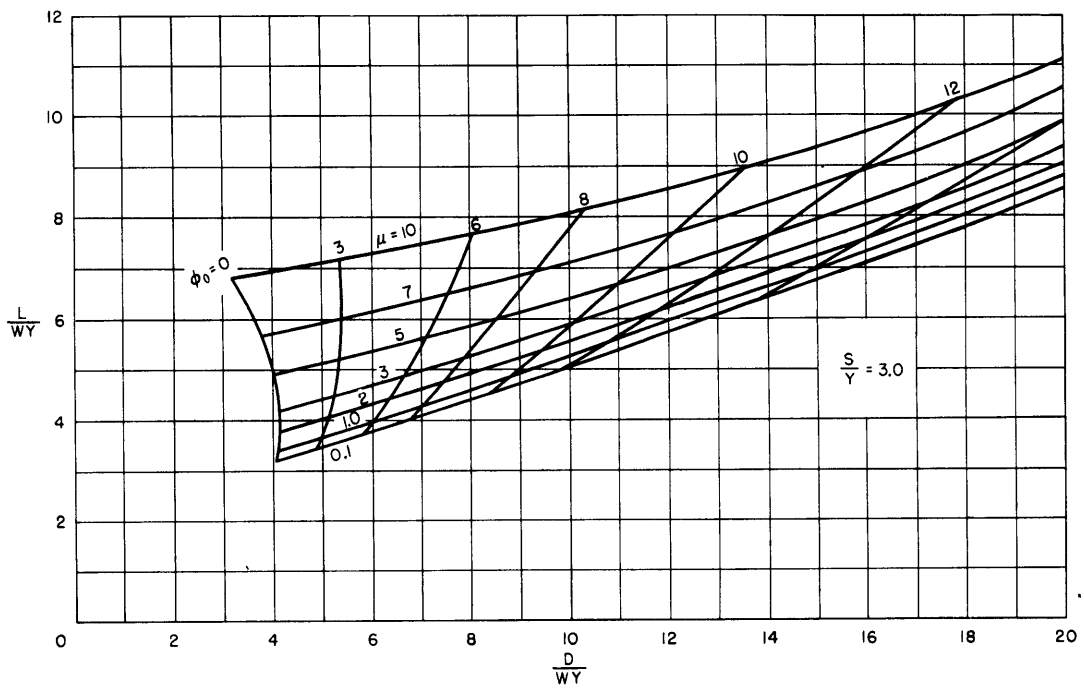

Figure 4 - Forces in an Anchor Cable for a Scope of 3.0

Expressed in functional form, Figure 2 gives a graphical presentation of the functions

$$\frac{L}{WY} = E\left(\frac{D}{WY}, \mu\right)$$

and

$$\frac{Y}{S} = F\left(\frac{D}{WY}, \mu\right)$$

when the angle of the anchor line at the anchor, ϕ_0, is zero. Here μ is a current parameter, which is defined in Figure 1 and discussed in a following section. When the drag of the ship, D, the weight per unit length of the line in water, W, the depth of water, Y, and the value of μ are given, then, from Figure 2, the values of Y/S and L/WY can be read, and hence the length of cable, S, and the corresponding downward load L at the ship can be determined such that $\phi_0 = 0$.

When the depth of water is very great, it may be impracticable to pay out the length of line required to obtain a zero angle at the anchor. In any case, however, it is desirable to keep the inclination of the cable and its tension at the anchor as small as possible. Consequently, for preliminary design purposes, it is desirable to have a method for determining the tension and angle at the anchor for various assumed types and lengths of anchor cable.

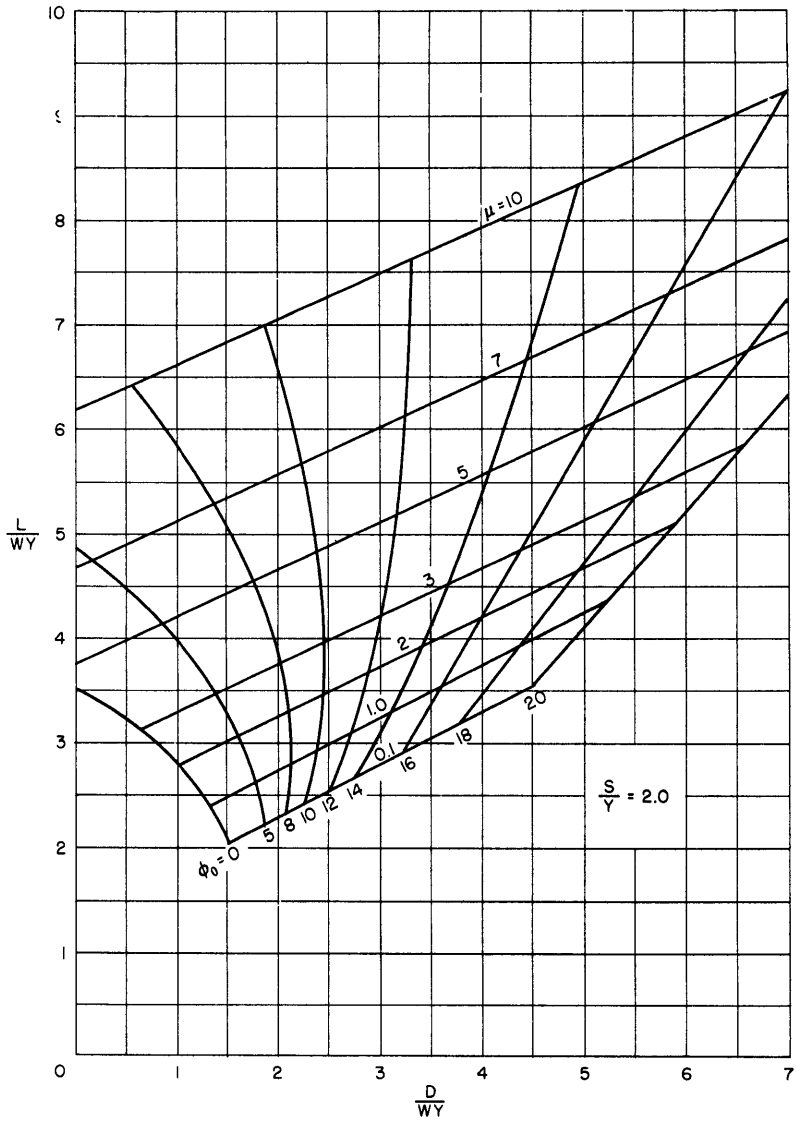

Figure 5 - Forces in an Anchor Cable for a Scope of 2.0

Figures 3, 4, and 5 were prepared for this purpose. The components of the tension at the ship are also included in these figures to ascertain whether the safe working load of the cable is exceeded.

Figure 3 is a graphical presentation of the functions

$$\frac{L}{WY} = G\left(\frac{D}{WY}, \mu\right)$$

$$\phi_0 = H\left(\frac{D}{WY}, \mu\right)$$

for $S/Y = 5$. Figures 4 and 5 present the same function for $S/Y = 3$ and 2 respectively. Hence, when D, W, Y, μ, and S are given, L and ϕ_0 can be determined.

The tension at the ship, T, is determined from the length of the radius vector T/WY to the point D/WY, L/WY. The tension at the anchor, T_0, is then given by

$$T_0 = T - WY \qquad [1a]$$

or

$$\frac{T_0}{WY} = \frac{T}{WY} - 1 \qquad [1b]$$

In using the charts to solve anchor problems it will be necessary frequently to estimate ship drags, to determine the weight per foot and the safe working load of anchor cable of given types, and to compute the current parameter μ and the required holding power. To facilitate the use of the charts, simple approximate expressions for these quantities have been derived and are assembled in the following section.

ESTIMATE OF SHIP DRAG

The drag of a ship due to current can be calculated from the approximate formula

$$D_c = 0.12\, V^2 \sqrt{l\Delta}$$

where D_c is the drag due to current, in pounds,
l is the length of the ship at the waterline, in feet,
Δ is the displacement of the ship, in tons, and
V is the current speed in knots.

The drag due to wind can be calculated from an approximate formula proposed by Captain E.F. Eggert, USN (Retired),

$$D_a = 0.0022\, B^2 V_a^2$$

where D_a is the drag due to wind, in pounds,
B is the beam of the ship, in feet, and
V_a is the wind speed in knots.

The drag of the ship will be greatest when V_a and V are in the same direction. In this case the drags will be additive and the total drag D is

$$D = D_a + D_c = 0.0022\, B^2 V_a^2 + 0.12\, V^2 \sqrt{l\Delta} \qquad [2]$$

EVALUATION OF CURRENT PARAMETER

The parameter μ is defined as

$$\mu = \frac{R}{W}$$

where R is the drag per unit length of the line when the cable is normal to the stream and W is the weight per unit length of the line in water.

The following recommended values of R are based on tests made at the Taylor Model Basin:

$$R = 0.34V^2d \text{ pound per foot, for wire rope} \qquad [3a]$$

$$R = 0.20V^2h \text{ pound per foot, for chain} \qquad [3b]$$

where d is the diameter of the cable in inches and h is the outside width of a link of chain in inches.

If anchor cables of different sizes were geometrically similar, W would be proportional to the square of the width of a link. The data in engineering handbooks and manufacturers' catalogues, corrected to give weight in water, show a small variation. Approximately, however,

$$W = 1.40d^2 \text{ pound per foot, for wire rope} \qquad [4a]$$

$$W = 0.64h^2 \text{ pound per foot, for chain} \qquad [4b]$$

From Equations [3] and [4], the expressions for μ can be written as

$$\mu = \frac{0.34d}{1.40d^2}V^2 = 0.24\frac{V^2}{d} \text{ for wire rope} \qquad [5a]$$

and

$$\mu = \frac{0.20h}{0.64h^2}V^2 = 0.31\frac{V^2}{h} \text{ for chain} \qquad [5b]$$

BREAKING STRENGTH AND SAFE WORKING LOAD OF CHAIN AND WIRE ROPE

Theoretically, the breaking strength of geometrically similar cables should vary as the square of the width of a link. However, data in handbooks and catalogues show a small variation with size. The following approximate values for the breaking strength T_B are recommended:

$$T_B = 70{,}000d^2 \text{ pounds, for plow-steel wire rope}$$

$$T_B = 4{,}000h^2 \text{ pounds, for forged stud-link anchor chain}$$

Then, from Equations [4a] and [4b]

$$\frac{T_B}{W} = 50{,}000 \text{ feet for plow-steel wire rope} \qquad [6a]$$

and

$$\frac{T_B}{W} = 6250 \text{ feet for forged stud-link anchor chain} \qquad [6b]$$

Based on a factor of safety of approximately 3, the values of the ratio of the safe working load T_s to W may be taken as

$$\frac{T_s}{W} = 16{,}000 \text{ feet for plow-steel wire rope} \qquad [7a]$$

and

$$\frac{T_s}{W} = 2000 \text{ feet for forged stud-link anchor chain} \qquad [7b]$$

When Y is given, the ratio T_s/WY can be computed from one of the Equations [7]. A circle of radius T_s/WY on any of the polar diagrams determines the region within which the anchor-line tensions are under the safe working load but outside of which they are unsafe.

REQUIRED HOLDING POWER

The holding power of an anchor falls off rapidly as the angle of the anchor line with the horizontal at the anchor is increased from zero. The variation of the holding power with this angle has been investigated (2), and the results are reproduced in Figure 13 of Reference (3). Expressed analytically, the ratio of the holding power H_{ϕ_0} at a finite angle ϕ_0 to the holding power H at $\phi_0 = 0$ is given by the formula

$$\frac{H_{\phi_0}}{H} = 1 - 0.0174\,\phi_0$$

or

$$H = \frac{H_{\phi_0}}{1 - 0.0174\,\phi_0} \qquad [8]$$

where ϕ_0 is in degrees. In using Equation [8] to design an anchor, H_{ϕ_0} is set equal to the maximum value of T_0 that is anticipated.

APPLICATION OF CHARTS

When the drag of the ship, D, the weight per unit length of cable in water, W, the depth of the water, Y, the speed of the current, V, the type and size of the anchor cable have been given, and the value of μ has been computed, the charts can be applied directly. The length of line and the tension corresponding to a zero angle of the cable with the horizontal at the anchor can then be determined from Figure 2. If this indicates that the length of cable for $\phi_0 = 0$ is impracticably long, then Figures 3, 4, and 5 can be applied to find the tensions in the line and the angle of the cable at the anchor for lengths of line corresponding to $S/Y = 5$, 3, and 2. The points spotted as solutions on the charts should be examined to ascertain whether they lie within the circle of safe working loads.

In an anchor problem where a suitable size of anchor cable must be determined it will be necessary to repeat the foregoing procedure for various

assumed sizes of cable. The optimum solution is the size of cable which will be loaded up to the safe working load under the given conditions.

EXAMPLE

Consider a ship whose displacement is 2000 tons, whose length is 360 feet, and whose beam is 40 feet, anchored in water 300 fathoms deep. Assume a 4-knot tide and a 70-knot wind, both in the same direction. It is desired to find the diameter and length of a wire-rope anchor cable and the size of anchor required. From Equation [2], the drag of the ship is

$$D = 0.0022 \times (40)^2 \times 4900 + 0.12 \times 16\sqrt{360 \times 2000}$$

$$= 17,200 + 1630 = 18,830 \text{ pounds}$$

The forces on the anchor will be examined for cable diameters of 1, 1.5, 2, and 2.5 inches. For convenience in the subsequent calculation, the values of W, WY, μ, and D/WY for these diameters and for Y = 1800 feet have been computed from Equations [4a] and [5a] and assembled in Table 1. From Equations [6a] and [7a] the values of T_B/WY and T_s/WY are found to be 27.8 and 8.89 respectively for all diameters.

TABLE 1

Values of W, WY, μ, and $\frac{D}{WY}$

d inches	W pounds per foot	WY pounds	μ	$\frac{D}{WY}$
1.0	1.40	2,520	3.84	7.47
1.5	3.15	5,670	2.56	3.32
2.0	5.60	10,080	1.92	1.87
2.5	8.75	15,740	1.54	1.20

Figure 6 illustrates the procedure for d = 1 inch and S/Y = 3, i.e., for the chart of Figure 4. The point corresponding to μ = 3.84 and D/WY = 7.47 is spotted on the chart. The values of L/WY = 5.4 and ϕ_0 = 7.0 degrees are then read by interpolation. The length of the radius vector to the point is readily obtained by swinging an arc with center at the origin back to the D/WY axis and reading the value at the intersection of the arc with this axis. This gives T/WY = 9.25 and then, from Equation [1a], T_0 = 20,800 pounds. The circle of safe working loads with radius T_s/WY = 8.89 is also shown in Figure 6. The factor of safety, T_B/T, is computed as the ratio of T_B/WY to T/WY where

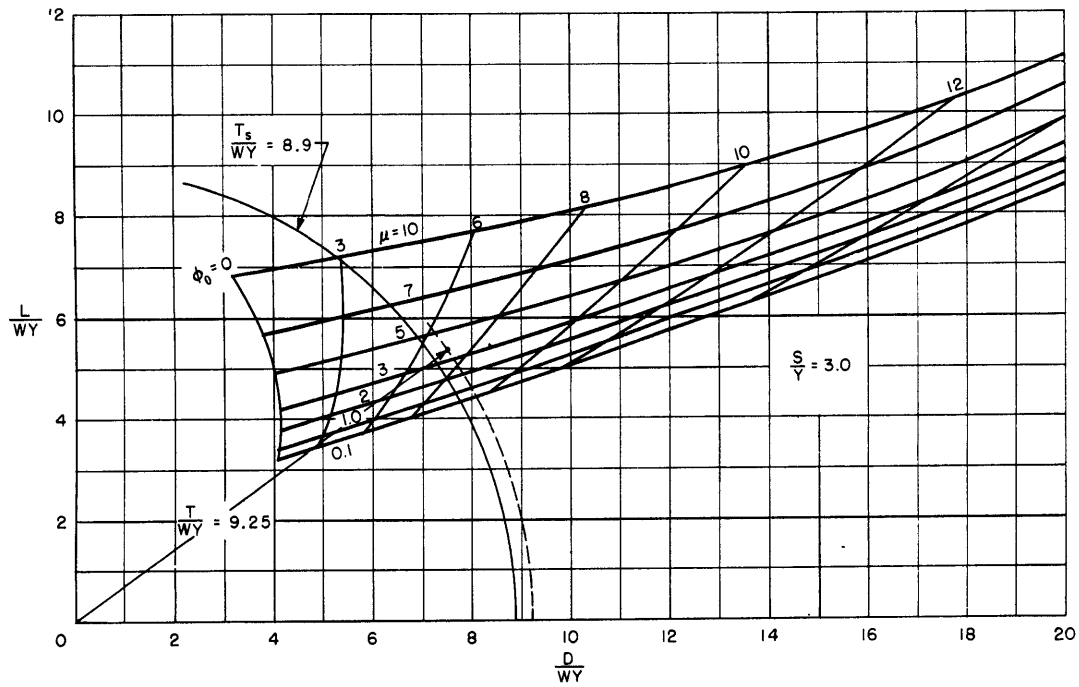

Figure 6 - Illustration of Application of Figure 4 for an Anchor Cable of 1-Inch Diameter and a Scope of 3.0

$$\frac{T_B}{WY} = \frac{50,000}{1800} = 27.8$$

Hence

$$\frac{T_B}{T} = \frac{27.8}{9.25} = 3.01$$

The holding power required, computed from Equation [8], is

$$H = \frac{20,800}{1 - 0.0174 \times 7} = 23,700 \text{ pounds}$$

The procedure for using the curves in Figure 2 is the same as that for the curves in Figures 3, 4, and 5 except that $\phi_0 = 0$ is given initially and the value of S/Y is read by interpolation.

These values and the results of similar calculations for other lengths and diameters for this example are summarized in Table 2.

It will be assumed that it is impracticable to use more than 3600 feet of cable, or an S/Y greater than 2.0. For an S/Y of 2, Table 2 shows that the factor of safety increases almost linearly with diameter of cable. The holding power, however, reaches a minimum of about 30,700 pounds with a cable of 2-inch diameter. If a safety factor of 3.1 is assumed to be adequate, interpolation from Table 2 indicates that the ship could be anchored with 3600 feet of 17/16-inch wire rope with an anchor having a holding power

TABLE 2

Holding Power and Safety Factors for Various Lengths and Diameters of Wire Rope for Conditions of Example

ϕ_0 degrees	$\frac{S}{Y}$	$\frac{L}{WY}$	$\frac{T}{WY}$	T_0 pounds	Safety Factor	Holding Power Required pounds
d = 1.0 Inch						
0	3.8	5.10	9.05	20,200	3.08	20,200
7	3	5.4	9.25	20,800	3.01	23,700
21	2	6.55	9.94	22,500	2.80	35,500
d = 1.5 Inch						
0	2.75	3.80	5.04	22,900	5.52	22,900
13.3	2	4.16	5.30	24,400	5.24	31,700
d = 2.0 Inches						
0	2.27	3.10	3.62	26,400	7.68	26,400
7.5	2	3.15	3.65	26,700	7.61	30,700
d = 2.5 Inches						
0.5	2	2.70	2.96	30,900	9.40	31,200

of 35,000 pounds. This would require a lightweight (LWT) anchor of about 1750 pounds if the experimental value for the holding power of a lightweight anchor as recommended by the Bureau of Ships, 20 pounds per pound of anchor weight, is assumed. A safety factor of 5 would require a cable of 1.5-inch diameter with a 1580-pound anchor.

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(1) Telephone conversation of 23 March 1945 between Comdr. P.W. Snyder, USN, BuShips, Hull Design (440), and Comdr. E.A. Wright, USN, David Taylor Model Basin.

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