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FREE-FALL TEST OF A MODEL OF THE GATE OF AN ARD12-CLASS FLOATING DRYDOCK

by

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PERSONNEL

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NOTATION

Symbol	Description	Units	Dimensions in Force-Length-Time System
k	Torsional spring constant	Pound-feet per radian	FL
f_n	Natural frequency with damping	Cycles per second	T^{-1}
c	Torsional damping torque per unit of angular velocity	Pound-feet per radian per second	FLT
c_c	Critical torsional damping constant	Pound-feet per radian per second	FLT
I	Effective moment of inertia	Slug-feet^2	FLT^2
Q	Torque	Pound-feet	FL
T	Period	Seconds	T
t	Time	Seconds	T
β_1	Maximum value of first amplitude	Radians	
β_2	Maximum value of second amplitude	Radians	
W	Weight of gate in water	Pounds	F
W_a	Weight of gate in air	Pounds	F
$\bar{x}$	Distance from hinge to center of gravity	Feet	L
ω	Angular velocity	Radians per second	T^{-1}
θ	Angular displacement of gate measured counterclockwise from vertical	Radians	
θ_T	Total angular displacement of spring	Radians	
θ_s	Angular displacement of spring beyond that produced by weight of gate	Radians	
θ_1	Angle of gate at horizontal, measured counterclockwise from vertical	Radians	
α_0	Initial angle of gate, measured clockwise from vertical	Degrees	
α_1	Angle of gate at horizontal, measured clockwise from vertical	Degrees	
α_2	Final angle of gate, measured clockwise from vertical	Degrees	
δ	Logarithmic decrement	Numeric	

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FREE-FALL TEST OF A MODEL OF THE GATE OF AN ARD12-CLASS FLOATING DRYDOCK

ABSTRACT

The data obtained in three series of tests made on a model of the stern gate of an ARD12-Class floating drydock are presented and evaluated. In the first series of tests, the angular velocities of the freely falling gate at various dock drafts are determined. In the second series, the value of the kinetic energy of the freely falling submerged gate at the horizontal position is derived. In the third series, the effect of adding a fin at the top of the gate is investigated.

Comparisons are made between the factors affecting the angular velocity and kinetic energy of the falling gate submerged in water and those affecting the angular velocity and kinetic energy of the falling gate in a vacuum.

INTRODUCTION

The Bureau of Yards and Docks of the Navy Department designed and built several large floating drydocks suitable for rapid transportation to, and for immediate readiness at, advanced fleet bases. These docks have a ship-shaped hull, the interior of which forms the docking chamber. A single gate in the stern affords access to the docking chamber. This gate is hinged at the bottom and when closed forms a watertight barrier to the sea. The dock is raised by pumping the water out of the docking chamber, the wing-wall tanks, and the cellular double-bottom tanks. The gate is opened and closed by two hoisting roller chains attached to the gate at the sides near the upper edge.

One class of these docks, the ARD12 Class, has an overall length of 488 feet 7 inches, a beam of 81 feet, a depth from base line to wing-wall deck of 38 feet, and a lifting capacity of approximately 5600 tons. The stern gate on these docks is 61 feet wide and 25 feet deep, and has a total weight of approximately 34 tons.

In the operation of these gates several failures occurred when the hoisting chains carried away and the gate, falling freely, acquired sufficient momentum to swing beyond the allowed free travel and to snap the hinges. To prevent recurrence of such failures, the Bureau of Yards and Docks desired to design an emergency stopping device consisting of heavy wire cables and spring buffers. This device was to be capable of stopping the gate in a horizontal position when the gate falls under its own weight from the closed position.

The size of cables and fittings required for this device depends on the momentum or the kinetic energy which the gate acquires in its free fall.

For the parallel problem of dropping the gate in air, these quantities can be readily found by theoretical calculations because the damping action of air is negligibly small. The damping action of water, however, is so considerable as to complicate the problem materially. The simplest way to determine this damping action was by model test.

The David Taylor Model Basin was requested by the Bureau of Yards and Docks (1)* to carry out model tests for this purpose. These tests, with their results and analysis, are described in this report.

TEST SETUP AND PROCEDURE

The tests were conducted on a 1:20-scale model of the stern gate of an ARD12-Class floating drydock. The model of the gate was constructed of sheet metal, directly similar to its prototype with respect to size, shape, weight, and construction. As seen in Figure 1, the gate consists of a curved

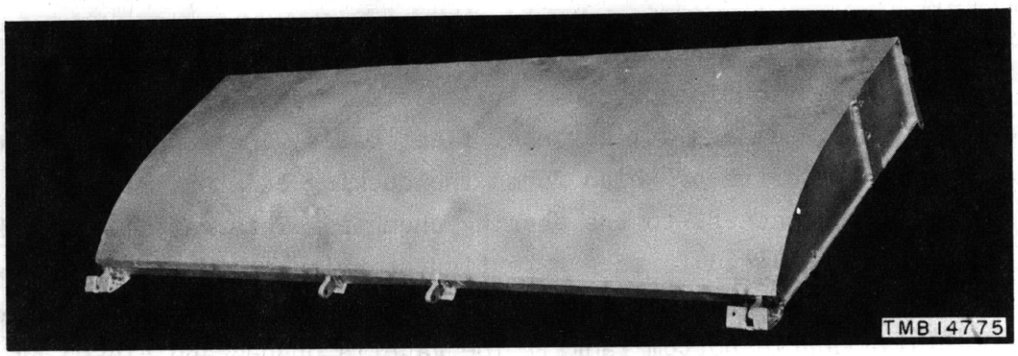

Figure 1a - View Showing Curved Frontal Surface and Reinforced Side of the Stern-Gate Model

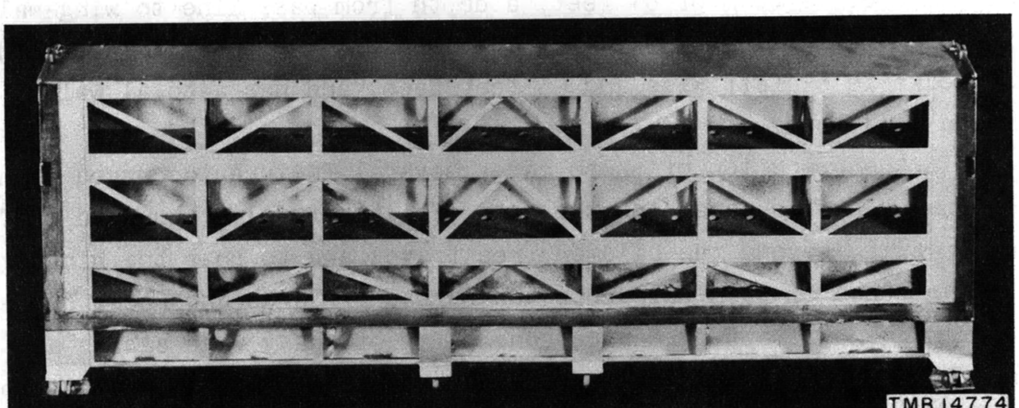

Figure 1b - View Showing Internal Construction

Figure 1 - Photographs of the Model Gate

* Numbers in parentheses indicate references on page 17 of this report.

plate stiffened by girders on the back, two side plates, and a top plate. The inside edges of the gate seat tightly against rubber gaskets mounted in the wing walls of the dock to form a watertight seal. The girders contain holes to allow free circulation of the water within the gate when the dock is submerged. The model gate was installed in a model stern section which represented an equivalent portion of the full-scale drydock. The stern section of the model, both the inside and the outside, was constructed similar to the prototype. A semicircular scale graduated in degrees was secured to the starboard wall of the stern section in a fore-and-aft plane, with the center of the circle lying on the axis of rotation of the gate. The common center of the hinges of the gate was marked on the outside of the starboard wall so that a straight line passing through this point and the intersection of the frontal face and the top of the gate would accurately indicate the angular position of the gate. The arrangement of the starboard wall, the gate, and the scale are shown in Figure 2.

The stern section containing the gate was placed in the transparent-wall tank at the Taylor Model Basin with the planes of the scale and the side of the gate parallel to the wall of the tank. A motion-picture camera was placed normal to the wall of the tank for the purpose of recording the movement of the gate. The time record was provided by a clock with a revolving face that was calibrated in hundredths of a second. An image of the clock

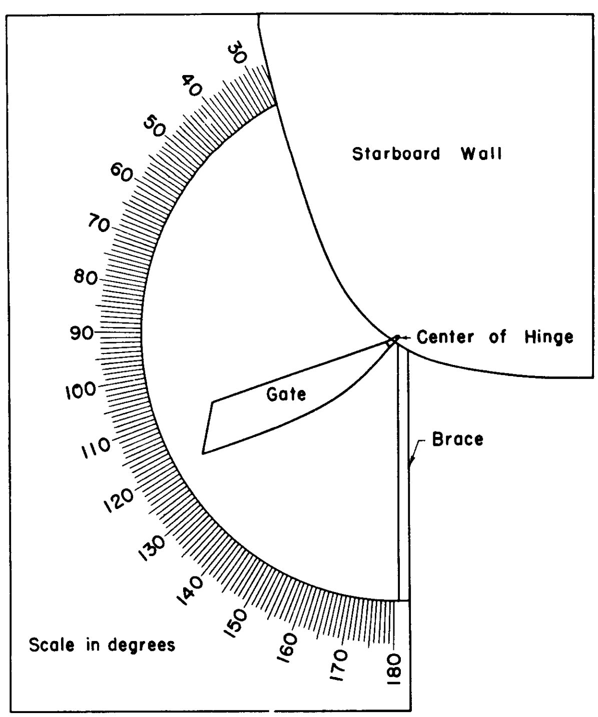

Figure 2 - Arrangement of Starboard Wall, Gate, and Scale for Test

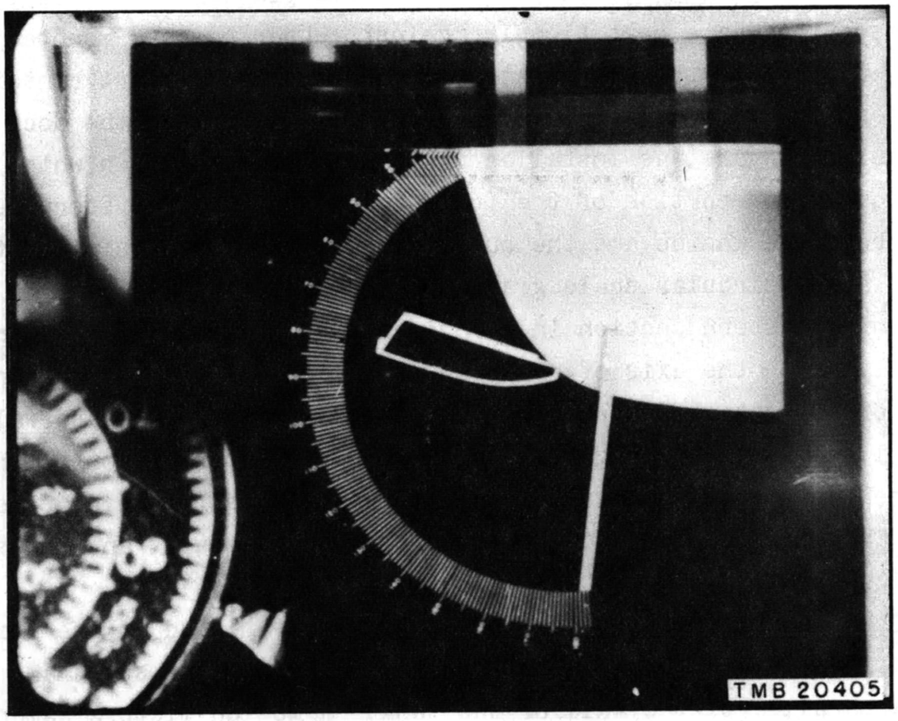

Figure 3 - Sample Record of Data Obtained in the Transparent-Wall Tank, Showing an Instantaneous Position of the Gate and a Clock Recording the Time

was superposed on each frame of the film by an auxiliary lens. A sample of the record obtained is shown in Figure 3.

In the first series of tests, the purpose of which was to obtain the angular velocity, the gate was permitted to fall freely from the closed position, in which it was held against the gaskets, to a position in which it hung vertically by its hinges. These tests were repeated at drafts* equivalent to 30, 25, 20, 15, and 10 feet on the full-scale drydock with the water level equalized on both sides of the gate.

In the second series of tests, the gate was suspended by a 1/16-inch cable secured to the top at one side of the gate. A spring was inserted at the top of the cable and hooked to a pad eye attached to a piece of 4-inch by 4-inch lumber extending across the top of the transparent-wall tank. The cable was adjusted so that there would be no tension in the spring until the gate reached the horizontal position,** i.e., the gate was allowed to fall freely from its closed position against the gaskets to the horizontal position. Several cycles of the spring oscillations were recorded by the motion-picture

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* The draft was measured from the base line of the dock.

** The gate is at the horizontal position when its inside edge is horizontal. The angle of the gate, measured as explained on page 3, is 103 degrees at this position.

camera. The tests were made first with the gate submerged at a dock draft equivalent to 30 feet full scale and were then repeated for a free fall in air.

A third series of tests was later conducted to determine the effect of adding a single fin which projected normally at the top of the after or leading surface of the gate and extended laterally for the approximate width of the gate. Two fins of the same overall size were designed for this purpose. The first fin was constructed from a solid strip of sheet brass and the second from sheet brass perforated with small holes, which were about 0.07 inch in diameter on the model. These fins will be referred to as Fin 1 and Fin 2, respectively. As shown in Figure 4, the top of the gate was provided with three studs which passed through holes in the fins. These studs were threaded to facilitate the changing of fins. The gate equipped with each of these fins separately was allowed to fall freely as in the previous series of tests at a dock draft aft equivalent to 30 feet full scale.

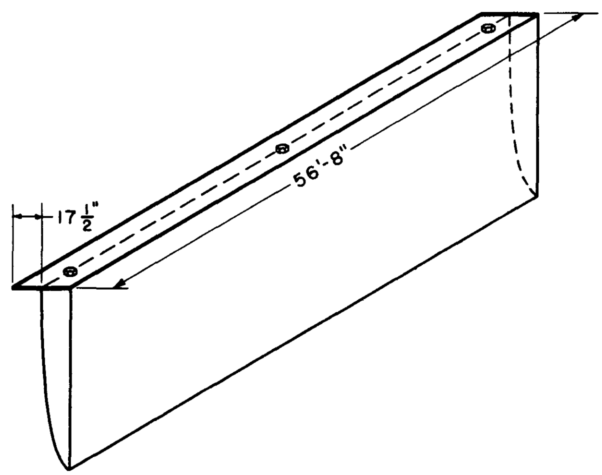

Figure 4 - Sketch Showing Fin Attached to Top of Gate

All dimensions are for the full-scale equivalent.

PRESENTATION OF DATA

The characteristics and dimensions of the gate for the model and the prototype are shown in Table 1.

The data for the first series of tests are plotted as angular displacement on a basis of time in Figures 5, 6, 7, 8, and 9. These curves, together with the derived curves of angular velocity and angular acceleration, are presented in ship dimensions. The values for these curves were obtained by expanding the values obtained in the model tests according to Froude's law of similitude. In Figure 10 angular-velocity curves for the gate equipped with each of the fins are shown for comparison.

Data for the second series of tests are shown in Figure 11 as curves of spring oscillations for two springs of different stiffness. These curves are presented for analysis and are plotted in model dimensions. Similar curves for the gate equipped with fins are given in Figure 12.

A detailed analysis of these data appears in Appendixes 1, 2, and 3; results computed from the data will be found in the "Discussion of Results."

TABLE 1

Characteristics and Dimensions of the Stern Gate

	Model	Prototype
Weight of gate in air, pounds	9.516	76,128
Weight of gate in water, pounds	7.69	61,520
Distance to center of gravity from center of hinges measured along line through center of hinge and top forward edge of gate, feet	0.558	11.16
Distance of point of support from center of hinges measured along line through point of support, feet	1.063	21.26
Distance of center of percussion from center of hinges measured along line through center of hinge and top forward edge of gate, feet	0.8821	17.64
Projected area of leading surface, square feet	3.40	1360
Moment of inertia of gate in air about an axis through the center of the hinges, slug-feet^2	0.1457	466,240
Height of center of gate hinges above base line of dock, feet	0.294	5.88

DISCUSSION OF RESULTS

The principal characteristics of the motion of the gate are apparent in Figures 5 to 9. When the gate is initially submerged, the angular velocity attained by it at the horizontal position is unaffected by the draft of the drydock and is equal to 0.222 radian per second full scale. At the 20-foot draft, the top of the gate is initially 5 feet above the surface of the water, yet the gate attains the same angular velocity. At lesser drafts, the angular velocity diminishes at the horizontal to 0.193 radian per second for the 15-foot draft and 0.090 radian per second for the 10-foot draft.

In the initial stages of the fall, the characteristics of the velocity curves differ greatly when the gate is not completely submerged. Since the speed of the gate is greatly retarded in water by damping and other forces, the initial velocity of the gate would be expected to increase with decrease of draft. This expectation was substantiated by the test results.

The second portion of the velocity curves corresponds to the time during which the compartments in the part of the gate which was initially unsubmerged are being flooded. The gate when empty possesses a great deal of buoyancy. It will not sink at an appreciable rate until it is approximately 95 per cent filled. Thus in Figures 7 to 9 it is seen that the velocity for this portion of the curve would be zero except for a certain amount of bouncing caused first by the initial impetus and later by the water surging into the compartments of the gate.

(Text continued on page 11)

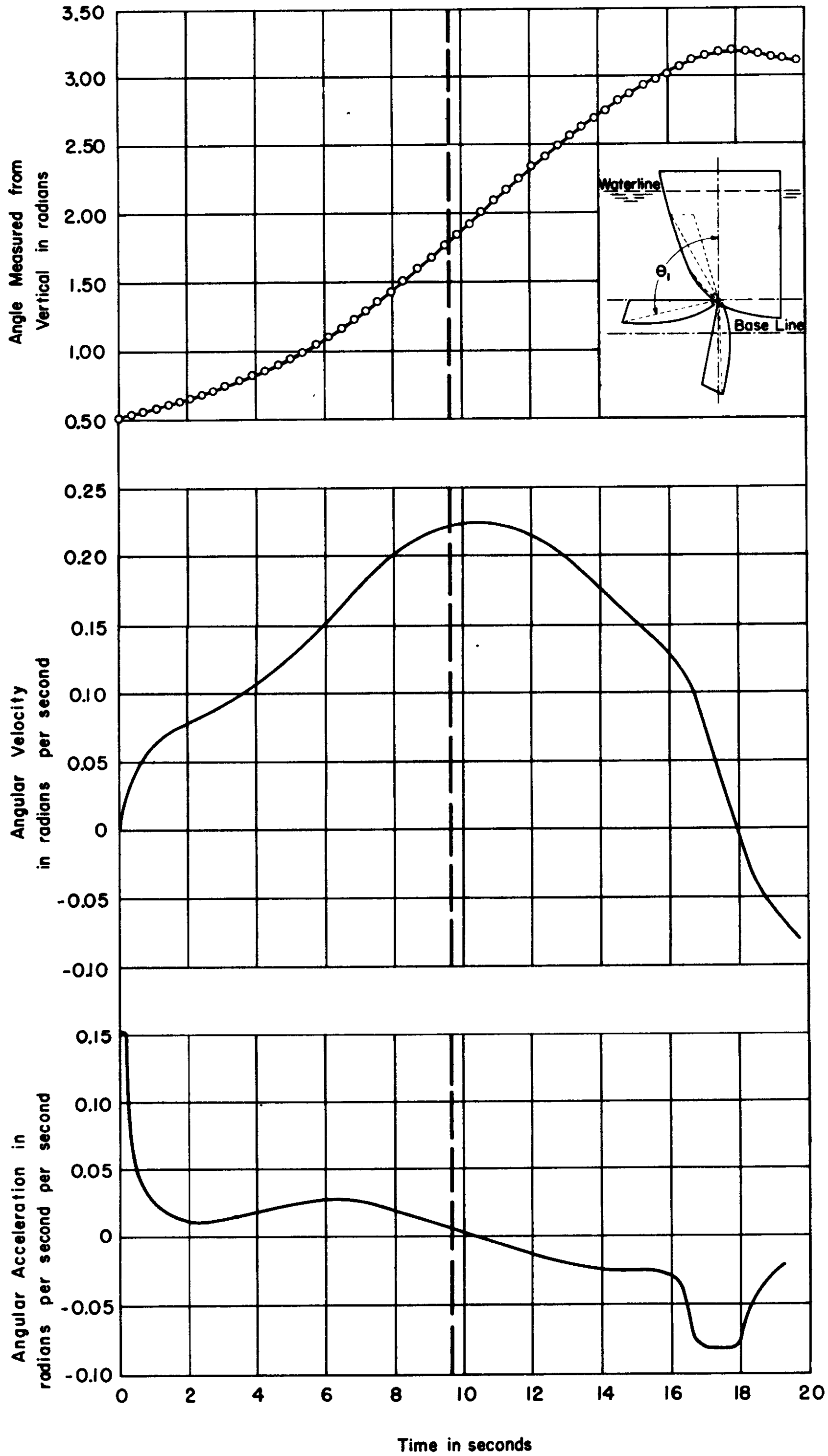

Figure 5 - Angular Displacement, Angular Velocity, and Angular Acceleration Plotted against Time for Full-Scale Gate at a Dock Draft of 30 Feet

The broken line represents the time at which the gate reached its horizontal position; this position is indicated by θ_1 in the sketch.

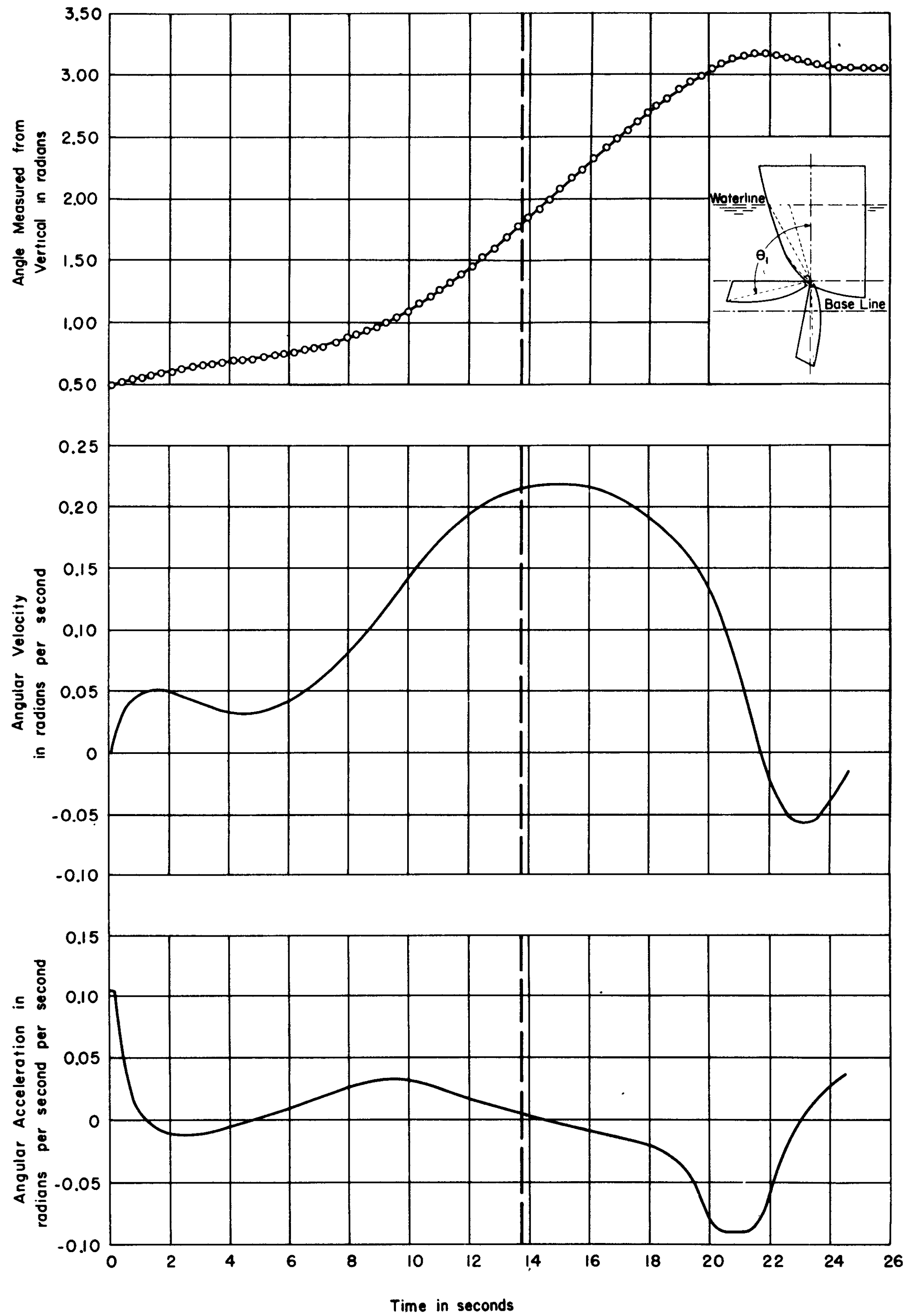

Figure 6 - Angular Displacement, Angular Velocity, and Angular Acceleration Plotted against Time for Full-Scale Gate at a Dock Draft of 25 Feet

The broken line represents the time at which the gate reached its horizontal position; this position is indicated by θ_1 in the sketch.

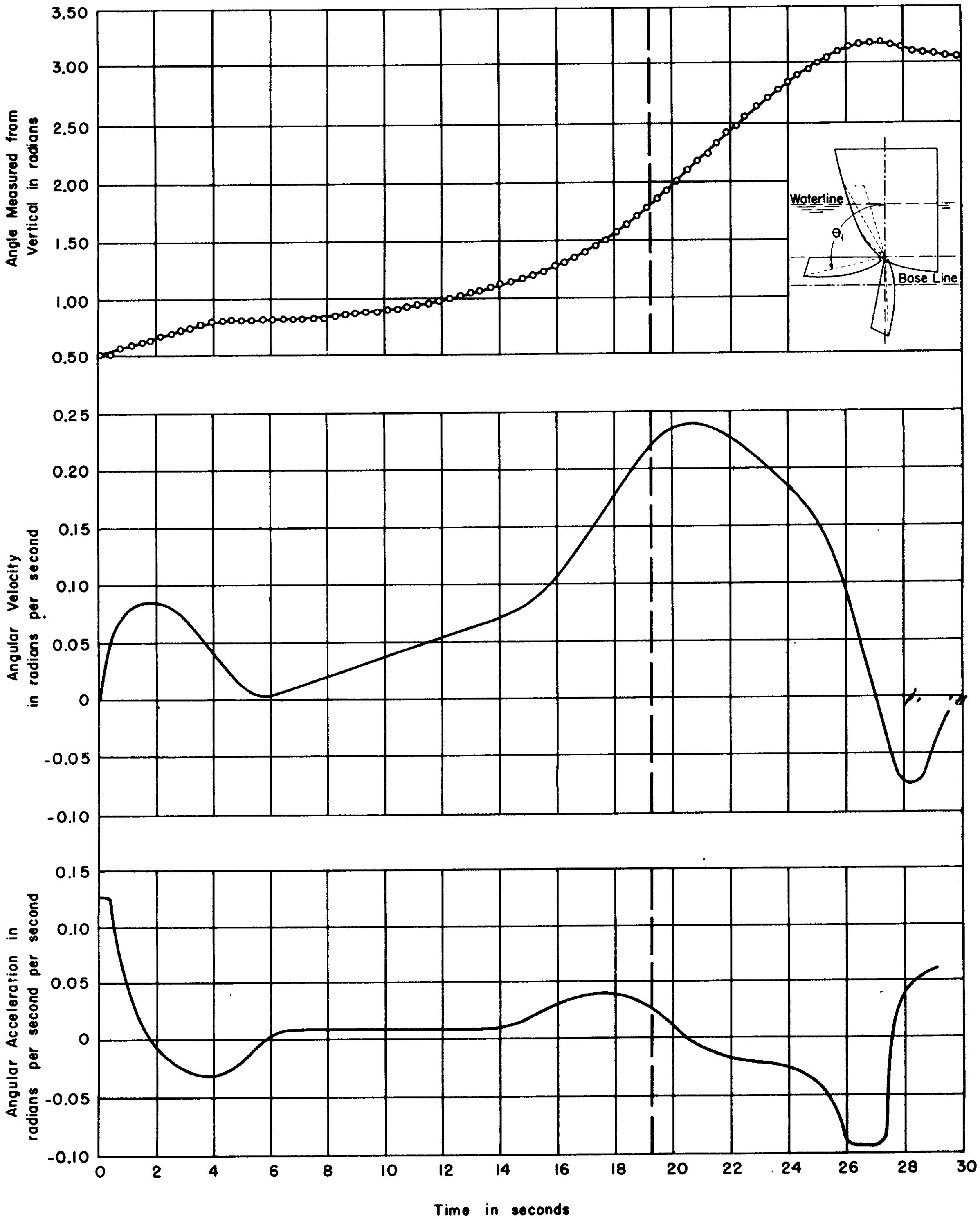

Figure 7 - Angular Displacement, Angular Velccity, and Angular Acceleration Plotted against Time for Full-Scale Gate at a Dock Draft of 20 Feet

The broken line represents the time at which the gate reached its horizontal position; this position is indicated by θ_1 in the sketch.

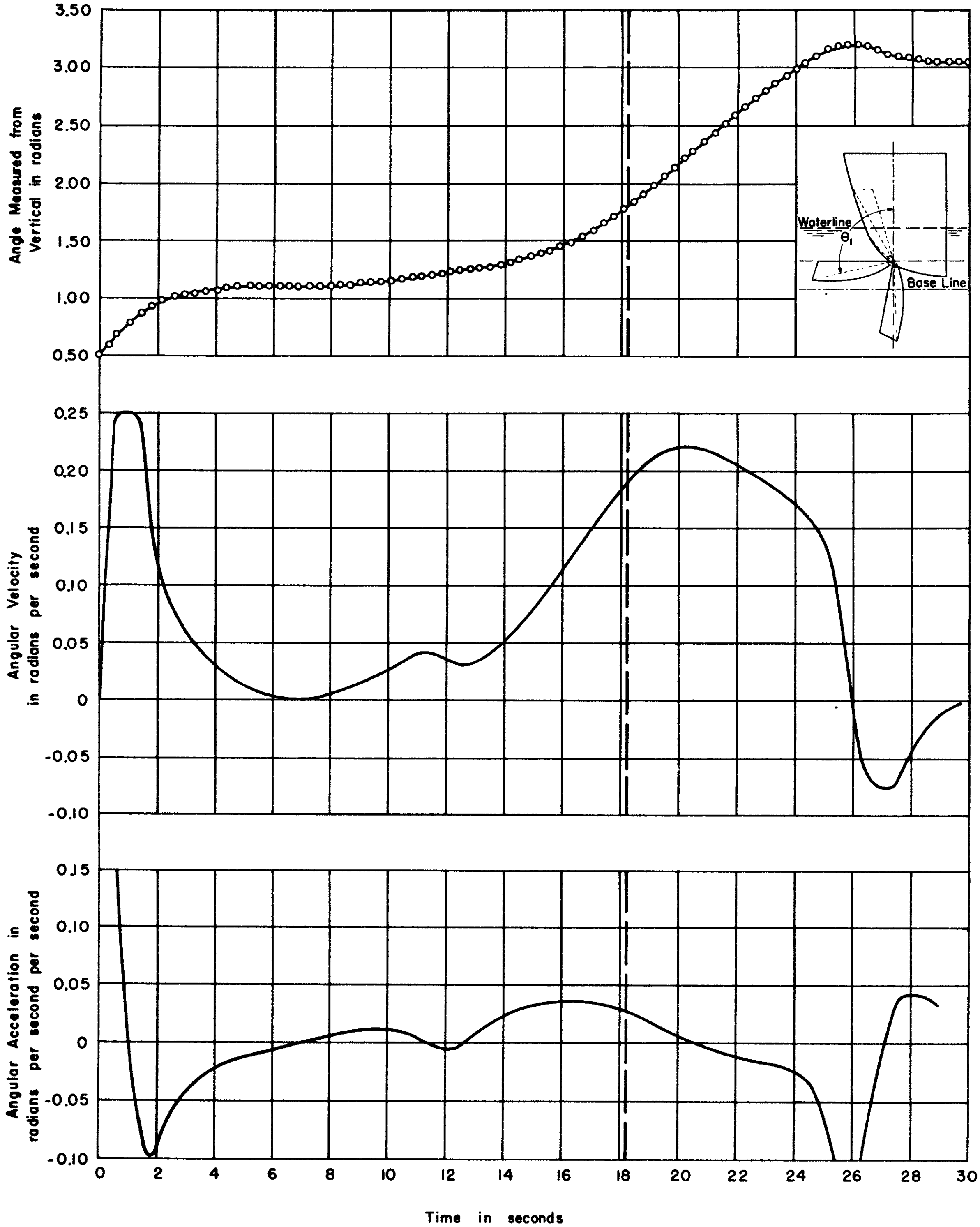

Figure 8 - Angular Displacement, Angular Velocity, and Angular Acceleration Plotted against Time for Full-Scale Gate at a Dock Draft of 15 Feet

The broken line represents the time at which the gate reached its horizontal position; this position is indicated by θ_1 in the sketch.

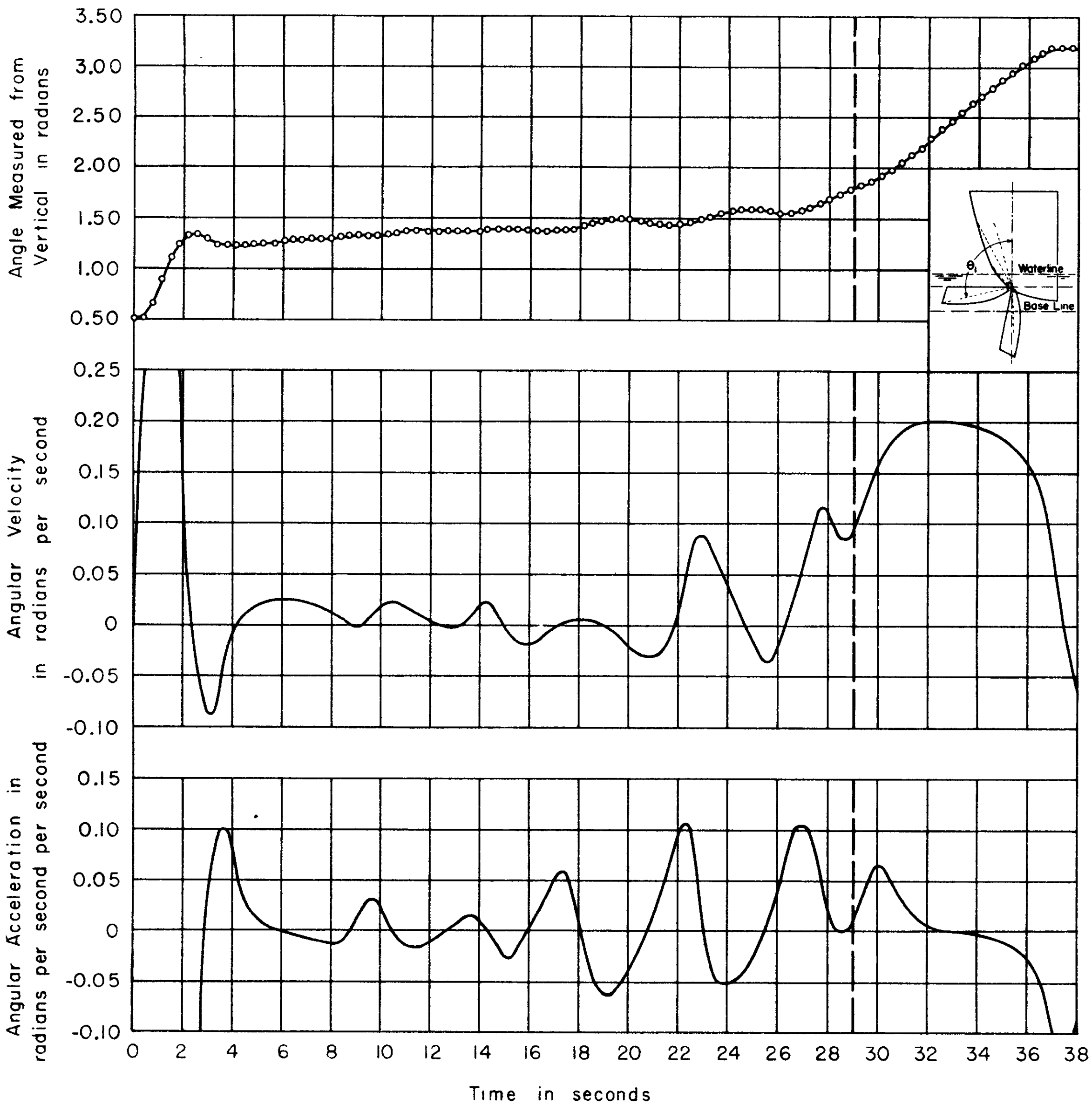

Figure 9 - Angular Displacement, Angular Velocity, and Angular Acceleration Plotted against Time for Full-Scale Gate at a Dock Draft of 10 Feet

The broken line represents the time at which the gate reached its horizontal position; this position is indicated by θ_1 in the sketch.

The third part of the velocity curve corresponds to the time when the gate passes the horizontal position. The maximum angular velocity is attained shortly after the horizontal, and diminishes to zero approximately at the vertical, beyond which the gate is restrained by its hinges and cannot rotate.

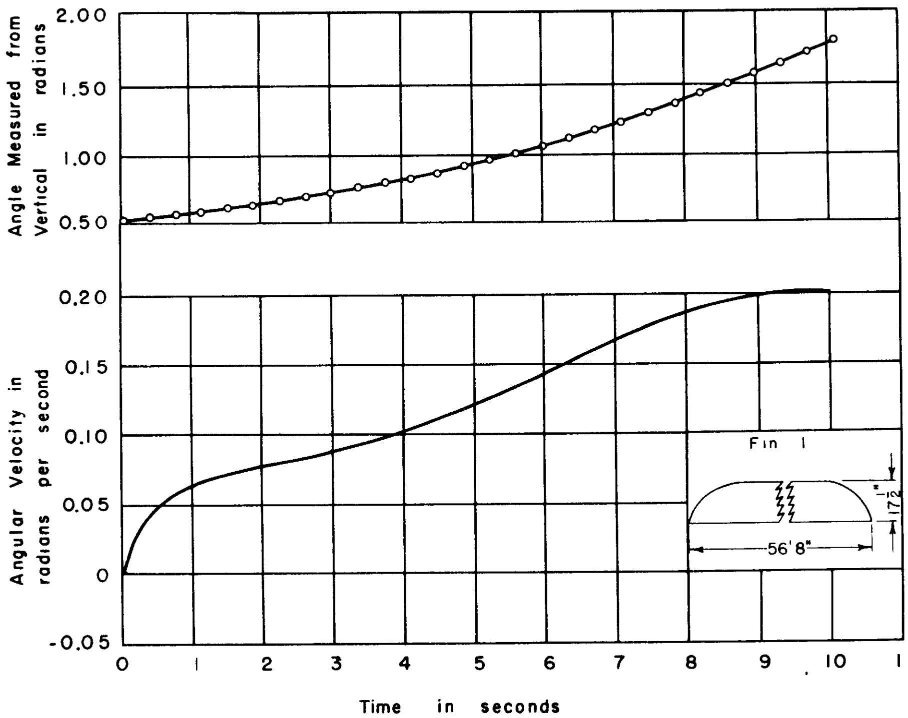

Figure 10a - Fin 1 Attached

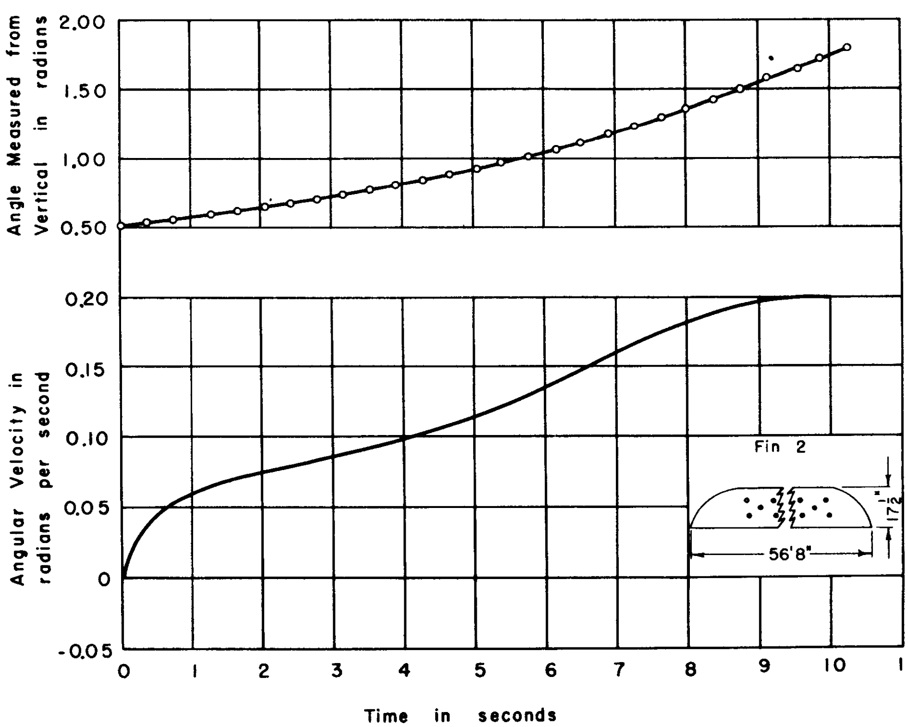

Figure 10b - Fin 2 Attached

Figure 10 - Angular Displacement and Angular Velocity Plotted against Time, for Full-Scale Gate with Fin Attached, at a Dock Draft of 30 Feet

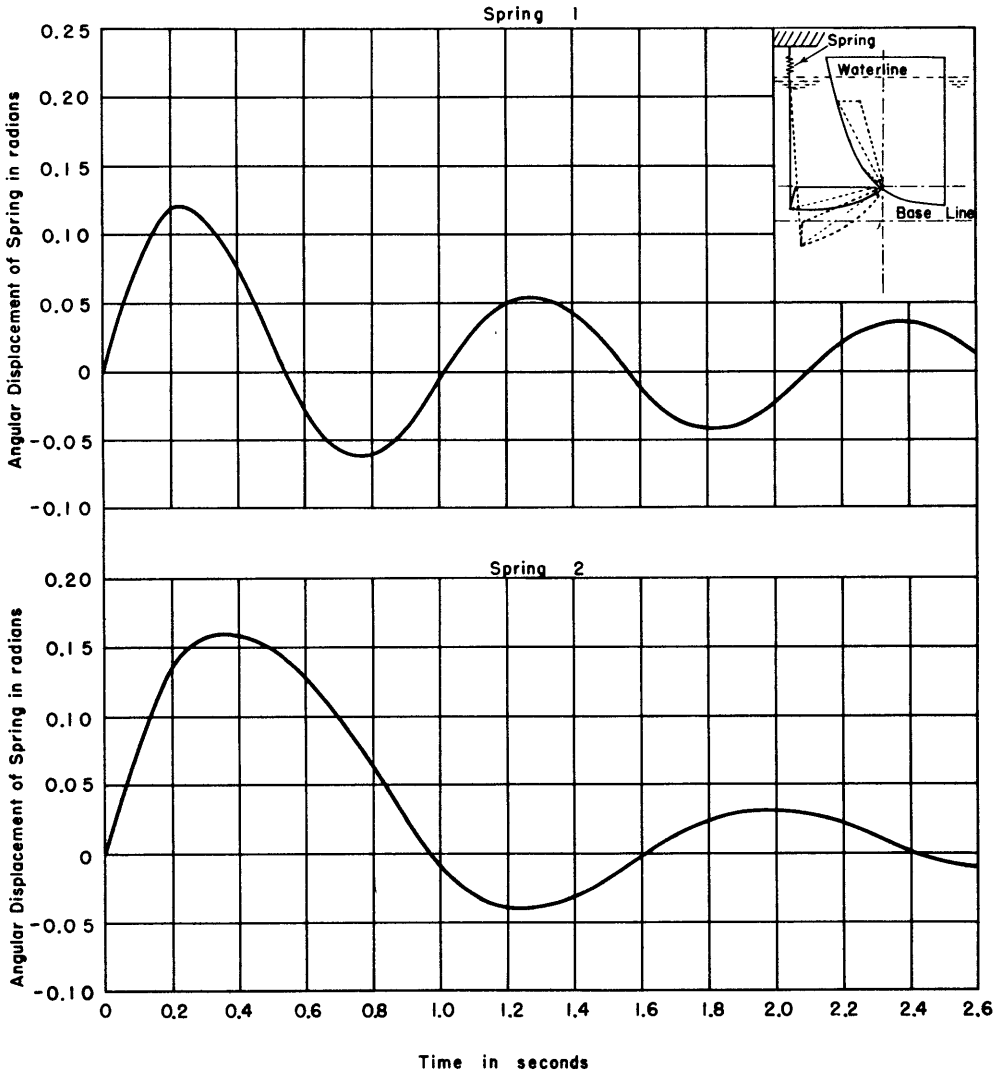

Figure 11 - Oscillations of Springs with Model Gate Attached, Plotted against Time, for a Dock Draft of 30 Feet

The torsional spring constant of Spring 1 is 86.20 pound-feet per radian; that of Spring 2 is 35.54 pound-feet per radian.

The gate when submerged in water acquires an angular velocity at the horizontal position which can be expressed as a percentage of the angular velocity of the gate at the same position when it falls freely in a vacuum. The angular velocity in a vacuum would be 1.996 radian per second, which shows the velocity in water to be 11.1 per cent of that in a vacuum.

The kinetic energy is directly proportional to the square of the angular velocity. Therefore, if the velocity can be decreased, the kinetic energy will be materially diminished. At the lesser drafts, the angular velocity of the gate at the horizontal position is smaller. Lowering the gate at a reduced dock draft can be excluded as a possible solution for decreasing the

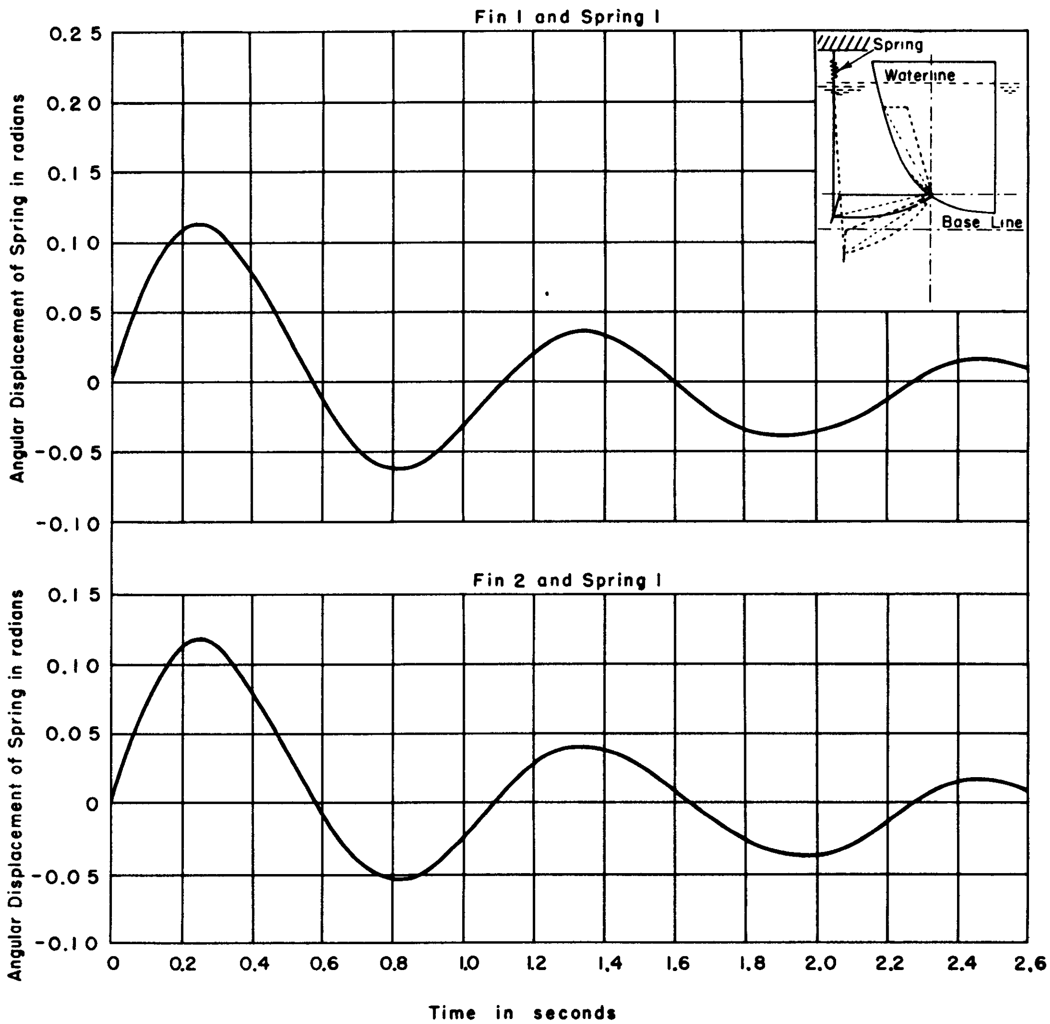

Figure 12 - Oscillations of Springs with Finned Model Gate, Plotted against Time, for a Dock Draft of 30 Feet

kinetic energy at the horizontal position of the gate as it is apparent that the much higher initial acceleration at the lesser drafts would produce a large impact force when the gate struck the water. In addition, since the gate is buoyant unless the gate compartments are almost filled with water, the gate would float on the water surface and would be subjected to wave action, which would be undesirable for several reasons.

As a method for reducing the angular velocity of the gate while it is submerged, it was suggested that a fin extending outward from the gate be added. This fin would increase the damping, which in turn would reduce the speed of the gate. The fin would act on a principle similar to that of the bilge keels on a ship. Figure 10 shows the angular velocity acquired by the gate equipped successively with two different types of fins when the dock was at a draft of 30 feet. With each type of fins the angular velocity was 0.201 radian per second. This represents a 10 per cent decrease from the velocity previously obtained. The effectiveness of this reduction in velocity is lessened, however, by a concurrent increase in what will be called in this report the *effective moment of inertia*, a term which is defined in the second paragraph following.

A detailed analysis for the determination of the kinetic energy which is not absorbed in the water cannot be made from the velocity curves alone since the moment of inertia of the gate with entrained water is unknown. To determine this moment of inertia and to determine also the amount of energy dissipated in the water by damping and other forces necessitated additional tests.

The moment of inertia of the gate in air is greatly supplemented by the inertia of the masses of water actuated by the gate. The moment of inertia of the moving mass of water has been termed the *virtual moment of inertia* and may be treated, for practical purposes, as the moment of inertia for an equivalent rigid body. For purposes of analysis, these two moments of inertia may be added to form the *effective moment of inertia*.

Once the effective moment of inertia is determined, it can be used in conjunction with the angular-velocity curves to compute the kinetic energy at any stage of the free fall.

Two methods have been used to determine the effective moment of inertia. In the first method the effective moment of inertia is determined by an analysis of damped-vibration curves; this analysis and the results obtained with it are given in Appendix 1. In the second method the effective moment of inertia is obtained by using energy equations; these equations are derived and the computations are performed in Appendix 2.

The experiments were conducted with two different springs to eliminate the effect of the stiffness of the springs, if any, from the computations. In addition, the effective moment of inertia was calculated by the first method for the gate with each type of fin attached. Table 2 gives the values of the effective moment of inertia obtained by these two methods for each spring. It can be seen that the two methods give results which are in very close agreement.

The four values of effective moment of inertia for the gate without fins can be averaged to give 2.173 slug-feet^2 for the model or 6,954,000 slug-feet^2 for the prototype. Compared with the value of 466,000 slug-feet^2 for the moment of inertia of the gate in a vacuum, this figure shows that the moment of inertia of the gate in a vacuum is only 6.7 per cent of that in water.

The influence of a fin on the effective moment of inertia of the gate can be seen in Table 2. The purpose of the holes in Fin 2 was to diminish

TABLE 2

Values of Effective Moment of Inertia for Model

	Effective Mass Moment of Inertia I, slug-feet^2	
	Obtained from Damped-Vibration Curves	Obtained from Energy Equations
Spring 1	2.208	2.120
Spring 2	2.186	2.177
Spring 1 with Fin 1 on Gate	2.629	
Spring 1 with Fin 2 on Gate	2.508	

the effective moment of inertia. Fin 1 increases the effective moment of inertia about 21 per cent; Fin 2 increases it 15 per cent.

The formula for kinetic energy is

$$K.E. = \frac{1}{2} I\omega^2$$

where I is the effective mass moment of inertia and ω is the angular velocity. The values of effective moment of inertia can be substituted in this formula to arrive at the values of unabsorbed kinetic energy in the system. These results, together with the value for kinetic energy of the gate falling in a vacuum, are shown in Table 3.

From Table 3 it is seen that the kinetic energy of the gate in water at the horizontal position is 18.5 per cent of that in a vacuum. Since the kinetic energy of the gate in air can be calculated easily, this figure can be used as a basis for future design of gates of this type.

TABLE 3

Values of Kinetic Energy of Gate at Horizontal Position

	Kinetic Energy, foot-pounds	
	Model	Prototype
In Vacuum	5.800	928,000
Submerged	1.071	171,360
Submerged with Fin 1	1.065	170,400
Submerged with Fin 2	1.016	162,560

CORRELATION WITH FULL-SCALE TESTS

On 20 December 1944, the Bureau of Yards and Docks reported orally that free-fall tests on the stern gate of a full-scale dock had been made. The full-scale gate, which was completely submerged at the closed position, was allowed to fall freely to the horizontal position. The velocity attained at this position was 0.217 radian per second as compared with 0.220 radian per second obtained in the model tests.

REFERENCES

(1) BuDocks letter N16-5 of 18 July 1944 to TMB.

Original data are filed in the TMB Record Vault under Miscellaneous 1070. All correspondence on this project can be found on TMB File N16-5.

APPENDIX 1

DETERMINATION OF EFFECTIVE MOMENT OF INERTIA OF GATE FROM DAMPED-VIBRATION CURVES

In the vibrating system shown in Figure 13, the torque Q caused by the rotating gate and its entrained mass of water acts on the spring to set it in motion. After the initial disturbance, the differential equation of the system becomes •

$$I \frac{d^2\theta}{dt^2} + c\frac{d\theta}{dt} + k\theta = 0 \qquad [1]$$

where I is the effective mass moment of inertia in slug-feet^2,

θ is the displacement of the gate in radians,

t is the time in seconds,

c is the torsional damping torque per unit of angular velocity in pound-feet per radian per second, and

k is the torsional spring constant in pound-feet per radian.

If $\frac{c^2}{4I^2} < \frac{k}{I}$, a damped oscillation will occur having a frequency

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{I} - \frac{c^2}{4I^2}}$$

or a period

$$T = \frac{2\pi}{\sqrt{\frac{k}{I} - \frac{c^2}{4I^2}}} = \frac{4\pi I}{\sqrt{4kI - c^2}} \qquad [2]$$

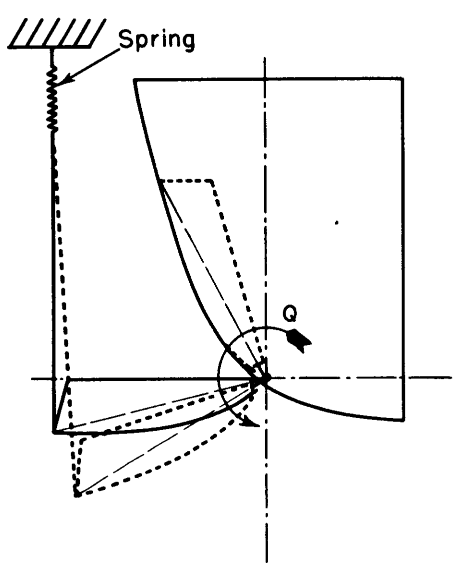

Figure 13 - Diagram Showing Successive Positions of the Gate for Analysis of Damped-Vibration Curves

The logarithmic decrement is defined as the natural logarithm of the ratio of two successive amplitudes where there is less than critical damping. In terms of critical damping, the logarithmic decrement

$$\delta = \log_e \beta_1 - \log_e \beta_2 = 2\pi \frac{c}{c_c} \qquad [3]$$

where β_1 and β_2 are the maximum values of the first and second amplitudes of vibration of the spring in radians, and c_c is the critical damping constant in pound-feet per radian per second.

For critical damping

$$\frac{c^2}{4I^2} = \frac{k}{I}$$

and

$$c_c = 2\sqrt{Ik} \qquad [4]$$

Then

$$\log_e \beta_1 - \log_e \beta_2 = \frac{\pi c}{\sqrt{Ik}} \qquad [5]$$

From Equation [2]

$$k = \frac{4\pi^2 I}{T^2} - \frac{c^2}{4I}$$

Equation [5] now becomes

$$\log_e \beta_1 - \log_e \beta_2 = \frac{\pi c}{\sqrt{\frac{4\pi^2 I^2}{T^2} - \frac{c^2}{4}}}$$

Neglecting $\frac{c^2}{4}$

$$\log_e \beta_1 - \log_e \beta_2 = \frac{Tc}{2I} \qquad [6]$$

$$\frac{c}{I} = \frac{2}{T}(\log_e \beta_1 - \log_e \beta_2) = a \qquad [7]$$

where a is a constant. Equation [2] may now be expressed as

$$T^2 = \frac{16\pi^2 I^2}{4Ik - c^2}$$

Then

$$I = \frac{4kT^2}{16\pi^2 + a^2T^2} \qquad [8]$$

The magnitude of I in slug-feet^2 can be found by substituting the known values, β_1, β_2, T, and k, in Equations [7] and [8]. Table 4 gives the data used in computing the effective moment of inertia and the results of this computation.

TABLE 4

Values of Effective Moment of Inertia for Model, Obtained from Damped-Vibration Curves

	Spring 1	Spring 2	Spring 1 with Fin 1 on Gate	Spring 1 with Fin 2 on Gate
β_1, radians	0.121	0.159	0.115	0.118
β_2, radians	0.054	0.032	0.037	0.041
T, seconds	1.015	1.610	1.115	1.090
$\text{Log}_e \frac{\beta_1}{\beta_2}$	0.8068	1.6031	1.1341	1.0571
k, pound-feet per radian	86.20	35.54	86.20	86.20
I, slug-feet^2	2.208	2.186	2.629	2.508

APPENDIX 2

DETERMINATION OF EFFECTIVE MOMENT OF INERTIA OF GATE BY USE OF ENERGY EQUATIONS

Consider the various positions of the gate as shown in Figure 14. Position 0 is the initial position of the gate. Position 1 is the position of the gate at the instant before tension occurs in the spring. Position 2 is the position of the gate at the maximum extension of the spring.

Let W be the weight of the gate in water in pounds, ω the angular velocity of the gate in radians per second, θ_T the displacement of the spring in radians, and $\bar{x}$ and the α's as shown in Figure 14. Then it can be readily seen that the energy at each of these positions is as follows:

At Position 0

Potential energy of gate $= W(\bar{x} - \bar{x}\cos\alpha_0)$

At Position 1

Potential energy of gate $= W(\bar{x} - \bar{x}\cos\alpha_1)$

Kinetic energy of moving gate and entrained water $= \frac{1}{2} I\omega^2$

At Position 2

Potential energy of gate $= W(\bar{x} - \bar{x}\cos\alpha_2)$

Elastic energy in spring $= \frac{k\theta_T^2}{2}$

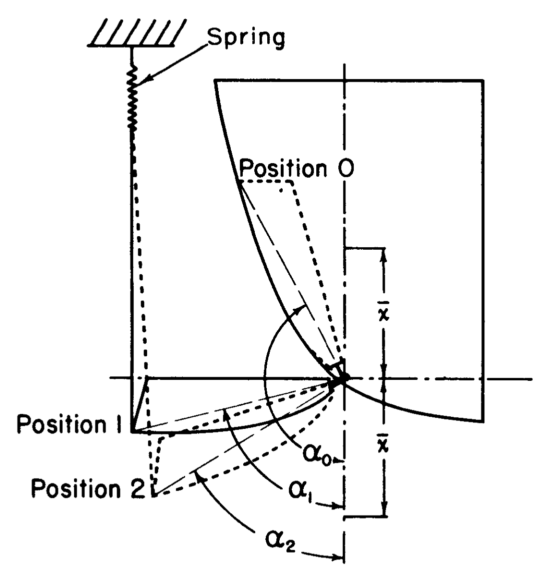

Figure 14 - Diagram Showing Successive Positions of the Gate as Stages of Potential Energy

Between Positions 0 and 1 a quantity of energy is dissipated in damping. If the magnitude of this energy loss is considered to vary as the square of the angular velocity, then

$$D_{0,1} = c\int_0^1 \omega^2\, d\theta$$

where $D_{0,1}$ is the loss of energy between Positions 0 and 1. Similarly, the energy dissipated in damping between Positions 1 and 2 is equal to

$$D_{1,2} = c\int_1^2 \omega^2\, d\theta$$

According to the law of conservation of energy, the following energy relationship exists between Positions 0 and 2:

$$W(\bar{x} - \bar{x}\cos\alpha_0) - (D_{0,1} + D_{1,2}) = W(\bar{x} - \bar{x}\cos\alpha_2) + \frac{k\theta_T^2}{2}$$

$$W\bar{x}(\cos\alpha_2 - \cos\alpha_0) - \frac{k\theta_T^2}{2} = D_{0,1} + D_{1,2} \qquad [9]$$

Between Positions 0 and 1 the energy equation may be stated as follows:

$$W(\bar{x} - \bar{x}\cos\alpha_0) - D_{0,1} = W(\bar{x} - \bar{x}\cos\alpha_1) + \frac{1}{2}I\omega^2$$

$$W\bar{x}(\cos\alpha_1 - \cos\alpha_0) = \frac{1}{2}I\omega^2 + D_{0,1}$$

$$I = \frac{2}{\omega^2}\left[W\bar{x}(\cos\alpha_1 - \cos\alpha_0) - D_{0,1}\right] \qquad [10]$$

TABLE 5

Values of Effective Moment of Inertia for Model, Obtained from Energy Equations

	Spring 1	Spring 2
k, pound-feet per radian	86.20	35.54
θ_T, radians	0.171	0.270
$\frac{k\theta_T^2}{2}$, foot-pounds	1.260	1.295
α_0, degrees	149.5	149.5
α_1, degrees	78.8	80.4
α_2, degrees	69.0	64.9
$\cos\alpha_0$	-0.86163	-0.86163
$\cos\alpha_1$	0.19423	0.16677
$\cos\alpha_2$	0.38386	0.46843
W, pounds	7.69	7.69
$\bar{x}$, feet	0.558	0.558
$W\bar{x}(\cos\alpha_2 - \cos\alpha_0) - \frac{k\theta_T^2}{2}$, foot-pounds	4.084	4.284
$\int_0^1 \omega^2\, d\theta$	0.655	0.632
$\int_1^2 \omega^2\, d\theta$	0.1092	0.1812
$\frac{D_{1,2}}{D_{0,1}}$	0.1660	0.2844
$D_{0,1}$, foot-pounds	3.509	3.335
$W\bar{x}(\cos\alpha_1 - \cos\alpha_0)$, foot-pounds	4.531	4.413
ω^2	0.980	0.990
I, slug-feet^2	2.120	2.177

If the damping coefficient is now assumed to be the same on deceleration as it is on acceleration of the gate, then $D_{1,2}$ may be expressed as a ratio of $D_{0,1}$, or

$$\frac{D_{1,2}}{D_{0,1}} = \frac{c\int_1^2 \omega^2\,d\theta}{c\int_0^1 \omega^2\,d\theta} = R$$

where R is a constant. These integrals can be evaluated by numerical integration of the data. Thus, substituting in Equation [9], we obtain

$$D_{0,1}(1 + R) = W\bar{x}(\cos\alpha_2 - \cos\alpha_0) - \frac{k\theta_T{}^2}{2} \qquad [11]$$

Since $D_{0,1}$ is the only unknown in this equation, its value can be readily found. This value can be inserted in Equation [10] to determine the value of I.

The data used in the computation of I and the results of this computation are given in Table 5.

APPENDIX 3

EQUATIONS FOR GATE FALLING FREELY IN A VACUUM

When the gate is permitted to fall freely in a vacuum, the energy dissipated in damping becomes zero. Therefore the energy between Positions 0 and 2 is expressed by the equation

$$W_a \bar{x}(\cos\alpha_2 - \cos\alpha_0) = \frac{k\theta_T^2}{2}$$

Between Positions 0 and 1 the energy may be expressed by

$$W_a(\cos\alpha_1 - \cos\alpha_0) = \frac{1}{2} I\omega^2$$

where W_a is the weight of the gate in air. If θ_s is taken here as the extension of the spring beyond that which is produced by the weight of the gate, then

$$\frac{k\theta_s^2}{2} = \frac{1}{2} I\omega^2 = W_a(\cos\alpha_1 - \cos\alpha_0)$$

From experimental values

$$\frac{k\theta_s^2}{2} = \frac{89.44 \times (0.3406)^2}{2} = 5.188 \text{ foot-pounds}$$

Using an equivalent drop for theoretical calculations

$$W_a(\cos\alpha_1 - \cos\alpha_0) = 9.516 \times 0.588(0.97830) = 5.195 \text{ foot-pounds}$$

which shows a close check between the theoretical and experimental values.

At the horizontal position of the gate, using the theoretical formula, the kinetic energy

$$\frac{1}{2} I\omega^2 = 5.80 \text{ foot-pounds}$$

and the angular velocity

$$\omega = 8.925 \text{ radians per second}$$