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MIGRATION OF UNDERWATER GAS GLOBES DUE TO GRAVITY AND NEIGHBORING SURFACES

by

Prof. E.H. Kennard

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NOTATION

c_X, c_Y, c_Z	Cosines of the angles between the upward vertical and the X, Y, Z axes, respectively
D	Depth of water
D'	Hydrostatic pressure, including atmospheric pressure, expressed as an equivalent depth of water
g	Acceleration due to gravity
g_X, g_Y, g_Z	Components of the acceleration due to gravity in the directions of the X, Y, Z axes; $g_X = -gc_X$, $g_Y = -gc_Y$, $g_Z = -gc_Z$
H	Vertical component of Q
I	Impulse per unit area or $\int p\,dt$
$I_{0.2}$	Impulse per unit area due to the peak of pressure included between the instants at which p has one-fifth of its intervening maximum value
M, N_X, N_Y, N_Z	Functions of the distances from certain surfaces, varying from case to case
p	Pressure in the water
p_0	Hydrostatic pressure, including atmospheric pressure, at the level at which a charge is detonated
p_A	Hydrostatic pressure p_0 expressed in atmospheres
p_g	Pressure of the gas in a gas globe
Q	Linear displacement of the center of the gas globe from the time of detonation until the time of the first peak recompression
R	Radius of the gas globe
R_0	Radius of gas globe when its pressure equals hydrostatic pressure at the level of its center
R_2	Maximum radius of the gas globe
r	Distance from the center of the gas globe to a point in the water
S	Horizontal component of Q
T_1	Period of first oscillation, up to first peak recompression
T_{10}	Value of T_1 at the same level if no bounding surface is near
t	Time
W	Weight of charge
X, Y, Z	Coordinates of the center of the gas globe
X_0, Y_0, Z_0	Values of X, Y, Z at instant of detonation of charge
z_0	A dimensionless coefficient referring to depth below the surface and occurring only in Figure 5
γ	Ratio of the specific heat of the gas in the gas globe at constant pressure to its specific heat at constant volume

DIGEST*

The gas globe formed by an underwater explosion not only pulsates in size but also usually changes position as each pulsation occurs (1).** This migration may be of importance because the first recompression or contraction of the globe thereby comes to be centered at a point different from that of the initial detonation, and the location of the point of recompression influences the damage that may be done by the associated secondary pulse of pressure.

Accurate solution of the general equations of the motion of the gas globe set up by Herring (4) in this country and by G.I. Taylor and others (5) (6) (7) (8) in England is, in general, possible only by numerical solution of the differential equations; it has not yet been found possible to embark on such an enterprise in this country because of the labor involved. For purposes of analysis, calculations of the motion of the gas globe have been made by an approximate method. The assumptions underlying this method are set forth in the first part of the report, together with a discussion of the apparently conflicting and paradoxical motions of the gas globe under various conditions.

For example, in free water of unlimited extent the gas globe rises because of its buoyancy. Near a rigid boundary such as a vertical wall, the globe is attracted toward the boundary, as shown in Figure 2. Near the surface of the water, the gas globe is repelled from the free surface. However, although the action of gravity is always present to cause the globe to rise through the water by virtue of its buoyancy, the attraction of a rigid bottom in shallow water, or the repulsion from the free surface of the water, may decrease the

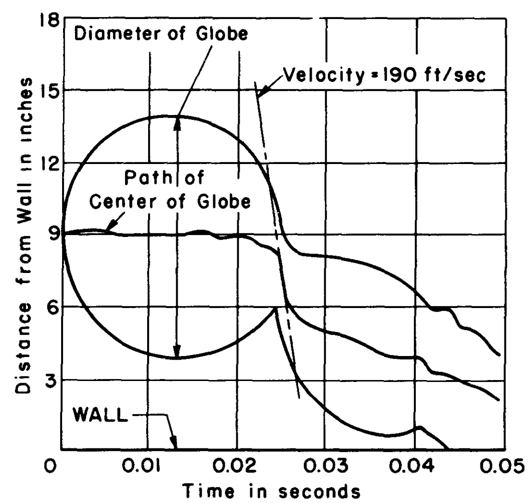

Figure 2 - Curves of Size and Position of a Gas Globe near a Vertical Wall

The charge was fired 9 inches from a rigid vertical wall. Note that the velocity of the center of the globe is greatest during the compression phases.

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* This digest is a condensation of the text of the report, containing a description of all essential features and giving the principal results. It is prepared and included for the benefit of those who cannot spare the time to read the whole report.

** Numbers in parentheses indicate references on page 33 of this report.

rise due to gravity; in certain cases, they may actually produce a downward motion of the globe.

The results found by acceptance of the assumptions given and by use of the approximations are summarized in a series of formulas and curves. Many of the results are stated in terms of the radius R_2 of the globe at maximum expansion. For tetryl or TNT, this is estimated from observation as

$$R_2 = 4.1\left(\frac{W}{p_A}\right)^{\frac{1}{3}} \qquad [18]$$

where R_2 is in feet,

W is the charge weight in pounds, and

p_A is the total pressure, including atmospheric, expressed in atmospheres.

Figure 4 presents this information graphically.

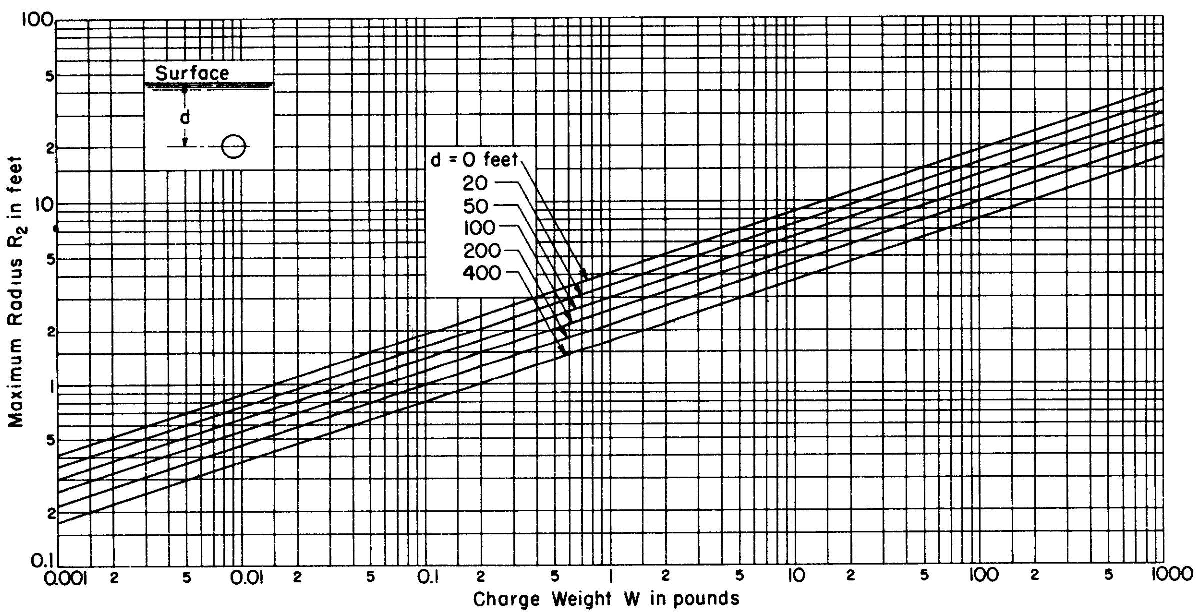

Figure 4 - Maximum Radius R_2 on First Expansion of the Gas Globe Produced by a Charge of W Pounds of Tetryl or TNT

The center of the globe is d feet below the surface of the sea; lines are drawn for several values of d; see Equation [18] at top of page.

Vertical migration due to gravity alone is a special case treated first by the approximate method. By neglecting the action of the gas in the globe, it becomes possible to express the solution in dimensionless terms, and this is done for four depths in Figure 5 on page 13, and in Table 1 on page 13. The minimum depth below the surface at which recompression will occur is assumed to be equal to the sum of the maximum radius and the rise to the time of the first peak. The results are shown in Figure 6.

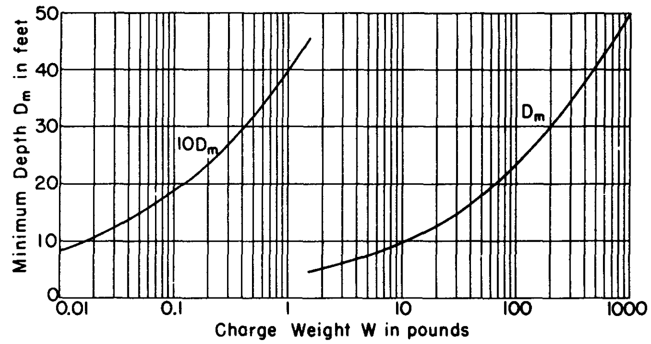

Figure 6 - Curve Giving a Rough Estimate of the Minimum Depth D_m below the Surface at which a Charge W may be Detonated without Blowing through the Surface before Undergoing Recompression

The migration caused by the proximity of a surface, in the absence of gravity, is shown in Figure 7 on page 16, expressed as a fraction of R_2. The combined effect of gravity and a vertical wall for a single small charge is shown in Figures 9 and 10. Curves for estimating the rise of the gas globe under a free surface and above a rigid horizontal bottom are given in Figure 11 and 12 respectively, page 20, for a wide range of charges.

In Figures 15 and 16 on page 26, the two components of migration, vertical and horizontal, are given for the combined effect of gravitation, a free surface, and a vertical wall. As the weight of the charge increases, the gravitational effect is shown to predominate; thus the downward motion of the globe from a small charge is reversed with an increasing charge in the

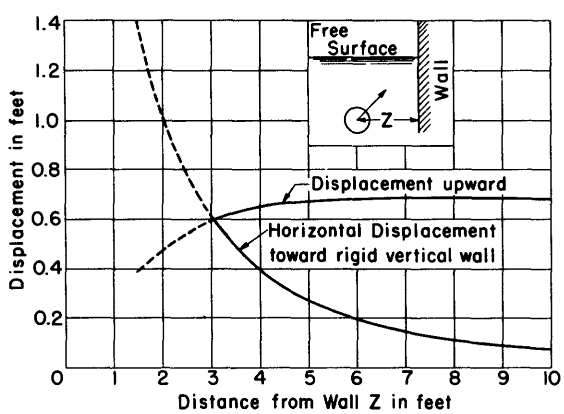

Figure 9 - Upward and Horizontal Components of Displacement of the Gas Globe from 3/4 Ounce of Tetryl or TNT

The charge is assumed to be detonated 10 feet below the surface of the water, far above the bottom, and Z feet from a rigid vertical wall. The curves show the displacement from the point of detonation up to the point of greatest recompression, according to approximate calculations.

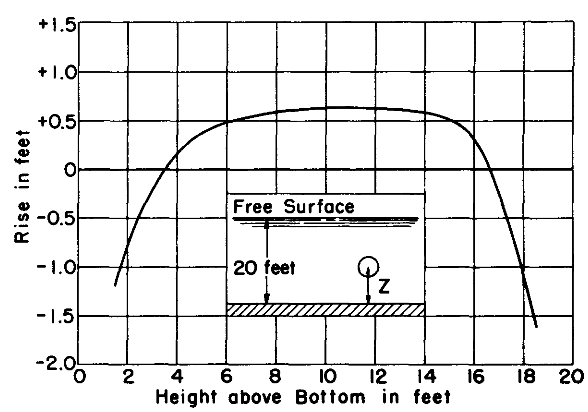

Figure 10 - Displacement of the Center of the Gas Globe from 3/4 Ounce of Tetryl or TNT, up to the Instant of Maximum Recompression

Detonation is assumed to occur at the height shown above a rigid horizontal bottom in water 20 feet deep. Positive ordinates represent a rise, negative ones a descent. The curve is based on approximate calculations.

cases studied, and the motion is upward for all charges over 1/2 pound. The horizontal motion diminishes with an increasing size of charge in the cases studied at all charges over 1/2 pound.

Finally, a rough estimate is made of the effect of the migration upon the pressure that is generated in the water by the recompression of the gas globe; curves are shown in Figure 17 on page 28. A migratory displacement equal to one-third of the maximum radius R_2 decreases the peak pressure by about one-third, whereas a displacement equal to R_2 reduces the peak pressure to one-tenth of its value for zero migration. The impulse, however, is much less affected by the migration.

Important cases remain to be taken up, in particular those in which the distance from the wall is less than $2R_2$, and in which the charge lies between two rigid surfaces, like the ground and a ship's bottom.

The conditions for exact similitude with respect to migration can not be reconciled with those governing the flow of destructive energy from a charge to a target, as applied in the nominal theory of TMB Report 492 (11). The application of model tests of migration effects to the prediction of full-scale phenomena is therefore subject to correction for scale effect and any direct expansion from a very small scale to full scale, without full knowledge of the scale effects, may lead to erroneous conclusions. The exact formulation of the scale effect corrections will form the subject of further work.

MIGRATION OF UNDERWATER GAS GLOBES DUE TO GRAVITY AND NEIGHBORING SURFACES

ABSTRACT

Approximate formulas are assembled, and illustrated by curves, for the migration of gas globes under water due to the action of gravity and of neighboring surfaces. In addition to the effect of a single surface, rigid or free, consideration is given to the combination of a free surface with a rigid bottom or a vertical wall. The general analytical procedure by which the formulas were obtained is described but most of the details are omitted.

INTRODUCTION

The gas globe formed by an underwater explosion not only pulsates in size but also usually changes position as each pulsation occurs (1).* This migration may be of importance because the first recompression or contraction of the globe thereby comes to be centered at a point different from that of the initial detonation, and the location of the point of recompression influences the damage that may be done by the associated secondary pulse of pressure. Measurements of the migration will be reported separately but a number of analytical results have been obtained, and these results will be assembled here for convenience of reference. Deductions of the formulas may be found elsewhere (2).

The motion of the water around the pulsating gas globe is sufficiently slow so that compression of the water can be neglected. Furthermore, good experimental support exists for the assumption that the globe remains approximately spherical during the larger part of the first cycle at least. For these reasons certain aspects of the migration are adequately covered by old investigations on the motion of spheres, which are summarized in Lamb's Hydrodynamics, Section 100 (3).

A thorough survey of the problem has been given recently by Herring (4) and numerical studies have been made, especially of the gravitational displacement, by Taylor and others in England (5) (6) (7) (8). Calculations by an approximate method have been made under the author's supervision at the David Taylor Model Basin. More extended calculations are in progress under the direction of Professor R. Courant of New York University; these will be described in a later report.

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* Numbers in parentheses indicate references on page 33 of this report.

GENERAL FEATURES OF THE MIGRATION OF A GAS GLOBE

The migration results from the action of gravity and from the effects of bounding surfaces such as a rigid wall or bottom or the free surface of the water; see Figure 1. These various actions are not simply superposed upon each other, because the extent of the migration is greatly increased by the periodic compression of the gas globe and the degree of compression is itself materially decreased when the rapidity of the migration becomes large. The migratory motion implies the existence of kinetic energy of translation in the surrounding water; this energy is abstracted from the energy of the radial motion, with the result that the inward motion of contraction ceases sooner than it would in the absence of the migration.

If the gas globe were fixed in size and far removed from all boundaries, it would simply rise with an acceleration of $2g$, or twice the ordinary acceleration due to gravity; for the water surrounding the gas globe is acted on by a buoyant force equal to the weight of the displaced water, and the effective mass of the water is only half of the mass of the displaced water for the type of motion that results in the upward displacement of the globe (3).

The effects of a bounding surface or wall in the neighborhood of the globe can be regarded as arising in the following manner: While the gas globe is compressed, the pressure in the water is positive, and this pressure is increased owing to the blocking effect of the wall. The pressure increase due to the wall is greatest between the gas globe and the wall, and the inequality of pressure thus produced has somewhat the same effect as if it were due to a gravitational field acting toward the wall. The gas globe then floats away from the wall in accelerated motion. During the expanded phase, on the other hand, the pressure is less than hydrostatic, and the deficit of pressure is greater on the side toward the wall, as it is relieved less on that side by the inflow of water. Thus during the expansion phase motion of the water is developed that acts to carry the gas globe toward the wall. The action on the globe throughout one cycle can be regarded as equivalent to a sort of buoyant force acting alternately away from the wall and toward it.

The action during the expansion phase predominates, however, both because this phase lasts longer than the compression phase, and because the buoyant action on a large globe is much greater than that on a small one. Since the first phase after detonation is one of positive pressure, the initial effect will be a slight displacement of the globe away from the wall; but thereafter the distribution of momentum in the water will always be such that the gas globe moves toward the wall, the momentum increasing in magnitude during each expansion phase and losing only a little during each compression. The actual velocity of the gas globe, however, will be greatest

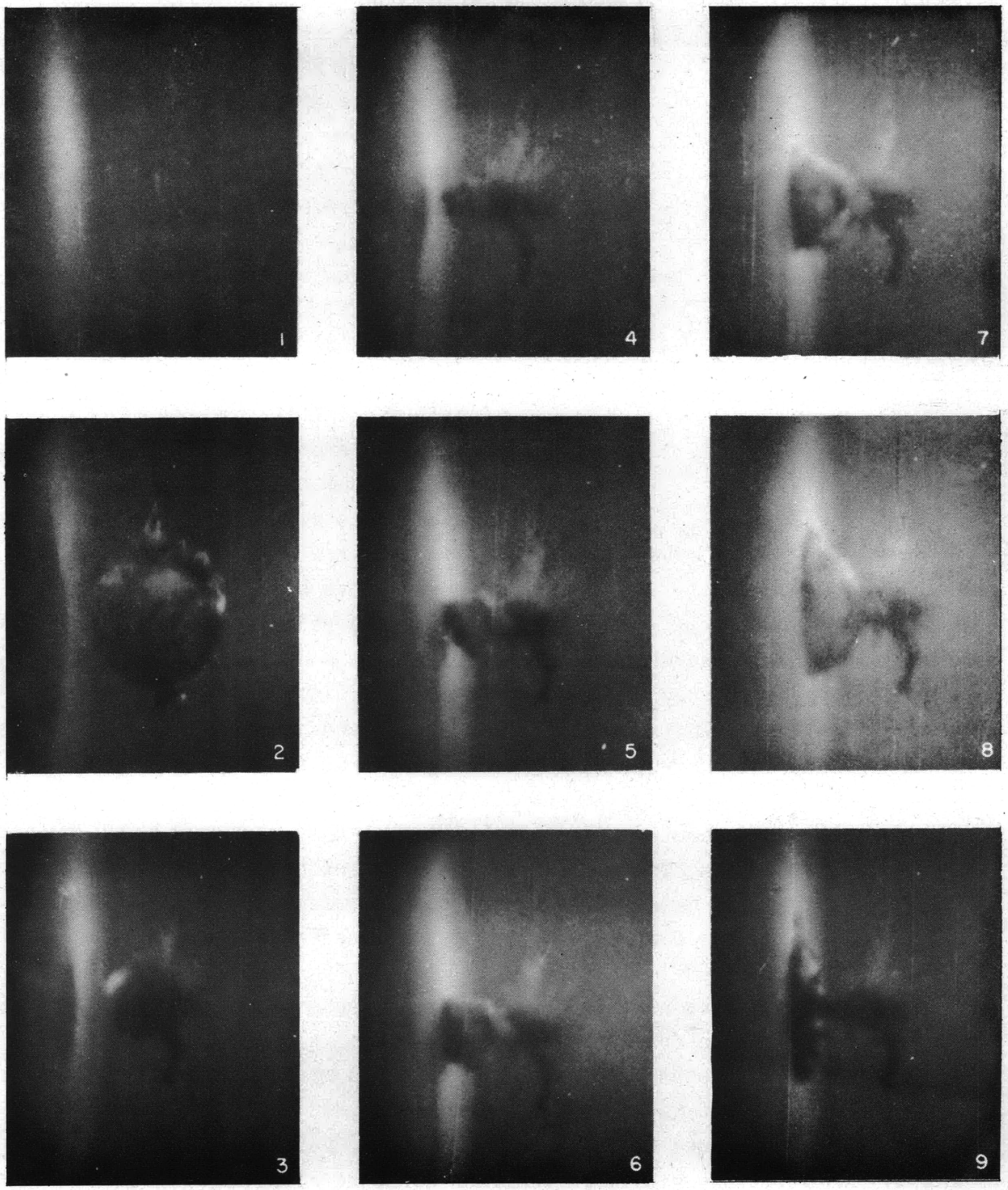

Figure 1 - High-Speed Photographs of a Gas Globe from a Charge Fired near a Wall

The charge was fired 6 inches from a rigid vertical wall shown at the left of the photographs. In Photograph 3 the globe begins to move toward the wall. Photograph 4 shows the peak of compression at the end of the first cycle. After reaching the wall, the globe continues to pulsate on it, as shown by Photographs 5 to 9.

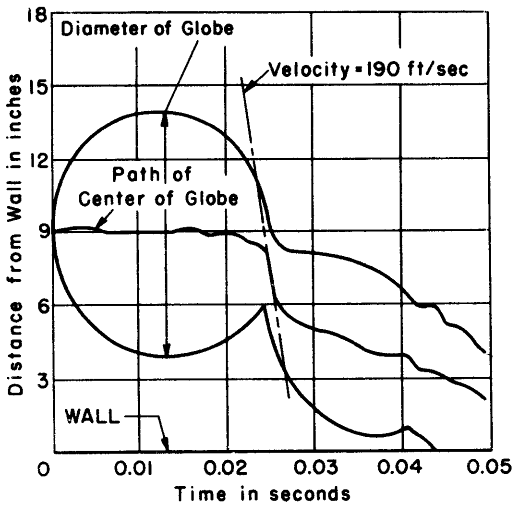

Figure 2 - Curves of Size and Position of a Gas Globe near a Vertical Wall

The charge was fired 9 inches from a rigid vertical wall. Note that the velocity of the center of the globe is greatest during the compression phases.

during the compression phase, when the momentum becomes concentrated into a comparatively small volume of water surrounding the globe. Thus the center of the gas globe moves continually toward the wall but advances chiefly in spurts during the compression phases, as shown in Figure 2.

The effect of a free surface, when the gas globe is not near enough to the surface to break through, is approximately opposite to that of a rigid surface. The initial expansion of the gas globe accelerates the water above it upward and that below it downward. While the globe is expanded, the pressure near it is very low, and this deficit of pressure acts so as to check and then reverse the radial motion. Because the pressure remains constant on the free surface, the deficit is less, or the pressure is greater, near the surface than it would be if there were additional water instead of air above the surface; and because of this relative excess of pressure, the water lying either above or below the gas globe is given an excess of momentum downward. During the next compression phase this momentum becomes concentrated in a much smaller volume of water and, provided the effect is not canceled by the simultaneous and opposed action of gravity, the globe is carried downward. Arguments from momentum in the water are dangerous, however; some further remarks on this subject are introduced at the end of this report.

By this action, the gas globe is attracted toward a rigid boundary but repelled from the free surface of the water. Although the action of gravity is always present, to cause the globe to rise through the water by virtue of its buoyant nature, attraction by the bottom in shallow water or repulsion from the free surface of the water may merely decrease the rise due to gravity, or it may actually produce a downward displacement of the globe.

The effect of a boundary should increase as the gas globe approaches the boundary. Under certain circumstances, however, observation shows that marked departures from sphericity of the globe may occur. Near a free surface, for example, part of the gas may blow through the surface while the remainder migrates down into the water; see Figure 3. Furthermore, a gas globe

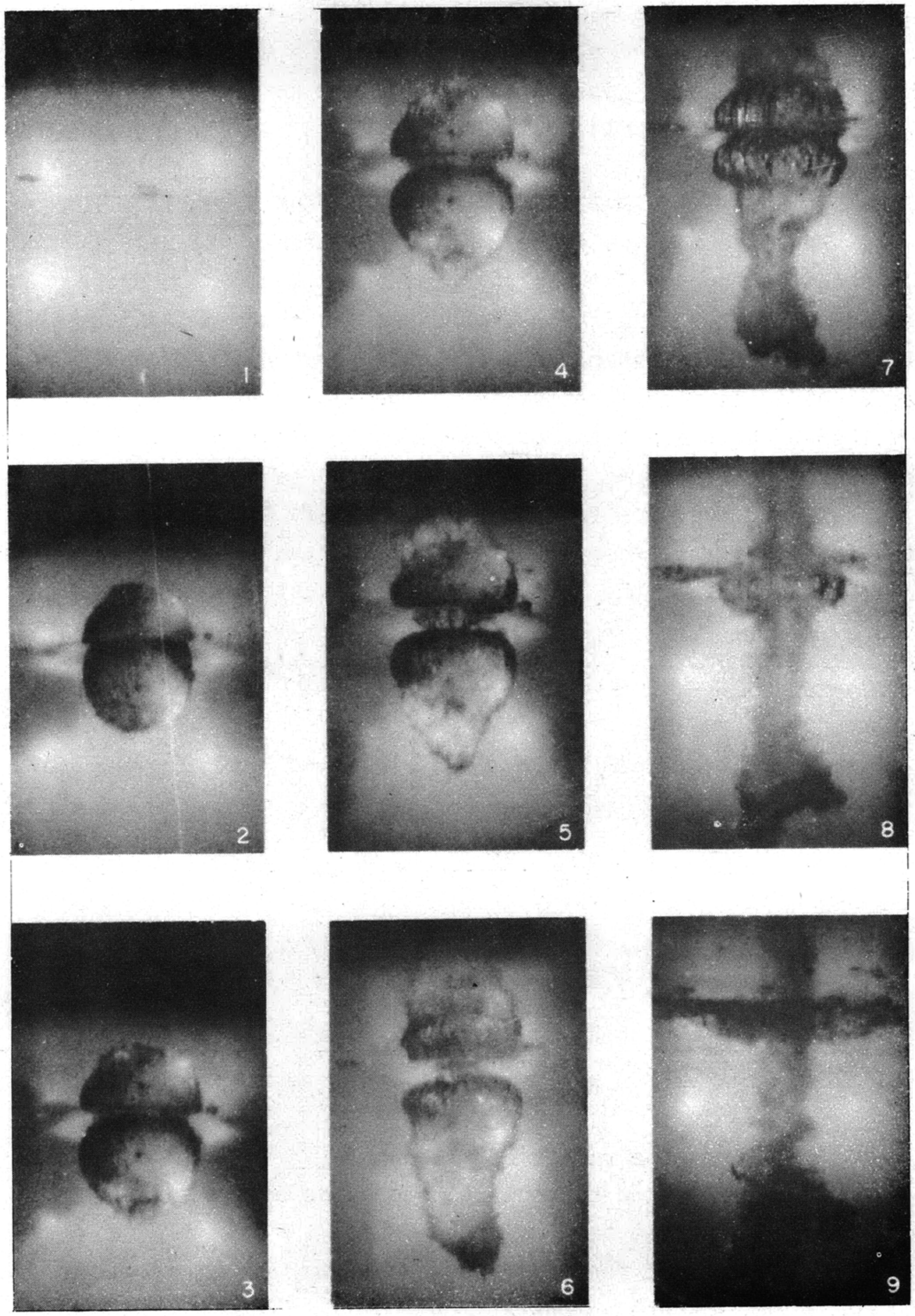

Figure 3 - High-Speed Photographs of a Gas Globe from a Charge Fired just under the Free Water Surface

The charge was fired 1 inch under the surface of the water. In Photograph 2 part of the gas has vented through the water surface. In Photographs 4 to 9 the lower part of the globe is moving downward. More gas appears to be venting through the surface on the second expansion shown in Photograph 7.

midway between two rigid boundaries may break in two, one half migrating each way (1). Such features can only be handled by a more complete analysis in which the shape of the globe is not limited by assumption but is left to the control of the hydrodynamic action.

DESCRIPTION OF THE MATHEMATICAL METHOD FOR DETERMINING MIGRATION

Provided the gas globe remains spherical, the effect of gravity alone is easily found. The analysis can be extended to include the effect of plane boundaries by using the method of images that is familiar in electrostatic theory, and by expanding in negative powers of the distances from the surfaces, as in Herring's report (4) The entire analysis, including a special method of approximation for the first oscillation, has been completed (2) but is not given here.

If the solution is required to satisfy the boundary conditions only as far as the inverse second powers of the distances from the boundaries, differential equations of the following type are obtained:

$$\dot{R}^2 = \frac{1}{1+\frac{MR}{2}}\left\{\frac{C}{R^3} + \frac{2}{\rho R^3}\int (p_g - p_0)\,R^2\,dR - \frac{1}{6}\left(\dot{X}^2+\dot{Y}^2+\dot{Z}^2\right) + \frac{1}{4}R^2\dot{R}\left(N_X\dot{X} + N_Y\dot{Y} + N_Z\dot{Z}\right) - \frac{2}{3}\left[g_X(X-X_0) + g_Y(Y-Y_0) + g_Z(Z-Z_0)\right]\right\} \qquad [1]$$

$$\dot{X} = \frac{3}{4}N_X R^2\dot{R} - \frac{3}{2}\frac{N_X}{R^3}\int_0^t R^4\dot{R}^2\,dt - \frac{2g_X}{R^3}\int_0^t R^3\,dt \qquad [2]$$

$$\dot{Y} = \frac{3}{4}N_Y R^2\dot{R} - \frac{3}{2}\frac{N_Y}{R^3}\int_0^t R^4\dot{R}^2\,dt - \frac{2g_Y}{R^3}\int_0^t R^3\,dt \qquad [3]$$

$$\dot{Z} = \frac{3}{4}N_Z R^2\dot{R} - \frac{3}{2}\frac{N_Z}{R^3}\int_0^t R^4\dot{R}^2\,dt - \frac{2g_Z}{R^3}\int_0^t R^3\,dt \qquad [4]$$

Here R is the radius of the bubble,

X, Y, Z are the cartesian coordinates of its center,

t is the time,

$\dot{R}, \dot{X}, \dot{Y}, \dot{Z}$ stand for dR/dt, dX/dt, dY/dt, dZ/dt, respectively,

X_0, Y_0, Z_0 denote values at $t = 0$,

ρ is the density of the water,

p_0 is the total hydrostatic pressure including atmospheric pressure, at the point X_0, Y_0, Z_0,

p_g is the pressure of the gas in the globe, supposed to be a known function of R,

g_X, g_Y, g_Z are components in the X, Y, Z directions of the gravitational acceleration g,

C is a constant depending on the initial conditions, and

M, N_X, N_Y, N_Z stand for simple functions of X, Y, and Z, depending upon the choice of axes and upon the nature and location of the boundaries.

An accurate solution of these equations can be effected only by numerical integration. This method has the disadvantage that many repetitions of the entire calculation are required to obtain results covering a wide variety of conditions.

For the first oscillation of the globe, on the other hand, formulas can be obtained which, although less accurate, are widely applicable. These formulas give the position of the gas globe at the peak of the first recompression, which is particularly important because it is the point of origin of the first secondary pulse of pressure.

The method of approximation is based upon the observation that, as the time t advances, the integrals in Equations [2], [3], and [4] grow chiefly while R is large, whereas, because of the factor $1/R^8$ preceding the integrals, they are effective in causing displacement of the gas globe chiefly while R is small. Approximate values for the displacement during the first recompression can be obtained, therefore, by substituting for these integrals in Equations [2], [3], and [4] constants equal to the values of the integrals at the instant of greatest compression. In calculating these values, on the other hand, an approximate value of dR/dt, obtained by neglecting certain terms in Equation [1], is sufficiently accurate. The same expression for dR/dt leads to a corrected value of the period.

The first period is thus found to be

$$T_1 = T_{10}(1 + 0.20\,MR_2) \qquad [5]$$

where T_{10} is the period when no bounding surfaces are near. Here M is the coefficient that occurs in Equation [1], and R_2 is the maximum radius of the gas globe during the first expansion; see Equation [18].

The approximate formulas obtained for the displacement of the center of the gas globe from the position of detonation X_0, Y_0, Z_0 to the point X_1, Y_1, Z_1 at which the next minimum radius occurs, may be written

$$X_1 - X_0 = \sqrt{6}\; B_X R_2 U \qquad [6a]$$

$$Y_1 - Y_0 = \sqrt{6}\; B_Y R_2 U \qquad [6b]$$

$$Z_1 - Z_0 = \sqrt{6}\; B_Z R_2 U \qquad [6c]$$

where

$$B_X = -\frac{1}{\sqrt{6}}\left(\frac{6\rho}{p_0}\ g_X Q R_2 + 3PN_X R_2^{\,2}\right) \qquad [7a]$$

$$B_Y = -\frac{1}{\sqrt{6}}\left(\frac{6\rho}{p_0}\ g_Y Q R_2 + 3PN_Y R_2^{\,2}\right) \qquad [7b]$$

$$B_Z = -\frac{1}{\sqrt{6}}\left(\frac{6\rho}{p_0}\ g_Z Q R_2 + 3PN_Z R_2^{\,2}\right) \qquad [7c]$$

Here N_X, N_Y, and N_Z are to be evaluated at the point X_0, Y_0, Z_0, and P, Q, and U represent the three integrals defined as follows:

$$P = \int_{y_1}^{1} y^4\left(1 + \frac{1}{2}MR_2 y\right)^{-\frac{1}{2}}\left[\frac{C'}{y^3} - \frac{1}{\gamma - 1}\left(\frac{R_0}{R_2}y\right)^{3\gamma} - 1\right]^{\frac{1}{2}} dy \qquad [8]$$

$$Q = \int_{y_1}^{1} y^3\left(1 + \frac{1}{2}MR_2 y\right)^{\frac{1}{2}}\left[\frac{C'}{y^3} - \frac{1}{\gamma - 1}\left(\frac{R_0}{R_2}y\right)^{3\gamma} - 1\right]^{-\frac{1}{2}} dy \qquad [9]$$

$$U = \int_{y_1'}^{1} \left(1 + \frac{1}{2}MR_2 y\right)^{\frac{1}{2}}\left[C''y^3 - \frac{1}{\gamma - 1}\left(\frac{R_0}{R_2}\right)^{3\gamma} y^{6-3\gamma} - y^6 - B^2\right]^{-\frac{1}{2}} dy \qquad [10]$$

in which

$$B^2 = B_X^{\,2} + B_Y^{\,2} + B_Z^{\,2} \qquad [11]$$

R_0 is the radius of the gas globe when the pressure of the gas equals the hydrostatic pressure at the level of its center, and C' and C'' have such values that the roots of the quantity in square brackets are in each case the same as the limits of integration, that is 1 and y_1 or y_1'. The gas is assumed to behave as an adiabatic ideal gas, in which the ratio of its specific heats is γ.

For gas globes due to underwater explosions, R_2/R_0 exceeds 2.5 and the term in γ has only a small influence upon the values of P and Q; this term represents the effect of the gas upon the motion during the expansion phase. If this term is dropped, P and Q are easily obtained as series in powers of MR_2:

$$P = 0.182(1 - 0.18\,MR_2 \cdots) \qquad [12a]$$

$$Q = 0.467(1 + 0.23\,MR_2 \cdots) \qquad [12b]$$

A curve for the integral U, defined in Equation [10], as a function of B^2 has been constructed by numerical integration, on the simplifying assumption that $\gamma = 4/3$. As B^2 increases, U decreases; the conversion of the energy of oscillation into translatory kinetic energy, as a result of gravity or of the presence of boundaries, checks the inward motion and thereby diminishes the extent of the compression, with a resulting decrease in U. For $R_2/R_0 = 2.65$, which seems to be within reason as an estimate for actual gas globes, the curve is represented closely by the formula

$$U = \frac{1.06}{\sqrt{B + \frac{0.009}{1 + 4000B^2}}} \qquad [13]$$

With the introduction of these approximate values of the integrals, Equations [6a, b, c], [7a, b, c], and [11] become, for sea water of specific gravity $\rho = 1.026$, if W is in pounds,

$$X_1 - X_0 = FB_X, \quad Y_1 - Y_0 = FB_Y, \quad Z_1 - Z_0 = FB_Z \qquad [14a, b, c]$$

where

$$F = \frac{2.60\ R_2}{\sqrt{B + \frac{0.009}{1 + 4000\ B^2}}} \qquad [15]$$

$$B = +\sqrt{B_X^2 + B_Y^2 + B_Z^2} \qquad [16]$$

$$B_X = 0.0346\,(1 + 0.23\,MR_2)\,c_X\,\frac{R_2}{p_A} - 0.223\,(1 - 0.18\,MR_2)\,N_X R_2^2 \qquad [17]$$

with two other sets of equations similar to Equations [15], [16], and [17], in which X is changed to Y or to Z, respectively. Here in the products MR_2, $N_X R_2^2$, $N_Y R_2^2$, $N_Z R_2^2$ it is sufficient to use the same unit of length in both factors, but elsewhere R_2 has been assumed to be expressed in feet; p_A is the hydrostatic pressure at the initial level of the center of the gas globe measured in atmospheres; and c_X, c_Y and c_Z are the cosines of the angles between the upward vertical and the X, Y and Z axes, respectively, so that $g_X = -gc_X$, $g_Y = -gc_Y$, $g_Z = -gc_Z$.

Here R_2, the maximum radius, may be assumed to vary as $(W/p_A)^{\frac{1}{3}}$. For tetryl or TNT a fair estimate seems to be

$$R_2 = 4.1\left(\frac{W}{p_A}\right)^{\frac{1}{3}} \text{ feet} \qquad [18]$$

where W is the weight of the charge in pounds and p_A is the total pressure in atmospheres. For tetryl the best experimental evidence available would replace 4.1 by 4.2, whereas for TNT, Figure 2 in Reference (7) gives 3.95. Equation [18] is plotted for several values of p_A in Figure 4; p_A is specified by

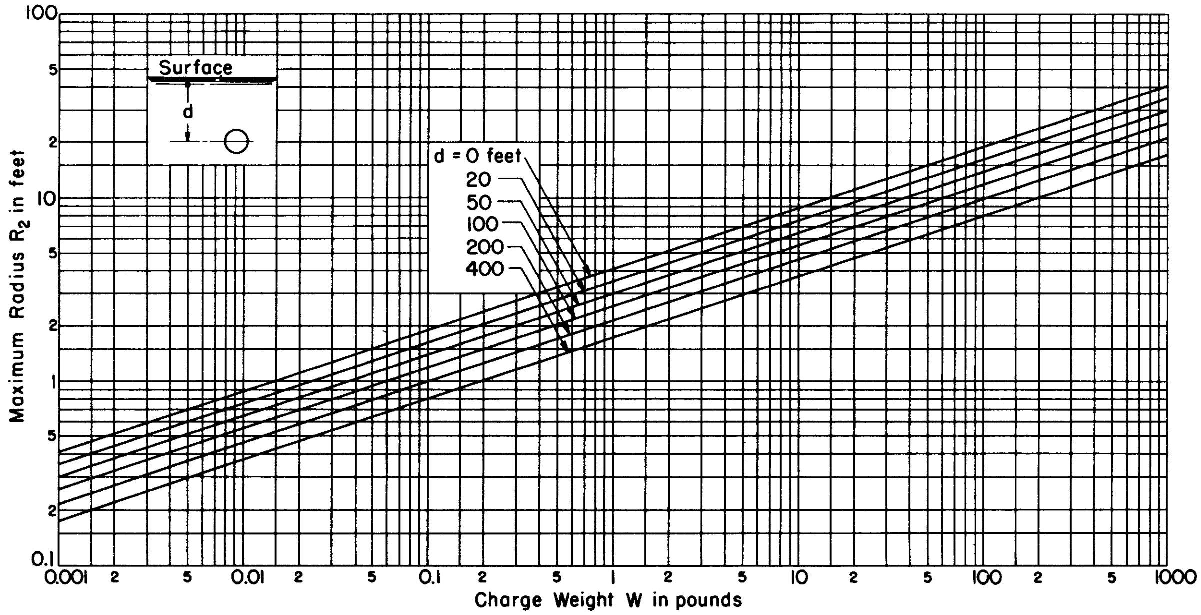

Figure 4 - Maximum Radius R_2 on First Expansion of the Gas Globe Produced by a Charge of W Pounds of Tetryl or TNT

The center of the globe is d feet below the surface of the sea; lines are drawn for several values of d; see Equation [18].

giving the equivalent depth d in feet below the surface of the sea, so that $p_A = 1 + d/33$.

With this value of R_2 and with M, N_X, N_Y, N_Z now expressed in terms of feet, Equations [15] and [17] become

$$F = \frac{10.7 \left(\frac{W}{p_A}\right)^{\frac{1}{3}}}{\sqrt{B + \frac{0.009}{1 + 4000\, B^2}}} \qquad [19]$$

$$B_X = 0.142\left[1 + 0.94\, M\left(\frac{W}{p_A}\right)^{\frac{1}{3}}\right] c_X \frac{W^{\frac{1}{3}}}{p_A^{\frac{4}{3}}} - 3.75\left[1 - 0.74\, M\left(\frac{W}{p_A}\right)^{\frac{1}{3}}\right] N_X \left(\frac{W}{p_A}\right)^{\frac{2}{3}} \qquad [20]$$

or, if the small term in M is dropped, approximately,

$$B_X = 0.142\, c_X \frac{W^{\frac{1}{3}}}{p_A^{\frac{4}{3}}} - 3.75\, N_X \left(\frac{W}{p_A}\right)^{\frac{2}{3}} \qquad [21]$$

with similar equations in Y and Z.

The accuracy of these approximate formulas is hard to estimate. Serious doubts arise as to their validity when the center of the gas globe comes closer to any bounding surface than $2R_2$ or twice its maximum radius. No correction has been made for the change in hydrostatic pressure as the gas globe rises or sinks. Furthermore, effects due to compressibility of the water, associated with the emission of acoustic radiation, have been neglected.

MIGRATION DUE TO GRAVITY ALONE

If X is taken vertically upward, Equations [1] and [2] become, when no boundary is near,

$$\dot{R}^2 = \frac{C}{R^3} + \frac{2}{\rho R^3} \int (p_g - p_0) R^2 dR - \frac{1}{6} \dot{X}^2 + \frac{2}{3} g (X - X_0) \qquad [22]$$

$$\dot{X} = \frac{2g}{R^3} \int_0^t R^3 dt \qquad [23]$$

Here g is the acceleration due to gravity.

Numerical integrations of these equations have been given by Taylor (5) and others (6) (7) (8) The effect of the gas, as represented by the occurrence of p_g in Equation [22], is usually not large; its smallness arises from the fact that, in practical cases of motion due to gravity alone, the inward motion of the water during each compression phase is arrested chiefly not by the gas but as a consequence of the conversion of radial kinetic energy of the water into kinetic energy of translational motion. That is, when the center of the gas globe is nearly stationary, the radial kinetic energy of the inrushing water becomes converted, at the instant of peak compression, entirely into energy of compression of the gas; but if translational motion of the gas globe occurs, part of the kinetic energy remains in the water in association with the translational motion. For this reason the inward radial motion is checked at a larger radius than when the globe is stationary. In migration due to gravity alone, nearly all of the energy usually thus remains in the water, and the motion during the compression phase is nearly the same as if no gas at all were present. Because of this conversion of the energy, the radial oscillations gradually die out, as the velocity of rise increases, especially if the hydrostatic pressure is very low or if the gas globe was produced by a large charge.

In UNDEX 10 (7), Figures 1 and 8, two plots are given, based upon numerical integrations, from which estimates of the rise due to gravity can be made for a wide range of charge weights and depths. These estimates agree within 8 per cent with values calculated from the convenient approximate formula

$$H = 4 \frac{\sqrt{W}}{p_A} \qquad [24]$$

where W is the weight of the charge in pounds,

p_A is the total hydrostatic pressure, including atmospheric pressure, expressed in atmospheres, and

H is the rise in feet from the point of detonation to the location of the next peak compression.

The total rise during the first compression and the re-expansion should be about $2H$. The charge may vary from 1 ounce to 1000 pounds, and the depth may be as much as 300 feet for the larger charges.

In using this formula it must be remembered that if the point of detonation is too close to the surface, the gases will blow through the surface, at least in large part, and no typical recompression can occur. This ought almost certainly to be the case if the point of detonation is closer to the surface than the maximum radius attained by the gas globe in its first expansion.

The formula for H as given by the approximate calculations of the present report may be found by putting $B_Y = B_Z = 0$, $c_X = 1$, $N_X = 0$ and dropping the term in B^2 in Equation [19], which turns out to be negligible in all interesting cases of purely gravitational action. Then Equations [14a], [16], [19], and [21] give $H = X_1 - X_0 = 4.0\sqrt{W}/p_A$, in exact agreement with Equation [24].

It will be noted that the rise H up to the first peak compression increases as the weight of the charge increases, and also as the hydrostatic pressure decreases. The increase results partly from the greater buoyant force on a larger gas globe and partly from an increase in the time occupied by the oscillation.

An interesting plot given in Reference (8) is reproduced in Figure 5. It shows the radius of the gas globe and the upward displacement of its center as functions of the time, for four different values of the initial hydrostatic pressure, as found by numerical integration. The plot applies approximately to any charge at suitable depths; small errors will remain owing to the fact that in the calculations no allowance was made for the gas pressure. It is interesting that at the smallest depth, $z_0 = 1$, the gravitational effect is so large that the radial motion is almost deadbeat, so that no succeeding pulses occur.

The scales in Figure 5 vary with W, the weight of the charge in pounds. Unity on the axis of ordinates represents L feet, and unity on the axis of abscissas represents T seconds, where L and T are given by the formulas

$$L = 10\,W^{\frac{1}{4}} \text{ feet}$$

$$T = 0.55\,W^{\frac{1}{8}} \text{ seconds}$$

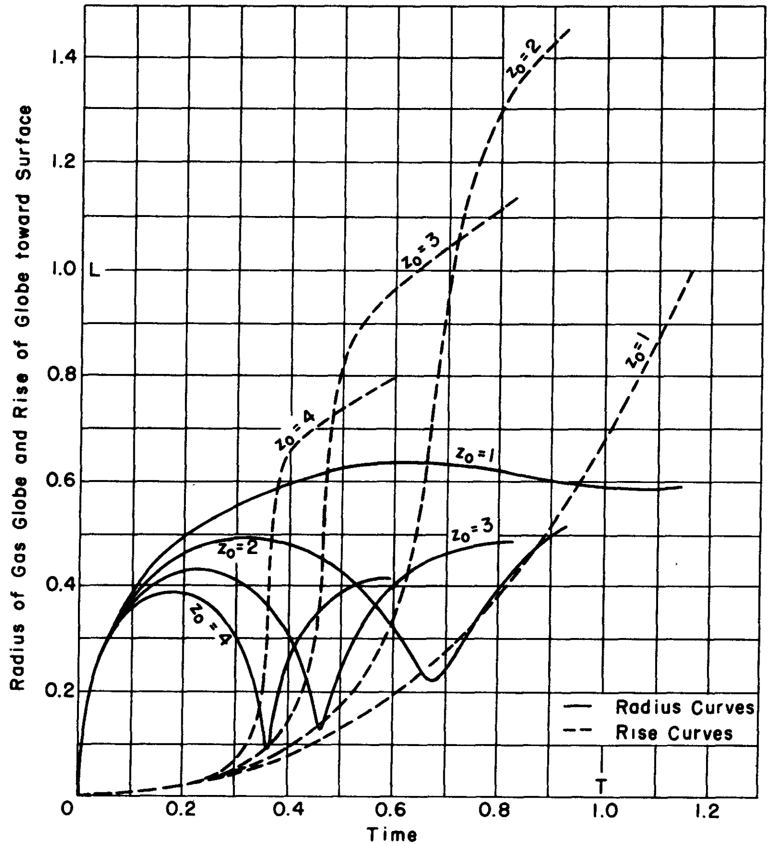

Figure 5 - Curves, Obtained by Numerical Integration, Showing Variation with Time of the Radius of the Gas Globe and the Rise of the Globe toward the Surface

For notation and units, see the text. This figure is copied from Reference (8).

TABLE 1

Values of the Coordinate Units for Figure 5

Charge pounds	L feet	T seconds	$z_0 L$*			
			$z_0 = 1$	$z_0 = 2$	$z_0 = 3$	$z_0 = 4$
1/16	5	0.39	5	10	15	20
1	10	0.55	10	20	30	40
50	26.6	0.90	27	53	80	106
100	31.6	0.98	32	63	95	126
300	41.6	1.12	42	83	125	166
1000	56.2	1.30	56	112	168	225

* Here $z_0 L$ is the total hydrostatic pressure in equivalent feet of sea water, where 33 feet = 1 atmosphere.

The four pairs of curves refer to an initial hydrostatic pressure, including atmospheric pressure, at the center of the globe equivalent to $z_0 L$ feet of sea water, or to L, $2L$, $3L$, and $4L$ feet, respectively. Some values are given in Table 1.

A question of interest in practice concerns the depth at which the globe from a given charge may be

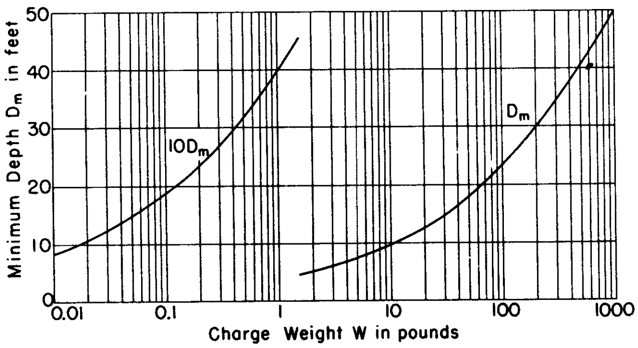

Figure 6 - Curve Giving a Rough Estimate of the Minimum Depth D_m below the Surface at which a Charge W may be Detonated without Blowing through the Surface before Undergoing Recompression

expected to execute a complete oscillation and emit a secondary pressure pulse during a first phase of recompression. If the charge is too near the surface, at least part of the gas will blow through the surface and no recompression of the full globe can occur, as illustrated in Figure 3 on page 5. The proper criterion is uncertain. It may be assumed tentatively as plausible, and as supported somewhat by experiment, that recompression will occur only when the depth exceeds both the maximum radius as calculated for a spherical gas globe and the calculated gravitational rise to the first peak compression.* The minimum depth D_m determined in this way is plotted on a basis of charge weight W in Figure 6. If the depth of detonation exceeds D_m, recompression of the gas globe should occur, although the emitted pulse of pressure may not be very effective if, because of the gravitational rise, the recompression occurs very near to the surface of the water.

GENERAL EFFECT OF SURFACES

A nearby surface limiting the body of water attracts or repels the gas globe. In a rough way this effect is superposed upon that of gravity, as is evident from the linear combination of the two terms in B_X, B_Y, B_Z, as in Equations [7a, b, c], [17], [20], [21]. Some interaction of the two effects arises, however, from the fact that the integral U in Equation [6a, b, c], or the quantity B defined by Equation [16], depends upon both effects.

Comparisons for charges of different weights are most easily made at distances from the limiting surface in proportion to the maximum radius R_2. Then at corresponding distances the factors N_X, N_Y, N_Z in Equations [7a, b, c], [17], [20], [21] actually vary as $1/R_2^2$, so that the corresponding surface terms in B_X, B_Y, B_Z do not vary at all, whereas under fixed hydrostatic pressure the gravity terms vary as R_2 or as $W^{\frac{1}{3}}$, where W is the weight of the charge. For this reason it turns out that, at distances of the order of $2R_2$

* For small charges, where the rise is decreased or even made negative by an effect of the free surface, D_m is determined by the maximum radius.

from the surface and at ordinary depths below the surface of the water, the effect of a bounding surface should predominate for small charges like detonators; for charges of a few ounces the two effects should be comparable in magnitude, and for charges of 100 pounds or more the gravity effect should usually predominate.

MIGRATION DUE TO A SURFACE IN THE ABSENCE OF GRAVITY

If Z denotes the distance from the surface, $N_X = N_Y = 0$, and it is found that

$$M = \pm \frac{1}{Z}, \quad N_Z = \pm \frac{1}{Z^2} \qquad [25a, b]$$

where the upper sign refers to a rigid surface and the lower sign to a free surface.

The period of the first oscillation will be approximately, from Equation [5]

$$T_1 = T_{10}\left(1 \pm 0.20 \frac{R_2}{Z}\right) \qquad [26]$$

in terms of the period T_{10} for $Z = \infty$ and the first maximum radius R_2. Thus a rigid boundary lengthens the period, a free surface decreases it. For $Z = 2R_2$, however, the change in the period is only 10 per cent.

If the gravitational effect is neglected, as is justifiable for the gas globe produced by a detonator under ordinary pressures, from Equations [14c], [15], [16], and the Z analog of Equation [17],

$$\frac{Z_1 - Z_0}{R_2} = \mp \frac{2.60\,B}{\sqrt{B + \dfrac{0.009}{1 + 4000\,B^2}}} \qquad [27]$$

$$B = 0.223\left(1 \mp 0.18 \frac{R_2}{Z}\right)\left(\frac{R_2}{Z}\right)^2 \qquad [28]$$

Here $Z_1 - Z_0$ represents the displacement of the gas globe, from the point of detonation up to the first peak recompression, measured positively away from the surface.

For R_2/Z near 1/2, approximately

$$\frac{Z_1 - Z_0}{R_2} = \mp 1.23 \frac{R_2}{Z} \qquad [28a]$$

whereas at very small R_2/Z, approximately

$$\frac{Z_1 - Z_0}{R_2} = \mp 6.1 \frac{R_2^2}{Z^2} \qquad [28b]$$

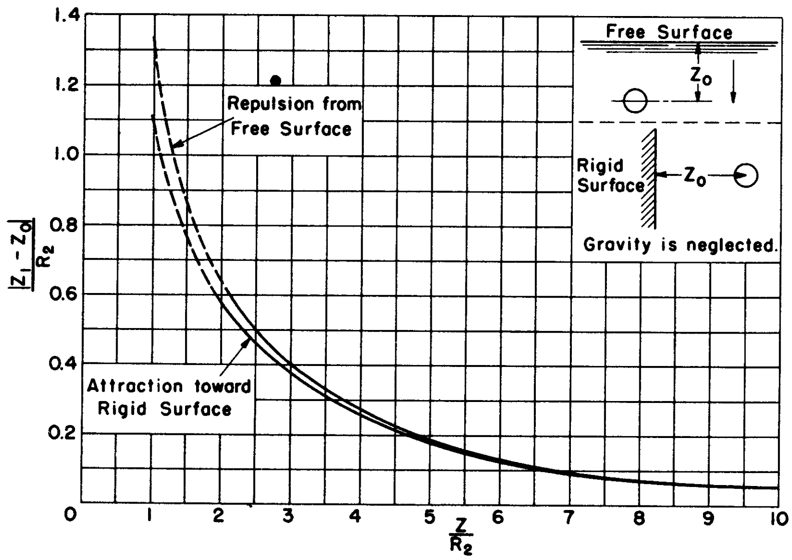

Figure 7 - Effect of a Single Surface on a Gas Globe when Gravity is Neglected

$|Z_1 - Z_0|$ denotes the displacement of the center during the interval until the first peak compression, Z_0 the distance of the point of detonation from the surface, R_2 the maximum radius during the first expansion.

Thus the effect of the surface should fall off with increasing distance Z at first nearly as $1/Z$, then more rapidly and ultimately as $1/Z^2$.

The sign indicates that a rigid surface (upper signs) should attract the gas globe, whereas a free surface (lower signs) should repel it, in agreement with observation. The two effects are nearly equal in magnitude, but the repulsion is a little greater.

Equation [27] is plotted in Figure 7. In using these formulas it must be remembered that R_2 varies with the hydrostatic pressure, as indicated in Equation [18]. The formulas probably become unreliable when $Z < 2R_2$; the corresponding parts of the curves are shown broken in Figure 7.

MIGRATION DUE TO GRAVITY AND A SINGLE SURFACE

Assume that the surface is rigid, and let its normal, drawn toward the gas globe, make an angle θ with the upward vertical; let Z denote distance of the center of the gas globe from the surface, and let X be another coordinate of the center measured parallel to the surface and more or less upward in a vertical plane; see Figure 8. Equations appropriate to this case are Equations [14a, c], [15], [16], and [17] and its analog in Z. Here $c_X = \sin\theta$, $c_Y = 0$, $c_Z = \cos\theta$; and the analysis indicates that

$$M = \frac{1}{Z}, \quad N_Z = \frac{1}{Z^2} \qquad [29a, b]$$

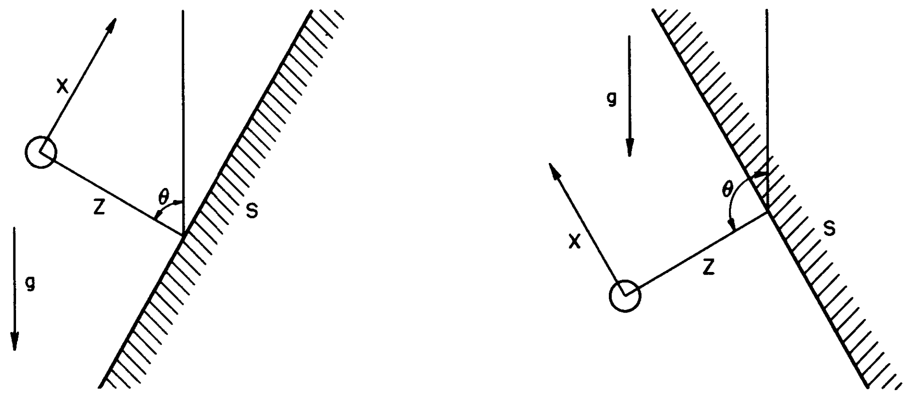

Figure 8 - Diagram Illustrating a Gas Globe under the Simultaneous Influence of Gravity and a Neighboring Rigid Surface S

The circle represents a globe of gas surrounded by water; the mass of water is bounded on one side by a rigid wall.

while $N_X = N_Y = 0$. Hence, for the displacement of the globe to the first peak recompression

$$X_1 - X_0 = FB_X, \quad Z_1 - Z_0 = FB_Z \qquad [30a, b]$$

$$F = \frac{2.60\, R_2}{\sqrt{B + \dfrac{0.009}{1 + 4000\, B^2}}} \qquad [31a]$$

$$B = \sqrt{B_X^{\,2} + B_Z^{\,2}} \qquad [31b]$$

$$B_X = 0.0346 \left[1 + 0.23\, \frac{R_2}{Z}\right] \frac{R_2}{p_A} \sin\theta \qquad [32]$$

$$B_Z = 0.0346 \left[1 + 0.23\, \frac{R_2}{Z}\right] \frac{R_2}{p_A} \cos\theta - 0.223 \left[1 - 0.18\, \frac{R_2}{Z}\right]\left(\frac{R_2}{Z}\right)^2 \qquad [33]$$

or, to a good approximation,

$$B_X = 0.0346\, \frac{R_2}{p_A} \sin\theta \qquad [34a]$$

$$B_Z = 0.0346\, \frac{R_2}{p_A} \cos\theta - 0.223 \left(\frac{R_2}{Z}\right)^2 \qquad [34b]$$

Here $X_1 - X_0$ and $Z_1 - Z_0$ represent components of the displacement measured positively in the direction of increasing X or Z.

The first period, from Equation [5], is

$$T_1 = T_{10}\left[1 + 0.20\, \frac{R_2}{Z}\right] \qquad [35]$$

In the formulas, values of W are in pounds and all values of X and Z in feet; p_A is the total hydrostatic pressure in atmospheres.

The case, $\theta = 0$, applies to a rigid horizontal bottom. The formulas for $\theta = \pi$, on the other hand, are found to apply to a free surface, with Z measured away from the surface and hence downward, provided changes are made corresponding to the assumption that, for the free surface,

$$M = -\frac{1}{Z}, \quad N_Z = -\frac{1}{Z^2} \qquad [36a, b]$$

in place of Equations [29a, b]. Hence for a rigid horizontal bottom and for the free surface of the water, the formulas can conveniently be written together as follows:

$$Z_1 - Z_0 = \frac{2.60\, B_Z R_2}{\sqrt{|B_Z| + \frac{0.009}{1 + 4000\, B_Z^2}}} \qquad [37]$$

$$B_Z = \pm\left\{0.0346\left[1 \pm 0.23\,\frac{R_2}{Z}\right]\frac{R_2}{p_A} - 0.223\left[1 \mp 0.18\,\frac{R_2}{Z}\right]\left(\frac{R_2}{Z}\right)^2\right\} \qquad [38]$$

or, to a good approximation

$$B_Z = \pm\left[0.0346\,\frac{R_2}{p_A} - 0.223\left(\frac{R_2}{Z}\right)^2\right] \qquad [38a]$$

The upper sign refers to the rigid bottom and the lower sign to the free surface. The symbol $|B_Z|$ denotes the numerical value of B_Z taken without regard to sign; and $Z_1 - Z_0$ represents in each case the displacement measured positively away from the surface.

The first period is

$$T_1 = T_{10}\left[1 \pm 0.20\,\frac{R_2}{Z}\right] \qquad [39]$$

These formulas probably become unreliable when

$$Z < 2R_2 = 8\left(\frac{W}{p_A}\right)^{\frac{1}{3}}$$

Because of the negative sign between the two parts of B_Z, the gravitational effect and the effect of the surface oppose each other in the case of a free surface or a rigid surface below the charge, whereas the two effects are in the same direction when a rigid surface lies above the charge. The gas globe from a *small* charge near the surface of the water sinks instead of rising.

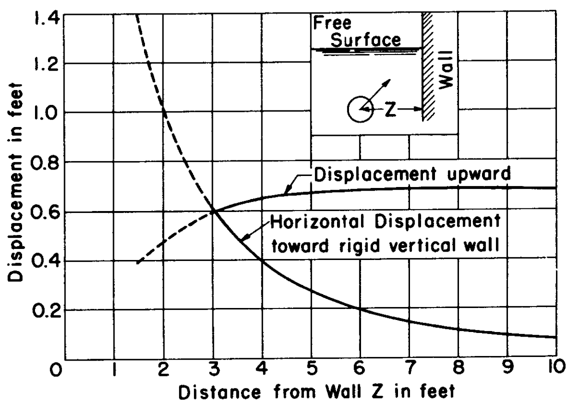

Figure 9 - Upward and Horizontal Components of Displacement of the Gas Globe from 3/4 Ounce of Tetryl or TNT

The charge is assumed to be detonated 10 feet below the surface of the water, far above the bottom, and Z feet from a rigid vertical wall. The curves show the displacement from the point of detonation up to the point of greatest recompression, according to approximate calculations.

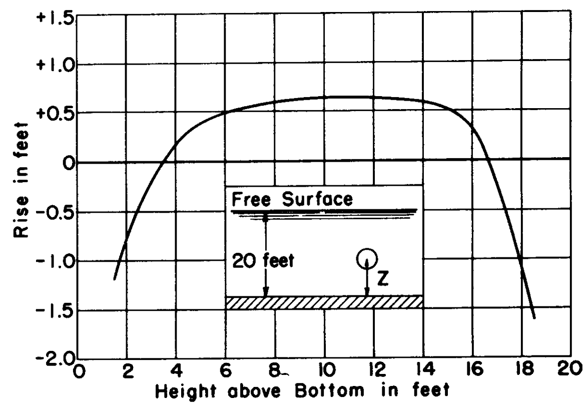

Figure 10 - Displacement of the Center of the Gas Globe from 3/4 Ounce of Tetryl or TNT, up to the Instant of Maximum Recompression

Detonation is assumed to occur at the height shown above a rigid horizontal bottom in water 20 feet deep. Positive ordinates represent a rise, negative ones a descent. The curve is based on approximate calculations.

The effects due to gravity and to the surface are almost additive but not entirely so, because of the occurrence of B or $|B_Z|$ in F. When gravity and the surface produce opposite effects, as in the case of a rigid bottom or a free surface, the net displacement is a little greater than the numerical difference of the values that the two displacements would have if they occurred singly. In such cases the gas globe contracts to a smaller radius than it would if only one effect occurred, and this decrease in the minimum radius increases the displacement. Otherwise, as in the case of a rigid wall located to one side of the globe or the rigid bottom of a boat above it, the two displacements are slightly decreased by their coexistence.

The formulas are illustrated in Figures 9 and 10, which refer to a 3/4-ounce charge of tetryl or TNT detonated at Z feet from a surface. In Figure 9 the surface is assumed to be a rigid vertical wall, the charge is detonated 10 feet below the surface of the water, and the bottom is assumed to lie much deeper. The vertical rise of the center of the gas globe and its horizontal displacement toward the wall, up to the point of maximum compression, are shown by curves. In Figure 10 curves are shown for the same charge Z feet above a horizontal rigid bottom in water 20 feet deep. It will be noted that the gas globe descends if formed less than about 3.5 feet from either the bottom or the free surface.

The formulas for a free surface and for a rigid bottom are plotted in general terms in Figures 11 and 12, as explained under the figures.

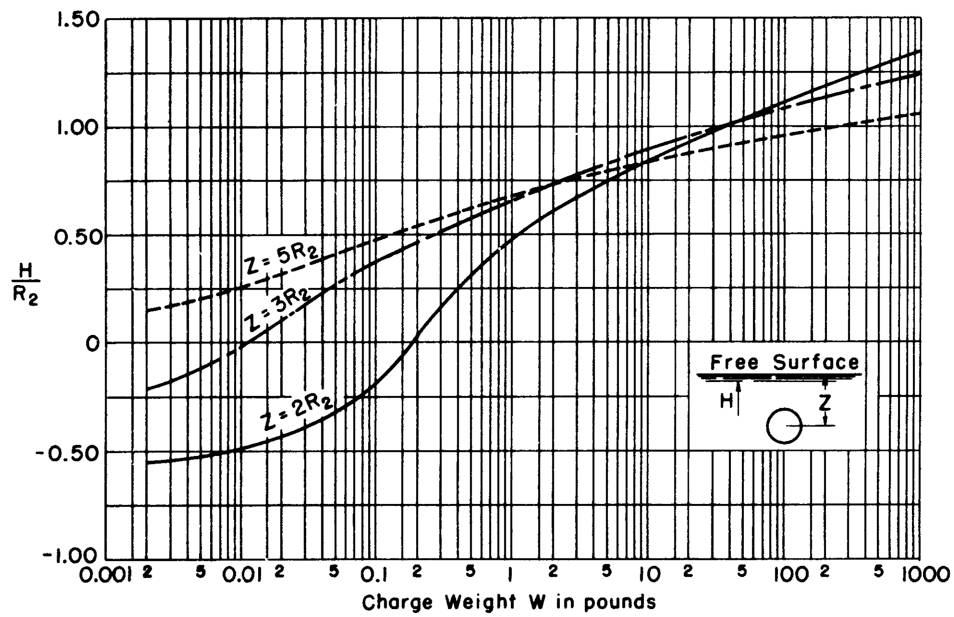

Figure 11 - Rise H of the Gas Globe Formed by W Pounds of Tetryl or TNT Detonated at a Depth Z below the Surface of the Sea of Infinite Depth

H is the rise of the center of the globe from the time of detonation up to the time of the first peak recompression; both H and Z are expressed in terms of R_2, the intervening maximum radius. Curves are shown for three values of Z. For R_2 see Figure 4 or Equation [18].

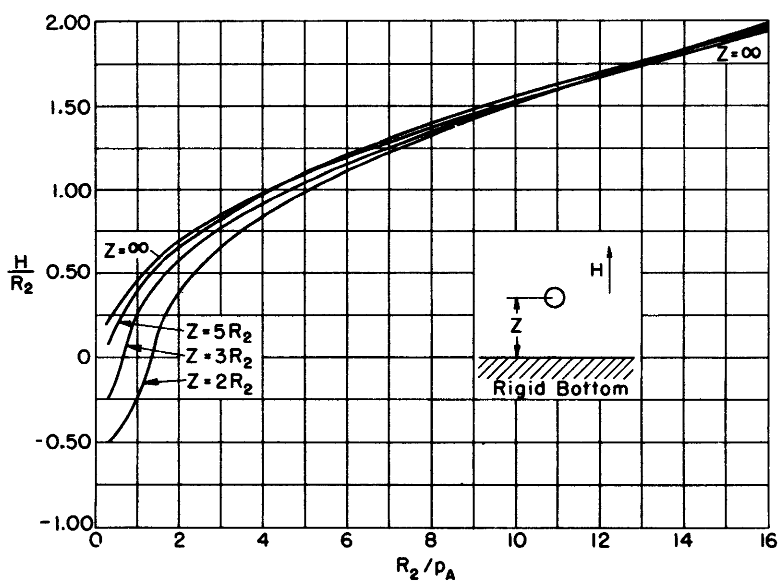

Figure 12 - Curves for Estimating the Rise H of a Gas Globe at a Distance Z above the Bottom of the Sea

The rise is from the time of detonation up to the time of the next peak recompression. R_2 is the intervening maximum radius in feet; p_A is the total hydrostatic pressure in atmospheres or $1 + d/33$ where d is the depth of water in feet at the point of detonation. Curves are shown for four values of Z/R_2. For R_2 see Figure 4 or Equation [18]. The curves are drawn on the assumption of infinite depth; they are fairly accurate if the gas globe is at least $5R_2$ below the surface.

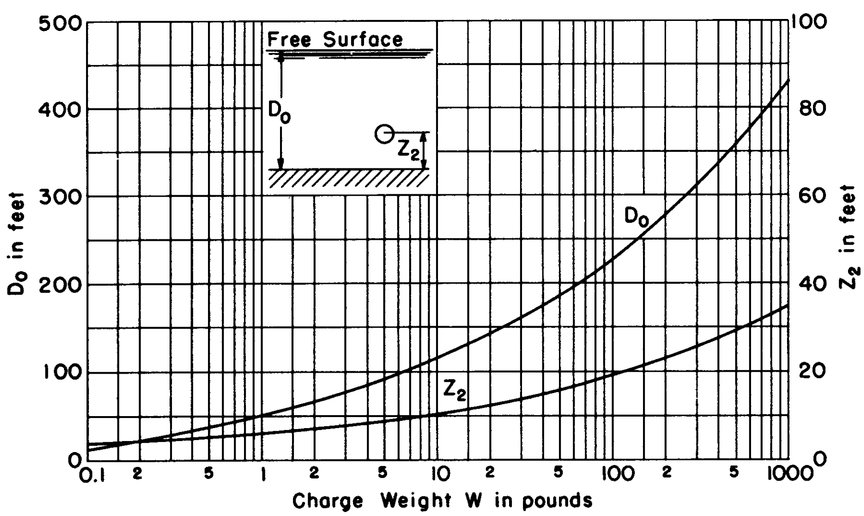

Figure 13 - Plot of the Critical Depth D_0 for Migration near the Bottom of the Sea

In sea water of depth less than D_0 feet, the gas globe should rise during the first recompression if detonation occurs at a distance $Z > Z_2$ above the bottom, where $Z_2 = 2R_2$; Z_2 is plotted in terms of a larger scale shown at the right. In water of depth greater than D_0, the globe should sink toward the bottom if $Z = Z_2$. For fair accuracy the gas globe should be at least $5R_2$ below the surface of the sea.

From Figure 11 it is seen that migration downward can be produced by a free surface only if the charge is less than 0.2 pound, provided detonation occurs at a depth at least as great as $2R_2$ below the surface. In the absence of more exact calculations, it may reasonably be surmised that the globe from 1 pound or more of TNT or tetryl should migrate upward, however close to the surface it may be formed.

The effect of the bottom is more complicated because the total hydrostatic pressure, as influenced by the depth of the water, enters as a new variable. In order to illustrate more concretely the implications of Figure 12, there is plotted on a basis of W in Figure 13 the depth of sea water D_0 at which $R_2/p_A = 1.33$; the value of Z when $Z = 2R_2$ in water of this depth is shown, on a different scale, as Z_2.

The formulas for D_0 and Z_2 are

$$D_0 = 33\left[\frac{107}{99}\left(\frac{3}{4}\ 4.1\right)^{\frac{3}{4}} W^{\frac{1}{4}} - 1\right], \quad Z_2 = 8.2\left(\frac{3}{4}\ 4.1\right)^{-\frac{1}{4}} W^{\frac{1}{4}}$$

In water shallower than D_0, the gas globe should rise if formed at a distance Z_2 or greater above the bottom; in water deeper than D_0, it should sink when it is formed at a distance equal to Z_2, and also at progressively greater distances as the depth of water is increased.

Unfortunately the approximate formulas become unreliable at those short distances which are of greatest practical interest; they should be fairly accurate if $Z \geqq 2R_2$.

It is particularly unfortunate that calculations do not exist for charges detonated on the bottom. They are made difficult by the inevitable distortion of the gas globe. As Z is diminished below $2R_2$, the attractive effect of the bottom should probably increase and then decrease again. This conclusion is based on the following ideal case. The water flow around a hemispherical charge lying with its flat face on a rigid bottom and detonated at its center should resemble half of the flow around a spherical charge of the same radius detonated in open water; gravity should, therefore, cause the gas globe from the hemisphere to rise. From the analytical results it may reasonably be surmised that the gas globe from 10 pounds or over, detonated on the bottom under any depth of water of practical interest, will probably rise during the first recompression.

MIGRATION OF A GAS GLOBE IN SHALLOW WATER

The combined effect of the free surface and of a parallel rigid bottom can be obtained by extending the method of images. If Z is taken to stand for the distance of the center of the gas globe above the bottom, it is found that only those changes need to be made in the formulas, as obtained for a rigid bottom alone, which correspond to the assumption, instead of [29a, b]

$$M = T_1 - \frac{1.39}{D}, \quad N_Z = S_2 \qquad [40a,\ b]$$

where D is the total depth of the water, 1.39 represents 2 log 2, and T_1 and S_2 stand for the series

$$T_1 = \frac{1}{Z} - \frac{1}{D-Z} - \frac{1}{D+Z} + \frac{1}{2D-Z} + \frac{1}{2D+Z} - \frac{1}{3D-Z} - \cdots$$

$$S_2 = \frac{1}{Z^2} + \frac{1}{(D-Z)^2} - \frac{1}{(D+Z)^2} - \frac{1}{(2D-Z)^2} + \frac{1}{(2D+Z)^2} + \cdots$$

Hence the displacement of the gas globe measured upward, during the first expansion and recompression, is

$$Z_1 - Z_0 = \frac{2.60\ B_Z R_2}{\sqrt{|B_Z| + \dfrac{0.009}{1 + 4000\,B_Z^2}}} \qquad [41]$$

$$B_Z = 0.0346\left[1 + 0.23\left(T_1 - \frac{1.39}{D}\right)R_2\right]\frac{R_2}{p_A} - 0.223\left[1 - 0.18\left(T_1 - \frac{1.39}{D}\right)R_2\right]S_2 R_2^2 \qquad [42]$$

or, very nearly

$$B_Z = 0.0346\,\frac{R_2}{p_A} - 0.223\,S_2 R_2^{\,2} \qquad [42a]$$

The first period, from Equation [5], is

$$T_1 = T_{10}\left[1 + 0.20\left(T_1 - \frac{1.39}{D}\right) R_2\right] \qquad [43]$$

Here distances are to be measured throughout in feet. The formulas become questionable if either boundary is closer than $2R_2$ or $8(W/p_A)^{\frac{1}{3}}$.

These formulas are the same as those obtained for the bottom alone except that $1/Z$ is replaced by $T_1 - 1.39/D$ and $1/Z^2$ is replaced by S_2. If the small term containing T_1 and D is omitted, it is clear that the displacement is the same as that due to a single surface at a distance Z_s such that $1/Z_s^2 = S_2$ or

$$Z_s = \frac{1}{\sqrt{S_2}}$$

In S_2 the effects of the bottom and of the free surface are added in a sort of quadratic fashion. If $Z = D/2$, so that the charge lies midway between the surface and the bottom,

$$T_1 = 0, \quad S_2 = \frac{2}{Z^2}\left(1 - \frac{1}{3^2} + \frac{1}{5^2} \cdots\right) = \frac{1.83}{Z^2}$$

Thus

$$Z_s = 0.74\,Z$$

so that the displacement is roughly the same as that when either surface alone is present at about three-fourths of the actual distance to either top or bottom. As the charge is moved toward either surface, however, the effect of the other surface rapidly decreases. Thus if $Z = 0.35\,D$ or $0.65\,D$, the effect is about the same as that due to the nearer surface acting alone at a distance 0.91 times its actual distance.

The effect of the free surface on the period somewhat exceeds that of the bottom. Hence when the charge is detonated midway between the two the period is shortened. The first period is

$$T_1 = T_{10}\left(1 - 0.28\,\frac{R_2}{D}\right)$$

EFFECT OF PROXIMITY TO A FREE SURFACE AND A VERTICAL WALL

The wall is supposed to be plane and rigid and to extend from the surface to a great depth. Let X denote the distance of the center of the

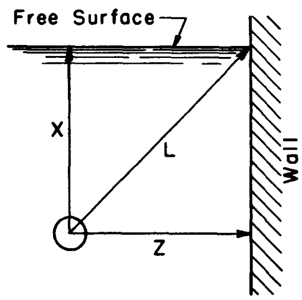

Figure 14 - Diagram Illustrating a Gas Globe near the Surface of the Water and also near a Vertical Rigid Wall

globe below the surface, measured downward, and Z its distance from the wall; see Figure 14. Then, in Equations [14a, c], [15], [16], [17] and its Z analog, clearly $c_X = -1$, $c_Y = c_Z = 0$. The analysis gives

$$M = \frac{1}{Z} - \frac{1}{X} - \frac{1}{L} \qquad [44a]$$

$$N_X = -\left(1 + \frac{X^3}{L^3}\right)\frac{1}{X^2} \qquad [44b]$$

$$N_Z = \left(1 - \frac{Z^3}{L^3}\right)\frac{1}{Z^2} \qquad [44c]$$

where $L = \sqrt{X^2 + Z^2}$, and $N_Y = 0$. Thus, for the displacement from the point of detonation to the point of peak compression,

$$X_1 - X_0 = \frac{2.60\left(\frac{W}{p_A}\right)^{\frac{1}{3}} B_X R_2}{\sqrt{B + \frac{0.009}{1 + 4000B^2}}} \qquad [45]$$

$$Z_1 - Z_0 = \frac{2.60\left(\frac{W}{p_A}\right)^{\frac{1}{3}} B_Z R_2}{\sqrt{B + \frac{0.009}{1 + 4000B^2}}} \qquad [46]$$

$$B = \sqrt{B_X{}^2 + B_Z{}^2} \qquad [47]$$

$$B_X = -0.0346\left[1 + 0.23\,MR_2\right]\frac{R_2}{p_A} + 0.223\left(1 + \frac{X^3}{L^3}\right)\left[1 - 0.18\,MR_2\right]\left(\frac{R_2}{X}\right)^2 \qquad [48]$$

$$B_Z = -0.223\left[1 - 0.18\,MR_2\right]\left(1 - \frac{Z^3}{L^3}\right)\left(\frac{R_2}{Z}\right)^2 \qquad [49]$$

or, very nearly,

$$B_X = -0.0346\,\frac{R_2}{p_A} + 0.223\left(1 + \frac{X^3}{L^3}\right)\left(\frac{R_2}{X}\right)^2 \qquad [50]$$

$$B_Z = -0.223\left(1 - \frac{Z^3}{L^3}\right)\left(\frac{R_2}{Z}\right)^2 \qquad [51]$$

Here $X_1 - X_0$ is the upward component of the displacement, while $Z_1 - Z_0$ is the horizontal component measured positively away from the wall.

The first period, from Equation [5], is

$$T_1 = T_{10}\left[1 + 0.20\left(\frac{1}{Z} - \frac{1}{X} - \frac{1}{L}\right)R_2\right] \qquad [52]$$

The formulas are probably unreliable when either X or Z is less than $2R_2$ or $8(W/p_A)^{\frac{1}{3}}$.

Here the additional terms containing L represent the principal effect of the interaction between the surfaces. Crudely speaking, the repulsive effect of the surface is increased by a factor $1 + X^3/L^3$, while the attractive action of the wall is decreased by a factor $1 - Z^2/L^3$, as compared to what these effects would be if the other surface were not present. The interaction between the two effects is greatest when $X = Z$. Then $L = \sqrt{2}X$ and the repulsion from the surface is increased in the ratio 1.35, while the attraction toward the wall is decreased in the ratio 0.65. These numbers will be somewhat modified, however, by the concomitant change in B.

On the period, the surface effect again predominates and results in a shortening; the first period is

$$T_1 = T_{10}\left[1 - 0.20\,\frac{R_2}{L}\right]$$

The most interesting feature in this case is the variation of the displacement with weight of charge. As the weight increases, the gravitational effect comes to predominate. In order to illustrate this fact, Figures 15 and 16 show curves of vertical displacement H and the horizontal displacement S toward the wall, for a charge detonated at several distances Z from the wall in combination with several distances X below the surface of the water, plotted against the charge weight W. These figures also serve to indicate qualitatively the relative magnitudes of the two displacements at shorter distances from the surface, where the numerical formulas become unreliable.

This case has some resemblance to that of a floating mine exploding near a ship. For the relatively slow motion involved in the production of migration a ship should function as a rigid obstacle. The ship extends downward, however, only to a limited depth. For this reason the attraction toward the ship should be considerably less, and the rise a little greater, than in the ideal case here considered.

PRESSURE IN THE WATER AS INFLUENCED BY THE MIGRATION

The pressure generated in the water by the recompression of the gas globe may be greatly altered by the migration. The general effect is complicated, as is illustrated by G.I. Taylor (5). The pressure will probably be further modified, however, in consequence of departures from spherical symmetry, so that calculations based upon the assumption of symmetry possess in most cases only a limited interest. For this reason the following rough method of estimating the pressure as modified by the occurrence of migration may be of interest.

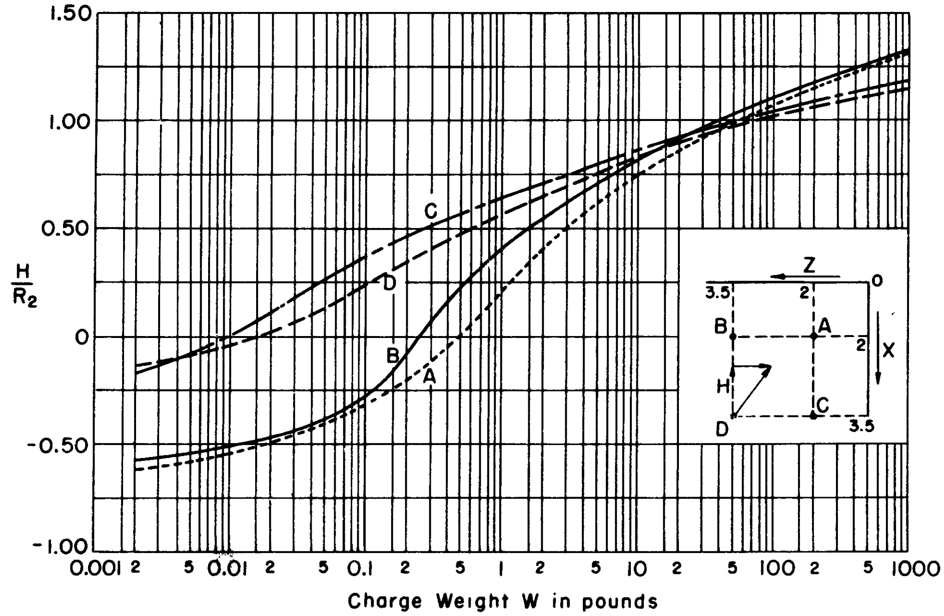

Figure 15 Vertical Rise *H* of a Gas Globe near the Surface of the Sea and near a Vertical Wall

The rise is from the time of detonation up to the time of the next peak recompression. R_2 is the intervening maximum radius. Curves are shown for 4 positions: (A) $X = Z = 2R_2$; (B) $X = 2R_2$, $Z = 3.5R_2$; (C) $X = 3.5R_2$, $Z = 2R_2$; (D) $X = 3.5R_2$, $Z = 3.5R_2$, where X is the distance of the point of detonation below the surface and Z the distance of this point from the wall. W is the weight of the charge in pounds. For R_2 see Figure 4 or Equation [18].

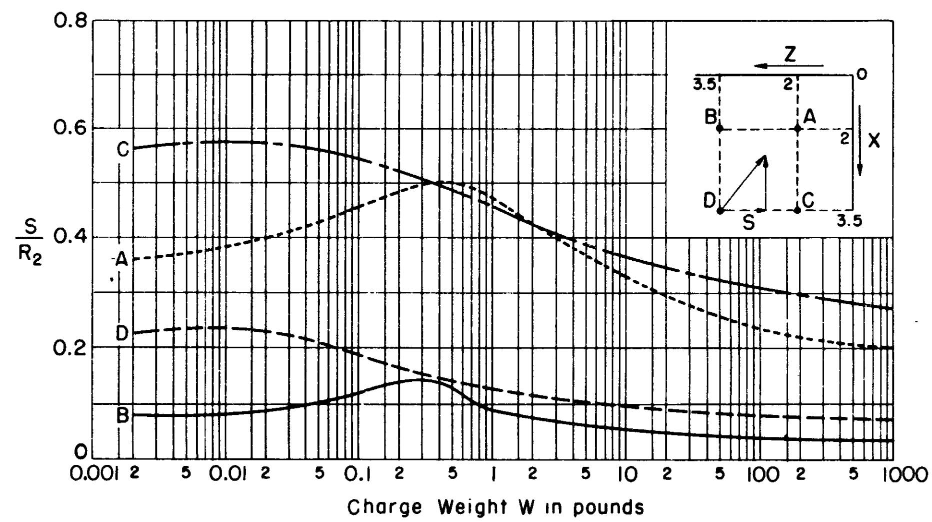

Figure 16 - Horizontal Component of Displacement *S* of the Gas Globe of Figure 15

The motion of the water can be resolved into three parts superposed upon each other, a spherically symmetrical part associated with the radial oscillations of the gas globe, a part caused by any bounding surfaces that may be present, and a part associated with the motion of migration. The pressure can then be divided into three corresponding parts, provided the Bernoulli term $\rho v^2/2$ is omitted, so that the pressure is simply proportional to the rate of change of the velocity potential. The part of the pressure that is associated directly with the migratory motion is then essentially of dipole character and hence falls off relatively rapidly with distance; it may therefore be dropped in a rough calculation, except near the gas globe. The part due to a bounding surface, if any, is simply the pressure due to the image of the gas globe in the surface and is easily allowed for on this basis. There remains then the part of the pressure that is associated with the radial motion. This part is altered by the migration because the radial motion is altered.

The radial part of the pressure is given by Equations [6] or [7] on page 45 of TMB Report 480 (10) with the omission of u^2; it may not be correctly given by Equation [8] of that report, however, in which the term in u_g^2 is not negligible and is influenced by the migratory motion. The pressure p at a point distant r from the center of the gas globe is thus

$$p = \frac{\rho}{r}\frac{d}{dt}(R^2\dot{R}) + p_0 \qquad [53]$$

where ρ is the density of the water and p_0 denotes the total hydrostatic pressure at the level of the gas globe. Only the phase of intense compression is of interest, hence Equation [1] of the present report can be simplified as before, and even the small term $MR/2$ can be omitted for the present purpose. Thus from Equation [1]

$$\dot{R}^2 = \frac{C}{R^3} + \frac{2}{\rho R^3}\int (p_g - p_0)\, R^2\, dR - \frac{(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2)}{6} \qquad [54]$$

The approximate values employed previously for $\dot{X}$, $\dot{Y}$, $\dot{Z}$ can then be inserted, and they may conveniently be expressed in terms of the linear displacement of the gas globe from the instant of detonation to the instant of peak recompression, which is

$$Q = \sqrt{(X_1 - X_0)^2 + (Y_1 - Y_0)^2 + (Z_1 - Z_0)^2} = \sqrt{H^2 + S^2} \qquad [55]$$

The pressure as thus estimated is found to depend on the ratio Q/R_2, where R_2 is the first maximum radius, and to be proportional to p_0R_2/r. The impulse $I = \int(p - p_0)dt$, is proportional to $R_2^2\sqrt{p_0}/r$. A single graph applicable

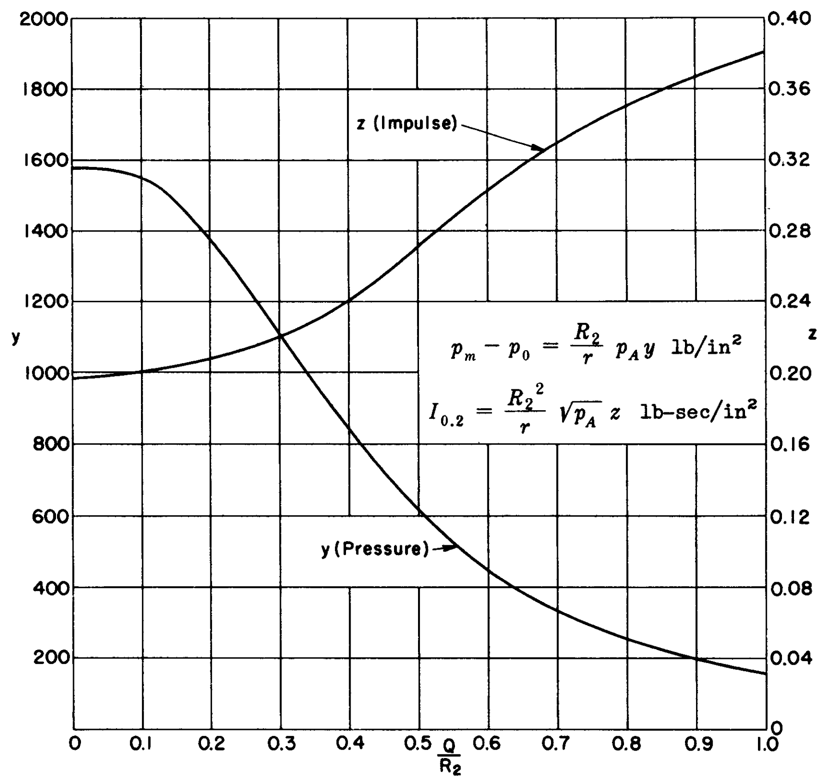

Figure 17 - Curves and Formulas for Estimating Roughly the Effect of Migration on the Pressure in the Water

p_m is the maximum pressure in the water at the instant of greatest recompression of the gas globe in pounds per square inch, p_0 is the total hydrostatic pressure in pounds per square inch and p_A is the same pressure expressed in atmospheres, $I_{0.2}$ is the peak impulse defined as $\int p\,dt$ integrated between points before and after the peak at which $p_m - p_0$ is one-fifth of its value at the peak, r denotes distance in feet from the center of the gas globe at the instant of maximum recompression, R_2 is the first maximum radius of the gas globe in feet, Q is the linear displacement of the gas globe from the point of detonation until the instant of greatest recompression, in feet.

to all migratory cases can be constructed, therefore, by plotting against Q/R_2 values of

$$\frac{p_m r}{p_A R_2}, \quad \frac{rI}{R_2^2 \sqrt{p_A}}$$

where p_A is the total hydrostatic pressure p_0 measured in atmospheres and p_m is the maximum excess of pressure above p_0. This is done in Figure 17. The impulse $I_{0.2}$ is taken between the two instants at which the excess of pressure $p - p_0$ is 1/5 of its intervening maximum value. The values of $I_{0.2}$ thus serve to give an idea of the estimated width of the pressure peak.

To use Figure 17, the value of the displacement Q is first estimated by use of formulas given previously; then values of y and z are read off the curves and p_m and $I_{0.2}$ are calculated from the formulas

$$p_m - p_0 = \frac{R_2}{r} p_A y \text{ pounds per square inch} \qquad [56a]$$

$$I_{0.2} = \frac{R_2^{\ 2}}{r} \sqrt{p_A}\, z \quad \text{pound-seconds per square inch} \qquad [56b]$$

in which r and R_2 are in feet, p_m in pounds per square inch and $I_{0.2}$ in pound-seconds per square inch.

If there is a plane bounding surface in the neighborhood, a correction is then to be added representing the effect of a phantom gas globe located at the mirror image in the surface of the actual one. The pressure and impulse due to the image are calculated from the same formulas as those due to the actual gas globe, with r made equal to the distance from the image. The total pressure and impulse are then the sum of these respective quantities for the actual globe and the image if the surface is rigid, or the difference if the surface is a free one. At the surface itself, the effect of the image is to double the excess of pressure above hydrostatic pressure on a rigid surface, or to keep the pressure at the hydrostatic level on a free surface.

From Figure 17 it is seen that, as the migration increases, the peak pressure decreases, but the width of the peak increases so that the impulse due to it is about constant. At $Q = R_2/3$, the peak pressure has decreased by about a third; the rate of decrease then becomes greater, so that at $Q = 1$ the peak pressure is only a tenth of what it would be in the absence of migration.

The increase in $I_{0.2}$ at large values of Q/R_2 in the figure arises from the fact that the total range of pressure becomes small and the peak, defined as extending from one-fifth of the maximum to one-fifth on the other side, comes to include almost the entire range of positive pressure. The ratio of $I_{0.2}$ to the total positive impulse increases from about 1/3 at Q = 0 to 4/5 at $Q = R_2$.

MIGRATION OF A GAS GLOBE IN A TROUGH OR BOX

The necessary formulas have been written out for a charge detonated inside a rectangular box partly filled with water; this obviously includes as special cases a deep well of rectangular cross section or a trough with parallel vertical sides. The series obtained in these cases are complicated, however, and for this reason no results will be cited here.

MIGRATION AND SIMILITUDE

In considering similitude as it applies to phenomena of migration, it should be noted that physical processes of four different types are involved and each process imposes its own characteristic requirements for similitude. These differing requirements are not entirely reconcilable. Thus, in order that similar motions may occur on different scales:

a. The laws of the non-compressive motion of water require that pressure differences and the squares of velocities shall vary in the same ratio, or

$$\Delta p \propto v^2$$

b. The intervention of gravity requires that all pressure differences shall vary, as do gravity heads, in proportion to the linear dimensions, or

$$\Delta p \propto L$$

where L is any convenient linear dimension.

c. If a confined mass of gas is present, its pressure must usually remain unchanged with scale, on the assumption that the mass of gas present is varied in proportion to the cube of the linear dimensions. Hence, all pressures must remain unchanged.

d. Certain boundary conditions, such as exposure to the atmosphere or the presence of cavitation, may fix the actual value of the pressure at certain points.

The hydrodynamical requirement a. is consistent with many types of similitude. The addition of the gravity requirement b. restricts the choice to one in which $v^2 \propto L$, as in ship model testing. Then, also, $\Delta p \propto v^2 \propto L$.

If the migration effect on a gas globe is large, the gas has little effect on the radial and translatory motion, so that the pressure in the gas globe may be assumed to be zero. The pressure at certain points is then fixed, as in d. To keep $\Delta p \propto L$, the atmospheric pressure must then be adjusted in proportion to L. Furthermore, since the kinetic energy in the water is proportional to $L^3 v^2$ or to L^4, and since this energy may be assumed to be a fixed fraction of the energy released by the charge, and since the latter energy is proportional to the weight of the charge W, it follows that W must be varied in proportion to L^4. Thus linear dimensions and pressures vary in proportion to $W^{\frac{1}{4}}$, and, since $v^2 \propto L$, velocities and times vary in proportion to $W^{\frac{1}{8}}$. The atmospheric pressure must also be varied in proportion to $W^{\frac{1}{4}}$; and the depth of the water must be varied in the same ratio if the depth is significant. In corresponding positions, the maximum radius R_2 and the migratory displacement of the gas globe likewise vary as $W^{\frac{1}{4}}$.

When the migratory displacement is small relative to R_2, the similitude is not exact, because the motion is then appreciably influenced by the pressure of the gas. The partial failure of similitude in this case is not apparent from the approximate equations written in this report, because these equations are based upon a fixed value of R_2/R_0, whereas in reality this ratio will vary somewhat with the hydrostatic pressure.

In any case, similitude of the type described does not extend to the dimensions of the charge nor to the shock wave. These features can be included only if gravitational effects are neglected. If that is done, requirements a, c, and d can be met by keeping all pressures and velocities the same at corresponding points, while linear dimensions and times are changed in proportion to $W^{\frac{1}{3}}$. The effects of the gas pressure are then correctly covered; and the similitude will hold for migration due solely to the presence of bounding surfaces. This is the type of similitude that is familiar in the discussion of underwater explosions with neglect of all gravitational effects.

The various conditions requiring study thus lead to different criteria for similitude, a situation which occurs also in other applications of the ship model testing method. Thus in model tests of ship propulsion it has long been the accepted practice to break down the model resistance into two parts which are stepped up to full-scale values by the use of different laws of similitude.

It is hoped that a similar procedure can eventually be established in the present case, so that migration effects observed on small scale can be made the basis for a correction of the results of direct scaling according to the nominal theory based on the solid angle subtended at the charge by the target, as explained in TMB Report 492 (11).

However, such a procedure is not yet possible. A study of migration from that point of view is being made and the results will be communicated in a later report.

NOTE ON MOMENTUM IN THE WATER

In thinking about the motion of gas globes it is often natural to resort to reasoning based upon the principle of momentum. Much greater care must be used, however, in applying the principle of momentum to the noncompressive motion of liquids than in applying the principle of energy. It is very easy to go astray and arrive at the wrong conclusion. The fundamental reason lies in the fact that the transmission of momentum involves only the pressure itself, whereas the transmission of energy depends upon both pressure and particle velocity; because of this difference, momentum in an incompressible liquid is more readily transmitted to great distances than is energy.

To illustrate the care that must be used in considerations of momentum, consider a sphere of the same mean density as water, so that it will remain suspended without rising or sinking. Let an upward force be applied to it, causing it to rise in accelerated motion. The sphere is thereby caused to press upward against the water in order to accelerate it; the water loads the sphere, in fact, with an equivalent mass equal to half the mass M of the displaced water, and an upward force $F = Ma/2$ must act on the water where a is the acceleration of the sphere. This force imparts upward momentum $\int F\,dt$ to the water.

Yet if the total amount of upward momentum is calculated from the usual formulas, for the water lying within any given distance r of the center of the globe, the result is zero. The water around the sides of the gas globe moves downward as that above and below it moves upward, and, as regards the water inside of any spherical surface concentric with the gas globe, the downward momentum on the sides just cancels the upward momentum above and below the sphere. The question thus arises, what has become of the upward momentum imparted to the water by the upward force F?

The paradox is redoubled by the following consideration. Since the sphere moves upward, water must on the whole move downward. The total momentum in the water must, therefore, be directed *downward,* not upward. It is easily shown that this downward momentum is, in fact, of magnitude $2\int F\,dt$.

The solution of the paradox is found upon careful consideration to lie in the occurrence of a decrease in the pressure near the bottom. As a result of the motion of the sphere, the upward force of the bottom on the water is decreased by $3F$. One-third of this decrease compensates for the lifting effect of the sphere caused by its upward motion and thereby absorbs the upward momentum given by it to the water; the remaining two-thirds of the decrease allows part of the downward force due to gravity to develop in the water downward momentum of magnitude $2\int F\,dt$.

These considerations are based, of course, upon the assumption of incompressible water. If the action is so rapid, or the body of water so large, that non-compressive theory is not adequate to describe the motion throughout, then part or all of the upward momentum given to the water will remain in it, although perhaps not in the neighborhood of the sphere.

The motion of the water around a moving spherical gas globe of fixed radius is exactly the same as around a sphere of equal size moving at the same rate, hence the same considerations apply to the motion of the gas globe. The statements made in this report have been carefully worded so as to be correct as they stand; caution must be used if the references to momentum are altered or extended.

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