THE DAVID TAYLOR MODEL BASIN

ELECTRONIC METHODS OF OBSERVATION
at the David W. Taylor Model Basin

MEASUREMENTS OF STEADY AND ALTERNATING STRESSES IN ROTATING SHAFTS

PART 2

January 1942

Report R-54

FOREWORD

This is the second report of the series prepared to make available to other experimental and research activities the results of developments at the David W. Taylor Model Basin in the application of electronic methods of observation to a great number of naval problems, other than those of communication.

The first report of the series explained the purpose in view. The present report covers specifically a record of the development of an electronic device which was necessary in the pursuit of work assigned to this activity and which, it is felt, will prove useful to other agencies.

The development of the device described in this report is not complete, but the device has been successfully operated on the full-scale trials of naval vessels.

DIGEST*

This report describes instrumentation and procedure which have been developed and utilized to make observations of steady and of alternating components of thrust and torque stresses in a rotating shaft. The method is not limited by considerations of size or scale, as much of the development work on this equipment has been undertaken on model scale as well as on full scale.

Measurement of alternating stresses on rapidly moving mechanical parts has been attempted in the past by using various lightweight mechanisms or by optical methods, but the results have been by no means satisfactory. This new type of thrust and torque measuring equipment is based upon the application of extremely light strain gages in intimate contact with those portions of the shaft in which it is desired to measure stresses while the shaft is rotating.

The strain gages are of the resistance-sensitive type,** cemented to the outer surface of the shaft in certain selected positions as shown in Figure 1 on page 2 of the report. The active element in this type of gage is a fine alloy wire 0.001-inch in diameter, conveniently mounted in multiple-W form on a sheet of paper which can be cemented to the part under test. These gages, because of their negligible weight, are practically inertialess and are instantaneous in their response to deformations occurring under them. The essential property of the gage is that the wire undergoes a change in resistance proportional to the strain in the wire and hence proportional to the strain in the member to which the gage is attached.

The gages are connected into a bridge circuit on the shaft, as shown in Figure 2 on page 4 of the report, in such manner as to minimize the effects of the resistances of the brushes, slip rings, and other contact resistances involved in bringing the circuits from the revolving shaft to the stationary indicators. The gages are located in pairs, two gages at each end of a shaft diameter, to balance out the effect of bending and to eliminate other undesired stresses. As the individual gages in the pairs are subjected to tension and compression simultaneously, connecting them together in adjacent arms of the bridge increases the sensitivity of the installation beyond that possible with an individual gage installation.

The signals from the electrical circuits on the rotating shaft are amplified before measurement of the signal in an oscillograph or galvanometer indicator. The frequency range is well within the capacity of the common string type of oscillograph with photographic recording on moving film.

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* This digest is included for the benefit of those readers who do not have time to read the full report or who do not care to follow the mathematical treatment in certain parts of the report.

** These are wire resistance strain gages of the type known as metalectric, trade designation SR-4, manufactured by the Baldwin Southwark Division of the Baldwin Locomotive Works, Paschall P.O., Philadelphia, Pennsylvania.

A cam or an interrupted slip ring may be provided on the shaft with a circuit connected to the oscillograph so as to indicate phase position of the shaft; in other words, phase angle on the rotating shaft may be correlated with the alternating components of thrust and torque.

The method developed permits harmonic analysis of the wave form as well as computation of the steady and the alternating components of thrust or torque.

Measurement of thrust and torque stresses by this method has the following advantages:

1. Use of a modulated carrier wave permits simultaneous records of steady components as well as alternating components of all frequencies.

2. Use of a complete bridge circuit on the shaft:
   (a) Reduces the effect of resistance variation of brushes and slip rings to a minimum.
   (b) Balances out to a first order of approximation the effect of gage temperature changes.
   (c) Balances out the effect of bending and other undesired stresses in the shaft.
   (d) Utilizes maximum sensitivity as all four bridge arms are active strain elements.

3. Use of gages which are practically inertialess introduces no inertia limitations as to frequency response for high-frequency vibrations.

4. Permits application of the method to any shaft in the field, with no previous preparation other than installation of slip rings and gages shortly before test.

5. Installation and use of the apparatus in no way modifies or injures existing shafting, and the parts may be left attached indefinitely.

6. Permits four, eight, or twelve simultaneous measurements.

7. Requires a negligible amount of space around or along the shaft.

On the other hand, the method requires personnel skilled in electronics to operate the equipment in the field, and its accuracy is subject to:

(a) Effects of temperature and humidity on the electronic equipment.

(b) Frequent and accurate zero balance readings, especially for steady components.

(c) Accuracy of modulus of rigidity of the shaft.

(d) Interference effects from adjacent wiring and equipment.

The present equipment leaves something to be desired in compactness and portability, but these features will doubtless be improved as development continues on this device.

ELECTRONIC METHODS OF OBSERVATION
at the David W. Taylor Model Basin

PART 2
MEASUREMENTS OF STEADY AND ALTERNATING STRESSES
IN ROTATING SHAFTS

ABSTRACT

The methods of electrical measurement of steady and alternating stresses in rotating shafts as employed at the David W. Taylor Model Basin are discussed and the mathematical formulas necessary for reduction of the data to usable form are given. The report describes certain specific equipment consisting of a bridge circuit on the rotating shaft, made up of four resistance strain-sensitive elements: an amplifier, a variable frequency oscillator, capacity and resistance balances and an oscillograph, with a moving-film camera type of recording apparatus. The report concludes with a short statement of the accuracy obtained with the method to date and with certain remarks concerning its future application to problems of this type.

The device described provides an accurate and practically inertialess type of stress measurement for rotating shafting for the experimental solution of torsional and longitudinal vibration problems. For many problems it is the only type of equipment fully developed which permits measurement of alternating components of stress.

INTRODUCTION

Measurement of alternating stresses on rapidly moving mechanical parts has long been desired for vibration analysis and fatigue strength calculations. This has been difficult to accomplish by mechanical means because of the following major problems:

1. Presence of inertia effects in mechanical linkages.
2. Transmission of the indication from a rotating or moving part to the indicator or observer.
3. Calibration of the indicating mechanism.

Various lightweight mechanisms have been devised which by optical or other means have met with some success in this type of measurement. Nearly all of these employ the principal of the strain gage which accurately measures the strain elongation over a certain base length, which when combined with the modulus of the material gives the stress desired, i.e., $E\Delta l/l$ = Stress. It is readily seen that such a gage mounted on one spot on a rotating shaft will be subject to temperature effects, bending stresses, and other inaccuracies.

The wire resistance sensitive type of electric strain gage, used in the form of a bridge, not only overcomes these limitations and disadvantages for dynamic or alternating measurements but permits a quite accurate simultaneous measurement of the accompanying static or steady components of stress.

APPARATUS

The active element in this type of gage is an extremely fine alloy wire 0.001 inch in diameter, conveniently mounted in multiple W form on paper for cementing to the part under test. An essential property of the gage is that it undergoes a change in resistance proportional to the strain in the wire and hence to the strain in the member under test, i.e.,

$$\frac{\Delta R}{R} = k \frac{\Delta l}{l} \qquad [1]$$

where $k \doteqdot 2$ for the type of gage used. Exact values of k are supplied by the manufacturer for each lot of gages, and the equation is valid for both tension and compression.

Resistance measurements are made by a form of Wheatstone bridge made up of four strain gages cemented to the shaft in appropriate positions and directions, as shown in Figure 1. A circumferential paper template is made which indicates the proper location of two sets of the gages diametrically opposite each other on the shaft and

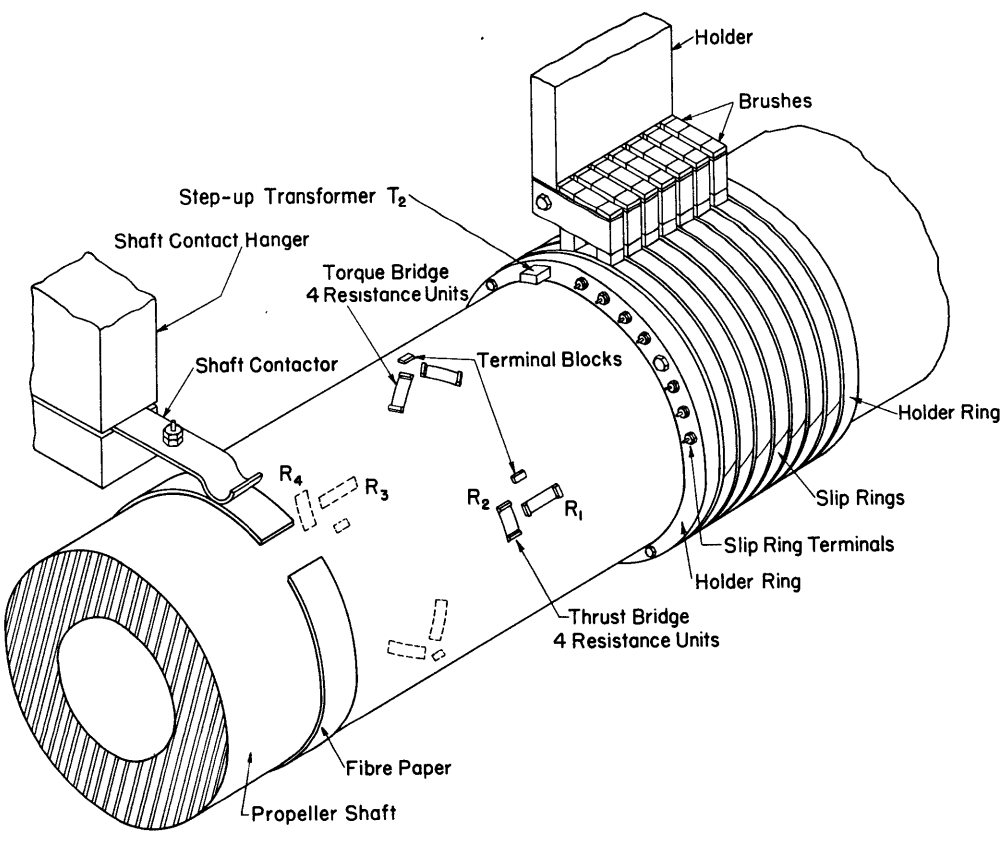


Figure 1 - Schematic Drawing of Torque and Thrust Measuring Apparatus

which orients them in their proper relations to the shaft axis depending upon their use for torque or thrust measurement. For torque measurement, the gages are mounted on 45-degree helices at right angles to each other, so that they measure the stresses in the principal directions of tension and compression of a shaft in pure torsion.

For thrust measurement, the gages are mounted at right angles to each other as before but with their axes respectively parallel to and perpendicular to the shaft axis. In this arrangement, the gages are again oriented for measurement of principal stresses of tension and compression, for either direction of thrust.

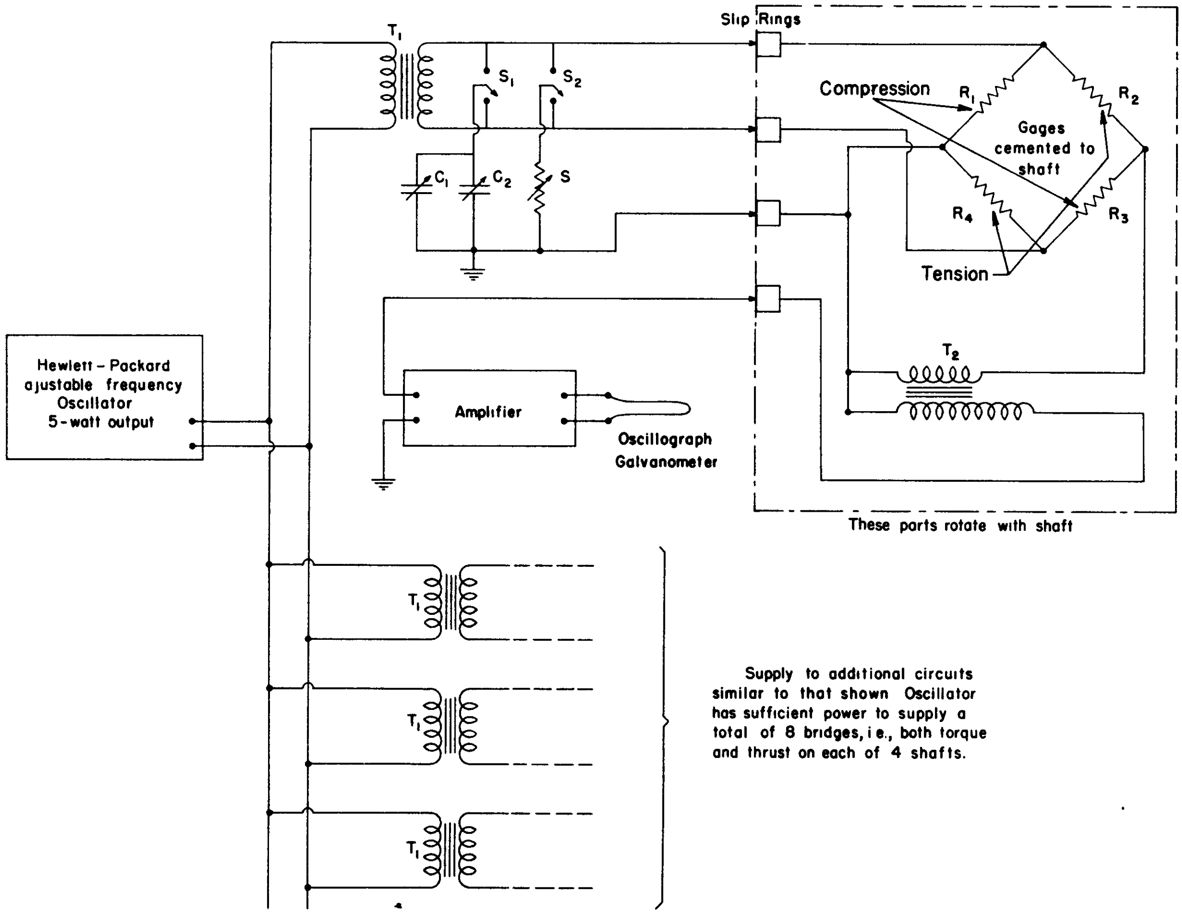
The four gages in each set are connected into a closed loop or bridge with external connections made to the slip rings from the corners of the bridge. Connecting the slip rings to the external leads eliminates the slip rings from the bridge circuit proper. The variable resistance of the brushes and rings has been a major source of trouble in previous designs in which part of the bridge circuit is on the rotating shaft and part is stationary.

Location of pairs of gages at both ends of a shaft diameter has the advantage of balancing out the effect of bending and other undesired stresses, as well as increasing the sensitivity by the employment of gages which are subjected to tension and compression respectively in adjacent arms of the bridge; this gives an overall increase in signal beyond that possible with any individual gage installation.

The shaft is prepared for application of the gages by removal of grease and paint, and choice of a smooth, regular surface to which the gages are applied in direct contact with the shaft. A quick-drying Duco or other insulating cement is used and continuous application of pressure is made during the early drying period to insure intimate adhesion of the gage wires to the shaft surface. The end connections are made to terminal blocks cemented to the shaft. The leads are then connected to the slip rings directly or through a step-up transformer to increase signal strength.

Gage resistances are so chosen that, when connected into the bridge circuit, a high resistance connected in parallel with one arm of the bridge through the slip rings is sufficient to bring the bridge into balance. This resistance is chosen in the order of 10,000 to 100,000 ohms as resistances in this range minimize the errors due to brush and contact resistance variations. This adjustable balancing resistor is then used in further measurement of the amount of resistance unbalance introduced into the bridge arms by the strains in the shaft.

Obviously, steady components of stress could be readily measured by a null method, but alternating stresses require the use of an oscillograph and a high-gain amplifier. In order to avoid the difficulties associated with amplifying some low frequencies encountered, of the order of 2 to 20 CPS, the bridge is usually supplied with an alternating current carrier wave. Stress variations in the shaft appear as a modulation of the a-c. output from the bridge. For accurate results, the carrier frequency should be at least ten times the frequency of the highest frequency vibration which is to be measured; for propeller shafts of naval vessels this usually requires a supply frequency between 200 and 700 CPS. The source is a 5-watt vacuum-tube



T_1 - 200-ohm to 120-ohm output transformers

T_2 - 120-ohm to 12.500-ohm input transformer

R_1, R_2, R_3, R_4 - Metalectric gages, Type A

S_1, S_2 - Single-pole, double-throw toggle switches

C_1 - Decade condenser box variable from zero to 0.01 mfd. in steps of 0.0001

C_2 - 0 to 140-mmf. continuously variable air condenser

S - Precision decade resistance box, 0 to 100,000 ohms in 1-ohm steps

Figure 2

oscillator. This power is sufficient to maintain about 8 volts across the input terminals of 8 bridges simultaneously. The gages are capable of withstanding a somewhat higher voltage, but it has been found that higher voltages affect the zero setting of the bridge, doubtless owing to second-order temperature effects. A step-up transformer, T_2 in Figure 2, is usually mounted on the shaft to raise the bridge output voltage relative to the electrical noise generated by the brushes. Condenser C_1, Figure 2, is a midget variable air condenser shunted by small fixed condensers as necessary to balance out stray capacities. Resistance shunt S is a precision decade resistance box with a range of 0 to 100,000 ohms by 1-ohm steps.

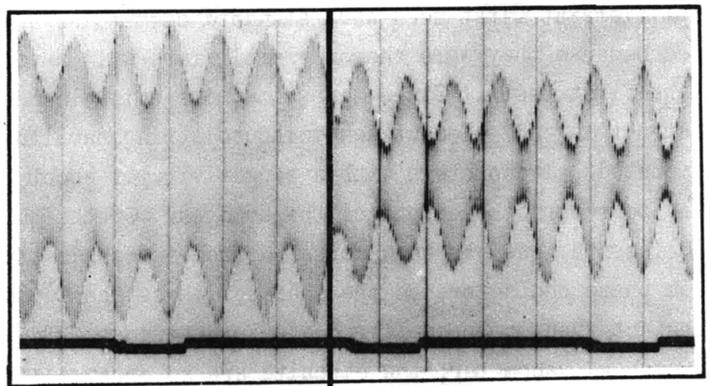
The amplifiers at first used were standard commercial speech amplifiers. They were employed because they were the only amplifiers with sufficient gain procurable in time for the first tests. Since that time more suitable amplifiers have been designed and constructed. A 7-element Westinghouse string oscillograph containing four vertically-mounted galvanometers with a sensitivity of about 2.3 milliamperes per inch has been used to record the output of the bridges. Of the three remaining horizontally mounted elements, one has been actuated from a seconds-timing contactor and the other two from phase contactors on the shafts. These shaft contactors are set so that they make contact when one propeller blade is uppermost. Two shaft contactors are connected to each galvanometer; the circuits are so arranged that one contactor produces twice the deflection recorded by the other. Any number up to four contactors can thus be identified on the record, using only two galvanometer elements. A 12-element Hathaway oscillograph has recently been acquired to permit simultaneous recording of thrust, torque, and phase measurements on four shafts.

PROCEDURE

The "zero balance" of the strain-gage bridge is obtained as follows: With the vessel dead in the water and all shafts turning slowly ahead, decade resistors S are varied until all bridges balance. Balance is indicated by minimum deflection on the observing screen of the oscillograph. The values of resistance necessary to accomplish this for each bridge are recorded. This measurement is repeated with all shafts stopped and is again repeated with all shafts turning slowly astern.

As soon as it is reported that the vessel is steady on a speed selected for measurement, the decade resistors and the amplifier gain-controls are readjusted to give a pattern on the viewing screen which seems favorable for measurement, i.e., a portion of the steady stresses is balanced out by adjusting the shunt resistors S. The gain controls are then set so that the deflections due to the alternating stresses take up most of the available space on the viewing screen. The pattern is then photographed. The portion of the steady stresses balanced out can then be computed from the resistance values recorded. In order to avoid dependence on a precise knowledge of amplifier gains in interpreting the oscillogram, a second exposure is then taken at slightly different settings of the decade resistors. Assuming that the amplifier gains and the steady components of thrust, or torque, remain constant in the interval, the amount of thrust, or torque, represented by the deflections on the photograph can then be calculated by solving a set of simultaneous equations as explained in the latter part of the sections following.

In early trials, it was found that on certain ships the thrust was subject to variations so slow as to cause possible errors in this procedure. This might have been due to ship motion or waves. For this reason the change in setting of the decade resistors required for calibration has been performed on the later trials while the oscillograph camera was operating, in order to cut the time interval between exposures to a minimum. A sample record is shown in Figure 3.



Record Obtained with Balance Resistor Set at "Zero" or "Balance" | Record Obtained by Moving the Balance Resistor to a New Setting

Figure 3

This procedure has the effect of directly calibrating the record in "ohms change per inch." It may also be used to transfer from the record to the balance resistor a portion of the steady component to insure adequate amplification of the alternating component.

COMPUTATION OF RESULTS

Let A be the cross-sectional area of the shaft in square inches

D be the outside diameter of the shaft in inches

d be the inside diameter of the shaft in inches

E be the Young's modulus of the shaft, in pounds per square inch

G be the shear modulus of the shaft, in pounds per square inch

m be Poisson's ratio of the shaft

Q be the torque in the shaft in pound-inches

S_v be the shearing stress in the shaft due to torque, in pounds per square inch

T be the thrust on the shaft, in pounds

R_1 to R_4 be the resistances of various gages, in ohms

S be the resistance of shunt decade resistor

k be the gage constant, as defined by Equation [1], the same for all gages chosen

δ be the specific resistance variation of gage = $\Delta R/R$

V be the input voltage to strain-gage bridge

V' be the output voltage of strain-gage bridge

In the case of a resistance-gage bridge arranged to measure thrust, two of the gages, R_1 and R_3 of Figure 1, are mounted parallel to the shaft axis.* The strain in the shaft where these gages are mounted is

* These two gages must be connected in opposite arms of the bridge, as R_1 and R_3 in Figure 2. In practice, they are mounted at opposite ends of the same shaft diameter so that the effects of any bending stresses cancel.

$$\frac{\Delta l}{l} = \frac{T}{AE}$$

As this strain is compressive for ahead thrust, and a compression reduces the gage resistance,

$$\delta = \frac{\Delta R}{R} = \frac{-kT}{AE}$$

The other two gages, R_2 and R_4 in Figure 2, are mounted circumferentially around the shaft. For an ahead thrust T they are subjected to a tensile strain

$$\frac{\Delta l'}{l'} = \frac{mT}{AE}$$

and hence undergo a resistance change

$$\delta_a = \frac{+mkT}{AE} = -m\delta$$

Thus

$$T = \frac{(-)AE\delta}{k} = \frac{AE\delta_a}{mk} \qquad [2]$$

In the case of a bridge arranged to measure torque, the gages are mounted along the two opposite helices on the shaft surface which make an angle of 45 degrees with the shaft axis. It is well known that the stress along one of these helices is a pure compression S_C and the stress along the other is a pure tension S_T, where

$$S_C = S_T = S_V$$

As these two helices cross each other at right angles the strains are respectively

$$\frac{\Delta l}{l} = \frac{S_C}{E} + \frac{mS_T}{E} = \frac{S_V}{E}(1+m)$$

$$\frac{\Delta l'}{l'} = \frac{S_T}{E} + \frac{mS_C}{E} = \frac{S_V}{E}(1+m)$$

and the resistance changes are

$$\delta = -\delta_a = \frac{kS_V}{E}(1+m)$$

From elementary theory of stresses

$$E = 2G(1+m)$$

$$Q = \frac{\pi}{16}\left(\frac{D^4-d^4}{D}\right)S_V$$

Whence

$$Q = \frac{\pi}{8}\left(\frac{D^4-d^4}{D}\right)\left(\frac{\delta G}{k}\right) \qquad [3]$$

It now remains to express δ in terms of the bridge output voltage and the shunt resistances necessary to balance the bridge before and after the strain is applied.

Assuming that the shunt S is applied across R_4,* Kirchoff's laws may be applied to the circuit of Figure 2. This gives (before applying any stress)

$$\frac{V'}{V} = \frac{R_2}{R_1 + R_2} - \frac{R_3}{R_3 + \frac{R_4 S}{R_4 + S}}$$

After applying a stress, R_1 becomes $R_1(1 - \delta)$; R_3 becomes $R_3(1 - \delta)$; R_2 becomes $R_2(1 + c\delta)$; R_4 becomes $R_4(1 + c\delta)$ where $c = m$ for a thrust bridge and $c = 1$ for a torque bridge, in accordance with the arrangements previously discussed. Therefore, quite generally,

$$\frac{V'}{V} = \frac{R_2(1+c\delta)}{R_1(1-\delta) + R_2(1+c\delta)} - \frac{R_3(1-\delta)}{R_3(1-\delta) + \frac{R_4 S(1+c\delta)}{R_4(1+c\delta)+S}}$$

$$\frac{V'}{V} = \frac{1}{\frac{R_1}{R_2}\left[\frac{1-\delta}{1+c\delta}\right] + 1} - \frac{1}{\frac{R_4}{R_3}\left[\frac{S}{R_4(1+c\delta)+S}\right]\frac{(1+c\delta)}{(1-\delta)} + 1}$$

For a chosen value of S, corresponding to a condition with a portion of the steady component balanced out, there exists a resistance change δ' for that stress such that the voltages V' become zero. Under these conditions the preceding equation reduces to

$$\frac{R_2(1+c\delta')}{R_1(1-\delta')} = \frac{R_3(1-\delta')}{\frac{R_4(1+c\delta')S}{R_4(1+c\delta')+S}} \qquad [4]$$

or

$$\frac{R_1}{R_2} = \frac{R_4}{R_3}\left[\frac{S}{R_4(1+c\delta')+S}\right]\frac{(1+c\delta')^2}{(1-\delta')^2}$$

Then, for some stress other than the stress for which the bridge is in balance

$$\frac{V'}{V} = \frac{1}{\frac{R_4}{R_3}\left[\frac{S}{R_4(1+c\delta')+S}\right]\frac{(1+c\delta')^2}{(1-\delta')^2}\frac{(1-\delta)}{(1+c\delta)} + 1} - \frac{1}{\frac{R_4}{R_3}\left[\frac{S}{R_4(1+c\delta)+S}\right]\frac{(1+c\delta)}{(1-\delta)} + 1}$$

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* If S were applied to some other branch R_1 the ultimate results would be the same, owing to the symmetry of the arrangement.

where δ corresponds to the stress actually existing and δ' corresponds to the stress for balance as already explained. This equation can be drastically simplified by approximation without introducing any significant errors.

To a degree of approximation far beyond the limits of experimental accuracy

$$\frac{R_4}{R_3}\left[\frac{S}{R_4(1+c\delta')+S}\right] \doteq \frac{R_4}{R_3}\left[\frac{S}{R_4(1+c\delta)+S}\right]$$

Substituting B equivalent to either of these quantities:

$$\frac{V'}{V} = \frac{1}{B\left(\frac{1+c\delta'}{1-\delta'}\right)^2\left(\frac{1-\delta}{1+c\delta}\right)+1} - \frac{1}{B\left(\frac{1+c\delta}{1-\delta}\right)+1}$$

$$= \frac{B\left[(1+c\delta)^2(1-\delta')^2-(1+c\delta')^2(1-\delta)^2\right]}{B^2(1+c\delta')^2(1-\delta)(1+c\delta)+B\left[(1+c\delta')^2(1-\delta)^2+(1+c\delta)^2(1-\delta')^2\right]+(1-\delta)(1+c\delta)(1-\delta')^2}$$

The resistance changes δ and δ' are always very small quantities. For example, if $k = 2$, a stress of 15,000 pounds per square inch in steel gives $\delta = 0.001$. The usual stress encountered in naval ships is of the order of 1000 pounds per square inch for thrust and about 7000 to 10,000 pounds per square inch for torsion. As δ cannot exceed 0.001 all terms of the second and higher degrees in the δ's can be discarded. This gives

$$\frac{V'}{V} = \frac{2B(1+c)(\delta-\delta')}{B^2\left[1+2c\delta'+(c-1)\delta\right]+2B\left[1+(c-1)(\delta+\delta')\right]+\left[1+(c-1)\delta-2\delta'\right]}$$

Approximating further,

$$\frac{V'}{V} = \frac{2B(1+c)(\delta-\delta')}{B^2+2B+1} \quad \text{or} \quad \delta-\delta' = \frac{V'}{2V(1+c)}\left[B+2+\frac{1}{B}\right] \qquad [5]$$

It will be remembered that $c = 1$ in the torsional case and $c = m \doteq 1/4$ in the thrust case. By substituting these values in the next to the last equation, it can be shown that there is no error at all in the last approximation if B equals 1, and only a very slight error if B is slightly different from 1. By choosing gages of nearly equal resistance for each set of four, B can be made equal to 1 within 2 or 3 per cent. This makes possible a further simplification, i.e.,

$$\delta-\delta' = \frac{2V'}{V(1+c)} \qquad [6]$$

The error involved in this last approximation is less than one part in ten thousand for B between 0.95 and 1.05.

Equation 6 shows the output voltage to be a linear function of percentage change of resistance from an initial condition of balance. In general the initial or primary reference condition is "zero balance", i.e., no torque or thrust. In this case $\delta' = 0$ and

$$\delta = \frac{2V'}{V(1+c)}$$

Let S_0 be the value of the shunt at zero balance and zero output voltage. Then from Equation [4] with $\delta' = 0$

$$\frac{R_2}{R_1} = \frac{R_3}{\dfrac{R_4 S_0}{R_4 + S_0}}$$

Substituting back into Equation [4]

$$\frac{(R_3 \; 1 + c\delta')}{\left(\dfrac{R_4 S_0}{R_4 + S_0}\right)(1 - \delta)} = \frac{R_3(1 - \delta')}{\left(\dfrac{R_4 S}{R_4(1 + c\delta') + S'}\right)(1 + c\delta')}$$

$$\left(\frac{1 + c\delta'}{1 - \delta'}\right)^2 = \frac{R_3}{\dfrac{R_4 S}{R_4(1 + c\delta') + S}} \quad \frac{\dfrac{R_4 S_0}{R_4 + S_0}}{R_3} = \frac{[R_4(1 + c\delta') + S]S_0}{(R_4 + S_0)S}$$

Neglecting terms of second order of δ':

$$(R_4 + S_0)\, S + 2c\delta'(R_4 + S_0)\, S \doteqdot [R_4(1 + c\delta') + S]\, S_0 - 2\delta'\, [R_4 + S]\, S_0$$

$$2\delta'\left[cR_4 S + cS_0 S + R_4 S_0 + S S_0 - \frac{cR_4 S_0}{2}\right] = (R_4 + S)\, S_0 - (R_4 + S_0)\, S$$

$$\delta' = \frac{R_4(S_0 - S)}{2\left(S\, S_0(1 + c) + R_4\left[S_0\left(1 - \frac{c}{2}\right) + cS\right]\right)} \qquad [7]$$

$$\delta \doteqdot \frac{R_4(S_0 - S)}{2(1 + c)S_0 S} \qquad [8]$$

since S_0 and S are ordinarily several hundred times greater than R_4.

Equations [6] and [8] are sufficient to evaluate the resistance changes, provided the ratio of bridge output to bridge input voltage V'/V, can be measured. The measurement of this quantity, however, involves the amplifier gain, which can not be depended upon to remain constant unless special precautions are taken when designing the amplifier. As previously noted, this difficulty can be eliminated by taking two records of each stress.

Let S_1 be the resistance of shunt during the first measurement

S_2 be the resistance of shunt during the second measurement, etc.

Then from Equations [6] and [8]

$$\delta_1 - \delta_1' = \frac{2V_1'}{V(1 + c)} \quad ; \quad \delta_2 - \delta_2' = \frac{2V_2'}{V(1 + c)}$$

$$\delta_1' = \pm\frac{R_4(S_0 - S_1)}{2(1 + c)S_0 S_1} \quad ; \quad \delta_2' = \pm\frac{R_4(S_0 - S_2)}{2(1 + c)S_0 S_2}$$

where the δ's and V's represent average values.

If the average thrust remains constant during the short time required to make both records

$$\delta_1 = \delta_2 = \delta$$

If the amplifier gain and the applied voltage V remain constant during the same time

$$\delta_1' = \pm h y_1 \; ; \quad \delta_2' = \pm h y_2$$

where y is the average deflection on the oscillograph, h is a calibration constant of the apparatus and the sign depends upon whether the strain for which the bridge is in balance is greater than or less than the existing strain.
Then

$$\pm h y_1 + \frac{R_4 (S_0 - S_1)}{2(1+c) S_0 S_1} = \delta \qquad [9]$$

$$\pm h y_2 + \frac{R_4 (S_0 - S_2)}{2(1+c) S_0 S_2} = \delta \qquad [10]$$

Subtracting Equation [10] from Equation [9] and solving for h gives

$$h = \left(\frac{1}{\pm y_1 \mp y_2}\right)\left[\frac{R_4(S_0 - S_1)}{2(1+c)S_0 S_1} - \frac{R_4(S_0 - S_2)}{2(1+c)S_0 S_2}\right]$$

$$= \frac{R_4}{2(1+c)S_0(\pm y_1 \mp y_2)}\left[\frac{S_0 S_2 - S_1 S_2 - S_0 S_1 + S_1 S_2}{S_1 S_2}\right]$$

$$h = \frac{R_4}{2(1+c)(\pm y_1 \mp y_2)}\left(\frac{S_2 - S_1}{S_1 S_2}\right) \qquad [11]$$

The double signs are completely independent so that four possible h's immediately result. The proper one is chosen by substituting back in Equations [9] and [10]. (Obviously the sign of the first term in Equation [9] must be the same as the sign of y_1 in Equation [11]; and the sign of the first term in Equation [10] must be opposite to the sign of y_2 chosen.)

The value of h which gives the same value of δ for both equations is of course the correct one; the value of δ then found gives the steady component of thrust or torque when applied to Equation [2] or [3]. Alternating components are found from h, the instantaneous deflection of the oscillograph, and Equations [2] or [3].

CONCLUSIONS

The accuracy of the method of shaft stress measurement described here depends largely on the accuracy of the gage characteristics, permanence of adhesion of gages, freedom from temperature and humidity effects, and reduction of inductive and capacitative pickup from extraneous sources.

While it is possible with laboratory procedures to obtain accuracy to 3 per cent or better it is believed that such accuracy has not been obtained to date with measurements in the field. Additional tests are now in progress to improve the accuracy and to determine the field accuracy that can be expected.

While the preceding remarks apply to steady measurements of thrust stresses in general, it must be emphasized that measurement of torsional stresses is inherently more accurate than measurement of thrust forces. This is due to the fact that the average torque stress in naval vessels is approximately ten times the average thrust stress.

As the magnitude of the principal errors in pounds per square inch is nearly independent of the magnitude of the stress being measured, the percentage error in torque measurement by this method should be only about one-tenth that for thrust measurement.

Measurements of alternating components of either of these types of stress are inherently more accurate than measurements of steady components because of the relative freedom of the former from such effects as zero drift due to temperature, humidity or other causes.

Though it may be possible to refine this method sufficiently to permit accurate measurement of steady thrust as well as steady torque it is evident that the most advantageous application of this technique is in the field of measurement of alternating thrust and torque stresses in a rotating shaft.

It may be definitely stated that this method of measurement is the only one sufficiently developed at this time to permit independent or simultaneous measurement of alternating and steady components of thrust and torque unlimited by inertia or frequency considerations in the range of ship vibrations.

PERSONNEL

Development of this type of thrust and torque measuring equipment is primarily the work of Mr. W. F. Curtis and Mr. W. J. Sette of the Model Basin staff, though some of the early ideas were contributed by Professors A. V. DeForest and A. C. Ruge of the Massachusetts Institute of Technology. This development is allied with the propeller-excited vibration project of the Water Tunnel Section under the direction of Lt. Comdr. A. G. Mumma, USN.