THE DAVID TAYLOR MODEL BASIN

THE VALUE OF MATHEMATICS IN RESEARCH AT THE DAVID W. TAYLOR MODEL BASIN

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The desirability of employing additional scientific workers having a wide mathematical education for research at the David W. Taylor Model Basin seems to be self-evident and no further comments are necessary if the question is viewed from this general point of view. If, however, the nature of problems considered by the Taylor Model Basin is analyzed to some extent, a few more specific comments can be formulated. This constitutes the subject of this report.

The David W. Taylor Model Basin is concerned with improvements in the design of naval vessels as far as their propulsive, maneuvering, tactical, and structural features are concerned. This requires a series of experimental and theoretical researches which are generally interrelated by the requirements and conditions of each individual problem. In a given problem it is generally impossible to ascertain in advance to what extent a mathematical treatment is sufficient and to what extent experiment should be brought into play. In some cases, such as in ship model towing tests, experiment seems to be a governing factor, and mathematics recedes to a rather secondary role; in case of other problems such as vibration, stress distribution, etc., mathematics occupies a more prominent role; finally there are problems, such as the turning of ships, in which the application of mathematical methods is retarded by the lack of important experimental data without which mathematical analysis cannot be applied. Viewed from this angle the problems, at least the most complicated ones, of the Taylor Model Basin represent in many instances a gradual bridging of the gap between experiment and theory. In a number of problems this process of bridging the gap is relatively slow, partly in view of the inherent difficulties of the problems involved and partly on account of the difficulties and length of time sometimes required to obtain the requisite experimental information, particularly from the full-scale tests.

A mathematical physicist who is confronted with a problem of this kind finds himself from the very beginning of his work in a rather embarrassing situation. The definiteness of problems to which he grew accustomed in his academic work does not exist in these naval problems; a considerable amount of data which he would like to have in order to start his differential equations is not available, and he cannot wait until it will be available. It is necessary thus to start an approximate theory, to form a provisional hypothesis in the hope that in this manner he will reach at least a *first approximation* rather than a wrong guess. A broad mathematical training helps considerably in such a case; as mathematicians say, it is a *necessary* condition, but by no means a *sufficient* one.

Problems of naval design, particularly those involving the ship as a whole, are of such extreme complexity on account of so many variables that unless one is prepared to simplify the problem by neglecting a considerable number of these variables, or parameters, the mathematical end of the problem simply cannot be started at all. At this point a mathematical expert encounters a most difficult point in his line of attack; which parameters can he safely set aside in the beginning of the work? If the answer depends solely on the order of magnitude of the different factors, this is not yet a difficult problem provided that this order of magnitude can be ascertained from the experimental data and that the problem is considered within the range in which it does not exhibit any critical or "threshold" conditions; or, if one wishes to express it mathematically, when a reasonable amount of "linearization" is permissible.

If the mathematician is lucky enough to simplify the problem in this way, the first approximation is thus obtained; if not, he simply makes a wrong guess and from this moment on theory and practice begin to diverge. He has to have the courage in this case to admit that he was wrong and to apply some other methods of procedure.

It appears, thus, that in addition to a broad mathematical training, a mathematical expert called on to attack such complicated problems must possess also a kind of a combined physico-mathematical common sense - simplifying the problem when it can be simplified and facing difficult situations when a simplification is dangerous on account of the possibility of losing contact with the practical problem. In this connection an ample mathematical training is helpful in so far as a man possessing it can rapidly change his mathematical tactics in his endeavor to conquer the difficulties which he is facing; this, however, is not a sufficient condition unless it is coupled with a searching mind, initiative, perseverence and absence of any preconceived ideas which might force him to subordinate a given problem to mathematics rather than to subordinate the latter to the former. These qualities probably cannot be acquired in school; they are undoubtedly inherent in an individual and when a man possesses them, such a man will be particularly suited for attacking the problems within the scope of activity of the Taylor Model Basin.

The general considerations mentioned previously are obvious, although perhaps too general, and mathematicians might therefore require an illustration of these difficulties in a few concrete examples. For this reason a few of these particularly difficult problems, presenting the gaps between theory and practice, are briefly indicated.

Example 1. Behavior of a Ship in a Seaway

About eighty years ago Froude presented a theory of rolling and pitching of a ship among waves, in which he assumed that the ship is a rigid body and the seaway is a regular trochoidal pattern of waves. This theory served its purpose well for many years in connection with ships of early design (low stability). With the advent of modern high-stability ships, departures from Froude's theory became more pronounced. There exists no theory, so far, capable either of replacing or improving the Froude theory at present.

A mathematical physicist approaching this problem feels vaguely that perhaps the main source of difficulty in this case is due to the fact that the problem was over-idealized. Perhaps instead of dealing with a simplified mathematical picture of a trochoidal seaway one has to face the more difficult situation of a certain *random distribution* of isolated impulses and to try to form a kind of a statistical theory of action of seaway on a ship more in conformity with actual observations. On the experimental side a more detailed study of ship's motion at sea with modern improved instruments thus appears to be unavoidable if a solution of this problem is to be attempted. On the mathematical side these observations are to be fitted into a mathematical framework of a certain statistical distribution of impulses both in time and in space (along the length of the hull). Furthermore, the ship under such circumstances can hardly be considered as a rigid body as assumed by Froude, but must be treated as a certain complicated elastic structure acted upon by such erratic impulses. How a useful theory could be developed starting from these more general assumptions is yet unknown but it is clearly indicated as far as can be seen at present.

In this particular problem a mathematical physicist will have to grapple with many difficult problems such as the statistical theory of a random distribution of impulses, their action on the ship, an exceedingly complicated elastic structure, difficult problems of transition between the phenomena governed by partial differential equations of an elliptic type (stress distribution in the ship caused by the "organized" average action of the seaway) and those governed by equations of a hyperbolic type (propagated stress waves due to shock excitation of the ship by individual waves), and possibly many others. It is already evident from this example that this process of bridging the gap from the mathematical end is a problem of extreme difficulties, and that progress is certain to be slow.

Example 2. Steering and Turning of Ships

This problem is a very old one; original theories were somewhat over-simplified. With the advent of high-speed ships of modern design these theories were found to be more and more at variance with observations. No theory capable of predetermining the tactical characteristics of ships exists at present. This situation is due partially to lack of the necessary data with which a theoretical study could be started and partially to the great complexity of the theoretical problem as such even if certain reasonable simplifications are made and the experimental data are available. In this particular problem the tendency previously mentioned for a simplification of the problem, though desirable, is at the same time very treacherous. Some results can, in fact, be predicted theoretically on the basis of such a "linearization" of the problem; for example, drift angle oscillation accompanied by oscillation of the angle of heel on turning. On the other hand, there seems to be no means of predicting on the basis of this theory the existence of rather mysterious phenomena of instability, of an extraordinarily large effect of even minute difference in the design of two otherwise identical ships, as far as their turning characteristics are concerned. From a mathematical standpoint part of these difficulties seem to be due to the fact that, while the linearization of the differential equation for the radial motion seems to be legitimate by the nature of hydrodynamical forces involved, this is not the case for the azimuthal motion about the center of gravity; in fact, a linearization of the differential equation in this second case entails a *loss of an essential feature* of the dynamical phenomenon. A mathematical physicist in such a case (assuming that he is in possession of all the necessary experimental data that may exist) is confronted with a more difficult problem of a non-linear differential equation with complicated conditions of stability in vicinity of its singularities, where, in fact, an insignificant change in one parameter is capable of producing widely different results.

Example 3. Impact Excitation of Mechanical Structures

This problem is under study at present at the Taylor Model Basin in connection with impact transients of a very short duration, such as those produced by projectiles on armor and armor backing. Experimental studies of these phenomena are in progress; there are considerable difficulties in this connection, particularly during the initial period when the disturbance grows from zero to a maximum which is, generally, of an extremely short duration, probably of the order of a few microseconds. The experimental data are thus not quite reliable, particularly for this initial phase of the transient load. The problem splits itself logically into two partial problems:

(a) Behavior of material in the immediate vicinity of impact.

(b) Response of the structure as a whole to shock excitation.

Problem (a) concerns the designer of armor rather than the Taylor Model Basin, but problem (b) falls within the scope of the Taylor Model Basin activities.

Aside from uncertainty in the experimental data, the methods of mathematical analysis are not yet quite clear. Approximating a complicated structure, for example a barbette, a turret, or even a simple plate with suitable boundaries, by a system with one degree of freedom as was attempted in the early part of the work, is hardly justified. It might be justified perhaps to some extent in problems with relatively simple boundaries in which the separation of variables in the partial differential equation leads to a relatively simple ordinary differential equation which might be investigated *per se* by an analogue of some kind.

Unfortunately in all practical problems the boundaries generally are so complicated that no simple reduction of the actual problem to an equivalent one-degree-of-freedom problem is possible. One is thus obliged to face the difficult problem rather than to simplify it too much and thereby to lose the necessary contact between the theory and the experimental facts.

There is another difficulty. If one assumes that the duration of the impact is of the same order as the time interval necessary for stress waves radiating from the center of impact to reach the boundaries, the phenomenon is "blurred" to some extent. It seems to be impossible to specify in this case how the energy content of the impact settles ultimately between the normal modes which are of interest in problem (b). In order to be able to approach this subject and to separate the transient period when the phenomenon of propagation (partial differential equation of a hyperbolic type) is predominant from that of a quasi-steady state (partial differential equation of an elliptic type), one has to be able to produce impacts of an extremely short duration (of the order of time necessary for stress waves to reach the boundaries). An attempt is being made to produce such impacts electrically with a known time curve of impact, as recorded by a cathode-ray oscillograph. If this attempt is successful the mechanism of impact as well as the effect of the boundaries on excitation of normal modes perhaps will be better understood.

If one assumes, therefore, that a stage is finally reached when the time curve of impact of this kind is known sufficiently, a Fourier resolution can be attempted. The mathematical problem will be still a rather complicated one for the following reasons:

1. In Naval problems which involve plates, turrets, and barbettes, the medium is dispersive. Stress waves of different frequencies travel with different speeds. A given time curve of impact at $t = 0$ at the center of impact undergoes a deformation as the disturbance travels outward towards the boundaries.

2. Little is known as yet of the effects of boundaries. Will it be advantageous to have rigid boundaries, or elastic ones, or semi-elastic and semi-plastic? How will this circumstance eventually react on the distribution of energy between normal modes of the ultimate motion?

3. What is the effect of the third dimension (thickness)? There is some evidence that the third dimension complicates the phenomena to some extent. Thus for instance it happens sometimes that when the material withstands the impact *on the side* of the impact, it is damaged *on the opposite side*, evidencing thus the existence of stress waves through the thickness (i.e., at right angles to the conventional propagation of waves in a two-dimensional medium). If such is the case one might expect the appearance of a much higher frequency of vibration in this direction, which might account for the phenomenon of "multiple impacts," i.e., modulation of the original impact by this frequency. This effect has been suspected but there is no definite evidence that it actually exists, because such a frequency would be beyond the limit of detection by measuring instruments in their present form.

It may be seen that this particular naval problem imposed by actual service conditions is again of the nature of "bridging the gap." Much still remains to be done on the experimental end, but there are great difficulties to be overcome on the mathematical end as well.

It is to be expected that the mathematical end of the problem will become more and more definite with each new step in experimental knowledge of the problem.

Example 4. Resistance of Hull Structure to Explosions

This problem for obvious reasons is of a fundamental importance in improving the design of naval vessels. In order to be able to formulate any specific recommendations in connection with these improvements the mechanism of propagation of explosive waves in fluids and their effect on the hull must be better understood in light of experimental data available. These experimental data are generally collected by means of various gages registering pressure against time.

One of the outstanding difficulties in this case lies in the interpretation of records so obtained. Any gage inevitably brings its own features into the record so that unless the characteristics of the gage itself are taken into account by means of a suitable mathematical analysis the interpretation of these records is somewhat dubious. It appears thus that the mathematical analysis in this case centers on two more major problems.

(c) Analysis of records with a view to separating the effects due to the gage from the phenomenon which the gage is supposed to record.

(d) The investigation of the nature of the disturbance in the three-dimensional medium with boundaries at the free surface of water and in the vicinity of the hull on which the wave is impinging.

The first part of the problem is by no means a simple one in view of the fact that the initial stage, when the pressure rises from its initial non-disturbed value up to the maximum occurs probably within a time interval of a few microseconds; the existing gages are hardly capable of recording this initial stage so that certain mathematical assumptions are necessary to supplement this lack of data. For the final stage of the phenomenon, which lasts a relatively long time (of the order of 1000 microseconds, perhaps, or more) the records are generally available but their interpretation is complicated by a simultaneous presence of free oscillations of higher modes in the gage itself. A mathematical analysis of this problem may throw light on the performance of the various gages and perhaps will suggest further improvements in their design.

Once the problem of separating the gage performance from the record is satisfactorily solved, the mechanism of propagation of explosive disturbances will be better understood and the major part (d) of the problem can then be approached with a greater certainty.

A mathematical analysis recently made in connection with propagation of discontinuities in compressible gases reveals that the phenomenon is apparently governed by a non-linear partial differential equation of a hyperbolic type with boundaries determined by the surface of discontinuity. This equation can be integrated in a particular case when the law of the perfect gas is assumed. It is not certain at this time whether this problem can be solved for explosions in water in view of the fact that the equation of state in this case remains somewhat uncertain. Experimental data are thus necessary for suitable guidance of future analytical investigations on this subject. The "bridging the gap" process mentioned previously at several points in this discussion is particularly and distinctly defined in this particular problem: any improvement in measuring instruments will undoubtedly contribute to greater certainty of results derived analytically; conversely, a better theoretical understanding of the phenomena involved will guide future improvements in the design of these instruments so as to be able to obtain a more reliable and direct interpretation of records without too many corrections and calculations.

The four examples given are probably sufficient as illustrations of difficulties of the problems which are in the process of a gradual solution at the Taylor Model Basin. The more the gap between theory and experimental data is bridged, the more this particular problem approaches the state of an exact engineering *science*: the less it is, the more it has the appearance of an *art*.

The role of a mathematical physicist in this process is to help to build the span of the bridge from the mathematical end. In so doing he should be careful that he is directing his end of the bridge so as to meet the other half; in other words, his activity should be guided by the experimental data at all times.

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