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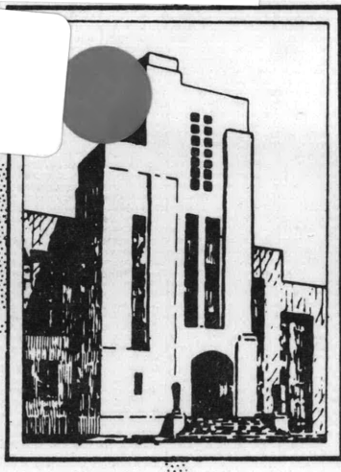

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OPTIMUM SUPERCAVITATING SECTIONS

by

W.B. Morgan

HYDROMECHANICS LABORATORY

RESEARCH AND DEVELOPMENT REPORT

August 1957

Report C-856

SUMMARY

STATEMENT OF PROBLEM

When using supercavitating (SC) sections, two problems that arise are the determination of the minimum drag-lift ratio and the section modulus. This study investigates the minimum drag-lift ratio of a number of SC sections.

CONCLUSIONS

1. For the Tulin SC section with the prescribed limits for angle of attack, the optimum section occurs at a design lift coefficient of 0.16.

2. Of the five sections investigated, the Johnson five-term section theoretically has the least drag for a given lift.

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NOTATION

C_D	Total drag coefficient
$(C_D)_f$	Friction drag coefficient
$(C_D)_P$	Cavitation drag coefficient
C_L	Section lift coefficient
D	Drag
L	Lift per unit span
ℓ	Chord length
p	Static pressure
p_c	Cavity pressure
R_e	Reynolds number $(U\ell/\nu)$
U	Free-stream velocity
α	Design angle of attack
ϵ	Drag-lift ratio
η	Propeller efficiency
η_i	Propeller ideal efficiency
η_ϵ	Propeller friction loss
λ_i	Propeller ideal advance ratio
ν	Kinematic viscosity
ρ	Density of the fluid
σ	Cavitation number $\left(\dfrac{p - p_c}{\frac{1}{2}\rho U^2}\right)$

ABSTRACT

If both the cavitation and friction drag of a supercavitating (SC) section are considered, a minimum drag-lift ratio occurs at a specific lift coefficient. This lift coefficient is mainly dependent on the type of SC section being considered.

INTRODUCTION

Considerable interest has been shown recently in the use of fully cavitating sections (SC) for high-speed lifting surfaces. This interest has been brought about by the increasing speeds of naval vessels and underwater missiles where the suppression of cavitation is difficult or impossible. The feasibility of the use of such sections for propellers has been shown in Reference 1,* where the results of a number of propellers using Tulin's[2] low-drag SC section are given.

Two problems that arise in the use of these sections are the determination of the minimum drag-lift ratio and the section modulus. These problems are interrelated because of the necessity of approximating the section length and camber without being able to easily determine the effect on the drag-lift ratio and section stress. It is known that when only the potential or cavitation drag is considered, the drag-lift ratio decreases with decreasing lift coefficient. However, when the frictional drag is also included, a minimum drag-lift ratio occurs at a specific coefficient of lift.

This report gives the theoretically derived lift coefficient for minimum drag-lift ratios for a number of SC sections operating at

*References are listed on page 12.

zero cavitation number. The sections investigated are the flat plate, the section derived by Tulin having all its lift developed through camber, the same section having part of its lift developed through camber and part through angle of attack, and the three-term and five-term sections derived by Johnson.[3] A simplified method for obtaining the stress of Tulin's SC section is given in Reference 4.

GENERAL CONSIDERATIONS

For a two-dimensional section of unit span, the lift coefficient is given by

$$C_L = \frac{L}{(\rho/2)\, U^2 \ell} \tag{1}$$

where C_L is the section lift coefficient,

L is the lift per unit span,

ℓ is the chord or section length,

U is the free-stream velocity, and

ρ is the density of the fluid.

For any given design, the design lift is a constant for a given velocity, therefore, the lift coefficient and the chord are dependent variables and can be written as

$$C_L \ell = \frac{L}{(\rho/2)\, U^2} \tag{2}$$

This value of $C_L \ell$ is the essential design coefficient for a hydrofoil section. The relationship between the lift coefficient C_L and ℓ will determine the section performance. The optimum section occurs when the drag-lift ratio ϵ is a minimum.

$$\epsilon = \frac{D}{L} = \frac{C_D}{C_L} \tag{3}$$

where D is the drag and C_D is the drag coefficient.

The drag coefficient C_D is dependent on the cavitation or potential drag coefficient $(C_D)_p$ and the friction drag coefficient $(C_D)_f$. The cavitation drag is a function of the coefficient of lift, the shape of the section, and the design angle of attack (α). The friction drag is a function of Reynolds number R_e, where

$$R_e = \frac{U \ell}{\nu} \tag{4}$$

and ν is the kinematic viscosity. Once the type of section and the design angle of attack α are established, the section drag becomes dependent on the lift coefficient C_L and α. It should be noted that the design angle of attack α is an angle in excess of the angle of attack necessary to obtain "shock-free entry."

The general shape of one SC section has been established in Reference 2 for zero cavitation number. In this work the section face or pressure side is given by the ordinates of the camber line and the back or suction side by the ordinates of the cavity thickness. Thus the SC section is assumed to be a thin airfoil with the necessary thickness distribution obtained by filling in the cavity on the back of the section. These ordinates are dependent on the coefficient of lift, design angle of attack, and section length.

The smaller the angle of attack the lower the drag-lift ratio; therefore, for optimum performance, this angle should be as small as possible. However, when considering the practical applications of SC sections, a minimum angle is necessary to prevent face cavitation and stress failure. From Tulin's results the thickness ordinates become negative if

$$\alpha < 4.55\ C_L \tag{5}$$

where α is in degrees. Thus for strength purposes α must be greater than the value given by Equation (5). The exact value depends on the design lift and stress limitations. The establishment of a lower limit in order to prevent face cavitation is somewhat more involved because the pressure distribution on the face of the section must first be calculated. Experimental results[1] have indicated that the minimum α to prevent face cavitation must be of the order of 10 C_L, or

$$\alpha \geqslant 10\ C_L \tag{6}$$

For specific applications this value may be somewhat low since local variations in the inflow velocity could cause the effective α to be reduced. Also at low values of C_L ($C_L < 0.1$), the section may become exceedingly thin near the nose. Even though theoretically the section may be strong enough, this thickness could lead to difficulty from vibration and in manufacturing. Consequently, arbitrary limits have been established for α for the various ranges of lift coefficients. These are:

$$\alpha = 36.5\ C_L \quad \text{for} \quad 0 < C_L \leqslant 0.0548 \tag{7a}$$

$$\alpha = 2\ \text{deg} \quad \text{for} \quad 0.0548 \leqslant C_L \leqslant 0.2 \tag{7b}$$

$$\alpha = 10\ C_L \quad \text{for} \quad 0.2 \leqslant C_L \tag{7c}$$

In the above limits where $C_L \leq 0.0548$, all the lift is taken by angle of attack and the hydrofoil becomes a flat plate.

The cavitation number is assumed to be zero in determining the optimum section. This assumption is necessary because of the difficulty of the calculations at finite cavitation numbers. However, justification for assuming $\sigma = 0$ can be made due to the fact that for $\sigma < 0.1$, it can be shown theoretically[5] that the drag-lift ratio is practically constant. Cavitation numbers of less than 0.05 are of particular interest for propeller blade sections.

CALCULATION OF THE OPTIMUM SECTION

THE FLAT-PLATE HYDROFOIL

Previously, it was stated that for optimum performance the drag-lift ratio must be a minimum. To calculate this ratio the coefficient of drag of the section must first be derived. The total drag is composed of the cavitation drag $(C_D)_P$ and the friction drag $(C_D)_f$.

$$C_D = (C_D)_P + (C_D)_f \tag{8}$$

For $\sigma = 0$, the cavitation drag $(C_D)_P$ of a flat plate is given by Rayleigh's theory of oblique lamina[6]

$$(C_D)_P = \frac{2\pi \sin^2\alpha}{4+\pi\sin\alpha} = C_L \tan\alpha \tag{9}$$

If α is less than 10 degrees ($\alpha \doteq \sin\alpha$), then Equation (9) reduces to

$$(C_D)_P \doteq \frac{2\pi\left(\frac{\alpha}{57.3}\right)^2}{4+\pi\left(\frac{\alpha}{57.3}\right)} = \left(\frac{\alpha}{57.3}\right) C_L \tag{10}$$

and using the linearized value for C_L $\left(C_L = \frac{\pi}{2}\frac{\alpha}{57.3}\right)$,

$(C_D)_P$ becomes

$$(C_D)_P \doteq \frac{\pi}{2}\left(\frac{\alpha}{57.3}\right)^2 \qquad (11)$$

For finite cavitation numbers the results are considerably more complicated. However, it has been shown[7] that for small cavitation numbers

$$(C_D)_P \doteq \frac{\pi \sin^2\alpha}{4+\pi\sin\alpha}(1+\sigma) \qquad (12)$$

The lift for a flat plate at zero cavitation number is given by

$$C_L = (C_D)_P \cot\alpha = \frac{2\pi\sin\alpha\cos\alpha}{4+\pi\sin\alpha} \qquad (13)$$

and when $\alpha \doteq \sin\alpha$ and $\cos\alpha \doteq 1$, then

$$C_L \doteq \frac{2\pi\left(\frac{\alpha}{57.3}\right)}{4+\pi\left(\frac{\alpha}{57.3}\right)} \qquad (14)$$

Eliminating the design angle of attack α from Equations (10) and (14) results in an equation for $(C_D)_P$ involving C_L as the variable.

$$(C_D)_P = \frac{4}{\pi}\,\frac{C_L^2}{(2-C_L)}\ \text{for}\ C_L < 0.241 \qquad (15)$$

The frictional drag coefficient can be estimated using the Prandtl-Schlichting formula.[8] For high angles of attack the frictional drag is negligible compared to the cavitation drag. However, for more reasonable angles of attack the frictional drag cannot be neglected. For Reynolds numbers between 5×10^5 and 5×10^9, the frictional drag coefficient is given by

$$(C_D)_f = \frac{0.455}{(\log R_e)^{2.58}} \qquad (16)$$

The total drag is then given by

$$C_D = \frac{4}{\pi} \frac{C_L^2}{(2 - C_L)} + \frac{0.455}{\left(\log \frac{U\ell}{\nu}\right)^{2.58}} \qquad (17)$$

Since the drag-lift ratio is desired, this equation must be divided by C_L

$$\epsilon = \frac{C_D}{C_L} = \frac{4}{\pi} \frac{C_L}{(2-C_L)} + \frac{0.455}{C_L \left(\log \frac{U\ell}{\nu}\right)^{2.58}}, \text{ for } C_L < 0.241 \qquad (18)$$

For any given design, the lift L, U, and ν are known. The coefficient $C_L \ell$ is then a constant and by assuming different values of C_L, corresponding values of ℓ are obtained. Thus Equation (18) can be evaluated for each assumed value of C_L. This evaluation will show that there is an optimum relationship between C_L and ℓ, this optimum being reached when ϵ is at a minimum. Calculations were made varying $C_L \ell$ between 0.1 and 3.0 and U between 75 fps and 2500 fps. The section was assumed to be operating in salt water, where $\nu = 1.2817 \times 10^{-5}$ ft^2/sec at 59 degrees F.

Results of these calculations are plotted in Figure 1 where it is shown that the minimum ϵ occurs for all combinations of $C_L \ell$ and U at a C_L of approximately 0.05.

TULIN'S SC SECTION (DESIGN ANGLE ZERO)

The optimum of Tulin's SC section, when all the lift is developed through camber, is not necessarily of practical interest because of the susceptibility to face cavitation and because the cavity thickness becomes negative. However, it is interesting to consider this condition since it denotes the theoretical optimum for this class of section.

Calculation of this optimum is approached by the same method as used for the flat plate. The friction drag coefficient is assumed to be the same as for the flat plate since the length of the face surface is approximately equal to the chord of the section. The cavitation drag coefficient is given by[2]

$$(C_D)_P = \frac{(0.4\ C_L + 0.0164\alpha)^2}{1.5708} \tag{19}$$

and when all the lift is developed through camber ($\alpha = 0$), then

$$(C_D)_P = 0.1019\ C_L^2 \tag{20}$$

When the friction drag is included, the drag-lift ratio is

$$\epsilon = 0.1019\ C_L + \frac{0.455}{C_L\left(\log \frac{U\ell}{\nu}\right)^{2.58}} \tag{21}$$

Again assuming $\nu = 1.2817 \times 10^{-5}$ and varying $C_L\ell$ between 0.01 and 3.0 and U between 75 fps and 2500 fps, the drag-lift ratio ϵ can be calculated. Figure 2 gives the results of these calculations. Note that the minimum varies over a greater range of C_L than the minimum for a flat plate, i.e., from $C_L = 0.11$ to $C_L = 0.19$.

TULIN'S SC SECTION WITH ANGLE OF ATTACK

For practical applications, it is of considerable interest to examine the case when part of the lift is developed through angle of attack. The angle of attack α for different C_L values was calculated in accordance with the arbitrary values given by Equation (7). Using Equation (19) for the drag in conjunction with Equation (7) for the design angle of attack, the cavitation drag coefficient becomes

$$(C_D)_p = \frac{4}{\pi} \frac{C_L^2}{(2 - C_L)} \doteq 0.6366\, C_L^2 \quad \text{for} \quad 0 < C_L \leqslant 0.0548 \qquad (22a)$$

$$(C_D)_p = \frac{(0.4\, C_L + 0.0328)^2}{1.5708} \quad \text{for } 0.0548 \leqslant C_L \leqslant 0.2 \qquad (22b)$$

$$(C_D)_p = 0.2025\, C_L^2 \quad \text{for} \quad 0.2 \leqslant C_L \qquad (22c)$$

When the friction drag coefficient is added to Equations (22) the drag-lift ratio becomes

$$\epsilon = 0.6366\, C_L + \frac{0.455}{C_L \left(\log \frac{U\ell}{\nu}\right)^{2.58}} \quad \text{for} \quad 0 < C_L \leqslant 0.0548 \qquad (23a)$$

$$\epsilon = \frac{(0.4\, C_L + 0.0328)^2}{1.5708\, C_L} + \frac{0.455}{C_L \left(\log \frac{U\ell}{\nu}\right)^{2.58}} \qquad (23b)$$

$$\text{for} \quad 0.0548 \leqslant C_L \leqslant 0.2$$

$$\epsilon = 0.2025\, C_L + \frac{0.455}{C_L \left(\log \frac{U\ell}{\nu}\right)^{2.58}} \quad \text{for} \quad 0.2 \leqslant C_L \qquad (23c)$$

The results of the evaluation of the above equations for ϵ for $C_L \ell$ from 0.01 to 3.0 and U from 75 fps to 2500 fps are plotted in Figure 3. From this plot, it can be concluded that the optimum section occurs at a C_L very close to 0.16 for all combinations of $C_L \ell$ and U that were investigated.

THREE- AND FIVE-TERM SECTIONS

Tulin derived a family of sections by considering a two-term expansion. Johnson[3] expanded this work to describe a section with three terms and one with five terms. For comparison, the drag-lift ratio of each of these sections, when all the lift is developed through camber, was evaluated for a C_L of 0.1 and U of 1000 fps. The equations for ϵ for these sections are:

For three-term expansion

$$\epsilon = 0.07074\, C_L + \frac{0.455}{C_L \left(\log \frac{U\ell}{\nu}\right)^{2.58}} \tag{24}$$

For five-term expansion

$$\epsilon = 0.0573\, C_L + \frac{0.455}{C_L \left(\log \frac{U\ell}{\nu}\right)^{2.58}} \tag{25}$$

Results of the evaluations are shown in Figure 4 with the other sections at the same value of C_L and U. It will be noted that theoretically the Johnson five-term section has a much lower drag than any other section, however, its shape is such that it may be more susceptible to face cavitation. Therefore, this section cannot be properly evaluated until experimental results are available.

APPLICATION OF THE OPTIMUM SECTION TO PROPELLER DESIGN

Propeller efficiency η can be considered to be composed of the ideal efficiency η_i, corrected for the friction losses η_ϵ, where $\eta = \eta_i \eta_\epsilon$. The friction loss depends on the drag-lift ratio of the section and the ideal advance ratio λ_i. By assuming that the average ϵ

and λ_i for a propeller occur at the 0.7 radius of the propeller, the friction losses can be approximated by the following equation:

$$\eta_\epsilon \doteq \frac{1-2\epsilon\lambda_i}{1+\frac{2}{3}\frac{\epsilon}{\lambda_i}} \qquad (26)$$

This equation has been evaluated and plotted[9] for different values of ϵ and λ_i (Figure 5) where it is seen that at $\lambda_i = 0.5$, a 1-percent change in $(1 - \epsilon)$ causes a change of approximately 2 percent in η_ϵ. At other values of λ_i this effect is greater. As seen in Figures 4 and 5, the difference in the drag-lift ratio of the different sections can have a great effect on propeller efficiency.

In the present SC propeller design method[1] it is necessary to assume a blade outline. However, it is now possible to obtain the optimum lift coefficient from Figure 3 where it can be assumed that the coefficient of lift C_L is approximately 0.16 and α is approximately 2 degrees. If a check on the strength shows the propeller to be over stressed, the angle of attack can be increased. The section lengths must finally be adjusted to give a faired blade outline.

Experimental results, however, have shown that the theoretical value of drag-lift ratio given by Equation (23) is approximately 25 percent low. Taking this into consideration in order to estimate the efficiency of a propeller, Equation (23b) was evaluated at $C_L = 0.16$ for various blade lengths ℓ and velocities U. Figure 6 is a plot of the drag-lift ratio against velocity with $C_L\ell$ as a parameter. This plot can then be used in conjunction with Figure 5 and the ideal efficiency of the propeller to estimate the actual propeller efficiency.

CONCLUSIONS

1. Each type of SC section has an optimum point, at minimum drag-lift ratio, which is principally dependent on the lift coefficient of the section. For the Tulin SC section with the prescribed limits for angle of attack, the optimum section occurs at a design lift coefficient of 0.16.

2. Of the five sections investigated the Johnson five-term section theoretically has the least drag for a given lift. However, practical application of this section will have to await experimental confirmation.

3. Due to the large variance possible in the drag-lift ratio when using SC sections, the propeller efficiency may vary from the optimum by several percent.

REFERENCES

1. Tachmindji, A., et al, "The Design and Performance of Supercavitating Propellers," David Taylor Model Basin Report C-807 (Feb 1957) CONFIDENTIAL.

2. Tulin, M. and Burkhart, M., "Linearized Theory for Flows About Lifting Foils at Zero Cavitation Numbers," David Taylor Model Basin Report C-638 (Feb 1955) CONFIDENTIAL.

3. Johnson, V., "Theoretical Determination of Low-Drag Supercavitating Hydrofoils," NACA RML57G11a (in preparation) CONFIDENTIAL.

4. Morgan, W., "Centroid and Moment of Inertia of a Supercavitating Section," David Taylor Model Basin Report C-857 (in preparation) CONFIDENTIAL.

5. Waid, R. and Lindberg, Z., "Experimental and Theoretical Investigations of a Supercavitating Hydrofoil," California Institute of Technology Report 47-8 (Apr 1957).

6. Lamb, H., "Hydrodynamics," Sixth Edition, Dover Publications, New York (1945).

7. Wu, T.Y., "A Free-Streamline Theory for Two-Dimensional Fully Cavitated Hydrofoils," California Institute of Technology Report 21-17 (Jul 1955).

8. Schlichting, H., "Boundary Layer Theory," NACA TM 1218 (1949).

9. Lerbs, H., "The Present Status of Theoretical Research on Ship Propellers with Respect to Its Technical Application," (Der Stand Der Forschung Über Der Schiffspropeller Im Hinblick Auf Die Technische Berechnung, Werft-Reederei-Hafen, Vol. 23, 15 Feb 1942) David Taylor Model Basin Translation 243 (Jan 1952).

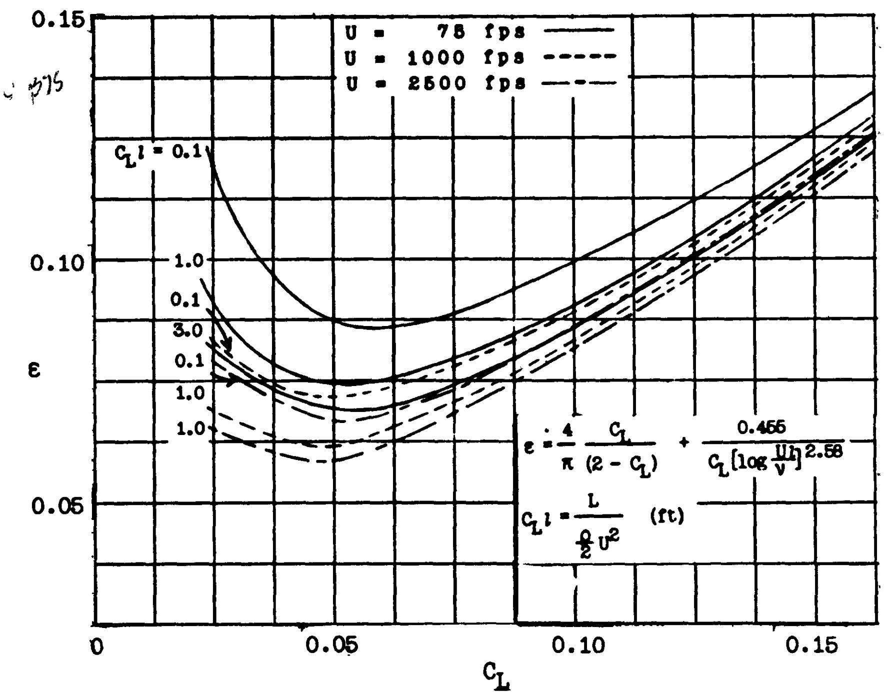

Figure 1 - Drag-Lift Ratio (ε) Versus the Lift Coefficient (C_L) for a Flat-Plate

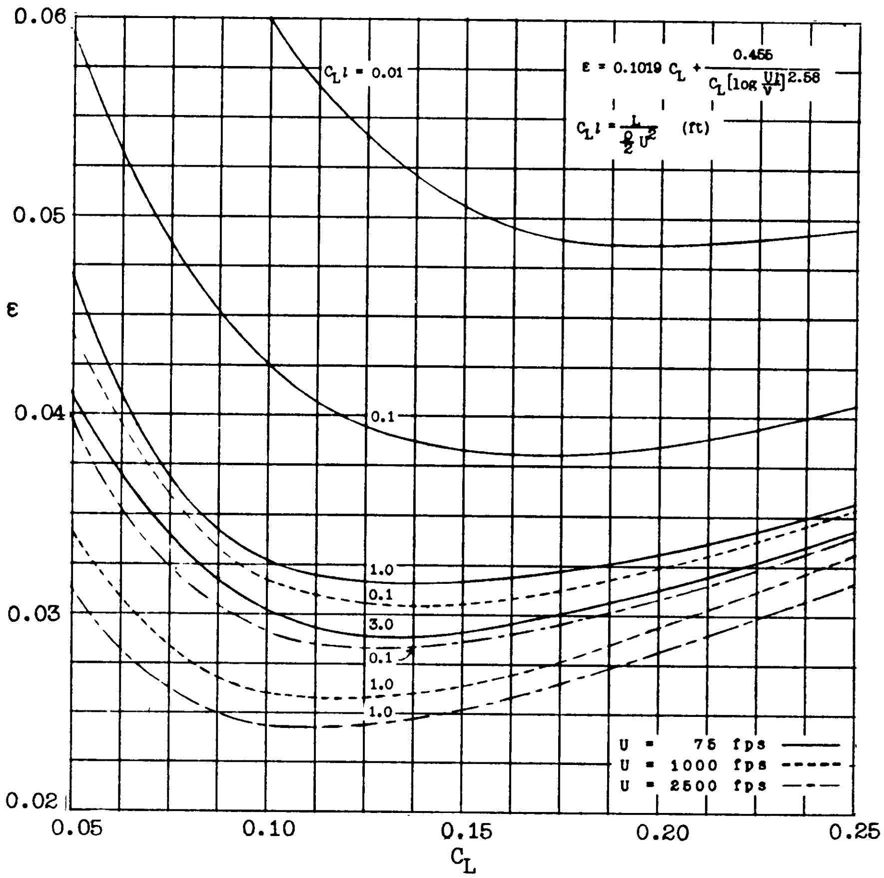

Figure 2 - Drag-Lift Ratio (ε) Versus the Lift Coefficient (C_L) for Tulin's SC Section (α = 0)

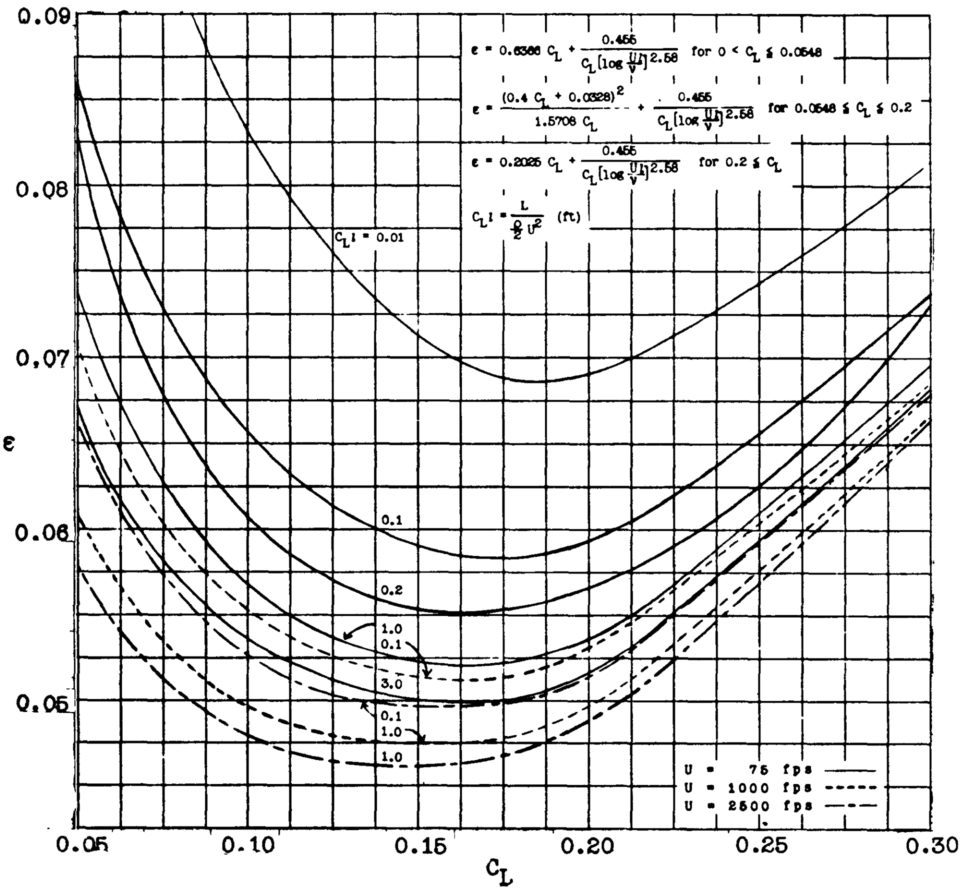

Figure 3 - Drag-Lift Ratio (ε) Versus the Lift Coefficient (C_L) for Tulin's SC Section with Angle of Attack

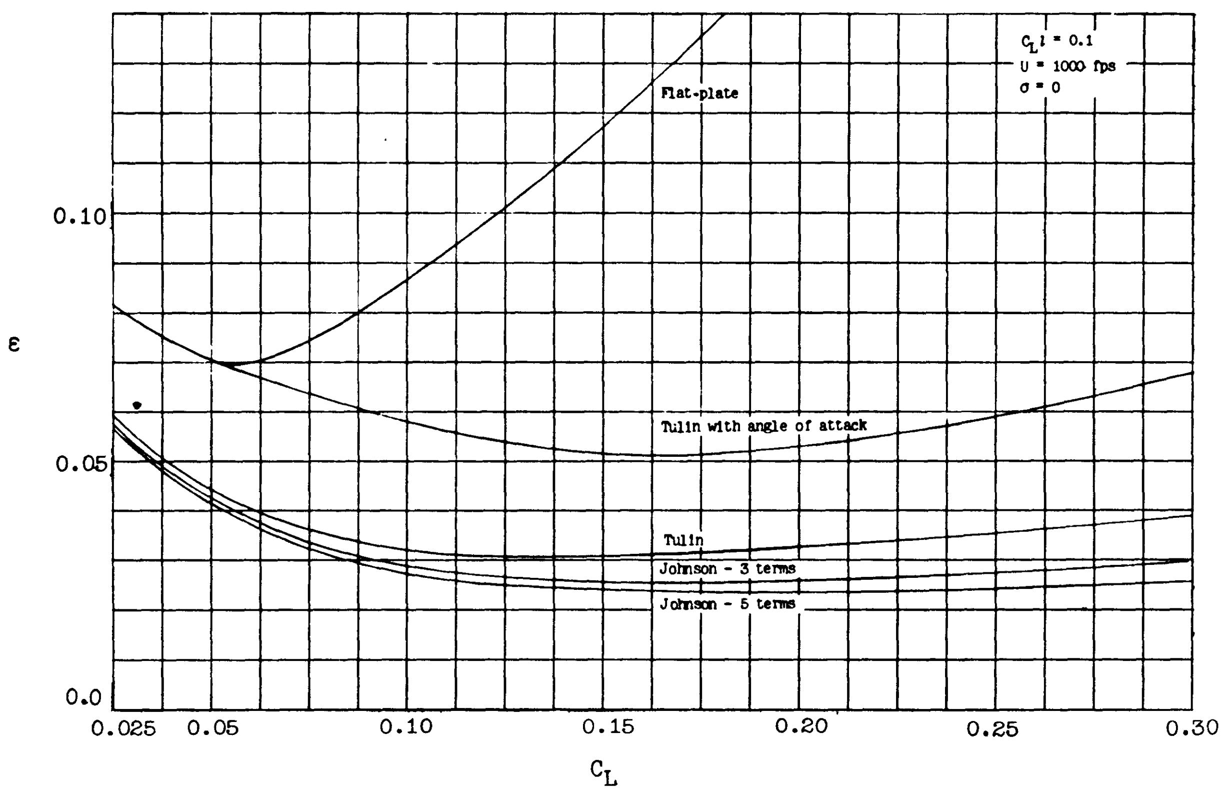

Figure 4 - Drag-Lift Ratio (ε) Versus the Lift Coefficient (C_L) for a Number of SC Sections

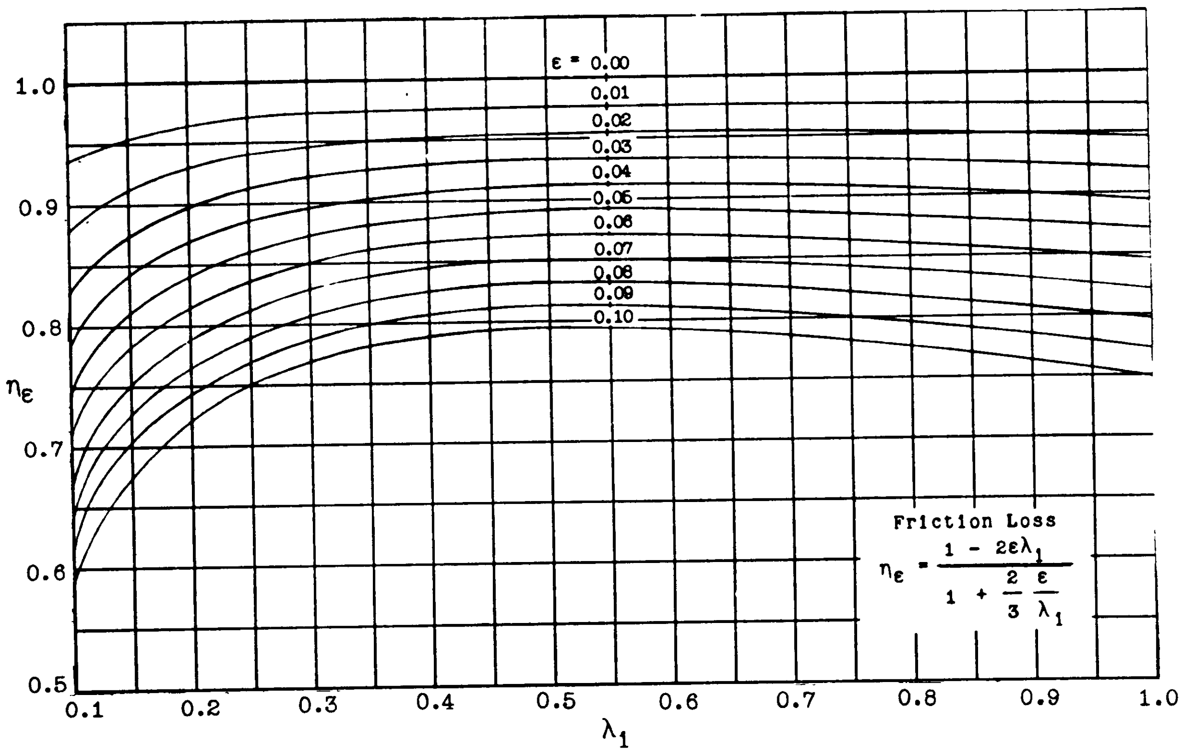

Figure 5 - Propeller Friction Loss

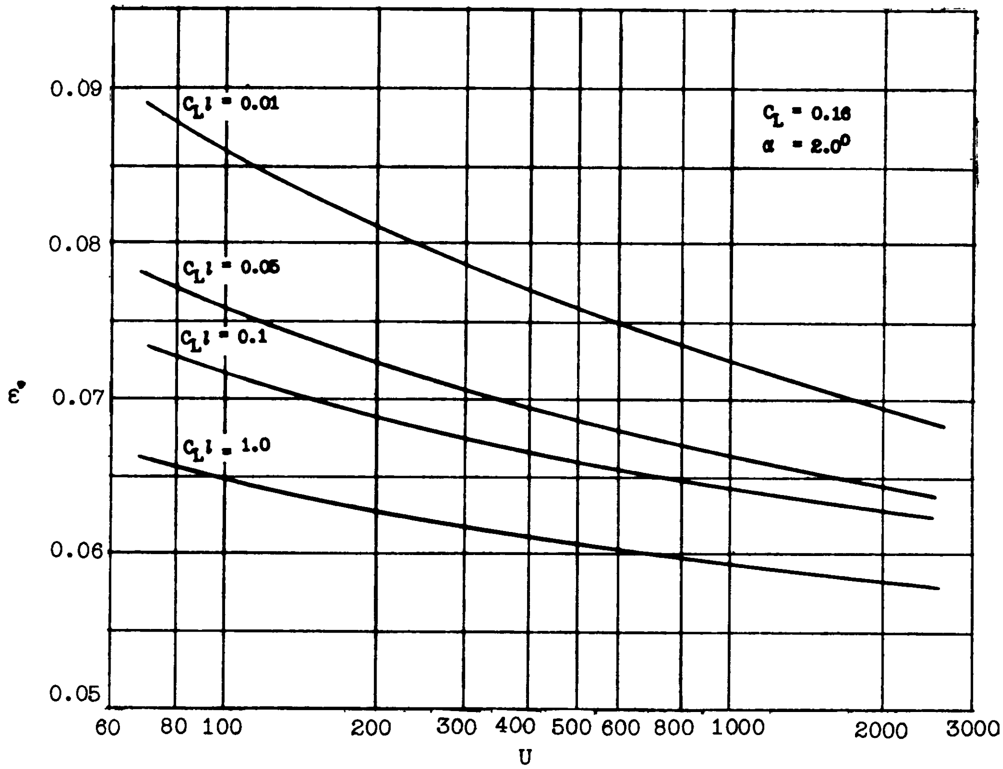

Figure 6 - Drag-Lift Ratios of Optimum SC Section

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