NAVY DEPARTMENT
THE DAVID W. TAYLOR MODEL BASIN
WASHINGTON 7, D.C.

ON THE STRUCTURAL DESIGN OF THE MIDSHIP SECTION

by

M. St. Denis

October 1954

Report C-555
ON THE STRUCTURAL DESIGN OF THE MIDSHP SECTION

by

M. St. Denis

"This document contains information affecting the national defense of the United States within the meaning of the Espionage Laws, Title 18, U.S. C., Sections 793 and 794. The transmission or the revelation of its contents in any manner to an unauthorized person is prohibited by law."

"Reproduction of this document in any form by other than naval activities is not authorized except by special approval of the Secretary of the Navy or the Chief of Naval Operations as appropriate."

October 1954

Report C-555

NS 731-037
The procedure expounded in this report was developed by the author as the result of design studies carried out particularly over a period of nine years while at the Bureau of Ships. This period, which included the war years, was one of fervid engineering development. The enormous expansion of the Navy from a military arm capable of securing one ocean to a striking force capable of maintaining superiority in every sea resulted in the continual preparation of new designs of naval vessels capable of carrying out effectively the ever-changing tasks introduced by the rapidly evolving strategic and tactical aspects of modern warfare.

In carrying out his assigned responsibilities, which included the structural design of the fighting vessels of the Navy, the author soon discovered that if he were to do his work in an efficient manner, in spite of the never-ebbing pressure imposed by the military urgency, it was necessary that the work of design follow a definite and orderly pattern. Such a pattern leading to the synthesis of the structural elements to form a vessel’s midship section is presented in this report.

Herein are set down both design criteria and the procedure to follow in selecting the structure to satisfy them. In establishing these criteria, the most valuable source of knowledge was the reports of structural damage received in the Bureau of Ships from the Commanding Officers of the vessels that had met with distress as the result of operations in heavy weather. Analysis of all such reports that came to the author’s attention indicated that in every instance of failure traceable to a design weakness, the structure did not measure up to the criteria set forth herein.

The author’s primary aim in writing this report is not to present some method of structural analysis but, rather a method of structural synthesis. The report gives a procedure for proportioning the various elements of a structural entity so that the whole fulfills efficiently its specific purpose. In the author’s opinion, design synthesis has suffered from altogether too gross neglect; it has rarely been treated in professional papers. This is unfortunate, for the synthetic work is far more important than the analytic—a point that may be argued by analogy. In music a complete knowledge of harmony (almost an analytic science) will not insure the composition of even the simplest melody (essentially a synthetic art). The same is true also in engineering.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>1</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>PART I - GENERAL THEORIES</td>
<td></td>
</tr>
<tr>
<td>PRELIMINARY CONSIDERATIONS</td>
<td>2</td>
</tr>
<tr>
<td>EXTERNAL LOADINGS</td>
<td>3</td>
</tr>
<tr>
<td>WAVE CHARACTERISTICS</td>
<td>4</td>
</tr>
<tr>
<td>ELASTICITY AND ELASTIC STABILITY</td>
<td>7</td>
</tr>
<tr>
<td>DUCTILITY</td>
<td>9</td>
</tr>
<tr>
<td>THE THREE TYPES OF STRUCTURE</td>
<td>10</td>
</tr>
<tr>
<td>PLATE STRESSES</td>
<td></td>
</tr>
<tr>
<td>The Plate under Normal Loading Alone</td>
<td>13</td>
</tr>
<tr>
<td>The Plate under Compression and Normal Loading</td>
<td>13</td>
</tr>
<tr>
<td>EFFECT OF PLATE CURVATURE</td>
<td>17</td>
</tr>
<tr>
<td>CROSS-STIFFENED PLATING</td>
<td>17</td>
</tr>
<tr>
<td>EFFECTIVE WIDTH OF PLATING</td>
<td>28</td>
</tr>
<tr>
<td>The Stability Problem</td>
<td>28</td>
</tr>
<tr>
<td>The Elasticity Problem</td>
<td>30</td>
</tr>
<tr>
<td>ADEQUACY OF STIFFENERS</td>
<td>31</td>
</tr>
<tr>
<td>COMBINED STRESSES - INTERACTION</td>
<td>32</td>
</tr>
<tr>
<td>DISCONTINUITIES - STRESS CONCENTRATION</td>
<td>34</td>
</tr>
<tr>
<td>TRANSVERSE FRAMING</td>
<td>38</td>
</tr>
<tr>
<td>The Method of Moment Distribution</td>
<td>38</td>
</tr>
<tr>
<td>The Method of Strain Energy</td>
<td>39</td>
</tr>
<tr>
<td>Assumptions</td>
<td>39</td>
</tr>
<tr>
<td>EMPIRICAL RESULTS</td>
<td>41</td>
</tr>
<tr>
<td>EXPERIMENTAL RESULTS</td>
<td>41</td>
</tr>
<tr>
<td>FACTORS OF SAFETY</td>
<td>42</td>
</tr>
<tr>
<td>DESIGN CRITERIA</td>
<td>43</td>
</tr>
<tr>
<td>Conditions of Loading</td>
<td>43</td>
</tr>
<tr>
<td>Conditions of Buoyancy</td>
<td>43</td>
</tr>
<tr>
<td>Hydrostatic Loads</td>
<td>43</td>
</tr>
<tr>
<td>Allowable Stresses</td>
<td>44</td>
</tr>
<tr>
<td>PART II - PROCEDURE FOR THE DESIGN DEVELOPMENT</td>
<td></td>
</tr>
<tr>
<td>PRESENTATION</td>
<td>46</td>
</tr>
<tr>
<td>PREREQUISITE DATA</td>
<td>46</td>
</tr>
<tr>
<td>Topic</td>
<td>Page</td>
</tr>
<tr>
<td>-----------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>STRUCTURAL ARRANGEMENT</td>
<td>48</td>
</tr>
<tr>
<td>Longitudinal Bending Moment</td>
<td>49</td>
</tr>
<tr>
<td>Required Midship Section Moduli</td>
<td>50</td>
</tr>
<tr>
<td>Bottom Plating</td>
<td>50</td>
</tr>
<tr>
<td>Side Plating at Neutral Axis</td>
<td>53</td>
</tr>
<tr>
<td>Side Plating at Main Deck - Sheer Strake</td>
<td>55</td>
</tr>
<tr>
<td>Main Deck Plating</td>
<td>56</td>
</tr>
<tr>
<td>Inner Bottom Plating at Center - Rider Plate</td>
<td>58</td>
</tr>
<tr>
<td>Inner Bottom Side Plating</td>
<td>59</td>
</tr>
<tr>
<td>Second Deck Plating</td>
<td>60</td>
</tr>
<tr>
<td>Nonwatertight Double-Bottom Longitudinals</td>
<td>61</td>
</tr>
<tr>
<td>Watertight Double-Bottom Longitudinals</td>
<td>62</td>
</tr>
<tr>
<td>Shell Stringers Below Second Deck</td>
<td>63</td>
</tr>
<tr>
<td>Shell Stringers Above Second Deck</td>
<td>66</td>
</tr>
<tr>
<td>Inner Bottom Stringers</td>
<td>66</td>
</tr>
<tr>
<td>Main Deck Longitudinals</td>
<td>67</td>
</tr>
<tr>
<td>Second Deck Longitudinals</td>
<td>68</td>
</tr>
<tr>
<td>Moment of Inertia of Midship Section</td>
<td>69</td>
</tr>
<tr>
<td>Nonwatertight Floors</td>
<td>69</td>
</tr>
<tr>
<td>Main Deck Transverse Girder and Web Frame</td>
<td>74</td>
</tr>
<tr>
<td>Stanchions - Second to Main Deck</td>
<td>77</td>
</tr>
<tr>
<td>Second Deck Transverse Girder</td>
<td>78</td>
</tr>
<tr>
<td>Stanchion Below Second Deck</td>
<td>79</td>
</tr>
<tr>
<td>Survey</td>
<td>80</td>
</tr>
<tr>
<td>Transverse Strength</td>
<td>80</td>
</tr>
<tr>
<td>Tertiary Stresses in Bottom Plating</td>
<td>93</td>
</tr>
<tr>
<td>Interaction in Floor Plating</td>
<td>95</td>
</tr>
<tr>
<td>CONCLUDING REMARKS</td>
<td>97</td>
</tr>
<tr>
<td>ACKNOWLEDGMENT</td>
<td>99</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>99</td>
</tr>
</tbody>
</table>
### NOTATION

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Cross-sectional area</td>
</tr>
<tr>
<td>$a$</td>
<td>Sectional area</td>
</tr>
<tr>
<td>$b$</td>
<td>Constant</td>
</tr>
<tr>
<td>$B$</td>
<td>Beam of vessel</td>
</tr>
<tr>
<td>$E$</td>
<td>Modulus of elasticity</td>
</tr>
<tr>
<td>$F$</td>
<td>Normal force</td>
</tr>
<tr>
<td>$f$</td>
<td>Safety factor (ratio of applied to yield or critical stress intensity)</td>
</tr>
<tr>
<td>$H$</td>
<td>Draft of vessel</td>
</tr>
<tr>
<td>$h$</td>
<td>Thickness of plating</td>
</tr>
<tr>
<td>$i$</td>
<td>Unit stiffness</td>
</tr>
<tr>
<td>$k$</td>
<td>Coefficient</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of vessel</td>
</tr>
<tr>
<td>$l$</td>
<td>Length of beam</td>
</tr>
<tr>
<td>$l_1$</td>
<td>Length of opening</td>
</tr>
<tr>
<td>$D$</td>
<td>Depth of vessel</td>
</tr>
<tr>
<td>$E_h$</td>
<td>Flexural rigidity of plating - $D = \frac{E h^3}{12(1 - \mu^2)}$</td>
</tr>
</tbody>
</table>

Where:
- $E$ is the modulus of elasticity, $b$ is a constant, $h$ is the thickness of plating, and $D$ is the depth of vessel.
M  Bending moment
m  Coefficient
n  Number
P  Axial compressive force
p  Normal pressure
Q  Shearing force
R  Stress ratio
    Constant
r  Ratio
s  Spacing of stiffeners
    Variable of integration along path
    Station
u  Abscissa
V  Vertical force
v  Ordinate
W  Work
w  Deflection
    Weight per unit length
y  Vertical distance
Z  Section modulus

α  Thickness of plating at connection angle (normal with horizontal)
β  Aspect ratio
    Central angle
γ  Inertia factor - γ = B/bD
Δ  Displacement
δ  Area factor - δ = A/bh
η  Torsion coefficient
θ  Coefficient
\begin{itemize}
\item $\lambda$ Wavelength
\item $\mu$ Poisson's ratio
\item Displacement coefficient
\item $\pi$ 3.1416
\item $\rho$ Virtual side ratio
\item Radius of gyration
\item $\sigma$ Axial (normal) stress intensity
\item $\tau$ Shearing stress intensity
\item $\phi$ Coefficient
\item $\chi$ Heading angle
\item $\psi$ Coefficient
\end{itemize}

Subscripts

\begin{itemize}
\item $a$ in $a$ (long) direction
\item act Actual
\item all Allowable
\item $b$ In $b$ (short) direction
\item Bending
\item $c$ Compressive
\item $cr$ Critical
\item $e$ Effective
\item $f$ Full load
\item $h$ Hoggling
\item $i$ Indeterminate
\item Specific
\item Inner bottom
\item $L$ At length $L$
\item Longitudinal
\item $l$ Light load
\end{itemize}
### Subscripts (continued)

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{(max)}$</td>
<td>Maximum</td>
</tr>
</tbody>
</table>
| $n$ | Total (plating and stiffener)  
Referring to point $n$ |
| $o$ | Origin  
Reference point or section |
| $p$ | Plating alone |
| req. | Required |
| $s$ | Shear  
Shell  
Sagging  
Static |
| $T$ | Transverse (floor) |
| $x$ | In $x$-direction (longitudinally or along principal stress) |
| YP | Yield point |
| $y$ | In $y$-direction (transversely or across principal stress) |
| $\sigma$ | Tension or compression |
| $\tau$ | Shearing |
| 1 | Primary |
| 2 | Secondary |
| 3 | Tertiary |
ABSTRACT

A consistent and integrated procedure is presented for carrying out systematically the structural design of the midship section of a naval vessel. The report is written in two parts. In the first part the problem is considered in a general manner and the specific theories and methods used in the procedure are introduced. In the second part an illustrative example is worked out for an idealized vessel embodying the simplest possible structure sufficient to illustrate all the points discussed.

INTRODUCTION

Undoubtedly, the design of the midship section is the most important structural problem in naval architecture. Since the hull scantlings for a vessel’s full length are generally derived from those obtaining amidships, a correctly conceived and developed midship section insures, to a large degree, adequate strength for the whole vessel. On the other hand, a poorly proportioned section results either in structural waste or in structural distress (possibly fatal) during the lifetime of the ship. In view of the importance of the problem, it is not surprising to find that a systematic procedure for designing ab initio the midship section of naval vessels has never been presented. Practically all literature on the strength of ships deals only with the analysis of some one or other structural component. Even in the masterly treatise by Hovgaard, all the structural elements are discussed as if they were independent of each other and, therefore, uncorrelated.¹ There is no integration. The synthesis is never carried out.

This integration of separate elements into a consistent structure is left to the designer and, in actual practice, experience, intuition and a sense of proportion play a greater role to this end than is generally conceded. The responsibility attached to designing a midship section is so great that calculations are usually made only to confirm preconceived ideas. When guides are available in the form of designs of successful vessels intended for the same (or essentially the same) service, this work of calculation may at times amount to no more than one of simple comparison.* This is a safe, even if unimaginative, procedure. But it is a procedure having serious limitations as well. The lapse of years from the inception of a design until the time the completed vessel has had several successful voyages in heavy weather is so great that the vessel is likely to have become obsolescent as the result of the technical developments that have taken place in the interim. At best, comparative design is design by hindsight. Wonderful though such a faculty is, it should find little application in a creative

¹References are listed on page 99.

*In the case of merchant vessels, the guides are given by the rules and regulations of the classification societies. The work of calculation, then, reduces to one of bookkeeping.
endeavor. Here vision and imagination are the paramount requisites. The philosophy of the successful designer is not one of inspiration from the past but of creation for the future. The lessons of experience are not discarded and forgotten, but the role of imagination is heightened. And with it the importance of theory. For only the use of properly developed theory allows greater freedom of design and enhances progress.

Today we are at the threshold of far-reaching changes in ship design. In view of the new tactical functions that naval vessels are required to perform, in view of the ever-increasing importance of naval auxiliaries, and in view of the development in this country of passenger superliners for the North Atlantic trade, it may be timely and advantageous to survey the present state of the art of designing the most important structural assembly of a ship—her midship section.

In order to bring out the correlation that must exist between the various phases of the design, a procedure will be given for carrying out systematically the structural development of the midship section. This is intended to be a consistent and integrated procedure by which the most important structural elements in a ship can be readily determined, starting with a minimum amount of information.

The author hopes that the practicing naval architect will find the procedure set forth in this paper useful and that he will be able to save some effort in the routine development of a design. He further hopes that the research scientist will gain an understanding of the designer's problem and will learn along what lines further knowledge is required.

The author is well aware of his own limitations when discussing so difficult a subject as that of this presentation. Yet he does not hesitate to set down what he knows in the hope that with the kernel of knowledge thus provided, others will find it easier to criticize and improve the work presented, to the ultimate benefit of all.

PART I - GENERAL THEORIES

PRELIMINARY CONSIDERATIONS

The design of a vessel subject to the forces of the elements is a most complex undertaking. First it becomes necessary to estimate the external forces acting upon the structural element being designed. This is not easily accomplished since the forces are transient in character and neither adequate theory nor sufficient experimental data are available for dealing with this phase. These transient forces must then be converted into equivalent static loadings upon the structure which is then designed in accordance with theory or empiricism to conform to the imposed criteria.

The theories available for this work are not always rigorous, but they do afford a sound basis for engineering judgment. From a theory one can expect no more.

For a better appreciation of the procedure illustrated in this text, certain aspects of design will be discussed in advance. Theories of design will not be fully expounded herein but reference will be made to a generally available source where the full exposition may be found.
EXTERNAL LOADINGS

According to the procedure in use at present, all dynamically applied external loadings are at first reduced to equivalent static loadings. This reduction is accomplished in one of three ways:

A. Theoretically, as in the design of structures subject to wave pounding, gun blast or airplane landing. With the exception of the special case of the aircraft carrier (a case not considered herein), such transient loadings that can be dealt with in this manner do not occur amidships. A brief exposé on the manner of dealing theoretically with such transients has been given by the author.2

B. Empirically, as the result of experience evaluated by simple theory. This has been in the past, and remains at present, the fashionable procedure. It is a dangerous procedure, however, when dealing with problems beyond intimate experience. The dynamic problem par excellence reduced empirically to a static one is the determination of the maximum longitudinal bending moment when the vessel is in a seaway. The problem is reduced simply to that of poising the vessel in static balance on waves of properly chosen characteristics.

C. Arbitrarily, as in the estimate of the live loads acting on a deck. Here a simple figure (based on previously accumulated actual data whenever possible) is given to represent both the weight of equipment and the effect thereon of the forces due to heaving, pitching, and rolling. Arbitrary loadings should be used only when there is insufficient information on hand to estimate more closely the actual loading and when even a large error in estimation will not have a correspondingly large effect on the resultant structure. Providing the foregoing is true, the only test one can apply to arbitrary loadings is to inquire whether or not they are reasonable.

In any event, the reduction of a dynamic problem to a static one requires a knowledge of the time history of the loading. Such knowledge is practically nonexistent. The reasons for this are twofold: First, the theoretical approach, even in the few simple cases where attempted, runs into serious difficulties and requires a disproportionate amount of time. Second, until quite recently experimental studies have suffered from the lack of adequate testing equipment and reliable testing technique. Even now, though equipment is available and techniques developed, the large cost of obtaining the data and performing the analysis is a major obstacle except in an exceptionally small number of cases.

Recent years have seen the publication of only two attempts to find experimentally a correlation between the action of the sea on a vessel and the resulting strains in her structure. The first of these trials was carried out by Schnadel on the MS SAN FRANCISCO.3 The second was undertaken by the Admiralty Welding Committee on the SS OCEAN VULCAN.4,5

A third trial was carried out recently by the David Taylor-Model Basin on the USCGC CASCO, and a report thereon by Jasper is in course of publication.6
WAVE CHARACTERISTICS

In contrast to the arbitrary loadings mentioned in the preceding section, the choice of the proper wave characteristics to use is of paramount importance, for the greater part of the structural elements of the midship section are absolutely dependent on this choice. Here the unwise selection of wave proportions will result either in waste of material or in the risk of serious damage. As usual, where the risk is greatest, the development is least.

For much too long, proportioning of a vessel's structure has been based on the loading that results when she is statically poised on a standard trochoidal wave whose length is the same as the ship's and whose height is one-twentieth of its length. It should be evident, upon a little thought, that a constant ratio of height to length cannot hold for all waves. The inconsistencies resulting from such an unrealistic assumption are somewhat lessened by the proper choice of an allowable longitudinal bending stress conveniently varying with length of vessel:

$$\sigma_L = \sigma_0(a + bL) \tag{1}$$

where $\sigma_L$ is the allowable longitudinal bending stress for a vessel of length $L$,

$\sigma_0$ is the allowable longitudinal bending stress for a vessel of zero length, and

$a, b$ are arbitrary constants.

But such artificialities can logically be used only in the crudest design approximations. They cannot be allowed if optimum utilization of all material is to be achieved.

Recently a height of wave varying as $1.1 \sqrt{L}$ has been used, especially when designing vessels under 484 ft in length (at which length $L/20 = 1.1 \sqrt{L}$). The same objection holds. Here also the wave height is assumed to increase monotonically with length. Such is not the case in reality.

According to observations made in the Northern Pacific Ocean by the Scripps Institution of Oceanography and by the University of California, it is the ratio of length to height of wave that increases steadily as a function of wave length; see Figure 1.\textsuperscript{7,8} The curve of wave height against wave length (Figure 2), reaches a peak between 450 and 600 ft and then decreases. In this figure three curves are shown:

**Curve A** - Defining the most probable relationship of wave height to wave length to be found in northern oceans if the observations are extended through a whole year.

**Curve B** - Defining the relationship of extreme wave height to wave length to be found in northern oceans if the observations are extended through many years.
Curve C - Defining a proposed relationship to be used in ship design.

The first two curves are based on the observations referred to; the third curve assumes that the length of the wave ($\lambda$) equals the length of the ship ($L$). The wave height given by...
this third curve is approximately the mean of Curves A and B up to the peak value of 34.0 ft, corresponding to a wave length of about 550 ft. Beyond this point, the wave height is assumed constant for design purposes since this condition results, as a first approximation, when a vessel whose length is greater than 550 ft runs diagonally in waves shorter than her length. The offsets of this proposed curve are given by the empirical formula

\[ h = 34 \sin \frac{\pi \lambda}{1100}, \quad 0 < \lambda < 550 \text{ ft} \]

\[ h = 34 \quad \lambda > 550 \text{ ft} \]

which has the following specific values:

<table>
<thead>
<tr>
<th>Wave Length ft</th>
<th>Wave Height ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>18.5</td>
</tr>
<tr>
<td>250</td>
<td>22.2</td>
</tr>
<tr>
<td>300</td>
<td>25.7</td>
</tr>
<tr>
<td>350</td>
<td>28.6</td>
</tr>
<tr>
<td>400</td>
<td>30.9</td>
</tr>
<tr>
<td>450</td>
<td>32.6</td>
</tr>
<tr>
<td>500</td>
<td>33.7</td>
</tr>
<tr>
<td>550 and above</td>
<td>34.0</td>
</tr>
</tbody>
</table>

It is hardly possible here to enter fully into a justification of Curve C as a basis of design, but one may argue:

a. That the proposed curve is based on actual observations and constitutes a compromise between waves a vessel is almost certain to encounter during her lifetime and waves she may never encounter.

b. That for almost all designs it is a more severe criterion than the traditional \( \frac{L}{20} \) and the \( 1.1 \sqrt{L} \) waves.

c. That its use permits the adoption of allowable stresses independent of ship's length.

A comparison of the proposed relationship of wave proportions to those obtaining for the \( \frac{L}{20} \) and the \( 1.1 \sqrt{L} \) waves is given in Figure 3.

It might be well to point out that the adoption of more extreme proportions of wave height to wave length than given by Curve C (as, e.g., those obtaining for Curve B) does not necessarily result in a corresponding increase in longitudinal bending moment, for this moment is not always directly proportional to wave height. When the crest of a wave extends above the freeboard deck of the vessel, the buoyancy curve tends to approach more nearly the weight curve with consequent reduction in longitudinal bending moment. For the vessel considered in this paper, the maximum longitudinal bending moment will be induced by a wave of the same length as the ship and between 37 and 38 ft in height. On the basis of the proposed
relationship, the wave height to be used in the calculations is 34 ft. A wave height of 43 ft as given by the Curve B would result in a reduced longitudinal moment.

For a vessel longer than 550 ft, the largest longitudinal bending moments will be induced by waves shorter than her length. It will be assumed that such a vessel encounters waves 550 ft by 34.0 ft and that her orientation with respect to their crests is given by

\[ \chi = \arcsin \left( \frac{A}{L} \right) \]  

[3]

This orientation has the effect of expanding the length of the wave to the length of the ship while keeping the wave height constant.

ELASTICITY AND ELASTIC STABILITY

When the external loadings on a structure have been determined it then becomes necessary to determine the actual scantlings of the structure itself. In this step use is made of the theories of elasticity and of elastic stability. The first deals with the condition of stress existing in a fully elastic body under the action of externally applied forces when the deformations produced by these remain small compared with the dimensions of the body. The second deals with the determination of the stress at which the deformations in a fully elastic body cease to be small and become most sensitive to the least change in magnitude of the applied loading.
In determining the scantlings for a member, the requirement of adequate strength (theory of elasticity) must always be fulfilled. In addition, for those members subject to compression or shear, requirements of adequate stability (theory of elastic stability) must be satisfied simultaneously. When requirements, for both adequate strength and stability, need to be satisfied, it is easier to fulfill the latter first since it is more easily expressed. Thus, for a panel of plating, the critical stress intensity for any unidirectional loading is given simply by

$$\sigma_{cr} = k \cdot \frac{\pi^2 D}{b^2 h}$$

where $b$ is the width of plate,
$h$ is thickness of plate, and
$D$ is the flexural rigidity of plate $= \frac{E h^3}{12(1 - \mu^2)}$

where $E$ is the modulus of elasticity of material,
$\mu$ is Poisson’s ratio, and
$k$ is a coefficient depending only upon the boundary conditions and aspect ratio of the plate.

The requirement that

$$\sigma_{cr} \geq \sigma_m$$

where $\sigma_m$ is a limiting stress intensity defined in the Design Criteria, is simply expressed as

$$\frac{b}{h} \leq \sqrt{\frac{k \mu^2 E}{12(1 - \mu^2) \sigma_m}}$$

and one needs only apply the proper values for $k$, $E$, and $\sigma_m$.

Equation [5] requires some explanation. Actually $\sigma_{cr}$ can never exceed the yield point of the material, $\sigma_{yp}$, because of the influence of the yield point of the material upon the critical strength. Equation [4] does not reflect this influence and is consequently unreliable for predicting the actual point of initial buckling. Still this "Euler" critical stress (so termed because it depends solely upon the geometry of the structure and not on the strength of the material) does afford a convenient basis for design since the coefficient $k$ has been evaluated already for a number of different boundary conditions. In contrast to this, the effect of the yield point on the critical strength has been evaluated only for the simply supported plate loaded along two edges.

But it might be well to warn the reader that the theories of elasticity and of elastic stability will not solve all the structural problems. Indeed, solutions by these theories can be obtained only upon acceptance of powerful limitations imposed by the underlying assumptions.
These are:

a. That the material is elastic. The yield point of the material consequently introduces a limitation beyond which neither theory applies.

b. That the material behaves linearly (obeys Hooke's law). The principle of superposition thus applies. For shipbuilding steel this assumption is one of opportunity and convenience, not of reality.

c. That the structure is continuous. At first impression it appears as if this assumption might be fulfilled for welded assemblies. But the generally poorer lining up of welded structural members makes this assumption even less valid than for riveted construction.

In addition to these limitations, theoretical solutions are only available for certain definite boundary conditions which rarely, if ever, apply exactly to the problem on hand.

Yet, despite all these restrictions, the naval architect will always find the theories of elasticity and of elastic stability to be his most powerful tools of analysis.

**DUCTILITY**

Although all calculations of scantlings are based on the supposition that the material behaves elastically, no one would ever entertain the thought of building a ship from purely elastic material (cast iron or glass) meeting the requirement of minimum yield strength. This is a curious paradox.

The material specifications require that shipbuilding steel have a certain amount of ductility, expressed as a percentage elongation at rupture of the standard tensile test specimen (from 22 to 25 per cent on a 2-in. base length). Yet having obtained it, the naval architect does not use it in his work. The reasons are, perhaps, twofold. First, there is as yet no manageable theory for analyzing a complex, ductile structure. Second, the stress history of every structural element cannot be known. Without this, a prediction of the absolute stress at any point is simply not possible.

The theory of plasticity and the theory of limit design have been used in recent years with remarkable results, especially in the design of watertight bulkheads and other structures intended to be stressed beyond the yield point of the material. But, as presently developed only relatively simple problems can be dealt with in this manner. For the analysis of a vessel in which the load on each element is complex and where the far from simple structure is highly redundant and subject to buckling, these theories, although promising, require considerable further development.

Determination of the final stress condition in a structural element is severely complicated by another factor. This condition is the grand resultant of all the separate stress fields induced by the rolling, cutting and welding of the material; by changes in temperature, loading, and constraints during the periods of fabrication and erection; by changes in the support reaction during building, launching, and docking; and, finally when sailing, by the behavior
of the natural elements of wind and sea. Only the last is calculated. The remainder are ignored. Yet, for all that, they continue to exist.

And this is where enters ductility. It assumes responsibility for all the unknown stress components which the available theories must perforce neglect. That there are so many unknowns to be taken care of in this manner is irrefutable evidence that ship design is, after all, the most difficult structural problem in the world.

THE THREE TYPES OF STRUCTURE

The deformations of a vessel's structure cannot be conveniently referred to an absolute system of reference (fixed in space). It is usual, therefore, to choose as reference an opportune system which, for the case at hand, effectively replaces the absolute system yet permits considerable simplification in the analysis. Where this possibility exists, all deformations in a structural element or assembly under the action of a set of applied forces are thus measured relative to another element or assembly. The only condition to be met in selecting this second (reference) system is that under the action of the same set of applied forces, its deformations will have only a negligible effect upon the stresses resulting in the first (analyzed) system.

Whether the deformations in a structure are appreciable or negligible depends on:

a. The loading imposed.

b. The rigidity of the structure in the plane of loading.

According to this, all structural elements or assemblies will fall into one of the following three groups or types:

1. Structure of quasi-infinite rigidity in the plane of loading.

2. Structure of finite rigidity or flexibility in the plane of loading.

3. Structure of small rigidity (extreme flexibility) in the plane of loading.

For conciseness these structures may be termed primary, secondary, and tertiary, respectively. Their comparative characteristics are given in Table 1. It might be well to illustrate.

A strake of bottom plating is "tertiary structure" when it is considered as a simple panel of plating whose sole resisting action consists of transferring the external hydrostatic pressure from its surface to its boundaries where it is then imposed upon a first system of supporting structure. This first supporting system consists of frames, floors, and longitudinals, is termed "secondary structure," and constitutes the reference system for measuring the deformations of the bottom plating. The maximum deformation of tertiary structure in the direction of loading (maximum deflection) is of the same order as its depth (thickness).

The deformations of secondary structure, under the same applied loading, are measured relative to a second system of supporting structure. This second supporting system consists here of transverse or longitudinal bulkheads and shell plating and is termed "primary structure."
TABLE 1

Comparison of Primary, Secondary, and Tertiary Structure

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Primary Structure</th>
<th>Secondary Structure</th>
<th>Tertiary Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigidity in plane of loading</td>
<td>Quasi-infinite</td>
<td>Finite</td>
<td>Very small</td>
</tr>
<tr>
<td>Loading</td>
<td>In plane of structure</td>
<td>Normal to structure</td>
<td>Normal to structure</td>
</tr>
<tr>
<td>Stresses</td>
<td>Primary – (\sigma_1)</td>
<td>Secondary – (\sigma_2)</td>
<td>Tertiary – (\sigma_3)</td>
</tr>
<tr>
<td></td>
<td>Tension, compression, and shear</td>
<td>Bending and shear</td>
<td>Bending and shear; membrane</td>
</tr>
<tr>
<td>Type of Structure</td>
<td>Shell, bulkheads, decks,</td>
<td>Only stiffened structure - Shell,</td>
<td>All unstiffened plating loaded normally</td>
</tr>
<tr>
<td></td>
<td>inner bottom - loaded in their plane</td>
<td>bulkheads, decks, double bottom, etc.*</td>
<td></td>
</tr>
<tr>
<td>Boundaries determined by</td>
<td>Undetermined</td>
<td>Primary Structure. Locus of zero</td>
<td>Secondary Structure</td>
</tr>
<tr>
<td></td>
<td></td>
<td>bending moment.</td>
<td></td>
</tr>
</tbody>
</table>

For other loadings it may consist of decks and platforms as well. The maximum deformation of secondary structure in the direction of loading is of a first smaller order than its depth. In this instance the strake of bottom plating becomes also secondary structure when considered as the flange of floors or longitudinals.

Of course, the chain is evident. The same loading deforms the primary structure also. But now there is no further supporting system of reference, and all one can obtain is the change in deformations between two different loadings. The maximum deformation of primary structure in the direction of loading is of a second smaller order than its depth.

Perhaps it is necessary to restate the idea in a different form, and, in this connection, reference is made to Figure 4.

The particular appellation given to a structural element or assembly depends essentially on its relative size. An unstiffened panel of plating considered as a unit apart from all other elements of a ship is tertiary structure. A structural assembly spanning a whole bay and extending from shell to shell, deck to deck, or bulkhead to bulkhead is secondary structure. When considered in its entirety as a unit, the ship is primary structure. Depending then on the structure to be designed or analyzed, the same component may be considered as being in turn either tertiary or secondary or primary structure.

This idea of classifying ship's structures under three basic types leads to a significant simplification of work. Such a conception leads to the possibility of cutting adrift certain
Figure 4 - The Three Types of Structure

$\delta = \text{Deflection of Structure (Not to Scale)}$
structural components or assemblies from the rest of the ship so that they may be designed separately and independently of the rest of the structure.

A final point. By correspondence, the stresses in primary structure are termed primary stresses; in secondary structure, secondary stresses; and in tertiary structure, tertiary stresses. Their intensities are represented herein by \( \sigma_1 \), \( \sigma_2 \), and \( \sigma_3 \), respectively. In this discussion it will be more convenient at times to use this nomenclature instead of a less rigorous one. Thus, plate bending stresses will be referred to as tertiary stresses and the longitudinal bending stresses as primary stresses. The fact that such a nomenclature conveys other meanings in other applications is of no significance, providing it makes for clarity here. The important thing is not the nomenclature but the idea behind this division of structure into types.

The absolute stress intensity at any point is obtained by the simple superposition of primary, secondary, and tertiary stress intensities.

PLATE STRESSES

In shipbuilding the plating is usually of such thickness that diaphragm stresses may be ignored. This simplification is conservative since, if diaphragm action were considered, the stresses for a given load would be somewhat less. For plating subjected to normal loading alone, the solution is extremely simple. If tensile stresses are added in the plane of the plating and acting along either or both pairs of sides, the resultant stresses are obtained by superposition. However, if the plating is subjected simultaneously to normal and compressive loadings, the problem is considerably more complicated. These cases will be treated separately.

THE PLATE UNDER NORMAL LOADING ALONE

In all cases the bending stress intensities in a rectangular plate loaded normally are given by

\[
\sigma = 5.46 k p \left( \frac{b}{h} \right)^2
\]

[7]

where \( p \) is the normal pressure on the plate and \( k \) is a coefficient depending only on the boundary conditions, the aspect ratio of the plating, and the point of measurement of the stress.

Curves for \( k \)-values for four combinations of boundary conditions and a full range of aspect ratios are given by Schade.\(^1\)\(^1\)

THE PLATE UNDER COMPRESSION AND NORMAL LOADING

Solutions to this problem have been presented by Bengston for both the case of the rectangular plate simply supported along its boundaries and that of the rectangular plate fixed along its boundaries.\(^1\)\(^2\) Solutions for elastically restrained boundaries (intermediate degrees of fixity) are not yet available. Indeed, even the solutions for fixed boundaries as developed are subject to strong limitations.
In applying this work, the question always arises as to what boundary conditions to assume. For the bottom plating in a ship, where the normal loading is large and always present, the condition of fixity along all boundaries is approached because of the symmetry of the loading and can be assumed at least within the limitations of the theory (aspect ratio approaching unity). However, when the normal loading is small, as in decks, it is preferable to assume simply supported boundaries. The magnitude of the normal loading at which this change in boundary conditions takes place has not been determined as yet.

The Simply Supported Plate

The procedure for obtaining the plate stresses in the simply supported plate is outlined as follows:

a. Calculate the deflection $w_0$ at the center of the plate by the formula:

$$\frac{w_0^3 C}{a^2} - \frac{w_0}{E} \left[ \frac{\pi^2 (1 - \mu^2)}{2} \left( \frac{b}{a} \sigma_x + \frac{a}{b} \sigma_y \right) - \frac{ERh^2}{a^2} \right] = \frac{8 p a b (1 - \mu^2)}{\pi^2 Eh}$$

in which

$$C = 3.24 \frac{a^3}{b^3} + 3.24 \frac{b}{a} + 0.92 \frac{a}{b}$$

$$R = \frac{a^4}{24} \frac{a}{b} \left( \frac{a^2}{b^2} + \frac{b^2}{a^2} + 2 \right)$$

and where $\sigma_x$ is the average compressive stress intensity in the $x$-direction across side $b$ and $\sigma_y$ is the average compressive stress intensity in the $y$-direction across side $a$.

b. Determine the maximum bending stress intensities at the center of the plate by the relations:

In the $x$-direction

$$\sigma_{xb} = \frac{6\pi^2 Dw_0}{a^2 h^2} \left( 1 + \mu \frac{a^2}{b^2} \right)$$

In the $y$-direction

$$\sigma_{yb} = \frac{6\pi^2 Dw_0}{b^2 h} \left( \frac{a^2}{b^2} + \mu \right)$$

C. Combine the axial and bending stress intensities by superposition. For the scantlings used in shipbuilding, the reduction in axial stress at the center of the sides is negligible.
The Fixed Plate

The procedure for obtaining the plate stresses in the fixed plate is as follows:

a. Calculate the deflection $w_0$ at the center of the plate by the formula

$$\frac{w_0^3 C}{a^2} - \frac{w_0}{E} \left[ \frac{3\pi^2}{8} (1 - \mu^2) \left( \frac{b}{a} \sigma_x + \frac{a}{b} \sigma_y \right) - \frac{E T h^2}{a^2} \right] = \frac{p a b}{2 E h} (1 - \mu^2)$$

in which

$$C = 3.78 \frac{a^3}{b^3} + 3.78 \frac{b}{a} + 1.64 \frac{a}{b}$$

and

$$T = \frac{\pi^4}{24} \cdot \frac{a}{b} \left( \frac{3 a^2}{b^2} + 3 \frac{b^2}{a^2} + 2 \right)$$

b. Obtain the maximum bending stress intensities (at the middle of the sides) when the compressive stresses are of such magnitude as to cause buckling. These are given:

In the $x$-direction by

$$\sigma_{xb} = \frac{12 \pi^2 D w_0}{a^2 h^2}$$

In the $y$-direction by

$$\sigma_{yb} = \frac{12 \pi^2 D w_0}{b^2 h^2}$$

c. Obtain the maximum bending stress intensities when there are no compressive stresses in the plating. These are given by

$$\sigma'_{xb}, \sigma'_{yb} = \frac{\phi D w_0}{a^2 h^2}$$

where $a$ is here the smallest side and $\phi$ is a function of the aspect ratio given in the following table.
d. Determine the critical buckling stress intensity from the equation

\[
\left( \sigma_x + \frac{a^2}{b^2} \sigma_y \right)_{cr} = \frac{4 \pi^2 D}{3 h b^2} \left( 3 \frac{a^2}{b^2} + 3 \frac{b^2}{a^2} + 2 \right)
\]  

[19]

e. Obtain the expression based on the actual stress intensities

\[
\left( \sigma_x + \frac{a^2}{b^2} \sigma_y \right)_{act}
\]  

[20]

f. Obtain the ratio

\[
r = \left( \frac{\sigma_x + \frac{a^2}{b^2} \sigma_y}{\sigma_x + \frac{a^2}{b^2} \sigma_y} \right)_{act} \left( \frac{\sigma_x + \frac{a^2}{b^2} \sigma_y}{\sigma_x + \frac{a^2}{b^2} \sigma_y} \right)_{cr}
\]  

[21]

g. Interpolate between Steps b and c in accordance with this ratio

\[
\sigma_{xb} = \sigma_{xb}' + r(\sigma_{xb} - \sigma_{xb}')
\]  

[22]

\[
\sigma_{yb} = \sigma_{yb}' + r(\sigma_{yb} - \sigma_{yb}')
\]  

[23]

h. Combine the axial and bending stress intensities by superposition. Again, the reduction in axial stress at the center of the sides is negligible.
EFFECT OF PLATE CURVATURE

In the preceding section, the assumption was made that the plating was plane, and the critical and bending stresses were obtained on this assumption. Since in a ship a considerable number of plates are curved, we should consider the effect of this curvature upon the stresses. For plates simply supported along all edges and loaded in the direction of the generators, we find from Equation [13] that curvature increases the critical stress by an amount

$$\Delta \sigma_{cr} = \frac{E \beta^2}{4 \pi^2}$$  \hspace{1cm} [24]

where $\beta$ is the central angle of the shell in radians. This is evaluated for some values of $\beta$ in the following table.

<table>
<thead>
<tr>
<th>Central Angle, $\beta$ (degrees)</th>
<th>Increase in Critical Stress from Curvature, $\Delta \sigma_{cr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
<td>900</td>
</tr>
<tr>
<td>3</td>
<td>2,100</td>
</tr>
<tr>
<td>4</td>
<td>3,700</td>
</tr>
<tr>
<td>5</td>
<td>5,800</td>
</tr>
<tr>
<td>6</td>
<td>8,300</td>
</tr>
<tr>
<td>7</td>
<td>11,300</td>
</tr>
<tr>
<td>8</td>
<td>15,200</td>
</tr>
<tr>
<td>9</td>
<td>18,600</td>
</tr>
<tr>
<td>10</td>
<td>23,000</td>
</tr>
</tbody>
</table>

This increase in critical stress from curvature can be quite appreciable, especially in the case of vessels having generous curvature of lines. But the boundary conditions do not fit the specific applications we have in mind (the bottom shell plating, for example, is assumed fixed along the edges), and it is not yet possible to assess the effect of constraining moments along the boundaries upon the critical stress, especially if the curved plate is simultaneously loaded across the straight edges (generators). Such conservatism is generally acceptable when designing vessels of normal scantlings. However, in the case of vessels having light scantlings, such as destroyers, it is desirable to allow for the increase in critical stress from plate curvature on the basis of the foregoing table.

CROSS-STIFFENED PLATING

An elegant solution to the problem of cross-stiffened plating under normal loading alone has been presented by Schade in the quoted Reference 11. The presentation is in the form of extreme simplicity which the designer appreciates.
According to Schade's method, a rectangular panel of cross-stiffened plating is considered to be an orthotropic plate, i.e., a plate whose elastic properties along one axis differ from those along another axis orthogonal thereto. The axes are conveniently chosen in the direction of the stiffeners.

Two parameters are at first determined:

a. A torsion coefficient

\[ \eta = \sqrt{\frac{I_{pa}}{I_{na}} \cdot \frac{I_{pb}}{I_{nb}}} \]  \[25\]

b. A virtual side ratio

\[ \rho = \frac{a}{b} \sqrt{\frac{i_b}{i_a}} \]  \[26\]

where

- \( a \) is the length of the rectangle,
- \( b \) is the width of the rectangle, and
- \( i_a, i_b \) indicate unit stiffness in long and short directions, respectively.

Unit stiffness is simply total stiffness divided by stiffener spacing.

Also

- \( s_a, s_b \) indicate spacing of long and short stiffeners, respectively,
- \( I_{na}, I_{nb} \) indicate total stiffness (moment of inertia) including effective width of plating of all long and short stiffeners, and
- \( I_{pa}, I_{pb} \) indicate total stiffness (moment of inertia) of effective width of plating only working with long and short stiffeners.

Generally,

\[ i_a = \frac{I_{na}}{s_a}, \ldots \]

With these parameters one may obtain from Figures 5 to 13

a. The deflection at center of panel,

b. The bending stress intensities in plating and free flanges.

c. The shear stress intensities in the webs.

In entering these charts, one should be careful to distinguish between stresses in the plating (capable of torsional rigidity) and stresses in the free flanges (devoid of torsional rigidity).
Figure 5 - Deflection at Center
Figure 6 - Field Bending Stress in Plating in Long Direction

General Formula: $\sigma = K \frac{p b^2 t_a}{\sqrt{L_b}}$

Symbol @ indicates location of stress
Poissons ratio $\mu$ assumed 0.3

For unstiffened plates (Type "D"), the plate formula: $\sigma = \frac{5.46 t_p}{b}$ applies

Note: $\rho - \frac{a}{b} \sqrt{\frac{E}{t_b}}$
Figure 7 - Field Bending Stress in Free Flanges in Long Direction

Formula: \( \sigma = K \frac{p M_b}{\sqrt{a^2 b}} \)

Note: \( \rho = \frac{a}{b} \sqrt{\frac{a}{i_a}} \)

Symbol \( \bigcirc \) indicates location of stress

Poissons ratio \( \mu \) assumed 0.00
General Formula: \( \sigma = K \frac{p^{0.5} t_b}{l_b} \)

Symbol (●) indicates location of stress
Poisson ratio \( \mu \) assumed 0.3

For unstiffened plates (Type "D"), the plate formula: \( \sigma = 5.46 K_p \left( \frac{b}{l_b} \right)^{0.5} \) applies

Figure 8 - Field Bending Stress in Plating in Short Direction
Figure 9 - Field Bending Stress in Free Flanges in Short Direction

Formula: \( \sigma = K \frac{E b^4 c_s}{i_b} \)

Symbol \( \circ \) indicates location of stress
Poisson's ratio \( \mu \) assumed 0.0
Figure 10 - Support Bending Stress in Plating
Figure 11 - Support Bending Stress in Free Flanges
Figure 12 - Shear Stress in Long Webs

General Formula: $T = K \cdot \frac{p b l a}{A_o \sqrt{a b}}$

Symbol $\bigcirc$ indicates location of stress

For unstiffened plates (Type "D"), the shear load per unit width $= K p b$
Figure 13 - Shear Stress in Short Webs

General Formula: \( \tau = K \frac{p b l_1}{A b l_0} \)

Symbol \( \bigcirc \) indicates location of stress

For unstiffened plates (Type "D"), the shear load per unit width = \( Kpb \)

Note: \( \rho = \frac{a}{b} \sqrt{\frac{1}{E_i}} \)
EFFECTIVE WIDTH OF PLATING

An investigation of frequent occurrence in stiffened plating is the amount of flange material on each side of a stiffener that can be considered effective in calculations of strength. Here a distinction must be made between the problem of elasticity and that of elastic stability. The first occurs when stiffened plating is subjected to normal loading alone, the second when stiffened plating is subjected to a compressive load in the plane of the plating.

THE STABILITY PROBLEM

The behavior of stiffened plating subjected to a uniform compressive load in the plane of the plating has received considerable attention, especially because of its applications in the aeronautical field. The presentation herein is based on Bengston's work. Again we consider the two cases of the rectangular plate simply supported at all boundaries and the rectangular plate with fixed boundaries.

The Simply Supported Plate

The effective width $b_e$ is given by

$$\frac{b_e}{b} = \frac{1}{1 + \psi \left( \frac{b}{a} \right) \left( 1 - \frac{\sigma_{cr}}{\sigma_x} \right)} \quad [27]$$

where $\psi$ is a coefficient varying with aspect ratio such that

- $\psi = 1.88$ for $a/b$ approaching zero,
- $\psi = 1.07$ for $a/b = 1$, and

$$\sigma_{cr} = \left( \sigma_x + \frac{a^2}{b^2} \sigma_y \right)_{cr} = \frac{\pi^2 D \left( \frac{b}{a} + \frac{b}{a} \right)^2}{h b^2} \quad [28]$$

For convenience, values of $b_e/b$ are plotted against $\sigma_x/\sigma_{cr}$ for various values of $a/b$ in Figure 14.

The Fixed Plate

The effective width is given here by an expression similar to Equation [27]

$$\frac{b_e}{b} = \frac{1}{1 + \theta \left( \frac{b}{a} \right) \left( 1 - \frac{\sigma_{cr}}{\sigma_x} \right)} \quad [29]$$
where $\theta$ is again a coefficient varying with aspect ratio such that

\[
\theta = 0.906 \text{ for } a/b \text{ approaching zero}
\]

\[
\theta = 0.483 \text{ for } a/b = 1, \text{ and}
\]

\[
\sigma_{cr} = \left(\frac{\sigma_x + \frac{a^2}{b^2} \sigma_y}{\sigma_x}ight)_{cr} = \frac{4\pi^2D}{3h^2} \left(\frac{3}{a^2} + \frac{3}{b^2} + 2\right)
\]

Since the expressions for the fixed plate are developed from a single wave system, they are more limited in their application than the similar expressions for the simply supported plate. In general, Equations [29] and [30] will hold

a. For large variations in $\sigma_x$ and $\sigma_y$ when $a \approx b$

b. For large variations in $a/b$ when $\sigma_y = 0$

In Figure 15, values of $b_e/b$ are plotted against $\sigma_x/\sigma_{cr}$ for $a/b = 1$ and $1/2$.

---

**Figure 14 - Simply Supported Plate**

*Effective width $b_e/b$ based on uniform stress $\sigma_x$.*

**Figure 15 - Fixed Plate**

*Effective width $b_e/b$ based on uniform stress $\sigma_x$. Single wave system.*

It should be noted that the effective width of plating is determined upon considerations of elastic stability only when the unit compressive stress in the plane of the plating exceeds the critical strength thereof. This will occur in the design of transversely framed vessels, of ordinary watertight bulkheads, and of superstructure. No application will be made of the stability formulas [27] and [29] in the design work expounded herein. The formulation is given only for the sake of completeness.
THE ELASTICITY PROBLEM

This aspect has been studied by Timoshenko, Vedeler, and Boyd. The second author gives the following expression for the effective flange width of a box-shaped beam of length $l$ and width $b$ subjected to a sinusoidal bending moment (see Figure 16)

$$\frac{b_e}{b} = \frac{1 + \sinh \pi \beta}{\pi \beta} \frac{\pi \beta}{1 + \cosh \pi \beta}$$

where $\beta = \frac{b}{l}$.

This expression is based on the assumption that the bending moment curve passes through zero at the ends. When this is not the case, the distance between points of zero bending moment should be substituted for the length $l$ of the beam. This substitution introduces only a small error.

![Figure 16 - Effective Width of Flange Plating](image)

Curve A

Curve B

Definition of Length $l$ for Constrained and Simple Supports

Curve $A - \beta = \frac{b}{l}$, $b$ = spacing of stiffeners, and $l$ = virtual span, see diagram.

If the bending moment curve departs somewhat from the sinusoidal, the effective width varies roughly in proportion to the areas under the bending moment curves.

This calculation for effective width is valid for the tension side and also for the compression side when the maximum stress is below the critical. The case in which the critical...
compressive stress is exceeded has been treated by Schnadel, see Vedeler. This case, however, will not be considered here, since it does not occur in longitudinally framed vessels.

It may be of interest to note that problems of shear lag in decks can also be solved readily by reference to Figure 16. Since here $B = B/L$ ($B =$ beam, $L =$ length of ship) and is around 0.1, the difference in stress between the average and the extremes (maximum at deck edge, minimum at center of deck) is approximated by

$$\Delta \sigma = \left( \frac{b - b'_{av}}{b} \right) \sigma_{\text{average}}$$

and unless $\sigma > \sigma_{cr}$, this difference is negligible. Accordingly, the strength decks of longitudinally framed vessels should be made of uniform thickness across the section, for in this case the maximum compressive stress intensity is always less than the critical. The stringer plate need not be increased in thickness. On the other hand, in transversely framed vessels where the maximum unit compressive stress exceeds the critical, the deck edge stress (at gunwale) can be considerably larger than the average for the whole deck. A thicker stringer plate is consequently required.

ADEQUACY OF STIFFENERS

When stiffeners are used to increase the critical compressive strength of the plating to which they are attached, it is necessary to determine the number and size of stiffeners to be used. The theory required to answer these queries has been discussed by Timoshenko.

Stiffening should be applied only in the direction of the compressive force. Stiffening applied transversely thereto may prove to be inadequate regardless of size. This latter arrangement of stiffeners will not be considered.

If we represent the critical strength of the unstiffened plating by $\sigma_{cr}$ then, to insure that after stiffening the critical strength $\sigma'_{cr}$ will exceed the limiting stress $\sigma_m$, i.e.,

$$\sigma'_{cr} > \sigma_m$$

the number of stiffeners $n$ in the direction of compression is given by

$$n > \sqrt{\frac{\sigma_m}{\sigma_{cr}}} - 1$$

and the critical strength of the stiffened plating by

$$\sigma'_{cr} = \frac{\pi^2 D}{b^2 h} \frac{(1 + \beta^2)^2 + 2 \Sigma_i \gamma_i \sin^2 \frac{\pi c_i}{b}}{\beta^2 \left(1 + 2 \Sigma_i \delta_i \sin^2 \frac{\pi c_i}{b} \right)}$$

where $b$ is the width of plating,

$\beta$ is equal to $a/b$,

$D$ is the flexural rigidity of the plating,

$\gamma$ equals $B/bD$ and is an inertia factor,
\( \delta \) equals \( A/bh \) and is an area factor, 
\( c \) is the spacing of a stiffener from the edge of the plate, and 
\( i \) is a subscript denoting that the value of the quantity applies to the specific stiffener \( i \).

Also \( a \) is the length of plating in direction of compression, 
\( l \) is the moment of inertia of the stiffener about an axis in the plane of the plating, 
\( B \) is equal to \( EI \), 
\( V \) equals \( B/bD \) and is an inertia factor, and 
\( A \) is the cross-sectional area of the stiffener.

The size of the stiffener should be so chosen as to make Equation [33] hold.

The stiffeners are not only required to be adequate in number (Equation [34]) and to have sufficient area and inertia (Equations [33] and [35]), but they must be otherwise proportioned so as to have adequate elastic stability of themselves. The fulfillment of this requirement is discussed by Windenburg\(^1\) and is presented in a form suitable for design work in Reference 20, from which Figure 17 is reproduced. The safe length within which a stiffener of given proportions will not buckle is given by the flange width multiplied by a factor \( K \). This factor depends primarily on \( \sigma_{yp} \) \( \text{or} \) \( \tau_{yp} \), and the ratio of flange width to depth.

**COMBINED STRESSES - INTERACTION**

So far we have dealt with only a single condition of loading. But it is natural to inquire what happens when several conditions of loading are superposed. This case is analyzed most readily by means of the method of stress ratios described in References 21 and 22. According to this method it is only necessary to satisfy a relationship of the form:

\[
R_k^k + R_c^l + R_b^m + R_s^n = \frac{1}{f}
\]  

[36]

where \( R_k, R_c, R_b, R_s \) are stress ratios, each referring to a separate condition of simple loading in tension, compression, bending and shear given by

\[
R = \frac{\text{intensity of applied stress (} \sigma_{act} \text{ or } \tau_{act} \text{)}}{\text{yield or critical stress intensity (} \sigma_{yp} \text{ or } \tau_{yp} \text{) for simple loading}}
\]  

[37]

\( k, l, m, n \) are empirically derived exponents, and \( f \) is a factor of safety.

When a unit critical stress \( \sigma_{cr} \text{ or } \tau_{cr} \) is less than the corresponding yield strength of the material \( \sigma_{yp} \text{ or } \tau_{yp} \), it should replace such in the expression. The following table of relations applicable to a rectangular plate will prove useful:
1. Plate under compression in both directions
\[ R_c + R_c = \frac{1}{f} \]  
[38]

2. Plate under combined shear and compression - Plate infinitely long, stressed in compression across short sides
\[ R_s^{1.5} + R_c = \frac{1}{f} \]  
[39]

3. Simply supported plate under combined bending and compression
\[ R_s^{1.75} + R_c = \frac{1}{f} \]  
[40]

4. Simply supported plate under combined bending and shear
\[ R_s^2 + R_s^2 = \frac{1}{f} \]  
[41]

![Figure 17 - Proportions of Stiffeners to Prevent Tripping](image)

Length of stiffener beyond which stiffener must be supported against tripping is \( s = kw \). From BuShips pamphlet 017969 "Design Data for Tee Stiffeners - Proportions for Lateral Stability and Requirements for Lateral Support to Prevent Tripping."
DISCONTINUITIES - STRESS CONCENTRATION

There are two types of discontinuities—those arising from design features and therefore necessary for good reasons, and those arising from workmanship and hence to some extent avoidable. We will not concern ourselves with the latter. Since design analysis cannot readily account for the vagaries of workmanship, these are always ignored and the factor of safety is made to account for them in some mysterious fashion. The effects of discontinuities arising from workmanship are lessened through careful inspection in accordance with reasonable standards. But it is well to remember that the most carefully refined design is jeopardized by poor standards and laxity in inspection.

Design discontinuities arise through

a. The tapering in thickness of plating,
b. The introduction of openings, and
c. The arrangement of the structure.

A typical example of the last is the sudden termination of longitudinal bulkheads and inner bottom at transverse bulkheads and decks. This type of discontinuity can have extremely serious consequences, yet it takes so many varied forms that specific rules for each case cannot well be laid down in advance. Although, in the final analysis, design of discontinuities is detail design, its importance can never be sufficiently emphasized. For when a serious failure has occurred and upon investigation the possibilities of poor material and poor workmanship have been eliminated, it is rarely that one can trace the failure to anything else but a matter of poor detail design. Here alertness and attention will pay off. A few simple rules, while far from complete, may be of help.

a. Endeavor to design the main structure of the ship to avoid imposing any concentrated loadings thereon.

b. If (a) is not feasible, endeavor to arrange the material so that under such heavy concentrated loadings it will be stressed axially (pure tension or compression) or in shear rather than in bending.

c. Insure that the structure changes section slowly in the direction of stress.

d. If the previous requirement cannot be satisfied, then crack-stoppers (usually riveted seams) should be provided to arrest the possible development of a failure.

Discontinuities arising from the tapering in thickness of plating are more simply dealt with. It is usual to specify that in the longitudinal direction the tapering in thickness between adjacent plates shall not exceed 10 lb or one-third of the thickness. The effects are not as serious transversely because of the reduced magnitude of the stresses in this direction, and although a similar specification can be laid down, in practice this has not yet been done.
The effects of openings in structural members are:

a. To raise the average stress level at the section where they occur and for some distance either side in the direction of stress.

b. To introduce extremely high stress gradients (concentrations) in the immediate vicinity of the opening.

The only way at present available to lessen these highly localized stress concentrations is by good design of the openings and the provision of suitable reinforcement. Yet this method has its limitations, for even the optimum reinforcement will not reduce the stress concentrations to a value less than 1.7 times the average stress at the section. Consequently, it is to be expected that the yield point of the material will be exceeded in certain small localized areas. The significance of this in design is yet to be fully evaluated.

One way to fulfill the requirement of minimum stress concentration around any single opening is by the use of circular or elliptical openings; or, where this is not feasible, by the use of rectangular openings having the least possible width (measured across the direction of the greatest stress). The corner radii of these openings should be as large as practicable, but in no case less than $\frac{1}{8}$ the width of the opening. Another way is the installation of a reinforcing bar along the periphery of the opening, the dimensions of this reinforcing bar to be in accordance with Figures 18 and 19.

Based on experiments by Bruhn\textsuperscript{23}, the structure rendered ineffective by an opening is assumed to extend triangularly a distance of $2w$ (where $w$ is the width of opening) both fore and aft in the direction of stress, see Figure 20. The plating in this ineffective area is assumed to be unstressed by forces acting in the plane of the plating and imposed at a distance from the opening.

The maximum shearing stress developed at the corners of the uptake openings and hatches can be estimated by the method expounded in Reference 1, p. 59, \textit{et seq.} according to which this terminal shearing stress intensity is given (in our notation) by

\[
\tau_{\text{max}} = \frac{\sigma_1 \tanh ml}{2m \mu E}
\]

where $\sigma_1$ is the average primary stress intensity in way of opening,

$l$ is the length of opening, and

$\mu$ is the displacement coefficient assumed at $0.5 \times 10^{-6}$ for MS and HTS

Also $A$ is the sectional area—on one side—of continuous plating,

$a$ is the sectional area—on one side—of discontinuous plating,

$\alpha$ is the thickness at connection, and

\[
\sqrt{\frac{\alpha(A+a)}{aA \mu E}} = m
\]

is the coefficient independent of load.
Example: 15 in. x 21 in. opening in 1/2 in. web plate - Use 2 1/2 x 1/2 in. flat bar.
24 in. clear circular opening in 7/16 in. web plate - Use 3 in. x 1/2 in. flat bar.

When Point Falls Above Diagonal, Thickness of Ring is Equal to Web Thickness

Figure 18 - Chart for Selection of Flat Bar Reinforcing Rings for Transverse Framing
Figure 19 - Chart for Selection of Flat Bar Reinforcing for Openings in Shell, Inner Bottom, Decks and Longitudinal Framing

When Point Falls Above Diagonal, Thickness of Ring is Equal to Web Thickness
TRANSVERSE FRAMING

The two current methods for the calculation of transverse framing are the method of moment distribution developed by Cross\textsuperscript{24} and the method of strain energy, whose application to ship's structures was first expounded by Bruhn\textsuperscript{25,26} and later by Hovgaard.\textsuperscript{1} In this report the discussion of these methods will be limited to some comments on their application to ship design, and the reader is invited to consult the references for a fuller understanding.

THE METHOD OF MOMENT DISTRIBUTION

This method, based on the theory of the elastic curve, lends itself readily to the solution of problems of continuous structures involving prismatic members. It is rapidly carried out and self-checking, and is, therefore, particularly convenient in the design of transverse and longitudinal deck framing.
THE METHOD OF STRAIN ENERGY

When framing is no longer prismatic, as, say, the transverse framing of slender ships, this method is the only reasonable alternative. In analysis we find that this method is used in two specific forms termed the method of least work and the method of column analogy. These two forms are developments of the same theory and differ only slightly. Yet in cases involving several redundants, the column analogy is to be preferred since, contrary to the least work method, it does not involve the clumsy solution of simultaneous algebraic equations.

ASSUMPTIONS

Of greater importance than the method to be used are the assumptions that must be made regarding the boundary conditions. It must be remembered that transverse framing is secondary structure. For analysis purposes, all deformations in secondary structure are referred to some primary structure which is assumed fixed in space and incapable of any deformation, and which therefore provides a fixed reference point. This repetition is not idle. The restatement is believed necessary in view of the many erroneous procedures developed in the past for analyzing the transverse framing. It is in this respect that the methods expounded in References 1 and 25 are incorrect; both assume that the frame ring is cut adrift from the longitudinal primary structure (Bile's hypothesis). But horizontal discontinuous motion of decks and vertical discontinuous motion of shell and bulkheads are allowed by this assumption. This is utterly unrealistic.

The assumption that for this analysis primary structure remains fixed in space is not only more consistent with the facts but permits an enormous simplification in the work. One need no longer consider a complete transverse ring of framing. The structure can be broken down into separate entities between consecutive intersections of primary members. Thus, in the example chosen for illustration (see Figure 23), the complex transverse ring is reduced to six simpler separate structures:

a. The arch of bottom structure below the second deck,
b. The second deck transverse.

Figure 22 - Nonwatertight Floor Detail
c. The main deck transverse
d. The web frame between second and main decks.
e. The stanchion between main and second decks.
f. The stanchion between second deck and keel.

When structure is so reduced, one must be careful, however, to apply the correct boundary conditions and not to overlook any force or moment.

In effecting a simplification as discussed in the preceding paragraph, two simple rules may be of assistance. The first is stated as follows: When there is a great disparity in the rigidity between adjacent spans of a continuous stiffener, the more flexible member may be assumed fixed and the more rigid member hinged at the common point of support. Each member can then be considered separately.

![Diagram of Midship Section](image)

Figure 23 - Geometry of Midship Section
The second rule is somewhat similar: When there is a great disparity in the rigidity of two beams connected by a stanchion, the more flexible beam may be assumed fully supported, the more rigid unaffected by the presence of the stanchion. Both of these rules will be illustrated later.

A final comment on transverse framing. The refinement of the assumptions used should depend on the magnitude of the stresses to be determined. Where transverse bending stresses are relatively low, as in bottom structure, relatively crude assumptions are justifiable.

**EMPIRICAL RESULTS**

In the design of indeterminate structures, it is necessary to estimate in advance the structure to be analyzed. In doing so, it is quite often convenient to make use of empirical results. As discussed on page 3 a particular case arises in estimating the longitudinal bending moments acting on the vessel, for it is the relation of these moments to the allowable primary stresses that gives us the section moduli to which we must design the section.

As is well known, the longitudinal bending moment is estimated by the formula

\[ M = \frac{\Delta L}{c} \]  

where \( M \) is the bending moment, 
\( \Delta \) is the displacement of the ship, 
\( L \) is the length of the ship, and 
\( c \) is an empirical coefficient depending on the load acting on the ship's girder, i.e., on the weight and buoyancy distributions.

As previously stated, the weight depends essentially on the type of vessel, the buoyancy distribution on the shape of the hull and the proportions and location of the wave upon which the vessel is poised. This coefficient varies only over a narrow range and, if data are available for other similar vessels, it can be chosen with some accuracy for the design on hand.

But valid though this formula is for estimating the bending moment on the structure, it is not considered sufficiently reliable for the final design. Accordingly, upon completion of the first design estimate, it is necessary to carry out a complete longitudinal strength calculation to confirm or modify the structure developed on such an empirical basis.

**EXPERIMENTAL RESULTS**

The only reliable means for checking the accuracy of a theory of design, or the limitations thereof, is by experiment. When no theory exists, experiment is the only means by which a design may be developed.
Some structures are well beyond any but the crudest of analyses. (Indeed, only the simplest of structures can be accurately analyzed.) This applies when the material is intentionally stressed beyond the yield point or critical strength or when the geometry of the elements is such that simple stress fields no longer obtain.

A representative case is the design of nonwatertight floors. No accurate analysis can even be attempted because of the openings cut for drainage and accessibility. The choice of floor scantlings is based on experiment and on comparisons with a simple analyzable structure.

Since tests indicate that a nonwatertight floor of the type shown in Figure 22 is not inferior to the solid plate of the same thickness and that the solid plate behaves as if it were fixed at the boundaries, the dimensions of the solid plate are calculated and the nonwatertight floor is substituted.

Such a procedure is extremely simple. It may also be extremely dangerous.

FACTORS OF SAFETY

If all our theories were fully correct and if the assumptions on which they are based corresponded to reality, there would be no need for any margin of safety and the factor of safety would, consequently, equal unity. The use of a factor of safety greater than unity, therefore, reflects the degree of confidence of the designer in his work, for actually it is nothing but a discount of the assumptions he has made and of the theory he has used. Since a factor of safety is then only an expression of faith and experience, it is rather personal and arbitrary. It cannot always be argued and its value is sometimes admitted or accepted even without justification.

In this report the factors of safety are assumed to be functions only of the condition of stress existing in the structure to which applied. They are based either on the yield strength of the material or the critical strength of the plating or stiffener, whichever is lower.

Two conditions of stress are considered. The first obtains when there is essentially no stress gradient across the full depth of a plate or shape and results when only primary and secondary stresses are combined. The second obtains when such a stress gradient exists and is large and results when primary, secondary, and tertiary stresses are combined.

Corresponding to these two conditions, the factors of safety used in this report for the purpose of illustrating the design procedure to be followed are:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Factor of Safety</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combined primary and secondary stresses</td>
<td>1.25</td>
</tr>
<tr>
<td>Combined primary, secondary, and tertiary stresses</td>
<td>1.00</td>
</tr>
</tbody>
</table>
The reduced factor of safety proposed for the case of the full combination of stresses (primary plus secondary plus tertiary) is justified on the basis of the high localization of stress that obtains in this condition.

**DESIGN CRITERIA**

Design experience is reflected in the criteria which the naval architect sets down and follows in his design. Since a design will be carried out for the purpose of illustrating the points already discussed, it is convenient to state them in advance. Only such design criteria as affect the design of the midship section will be given here.

**CONDITIONS OF LOADING**

The standard loading condition for the calculation of both hogging and sagging shall be the full load condition.

A special condition for both hogging and sagging shall be calculated based upon the full load condition but so modified as to give the maximum bending moment. For sagging, this condition shall correspond to full load with all tankage and stores beyond the quarterpoints removed.

**CONDITIONS OF BUOYANCY**

The vessel shall be assumed poised in static equilibrium on a trochoidal wave whose length is equal to that of the ship and whose height is given by Curve C, Figure 2.

**HYDROSTATIC LOADS**

**Main Deck**

The main deck outside the deckhouse structure shall be designed to carry a head of 4 ft of water.

**Bottom Structure**

The maximum hydrostatic head to which the bottom structure shall be designed is that occurring at the passage of the crest of the wave. This is approximated by the following formula

\[ H_m = H_f + 0.4h \]  \[44\]

where \( H_m \) is the maximum hydrostatic head in ft above the baseline in way of a wave crest, \( H_f \) is the full load draft in ft, and \( h \) is the wave height (crest to trough) in ft.
Shell Plating and Framing

When a greater hydrostatic head than given in the preceding paragraph is obtained at any point of the shell plating and framing by rolling the vessel 30 deg. in still water, this greater head shall be used in the calculation. Shell plating and framing shall be adequate to withstand a hydrostatic head to the margin line in still water.

Decks (Other than Main) and Transverse Bulkheads

The design hydrostatic head amidships for decks (other than main) and for the main transverse bulkheads shall be the bulkhead deck.

Platforms, Flats, and Longitudinal Bulkheads

Platforms, flats, and longitudinal bulkheads forming boundaries of vital spaces shall, in general, be designed to the hydrostatic heads given in the preceding paragraphs.

Tank Boundaries

All tank boundaries shall be designed for a head of the densest liquid they are to contain, to the top of the overflow pipe or to the highest point the liquid will rise in service, whichever is greater.

Live Loads

Decks, platforms, and flats shall be designed to the hydrostatic loading given in the preceding paragraphs or to the following schedule of live loads, whichever is greater.

a. Living spaces, offices and passages on the main deck and above - 75 psf.
b. Stowage spaces on main deck and above - 200 psf.
c. Living spaces below main deck - 100 psf.
d. Offices and control spaces below main deck - 150 psf.
e. Shop spaces - 200 psf.
f. Storerooms - 300 psf., etc.

ALLOWABLE STRESSES

Primary Stresses

The scantlings of the vessel shall be such as to give maximum primary stresses in both tension and compression for the vessel in upright condition as follows:
Primary stresses shall also be calculated for the vessel heeled to the angle corresponding to maximum stresses. In this case the allowable primary stress intensities given may be exceeded by 5 percent.

Secondary Stresses

The secondary stress intensities in any specific structure of the vessel subjected to continuous loading shall not exceed the following values:

<table>
<thead>
<tr>
<th>Material</th>
<th>Intensity of Secondary Stress (psi)</th>
<th>Factor of Safety on Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medium steel (σ&lt;sub&gt;YP&lt;/sub&gt; = 35,000)</td>
<td>28,000</td>
<td>1.25</td>
</tr>
<tr>
<td>High Tensile Steel (σ&lt;sub&gt;YP&lt;/sub&gt; = 45,000)</td>
<td>36,000</td>
<td>1.28</td>
</tr>
</tbody>
</table>

The foregoing values are for tensile and compressive stresses. The allowable shear stresses are 0.6 of these values.

Where primary stresses act in conjunction with secondary stresses, the algebraic sum of both unit stresses shall not exceed these values.

Tertiary Stresses

The tertiary stresses at any point shall be such that the sum total of primary, secondary, and tertiary stress intensities shall not exceed the yield point of the material, i.e., 35,000 psi for medium steel and 45,000 psi for high tensile steel.

Combined Stresses

In combining stresses, it shall be insured that only simultaneously occurring stresses are considered and that the attitude of the vessel is the same for all stresses in question.

Buckling Strength

In general the critical buckling strength of any panel of plating or stiffener shall not be less than the yield strength of the material except for structure not stressed fully up to the
allowable limit. In this case it is only necessary to insure that the critical buckling strength be 25 percent higher than the combined primary and secondary stresses, i.e., \( \sigma_{cr} = \sigma_{YP} \) or \( 1.25(\sigma_1 + \sigma_2) \) or whichever is less. This corresponds to \( \sigma_m \) in Equations [5], [6], [33], and [34].

**PART II - PROCEDURE FOR THE DESIGN DEVELOPMENT**

**PRESENTATION**

Two steps are required to carry out fully the design of the midship section, as they are for all important indeterminate structures. The first step is to enable the structure to be developed in sufficient detail so that a weight estimate may be made and the longitudinal bending moments calculated. This first development is only approximate. It is essential that it be carried out quickly so that other phases of design are not retarded. But the structure resulting from this first approximation should be so near the structure finally desired that when a careful and refined analysis of the design is made with calculated values of stresses and weights replacing the estimated ones, only minor modifications will be indicated.

The procedure for carrying out the first analysis for a naval vessel is illustrated in the pages that follow. The vessel chosen for this purpose is of no specific existing type. It is merely an idealized vessel embodying the simplest possible structure sufficient to illustrate all the points discussed.

We will concern ourselves only with the problem of strength and on this basis will obtain the minimum scantlings resulting in a well-balanced and structurally sound midship section. In actual practice, the task is not so simple; there are usually over-riding requirements. An armor deck, for example, would not only change entirely the size and disposition of its supporting structure but would materially affect the structure in the whole section.

The design of superstructure will not be discussed. If we consider for a moment the large differences between the arrangement of superstructure on a passenger liner, a cargo vessel, a cruiser, an aircraft carrier, etc., we realize that the problem is much too varied to be included as part of this report.

As is customary in naval design no allowances will be made for mill tolerances, nor will allowances be made for corrosion, with a possible exception in the case of the flat keel.

**PREREQUISITE DATA**

Before the structural design of the midship section can be undertaken successfully, certain essential information is required. This is obtained in advance from considerations other than strength and consists of:

1. The full load displacement (\( \Delta \)) - determined on the basis of the minimum size of vessel compatible with the design requirements to be fulfilled.
2. The light displacement ($\Delta_l$) - determined empirically as a proportion of the full load displacement.

3. The principal dimensions: Length ($L$), beam ($B$), draft ($H$), and depth ($D$) - determined essentially upon considerations of powering, stability, available depth of water for operation and freeboard (reserve buoyancy), respectively.

4. The outline of the midship sectional area - determined primarily from considerations of powering, protection and stability.

5. The outline of the inner bottom (where such exists) - determined from the required tankage capacity and, in turn, from the cruising radius. The depth of double bottom should not be less than 3 ft, where practicable, for accessibility in working.

6. The outline of decks and the possible location of stanchions - determined from the vessel's arrangement.

7. The spacing of the transverse machinery bulkheads - determined from the size and arrangement of the power plant.

8. The spacings of webs and floors - these are submultiples of the machinery bulkhead spacings, whenever possible, and are chosen empirically to suit the structural and machinery arrangements and to result in a minimum weight of structure.

9. The material. All structure is ordinarily of medium steel (MS). High tensile steel (HTS) is used, if available, where necessary for lightness.

10. Military requirements. The characteristics of a naval vessel embody over-riding military requirements as, e.g., protective plating, which must be satisfied. These will not be discussed here.

The prerequisite data for the illustrative design are assumed as follows:

$\Delta = 9500$ tons - full load

$\Delta_l = 4500$ tons

$L = 575$ ft

$B = 55.50$ ft

$H = 17.75$ ft at full load

$D = 35.00$ ft at centerline

Spacing of the transverse machinery bulkheads = 42.00 ft

Web spacing = 14.00 ft

Floor spacing = 7.00 ft

Material: MS or HTS

See Figure 23 for the geometric outline of the shell, inner bottom, decks and for the possible location of stanchions.

In the discussion that follows, dimensions of length and breadth are given in feet (ft) or inches (in.), thicknesses are given in inches (in.) or pounds (lb), external loadings are given in kips, and internal stresses are given in pounds per square inch (psi) unless stated otherwise.
STRUCTURAL ARRANGEMENT

Before entering into the calculations, a tentative arrangement is laid out of shell and inner bottom plating and shell and deck longitudinals and stringers in order to establish possible plate widths and spacings for stiffening structure. At this point an attempt is made to fulfill all possible practical requirements concerning the erection, assembly, and interference of structure. In our case, the arrangement in Figure 24 results based, in part, upon the following considerations:

With the exception of the flat keel, plating along the curved bottom and bilge is to be 96 in. wide because of the rapid convergence of seams as one departs fore and aft from the midship section.

![Figure 24 - Practical Considerations Affecting the Design of the Midship Section](image-url)
All other plating is to be 72 or 84 in. wide wherever possible. The flat keel is to be 72 in. wide.

Double bottom plate longitudinals are to be spaced uniformly along the shell.

Main and second deck longitudinals are to line up vertically.

Plate longitudinals are not always installed along the full girth of the double bottom. When the double bottom increases appreciably in depth above the bilge, it is often more economical to run stringers for the support of shell and inner bottom. The location of the point of transition where the change-over may be made depends on comparative studies of weight economy. It should be noted that longitudinal structure around mid-depth contributes little to primary strength.

For the first step of development, we assume that the uppermost plate longitudinal is fitted in the region where the shell becomes vertical. Studies of seam interferences indicate that for the chosen location (girth from centerline equals 420 in.) we may have 8, 9 or 10 longitudinals spaced uniformly 52.5, 46.7 and 42.0 in., respectively. From this point on, the structure will be investigated for the 46.7 in. spacing. Parallel investigations for other justifiable spacings should be carried out concurrently.

**LONGITUDINAL BENDING MOMENT**

The first approximation to the longitudinal bending moment is obtained empirically by use of Equation [43]. A tabulation is kept of the bending moment constant c for all previously designed vessels. Based upon this tabulation, c is estimated for both the hogging and sagging conditions. Calculations will be here carried out only for the full load condition. Any other condition of loading as, e.g., the special load condition, involves at this point merely the substitution of the proper value of c. Then

- Full load hogging: \( c_h = 28 \)
- Full load sagging: \( c_s = 25 \)

The corresponding maximum bending moments in foot-ton units are:

**Hogging:**

\[
M_h = \frac{(9500)(575)}{28} = 195,000
\]

**Sagging:**

\[
M_s = \frac{(9500)(575)}{25} = 218,000
\]

For the final analysis of the structure, these estimated bending moments must be confirmed by a longitudinal strength calculation.
REQUIRED MIDSHIP SECTION MODULI

The maximum allowable primary tensile and compressive stress intensities in the ship's plating are 19,000 psi for MS and 24,000 psi for HTS (see Design Criteria). The following midship section moduli, $Z$ (given in inch$^2$-feet) are therefore required:

\[
\begin{align*}
\text{MS} & \quad \text{Hogging} \quad \frac{(195,000)(2240)}{19,000} = 22,800 \\
& \quad \text{Sagging} \quad \frac{(218,000)(2240)}{19,000} = 25,700 \\
\text{HTS} & \quad \text{Hogging} \quad \frac{(195,000)(2240)}{24,000} = 18,200 \\
& \quad \text{Sagging} \quad \frac{(218,000)(2240)}{24,000} = 20,300
\end{align*}
\]

At this point, because of the permissible choice of material, we have four possible solutions. There is no way of telling in advance which combination of MS and HTS gives the most desirable arrangement. This can only be answered after trial analyses which should be carried on simultaneously for each combination. In this report we will concern ourselves with the design embodying an HTS main deck and an MS bottom. This is a disposition of material often used and for us it has the advantage of illustrating the slightly more complicated handling of materials of different yield strengths in the same design. There is no guarantee that such an arrangement will be the optimum.

We will therefore endeavor to obtain the following midship section moduli:

To deck: $Z_d = 20,300$ in$^2$ft

To keel: $Z_k = 25,700$ in$^2$ft

BOTTOM PLATING

The analysis is carried out for the starboard strake (Strake A).

Tentative Schedule of Longitudinal Stresses.

<table>
<thead>
<tr>
<th>Stress</th>
<th>Magnitude psi</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary $\sigma_1$</td>
<td>19,000</td>
<td>Design Criteria</td>
</tr>
<tr>
<td>Secondary $\sigma_2$</td>
<td>2,000</td>
<td>Empirically, to be verified</td>
</tr>
<tr>
<td>Tertiary $\sigma_3$</td>
<td>14,000</td>
<td>Balance from $\sigma_1 + \sigma_2 + \sigma_3 = \sigma_Y$</td>
</tr>
<tr>
<td></td>
<td>35,000</td>
<td>$\sigma_{YP} = \text{yield strength}$</td>
</tr>
</tbody>
</table>

CONFIDENTIAL
Tentative Schedule of Transverse Stresses. Similarly:

<table>
<thead>
<tr>
<th>Stress</th>
<th>Magnitude psi</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary - $\sigma_1$</td>
<td>None</td>
<td>Empirically, to be verified</td>
</tr>
<tr>
<td>Secondary - $\sigma_2$</td>
<td>3,000</td>
<td>Balance available</td>
</tr>
<tr>
<td>Tertiary - $\sigma_3$</td>
<td>32,000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>35,000</td>
<td>$\sigma_{YP}$</td>
</tr>
</tbody>
</table>

External Loading

Full load draft, $H_f = 17.75$ ft

Wave height (Figure 2), $h = 34.00$ ft

Equation [44] gives the height of crest of wave above base line.

$$H_m = H_f + 0.4h = 17.75 + (0.4)(34.00) = 31.45 \text{ ft}$$

or, expressed as pressure:

$$p = (0.445)H_m = (0.445)(31.45) = 14.00 \text{ psi}$$

Thickness of Bottom Plating on Basis of Bending Stresses. If the assumption is tentatively made that the primary and secondary stresses are not acting, the tertiary unit stresses are given by Equation [7].

$$\sigma_3 = 5.46 \text{ kp} \left( \frac{b}{h} \right)$$

or solving for $b/h$

$$\left( \frac{b}{h} \right) = \sqrt[5.46 \text{ kp}]{\frac{\sigma_3}{5.46 \text{ kp}}}$$

For floors spaced 84 in. and longitudinals spaced 46.7 in. we obtain from Figure 10

$k = 0.0627$ in the longitudinal direction, and

$k = 0.0892$ in the transverse direction.

Then

$$\frac{b}{h} \leq \sqrt[5.46 \text{ kp}(0.0627)(14.00)]{14,000} = 54$$
and

\[
\frac{b}{h} \leq \sqrt{\frac{32,000}{(5.46)(0.0892)(14.00)}} = 68
\]

The stresses in the longitudinal direction govern. For \( b = 46.7 \) in., \( h \geq 46.7/54 = 0.865 \) in.

**Critical Strength of Plating.** The following assumptions are made:

1. That the plate is loaded only in the longitudinal direction. (The case of the plate loaded in both longitudinal and transverse directions will be treated later.)

2. That the plating is fixed along all edges.

According to the design criteria, the bottom plating shall be designed so that its critical strength will exceed by 25 per cent the sum of the primary and secondary stresses, i.e.,

\[
\sigma_{cr} \geq 1.25 (\sigma_1 + \sigma_2) = 1.25 (21,000) = 26,250 \text{ psi}
\]

Equation [6] gives the condition to be fulfilled. For a plate fixed along its four edges and an aspect ratio \( a/b = 84/46.7 = 1.8 \), Reference 13 gives \( k = 8.1 \) (page 365) and we have

\[
\frac{b}{h} = \sqrt{\frac{8.1 \pi^2 (30 \times 10^6)}{(10.9)(26,250)}} = 91
\]

where \( 10.9 = 12(1 - \mu^2) \).

The critical strength of the plating does not, as a consequence, govern.

**Basic Thickness of Bottom Plating.** The basic thickness of the bottom plating is determined on the basis of the stresses in the longitudinal direction which give \( h \geq 0.865 \) in. Let \( h = 0.875 \) in.

**Approximate Tertiary Stresses.**

\[
\sigma_3 = 5.46 \times 10^6 \left(\frac{46.7}{0.875}\right)^2
\]

<table>
<thead>
<tr>
<th>Stress</th>
<th>Direction</th>
<th>( k ) Figs. 6,8,10</th>
<th>( \sigma_3 ) psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field Bending</td>
<td>Long (fore and aft)</td>
<td>0.0191</td>
<td>3,900</td>
</tr>
<tr>
<td>Field Bending</td>
<td>Short (athwartship)</td>
<td>0.0441</td>
<td>9,800</td>
</tr>
<tr>
<td>Support Bending</td>
<td>Long (fore and aft)</td>
<td>0.0627</td>
<td>13,700</td>
</tr>
<tr>
<td>Support Bending</td>
<td>Short (athwartship)</td>
<td>0.0892</td>
<td>19,600</td>
</tr>
</tbody>
</table>

Basic thickness of bottom plating = 0.875 in. (35.7 lb)

Basic spacing of double bottom longitudinals = 46.7 in.
Thickenees of Flat Keel. Because of the interference of docking blocks, the preservation of the flat keel is somewhat more difficult than that of the rest of the shell plating. The practice is, therefore, to allow some additional thickness as a margin against corrosion. Of course, this additional thickness should not be figured in the calculations, but it always is because of the convenience to have a uniform procedure, because different allowances are made for different classes of vessels and because its effect on the primary stresses is quite small. Allowing 0.125 in. (5.1 lb) extra thickness for corrosion, we have:

\[
\text{Thickness of flat keel} = 1.00 \text{ in. (40.8 lb)}
\]

SIDE PLATING AT NEUTRAL AXIS

The analysis is carried out for Strake E.

Vessel upright - crest of wave amidships

Tentative Schedule of Longitudinal Stresses

<table>
<thead>
<tr>
<th>Stress</th>
<th>Magnitude</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary - (\sigma_1)</td>
<td>None</td>
<td>Empirically, to be verified</td>
</tr>
<tr>
<td>Secondary - (\sigma_2)</td>
<td>2,000</td>
<td>Balance available</td>
</tr>
<tr>
<td>Tertiary - (\sigma_3)</td>
<td>33,000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>35,000 = (\sigma_{YP})</td>
<td></td>
</tr>
</tbody>
</table>

Tentative Schedule of Transverse Stresses

<table>
<thead>
<tr>
<th>Stress</th>
<th>Magnitude</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary - (\sigma_1)</td>
<td>None</td>
<td>Empirically, to be verified</td>
</tr>
<tr>
<td>Secondary - (\sigma_2)</td>
<td>3,000</td>
<td>Balance available</td>
</tr>
<tr>
<td>Tertiary - (\sigma_3)</td>
<td>32,000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>35,000 = (\sigma_{YP})</td>
<td></td>
</tr>
</tbody>
</table>

Approximate Location of Neutral Axis. If the maximum allowable primary stress intensity of 24,000 psi in the main deck and 19,000 psi in the keel are attained simultaneously, the neutral axis is located

\[
\frac{19,000}{19,000 + 24,000} = 15.45 \text{ ft above the baseline} = y_0
\]
External Loading on Plating

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of wave crest above baseline</td>
<td>31.45 ft</td>
</tr>
<tr>
<td>Neutral axis above baseline</td>
<td>15.45 ft</td>
</tr>
<tr>
<td>Hydrostatic head on plating</td>
<td>16.00 ft</td>
</tr>
</tbody>
</table>

Vessel inclined 30 deg. in still water

See diagram, Figure 25, which gives for this condition a hydrostatic head on plating equal to 15.95 ft, i.e., less severe than the previous one. Using, therefore, a head of 16.00 ft we have

\[ p = (0.445)(16.00) = 7.13 \text{ psi} \]

**Figure 25 - Estimated Hydrostatic Head on Side Plating**
Thickness of Side Plating. Solving Equation [7] for \( h \) we have

\[
h = \sqrt{\frac{5.46 k p b^2}{\alpha_3}}
\]

Because of the absence of primary stresses and the small values of the estimated secondary stresses, almost equal values of tertiary stresses result. The thickness of plating is, consequently, determined by the stresses in the short direction. Again, as for the bottom plating, \( k = 0.0892 \) and

\[
h \geq \sqrt{\frac{(5.46)(0.0892)(7.13)(46.7)^2}{82,000}} = 0.49 \text{ in.}
\]

Let \( h = 0.50 \text{ in.} \)

Critical Strength of Plating. At the neutral axis the vessel experiences very low primary and secondary stresses even in the inclined position. The condition that the critical strength of the plating shall exceed their sum is, consequently, not an important one. It is not necessary therefore to assess the critical strength.

Reduce thickness of shell plating from 0.875 in. (35.7 lb) at bilge to 0.50 in (20.4 lb) at neutral axis.

SIDE PLATING AT MAIN DECK - SHEER STRAKE

The analysis is carried out for the sheer strake. Because of the low normal loading, the plating is designed on the basis of critical strength alone.

Critical Strength of Plating. The assumptions are made that the plating is loaded only in the longitudinal direction and that the plating is simply supported along all edges. The plating is to be designed so that

\[
\sigma_{cr} = 1.25(\sigma_1 + \sigma_2)
\]

The tentative schedule of longitudinal stresses is

<table>
<thead>
<tr>
<th>Stress</th>
<th>Magnitude</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary - ( \sigma_1 )</td>
<td>24,000</td>
<td>Design Criteria</td>
</tr>
<tr>
<td>Secondary - ( \sigma_2 )</td>
<td>3,000</td>
<td>Empirically, to be verified</td>
</tr>
<tr>
<td>Tertiary - ( \sigma_3 )</td>
<td>18,000</td>
<td>Balance available</td>
</tr>
<tr>
<td></td>
<td>45,000 = ( \sigma_{YP} )</td>
<td></td>
</tr>
</tbody>
</table>
Consequently

\[ \sigma_{cr} = 1.25 \left( 24,000 + 3000 \right) = 33,750 \text{ psi} \]

Equation [6] is used with the value \( k = 4.00 \) (this is valid for all values of \( a/b > 0.7 \)). Then

\[ \frac{b}{h} \leq \sqrt{\frac{4.00 \pi^2 (30 \times 10^6)}{\left(10.9\right)(33,750)}} = 57 \]

For one shell stringer at mid-depth between second and main deck, \( b = 51 \text{ in.}, h = 0.90 \text{ in.} \)
For two shell stringers at the third points, \( b = 34 \text{ in.}, h = 0.60 \text{ in.} \) Select latter arrangement.
Let \( h = 0.625 \text{ in.} \)

Discussion. The assumption of loading in only the longitudinal direction needs be defended somewhat. When the transverse stress component is small, such an assumption is justifiable on the basis of simplicity. When it is tensile, such an assumption is conservative. But when the transverse stress component is compressive and large, such an assumption may lead to serious error. There is some mitigation, however. The plate is never simply supported at the boundaries. Actually some fixity always exists. This fixity has the effect of increasing the value of \( k \) and, consequently, the allowable width-thickness ratios. In the first step, these counterbalancing effects may not need to be evaluated. In the final synthesis they should not be neglected.

Sheer strake to be 0.625 in. (25.5 lb)

Plating to be tapered from 0.625 in. (25.5 lb) at sheer strake to 0.50 in. (20.4 lb) at neutral axis.

MAIN DECK PLATING

Critical Strength of Plating. The same assumptions are made as for the sheer strake. The same condition, consequently, holds, namely

\[ \frac{b}{h} \leq 57 \]

For \( b = 36 \text{ in.}, h = 0.633 \text{ in.} \) Let \( h = 0.625 \text{ in.} \)

But here, in contrast to the sheer strake, we may have a relatively high transverse stress component. The size of plating estimated on the basis of unidirectional loading may need be revised when the stresses in the transverse structure have been evaluated.
Strake A is in way of all uptake openings and cannot be stressed to its design limit. The rule illustrated in Figure 20 provides a means for estimating the permissible reduction in thickness. The shaded area in Figure 21 defines the amount of plating effective in resisting the longitudinal stresses for the vessel under consideration. The ratio of the shaded area to the full area is 0.57. By using \(0.57(1.25)(\sigma_1 + \sigma_2)\) instead of \(\sigma_{YP}\) in Equation [6], the result is

\[
\frac{b}{h} = \sqrt{\frac{4.00 \pi^2 (30 \times 10^6)}{(10.9)(0.57)(33.750)}} = 75
\]

and since \(b = 33\) in., \(h = 0.438\) in.

Because of the relatively low normal loading (hydrostatic head = 4 ft), tertiary stresses need not be considered.

Maximum Shearing Stress at Uptake Openings. It is of interest to calculate the maximum shearing stress at the corner of the uptake openings. To this end we apply Formula [42] and substitute therein the following numerical values:

\[
\begin{align*}
\sigma_1 &= 24,000 \text{ psi} \\
l &= 28 \text{ ft} = 336 \text{ in.} \\
\mu &= 0.5 \times 10^{-6} \\
A &= (0.625)(288) = 180 \text{ in.} \\
a &= (2.75)(12)(0.438) = 14.5 \text{ in.} \\
\alpha &= 0.438 \text{ in.} \\
E &= 30 \times 10^6 \text{ psi} \\
m &= \sqrt{\frac{(0.438)(180 + 14.5)}{(14.5)(180)(30 \times 10^6)(0.5 \times 10^{-6})}} = 0.0465 \\
\tau_{max} &= \frac{24,000 \tanh(0.0465)(336)}{2(0.0465)(0.5 \times 10^{-6})(30 \times 10^6)} = \frac{24,000 \tanh 7.8}{0.698} = 34,400 \text{ psi}
\end{align*}
\]

This unit stress is approximately equal to the yield point in shear of the material (HTS). It is considered acceptable because it is localized and because of the beneficial effect of the fillets at the corners of the openings whereby the actual value of this maximum stress is lower than the calculated one.

Strake A to be 0.438 in. (17.85 lb.), remaining strakes to be 0.825 in. (25.5 lb.)
INNER BOTTOM PLATING AT CENTER - RIDER PLATE

Critical Strength of Plating. The inner bottom being internal structure, the requirement to be satisfied is, according to the design criteria:

\[ \sigma_{cr} = 1.25(\sigma_1 + \sigma_2) \]

The maximum allowable primary stress intensity of 19,000 psi in the bottom shell reduces to

\[ \left(\frac{12.36}{15.45}\right) 19,000 = 15,200 \text{ psi} \]

at the rider plate, and if we allow the same secondary stresses as for the shell, we obtain, on the assumption that the plating is loaded only in the longitudinal direction (transverse component is tensile):

\[ \sigma_{cr} = 1.25(15,200 + 2,000) = 21,500 \text{ psi} \]

Here the boundaries must be assumed simply supported for the normal pressure may not be acting. Using then the value \( k = 4.00 \) in Equation [6] and replacing \( \sigma_{YP} \) by \( \sigma_{cr} \)

\[ \frac{b}{h} \leq \sqrt{\frac{4.00 \pi^2 (30 \times 10)}{(10.9)(21,500)}} = 71 \]

Normal Loading. The inner bottom is to be designed for a test head to the second deck (26.75 ft above the baseline). This gives

\[ p = (0.445)(26.75 - 3.0) = 10.6 \text{ psi} \]

Tertiary Stresses. Based on the primary stress given above and on the same secondary stresses as for the shell, the following allowable tertiary stresses obtain:

Longitudinally - \( \sigma_3 = 17,800 \text{ psi} \)

Transversely - \( \sigma_3 = 32,000 \text{ psi} \)

and the corresponding width-thickness ratios are \( (a/b = 1.9) \):

Longitudinally

\[ \frac{b}{h} \leq \sqrt{\frac{17,700}{(5.48)(0.0627)(10.6)}} = 70 \]

Transversely

\[ \frac{b}{h} \leq \sqrt{\frac{32,000}{(5.48)(0.0903)(10.2)}} = 80 \]
The most severe requirement for allowable stresses is, thus, approximately the same as that for buckling. For \( b = 44 \) in., this gives \( h = 0.63 \) in. Let \( h = 0.625 \) in.

Rider plate to be 0.625 in. (25.5 lb)

INNER BOTTOM SIDE PLATING

Investigate the plating between the sixth and seventh longitudinals. The center of this panel of plating is 9.0 ft above the baseline resulting in an external loading of

\[
p = (0.445)(26.75 - 9.0) = 7.90 \text{ psi}
\]

and a primary stress intensity of

\[
\left( \frac{6.36}{15.45} \right) 19,000 = 7,800 \text{ psi}
\]

Carrying out the same procedure as for the rider plate we obtain the following width-thickness ratios \((a/b = 2.2):\)

To insure critical strength -

\[
\frac{b}{h} \leq \sqrt{\frac{4.00 \pi^2 (30 \times 10)}{(10.9)(9,800)}} = 105
\]

To insure allowable stresses longitudinally -

\[
\frac{b}{h} \leq \sqrt{\frac{25,100}{(5.46)(0.0627)(7.90)}} = 96
\]

To insure allowable stresses transversely -

\[
\frac{b}{h} \leq \sqrt{\frac{32,000}{(5.46)(0.0916)(7.90)}} = 90
\]

The last requirement governs and for \( b = 38 \) in., \( h = 0.43 \) in. Let \( h = 0.438 \) in.

Inner bottom plating to be tapered in thickness from 0.625 in. (25.5 lb) at the rider plate to 0.438 in. (17.85 lb) in the region between longitudinals 6 and 7.
SECOND DECK PLATING

The procedure for designing the second deck plating is the same as that for the inner bottom.

For simplicity, the normal loading is assumed to be 200 psf, uniformly distributed over the whole deck, based on the design criteria and arrangement of spaces, giving \( p = 1.39 \) psi.

In the longitudinal direction, the primary stress is given by proportion based on the maximum allowable primary stress in the main deck plating:

\[
\sigma_1 = \left( \frac{26.75 - 15.45}{35.00 - 15.45} \right) 24,000 = 13,900 \text{ psi}
\]

The secondary stresses are appreciable, both longitudinally and transversely because here we have the more flexible single plate and stiffener combination instead of the more rigid double plate and stiffener as obtains for the double bottom. From previous examples we may tentatively allow \( a_2 = 0.25 a_{yp} \), subject to verification later.

The criterion is then

\[
\sigma_{cr} = 1.25 (\sigma_1 + \sigma_2) = 1.25 (13,900 + 8,800) = 28,400 \text{ psi}
\]

and the width-thickness ratio becomes on the assumption of simply supported boundaries

\[
\frac{b}{h} \leq \sqrt{\frac{4.00 \pi^2 (30 \times 10)}{(10.9)(28,400)}} = 62
\]

In determining the width-thickness ratios required to insure allowable total stresses we have for large \( a/b \):

Longitudinally - allowable unit tertiary stress \( \sigma_3 = 35,000 - 13,900 - 8,800 = 12,300 \) psi and, corresponding thereto,

\[
\frac{b}{h} \leq \sqrt{\frac{12,300}{(5.46)(0.0627)(1.39)}} = 160
\]

Transversely - \( \sigma_3 = 35,000 - 8,800 = 26,200 \) psi, and

\[
\frac{b}{h} \leq \sqrt{\frac{26,200}{(5.46)(0.0916)(1.39)}} = 194
\]

The first requirement governs. For \( b = 36 \text{ in.}, h = 0.58 \text{ in.} \) Try \( h = 0.563 \text{ in.} \).

Second deck plating to be 0.563 in. (23 lb)
NONWATERTIGHT DOUBLE-BOTTOM LONGITUDINALS

These will be designed according to the criterion

\[ \sigma_{cr} = 1.25(\sigma_1 + \sigma_2) \]

But since \( \sigma_2 \) is small and its average value for the depth of the longitudinal is zero, this equation can be reduced to the simpler

\[ \sigma_{cr} = 1.25 \sigma_1 \]

The maximum unit primary stress to which any longitudinal is subject is, by proportion

\[ \sigma_1 = 19,000 \cdot \left( \frac{y}{15.45} \right) \]

where \( y \) is the distance from the neutral axis to the center of depth of the longitudinal.

Consider Longitudinal 1:

\[ y = 13.86 \text{ ft}, \quad \sigma_1 = 17,100 \text{ psi} \]

\[ \sigma_{cr} = 1.25(17,100) = 21,400 \text{ psi} \]

On the assumption that the horizontal (longitudinal) edges are fixed and the vertical edges simply supported, \( k = 7.00 \) (since \( a/b > 0.7 \) where \( b \) is the loaded side). Using Equation [6] with \( \sigma_{cr} \) replacing \( \sigma_{YP} \)

\[ \frac{b}{h} \leq \sqrt{\frac{7.00 \pi^2 (30 \times 10)}{(10.9)(21,400)}} = 94 \]

For \( b = 36 \text{ in.} \), \( h = 0.383 \text{ in.} \)

Here the assumption of fixity along the horizontal edges is somewhat nearer the truth because of the relatively heavy shell and inner bottom plating to which the longitudinal is attached. However, unlike the usual assumption of simple supports, this assumption is not a conservative one and since the required thickness is slightly in excess of the available 0.375 in., it is, perhaps, best not to adopt this reduction, but to go all the way to the next heavier plate, for which \( h = 0.438 \text{ in.} \).

The remaining longitundinals are similarly figured with the following results:

| Longitudinal | \( h \) (in.) | \( \text{Calculated} \) | \( \text{Allowed} \) |
|--------------|---------------|------------------------|
| 1            | 0.38          | 0.438                  |
| 2            | 0.38          | 0.438                  |
| 3            | 0.37          | 0.438                  |
| 4            | 0.33          | 0.375                  |
| 5            | 0.30          | 0.313                  |
| 6            | 0.29          | 0.313                  |
| 7            | 0.07          | 0.250                  |
The bottom longitudinals 1 to 3 are given the same scantlings to allow for increased stresses when rolling. The remaining longitudinals are tapered in thickness. The minimum thickness of 0.25 in. is maintained.

### Thicknesses of Nonwatertight Double-Bottom Longitudinals

<table>
<thead>
<tr>
<th>Longitudinal</th>
<th>in.</th>
<th>lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 to 3</td>
<td>0.438</td>
<td>17.85</td>
</tr>
<tr>
<td>5</td>
<td>0.375</td>
<td>15.3</td>
</tr>
<tr>
<td>6, 7</td>
<td>0.313</td>
<td>12.75</td>
</tr>
<tr>
<td>9</td>
<td>0.25</td>
<td>10.2</td>
</tr>
</tbody>
</table>

### Watertight Double-Bottom Longitudinals

The center vertical keel, Longitudinals 4 and 8 are watertight and are to be designed for a hydrostatic head to the second deck (26.75 ft above baseline).

Consider the vertical keel.

### Schedule of Stresses in the Longitudinal Direction

<table>
<thead>
<tr>
<th>Stress</th>
<th>Magnitude (psi)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Primary Stress - $\sigma_1$</td>
<td>17,200</td>
<td>By proportion</td>
</tr>
<tr>
<td>Average Secondary Stress - $\sigma_2$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Allowable for Tertiary Stress - $\sigma_3$</td>
<td>17,800 = 35,000</td>
<td>Balance</td>
</tr>
</tbody>
</table>

### Schedule of Stresses in the Vertical Direction

Both the average primary and secondary stresses are zero. The allowable unit tertiary stress is, thus, 35,000 psi.

### Required Thickness

The same boundary conditions are assumed as for the nonwatertight longitudinals.

Entering Figures 6, 8 and 10, with an aspect ratio $a/b = 84/36 = 2.33$ we find

<table>
<thead>
<tr>
<th>Stress</th>
<th>Direction</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field Bending</td>
<td>Long (fore and aft)</td>
<td>0.0138</td>
</tr>
<tr>
<td>Field Bending</td>
<td>Short (vertical)</td>
<td>0.0460</td>
</tr>
<tr>
<td>Support Bending</td>
<td>Long (fore and aft)</td>
<td>0</td>
</tr>
<tr>
<td>Support Bending</td>
<td>Short (vertical)</td>
<td>0.0924</td>
</tr>
</tbody>
</table>
From the ratio of $k$ values and allowable tertiary stresses, it is evident that the support bending stresses in the short direction will be determining. Then, for a normal loading to the half depth of keel

$$p = (0.445)(28.75 - 1.5) = 11.2 \text{ psi}$$

we have, for $b = 36 \text{ in.}$ and solving Equation [7] for $h$

$$h = \sqrt[3]{\frac{(5.46)kp}{\sigma_3}} = \sqrt[3]{\frac{(5.46)(0.0924)(11.2)(36)}{35,000}} = 0.46 \text{ in.}$$

Let $h = 0.50 \text{ in.}$ Similarly:

<table>
<thead>
<tr>
<th>Longitudinal</th>
<th>$h$ (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculated</td>
</tr>
<tr>
<td>4</td>
<td>0.42</td>
</tr>
<tr>
<td>8</td>
<td>0.37</td>
</tr>
</tbody>
</table>

Thicknesses of Watertight Double-Bottom Longitudinals

<table>
<thead>
<tr>
<th></th>
<th>in.</th>
<th>lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>CVK</td>
<td>0.50</td>
<td>20.4</td>
</tr>
<tr>
<td>4</td>
<td>0.438</td>
<td>17.85</td>
</tr>
<tr>
<td>8</td>
<td>0.375</td>
<td>15.3</td>
</tr>
</tbody>
</table>

**SHELL STRINGERS BELOW SECOND DECK**

Above double-bottom Longitudinal 9, the shell will be stiffened by stringers. These will be in number and size sufficient to provide adequate rigidity to the panel of plating and to withstand the external hydrostatic head.

Number of Stiffeners Required. The critical strength of the unstiffened shell plating extending from Longitudinal 9 to second deck is given by Equation [4], on the assumption of simple supports at the boundaries, $k = 4.90$ for $a/b = 84/136$ and for $h = 0.5 \text{ in.}$ (the lowest value is used),

$$\frac{\pi^2D}{h} = 6,779,000$$

and

$$\sigma_{cr} = k \frac{\pi^2D}{b^2h} = 4.90 \frac{6,779,000}{(136)^2} = 1800 \text{ psi}$$
The required number of stiffeners is given by Equation [34] in which $\sigma_m$ is given by $1.25(\sigma_1 + \sigma_2)$. The primary stress intensity attains its maximum at the second deck level where $\sigma_1 = 13,900$ psi. Using this value of $\sigma_1$ and an estimated value of $\sigma_2 = 3000$ psi

$$\sigma_m = 1.25(13,900 + 3000) = 19,900 \text{ psi}$$

and

$$n > \sqrt{\frac{\sigma_m}{\sigma_{cr}}} - 1 = \sqrt{\frac{19,900}{1800}} - 1 = 2.24$$

Three stiffeners are therefore required.

**Critical Strength of Stiffened Plating.** The choice of adequate stiffener depends on a process of trial and error, where one tries to satisfy Equation [35] so that

$$\sigma_{cr} = \sigma_m = 19,900 \text{ psi}$$

Try a 6 in. x 4 in. x 8.5 lb I cut to T.

This stiffener has an area $A = 1.72$ sq. in. and a moment of inertia about the plating, $I = 35.3$ in. units. The following other values obtain.

$$B = EI = (30 \times 10^6)(35.3)$$

$$\beta = \frac{a}{b} = \frac{168}{136} = 1.23$$

$$D = 343,400$$

$$\gamma = \frac{B}{bD} = \frac{(30 \times 10^6)(35.3)}{(136)(343,000)} = 22.7$$

$$\delta = \frac{A}{bh} = \frac{1.72}{(136)(0.5)} = 0.0253$$

Values of $\sin^2 \frac{\pi c_i}{b}$ where $c_1 = 0.25b$, $c_2 = 0.5b$, etc. are

<table>
<thead>
<tr>
<th>$c$</th>
<th>$\sin^2 \frac{\pi c}{b}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>0.50</td>
</tr>
<tr>
<td>$c_2$</td>
<td>1.00</td>
</tr>
<tr>
<td>$c_3$</td>
<td>0.50</td>
</tr>
</tbody>
</table>

$$\sum \sin^2 \frac{\pi c}{b} = 2.00$$

$$\frac{\pi^2 D}{b^2 h} = \frac{6,780,000}{(136)^2} = 368$$

$$\beta^2 = (1.23)^2 = 1.51$$
\[(1 + \beta^2)^2 = (2.51)^2 = 6.3\]

\[
\sigma_{cr} = \frac{E}{b^2h} \left(1 + \beta^2 + 2 \sum i \sin \frac{\pi C_i}{b} \right) \frac{(1 + \beta^2) + 2 \sum i \sin \frac{\pi C_i}{b}}{\beta \left(1 + 2 \sum i \sin \frac{\pi C_i}{b} \right)}
\]

\[
= 368 \frac{6.3 + 2(22.7)(2.00)}{1.51 [1 + 2(0.0253)(2.00)]} = 21,500 \text{ psi}
\]

This is sufficient.

**Strength of Stringers.** The shell stringer must be adequate to withstand the hydrostatic head imposed either by the wave crest (31.45 ft above baseline) or by the rolling of the vessel (30 deg. in still water at full load draft). Of the two conditions the latter is the more severe in this case. The design of Stringer 10 is given.

Hydrostatic head to water level (vessel inclined 30 deg.) = 13.65 ft.

\[p = (0.445)(13.65) = 6.08 \text{ psi}\]

Span, \(l = 84 \text{ in.}\)

Effective span \(l_e = 0.577l = 48.3 \text{ in.} \) since by symmetry of loading the stringer is fixed at the supports.

Width \(b = \frac{136}{4} = 34 \text{ in.}\)

\[\beta = \frac{b}{l_e} = \frac{34}{48.3} = 0.705\]

Effective width from Figure 16 = (0.55)(34) = 18.7 in. for use in inertia calculations. Maximum bending moment (continuous beam)

\[M_F = \frac{p b l^2}{8} = \frac{(6.08)(34)(84)^2}{8} = 182,000 \text{ in-lb}\]

Allowable unit stress (primary plus secondary) = 28,000 psi

Estimated primary stress intensity (by proportion)

\[\sigma_1 = (19,000)\left(\frac{2.88}{15.45}\right) = 3500 \text{ psi}\]

Allowable \(\sigma_2 = 28,000 - 3500 = 24,500 \text{ psi}\)

Required section modulus.

\[Z = \frac{182,000}{24,500} = 7.45 \text{ in. units}\]

The 6 in. \(\times\) 4 in. \(\times\) 8.5 lb I cut to T section is inadequate. Use an 8 in. \(\times\) 4 in. \(\times\) 10 lb I cut to T.
Check for Stability of Section. Referring to Figure 17 we have

\[ \frac{W}{d} = \frac{4}{8} = 0.5 \text{ and } k = 26(MS) \]

The safe span is consequently 26(4) = 104 in. Since the actual span is 84 in., no intermediate lateral support is required.

**Shell stringers below second deck to be 8 in. x 4 in. x 10 lb I cut to T**

**SHELL STRINGERS ABOVE SECOND DECK**

Number and Size of Stiffeners Required. These are figured in the same manner as those below the second deck. The only difference is that now the dimensions \( a/b \) of the plating to be stiffened are 168 in. by 102 in. resulting in a \( k \) value of 4.00 and a critical strength of

\[ \sigma_c = 4,000 \frac{2D}{b^2h} = 4,000 \frac{10,590,000}{(102)^2} = 4100 \text{ psi} \]

for \( h = 0.625 \text{ in.} \) (25.5 lb).

The limiting stress intensity is given by

\[ \sigma_m = 1.25 (\sigma_1 + \sigma_2) \]

For an estimated \( \sigma_2 \) value in the plating of 3000 psi and the maximum allowable value of \( \sigma_1 \) of 24,000 psi for HTS, which would obtain at the top edge of the plate, \( \sigma_m \) becomes

\[ \sigma_m = 1.25 (24,000 + 3000) = 33,800 \text{ psi} \]

The required number of stiffeners is then

\[ n > \sqrt{\frac{33,800}{4100}} - 1 = 1.86 \]

or two stiffeners. The size of this, following the previous example, calculates to be 8 x 4 in. x 13 lb I cut to T with a critical strength after stiffening of 43,500 psi.

Because of the low hydrostatic head on the stringers, these need not be investigated for strength.

Check on Stability of Section. The same safe length obtains as for the stringers below the second deck, but now the actual span in 168 in. and intermediate lateral support is required at half span.

**Shell stringers above second deck to be 8 in. x 4 in. x 13 lb I cut to T**

**INNER BOTTOM STRINGERS**

Above double bottom Longitudinal 9, the inner bottom is also stiffened by stringers, and their size is determined similarly as for the stringers of the shell. The limiting stress
intensity $\sigma_m = 1.25 (\sigma_1 + \sigma_2)$ is estimated to be 13,000 psi and if the inner bottom plating is 0.312 in. (12.75 lb), four stiffeners are required. To reduce the number of inner bottom stringers to three so that they can be placed in the same plane with the shell stringers, the thickness of inner bottom plating must be increased to 0.375 in. The problem then arises whether to increase the thickness of inner bottom plating or the number of shell stringers. The latter solution is tentatively adopted. The size of stringer is retained.

Calculations of critical strength paralleling those for the shell stringers indicate that a 6 in. x 4 in. x 8.5 lb I section cut to T provides adequate stiffening.

Again, because of the low hydrostatic head acting on this part of the inner bottom, it is not necessary to investigate the strength of these stiffeners under normal loading. The check for stability indicates no necessity for intermediate lateral support.

*Inner bottom stringers to be 6 in. x 4 in. x 8.5 lb I cut to T*

**MAIN DECK LONGITUDINALS**

These will be designed, first to insure a critical strength for the stiffened deck in excess of the value of $\sigma_m = 1.25 (\sigma_1 + \sigma_2)$ obtaining at the main deck. This is the same as that obtained for the shell stringers above the second deck and equals 33,800 psi. Second, the main deck longitudinals will be designed to carry the hydrostatic loading on the main deck.

Critical Strength of Stiffened Plating. The procedure is in general similar to that used in the design of the shell stringers, although here the spacing of stiffeners is given and it only remains to calculate their size. There is, however, one important departure. The width of plating ($b$) to be used in the calculations is not the full deck width (666 in.). Because of the very small aspect ratio $(a/b = 168/666 = 1/4)$, the buckling pattern of the plating in the transverse direction is not given by a single wave but by four waves (2 crests and 2 hollows). The virtual width is consequently $666/4 = 166.5$ in. For convenience in calculation a virtual width of 180 in. may be assumed without significant error since this is exactly five times the longitudinal spacing.

A 10 in. x 4 in. x 15 lb I section cut to T is adequate. For this section the following values obtain, all in inch units:

\[
A = 3.32, \quad I = 176, \quad \beta = \frac{a}{b} = \frac{180}{168} = 1.07
\]

\[
\gamma = 25.3, \quad \delta = 0.0254, \quad \Sigma_i \sin^2 \frac{n\sigma_i}{b} = 2.50
\]

\[
i^2D/b^2h = 470, \quad \text{and the critical strength of the stiffened plating becomes:}
\]

\[
\sigma_{cr} = 470 \cdot \frac{4.63 + 2(25.3)(2.50)}{1.15 \cdot 1 + 2(0.0254)(2.50)} = 47,500 \text{ psi}
\]
Strength for Normal Hydrostatic Loading.

The hydrostatic head = 4.0 ft

\[ p = (0.445)(4) = 1.78 \text{ psi} \]

Weight of plating (30.6 lb) = 0.21 psi

Weight of stiffener = 0.03 psi

Total normal load = 2.02 psi

From design criteria

\[ \text{Allowable } (\sigma_1 + \sigma_2) = 36,000 \text{ psi} \]

\[ \text{Allowable } \sigma_1 = 24,000 \text{ psi} \]

Therefore allowable \( \sigma_2 \) = 12,000 psi

Bending moment (continuous structure)

\[ M = \frac{pbi^2}{12} = \frac{(2.02)(36)(168)}{12} = 171,000 \text{ in-lb} \]

Required section modulus

\[ Z_{\text{req.}} = \frac{171,000}{12,000} = 14.25 \text{ in. units} \]

To find the available section modulus, one first determines the effective width of plating associated with any stiffener. This is given in Figure 16. For \( b/l = 36/168 = 0.21 \), \( b_e/b = 0.94 \) and \( b_e = (0.94)(36) = 33.8 \text{ in.} \). The 10 in. x 4 in. x 15 lb I section cut to T in association with such an effective flange of plating has a section modulus of 18.8 in. units which again is adequate.

Check for Stability of Section. Based on Figure 17, we have

\[ \frac{W}{d} = \frac{4}{10} = 0.4 \text{ and } k = 21 \text{ (HTS)} \]

The safe span is therefore 21(4) = 84 in. Since the actual span is 168 in., an intermediate lateral support is required at half span.

Main deck longitudinals to be 10 in. x 4 in. x 15 lb I cut to T

SECOND DECK LONGITUDINALS

The design of the second deck longitudinals follows the same procedure as for the main deck longitudinals. Since the second deck is internal structure, the requirement is now that the critical strength of the stiffened plating.
\[ \sigma_{cr} = 1.25(\sigma_1 + \sigma_2) \]

which, by referring to the design of the second deck plating, we find to be 28,600 psi. The 10 in. x 4 in. x 15 lb I cut to T is sufficient.

The design for normal loading gives again the same section.

The check for stability against tripping results in no requirement of intermediate lateral support.

Second deck longitudinals to be 10 in. x 4 in. x 15 lb I cut to T

MOMENT OF INERTIA OF MIDSHIP SECTION

We now proceed to find if, with the scantlings developed, we obtain the required midship section moduli, i.e.,

\[ Z_d = 20,300 \text{ in}^2 \text{ ft} \]
\[ Z_k = 25,700 \text{ in}^2 \text{ ft} \]

The calculations are carried out in Table 2 based on the arrangement of longitudinal material given in Figure 26. The section moduli derived from these calculations are

\[ Z_d = 26,254 \text{ in}^2 \text{ ft} \]
\[ Z_k = 27,415 \text{ in}^2 \text{ ft} \]

Thus the first solution is only partially satisfactory. We do obtain a section for which the allowable primary stresses are not exceeded but which does not fully benefit by the HTS in the main deck. The section as developed is quite well balanced and if the parallel solution in which MS is used for both main deck and bottom structure retains this balance, it may well be the more desirable.

The parallel solution has not been carried out. The work would have consisted essentially of a duplication of the calculations illustrated and would have served no further purpose than to provide the specific conclusion discussed in the preceding paragraph. We leave this as an enjoyable exercise to the reader and proceed to the determination of the transverse structure.

NONWATERTIGHT FLOORS

In designing the nonwatertight floors (transverse shear members), it is convenient to consider a panel of double bottom structure bounded longitudinally by the transverse machinery space bulkheads and transversely by the locus of zero bending moment. This is estimated to be at a point one-third of the girth from the centerline to the second deck, or 14.60 ft from the centerline. The disposition of the structure is given in Figure 27.
### TABLE 2

Midship Section - Moment of Inertia Calculation - First Solution

Axis assumed at 20.00 ft above axis.

<table>
<thead>
<tr>
<th>Items</th>
<th>Scantlings</th>
<th>$A$ sq in.</th>
<th>$x$ ft</th>
<th>$A_x$</th>
<th>$A_x^2$</th>
<th>$I_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Above Axis:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shell - F - 72 in x 25.5 lb</td>
<td>0.625</td>
<td>45.0</td>
<td>4.4</td>
<td>198</td>
<td>371</td>
<td>132</td>
</tr>
<tr>
<td>Shell - G - 72 in x 30.6 lb</td>
<td>.750</td>
<td>54.0</td>
<td>10.4</td>
<td>562</td>
<td>5,345</td>
<td>164</td>
</tr>
<tr>
<td>Radius Plate - 23 in x 30.6 lb</td>
<td>.750</td>
<td>17.3</td>
<td>14.1</td>
<td>244</td>
<td>3,400</td>
<td></td>
</tr>
<tr>
<td>Main Deck - A - 33 in x 17.85 lb</td>
<td>.438</td>
<td>14.5</td>
<td>15.0</td>
<td>218</td>
<td>3,270</td>
<td></td>
</tr>
<tr>
<td>Main Deck - B to E - 288 in x 30.6 lb</td>
<td>.750</td>
<td>216.0</td>
<td>14.9</td>
<td>3,218</td>
<td>47,948</td>
<td></td>
</tr>
<tr>
<td>2nd Deck - 333 in x 23.0 lb</td>
<td>.563</td>
<td>187.5</td>
<td>6.7</td>
<td>1,256</td>
<td>8,415</td>
<td></td>
</tr>
<tr>
<td>Inner Bottom - F - 101 in x 12.75 lb</td>
<td>0.312</td>
<td>31.5</td>
<td>2.3</td>
<td>72</td>
<td>166</td>
<td>189</td>
</tr>
<tr>
<td>Shell Stringer - 8 in x 4 in x 13 lb I-T</td>
<td>2.8</td>
<td>12.0</td>
<td>34</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shell Stringer - 8 in x 4 in x 13 lb I-T</td>
<td>2.8</td>
<td>9.3</td>
<td>26</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shell Stringer - 8 in x 4 in x 10 lb I-T</td>
<td>2.1</td>
<td>4.3</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shell Stringer - 8 in x 4 in x 10 lb I-T</td>
<td>2.1</td>
<td>2.1</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inner Bottom Stringer - 6 in x 4 in x 8.5 lb I-T</td>
<td>1.7</td>
<td>4.3</td>
<td>7</td>
<td></td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Inner Bottom Stringer - 6 in x 4 in x 8.5 lb I-T</td>
<td>1.7</td>
<td>2.1</td>
<td>4</td>
<td></td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Main Deck Longitudinal - 10 in x 4 in x 15 lb I-T 8 at 3.43</td>
<td>27.4</td>
<td>14.6</td>
<td>400</td>
<td></td>
<td>5,840</td>
<td></td>
</tr>
<tr>
<td>2nd Deck Longitudinal - 10 in x 4 in x 15 lb I-T 8 at 3.43</td>
<td>27.4</td>
<td>6.1</td>
<td>167</td>
<td></td>
<td>1,019</td>
<td></td>
</tr>
<tr>
<td><strong>Total Above Axis</strong></td>
<td></td>
<td></td>
<td></td>
<td>633.8</td>
<td>6,419</td>
<td>77,549</td>
</tr>
<tr>
<td><strong>Below Axis:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flat Keel - 36 in x 40.8 lb</td>
<td>1.00</td>
<td>36.0</td>
<td>20.0</td>
<td>720</td>
<td>14,400</td>
<td></td>
</tr>
<tr>
<td>Shell A - 96 in x 35.7 lb</td>
<td>0.875</td>
<td>84.0</td>
<td>19.6</td>
<td>1,646</td>
<td>32,262</td>
<td></td>
</tr>
<tr>
<td>Shell B - 96 in x 35.7 lb</td>
<td>.875</td>
<td>84.0</td>
<td>18.3</td>
<td>1,537</td>
<td>28,127</td>
<td></td>
</tr>
<tr>
<td>Shell C - 96 in x 30.6 lb</td>
<td>.75</td>
<td>72.0</td>
<td>14.7</td>
<td>1,058</td>
<td>15,553</td>
<td></td>
</tr>
<tr>
<td>Shell D - 96 in x 25.5 lb</td>
<td>.625</td>
<td>59.4</td>
<td>9.0</td>
<td>535</td>
<td>4,815</td>
<td>215</td>
</tr>
<tr>
<td>Shell E - 84 in x 20.4 lb</td>
<td>.50</td>
<td>42.0</td>
<td>2.1</td>
<td>88</td>
<td>185</td>
<td>186</td>
</tr>
</tbody>
</table>

CMZ: -4
<table>
<thead>
<tr>
<th>Section Description</th>
<th>Weight (lb)</th>
<th>Area (in²)</th>
<th>Section Modulus (in⁴)</th>
<th>Torsional Constant (in⁷)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CVK - 36 in x 20.4 lb</td>
<td>.50</td>
<td>18.0</td>
<td>18.5</td>
<td>333</td>
</tr>
<tr>
<td>Longitudinal 1 - 36 in x 17.85 lb</td>
<td>.438</td>
<td>15.8</td>
<td>18.4</td>
<td>291</td>
</tr>
<tr>
<td>Longitudinal 2 - 36 in x 17.85 lb</td>
<td>.438</td>
<td>15.8</td>
<td>18.1</td>
<td>286</td>
</tr>
<tr>
<td>Longitudinal 3 - 36 in x 17.85 lb</td>
<td>.438</td>
<td>15.8</td>
<td>17.7</td>
<td>280</td>
</tr>
<tr>
<td>Longitudinal 4 - 36 in x 17.85 lb</td>
<td>.438</td>
<td>15.8</td>
<td>16.9</td>
<td>267</td>
</tr>
<tr>
<td>Longitudinal 5 - 36 in x 15.3 lb</td>
<td>.375</td>
<td>13.5</td>
<td>15.5</td>
<td>209</td>
</tr>
<tr>
<td>Longitudinal 6 - 36 in x 12.75 lb</td>
<td>.313</td>
<td>11.3</td>
<td>13.4</td>
<td>151</td>
</tr>
<tr>
<td>Longitudinal 7 - 38 in x 12.75 lb</td>
<td>.313</td>
<td>11.9</td>
<td>10.8</td>
<td>129</td>
</tr>
<tr>
<td>Longitudinal 8 - 44 in x 15.3 lb</td>
<td>.375</td>
<td>16.5</td>
<td>7.1</td>
<td>117</td>
</tr>
<tr>
<td>Longitudinal 9 - 48 in x 10.2 lb</td>
<td>.250</td>
<td>12.0</td>
<td>4.5</td>
<td>54</td>
</tr>
<tr>
<td>Rider Plate - 30 in x 25.5 lb</td>
<td>.625</td>
<td>18.7</td>
<td>17.0</td>
<td>318</td>
</tr>
<tr>
<td>Inner Bottom A - 72 in x 25.5 lb</td>
<td>.625</td>
<td>45.0</td>
<td>16.7</td>
<td>752</td>
</tr>
<tr>
<td>Inner Bottom B - 72 in x 23.0 lb</td>
<td>.563</td>
<td>40.5</td>
<td>16.0</td>
<td>648</td>
</tr>
<tr>
<td>Inner Bottom C - 62 in x 20.4 lb</td>
<td>.500</td>
<td>36.0</td>
<td>13.9</td>
<td>500</td>
</tr>
<tr>
<td>Inner Bottom D - 72 in x 17.85 lb</td>
<td>.438</td>
<td>31.5</td>
<td>10.2</td>
<td>321</td>
</tr>
<tr>
<td>Inner Bottom E - 72 in x 15.3 lb</td>
<td>0.375</td>
<td>27.0</td>
<td>4.8</td>
<td>130</td>
</tr>
<tr>
<td>Shell Stringer - 8 in x 4 in x 10 lb I-T</td>
<td>2.1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Shell Stringer - 8 in x 4 in x 10 lb I-T</td>
<td>2.1</td>
<td>2.2</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>Inner Bottom Stringer - 6 in x 4 in x 8.5 lb I-T</td>
<td>1.7</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Inner Bottom Stringer - 6 in x 4 in x 8.5 lb I-T</td>
<td>1.7</td>
<td>2.2</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td><strong>Total Below Axis</strong></td>
<td>730.1</td>
<td>10,378</td>
<td>168,427</td>
<td>516</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>1369.9</td>
<td>2.90</td>
<td>3,959</td>
<td>245,976</td>
</tr>
<tr>
<td><strong>Lever to keel</strong></td>
<td>20.00 - 2.90 + 0.08 = 17.18 ft.</td>
<td>Lever to deck 35.00 - 20.00 + 2.90 + 0.04 = 17.94 ft.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Z_d</strong></td>
<td>470,992</td>
<td>17.94</td>
<td>26,254</td>
<td>1,001</td>
</tr>
<tr>
<td><strong>Z_k</strong></td>
<td>470,992</td>
<td>17.18</td>
<td>27,415</td>
<td>246,977</td>
</tr>
<tr>
<td><strong>One side</strong></td>
<td>235,496</td>
<td>470,992</td>
<td>470,992</td>
<td>1,001</td>
</tr>
<tr>
<td><strong>Both sides</strong></td>
<td>470,992</td>
<td>470,992</td>
<td>470,992</td>
<td>1,001</td>
</tr>
</tbody>
</table>
The floors are designed to resist in association with the longitudinals the upward hydrostatic pressure on the bottom. The method is that of Reference 11.

Required gross area of floors (short webs):

\[ A_T = k \frac{p \delta s}{\tau_{\text{allow}}} \]

where \( k \) is given by Figure 13 and equals 0.35 for \( a/b = 42.00/29.20 = 1.44 \) psi and \( \eta = 1.00 \),

\( p \) is the hydrostatic pressure on bottom = 14.00 psi,

\( s \) is the spacing of floors = 84 in. and,

\( \tau_{\text{allow}} \) is the allowable unit shear stress = 16,000 psi.
Then

\[ A_T = 0.35 \frac{(14.00)(29.20)(12)(84)}{16,000} = 9.00 \text{ sq in} \]

For a 36 in. depth, \( h = 9.00/36 = 0.25 \text{ in.} \) This is the maximum thickness and will be adopted in the region where the bending moment becomes zero (Longitudinals 3 to 6). Between the vertical keel and Longitudinal 3 as well as beyond Longitudinal 6, this thickness will be reduced to 0.219 in. because of the decrease in shear.

The shear stress in the longitudinals can be obtained in the same manner. From Figure 12 we obtain \( k = 0.67 \) and for a value \( A_L = 36 \text{ in.} \times 0.437 \text{ in.} = 15.8 \text{ sq in.} \),

\[ \tau = k \frac{p b s}{A_L} = 0.67 \frac{(14.00)(29.20)(12)(4.7)}{15.8} = 9,700 \text{ psi} \]

The foregoing calculations are valid only for unlightened shear members. For the calculation of lightened, stiffened floors there is no other basis available but the experimental. As discussed under "Experimental Results" the floor outlined in Figure 21 is not inferior to the solid plate it replaces and is, therefore, simply substituted for it without elaborate rationalization.

At the second deck the floor needs to be increased in thickness to sustain the reaction of the transverse girder. This thickness is made equal to that of the web of the girder and the extent of this increase is twice the depth thereof. Looking forward to the section on the second deck girder, we find these values to be 0.455 in. and 21 in., respectively. The insert plate will then be 42 in. in depth and 0.438 in. (17.85 lb) in thickness.
The design of lightening holes will not be discussed herein except to state that their proportions are based on a lengthy compendium of simple rules aimed at limiting the depth of the hole to 40 percent of the depth of the inner bottom when shear is an important consideration. Thus, all lightening holes between the vertical keel and Longitudinal 9 are made \((0.4)(36 \text{ in.}) = 15 \text{ in. in diameter.}\)

Above Longitudinal 9 where, because of the greater depth of double bottom, shear ceases to be as critical, the depth of the lightening holes is adjusted so that the same net depth of web is retained, i.e., \(36 - 15 = 21 \text{ in.}\).

Flat bar reinforcement for the lightening holes is taken from Figure 18. For the 15 in. diameter holes, a section 1 1/2 in. \(\times\) 5/16 in. is required.

_**Nonwatertight floors between Longitudinal 3 and Longitudinal 6 to be 0.25 in. (10.2 lb)**_

_All other nonwatertight floors to be 0.219 in. (8.92 lb)_

_Insert plate at second deck to be 36 in. \(\times\) 0.375 in. (15.3 lb)_

_Lightening holes between centerline and Longitudinal 9 to be 15 in. in diameter_

_Lightening holes above Longitudinal 9 to allow a net depth of 21 in._

_Flat bar stiffening for the 15 in. diameter holes to be 1 1/2 in. \(\times\) 5/16 in._

_For the deeper holes to be given by Figure 18_

**MAIN DECK TRANSVERSE GIRDER AND WEB FRAME**

Since the main deck transverse girder and the web frame are prismatic members, it is convenient to use the method of moment distribution expounded in Reference 24 in the determination of their scantlings.

_Loadings on Main Deck. Almost the total loading on the transverse girder is imposed by the longitudinals as concentrated forces at discreet points along the span. Because of their relatively close spacing, however, it is convenient to assume that the loading is uniformly distributed in order to simplify the analysis._

Transverse girder spacing = 14 ft

Live load: (4 ft hydrostatic head = 256 psf)

\[w = (256)(14) = 3580 \text{ lb/ft}\]

Dead load: (based on 30.6 lb plating and 10 in. \(\times\) 4 in. \(\times\) 15 lb longitudinals)

\[w = 35.6 \times 14 = 500 \text{ lb/ft.}\]

Total load = 3580 + 500 = 4080 lb/ft = 4.08 kips/ft

CONFIDENTIAL
Loading on Web Frame. The greatest hydrostatic head on the web frame occurs when the vessel is inclined 30 deg. in still water, see Figure 28. Again, if we assume distributed instead of concentrated forces on the web frame, we find that the loading is almost triangular of an average value.

\[ w = (3.03 \text{ ft})(64)(14) = 2.71 \text{ kips/ft} \]

where 3.03 ft is the average head.

Fixed End Moments. A supporting stanchion is assumed 11.75 ft off the centerline, in line with the fourth main deck longitudinal. The fixed end moments \( M^F \) calculate as follows:

\[ M^F_{AB} = M^F_{BA} = \frac{wL^2}{12} = \frac{(4.08)(11.75)^2}{12} = 47.0 \text{ ft-kips} \]

\[ M^F_{BC} = M^F_{CB} = \frac{(4.08)(16)^2}{12} = 87.0 \text{ ft-kips} \]

\[ M^F_{CD} = 0.8 \frac{wL^2}{12} = 0.8 \frac{(2.71)(8.00)^2}{12} = 11.6 \text{ ft-kips} \]

\[ M^F_{DC} = 1.2 \frac{(2.71)(8.00)^2}{12} = 17.4 \text{ ft-kips} \]

Stiffness Factors

\[ K = I/l \ (I \text{ assumed equal to unity}) \]

\[ K_{AB} = K_{BA} = \frac{l}{11.75} = 0.085 \]

\[ K_{BC} = K_{CB} = \frac{l}{16.0} = 0.0625 \]

\[ K_{CD} = K_{DC} = \frac{l}{8.0} = 0.125 \]

Bending Moments. These are determined and plotted in Figure 28. The largest bending moment amounts to 77.5 ft-kips.

Required Section Moduli. Since no primary or secondary stresses act in the flanges of the girder and web frame, the minimum section modulus is given by dividing the maximum bending moment by the allowable unit stress of 28,000 psi. Thus,
Loading on Main Deck
4.0' Hydrostatic Head

Loading on Side Plating
Hydrostatic Head from Ship Inclined 30 degrees
in Still Water

Solution by Moment Distribution

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>0.085</td>
<td>0.085</td>
<td>0.0625</td>
<td>0.0625</td>
</tr>
<tr>
<td>$K/\Sigma K$</td>
<td>0.50</td>
<td>0.58</td>
<td>0.42</td>
<td>0.33</td>
</tr>
<tr>
<td>F.E.M.</td>
<td>-45.0</td>
<td>+45.0</td>
<td>-87.0</td>
<td>+87.0</td>
</tr>
<tr>
<td>First Distribution</td>
<td>0</td>
<td>+24.4</td>
<td>+17.6</td>
<td>-24.9</td>
</tr>
<tr>
<td>C.O.</td>
<td>+12.2</td>
<td>0</td>
<td>-12.4</td>
<td>-8.8</td>
</tr>
<tr>
<td>Second Distribution</td>
<td>0</td>
<td>+7.2</td>
<td>+5.2</td>
<td>-2.9</td>
</tr>
<tr>
<td>C.O.</td>
<td>+3.6</td>
<td>0</td>
<td>-1.4</td>
<td>+2.6</td>
</tr>
<tr>
<td>Third Distribution</td>
<td>0</td>
<td>+0.7</td>
<td>+0.7</td>
<td>-0.9</td>
</tr>
<tr>
<td>C.O.</td>
<td>+0.3</td>
<td>0</td>
<td>-0.4</td>
<td>+0.3</td>
</tr>
<tr>
<td>Fourth Distribution</td>
<td>0</td>
<td>+0.2</td>
<td>+0.4</td>
<td>-0.1</td>
</tr>
<tr>
<td>C.O.</td>
<td>+0.1</td>
<td>0</td>
<td>0</td>
<td>+0.1</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>-28.8</td>
<td>+77.5</td>
<td>-77.5</td>
<td>+70.0</td>
</tr>
</tbody>
</table>
Span A-C, \( Z_{\text{req.}} = \frac{77,500 \times 12}{28,000} = 33.2 \text{ in}^3 \)

Span C-D, \( Z_{\text{req.}} = \frac{69,900 \times 12}{28,000} = 30.0 \text{ in}^3 \)

For reasons of assembly and erection, the minimum depth of the transverse girder must be at least 4 in. greater than the longitudinals it supports, in this case, 14 in. The minimum depth of web frame is 12 in. for the same reason. Selecting the lightest 14 in. and 12 in. sections we have respectively:

For the girder - 14 in. \( \times 6\,\frac{3}{4} \text{ in.} \times 30 \text{ lb} \) I cut to T
For the web frame - 12 in. \( \times 6\,\frac{1}{2} \text{ in.} \times 25 \text{ lb} \) I cut to T

Effective Plate Flange. In determining the available section moduli, reference is made again to Figure 16 for estimating the width of effective plating. For the span A-B, \( l = 5.40 \text{ ft} \) (chord of positive moment). \( \beta = 5.40/14.00 = 0.386, b_e = 0.80 b = (0.8)(14)(12) = 134 \text{ in.} \) (For the span B-C, the effective width is somewhat larger.) The selected section in association with this amount of plating possesses more than adequate section modulus. The same conclusion is reached for the web frame.

Main deck transverse girder to be 14 in. \( \times 6\,\frac{3}{4} \text{ in.} \times 30 \text{ lb} \) I cut to T
Web frame to be 12 in. \( \times 6\,\frac{1}{2} \text{ in.} \times 25 \text{ lb} \) I cut to T

STANCHIONS - SECOND TO MAIN DECK

There are three stanchions, one on the centerline and one 11.75 ft off the centerline on either side. The somewhat more heavily loaded side longitudinals will be designed.

Referring to the design of the main deck transverse girder, the loading is approximately

\[ P = \frac{1}{2} (4.08)(11.75 + 16.0) = 61 \text{kips} \]

For \( l = 82 \text{ in.} \), \( \sigma_{\text{all}} = 20,000 \text{ psi} \) reduced for slenderness, a 5.563 in. O.D. \( \times 0.258 \text{ in.} \) tubing is satisfactory. For this (in inch units):

\[ A = 4.30, \quad \rho = 1.88, \quad \frac{l}{\rho} = 44 \]

\[ \sigma_{\text{all}} = 17,000 \text{ from Reference 28, and} \]

\[ P_{\text{all}} = (4.30)(17,000) = 75 \text{kips} \]

The centerline stanchion will be made the same size.

Stanchions second to main deck to be 5.563 in. O.D. \( \times 0.258 \text{ in.} \)
SECOND DECK TRANSVERSE GIRDER

As for the main deck transverse girder, the loading imposed by the longitudinals will be assumed uniformly distributed. In addition there is, however, the reaction of the stanchions supporting the main deck. The girder is assumed fully supported at the centerline by the stanchion in the machinery space and fully fixed against rotation at the inner bottom, see Figure 29. The girder spacing is 14 ft.

Uniform Load = 3.1 kips/ft.

Figure 29 - Second Deck Transverse Girder

Loading. Uniformly distributed -
Live load: (200) + plating (18) and stiffeners (5) = 223 psf.
\[ w = 14(223) = 3.1 \text{ kips/ft} \]
Concentrated (11.75 ft off centerline).
Stanchion reaction = 75 kips.

Bending Moments. Fixed end moment for uniform load

\[ M_{ED}^F = M_{DE}^F = \frac{wL^2}{12} = \frac{(3.1)(23.75)^2}{12} = 146 \text{ ft-kips} \]

Fixed end moment for concentrated load

\[ M_{ED}^F = \frac{a}{l} M^s = \frac{12.0}{23.75} M^s = 0.51 M^s \]

where \( M^s \) is the span moment.

\[ M^s = \frac{a}{l} \cdot P (1 - a) = (0.51)(75)(11.75) = 450 \text{ ft-kips} \]

\[ M_{ED}^F = (0.51)(450) = 229 \text{ ft-kips} \]

Full fixity is attained because of symmetry, consequently

\[ Z_{req.} = \frac{M_{max}^{\alpha}}{\sigma_{alt.}} = \frac{(229 + 146)(1000)(12)}{28,000} = 161 \]

Effective Plate Flange. To obtain the effective width of plating plot the bending moment curve. To do so obtain moment intercepts at center of span; \( M' \):

For uniform load -

\[ M' = 1.5 M^F = 1.5(146) = 219 \text{ ft-kips} \]

For concentrated load -

\[ M' = 2(M_{average}^F) = 2(225) = 450 \text{ ft-kips} \]

The chord of positive moment (Figure 29), \( l = 12.60 \text{ ft} \). Then \( \beta = 12.60/14.00 = 0.90 \) and from Figure 16, the effective width of plating \( b_e = 0.42 b = (0.42)(168) = 70.5 \text{ in.} \)

With this effective plate flange a 21 in. x 8 1/4 in. x 73 lb I cut to T is required to provide adequate section modulus.

Second deck transverse girder to be 21 in. x 8 1/4 in. x 73 lb I cut to T

STANCHION BELOW SECOND DECK

This is designed similarly to the stanchions above the second deck.

Loading from main deck -

\[ (4.08 \text{ kips/ft}) \frac{B}{2} = (4.08)(27.75) = 118 \text{ kips} \]
Loading from second deck -

3.1 kips/ft \times \text{net half span} \times (3.1)(23.75) = 74 \text{ kips}

Total load = 187 kips. \ l = 270 \text{ in.}

Try a 12.75 in. O.D. \times 0.33 tubing.

\[ A = 12.88, \ \frac{L}{\rho} = 61.5 \]

\[ \sigma_{all} = 14,300 \text{ from Reference 28} \]

\[ P_{all} = (14,300)(12.88) = 184 \text{ kips} \]

This is satisfactory.

Stanchion below second deck to be 12.75 in. O.D. \times 0.33 in. tubing

SURVEY

We have concluded the first approximation of the structure of the midship section. On the basis of the scantlings developed, it is now possible to proceed to the weight estimate, determination of the longitudinal bending moment and a more reliable second development of the structure. This is a lengthier undertaking to be carried out with refinement and care while the rest of the design work continues unhindered on the basis of the scantlings already determined.

The method of carrying out the longitudinal strength calculation is common knowledge and will not be discussed. It might be well, however, to illustrate three calculations of importance that are carried out as part of the second step, namely:

1. A determination of transverse strength.

2. A determination of the tertiary stresses in plating when primary and secondary stresses are also present.

3. A determination of interaction in plating subject to combined axial and shear stresses.

TRANSVERSE STRENGTH

When designing plating we have found it necessary, in the first step, to estimate the magnitude of secondary stresses. This was done empirically, by comparison to other vessels, and was subject to later verification. This verification will now be carried out for the double bottom.

Consider the scheme of the double bottom, Figure 30. It is supported against hydrostatic pressure by the shell and second deck, both of which are primary structure. Their intersection, consequently, is considered fixed in space for the purpose of assessing the
strength of the transverse (secondary) structure. The inertia of the backing structure (second deck transverse girder and web frame) is small compared to that of the double bottom. The latter can, therefore, be assumed hinged (free to rotate) at the point of support. The problem to solve, then, is that of the two-hinged arch.

Outline of Method. There is only one redundant force; the horizontal reaction $H_i$ at the support. The bending moment $M$ at any point along the neutral axis is given by

$$ M = M_s - H_i \cdot y $$

where $M$ is positive clockwise,

$H_i$ is positive to the left,

$y$ is the vertical distance from hinge to point, and is positive downwards, and

$M_s$ is the statical moment of all forces acting from the hinge to the point in question.
The work of bending in the complete arch is

$$ W = \frac{1}{E} \int \frac{M^2}{I} \, ds = \frac{1}{E} \int \frac{(M_s - H_i \cdot y)^2}{l} \, ds $$

where the symbol $\int$ means that the integration is made along the path from hinge to centerline, and the derivative of work with respect to the redundant $H_i$

$$ \frac{\partial W}{\partial H_i} = \frac{2}{E} \int \frac{M}{I} \cdot \frac{dM}{dH_i} \cdot ds = \frac{2}{E} \int \frac{M_s - H_i \cdot y}{l} \cdot y \, ds $$

$H_i$ is determined by setting this derivative equal to zero, i.e.,

$$ \int \frac{M_s - H_i \cdot y}{l} \cdot y \, ds = 0 $$

The integration is carried out graphically after having determined the values of $M_s$, $y$, and $I$ at specific stations along the path.

**Determination of Moment of Inertia.** The first step in calculating the moment of inertia of any section of the double bottom is to evaluate the width of effective plating. This presupposes a knowledge of the virtual span of the arch (location of zero bending moments). A simple rule is given: The bending moment curve goes through zero at a point located at a distance from the hinge equal to two-thirds of the girth from centerline to hinge.

For a girth $= 540$ in., $l = 2/3 (540) = 360$ in. The spacing of transverse $b = 84$ in., consequently $\beta = 84/360 = 0.234$ and, from Figure 16, $b_e = 0.92 b = 0.92(84) = 77.3$ in. Use this width of shell and inner bottom flange in the calculations.

The length along the neutral axis from the centerline to the second deck is subdivided into segments of equal length, except for the end segments whose length is one-half that of the others. For five full segments and two half-segments, the length of arc between stations (centers of segments) is

$$ ds = \frac{540}{6} = 90 \text{ in.} = 7.50 \text{ ft} $$

The origin of coordinates is taken at the intersection of second deck and neutral axis. The values of the coordinates of the stations and other geometrical properties of the arch are listed in Figure 31 together with the following information for each station:

- The moment of inertia ($I$) of the section.
- The section modulus to the shell ($Z_s$).
- The section modulus to the inner bottom ($Z_{ib}$).
The sectional area effective in tension and compression \((A_\sigma)\).

The sectional area effective in shear web area \((A_\gamma)\).

The method for obtaining these is well known and will not be touched upon.

Determination of the External Forces - General Notes. The condition assumed for analysis is that of maximum excess buoyancy, i.e., for the transverse frame under consideration, the externally applied buoyant force attains its maximum value and the internal weights are at their minimum. The vessel is upright, the crest of the wave is located amidships.

All forces acting on the transverse frame are assumed imposed at the intersection of a longitudinal with the neutral axis of the arch.

Hydrostatic forces are resolved into vertical and horizontal components at each point of application.

Fixed weights of structure and machinery are assumed to act vertically.

All tanks are assumed empty.

The resulting difference between the forces of buoyancy and weight is applied as a shear force in the vertical shell plating.

Hydrostatic Loading on Shell. The height of wave crest above baseline = 31.45 ft.

The hydrostatic head at any point \(n\) (intersection of longitudinal with neutral axis) is

\[ H_n = (31.45 - 26.75) + v_n \]

where 26.75 ft is the height of the second deck, i.e., axis of abscissae above baseline and \(v_n\) is the ordinate of point, see Figure 31. The abscissa is \(u_n\).

The normal force

\[ F_n = 7 \ b_n H_n (0.064) \]

where 7 ft is the spacing of transverse,

\(b_n\) is the width of shell plating supported by longitudinal \(n\) measured along shell, and

0.064 is the specific gravity of water in kips.

The components

\[ F_{nx} = F_n \cos \alpha_n, \quad F_{ny} = F_n \sin \alpha_n \]

where \(\alpha\) is the angle of normal with horizontal.

The coordinates \(u_n\) and \(v_n\) and the components \(F_{nx}\) and \(F_{ny}\) are also listed in Figure 31.
### Geometric Properties of Arch

<table>
<thead>
<tr>
<th>Station</th>
<th>$ds$</th>
<th>$x_s$</th>
<th>$y_s$</th>
<th>$l$</th>
<th>$Z_s$</th>
<th>$Z_{lb}$</th>
<th>$A_g$</th>
<th>$A_T$</th>
<th>$ds$</th>
<th>$y/l$</th>
<th>$y^2ds/l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.75</td>
<td>26.60</td>
<td>0</td>
<td>1.84</td>
<td>2,230*</td>
<td>1,035*</td>
<td>78.3</td>
<td>10.5</td>
<td>2.04</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>7.50</td>
<td>26.40</td>
<td>7.32</td>
<td>1.84</td>
<td>2,230</td>
<td>1,035</td>
<td>78.3</td>
<td>10.5</td>
<td>4.08</td>
<td>29.9</td>
<td>219</td>
</tr>
<tr>
<td>2</td>
<td>7.50</td>
<td>25.35</td>
<td>14.47</td>
<td>2.11</td>
<td>2,270</td>
<td>1,250</td>
<td>83.0</td>
<td>10.5</td>
<td>3.55</td>
<td>51.4</td>
<td>744</td>
</tr>
<tr>
<td>3</td>
<td>7.50</td>
<td>21.00</td>
<td>20.25</td>
<td>1.65</td>
<td>2,200</td>
<td>1,185</td>
<td>95.2</td>
<td>7.9</td>
<td>4.55</td>
<td>92.1</td>
<td>1865</td>
</tr>
<tr>
<td>4</td>
<td>7.50</td>
<td>14.60</td>
<td>23.78</td>
<td>1.79</td>
<td>2,370</td>
<td>1,440</td>
<td>114.</td>
<td>7.9</td>
<td>4.19</td>
<td>99.6</td>
<td>2368</td>
</tr>
<tr>
<td>5</td>
<td>7.50</td>
<td>7.40</td>
<td>25.02</td>
<td>1.92</td>
<td>2,390</td>
<td>1,610</td>
<td>119.</td>
<td>7.9</td>
<td>3.91</td>
<td>97.8</td>
<td>2447</td>
</tr>
<tr>
<td>6</td>
<td>3.75</td>
<td>0</td>
<td>25.35</td>
<td>2.05</td>
<td>2,560</td>
<td>1,700</td>
<td>139.</td>
<td>13.5</td>
<td>1.83</td>
<td>46.4</td>
<td>1176</td>
</tr>
</tbody>
</table>

* Insert plate not considered.

---

**External Loading on Arch**

**Figure 31 - Transverse Strength**

<table>
<thead>
<tr>
<th>Pt. $n$</th>
<th>$v_n$</th>
<th>$u_n$</th>
<th>$\alpha$</th>
<th>$F_{nx}$</th>
<th>$F_{ny}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>26.60</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>26.30</td>
<td>10.95</td>
<td>0</td>
<td>55.4</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>25.40</td>
<td>14.30</td>
<td>15.5</td>
<td>34.2</td>
<td>9.5</td>
</tr>
<tr>
<td>3</td>
<td>23.70</td>
<td>17.47</td>
<td>32.3</td>
<td>32.7</td>
<td>20.7</td>
</tr>
<tr>
<td>4</td>
<td>21.10</td>
<td>20.15</td>
<td>43.5</td>
<td>29.6</td>
<td>28.2</td>
</tr>
<tr>
<td>5</td>
<td>18.00</td>
<td>22.23</td>
<td>57.8</td>
<td>23.0</td>
<td>36.5</td>
</tr>
<tr>
<td>6</td>
<td>14.45</td>
<td>23.80</td>
<td>70.7</td>
<td>13.9</td>
<td>39.8</td>
</tr>
<tr>
<td>7</td>
<td>11.00</td>
<td>24.55</td>
<td>79.7</td>
<td>7.5</td>
<td>40.9</td>
</tr>
<tr>
<td>8</td>
<td>7.35</td>
<td>25.02</td>
<td>84.4</td>
<td>4.3</td>
<td>41.8</td>
</tr>
<tr>
<td>9</td>
<td>3.70</td>
<td>25.19</td>
<td>86.1</td>
<td>2.9</td>
<td>42.4</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>25.35</td>
<td>90.0</td>
<td>0</td>
<td>21.2</td>
</tr>
</tbody>
</table>

$\Sigma$ One Side = 351.8
Fixed Weights. If a reliable weight estimate is available, the fixed weights of structure and machinery are readily calculated.

If such an estimate is not yet available, the following procedure may be used and is justified because the fixed weights are appreciably smaller than the hydrostatic forces acting on the frame. A large error in estimating them will not result in a proportionate error in the final stresses.

The average weight per foot of fixed structure amidships is taken equal to

\[ w = k \frac{\Delta l}{L} \]

where \( \Delta l \) is the light displacement and \( k \) is a coefficient somewhat greater than unity obtained from other similar vessels. In the absence of any data, take \( k = 1 \).

Substituting our given values

\[ w = 1.0 \frac{(6500)(2.240)}{575} = 25.3 \text{ kips/ft} \]

Deduct weight of all structure not carried by the bottom:

- Deckhouse 0.45 kips/ft
- Main Deck 2.45 kips/ft
- Second Deck 1.90 kips/ft

4.80 kips/ft

The difference 25.3 - 4.8 = 20.5 kips/ft is the weight carried by the bottom and is assumed to be evenly distributed horizontally. Each longitudinal carries its share of the load in proportion to the horizontal projected width of the area it supports; see Figure 32. The weight components are also calculated in this figure.

Vertical Shear. The difference between the total upward hydrostatic force and the downward weight components is the vertical shear. From Figures 31 and 32 this is here equal to

\[ 351.8 - 72.0 = 279.8 \text{ kips (each side)} \]

and is carried by the vertical shell alone. For the vessel upright, there is no corresponding horizontal shear.

Summary of Forces Acting on Frame. From Figures 31 and 32
Load Supported by Each Longitudinal in 7-ft Bay

<table>
<thead>
<tr>
<th>$n$</th>
<th>$u_n$</th>
<th>$b_n$</th>
<th>$F_{ny}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>26.60</td>
<td>0</td>
<td>1.2</td>
</tr>
<tr>
<td>1</td>
<td>26.30</td>
<td>0.45</td>
<td>1.2</td>
</tr>
<tr>
<td>2</td>
<td>25.40</td>
<td>1.30</td>
<td>3.6</td>
</tr>
<tr>
<td>3</td>
<td>23.70</td>
<td>2.15</td>
<td>5.9</td>
</tr>
<tr>
<td>4</td>
<td>21.10</td>
<td>2.85</td>
<td>7.8</td>
</tr>
<tr>
<td>5</td>
<td>18.00</td>
<td>3.33</td>
<td>9.1</td>
</tr>
<tr>
<td>6</td>
<td>14.45</td>
<td>3.50</td>
<td>9.6</td>
</tr>
<tr>
<td>7</td>
<td>11.00</td>
<td>3.55</td>
<td>9.7</td>
</tr>
<tr>
<td>8</td>
<td>7.35</td>
<td>3.65</td>
<td>10.0</td>
</tr>
<tr>
<td>9</td>
<td>3.70</td>
<td>3.67</td>
<td>10.0</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>1.85</td>
<td>5.1</td>
</tr>
<tr>
<td>Σ</td>
<td>One Side</td>
<td></td>
<td>72.0</td>
</tr>
</tbody>
</table>

Figure 32 - Transverse Strength
Static Moment. The horizontal force (positive to the left) at any Station \( s \) is equal to the horizontal force at Station \( (s - 1) \) plus the sum of all the horizontal components of force acting between Stations \( (s - 1) \) and \( s \); Figure 33. Thus

\[
H_s = H_{s-1} + \sum_{s-1}^{s} \Delta F_{nx}
\]

The vertical force (positive upward) is similarly expressed as

\[
V_s = V_{s-1} + \sum_{s-1}^{s} \Delta F_{ny}
\]

The static moment (positive clockwise) becomes

\[
M_s = M_{s-1} - H_{s-1} (y_s - y_{s-1}) - V_{s-1} (x_s - x_{s-1}) - \sum_{s-1}^{s} \Delta F_{nx} (y_s - y_n) - \sum_{s-1}^{s} \Delta F_{ny} (u_n - x_s)
\]

The static moment calculation is carried out in Table 3 from which

<table>
<thead>
<tr>
<th>Station</th>
<th>Static Moment ( M_s ) ft-kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>94</td>
</tr>
<tr>
<td>3</td>
<td>604</td>
</tr>
<tr>
<td>4</td>
<td>1330</td>
</tr>
<tr>
<td>5</td>
<td>2001</td>
</tr>
<tr>
<td>6</td>
<td>2252</td>
</tr>
</tbody>
</table>
Expression for Static Moment

\[ M_s (t^+ + ) = M_{s-1} - H_{s-1} (y_s - y_{s-1}) - V_{s-1} (x_s - x_{s-1}) \]

\[ - \sum_{s=1}^{S} \Delta F_{nx} (y_s - y_n) - \sum_{s=1}^{S} \Delta F_{ny} (u_n - x_s) \]

\[ H_s (\pm) = H_{s-1} + \sum_{s=1}^{S} \Delta F_{nx} \]

\[ V_s (\uparrow +) = V_{s-1} + \sum_{s=1}^{S} \Delta F_{ny} \]

Expression for Shear and Axial Force

\[ H = H_i \text{ (Indeterminate)} + H_s \text{ (Static)} \]

\[ V = V_s \]

\[ P = H \sin \alpha - V \cos \alpha \]

\[ Q = H \cos \alpha + V \sin \alpha \]

Figure 33 - Transverse Strength
## TABLE 3

Transverse Strength - Determination of Static Moment

|   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
|   | x_s | y_s | x_s-1 | y_s-1 | u_n | v_n | u_n - x_s | v_n - y_s | F_n | H_s | M_{s-1} | F_{ny} | V_{s-1} | Δx | Δy | F_{nx} | H_{s-1} | V_{s-1} | H_{s-1} | M_{s-1} | F_{nx} | F_{ny} | Δy | H_{s-1} |
| 0 | 26.60 | 0 | - | - | 0 | 26.60 | 0 | - | - | 0 | 0 | -281.0 | -281.0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 26.40 | 7.32 | 26.60 | 0 | 0.20 | 7.32 | 0.05 | 7.15 | 0.95 | 3.52 | 55.4 | 0 | 0 | -281.0 | 0 | 0 | -295 | 195 | 195 | 94 | 2 |
| 2 | 25.35 | 14.47 | 26.40 | 7.32 | 0 | 26.30 | 10.95 | 0.95 | 3.52 | 55.4 | 0 | 0 | -281.0 | 0 | 0 | -295 | 195 | 195 | 94 | 2 |
| 3 | 21.00 | 20.25 | -25.35 | 14.47 | 0 | 23.70 | 17.47 | 4.35 | 5.78 | 2.70 | 2.78 | 32.7 | 20.7 | 28.2 | 0 | 0 | -271.5 | 66 | 518 | 1181 | 604 | 3 |
| 4 | 14.60 | 23.78 | 21.00 | 20.25 | 0 | 18.00 | 22.23 | 6.40 | 3.53 | 3.40 | 1.55 | 23.0 | 36.5 | 36 | 126 | 0 | 0 | -222.6 | 576 | 536 | 1424 | 1330 | 4 |
| 5 | 7.40 | 25.02 | 14.60 | 23.78 | 0 | 14.45 | 23.80 | 7.20 | 1.24 | 7.05 | 1.22 | 13.9 | 39.8 | 40.9 | 174.9 | 186.1 | 1302 | 217 | 1340 | 17 | 283 | 3 |
| 6 | 0 | 25.35 | 7.40 | 25.02 | 0 | 7.35 | 25.02 | 7.40 | 0.33 | 7.35 | 0.33 | 4.3 | 41.8 | 42.4 | 21.2 | 196.3 | 105.4 | 1973 | 65 | -780 | 1 | 307 | 0 |

\( x_s, y_s, x_s-1, y_s-1, u_n, v_n, u_n - x_s, v_n - y_s, F_{nx}, H_{s-1}, V_{s-1}, H_{s-1} \cdot Δy, F_{nx} \cdot Δy, H_{s-1} \cdot Δy, F_{ny} \cdot Δu, H_{s-1} \cdot Δy, M_{s-1} \)
Solution for Indeterminate Horizontal Reaction, \( H \). To solve for \( H \), we let

\[
\int \frac{M_s y \, ds}{I} = \int \frac{H_i y^2 \, ds}{I}
\]

The integration is carried out by summation as follows.

<table>
<thead>
<tr>
<th>Station ( s )</th>
<th>Static Moment ( M_s )</th>
<th>( \frac{y ds}{l} )</th>
<th>( \frac{M_s y ds}{l} )</th>
<th>( \frac{y^2 , ds}{l} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>29.9</td>
<td>0</td>
<td>219</td>
</tr>
<tr>
<td>2</td>
<td>94</td>
<td>51.4</td>
<td>4,800</td>
<td>744</td>
</tr>
<tr>
<td>3</td>
<td>604</td>
<td>92.1</td>
<td>55,600</td>
<td>1865</td>
</tr>
<tr>
<td>4</td>
<td>1330</td>
<td>99.6</td>
<td>132,500</td>
<td>2368</td>
</tr>
<tr>
<td>5</td>
<td>2001</td>
<td>97.8</td>
<td>195,700</td>
<td>2447</td>
</tr>
<tr>
<td>6</td>
<td>2252</td>
<td>46.4</td>
<td>104,500</td>
<td>1176</td>
</tr>
</tbody>
</table>

Then

\[ H_i = \frac{493,100}{8819} = 56.0 \text{ kips} \]

Solution for Bending Moment

\[ M = M_s - H_i y \]

<table>
<thead>
<tr>
<th>Station ( s )</th>
<th>( M_s )</th>
<th>( y )</th>
<th>( H_i \cdot y )</th>
<th>( M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>7.32</td>
<td>410</td>
<td>-410</td>
</tr>
<tr>
<td>2</td>
<td>94</td>
<td>14.47</td>
<td>810</td>
<td>-716</td>
</tr>
<tr>
<td>3</td>
<td>604</td>
<td>20.25</td>
<td>1134</td>
<td>-530</td>
</tr>
<tr>
<td>4</td>
<td>1330</td>
<td>23.78</td>
<td>1332</td>
<td>-2</td>
</tr>
<tr>
<td>5</td>
<td>2001</td>
<td>25.02</td>
<td>1401</td>
<td>+600</td>
</tr>
<tr>
<td>6</td>
<td>2252</td>
<td>25.35</td>
<td>1420</td>
<td>+832</td>
</tr>
</tbody>
</table>

The bending moment diagram, Figure 34, confirms the assumed location of the point of inflection. The calculated values of section moduli, therefore, stand as is.
Shear and Axial Forces. The compressive force $P$ at any station is given (see Figure 33, page 88) by

\[ P = H \sin \alpha - V \cos \alpha \]

The shearing force $Q$, by

\[ Q = H \cos \alpha + V \sin \alpha \]

where $H = H_i + H_s$ ($i$ is indeterminate and $s$ static), and $V = V_s$. For example, at Station 4

$H = 252.3$, $V = -186.1$, $\alpha = 77$ deg.

$H \sin \alpha = 245.5$, $H \cos \alpha = 56.7$

$V \sin \alpha = -181.3$, $V \cos \alpha = -41.9$

$P = 245.5 + 41.9 = 287.4$ kips

$Q = 56.7 - 181.3 = -124.6$ kips
The remaining forces are similarly calculated, giving:

<table>
<thead>
<tr>
<th>Station</th>
<th>P</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>+281.0</td>
<td>+56.0</td>
</tr>
<tr>
<td>1</td>
<td>+281.0</td>
<td>+145.6</td>
</tr>
<tr>
<td>2</td>
<td>+329.7</td>
<td>+90.7</td>
</tr>
<tr>
<td>3</td>
<td>+320.0</td>
<td>−16.7</td>
</tr>
<tr>
<td>4</td>
<td>+287.4</td>
<td>−124.6</td>
</tr>
<tr>
<td>5</td>
<td>+267.7</td>
<td>−82.5</td>
</tr>
<tr>
<td>6</td>
<td>+259.5</td>
<td>0</td>
</tr>
</tbody>
</table>

where + is direction assumed and P is compressive force.

Shear and Axial Stresses. Having determined P and Q, it is possible to obtain the axial and shear stress intensities based respectively on the gross areas $A_\sigma$ and $A_\tau$. The results are:

<table>
<thead>
<tr>
<th>Station</th>
<th>Areas - Plate</th>
<th>Forces</th>
<th>Stress Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A_\sigma$</td>
<td>$A_\tau$</td>
<td>$P$</td>
</tr>
<tr>
<td>0</td>
<td>78.3</td>
<td>10.5</td>
<td>281.0</td>
</tr>
<tr>
<td>1</td>
<td>78.3</td>
<td>10.5</td>
<td>281.0</td>
</tr>
<tr>
<td>2</td>
<td>83.0</td>
<td>10.5</td>
<td>329.7</td>
</tr>
<tr>
<td>3</td>
<td>95.2</td>
<td>7.9</td>
<td>320.0</td>
</tr>
<tr>
<td>4</td>
<td>114.0</td>
<td>7.9</td>
<td>287.4</td>
</tr>
<tr>
<td>5</td>
<td>119.0</td>
<td>7.9</td>
<td>267.7</td>
</tr>
<tr>
<td>6</td>
<td>139.0</td>
<td>13.5</td>
<td>259.5</td>
</tr>
</tbody>
</table>

Bending Stresses. These are simply given by $\sigma = \frac{12,000M}{Z}$, (12,000 = constant of conversion from ft kips to in-lb).

They are tabulated as follows:

<table>
<thead>
<tr>
<th>Station</th>
<th>$M$ ft-kips</th>
<th>$Z_{tb}$</th>
<th>$Z_s$</th>
<th>$\sigma_{ib}$</th>
<th>$\sigma_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1,035</td>
<td>2,230</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>−410</td>
<td>1,035</td>
<td>2,230</td>
<td>−4,800</td>
<td>+2,200</td>
</tr>
<tr>
<td>2</td>
<td>−716</td>
<td>1,250</td>
<td>2,270</td>
<td>−6,900</td>
<td>+3,800</td>
</tr>
<tr>
<td>3</td>
<td>−530</td>
<td>1,185</td>
<td>2,200</td>
<td>−5,400</td>
<td>+2,900</td>
</tr>
<tr>
<td>4</td>
<td>−12</td>
<td>1,440</td>
<td>2,370</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>+600</td>
<td>1,610</td>
<td>2,390</td>
<td>+4,500</td>
<td>−3,000</td>
</tr>
<tr>
<td>6</td>
<td>+832</td>
<td>1,700</td>
<td>2,560</td>
<td>+5,900</td>
<td>−3,200</td>
</tr>
</tbody>
</table>

+ means clockwise moment or tensile stress.
Combined Bending and Compressive Stresses. By adding to the unit bending stresses just determined the unit compressive axial stress in the arch, the combined stress intensities are obtained.

<table>
<thead>
<tr>
<th>Station</th>
<th>Bending Stress Intensity</th>
<th>Compressive Stress Intensity</th>
<th>Combined Stress Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>$\sigma_{ib}$</td>
<td>$\sigma_s$</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-3,600</td>
</tr>
<tr>
<td>1</td>
<td>-4,800</td>
<td>+2,200</td>
<td>-3,600</td>
</tr>
<tr>
<td>2</td>
<td>-6,900</td>
<td>+3,800</td>
<td>-4,000</td>
</tr>
<tr>
<td>3</td>
<td>-5,400</td>
<td>+2,900</td>
<td>-3,400</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>-2,500</td>
</tr>
<tr>
<td>5</td>
<td>+4,500</td>
<td>-3,000</td>
<td>-2,300</td>
</tr>
<tr>
<td>6</td>
<td>+5,900</td>
<td>-3,200</td>
<td>-1,900</td>
</tr>
</tbody>
</table>

This tabulation, together with that for the shear stresses, constitutes the results of the analysis of transverse strength.

TERTIARY STRESSES IN BOTTOM PLATING

The analysis of transverse strength permits us to proceed to a more accurate evaluation of the tertiary stresses in the structure. We will reconsider the starboard strake originally analyzed in the section on bottom plating. Refer to Figure 35.

![Figure 35 - Tertiary Stress in Bottom Plating](image)

Deflection. The same assumptions are made as previously except that now the magnitude of the secondary stress intensity is averaged at 5200 psi.


$$C_s = 3.78(0.172) + 3.78(1.80) + 1.64(0.556) = 8.36$$

$$T = \frac{\pi^4}{24} (0.556)[3(0.309) + 3(3.33) + 2] = 28.5$$
and Equation [13] becomes

\[
\frac{w_0^3 (8.36)}{2180} - \frac{w_0}{30 \times 10} \left\{ \frac{0.375(9.86)(0.91)[(1.80)(5200) + (0.556)(19000)]}{2180} \right\} - \frac{30 \times 10 (28.5)}{2180} = 0
\]

from which \( w_0 = 0.088 \) in.

**Bending Stresses.** When the compressive forces are sufficient to cause buckling, Equations [16] and [17] give

\[
\sigma_{yb} = \frac{12 \pi^2 (1.84 \times 10^6)}{(2180)(0.766)} = 11,500 \text{ psi (transversely)}
\]

\[
\sigma_{yb} = (\sigma_{yb})_x \cdot \frac{a}{b} = 11,500(0.309) = 3,600 \text{ psi (longitudinally)}
\]

When there is no compression in the plating, Equation [18] gives, using the proper values of \( \phi \),

\[
\sigma'_{xb} = \frac{\phi_x}{12 \pi^2} \cdot \sigma_{xb} = \frac{200}{118} (11,500) = 19,500 \text{ psi}
\]

\[
\sigma'_{yb} = \frac{\phi_y}{12 \pi^2 \sigma_{xb}} = \frac{141}{118} (11,500) = 13,700 \text{ psi}
\]

**Critical Buckling Stress.** From Equation [19]

\[
\left( \sigma_x + \frac{a^2}{b^2 \sigma_y} \right)_{cr} = \frac{4(9.86)(1.84 \times 10^6)}{(3(0.309) + 3(3.33) + 2) (3(0.875)(7060)} = 50,600 \text{ psi}
\]

**Actual Buckling Stress.** Equation [20] gives

\[
\left( \sigma_x + \frac{a^2}{b^2 \sigma_y} \right)_{act} = 5200 + (0.309)(19000) = 11,100 \text{ psi}
\]
For the ratio of actual to critical buckling stress intensity, see Equation [21]:

\[ r = \frac{11.100}{50,600} = 0.220 \]

Actual Tertiary Bending Stress. For transverse direction, Equation [22] gives:

\[ (\sigma_{x_b}) = 19,500 - (0.22)(19,500 - 11,500) = 17,700 \text{ psi} \]

(compare to 19,600 originally used.)

For longitudinal direction, Equation [23] gives:

\[ (\sigma_{y_b}) = 13,700 - (0.22)(13,700 - 3,600) = 11,500 \]

(compare with 13,700 originally used)

The step should be repeated for every panel in the shell innerbottom and decks.

INTERACTION IN FLOOR PLATING

The floors are subject to a particularly severe combination of bending and shear stresses at their attachment to shell or innerbottom, and they should be examined to insure that unduly high combined stresses are not developed.

The interaction formula, Equation [41], is used. The critical shear stress for a plate fixed along its edges is

\[ \tau_{cr} = k \frac{\pi^2 D}{b h} \]

where \( b \) is the smallest side and \( k = 15.5 \text{ for } a/b = 1, 11.6 \text{ for } a/b = 2, \) and \( 9.0 \text{ for } a/b = \infty \) (see Figure 36).

The yield stress, \( \tau_y = 20,000 \text{ psi (MS) } \)

Critical stresses in bending need not be considered since they will exceed the yield strength.

The floor panel assumed for analysis is located at Station 4, and we have for this panel the following actual unit stresses:

\[ \sigma_{ib} = -2,500, \quad \sigma_s = -2.500, \quad \tau = 15,800 \]

The critical shear stress intensity for \( h = 0.25, a = 43.8, b = 36, a/b = 1.22, k = 14.4 \) (from Figure 36) gives:

\[ \tau_{cr} = \frac{14.4(1,695,000)}{1296} = 18,800 \text{ psi} \]
Figure 36 - Critical Shear Strength of Plating Fixed along All Sides

\[ \tau_{cr} = \frac{k\sigma_y^2 D}{b^2 h} \]

Values of \( k \) from graph from Roarke (Reference 29). Formulas for stress and strain.

This is less than \( \tau_y \) and will, therefore, be used. Then substituting in Equation [41] for \( \sigma_y = 35,000 \).

\[ \left( \frac{2,500}{35,000} \right)^2 + \left( \frac{15,800}{18,800} \right)^2 = \frac{1}{f} \]

and

\[ f = 1.19 \]

This is insufficient and the floor plating in this region will need be increased in thickness. The next larger size of plating (\( h = 0.313 \) in. or 12.75 lb) is adequate.

Again, this step should be repeated for every panel.
CONCLUDING REMARKS

To the reader who has patiently reached this page I would like to address some concluding remarks lest he turn the closing leaf of this report with, perhaps, a distorted impression of its usefulness.

In spite of the endless assumptions made throughout the text, in spite of the stated limitations of the theories used, the reader may still feel that in designing a midship section (or indeed any other part of a ship's structure) it is only necessary to substitute numerical values into a given set of formulas. Nothing could be farther from the truth.

What I have attempted to present is only a simple procedure for structural analysis to be applied rapidly. For if a procedure, no matter how accurate, or clever, or appealing, takes more than a minimum amount of time, it cannot well be used.
TABLE 4

Properties of Steel Plate for a Strip One Inch Wide

<table>
<thead>
<tr>
<th>Weight</th>
<th>Thickness in.</th>
<th>$D_{Rh}$</th>
<th>$\frac{Eh^3}{12(1-\mu^2)}$</th>
<th>$\frac{\pi^2D}{h}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.28</td>
<td>0.0313</td>
<td>84</td>
<td>26,780</td>
<td></td>
</tr>
<tr>
<td>2.55</td>
<td>.0625</td>
<td>671</td>
<td>106,100</td>
<td></td>
</tr>
<tr>
<td>3.83</td>
<td>.0938</td>
<td>2,264</td>
<td>238,300</td>
<td></td>
</tr>
<tr>
<td>5.1</td>
<td>.1250</td>
<td>5,366</td>
<td>423,800</td>
<td></td>
</tr>
<tr>
<td>6.37</td>
<td>.1563</td>
<td>10,480</td>
<td>662,000</td>
<td></td>
</tr>
<tr>
<td>7.65</td>
<td>.1875</td>
<td>18,110</td>
<td>953,300</td>
<td></td>
</tr>
<tr>
<td>8.92</td>
<td>.2188</td>
<td>28,750</td>
<td>1,297,000</td>
<td></td>
</tr>
<tr>
<td>10.2</td>
<td>.2500</td>
<td>42,920</td>
<td>1,695,000</td>
<td></td>
</tr>
<tr>
<td>11.47</td>
<td>.2813</td>
<td>71,120</td>
<td>2,145,000</td>
<td></td>
</tr>
<tr>
<td>12.75</td>
<td>.3125</td>
<td>83,840</td>
<td>2,648,000</td>
<td></td>
</tr>
<tr>
<td>14.02</td>
<td>.3438</td>
<td>111,600</td>
<td>3,204,000</td>
<td></td>
</tr>
<tr>
<td>15.3</td>
<td>.3750</td>
<td>144,900</td>
<td>3,813,000</td>
<td></td>
</tr>
<tr>
<td>16.58</td>
<td>.4063</td>
<td>184,200</td>
<td>4,415,000</td>
<td></td>
</tr>
<tr>
<td>17.85</td>
<td>.4375</td>
<td>230,000</td>
<td>5,190,000</td>
<td></td>
</tr>
<tr>
<td>19.13</td>
<td>.4688</td>
<td>283,000</td>
<td>5,958,000</td>
<td></td>
</tr>
<tr>
<td>20.4</td>
<td>.5000</td>
<td>343,400</td>
<td>6,779,000</td>
<td></td>
</tr>
<tr>
<td>23.0</td>
<td>.5625</td>
<td>489,000</td>
<td>8,579,000</td>
<td></td>
</tr>
<tr>
<td>25.5</td>
<td>.6250</td>
<td>670,700</td>
<td>10,590,000</td>
<td></td>
</tr>
<tr>
<td>28.0</td>
<td>.6875</td>
<td>892,700</td>
<td>12,816,000</td>
<td></td>
</tr>
<tr>
<td>30.6</td>
<td>.7500</td>
<td>1,159,000</td>
<td>15,250,000</td>
<td></td>
</tr>
<tr>
<td>33.15</td>
<td>.8125</td>
<td>1,474,000</td>
<td>17,900,000</td>
<td></td>
</tr>
<tr>
<td>35.7</td>
<td>.8750</td>
<td>1,840,000</td>
<td>20,760,000</td>
<td></td>
</tr>
<tr>
<td>38.25</td>
<td>.9375</td>
<td>2,264,000</td>
<td>23,830,000</td>
<td></td>
</tr>
<tr>
<td>40.8</td>
<td>1.0000</td>
<td>2,747,000</td>
<td>27,110,000</td>
<td></td>
</tr>
<tr>
<td>45.9</td>
<td>1.1250</td>
<td>3,912,000</td>
<td>34,320,000</td>
<td></td>
</tr>
<tr>
<td>51.0</td>
<td>1.2500</td>
<td>5,366,000</td>
<td>42,370,000</td>
<td></td>
</tr>
<tr>
<td>56.1</td>
<td>1.3750</td>
<td>7,142,000</td>
<td>51,260,000</td>
<td></td>
</tr>
<tr>
<td>61.2</td>
<td>1.5000</td>
<td>9,272,000</td>
<td>61,010,000</td>
<td></td>
</tr>
<tr>
<td>66.3</td>
<td>1.6250</td>
<td>11,790,000</td>
<td>71,600,000</td>
<td></td>
</tr>
<tr>
<td>71.4</td>
<td>1.7500</td>
<td>14,720,000</td>
<td>83,030,000</td>
<td></td>
</tr>
<tr>
<td>76.5</td>
<td>1.8750</td>
<td>18,110,000</td>
<td>95,320,000</td>
<td></td>
</tr>
<tr>
<td>81.6</td>
<td>2.0000</td>
<td>21,980,000</td>
<td>108,500,000</td>
<td></td>
</tr>
</tbody>
</table>
Pressed by the constant necessity of obtaining solutions to a design that fluxes continuously because of developments and changes in requirements, the naval architect must have a tool that he can use quickly and upon which he can rely. For him, time is always of the essence—sometimes of the extreme essence. And yet he can never compromise with safety.

The procedure presented herein may aid him in saving time, it may aid him in obtaining tentative scantlings quickly. But it cannot tell him categorically whether his design is sound. No theory can do that—only judgment.

For in design there can never be any substitute for judgment. This is especially true in ship design where at present we have only a grossly inadequate knowledge of the problems ranging from the magnitude of the external forces acting on the hull to the ultimate behavior of the structure under the interaction of all the resulting fields of stress.

This is what makes the work so challenging and, let me repeat it, this is why ship design is truly the most difficult structural problem in the world.

ACKNOWLEDGMENT

I should like to express my appreciation to Mr. E.E. Johnson and Dr. F.H. Todd of the Taylor Model Basin for their critical review of this report and acknowledge my indebtedness for their many helpful suggestions.

REFERENCES


7. Scripps Institution of Oceanography Report Number 53, "Computed Wave Characteristics for a Three Year Period for a Station at 50 N - 150 W".


28. Navy Department General Specifications - Appendix 20, "Column Tables."
INITIAL DISTRIBUTION

<table>
<thead>
<tr>
<th>Serials</th>
<th>Serials</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-20</td>
<td>SUPSHIPINSORD, Pascagoula, Miss.</td>
</tr>
<tr>
<td>1-5</td>
<td>SUPSHIPINSORD, Quincy, Mass.</td>
</tr>
<tr>
<td>6</td>
<td>SUPSHIPINSORD, San Diego, Calif.</td>
</tr>
<tr>
<td>7</td>
<td>SUPSHIPINSORD, San Francisco, Calif.</td>
</tr>
<tr>
<td>8</td>
<td>SUPSHIPINSORD, Seattle, Wash.</td>
</tr>
<tr>
<td>9</td>
<td>CO, ONR Branch Office, New York, N.Y.</td>
</tr>
<tr>
<td>10-13</td>
<td>CO, ONR Branch Office, Boston, Mass.</td>
</tr>
<tr>
<td>14-17</td>
<td>CO, ONR Branch Office, Chicago, Ill.</td>
</tr>
<tr>
<td>20</td>
<td>CO, ONR Branch Office, Pasadena, Calif.</td>
</tr>
<tr>
<td>21</td>
<td>SUPSHIPINSORD, Bath, Me.</td>
</tr>
<tr>
<td>22</td>
<td>SUPSHIPINSORD, Camden, N.J.</td>
</tr>
<tr>
<td>23</td>
<td>SUPSHIPINSORD, Chicago, Ill.</td>
</tr>
<tr>
<td>24</td>
<td>SUPSHIPINSORD, Groton, Conn.</td>
</tr>
<tr>
<td>25</td>
<td>SUPSHIPINSORD, Jacksonville, Fla.</td>
</tr>
<tr>
<td>26</td>
<td>SUPSHIPINSORD, Long Beach, Calif.</td>
</tr>
<tr>
<td>27</td>
<td>SUPSHIPINSORD, New Orleans, La.</td>
</tr>
<tr>
<td>28</td>
<td>SUPSHIPINSORD, Newport News, Va.</td>
</tr>
<tr>
<td>29</td>
<td>SUPSHIPINSORD, New York, N.Y.</td>
</tr>
</tbody>
</table>
A consistent and integrated procedure is presented for carrying out systematically the structural design of the midship section of a naval vessel. The report is written in two parts. In the first part the problem is considered in a general manner and the specific theories and methods used in the procedure are introduced. In the second part an illustrative example is worked out for an idealized vessel embodying the simplest possible structure sufficient to illustrate all the points discussed.
A consistent and integrated procedure is presented for carrying out systematically the structural design of the midship section of a naval vessel. The report is written in two parts. In the first part the problem is considered in a general manner and the specific theories and methods used in the procedure are introduced. In the second part an illustrative example is worked out for an idealized vessel embodying the simplest possible structure sufficient to illustrate all the points discussed.