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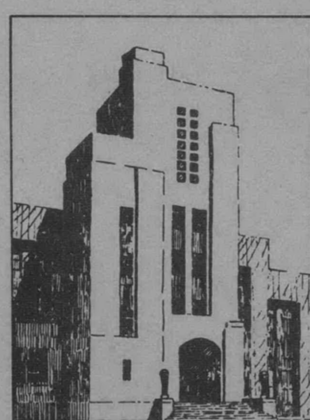
THE DESIGN OF CONTROL SURFACES FOR HYDRODYNAMIC APPLICATIONS

by

Leo F. Fehlner



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January 1951

Report C-358

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THE DESIGN OF CONTROL SURFACES FOR HYDRODYNAMIC APPLICATIONS

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Leo F. Fehlner

January 1951 Report C-358

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ABSTRACT

The design of control surfaces for maneuvering a vessel is discussed from the hydrodynamic standpoint. Control surfaces are divided into two main categories: (1) flapped and (2) all-movable. Recommended flapped designs are restricted to those having "overhang" balance. An example is given which demonstrates the remarkable superiority of the all-movable control surface over the flapped. The importance of designing for an elliptical spanwise load distribution is pointed out and means for obtaining it is given. A bibliography of reports applicable to the design of control surfaces is presented.

INTRODUCTION

This report concerns the design of control surfaces for maneuvering a vessel. For all practical purposes the hull of the vessel can be considered to be a rigid body, completely or partially streamlined, moving in a fluid. The hull is made to travel in a curved path by a hydrodynamic force on the hull itself acting towards the instantaneous center of curvature of its path, in opposition to the centrifugal force caused by the rotation of the vessel. This hydrodynamic force has its origin in the angle of attack of the hull to the fluid. For long bodies, such as the hulls of ships or submarines, this angle of attack varies appreciably along its length while turning, but in general the tightness of the turn is determined by the average magnitude of the angle of attack and the hull's effectiveness for producing hydrodynamic forces thereby.

It is the primary purpose of the control surfaces to cause the hull to have an adequate angle-of-attack range based on the desired maneuverability of the vessel. Accompanying any consideration of the primary purpose are two other important considerations: The additional resistance caused by the control surface extending into the stream and the torque required for turning the control surface.

The control surface causes the hull to have an angle of attack by developing a relatively small force at some distance from the center of gravity of the vessel. The moment thus applied to the hull turns it to an angle of attack. Obviously a suitable control surface is one that produces sufficient force to obtain sufficient controllability with a minimum amount of additional resistance and of torque required to turn the control surface itself.

From this point on, it is no longer necessary to discuss the hull with respect to the actual maneuver. The hull will only be considered as constituting a boundary near the control surface that may have complex secondary influences on the control surface. To arrive at a suitable control surface, it need only be considered as one which produces a sufficient hydrodynamic force at a given lever arm from the center of gravity of the vessel. These considerations are nondimensional in character. All that remains is to choose the actual size needed to control a given hull. To do this, a knowledge of the hull moments and the desired performance of the vessel is required as well as a detailed knowledge of the flow characteristics in that part of the flow to be occupied by the control surface. This phase of the problem will not be discussed in this report.

CHARACTERISTICS OF LOW ASPECT RATIO HYDROFOILS

In accordance with common practice,[37*] lift is defined as "the component of the resultant force perpendicular to the resultant velocity," and a coefficient of lift can be defined as

$$C_L = \frac{L}{\frac{1}{2}\rho U^2 A}$$

where L is the lift,

ρ is the mass density of the fluid,

U is the resultant velocity, and

A is the projected lifting area of the surface.

Similarly, drag is "the component of the resultant force aligned with the resultant velocity" and a drag coefficient can be defined as

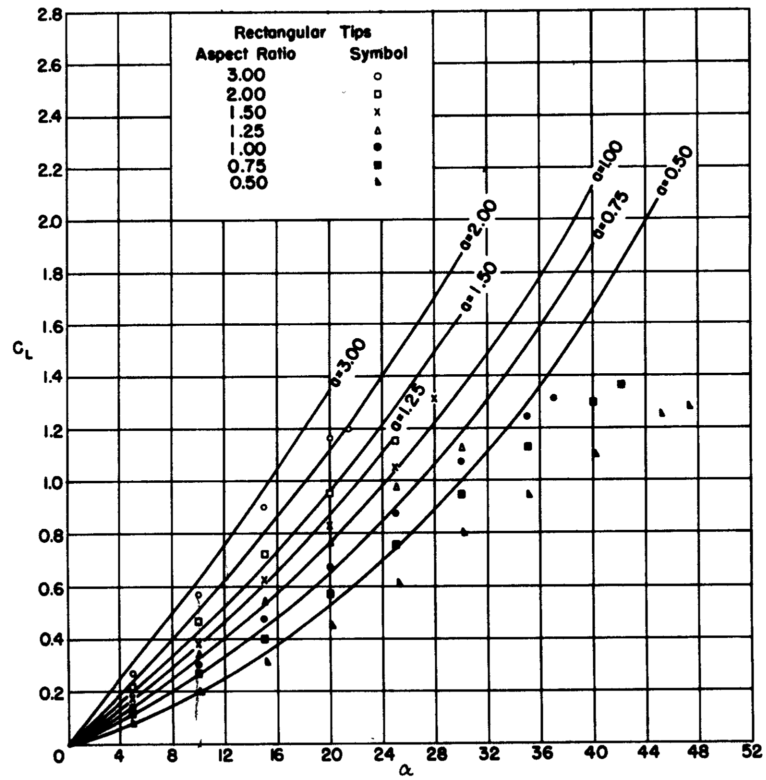
$$C_D = \frac{D}{\frac{1}{2}\rho U^2 A}$$

where D is the drag force.

Formulas giving the lift and induced-drag coefficients produced by lifting surfaces have been developed by many authors. Those based on the lifting-line theory produce accurate results for high aspect ratio but fail to check the data for low aspect ratio. Weinig[1] has developed formulas based on a concept of a lifting surface which check fairly well with what low-aspect-ratio data are available.[2,3] This check is shown in Figure 1.

Data obtained from wind-tunnel tests of airfoil section wings, as reported in Reference 3, are plotted in Figures 1a, 1b, and 1c, along with corresponding curves computed from Weinig's formulas (Equations [1] and [2]).

*References are listed on page 41.



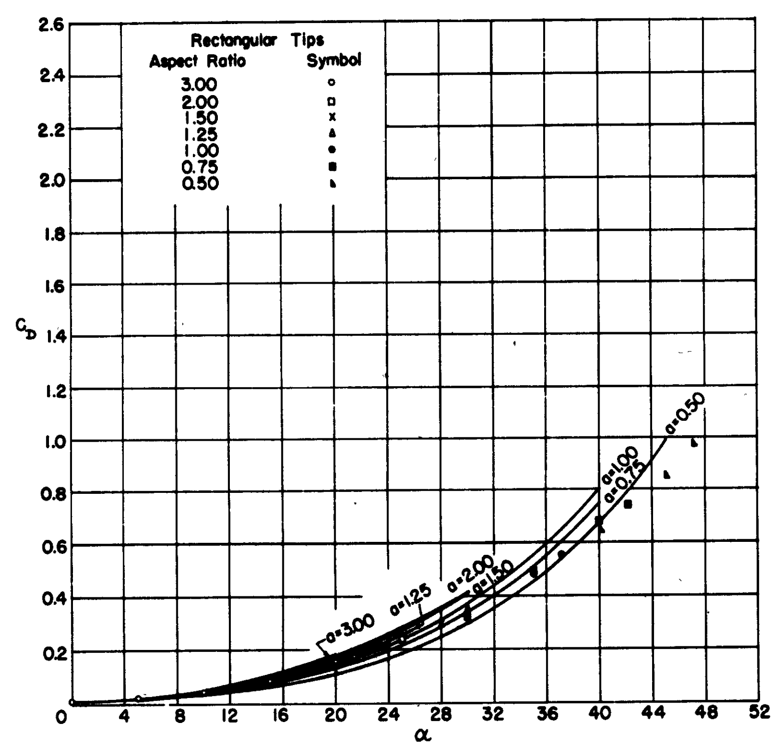


Figure 1a - Rectangular Plan Forms with Rectangular Spanwise Cross Section

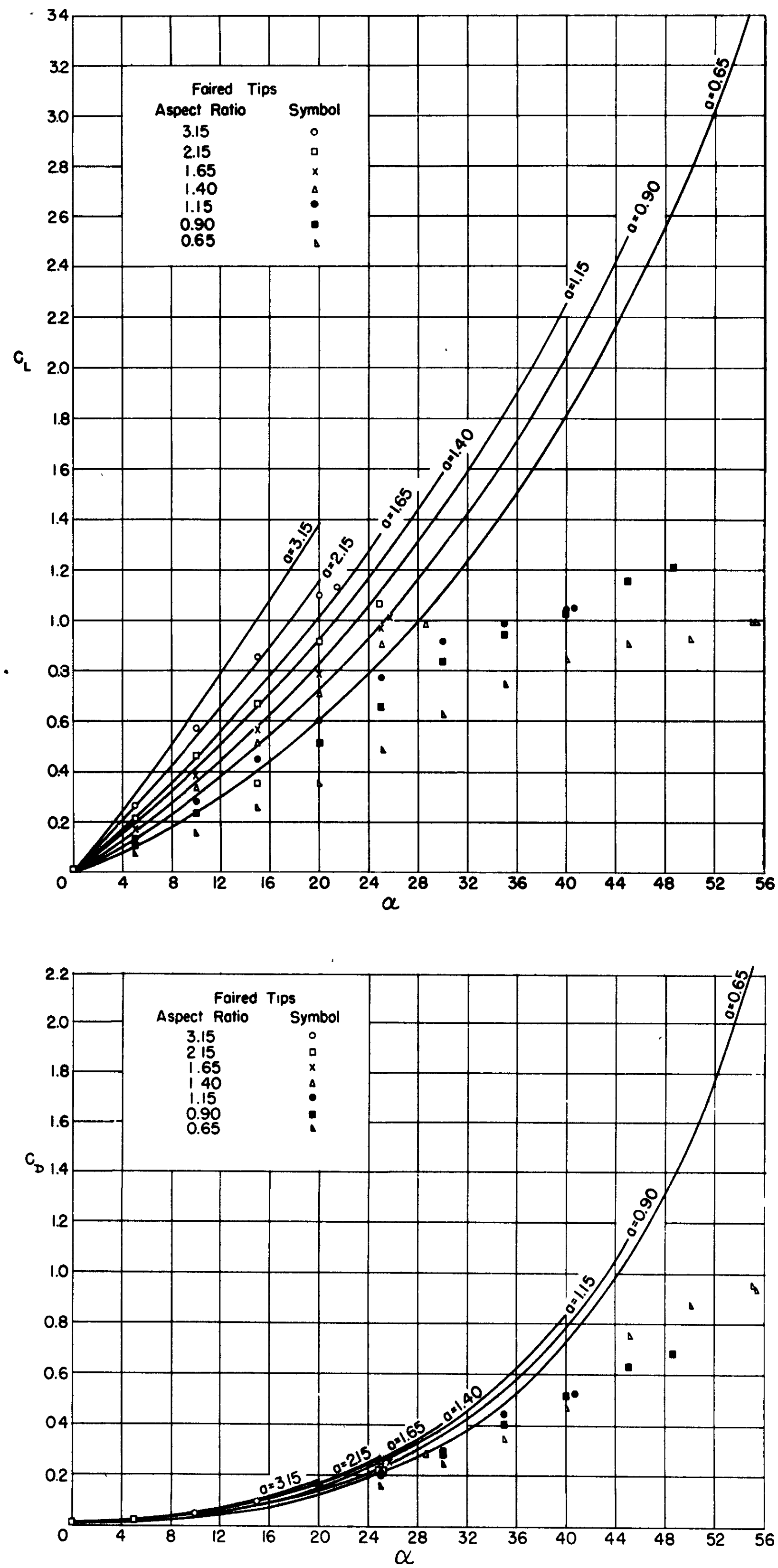


Figure 1b - Rectangular Plan Forms with Tips Faired by Adding Semi-Circle to the Rectangular Spanwise Cross Section

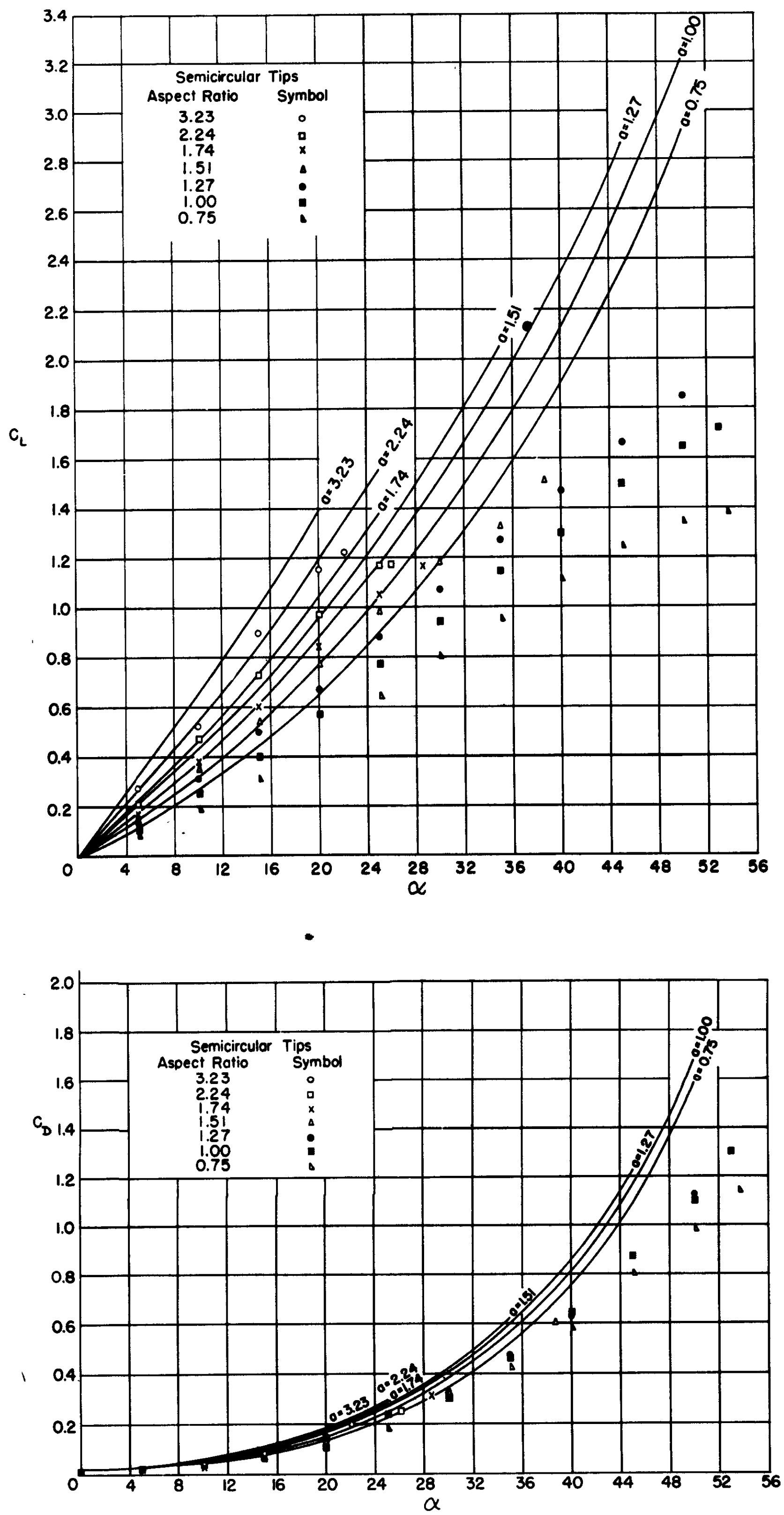


Figure 1c - Plan Forms Having Semi-Circular Tips

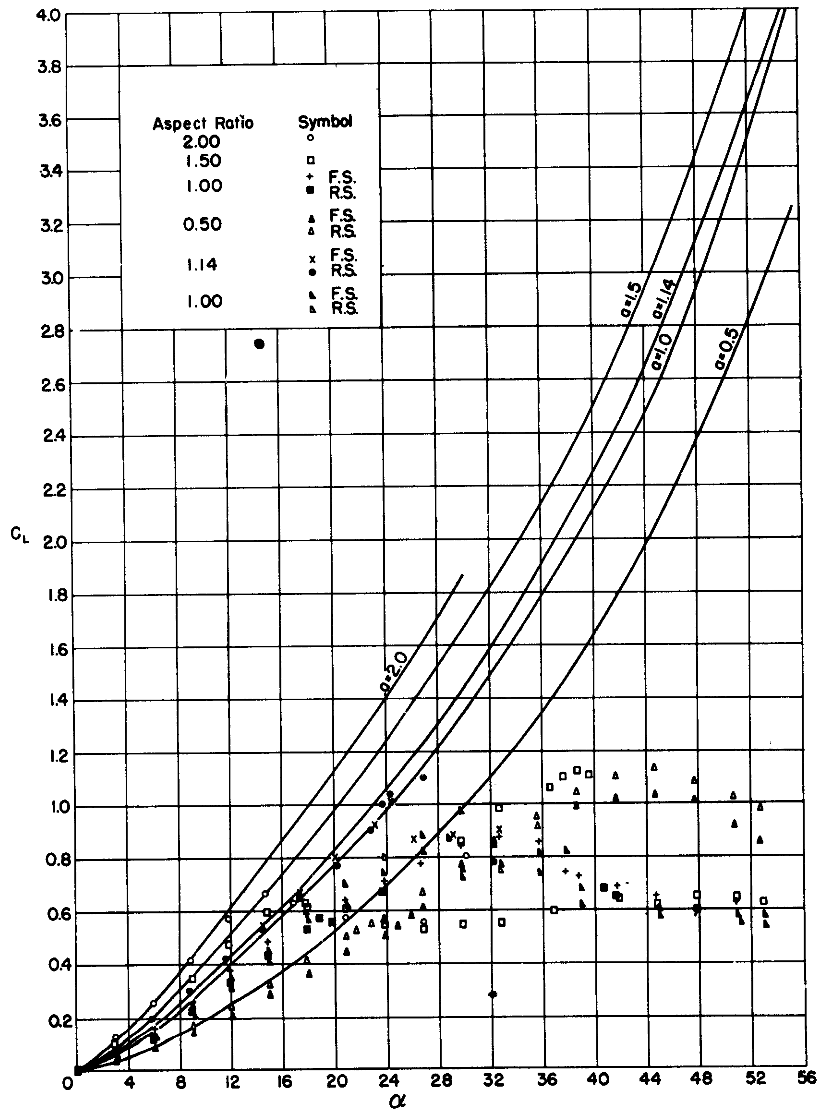


Figure 1d - Flat Plates

The lift and drag data plotted in Figure 1a were measured for wings of rectangular plan form and rectangular spanwise cross section; Figure 1b for wings essentially rectangular in plan form but with the tips faired by adding semicircular ends to the rectangular spanwise cross-section; and Figure 1c for wings with wing tips circular in plan form. The fit of Weinig's theoretical curves for lift coefficient computed for corresponding values of aspect ratio is best for wings with rectangular unfaired tips (Figure 1a). Above about 16°, the theoretical curves do not account for a falling off of the lift coefficient as the angle of attack becomes greater and therefore give values which are too great. The theoretical curves give values which are too great for the whole range of angles of attack for the wings with faired tips and circular

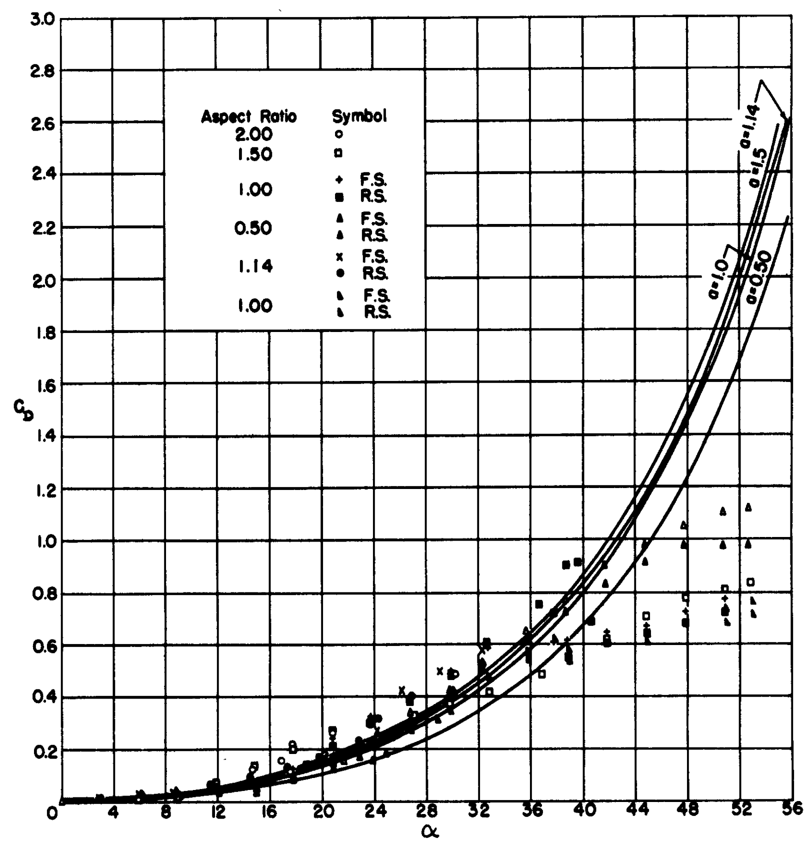


Figure 1d - Flat Plates

Figure 1 - The Comparison of Lift and Drag Measurements with Weinig's Theoretical Curves

tips (Figures 1b and 1c). Obviously, Weinig's theory does not adequately account for the effect of plan form and tip shape on the values of the lift coefficient although, quantitatively, the theoretical curves are of the right shape for angles of attack less than about 20°.

The same is generally true of the curves showing the drag coefficients. The fit is fairly good for wings with rectangular tips (Figure 1a) but is poor for faired tips and circular tips (Figures 1b and 1c). Computed values of induced drag are consistently too great at the larger angles of attack.

Although Weinig's theoretical formulas are the best developed to date for low aspect ratio, it is obvious that refinement of the theory or a new theory is needed to account for the lift and induced drag at large angles of attack and to account for the effects of plan form and tip shape.

The same conclusions can be reached from studying Figure 1d which shows data taken from Reference 2 for flat plates. The fit of the theoretical curves is reasonably good qualitatively up to a certain angle of attack but fails quantitatively to check any specific data by an appreciable percentage. This is not surprising in this case, however, since the theory is based on potential flow in which viscosity effects are disregarded whereas the flow about flat plates is markedly influenced by viscosity when at an angle of attack. One would expect a better fit for thin airfoil sections and this is actually the case.

The Weinig formula for lift coefficient is

$$C_L = \left(\frac{h}{1+h}\frac{2}{\pi}\tan|\alpha| + \frac{h}{1+h}\frac{1}{\cos\alpha}\frac{a}{2}\right) 2\pi \sin\alpha \qquad [1]$$

where α is the angle of attack,

a is the aspect ratio, and

h is equal to $\tanh \dfrac{1}{\frac{a}{2} + \frac{2}{\pi}\sin|\alpha|}$

and for induced drag coefficient

$$C_{D_i} = \frac{C_L^2}{\pi a + 4\sin|\alpha|} \qquad [2]$$

The aspect ratio, a, is defined by

$$a = \frac{b^2}{A}$$

where b is the span and A is the projected area.

These formulas for lift and induced drag, [1] and [2], are for isolated lifting surfaces. Control surfaces are never isolated and therefore a question arises as to what effect the proximity of the hull and other control surfaces has on lift and drag produced. The problem is easily solved in cases where a single control surface abuts with but very small clearance against a large surface that can be considered flat. The span, b, includes the span of the reflection and the area, A, includes the area of the reflection for the purposes of determining aspect ratio; see Figure 2.

Usually, however, the surface to which the control surface is attached is so curved that it cannot be considered flat. Very often also, horizontal surfaces and vertical surface are located close together. The effect of the horizontal surfaces on the lift and drag of the vertical surface and vice versa can usually be neglected since one usually is located fairly symmetrically in the plane of symmetry of the other. A rigorous analytical determination of the effect of the curvature of the hull is a very difficult

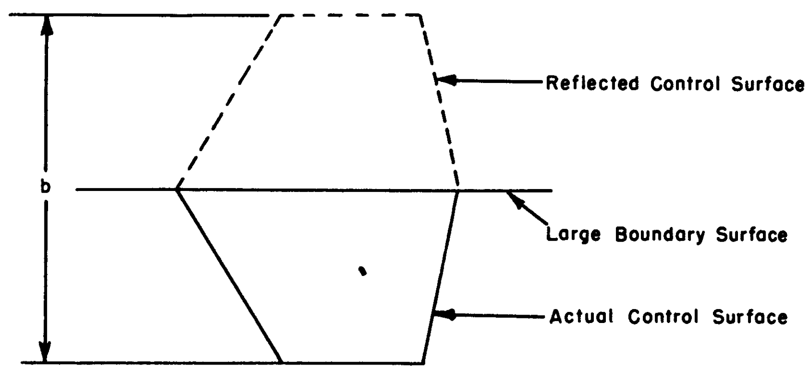


Figure 2

problem and for most specific cases is probably impossible. Some insight into the effect can be gained, however, by considering the idealized case of a control surface intersecting the axis of a cylindrical hull of infinite extent with equal area on each side of the cylinder. In Reference 37, a formula for the induced drag based on an elliptical spanwise distribution of lift is developed for such a configuration using lifting-line theory. The formula is

$$D_i = \frac{L^2}{4q\pi} \frac{1}{b^2\left(1-\frac{4R^2}{b^2}\right)^2} \quad [3]$$

where q is the dynamic pressure,

R is the radius of the cylindrical hull, and

b is the span measured from tip to tip.

The corresponding formula for induced drag based on lifting-line theory for an isolated lifting surface with elliptical spanwise load distribution is

$$D_i = \frac{L^2}{q\pi b_e^2} \quad [4]$$

where b_e is the span. Drawing an equivalence on the basis of the question, "With what span will an isolated surface give the same induced drag with the same lift as the surface-cylinder combination?" Equation [3] can be equated to Equation [4] with the result that the equivalent span is

$$b_e = b\left(1 - \frac{4R^2}{b^2}\right) \quad [5]$$

The question might also be asked, "What is the aspect ratio of an isolated surface which will develop the same lift at the same angle of attack as the surface of the surface-cylinder combination?" Only an approximate answer is immediately available even for the idealized combination. On the basis of a lifting surface having a rectangular spanwise load distribution, the following formula gives the lift of surface-cylinder combination[37]

$$L = qA\ 2\pi \sin\alpha \left(1 - \frac{4R^2}{b^2}\right) \qquad [6]$$

If it were assumed that the cylinder had about the same influence on the total lift when the spanwise distribution is elliptical, the total lift would be given by

$$L = \frac{a}{a+2}\, qA\ 2\pi \sin\alpha \left(1 - \frac{4R^2}{b^2}\right) \qquad [7]$$

The lift produced by an isolated surface using lifting-line theory for an elliptical spanwise distribution is

$$L = \frac{a_e}{a_e+2}\, qA_e\ 2\pi \sin\alpha \qquad [8]$$

Equating Equations [7] and [8] to obtain the equivalent aspect ratio

$$a_e = (a+2)\left(1 - \frac{4R^2}{b^2}\right) - 2 \qquad [9]$$

The equivalent area can also be found by substitution in the formula

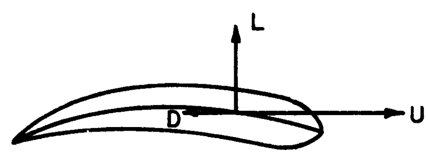
$$a_e = \frac{b_e^2}{A_e}$$

so that

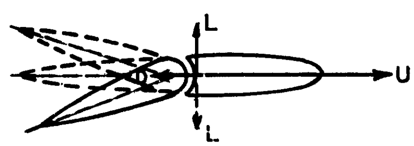
$$A_e = \frac{b^2\left(1 - \frac{4R^2}{b^2}\right)^2}{(a+2)\left(1 - \frac{4R^2}{b^2}\right) - 2} \qquad [10]$$

Although the equivalent span, aspect ratio and area are developed on the basis of lifting-line theory with approximations, it is suggested that these values be used in Weinig's formulas (Equations [1] and [2]) when a theoretical analysis is being made of an actual control-surface assembly. It would be well to check the accuracy of so doing by experiment.

A lifting surface in a fluid can generate its lift force in two ways: (1) By means of camber and (2) by means of an angle of attack relative to the fluid. Usually when camber is used to generate lift, the centerline of the cross-section of the lifting surface is continuously curved according to a certain law and since, after construction, this centerline curve is unalterable, the lift so generated is always in a given direction and of constant magnitude at a given speed. Now the flapped lifting surface provides a means of approximating the continuous camber by a series of straight lines (see Figure 3) and also of changing the magnitude and sense of the camber and therefore the size and direction of the lift force generated. Thus there exists a natural hydrodynamic classification of two types of control surfaces: (1) flapped, which makes primary use of camber and (2) all-movable, which makes use of angle of attack exclusively. The effectiveness of these two types for turning the vessel will now be discussed.



True Camber



Approximate Camber in Either Direction

Figure 3

FLAPPED CONTROL SURFACE

PRODUCTION OF LIFT AND DRAG

The effectiveness of a control surface is determined by a weighing of three characteristics according to the circumstances of the application. One characteristic is the maximum lift coefficient; the second is the drag characteristic —which can be further broken down to the drag at zero angle of attack and the drag at maximum lift; the third is the slope of the lift-coefficient curve plotted as a function of angle of attack. The first two characteristics, considered together, determine the size and plan-form of the control surface necessary to fulfill the severest demands made on it as estimated from the particular criterion for maneuverability chosen for the vessel. The method of determining actual size and plan-form from control-surface data in coefficient form is obvious and therefore need not be discussed here. The third characteristic is involved when questions of stability—with controls fixed, free, or automatically controlled—enter the problem or when maneuvers less than the severest are to be analyzed once the size and plan-form are fixed. For the purpose of demonstrating a method of taking into account this third characteristic, the slope effectiveness of a flapped hydrofoil section of infinite aspect ratio can be determined by finding what angle of attack, α_e, of an equivalent rigid section will produce the same lift as a flapped section with the flaps deflected to the angle δ; see Figure 4. The relation involved can be expressed as

$$\alpha_e = E\delta$$

where E can be called the slope effectiveness factor. In Figure 5, theoret-

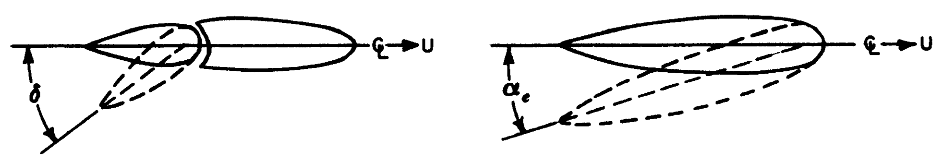


Figure 4 - Definition of α_e and δ

ical values of E are plotted as a function of the ratio of the flap chord to the total chord. The theoretical values are independent of aspect ratio[4] and are for flaps with the gap at the hinge sealed to prevent leakage of the flow through the hinge. Tests on flapped surfaces[5] with unsealed gaps for which a few data are plotted in Figure 5 show that the theoretical values are not exceeded. The loss in effectiveness due to unsealed hinge-gaps is very apparent although the data of Figure 5 were not obtained specifically to determine the effect of the gap. The loss in slope effectiveness is controlled largely by the shape of the gap and the kinematic viscosity of the fluid. When

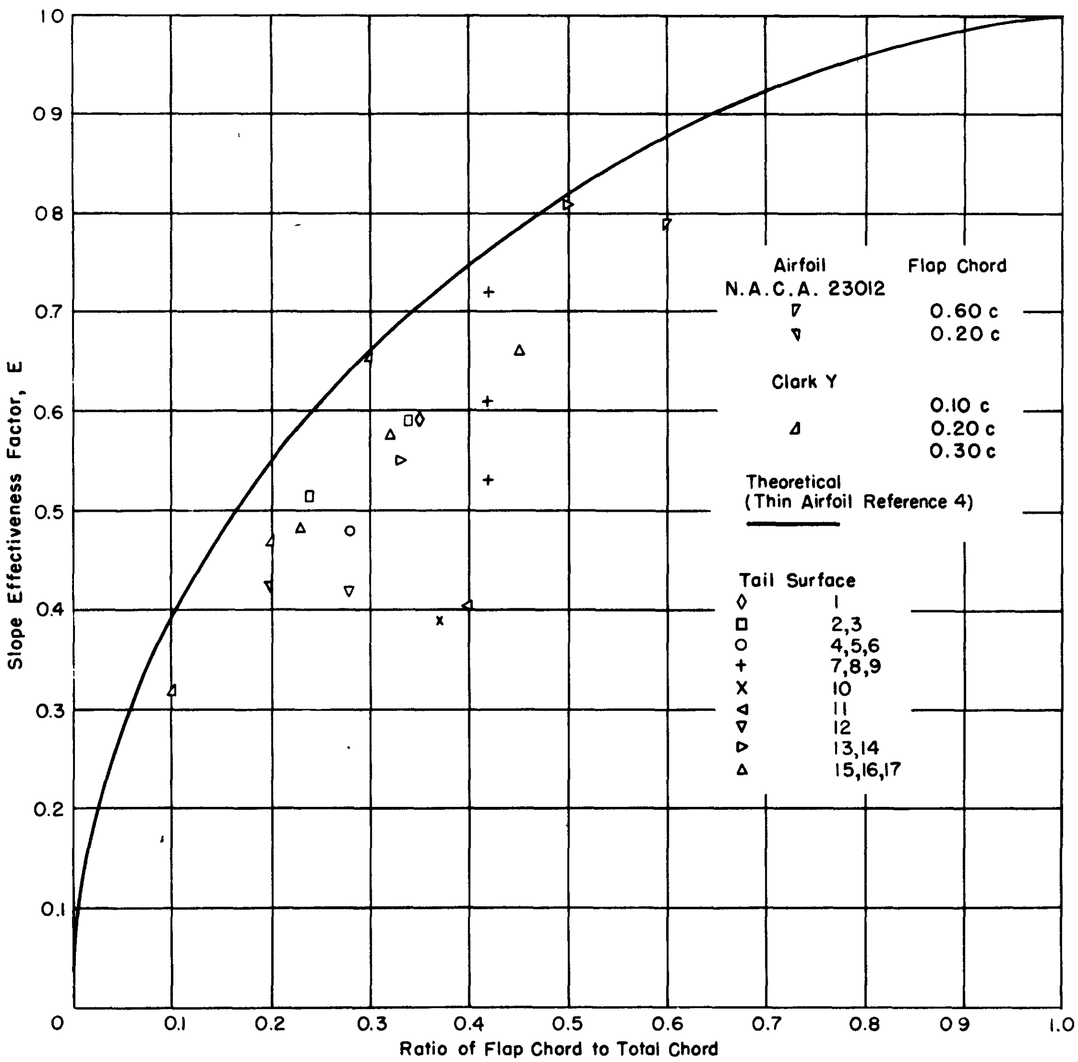


Figure 5 - The Slope Effectiveness of Flaps as a Function of Fraction of the Total Chord

The numbered tail surfaces refer to surfaces reported in Reference 5.

aerodynamic data are being considered in the design of control surfaces in water, the Reynolds number, therefore, at which the data were taken must be considered. If the Reynolds number is not sufficiently high the influence of the gap in water should be checked systematically by experiment. Sealing the gap presents a means of obtaining maximum effectiveness for a given flap but mechanical complications and maintenance difficulties of the seal quite often preclude its use. Nevertheless, even though sealed, E is never as large as unity for the flapped control surface. To demonstrate a possible consequence, a surface with a sealed 0.40-chord flap has a slope effectiveness of 0.748. To obtain equal lift forces, the flap deflection would have to be 1.337 times the deflection of the all-movable surface or the area would have to be 1.337

times as great. The increased area would result in increased drag and the increased angular deflection, in some cases where there is a limit on the control-surface area, may require deflections in excess of the useful range since this range is limited by stalling phenomena.

Although this particular comparison seems to favor the all-movable surface the slope effectivensss factor, E, does not account for phenomena by which the maximum lift of a control surface is limited. One limit on the lift produced by an all-movable surface is the stalling phenomenon which occurs at some angle of attack, which may be as large as 45 degrees for low aspect ratio. However, the chordwise distribution of lift due solely to angle of attack shows a large negative peak which is conducive to cavitation. Should cavitation be a serious problem, the flapped control surface could offer definite advantages since the flap tends to distribute the lift more evenly chordwise and therefore delays the inception of cavitation. A larger maximum lift therefore might be possible. However, as will be shown later, the all-movable control surface has definite advantages while the vessel is actually turning. Therefore, the choice of type of control surface must only be made after a careful consideration of the specific design problem in which the area of the surface, the maximum lift coefficient and the effectiveness for turning are considered.

EFFECTIVENESS FOR TURNING THE VESSEL

The following analysis of control-surface effectiveness is applicable in general to motions in either the horizontal (lateral) or vertical (longitudinal) plane. To avoid duplication of symbols in showing this applicability, a general nomenclature will be used where necessary. Lift, L, drag, D, and the resultant, R, are already general in that they are oriented with respect to the flow about the control surface and their coefficients are based on the resultant velocity at the control surface. The symbol X will be used for forces along the centerline of the vessel and no duplication of nomenclature arises. The symbol, r, will be used to indicate rotational velocity, β to indicate angle of flow orientation at the control surface and N to indicate forces normal to the centerline of the vessel. For specific application, these three must be interpreted with regard to the plane of motion under consideration.

The flapped control surface, by definition, involves a fixed portion ahead of the flap which cannot be oriented into the flow. When the vessel acquires a turning velocity, the orientation of the flow with respect to the control surface is changed. An angle of attack results which causes a force in opposition to the initial control force with a consequent reduction in rate of turning.

From Figure 6

$$\beta = \arctan \frac{rl}{u} - \epsilon \qquad [11]$$

and

$$\tan \gamma = \frac{C_D}{C_L} \qquad [12]$$

where r is the rotational velocity,

l is the distance from the center of pressure of the turning point of the vessel,

u is the velocity in the direction of the centerline of the vessel, and

ϵ is the angular deflection of the flow at the control surface caused by the influence of other components of the vessel.

The theoretical lift coefficient is given by Equation [1] and the theoretical induced drag coefficient by Equation [2]. The effective angle of attack of the flapped control surface for substitution in Equation [1] is

$$\alpha = E\delta - \beta \qquad [13]$$

and C_L is taken positive when α is positive. To the induced drag of the

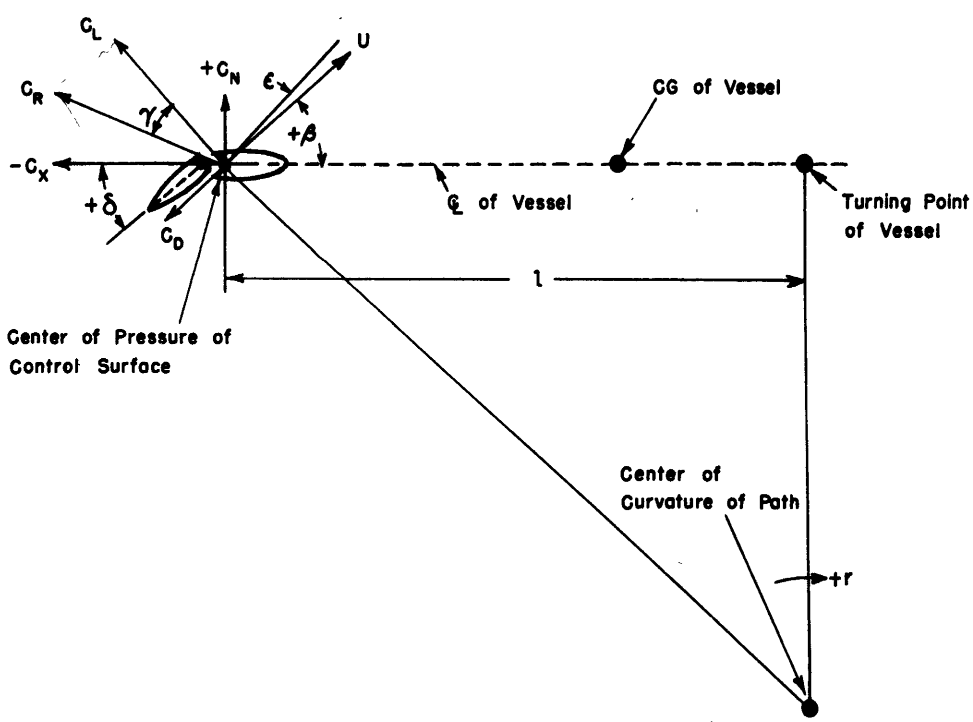


Figure 6 - Schematic Diagram Showing the Geometry of a Turn and the Forces on a Flapped Control Surface

flapped control surface must be added its frictional, or profile, drag coefficient so that the total drag coefficient is

$$C_D = C_{D_0} + C_{D_i} \qquad [14]$$

where C_{D_0} is the frictional (or profile) drag coefficient, and C_D is taken positive acting in the direction of the flow. Now, to turn a vessel, the control surface must produce a moment about the center of gravity of the vessel. If an X force is defined as a force in line with the centerline of the vessel, it can produce no turning moment about the center of gravity. If an N force is defined as a force normal to the centerline of the vessel, it is solely responsible for the moment. The N force is taken as positive when it tends to cause positive rotation of the hull and the X force is taken positive forward. Writing the N and X forces in coefficient form,

$$C_N = \frac{N}{\frac{1}{2}\rho U^2 A} \qquad C_X = \frac{X}{\frac{1}{2}\rho U^2 A}$$

where A is the projected area of the control surface. The range of values of C_N which may be produced by a given control surface is then an appropriate criterion for its effectiveness for turning the vessel. Similarly, if the desired range of the normal force, N, is known from an analysis of the required maneuverability of the vessel, the control surface can be designed to satisfy the requirements. For purposes of maneuverability, the X force is usually a secondary consideration since the available thrust is usually sufficient to overcome small changes in the total resistance of the vessel. For long range, however, it is important to keep C_X as small as possible.

The values of C_N and C_X produced by a given control surface are

$$C_N = C_L \cos\beta - C_D \sin\beta \qquad [15]$$

$$C_X = -(C_L \sin\beta + C_D \cos\beta) \qquad [16]$$

To make C_N a maximum and C_X a minimum, it is important to keep the drag of the control surface as small as possible. When the control surface is developing large lifts, the drag is mainly induced drag. Examination of Figure 6 and Equations [15] and [16] reveals the importance of reducing the drag of the control surface to a minimum to ensure effective enforcing of a turn, since the effect of drag is to rotate the resultant force astern, consequently reducing the magnitude of the N force and increasing the total resistance.

It is also obvious that the control surface will always be more effective in checking the turn than in enforcing it. For example, in checking the turn shown in Figure 6, β is positive and α is negative, making C_L negative which, according to Equation [15], causes the effect of the lift and drag forces to be additive and the N force increases negatively. The X force (Equation [16]) is reduced and even positive (forward) when $-C_L \sin \beta$ is greater than $C_D \cos \beta$. To reduce the induced drag of the control surface as much as possible the aspect ratio should be as high as possible and the plan-form should produce at least approximately an elliptical spanwise load distribution since it has been proved that the elliptical spanwise load distribution results in minimum induced drag.[6] Such a distribution is also advantageous to reduce the structural problem and to avoid cavitation.

TORQUE

The torque necessary to turn the flap of a flapped control surface is important in designing the supporting structure and the steering gear. Of equal importance are the forces on the flap at any angular position. The torque of flaps is best balanced by the use of "overhang" which is defined as that area ahead of the hinge line extending all along the span. This type of balance for flaps has displaced all others because of its versatility in producing practically any degree of balance as a function of both deflection and angle of attack. The "horn balance" (Reference 7), which has enjoyed some popularity, gives adverse increases in hinge moment with angle of attack. The versatility of the overhang balance arises because the torque and the forces are dependent on the shape of the flap, particularly that part ahead of the hinge line (i.e., the overhang) which can be varied in many ways. No single formula can be used satisfactorily to predict the forces and moments on the tremendous number of possible types. Reference must be made to the data resulting from tests of the type under consideration. The static moment produced by very many types of flaps equipped with many types of overhang balance has been measured by the NACA and reported in References 8 through 30. It might be noted that any degree of balance and practically any desired variation of torque with angle of flap deflection can be obtained by properly designing the shape of the flap section ahead of the hinge and the surfaces that control the variation in hinge gap with deflection. The same is true in water, but the quantitative effect of various arrangements should be checked for scale effect because the effect is largely dependent on Reynolds number which can become very large for the surface itself at high speeds in water.

ALL-MOVABLE CONTROL SURFACE

PRODUCTION OF LIFT AND DRAG

The all-movable control surface produces lift as a result of the angle of attack of the whole surface. Thus, on the curve of slope effectiveness, Figure 5, it is represented by the chord ratio of 1 giving an effectiveness of 1.

EFFECTIVENESS FOR TURNING THE VESSEL

Unlike the flapped control surface, the all-movable has no fixed portion and no inherent limit to the deflection relative to the centerline of the vessel. When the vessel starts to turn, the angle of attack can always be adjusted to the maximum relative to the flow angle, β, at the location of the control surface as shown in Figure 7.

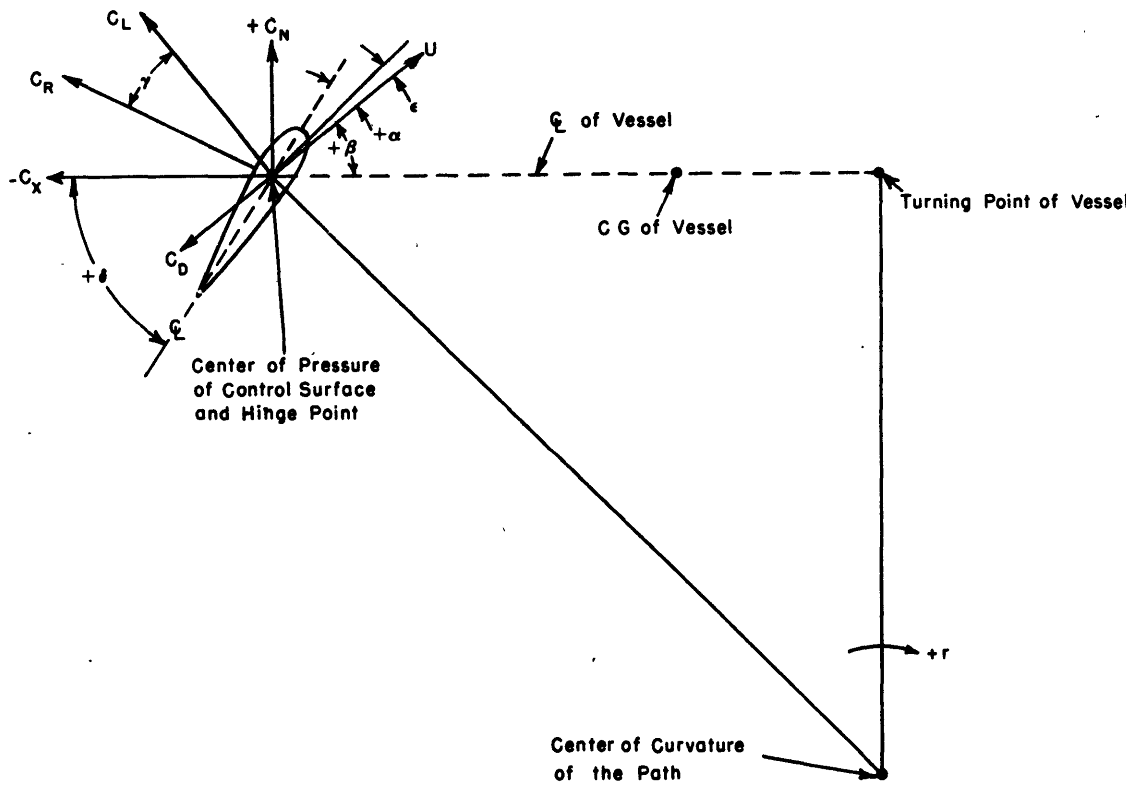


Figure 7 - Schematic Diagram Showing the Geometry of a Turn and the Forces on an All-Movable Control Surface

As before

$$\beta = \arctan \frac{rl}{u} - \epsilon \qquad [17]$$

and

$$\tan \gamma = \frac{C_D}{C_L} \qquad [18]$$

The theoretical lift coefficient developed by the control surface is given by Equation [1] and the theoretical induced drag coefficient by Equation [2]. The effective angle of attack of the all-movable control surface for substitution in Equation [1] is

$$\alpha = \delta - \beta \qquad [19]$$

where δ is the deflection of the control surface from the centerline of the vessel. To the induced drag coefficient must be added the profile drag coefficient, usually taken to be the drag coefficient at zero lift to obtain the total

$$C_D = C_{D_0} + C_{D_i} \qquad [20]$$

The N and X forces, which are used to establish the effectiveness for turning, are determined in the same manner as for the flapped control surface. Repeating the result here,

$$C_N = C_L \cos \beta - C_D \sin \beta \qquad [21]$$

$$C_X = -(C_L \sin \beta + C_D \cos \beta) \qquad [22]$$

With respect to the importance of keeping the drag a minimum, the same comment as was made for the flapped control surface applies.

TORQUE DUE TO STATIC MOMENT

The lift produced by the angle of attack of the all-movable control surface is the result of pressure distributed over the surface. The location of the center of pressure (or the location of the resulting lift force) varies between 30 and 45 percent of the chord from the leading edge of low-aspect-ratio surfaces for the range of angles of attack where the greatest lifts are developed.[3] The exact location depends on aspect ratio, angle of attack and plan-form. In order to predict suitable locations for the axis of rotation of a family of control surfaces, so as to minimize the static moment, calculations can be made by methods to be described later. The best location for a specific plan-form can best be determined from actual tests. Tests on a general series of plan-forms are planned from which it may be possible to choose a suitable location for the axis of rotation.

TORQUE DUE TO ANGULAR VELOCITY OF THE CONTROL SURFACE

The torque required for turning the all-movable control surface can be determined theoretically from the theory developed for studying the unsteady lift of airfoils. References 31, 32, 33, and 34 give the theory in considerable detail and can be referred to by those desiring to examine the complexities of the problem and to derive the maximum of refinement. For the present purpose, it is sufficient to state that the lift forces and the moment due to the turning velocity of the control surface are never greater at a given angle of attack than the lift and moment due to the given angle of attack having been held constant for a long time. This is due to the fact that there is some lag in producing the hydrodynamic-flow phenomena which take place when a lifting surface is deflected to produce lift. Therefore, if the mechanism which turns the control surface can produce torques at least equal at each angular deflection to the static moment produced under conditions of steady flow at that angle of attack, it will always be possible to move the control surface to any position.

TORQUE DUE TO ANGULAR ACCELERATION OF THE CONTROL SURFACE

In addition to accelerating the mass of the structure of the control surface to a turning velocity, the turning mechanism must be able to accelerate and decelerate what is known as the virtual additional mass. The concept of a virtual mass arises from the necessity for accelerating the flow of fluid associated with the control surface when the control surface itself is accelerated.

If the control surface is accelerated in turning about an axis in line with the half-chord, there is theoretically no hydrodynamic force developed by the acceleration but there is a moment. This moment has the magnitude (References 31 and 35)

$$T = -k' \frac{1}{48} \pi \rho c^4 b \frac{d\dot{\delta}}{dt} \qquad [23]$$

where T is the moment or torque,

ρ is the mass density of the fluid,

c is the chord,

b is the wetted span,

$\dot{\delta}$ is the turning velocity,

t is the time, and

k' is the correction for finite aspect ratio.

If the acceleration in turning takes place about an axis other than one in line with the half-chord, an additional torque is felt having the magnitude

$$T = -k \frac{1}{4}\pi\rho c^2 b l_1^2 \frac{d\dot{\delta}}{dt} \qquad [24]$$

where l_1 is the distance between the half-chord and the location of the axis of rotation, and k is the correction for finite aspect ratio.

Both of the factors, k and k', are mainly functions of aspect ratio and, to a much lesser degree, functions of plan-form and cross-sectional shape. Very few data are available relating k and k' to aspect ratio but the order of magnitude, at least, can be obtained from Figure 8 which is a reproduction in part from figures of Reference 34. Values used from Figure 8 should be sufficiently accurate for engineering purposes.

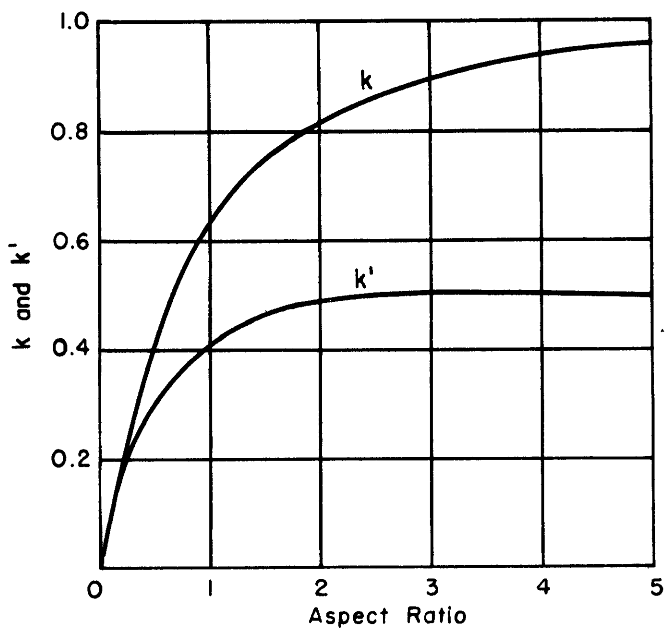


Figure 8 - The Variation of the Factors k and k' with Aspect Ratio for Use in Estimating the Virtual-Mass Effects

COMPARISON OF CONTROL EFFECTIVENESS OF FLAPPED AND ALL-MOVABLE CONTROL SURFACE

To demonstrate the relative effectiveness for turning of flapped and all-movable control surfaces, the following numerical example was calculated based on specifications for a hypothetical family of surfaces. These specifications are contained in Table 1. Although the tabulated specifications are hypothetical in that they are not measured values for any existing surfaces, they are realistic in that the values chosen are typical of what might be realized. Data on flapped, low-aspect-ratio control surfaces relative to the stalling characteristics are scarce but from what are available (Reference 5) and from data for infinite aspect ratio (References 8 through 30) it is estimated that the flap on an average will stall at about 30 degrees. What few data are available for the whole surface (References 2 and 3) indicate that the stall angle is very dependent on plan-form. Therefore arbitrary limits of 15°, 25°, 35°, and 45° were set for aspect ratios 4, 3, 2, and 1 respectively which fall within the average variation with aspect ratio.

TABLE 1

Specifications for Hypothetical Control Surfaces

Control Surface Type	Effective Aspect Ratio a	Flap Chord Ratio c_f/c	Flap Slope-Effectiveness Ratio E	Deflection Limit δ, degrees	Angle of Attack Limit α, degrees	Flow Angle Limit β, degrees
Flapped	1	0.20	0.550	30	-	45
Flapped	2	0.20	0.550	30	-	35
Flapped	3	0.20	0.550	30	-	25
Flapped	4	0.20	0.550	30	-	15
Flapped	1	0.50	0.818	30	-	45
Flapped	2	0.50	0.818	30	-	35
Flapped	3	0.50	0.818	30	-	25
Flapped	4	0.50	0.818	30	-	15
All-Movable	1	1	1	None	45	None
All-Movable	2	1	1	None	35	None
All-Movable	3	1	1	None	25	None
All-Movable	4	1	1	None	15	None

Calculation of the normal force coefficient, C_N, and the resistance coefficient, C_X, for the control surfaces specified by Table 1 are based on Weinig's formulas for lift and drag, Equations [1] and [2], because sufficient experimental data were not available. As pointed out above, these equations do not give accurate results for the complete range of angles of attack, but the comparison of the type of control surfaces is not jeopardized because the same formulas are used throughout and are qualitatively correct. The same comparative result would be obtained if experimental data were used instead of Equations [1] and [2].

The calculated values of C_N and C_X are shown all to the same scale in Figures 9, 10, and 11 as functions of the deflection of the control surface, δ, the flow angle, β, and the aspect ratio, a.

EFFECT OF ASPECT RATIO

In every case the effect of lowering the aspect ratio is to increase the size of the maximum normal force coefficient, C_N, available for control and also to increase the size of the corresponding resistance coefficient, C_X. The increase in C_X is at a greater rate than for C_N so that an advantage is noted for high aspect ratios when large normal forces are required. The advantage is realized by increasing the area of the high-aspect-ratio control surface to the point where it can produce with its smaller deflection actual normal forces that are equal to those of the low-aspect-ratio surface. However, at cruising speed when the control surfaces are deflected only a few degrees from zero to maintain trim for long periods of time, the resistance, (i.e., the minimum resistance) will be substantially greater. From the resistance standpoint it appears that, for most applications, it would be profitable to keep the aspect ratio as low as practicable considering other limitations. One limitation will be the structural problem that becomes worse as the chord becomes longer. Another limitation will be the necessity for maintaining a certain control-surface-lift slope to enforce a certain degree of static stability on the vessel without excessive surface areas. Therefore, for every design, there will be an optimum control surface area and aspect ratio which depend on structural, stability, maneuverability and cruising considerations.

EFFECT OF FLOW ANGLE AND CONTROL SURFACE DEFLECTION

The effect of an increase in the flow angle, β, is to decrease, in every case, the size of the normal force coefficient, C_N, available for enforcing a maneuver and to make the resistance coefficient, C_X, positive (i.e., forward). This effect is decidedly severe in the case of flapped control surfaces in comparison to the all-movable. It should be noted that, in the sign convention chosen, enforcing a turn is associated with the simultaneous existence of β and C_N of the same sign. Checking a turn is associated with β and C_N of opposite signs. On Figures 9, 10, and 11 this defines the areas useful for enforcing or checking a maneuver. Consider enforcing using controls

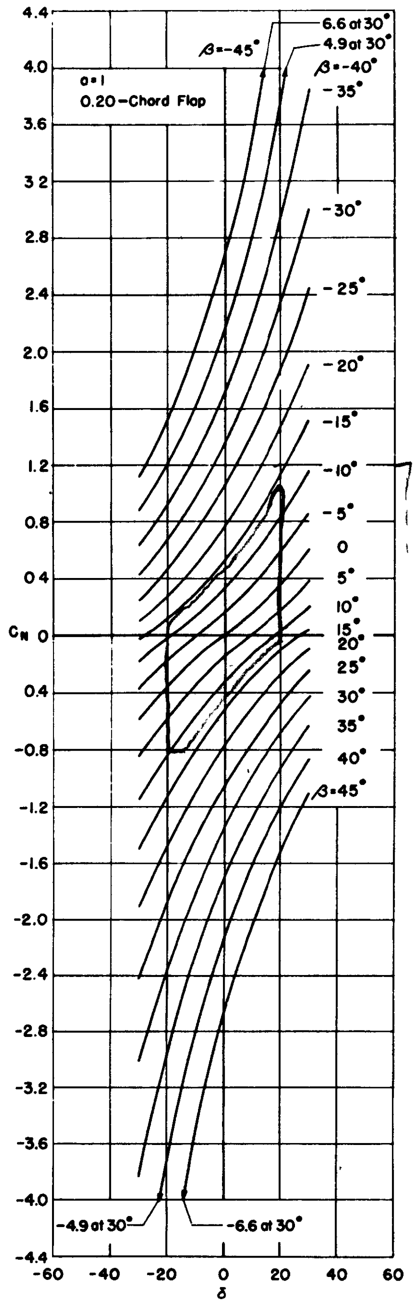


Figure 9a - Normal Force, Aspect Ratio 1

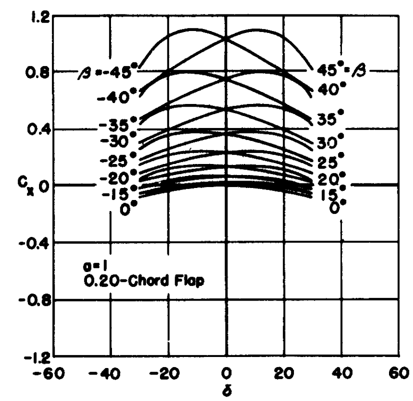


Figure 9b - Resistance, Aspect Ratio 1

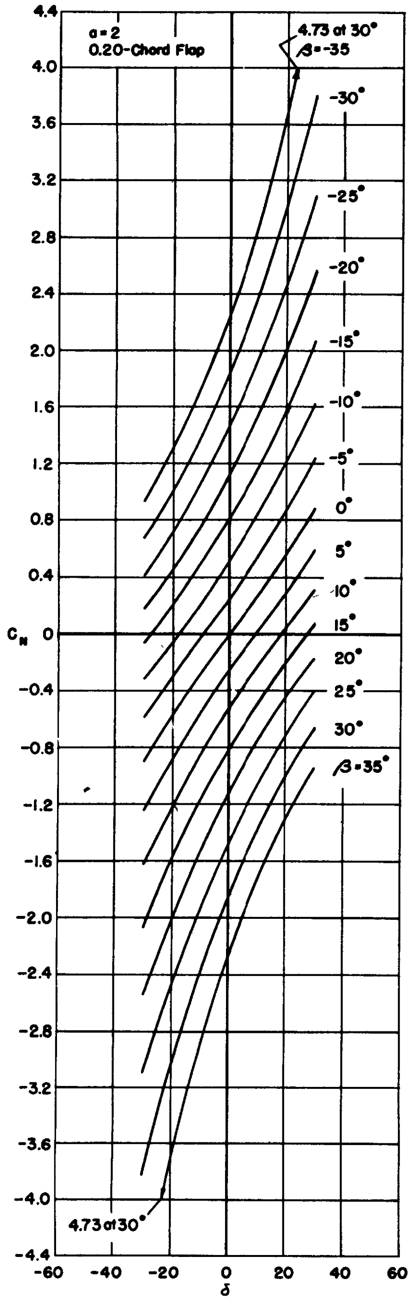


Figure 9c - Normal Force, Aspect Ratio 2

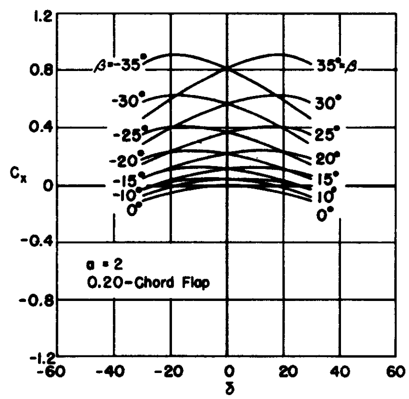


Figure 9d - Resistance, Aspect Ratio 2

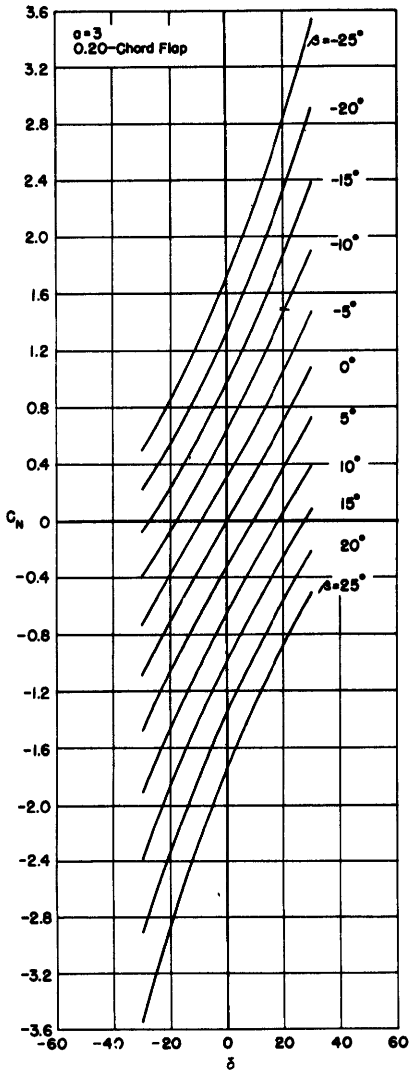


Figure 9e - Normal Force, Aspect Ratio 3

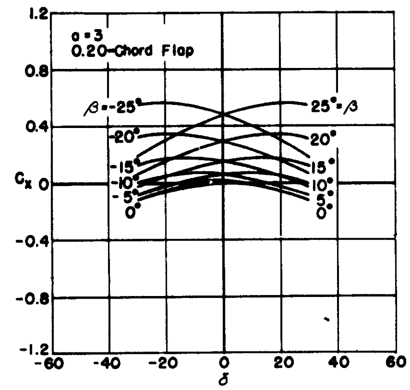


Figure 9f - Resistance, Aspect Ratio 3

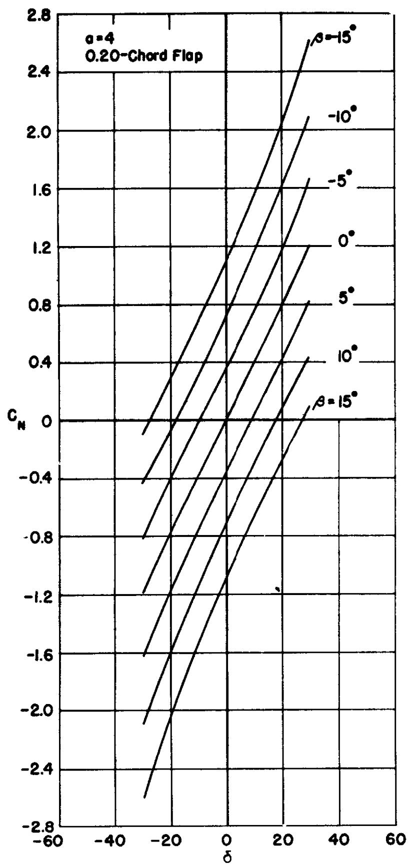


Figure 9g - Normal Force, Aspect Ratio 4

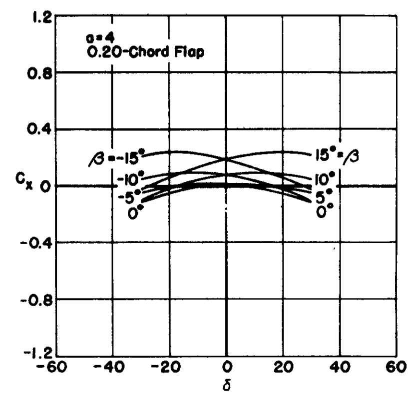


Figure 9h - Resistance, Aspect Ratio 4

Figure 9 - Theoretical Normal Force and Resistance Coefficients Produced by a Control Surface Having a 0.20-Chord Flap

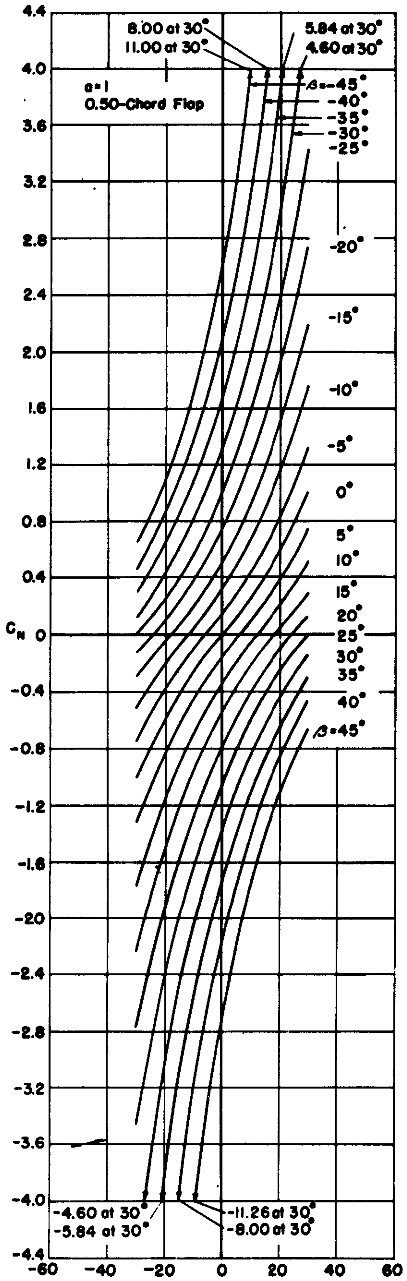


Figure 10a - Normal Force, Aspect Ratio 1

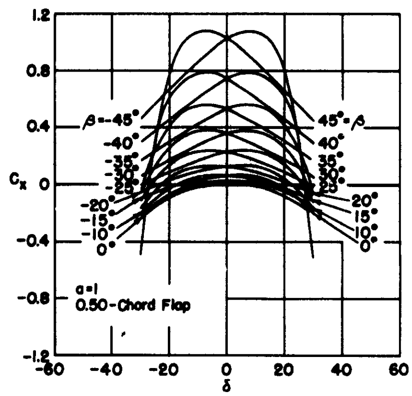


Figure 10b - Resistance, Aspect Ratio 1

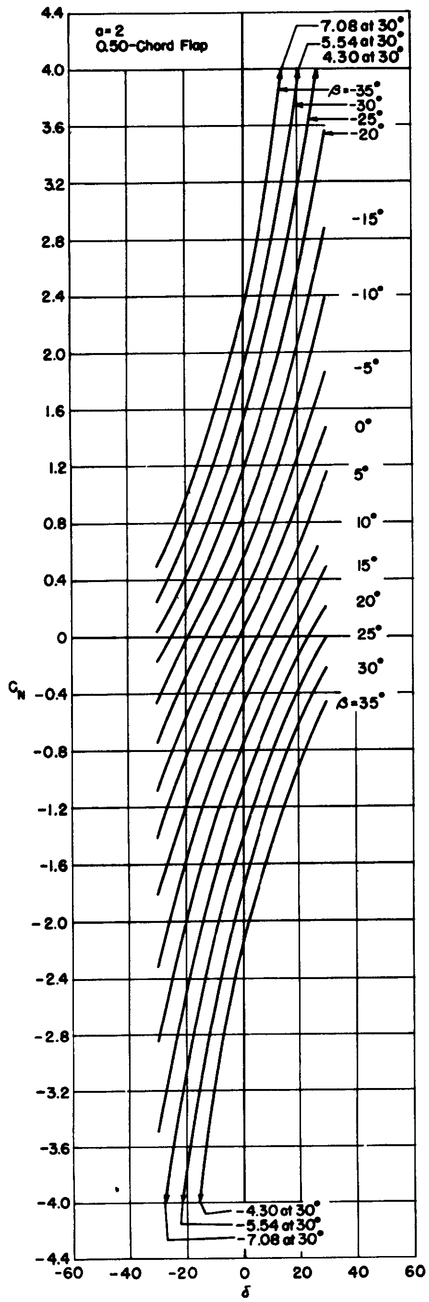


Figure 10c - Normal Force, Aspect Ratio 2

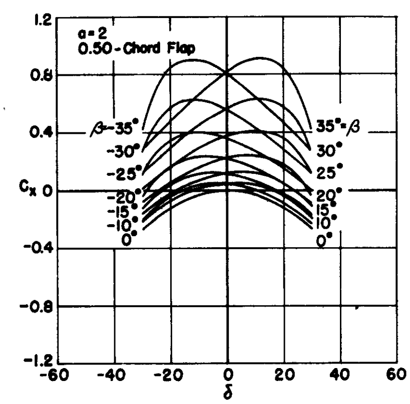


Figure 10d - Resistance, Aspect Ratio 2

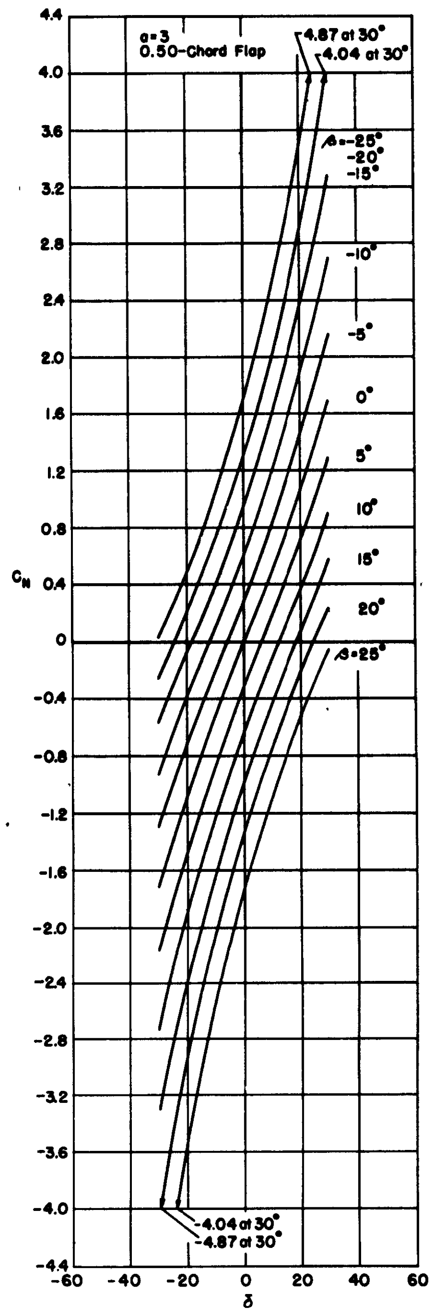


Figure 10e - Normal Force, Aspect Ratio 3

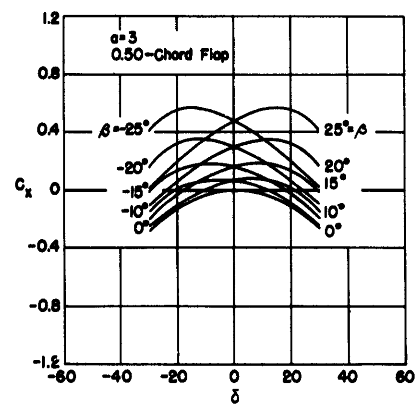


Figure 10f - Resistance, Aspect Ratio 3

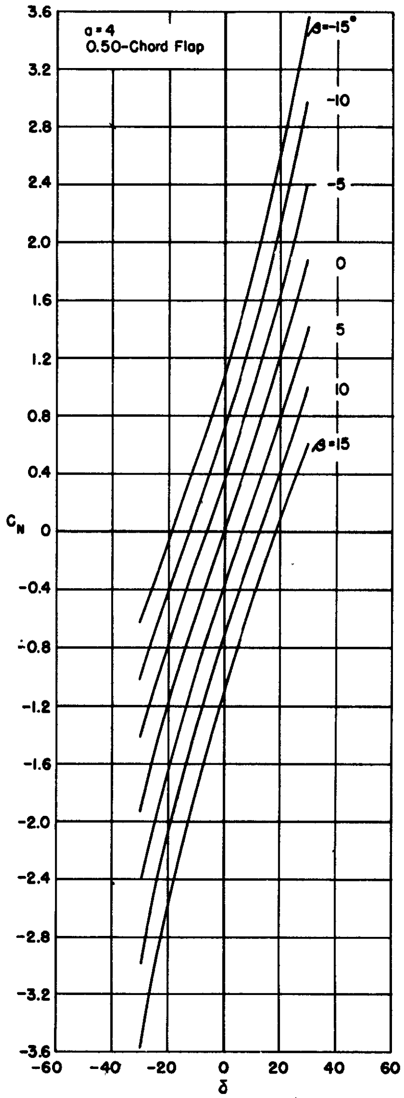


Figure 10g - Normal Force, Aspect Ratio 4

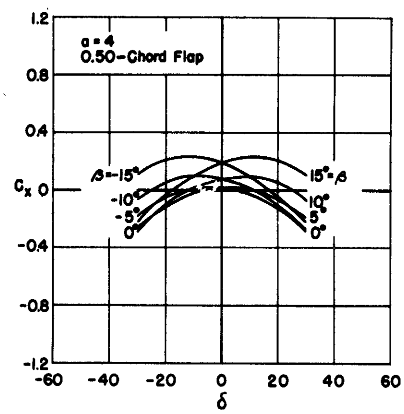


Figure 10h - Resistance, Aspect Ratio 4

Figure 10 - Theoretical Normal Force and Resistance Coefficients Produced by a Control Surface Having a 0.50-Chord Flap

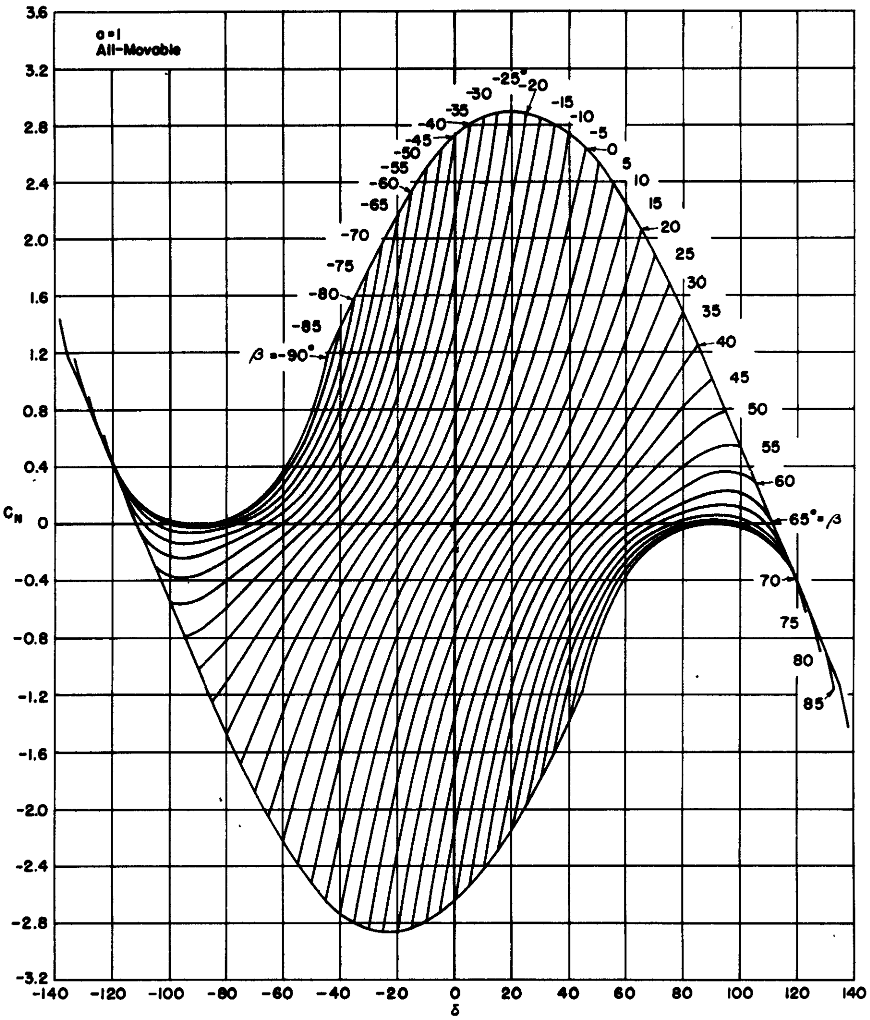


Figure 11a - Normal Force, Aspect Ratio 1

having aspect ratio of two, the area for 20 percent chord flaps (Figure 9), is bounded by $\beta = 0$, $C_N = 0$, and $\delta = +30$ and the maximum C_N contained therein is 0.84. For 50 percent chord flaps (Figure 10), the same boundaries apply but maximum C_N is 1.40. For the all-movable (Figure 11), the maximum C_N is 2.40 and the boundaries are $\beta = 0$, $C_N = 0$, and the values of δ which vary with β such that a deflection of 95° still give positive C_N when $\beta = 80°$. Thus the all-movable control surface is capable of keeping control over the most violent of maneuvers.

The area on Figures 9, 10, and 11 and the size of the maximum side force available for checking a maneuver are always larger than that available for enforcing it. This effect is unnecessarily extreme in the case of flapped control surfaces; the all-movable has much better balance of enforcing and

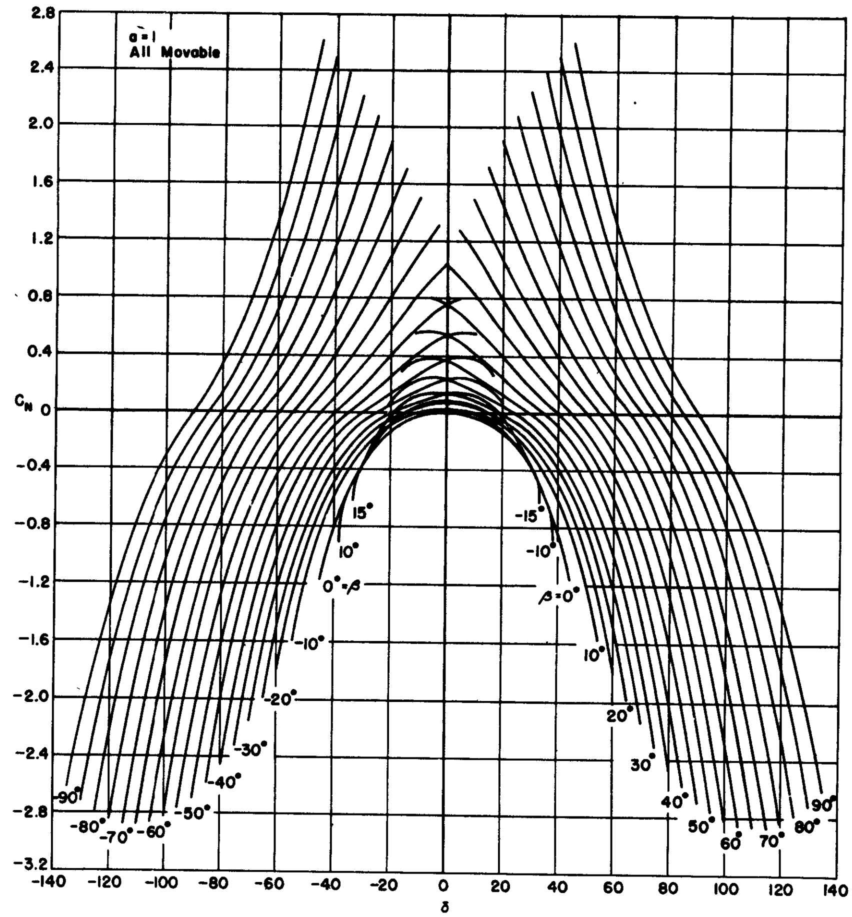


Figure 11b - Resistance, Aspect Ratio 1

checking areas. In neither case is it possible to enforce a maneuver that cannot be stopped unless the area of the control surface is in the first place improperly adjusted to the characteristics of the hull.

DISCUSSION

The outstanding observation to be made from Figure 11 is the large favorable range of all the control parameters that contribute to make the all-movable control surface the ideal one. There is no limit to its deflection, no practical limit to the flow angle at which normal force in the desired direction can be obtained and it can develop very large normal force coefficients for enforcing a maneuver. In a case where maneuverability is the controlling factor on the area, smaller control surface areas can be used and in cases where other factors, such as stability, are the controlling factor, more maneuverability can be gained.

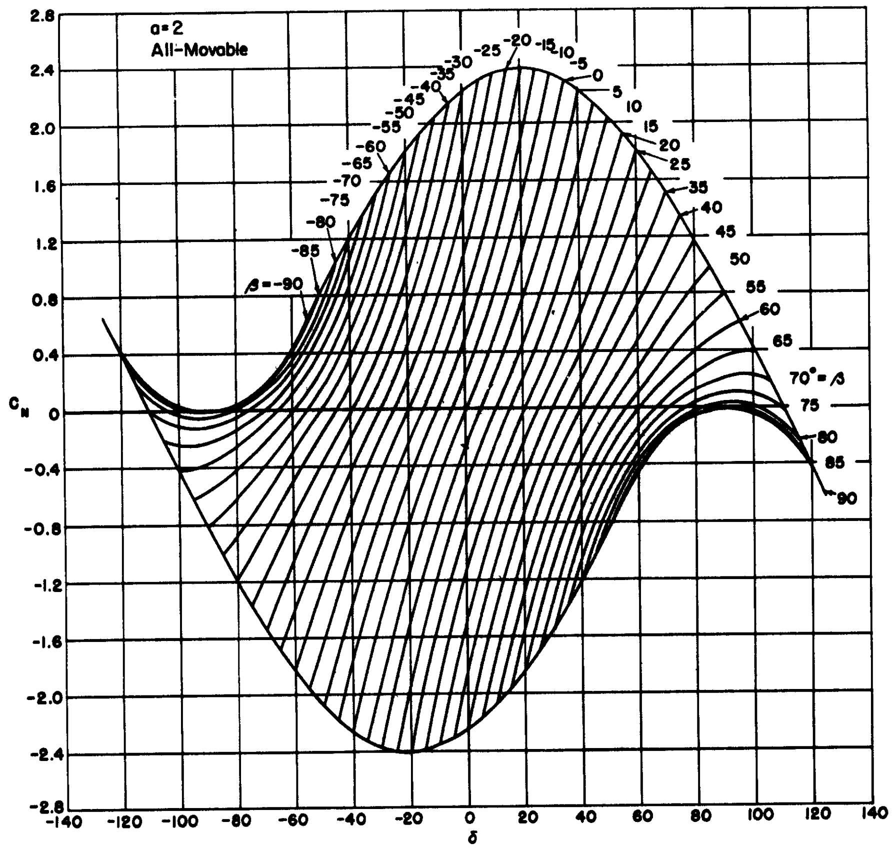


Figure 11c - Normal Force, Aspect Ratio 2

To take full advantage of the all-movable control surface, certain instrumentation might be incorporated. The instruments would measure the flow angle, β, and establish thereby a movable zero reference for a set of stops for the deflection of the control surface. The spread of the stops would be established by tests of the control surface and set so as to limit the deflection just short of the stall. As β changed, both stops would move together so that the helmsman would have available considerably more deflection. It would be well to indicate the position of the stops superimposed on the instrument that indicates control deflection. The helmsman could be given a "maximum maneuver setting" on his control panel, which when used would automatically keep the control surface against the stop until the helmsman releases it. In this way, the vessel could enforce and check a maneuver in the minimum time.

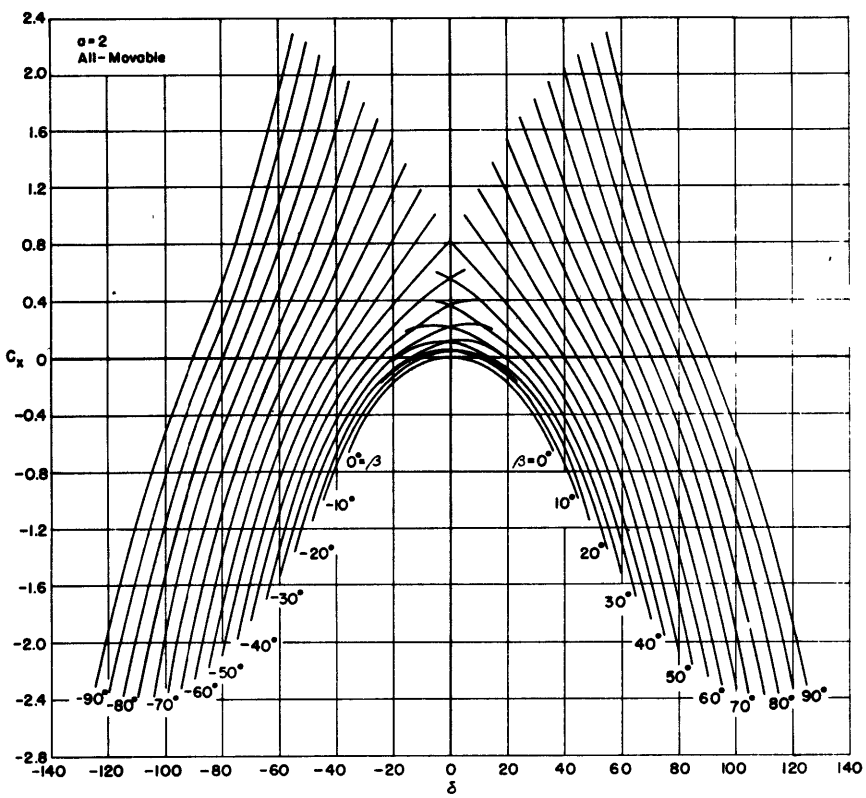


Figure 11d - Resistance, Aspect Ratio 2

EFFECT OF HULL PROXIMITY

The control surfaces are usually attached to the hull at a position near the stern and in general are quite small relative to the size of the hull. Because of its viscosity, the fluid near the hull is in a disturbed condition which is quite different from that of potential flow. The fluid so disturbed is called the boundary layer and the specific condition of flow in the boundary layer depends on the Reynolds number effective for the hull.

In any boundary layer however, two things are apparent: (1) Considering the flow through a cross section of the boundary layer the average velocity varies from zero right at the hull surface to the potential-flow velocity at the outer edge of the boundary layer. If the control surface is first designed to produce the elliptical spanwise load distribution in a flow, the velocity of which is uniform, a knowledge of the velocity distribution in the position of the control surface in the boundary layer permits the designer

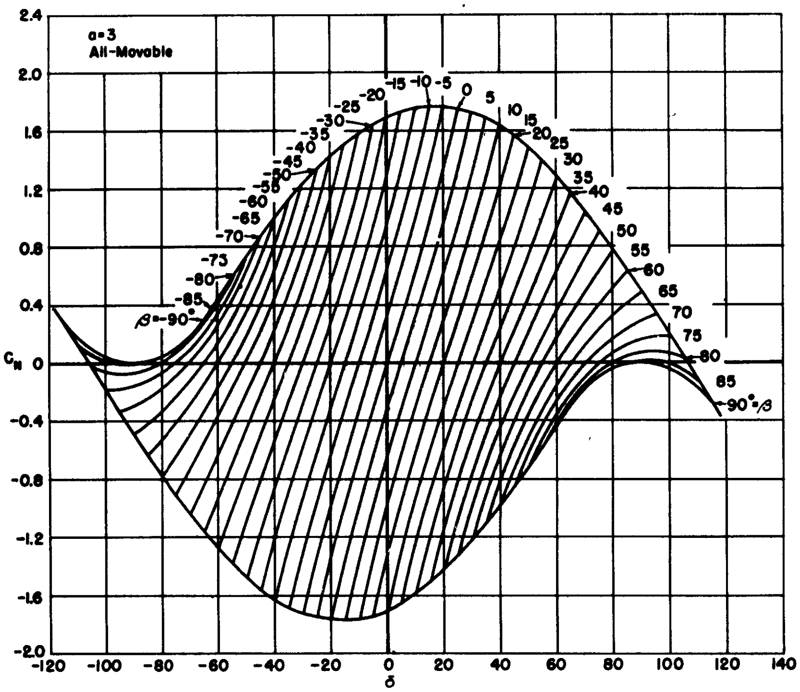


Figure 11e - Normal Force, Aspect Ratio 3

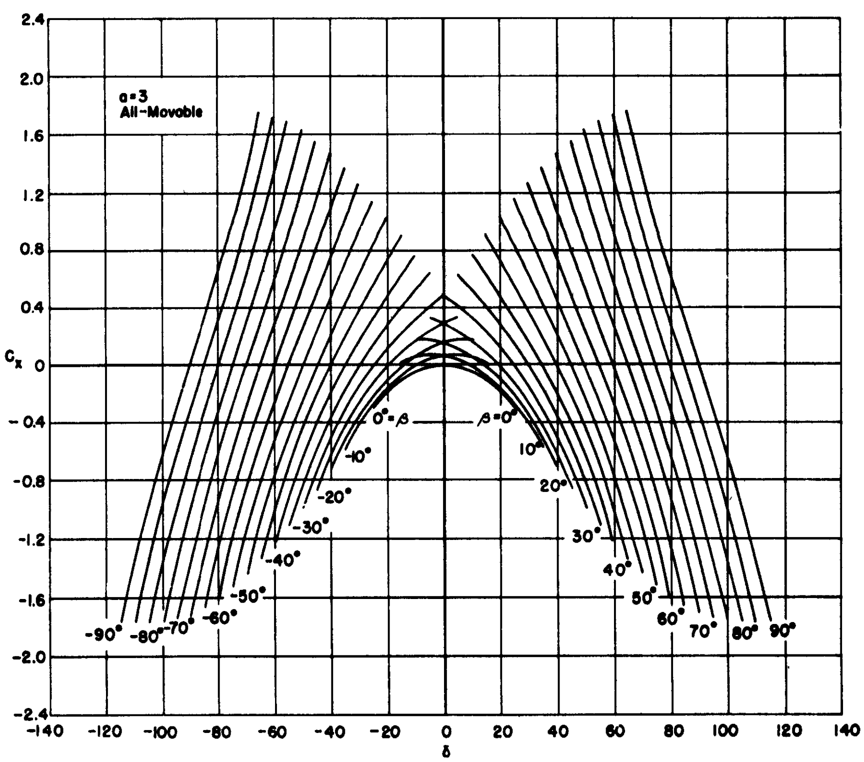


Figure 11f - Resistance, Aspect Ratio 3

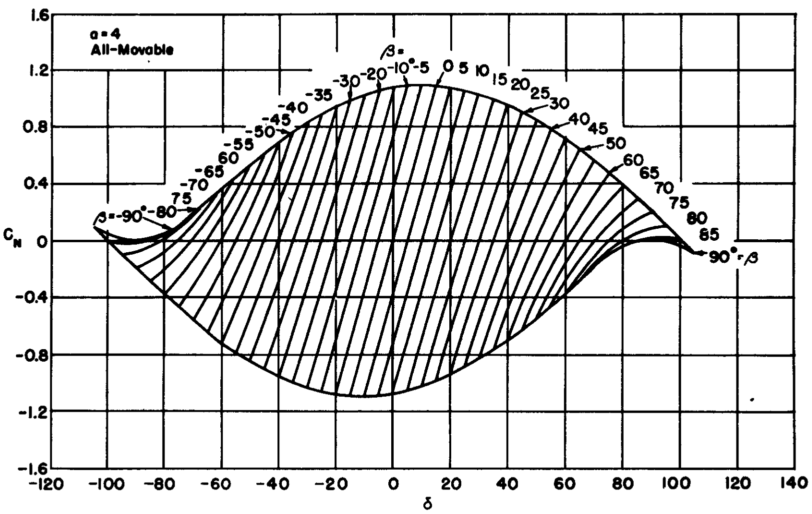


Figure 11g - Normal Force, Aspect Ratio 4

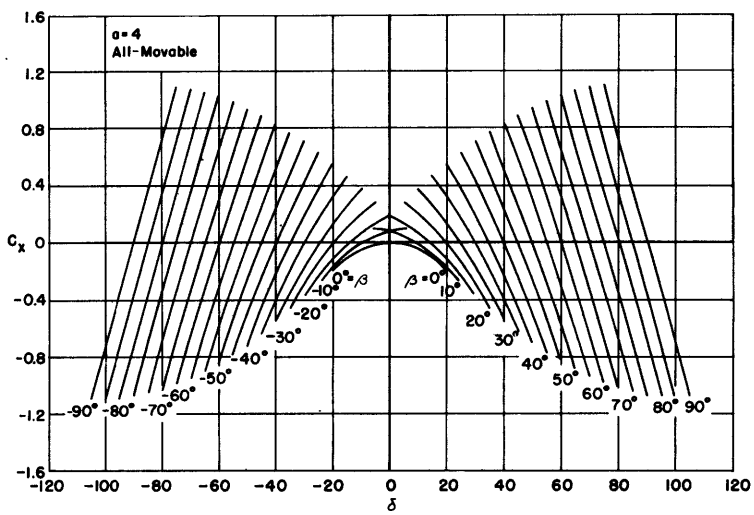


Figure 11h - Resistance, Aspect Ratio 4

Figure 11 - Theoretical Normal Force and Resistance Coefficients Produced by an All-Movable Control Surface

to alter the chord of each section of the control surface in inverse proportion to the square of the velocities. That is

$$c_N = c_1\left(\frac{U_0}{U}\right)^2$$

where c_N is the new chord,

c_1 is a chord of the first design based uniformly on U_0

U_0 is the free stream velocity, and

U is the actual velocity at the location of c_1

If the ratio, U_0/U, is shown to vary appreciably with U_0 at a given chord location, the design should be carried out for the cruising or full-speed conditions, whichever is more important for maneuvering.

(2) If the velocity at any point in the boundary layer aft of the transition point is examined in minute detail, it is observed that the velocity at that point fluctuates. This fluctuation is considered to be due to a random distribution of vorticity called turbulence. So long as the size of the individual vortices is small compared to the size of the control surface, wind-tunnel tests[35] indicate that the turbulence should have no appreciable effect on that part of the control surface which is submerged in the boundary layer other than to provide stimuli for causing the boundary layer of the control surface itself to be turbulent. The actual effect of the turbulent stream on the size of control forces produced by low-aspect-ratio control surfaces should be checked by tests.

EFFECT OF PROPELLER WAKE

The propeller increases the velocity of the fluid which passes through it. To take advantage of this increased velocity, often the control surfaces are located in the propeller wake. The advantage is especially apparent when the vessel is at rest, since as soon as the propeller begins to operate, a control force can be generated by a surface in its wake. However, because of the violent, large-scale turbulence which occurs in the wake of a propeller operating with no forward velocity, the forces resulting on the control surface do not submit readily to analysis. It is suggested that the condition of no forward velocity be omitted as a criteria for design.

When the vessel has some forward velocity, the wake behind the propeller should be less turbulent. In the design of the control surface account can be taken of the wake in the same manner as for the boundary layer if the average velocity distribution is known at the location of the control surface.

MEANS FOR OBTAINING CONTROL SURFACE CHARACTERISTICS

MEASUREMENT VERSUS CALCULATION TECHNIQUES

The importance of the pressure distribution, especially spanwise, was mentioned in regards to spanwise structural loading, cavitation and induced drag in the above text. To accurately appraise the merits of one control surface over another, therefore, it would be advantageous to determine their pressure distributions.

Because information relative to pressure distribution was lacking for surfaces of small aspect ratio, a general series of experimental-pressure distribution tests was planned from which it was hoped some empirical relation of the pressure distribution to the plan-form might be derived. The first four models of the series planned were constructed and the pressure distribution was measured in the David Taylor Model Basin 8-foot by 10-foot wind tunnel. The data accumulated will be the subject of a later report.

The procurement of data by experimental pressure distributions proved to be very costly in both time and currency. The data reduction is very lengthy and laborious. In searching for less laborious techniques for studying the effect of plan-form on pressure distribution, Reference 38 was discovered. Three methods of calculating the pressure distributions are compared with each other and with experiment in Reference 38 and data presented show that results of Falkner's and Weissinger's methods fit a large amount of experimental data very accurately. It is reported that Falkner's method takes 24 to 32 hours and Weissinger's method 2 1/2 to 3 hours. Weissinger's method results in spanwise distribution only but Falkner's gives both the spanwise and chordwise distribution. The accuracy is a little better than Weissinger's. It is recommended in Reference 38 that Weissinger's be used to obtain information on large families of surfaces and then if more detailed information is desired on one surface, Falkner's method be used. It is shown in Reference 39 that Weissinger's method can also be used to obtain the location of the center of pressure and the slope of the lift curve in some cases. However, force tests of control surfaces will be necessary to determine maximum lift and scale effect.

If the investigation into the relation of low-aspect-ratio planforms to the pressure distribution is continued, data will be obtained by calculation as recommended in Reference 38 rather than by experiment because by comparison the calculation methods are very efficient of time and cost with practically no sacrifice in accuracy.

PLAN FORMS FOR ELLIPTICAL SPANWISE LOADING

At the risk of repetition, it is well to point out again the importance of designing for an elliptical spanwise load distribution. Such a distribution reduces the induced drag, the structural bending moment and the critical cavitation index and also produces the maximum slope of the curve of lift plotted as a function of angle of attack.

An analysis is made in Reference 39 of the relation of spanwise loading to the plan-form. Many plan-forms are considered and charts are presented from which it is possible to locate the spanwise and chordwise location of the center of pressure for many combinations of aspect ratio, sweep and taper. It is also shown in Reference 39 that there is a relation between taper ratio and sweep which results in elliptical spanwise load distribution independent of aspect ratio. This relation is shown plotted in Figure 11 of Reference 39 and for convenience is reproduced here as Figure 12.

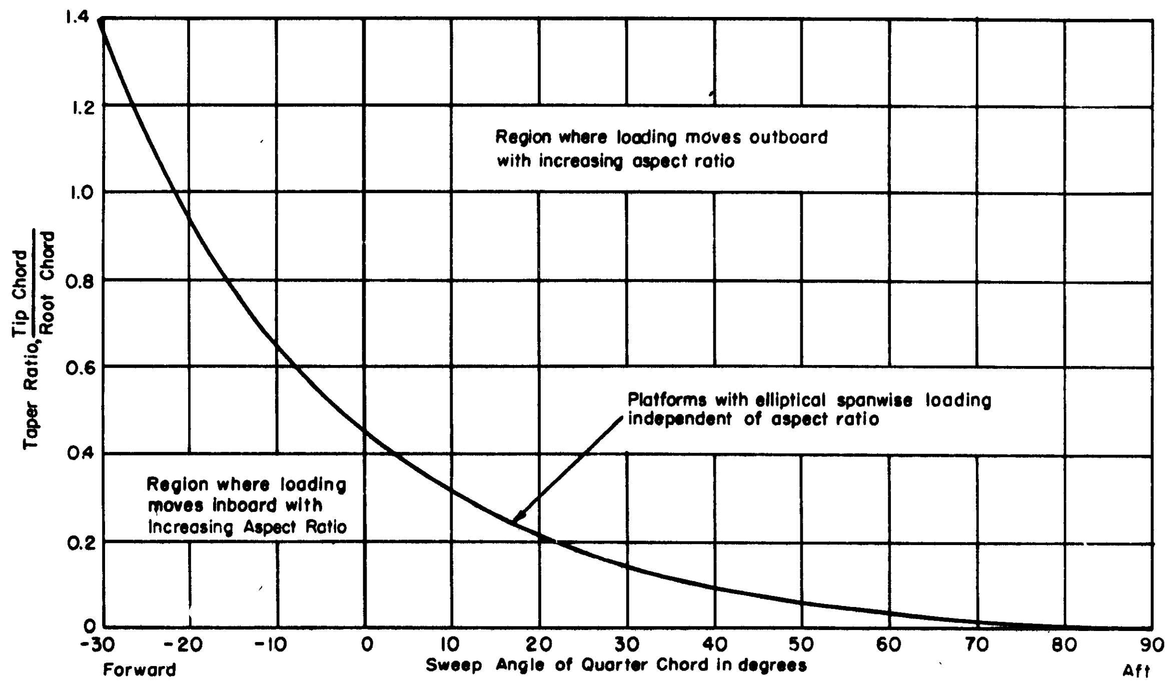


Figure 12 - Values of Taper and Sweep Which Give an Elliptical Spanwise Load Distribution

CONCLUDING REMARKS

Although the superiority of the all-movable control surfaces is demonstrated, there will be some circumstances where it will be impossible or grossly impractical to incorporate it in a design. In such instances flapped control surfaces are indicated and for these applications, flaps with overhang balance of forms similar to those described in References 8 through 30 are recommended. However, for vessels which are designed for maneuverability, it appears to be worth an extra effort to overcome any obstacle lying in the way of employing the all-movable control surface.

The recommendations and observations contained in this report are generally applicable to all cases where thin-airfoil theory applies. The application is generally considered to be admissible when the thickness ratio of the chord of the control surface is less than about 30 percent.

To improve further the precision with which control surfaces can be designed to meet various assigned requirements, it appears that future work should be applied to investigating the flow conditions at the potential location of a control surface on the vessel. Surveys of the fluid flow which give the velocity distribution and flow angle should be made while actually turning the hull at various angles of attack.

Additional work is also suggested on the control surfaces themselves so that finally there may exist a compilation of characteristics for many different plan forms of low-aspect-ratio surfaces. Such a compilation would provide a designer with design information on spanwise and chordwise loading for structural purposes, torque for control-engine design and lift and drag for consideration of stability and maneuverability.

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