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THE INFLUENCE OF METACENTRIC STABILITY ON THE DYNAMIC LONGITUDINAL STABILITY OF A SUBMARINE

SRD 542/46
NS 512-001

by

E. D. Hoyt and F. H. Imlay

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October 1948

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10 1957

NAVY DEPARTMENT
DAVID TAYLOR MODEL BASIN
WASHINGTON 7, D. C.

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THE INFLUENCE OF METACENTRIC STABILITY ON THE DYNAMIC LONGITUDINAL STABILITY OF A SUBMARINE

CONFIDENTIAL

by

E. D. Hoyt and F. H. Imlay

INTRODUCTION

In writing the equations for pitching motion of a submarine in submerged operation it has been customary to neglect the effect of metacentric stability. Recent operating experience at moderate speeds of the order of 10 knots has indicated, however, that this simplification does not adequately represent the behavior of the submarine for such speeds. In the present paper the equations of longitudinal motion have been written to include the effect of metacentric stability and these extended equations have been used in an analysis of the effect of metacentric stability on the pitching characteristics of a typical submarine operating at various speeds.

EQUATIONS OF MOTION

Assume, in Figure 1, a right-hand orthogonal system of reference axes fixed in space so that the origin O coincides with the location of the center of gravity of the

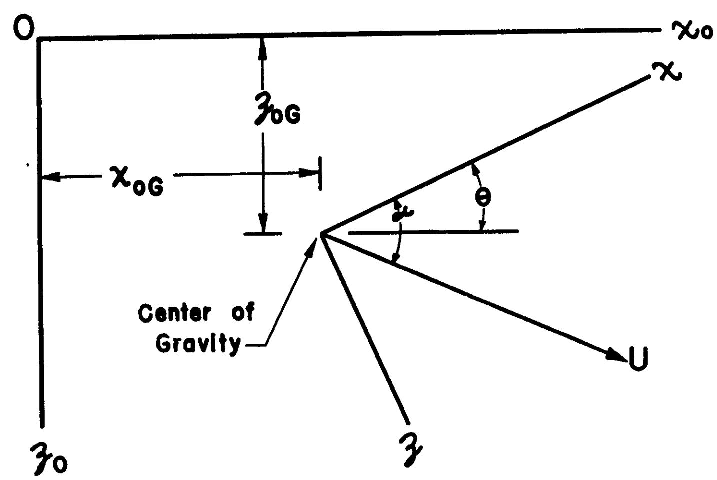


Figure 1.- Systems of Axes

submarine at zero time, and the plane defined by the x_o and z_o axes contains the direction of motion at zero time. The assumption will be made that the center of gravity remains in this plane during subsequent motion and that the progression of the center of gravity is in the general direction of the positive x_o axis. The positive direction of the z_o axis is vertically down. The subsequent motion of the submarine is completely defined by the coordinates x_{oG}, z_{oG} and the angle Θ (see Figure 1). In terms of the fixed-axes system the Newtonian equations of motion are

$$
\begin{aligned}
X_o &= m\,\ddot{x}_{oG} \\
Z_o &= m\,\ddot{z}_{oG} \\
M &= I_y\ddot{\theta}
\end{aligned}
\qquad [1]
$$

where X_o and Z_o are the total forces in the x_o and z_o directions respectively,

m is the mass of the submarine,

$\ddot{x}_{oG}$ and $\ddot{z}_{oG}$ are the second derivatives with respect to time t of x_{oG} and z_{oG} respectively,

M is the total moment about an axis through the center of gravity and parallel to the y_o axis, and

I_y is the moment of inertia of the submarine about the axis just mentioned.

The motion of the submarine is more conveniently expressed when referred to moving axes x and z in Figure 1. The moving axes form a right-hand orthogonal system with the origin located at the center of gravity of the submarine and the positive direction of the x axis is forward and parallel to the base line of the submarine. The y axis is coincident with the y_o axis at zero time and remains parallel thereto for subsequent motion. The instantaneous linear velocity of the origin of the moving axes is represented by the vector U and the orientation of the moving axes with respect to the direction of motion is given by the angle α, the positive sense of which is shown in Figure 1.

In order to convert the equations of motion from fixed axes to moving axes, the total forces X and Z in the x and z directions respectively are expressed in terms of X_o and Z_o as

$$\begin{aligned} X &= X_o \cos\theta - Z_o \sin\theta \\ Z &= Z_o \cos\theta + X_o \sin\theta \end{aligned} \qquad [2]$$

likewise

$$\begin{aligned} \dot{x}_{oG} &= u \cos\theta + w \sin\theta \\ \dot{z}_{oG} &= -u \sin\theta + w \cos\theta \end{aligned} \qquad [3]$$

where the dot above the symbol signifies the first derivative of the quantity with respect to time, and u and w are the components of U along x and z respectively.

Then

$$\begin{aligned} \ddot{x}_{oG} &= \dot{u} \cos\theta + \dot{w} \sin\theta - (u \sin\theta - w \cos\theta)\dot{\theta} \\ \ddot{z}_{oG} &= -\dot{u} \sin\theta + \dot{w} \cos\theta - (u \cos\theta + w \sin\theta)\dot{\theta} \end{aligned} \qquad [4]$$

Substituting Equation [4] in Equation [1] and inserting the resulting values of X_o and Z_o in Equation [2] gives

$$X = m\,(\dot{u} + w\dot{\theta})$$

$$Z = m\,(\dot{w} - u\dot{\theta})$$

These, plus the third member of Equation [1] , comprise the equations of motion with respect to moving axes. For completeness

$$X = m\,(\dot{u} + w\dot{\theta})$$
$$Z = m\,(\dot{w} - u\dot{\theta}) \qquad [5]$$
$$M = I_y\ddot{\theta}$$

The forces X and Z and the moment M in Equations [5] are assumed to have hydrodynamic, gravity, and buoyancy components which are functions of displacement, velocities, and accelerations. Thus

Neutral Buoyancy

$$X = F_x\,(u,w,\theta,\dot{u},\dot{w},\dot{\theta},\ddot{\theta}) \qquad [6]$$

with similar expressions for Z and M.

There will, in general, be equilibrium solutions of the equations of motion [5] for which the motion is along a straight line at constant trim. The equilibrium values will be designated u_1, w_1, θ_1, etc. The equilibrium values of $\dot{u}_1$, $\dot{w}_1$, $\dot{\theta}_1$, and $\ddot{\theta}_1$ are zero, and the equilibrium values of the other quantities satisfy the equations of motion. Thus, if the expressions for X, Z, and M (see Equation [6]) are substituted in Equations [5] and expanded in Taylor series about the equilibrium values, there result equations for the perturbations such as

$$\begin{aligned} & m\dot{u} + mw_1\dot{\theta} - (X_u)_1(u-u_1) - (X_w)_1(w-w_1) - (X_\theta)_1(\theta-\theta_1) \\ & -(X_{\dot{u}})_1\dot{u} - (X_{\dot{w}})_1\dot{w} - (X_{\dot{\theta}})_1\dot{\theta} - (X_{\ddot{\theta}})_1\ddot{\theta} = 0 \end{aligned} \qquad [7]$$

with similar expressions for Z and M. In Equation [7] $(X_u)_1$ denotes $\partial X/\partial u$ at the equilibrium condition, and similarly for the other symbols of this type.

The form of the perturbation equations can be simplified by some conventions of notation. The derivatives such as $(X_u)_1$, are all to be evaluated at the equilibrium condition so the portion $(\ )_1$ will be dropped. Further, the variables will be changed to variations from equilibrium values, writing u for $(u-u_1)$ and noting that $\frac{d}{dt}(u-u_1) = \dot{u}$, and proceeding similarly for the other variables. For sufficiently small disturbances X is an even function of w, $\dot{w}$, $\dot{\theta}$, and $\ddot{\theta}$; and Z and M are even functions of u and $\dot{u}$.

Thus

$$X_w = X_{\dot{w}} = X_{\dot{\theta}} = X_{\ddot{\theta}} = Z_u = Z_{\dot{u}} = M_u = M_{\dot{u}} = 0$$

The equations for the perturbations finally become

$$\begin{aligned} &-X_u u + (m-X_{\dot{u}})\dot{u} - X_\theta \theta + mw_1\dot{\theta} = 0 \\ &-Z_w w + (m-Z_{\dot{w}})\dot{w} - Z_\theta \theta - (mu_1 + Z_{\dot{\theta}})\dot{\theta} - Z_{\ddot{\theta}}\ddot{\theta} = 0 \qquad [8] \\ &-M_w w - M_{\dot{w}}\dot{w} - M_\theta \theta - M_{\dot{\theta}}\dot{\theta} + (I_y - M_{\ddot{\theta}})\ddot{\theta} = 0 \end{aligned}$$

If the submarine is assumed to be in neutral buoyancy and neutral trim when at rest, X_θ and Z_θ are zero and M_θ is given by

$$M_\theta = \Delta z_B \cos\theta_1 \qquad [9]$$

where Δ is the buoyancy force, positive upward and equal to the weight of the submarine and z_B is the distance along the z axis from the center of gravity to the center of buoyancy. Thus z_B is positive when the center of buoyancy is below the center of gravity (metacentric instability). Inserting these values for X_θ, Z_θ, and M_θ in Equations [8] and non-dimensionalizing by dividing the first two of Equations [8] by $\frac{1}{2}\rho\ell^2 u_1^2$ and the third by $\frac{1}{2}\rho\ell^3 u_1^2$ (where ρ is the density of the fluid and ℓ is the overall length of the submarine), leads to

$$\begin{aligned}
&-X_u'u' + (m'-X_{\dot u}')\dot u' && + m'\alpha_1\dot\theta' = 0 \\
&-Z_w'w' + (m'-Z_{\dot w}')\dot w' && - (m'+Z_{\dot\theta}')\dot\theta' - Z_{\ddot\theta}'\ddot\theta' = 0 \qquad [10] \\
&-M_w'w' - M_{\dot w}'\dot w' - (\Delta' z_B'\cos\theta_1)\theta && - M_{\dot\theta}'\dot\theta' + (I_y'-M_{\ddot\theta}')\ddot\theta' = 0
\end{aligned}$$

Equations [10] are the non-dimensional equations of longitudinal motion for a submarine in neutral buoyancy and trim. The primes in the equations are used to indicate non-dimensional forms of similar quantities appearing in Equations [8] and [9] ; and $\alpha_1 = w_1/u_1$. The dot or double dot over a quantity now signifies the first or second derivative, respectively, of the quantity with respect to non-dimensional time t', where $t' = \frac{tu_1}{\ell}$. Because Δ is a constant, Δ' in Equations [10] will vary with speed.

SOLUTIONS OF THE EQUATIONS OF MOTION

Equations [10] comprise a set of homogeneous linear differential equations in the variables u', w', and θ. The solutions to the equations are sums of terms of the form

$$u' = u_i e^{\sigma_i t'} \qquad w' = w_i e^{\sigma_i t'} \qquad \theta = \theta_i e^{\sigma_i t'}$$

where σ_i are the roots of the characteristic equation corresponding to Equations [10] , e = 2.718, and u_i, w_i, and θ_i are constants. Substitution of the typical solutions in the equations of motion leads to

$$- [X_u' - (m' - X_{\dot{u}}')\sigma]\, u' \qquad + m'\alpha_1\sigma\theta = 0$$

$$- [Z_w' - (m' - Z_{\dot{w}}')\sigma]\, w' \qquad - [(m' + Z_{\dot{\theta}}')\sigma + Z_{\ddot{\theta}}'\sigma^2]\theta = 0$$

$$- (M_w' + M_{\dot{w}}'\sigma)\, w' \qquad - [\Delta' z_B' \cos\theta_1 + M_{\dot{\theta}}'\sigma - (I_y' - M_{\ddot{\theta}}')\sigma^2]\theta = 0$$

The characteristic equation is

$$\begin{vmatrix} -[X_u' - (m' - X_{\dot{u}}')\sigma] & + 0 & + m'\alpha_1\sigma \\ 0 & -[Z_w' - (m' - Z_{\dot{w}}')\sigma] & -[(m' + Z_{\dot{\theta}}')\sigma + Z_{\ddot{\theta}}'\sigma^2] \\ 0 & -(M_w' + M_{\dot{w}}'\sigma) & -[\Delta' z_B'\cos\theta_1 + M_{\dot{\theta}}'\sigma - (I_y' - M_{\ddot{\theta}}')\sigma^2] \end{vmatrix} = 0$$

which reduces to

$$- [X_u' - (m' - X_{\dot{u}}')\sigma] \begin{vmatrix} -[Z_w' - (m' - Z_{\dot{w}}')\sigma] & -[(m' + Z_{\dot{\theta}}')\sigma + Z_{\ddot{\theta}}'\sigma^2] \\ -(M_w' + M_{\dot{w}}'\sigma) & -[\Delta' z_B'\cos\theta_1 + M_{\dot{\theta}}'\sigma - (I_y' - M_{\ddot{\theta}}')\sigma^2] \end{vmatrix} = 0 \qquad [11]$$

The first factor in Equation [11] furnishes the root

$$\sigma = \frac{X_u'}{m' - X_{\dot{u}}'}$$

which is always real and negative, and hence stable. The real problem of stability rests in the remaining factor which is a cubic in σ. Upon expansion this factor becomes

$$
\begin{aligned}
&[(m'-Z_{\dot{w}}')(I_y'-M_{\dot{q}}') - Z_{\dot{q}}'M_{\dot{w}}']\sigma^3 \\
&+ [-Z_w'(I_y'-M_{\dot{q}}') - (m'-Z_{\dot{w}}')M_q' - (m'+Z_q')M_{\dot{w}}'-Z_{\dot{q}}'M_w']\sigma^2 \qquad [12] \\
&+ [Z_w'M_q'-(m'-Z_{\dot{w}}')\Delta'z_B'\cos\theta_1 - (m'+Z_q')M_w']\sigma \\
&\qquad + Z_w'\Delta'z_B'\cos\theta_1 = 0
\end{aligned}
$$

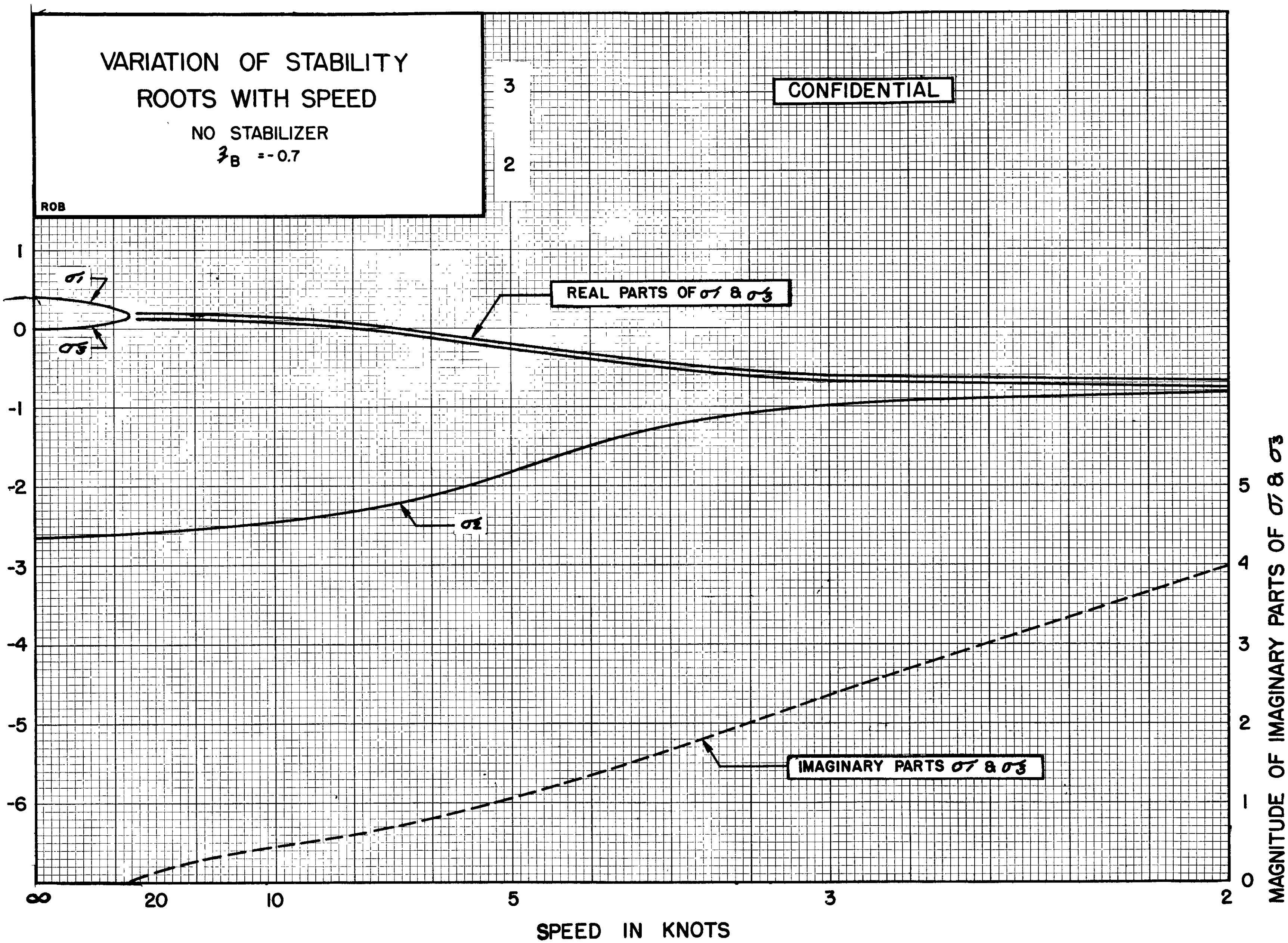
where $\dot{\theta}$ and $\ddot{\theta}$ have been replaced by the symbols q and $\dot{q}$ respectively.

EFFECT OF METACENTRIC STABILITY

The effect of metacentric stability on the dynamic stability in pitch of a submarine depends on the influence of the factor $\Delta' z_B' \cos\theta_1$ which appears in two places in Equation [12]. If metacentric stability is neglected there is no constant term in Equation [12] and hence the root $\sigma = 0$ can be factored out, signifying that the submarine in such a case is indifferent to the angle of its trajectory with respect to horizontal. With the introduction of a given amount of metacentric stability three finite roots are obtained, the values of which will vary with speed because the magnitude of Δ' in the factor $\Delta' z_B' \cos\theta_1$ changes with speed for a fixed amount of metacentric stability.

The amount of metacentric stability at a given speed can change, in a practical case, either through a change in buoyancy force Δ or a change in metacentric height z_B. Strictly speaking, a departure from the neutral buoyancy assumed in developing Equation [12] would lead to a different form of stability equation, describing motions about some new straight course with a different steady-state angular orientation of the body relative to the path. The exact expressions for X_θ , Z_θ , and M_θ would also be different. For sufficiently small changes in buoyancy, however, simplifying assumptions would reduce the new equations to those previously presented. Hence, a change in either buoyancy force or in metacentric height may be assumed to be described by changing the value of z_B' in the factor $\Delta' z_B' \cos \theta_1$.

As a means of evaluating the importance of metacentric stability in determining the values of the stability roots, Figures 2 to 5 have been prepared for a hypothetical submarine, either with no stabilizer or with a fixed stabilizing surface. Values of stability derivatives and mass factors for either stabilizer condition were chosen to be representative of current design practice and were inserted in Equation [12] to obtain the roots plotted in the figures. Two values of z_B , ± 0.7, were considered in the preparation of Figures 2 to 5. The value $z_B = +0.7$ represents a marked degree of metacentric instability, and the value -0.7 corresponds to a typical amount of metacentric stability.


VARIATION OF STABILITY ROOTS WITH SPEED
NO STABILIZER
ζ_B = -0.7
ROB
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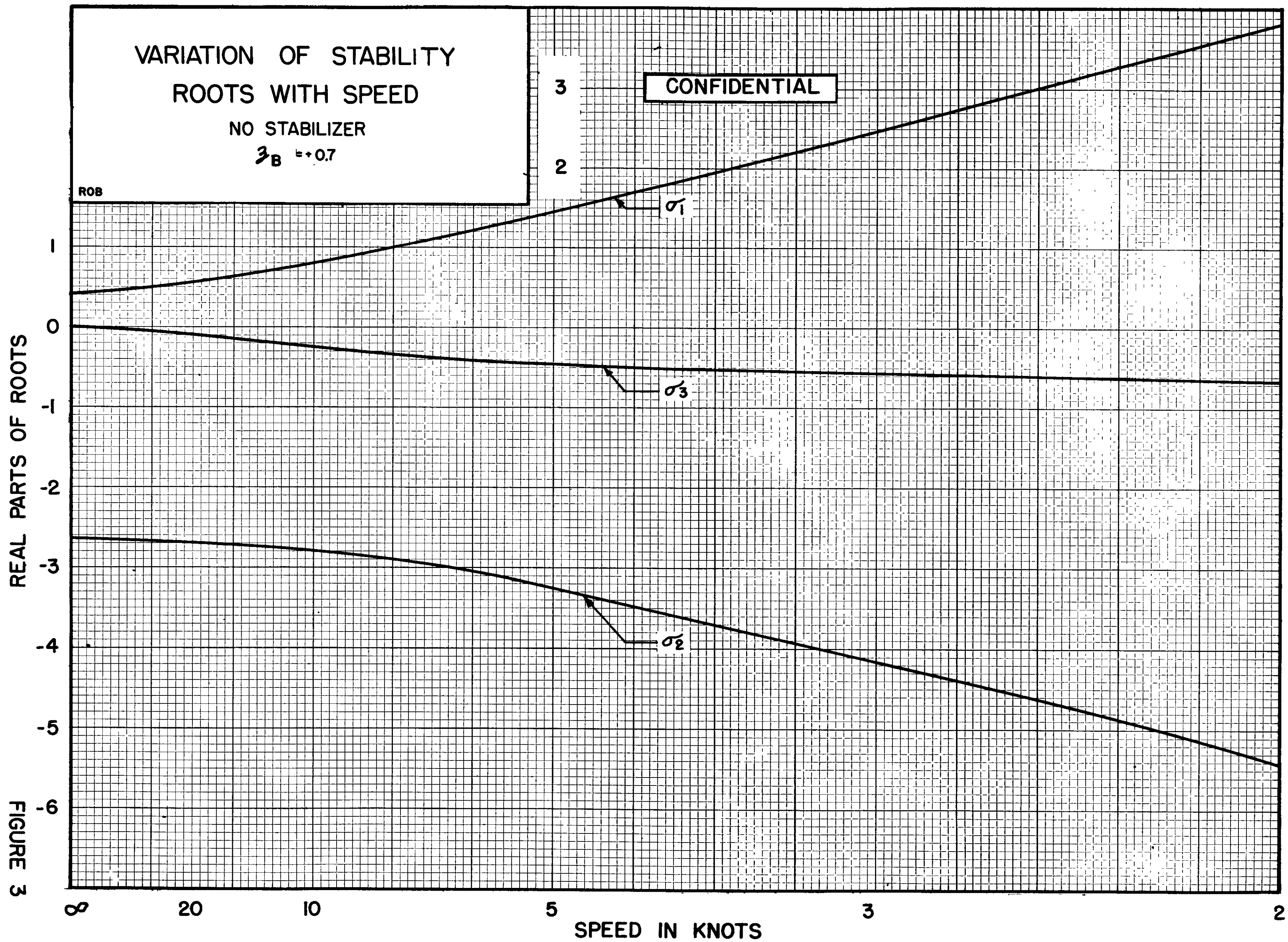
REAL PARTS OF $\sigma 1$ & $\sigma 3$
$\sigma 1$
$\sigma 3$
$\sigma 2$
IMAGINARY PARTS $\sigma 1$ & $\sigma 3$
REAL PARTS OF ROOTS
1
0
-1
-2
-3
-4
-5
-6
3
2
MAGNITUDE OF IMAGINARY PARTS OF $\sigma 1$ & $\sigma 3$
5
4
3
2
1
0
∞
20
10
5
3
2
SPEED IN KNOTS

FIGURE 2



VARIATION OF STABILITY
ROOTS WITH SPEED
NO STABILIZER
$\mathfrak{z}_B$ = + 0.7
ROB
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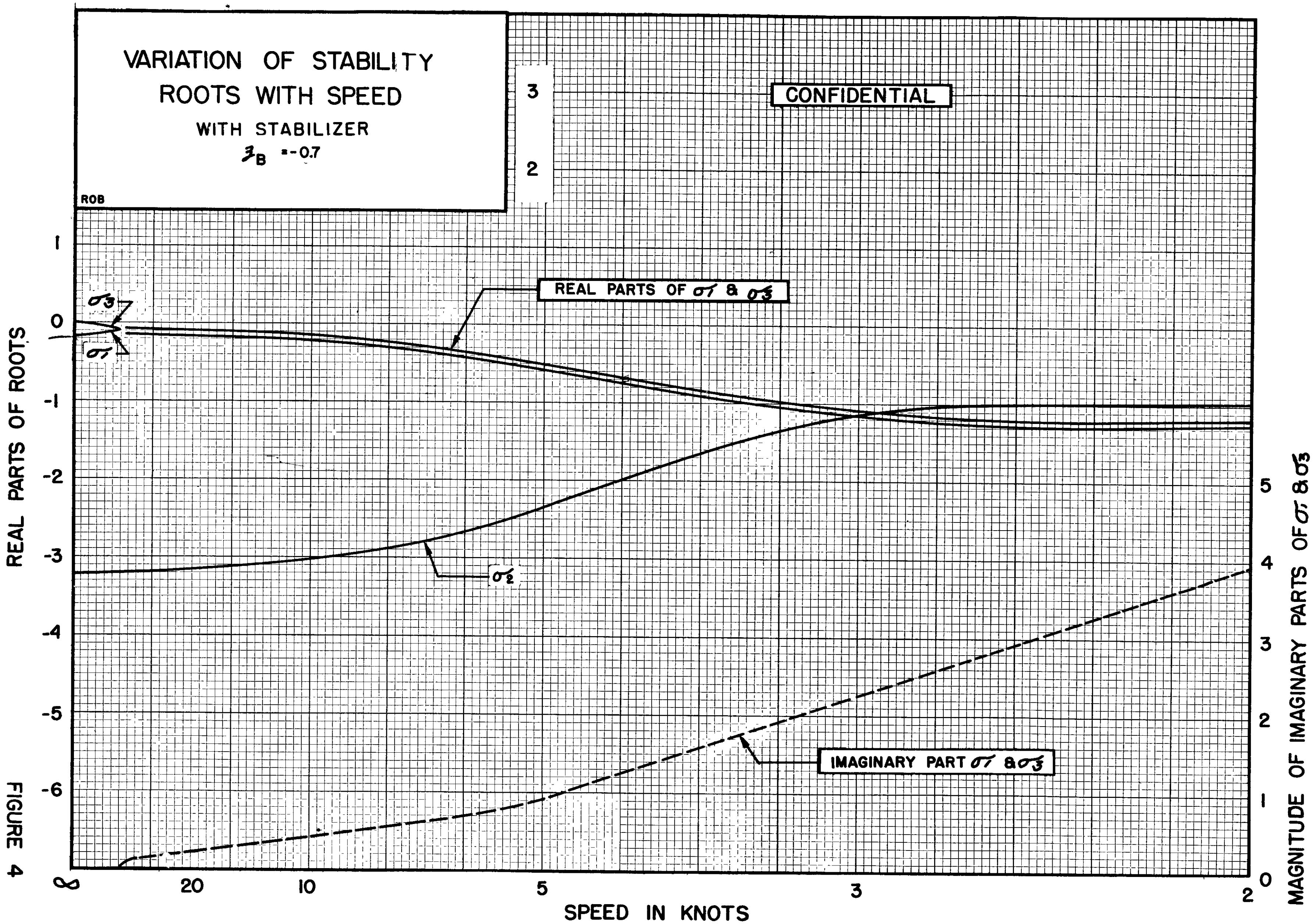
σ_1
σ_3
σ_2
REAL PARTS OF ROOTS
3
2
1
0
-1
-2
-3
-4
-5
-6
∞
20
10
5
3
2
SPEED IN KNOTS

FIGURE 3


VARIATION OF STABILITY
ROOTS WITH SPEED
WITH STABILIZER
$\mathfrak{z}_B = -0.7$
ROB
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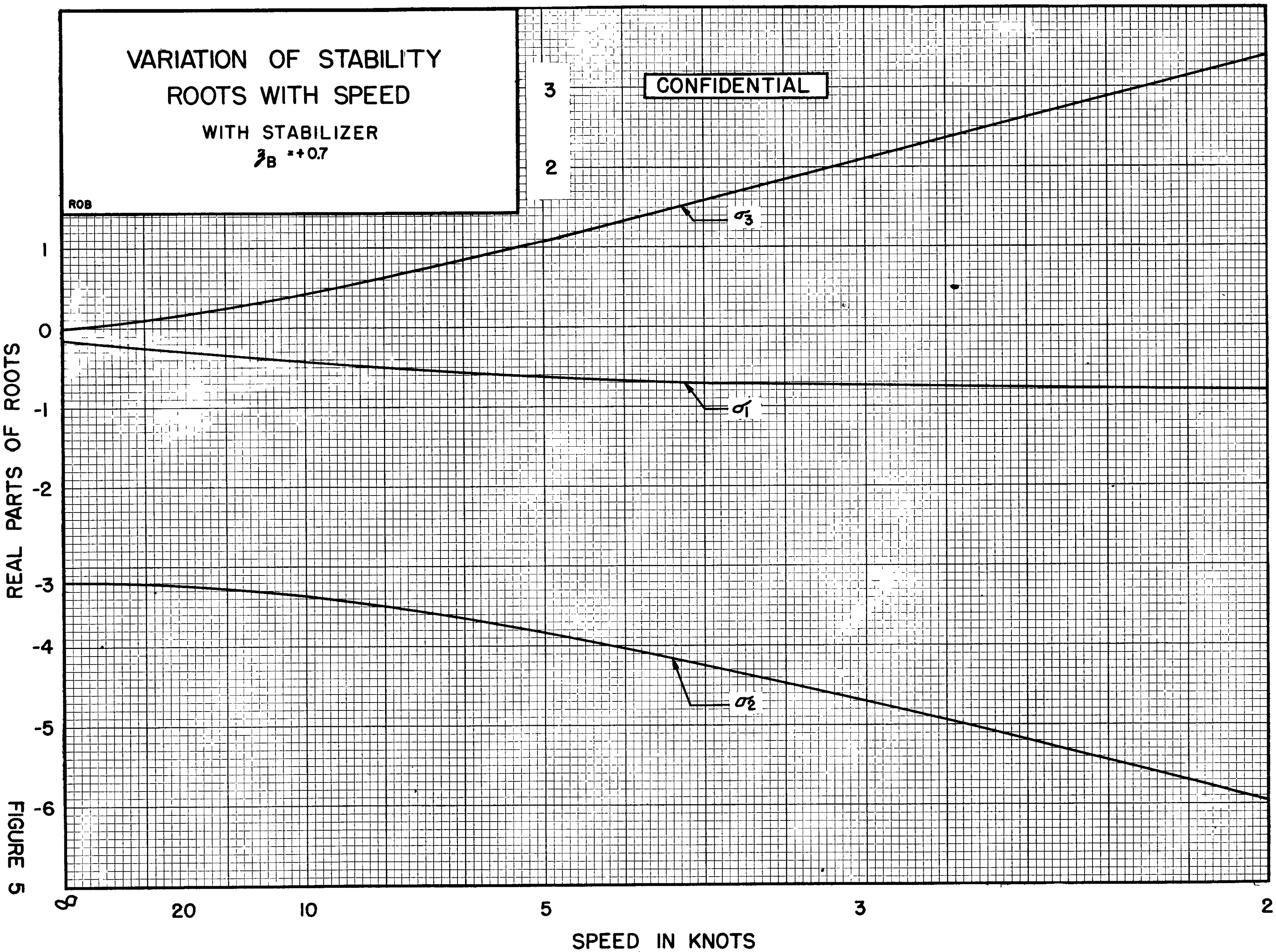
REAL PARTS OF σ_1 & σ_3
σ_3
σ_1
σ_2
IMAGINARY PART σ_1 & σ_3
REAL PARTS OF ROOTS
MAGNITUDE OF IMAGINARY PARTS OF σ_1 & σ_3
SPEED IN KNOTS
1
0
-1
-2
-3
-4
-5
-6
3
2
5
4
3
2
1
0
∞
20
10
5
3
2

FIGURE 4



VARIATION OF STABILITY ROOTS WITH SPEED
WITH STABILIZER
$\zeta_B = +0.7$
ROB
CONFIDENTIAL
3
2
1
0
-1
-2
-3
-4
-5
-6
σ_3
σ_1
σ_2
REAL PARTS OF ROOTS
∞
20
10
5
3
2
SPEED IN KNOTS

FIGURE 5

A comparison of Figures 2 and 3 show the variations of the stability roots σ with speed in knots that were obtained for two values of metacentric stability $z_B'' = -0.7$ and $+ 0.7$ respectively, for a submarine with no stabilizer. Consideration of Equation [12] shows that a plot of the stability roots for a third value of $z_B' = 0$ would consist of straight lines giving values of σ that remain constant with speed and of the same magnitude as those appearing at the left edge of Figures 2 or 3.

Before considering the changes brought about in the values of the stability roots by changes in the amount of metacentric stability it may be advantageous to examine Figure 2 in detail. Because some of the roots are real and some complex, the real roots have been shown in this and the following figures as a single solid line. Complex roots, which occur in conjugate pairs, have been separated into real and imaginary parts, the real part for the pair being represented by a double solid line and the magnitude of the imaginary parts (without regard to sign) being plotted as a dashed line. Thus, referring to Figure 2, at infinite speed three real roots are obtained. The values, one of which is zero, are the same as would be obtained if metacentric stability were neglected. The root σ_1 is positive and therefore unstable, and the root σ_2 has a large negative stable value. As the speed is decreased the roots are changed in value in such a way that when the speed has dropped to about 25 knots the values of σ_1 and σ_3 become equal and both are unstable. If the speed is further reduced, the two equal real roots are replaced by a pair of conjugate complex roots. Initially the real part is positive for the pair, signifying an unstable oscillation. When the speed drops below about 7 knots the real part becomes negative and the oscillation then becomes stable. The period of the oscillation, which is proportional to the reciprocal of the magnitude of the imaginary parts of the complex pair of roots, decreases with decreasing speed. Meanwhile, the large stable root σ_2 undergoes no pronounced changes in character as the speed is reduced, although a gradual decrease in stability of this mode is indicated.

Returning to a comparison of Figures 2 and 3, when z_B is changed from -0.7 to +0.7 there is a pronounced change in the variation of the roots with speed. There is no longer any coupling of the roots to form an oscillation at the lower speeds. Instead, the unstable root σ_1 becomes increasingly unstable as the speed decreases and the stable root σ_2 becomes more stable. The root σ_3, neutrally stable at infinite speed, is not notably affected at lower speeds, although it does become slightly stable. The effect of going from metacentric stability to metacentric instability may be summarized by stating that an overall decline in the stability of the submarine results which becomes more serious at the lower speeds, as would be expected.

The question arises as to whether a configuration that is stable without consideration of metacentric stability would exhibit the same improvement in stability characteristics with the introduction of metacentric stability as that portrayed in Figure 2 or the same lowering of stability when metacentric instability is present as that indicated in Figure 3. Consequently Figures 4 and 5 were prepared on the same basis as Figures 2 and 3 but for the submarine fitted with a fixed stabilizing surface so that at infinite speed (or, alternatively, with metacentric stability neglected) σ_1 was stable instead of unstable.

Comparisons of Figures 2 and 4 or 3 and 5 indicate that the influence of metacentric stability on the stability roots is generally the same whether or not the configuration is stable without consideration of metacentric stability. Examination of the variation of the roots in Figures 3 and 5, however, suggests that there may be some inconsistency in identifying the roots σ_1 and σ_3 for the two cases. Equation [12] will always yield one root of zero value at infinite speed and it appears reasonable to always assign the same identity to this root, which has been designated σ_3 in each case in the present investigation. Of the remaining roots, the root of smallest magnitude at infinite speed has been designated σ_1 in each case. A study of the various modes of disturbed motion for the submarine configurations treated in Figures 3 and 5 might remove the apparent inconsistency mentioned, but such a study represents a task of sufficient magnitude to constitute a separate investigatigation, the chief results of which would not bear too directly on the subject at hand.

CONCLUDING REMARKS

The present investigation shows that the influence of metacentric stability cannot be neglected in determining the dynamic longitudinal stability roots for any part of the speed range of a typical submarine of modern design. In general, if the submarine possesses metacentric stability the dynamic stability will be improved; and metacentric instability will cause a worsening of the dynamic stability characteristics, particularly at low speeds.

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