Report 4356

NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

Bethesda, Maryland 20034

MEASUREMENT OF THE ACOUSTIC PROPERTIES OF SOLID POLYMERS

by

William S. Cramer

APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED

SHIP ACOUSTICS DEPARTMENT
RESEARCH AND DEVELOPMENT REPORT

December 1974

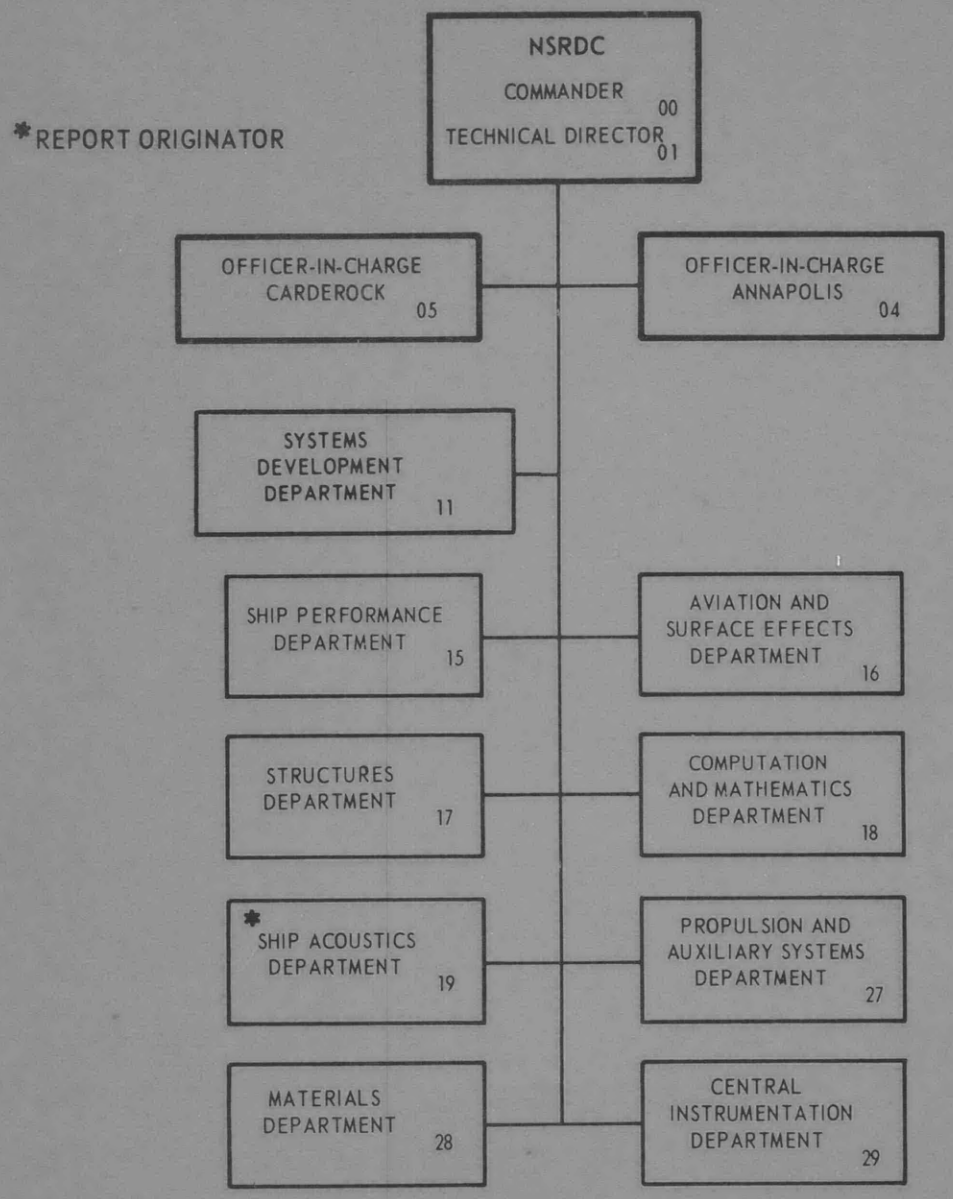
Report 4356

MEASUREMENT OF THE ACOUSTIC PROPERTIES OF SOLID POLYMERS

The Naval Ship Research and Development Center is a U. S. Navy center for laboratory effort directed at achieving improved sea and air vehicles. It was formed in March 1967 by merging the David Taylor Model Basin at Carderock, Maryland with the Marine Engineering Laboratory at Annapolis, Maryland.

Naval Ship Research and Development Center
Bethesda, Md. 20084

MAJOR NSRDC ORGANIZATIONAL COMPONENTS



NDW-NSRDC 3960/44 (REV. 8/71)
GPO 917-872

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: 4356

2. GOVT ACCESSION NO.:

3. RECIPIENT'S CATALOG NUMBER:

4. TITLE (and Subtitle): MEASUREMENT OF THE ACOUSTIC PROPERTIES OF SOLID POLYMERS

5. TYPE OF REPORT & PERIOD COVERED:

6. PERFORMING ORG. REPORT NUMBER:

7. AUTHOR(s): William S. Cramer

8. CONTRACT OR GRANT NUMBER(s):

9. PERFORMING ORGANIZATION NAME AND ADDRESS: Naval Ship Research and Development Center, Bethesda, Maryland 20084

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: See reverse side

11. CONTROLLING OFFICE NAME AND ADDRESS: Director of Laboratory Programs, Department of the Navy, Washington, D. C. 20360

12. REPORT DATE: December 1974

13. NUMBER OF PAGES: 67

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office):

15. SECURITY CLASS. (of this report): UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE:

16. DISTRIBUTION STATEMENT (of this Report): APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report):

18. SUPPLEMENTARY NOTES:

19. KEY WORDS (Continue on reverse side if necessary and identify by block number):

Polymers
Elastomers
Acoustic Properties
Dynamic Moduli

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The speed and attenuation of an acoustic wave in a homogeneous polymer can be measured directly in the laboratory at frequencies greater than 100 kilohertz. At lower frequencies, however, it is usually necessary to calculate the propagation properties from measurements of the dynamic elastic moduli and the density. Data on two of the three standard moduli (bulk, Young, and shear) are theoretically sufficient to describe any type

(Continued on reverse side)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE S/N 0102-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

(Block 10)

Task ZF 61 412 001
Project Number F 61 412
Program Element 62766N
Work Unit 1-1949-005

(Block 20 continued)

of acoustic propagation in a homogeneous material. Each modulus consists of a storage modulus, indicating the in-phase component of stress and strain, and a loss modulus, indicating the out-of-phase component. A wide variety of measurement techniques has been employed, depending on the type of modulus to be studied and on the frequency range of interest. A survey is presented of the commonly used techniques, and their relative advantages and limitations are discussed.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOTATION

a	Radius of rod
c	Speed of sound propagation (subscripts b, e, f, s, and t indicate, respectively, bulk, extensional, flexural, shear, and torsional waves)
E	Young's Modulus*
G	Shear Modulus*
K	Bulk Modulus*
k	Wavenumber (ω/c)
M	Plane Wave Modulus $\left(K + \frac{4}{3} G\right)^*$
m	General term for any modulus*
p	Acoustic pressure
Q	$2\pi \times \frac{\text{energy stored per cycle}}{\text{energy absorbed per cycle}}$
R	Real part of input impedance
r	Loss parameter, $\frac{\alpha c}{\omega}$ or $\frac{\alpha\pi}{2\pi}$
X	Imaginary part of input impedance
Z_i	Input impedance at the material-water interface ($Z_i = R + i X$)
Z_0	Complex characteristic acoustic impedance of material
Z_t	Impedance of backing to acoustic material
α	Attenuation in nepers per unit length
δ	Phase angle in radians between stress and strain
η	Loss factor ($\eta = \tan \delta$ or m''/m')
λ	Lamé constant or acoustic wavelength
μ	Lamé constant
ρ	Density
σ	Poisson's ratio
ω	Angular frequency

*The form of the moduli indicated here is the static modulus. The dynamic moduli are expressed in the form $m^* = m' + i\, m''$ or $m^* = m'(1 + i\,\eta)$. The primed term is called the storage modulus and the double prime the loss modulus.

ABSTRACT

The speed and attenuation of an acoustic wave in a homogeneous polymer can be measured directly in the laboratory at frequencies greater than 100 kilohertz. At lower frequencies, however, it is usually necessary to calculate the propagation properties from measurements of the dynamic elastic moduli and the density. Data on two of the three standard moduli (bulk, Young, and shear) are theoretically sufficient to describe any type of acoustic propagation in a homogeneous material. Each modulus consists of a storage modulus, indicating the in-phase component of stress and strain, and a loss modulus, indicating the out-of-phase component. A wide variety of measurements techniques has been employed, depending on the type of modulus to be studied and on the frequency range of interest. A survey is presented of the commonly used techniques, and their relative advantages and limitations are discussed.

ADMINISTRATIVE INFORMATION

The work reported herein was authorized and funded by the Director of Laboratory Programs under Task ZF 61 412 001, Project Number F 61 412, Program Element 62766N, and Work Unit 1-1949-005.

INTRODUCTION

In the development of many systems used in underwater acoustics, it is necessary, for proper design, to know the acoustic properties of the constituent materials. Due to the unlikelihood of published data being available on the materials of particular interest and the scarcity of measuring facilities, most investigators find it necessary to develop instrumentation and to carry out their own measurements. This report is offered as a guide in carrying out the measurements and in assessing the results. It addresses itself to the basic questions of what properties are measured, why they are measured, and what the principle techniques are for doing so. The descriptions are largely phenomenological, and references are made to the original publications for detailed analyses and instrumentation descriptions. The survey is considered to be reasonably complete although variations of basic techniques have been either deliberately or inadvertently omitted. Measurements that are likely to be of interest in Navy applications have been emphasized. Also, all known current or past work in this field done in Navy laboratories or under Navy sponsored projects has been mentioned.

In addition to references devoted to particular measurement techniques, the following general references are suggested. For a discussion at the intermediate level of sound

propagation in solids, the book by Kolsky[1] is useful. The relationship of the viscoelasticity of polymers to molecular structure and motion was thoroughly discussed by Ferry.[2] (The book included three chapters about measurement techniques.) For a general description of acoustic structures in such Navy applications as baffles, anechoic coatings, transparent windows, etc., in terms of the properties of their constituent materials, the sixth chapter of the book by Bobber[3] is suggested.

Data on the basic acoustic properties of materials are used not only to select materials for particular applications, or to predict the behavior of the materials used, but also to study their internal properties. By means of the acoustic signal an alternating mechanical stress is applied to the material, ranging in frequency from a fraction of a hertz to many megahertz. The resulting strain, as measured by the acoustic propagation properties, gives valuable information about the molecular structure of the material as a function of composition, preparation, fillers, frequency, temperature, static pressure, aging, plasticizers, etc. Acoustical measurements are a powerful tool in studying dynamic properties of materials that has not realized its full potential.

BASIC ACOUSTIC PROPERTIES OF HOMOGENEOUS MATERIALS

INTRODUCTION

Coatings or other structures for use in underwater acoustics are made of combinations of acoustically homogeneous materials. They range from a single layer of homogeneous material cemented to a backing to complicated multielement structures in which one or more element may be made of dispersions of materials in matrices. The dispersed materials range from gas-filled voids to metal particles or even to combinations of several dispersions. If the dimensions of the dispersed particles are small, compared with a wavelength, it is usually satisfactory to consider the composite as an acoustically homogeneous material having properties that can be measured directly. (Work has been done to relate the properties of the matrices and dispersed materials to those of the composite, and some theory is available.) In the low-kilohertz region, for example, a simple unicellular rubber with microscopic gas-filled voids would be considered homogeneous, and its properties could be measured directly; whereas, a rubber matrix with large voids, such as the German acoustic absorber ALBERICH, at most frequencies of interest must be treated in terms of the acoustic properties of the rubber and the shape and size of the voids.

---

[1]Kolsky, H., "Stress Waves in Solids," Oxford University Press, London (1953). A complete listing of references is given on pages 54–59.

[2]Ferry, J. D., "Viscoelastic Properties of Polymers," Second Edition, John Wiley and Sons, Inc., New York (1970).

[3]Bobber, R. J., "Underwater Electroacoustic Measurements," Naval Research Laboratory Underwater Sound Reference Division, Orlando, Fla. (Jul 1970).

It is quite difficult in some cases to predict with accuracy the acoustic properties of a multielement coating in terms of the basic properties of its constituent materials and its structure. However, this problem must be solved in each case to achieve optimum design and to avoid a time consuming and expensive evaluation program. If the exact relationship between the factors cannot be found, empirical relationships or even intuition furnishes some guidance but the more precise the relationship, the more satisfactory the results.

ELASTIC CONSTANTS

A logical development of the equations of acoustic propagation in a material is to assume an elemental rectangular parallelopiped in the material and consider all possible combinations of stress and strain on it; see Kolsky.[1] The result is Hooke's law in the form of a six by six matrix with 36 coefficients. If the material is assumed to be isotropic, 24 of these become zero, and the remainder are expressed by two basic terms λ and μ, which are called the Lamé constants. Only isotropic materials will be considered in this report.

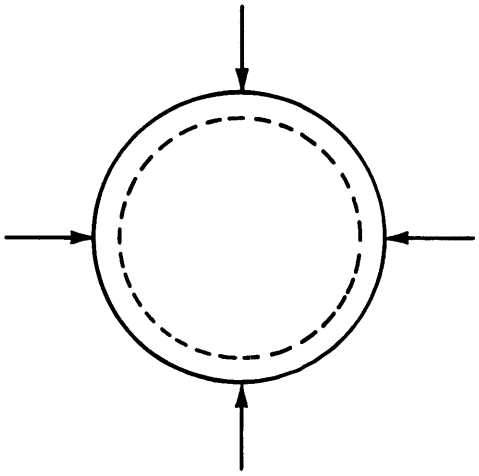
In most applications it is more convenient to describe the elastic properties of the material in terms of the various elastic moduli, instead of the Lamé constants, which directly relate the applied stress to resulting strains. The standard moduli, which will be defined in the next section, are *bulk* K, *shear* G, and *Young's* E. If we know any *two* of these, we can obtain the information we must know about the elastic properties of a given material, viz., its ability to change its volume without changing its shape *and* to change its shape without changing the volume. This point should be stressed. The two properties are independent and must be measured separately.

The moduli are listed and defined as follows.

Bulk Modulus

This measures directly the ability of a material to change its volume without changing its shape. It is defined in Figure 1a, where p is the pressure applied equally to all surfaces, ΔV is the change in volume as a result of the applied stress, and V is the original volume; the ratio $\Delta V/V$ is called the dilatational strain.*

*This strain and those for the other moduli are considered to be small enough for Hooke's law to apply.



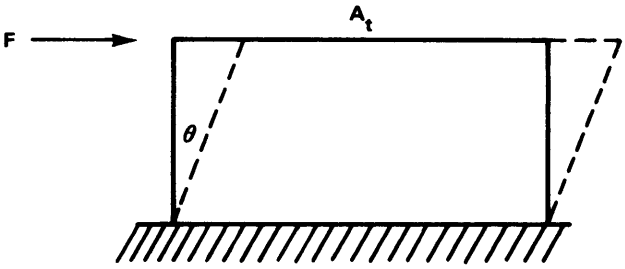
p = PRESSURE

V = VOLUME

ΔV = CHANGE IN VOLUME

$K = \frac{p}{\Delta V/V}$

Figure 1a – Bulk Modulus (K)



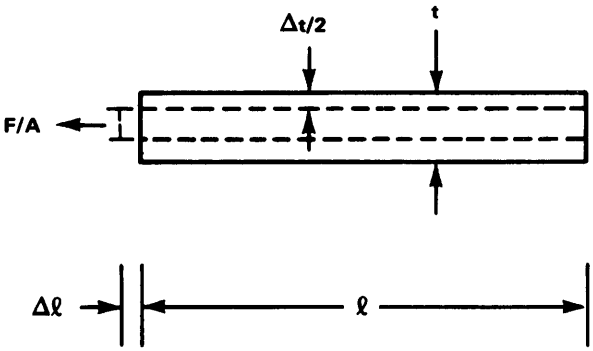
F = FORCE

A_t = TANGENTIAL AREA

θ = SHEAR ANGLE IN RADIANS

$G = \frac{F/A_t}{\theta}$

Figure 1b – Shear Modulus (G)



F = FORCE

A = CROSS SECTIONAL AREA

ℓ = LENGTH

t = THICKNESS

$E = \frac{F/A}{\Delta \ell/\ell}$

$\sigma = \frac{\Delta t/t}{\Delta \ell/\ell}$

Figure 1c – Young's Modulus (E) and Poisson's Ratio (σ)

Figure 1 – Elastic Moduli

Shear Modulus

This modulus describes the ability of a material to change its shape without changing volume. It is defined in Figure 1b, where F is the applied force, A_t is the tangential area, and θ is the shear angle of strain in radians.

Young's Modulus

This modulus is probably the best known and easiest to measure. It deals with long thin rods, or strips, with no lateral constraints as shown in Figure 1c. In this case both volume *and* shape change on application of axial stress. If the stress is compressive, instead of extensional, a strain results which is also governed by Young's modulus. For some purposes it is convenient to refer to Poisson's ratio σ, which is defined in the figure; this, too, is determined by both volume and shear strains. When $G \ll K$ as in elastomers, there is no significant volume increase on extension, and σ is approximately equal to one-half. If the volume were to increase enough to reduce significantly the lateral contraction of the rod, σ would approach zero. Poisson's ratio for the elastomeric materials ranges from 0.48 to very close to 0.50; for Lucite, it is 0.40; for steel it is 0.28. According to Ferry (page 25 of Reference 2) the minimum value of σ ordinarily observed is about 0.2 for homogeneous, isotropic materials; however, it may be smaller for certain heterogeneous materials such as cork or sponge rubber.

So far, six terms, i.e., λ, μ, K, G, E, and σ, have been introduced to describe elastic properties of a homogeneous material. (The Lamé constant μ is the same as the shear modulus G.) Data on *any* two of these are sufficient to completely describe the properties of a homogeneous material under the given conditions although practical considerations make some combinations better to work with than others as will be discussed later. Table 1 is presented, showing each parameter in terms of two of the standard moduli K, G, and E or the Poisson's ratio σ and one of the moduli.

The units of the moduli are force per unit of area and Poisson's ratio is dimensionless. The symbolism used follows Ferry; however, we should note the frequent use in the literature of B for bulk modulus (Kolsky calls it k), μ for shear modulus (this is logical because it is the symbol of the equivalent Lamé constant; however, I prefer not to mix the Greek and Latin alphabets in designating moduli), and Y for Young's modulus. Poisson's ratio is sometimes designated ν or μ. In engineering practice the units of the moduli are pounds per square inch. In most basic studies it is dynes per square centimeter or, in the mks system, newtons per square meter.* The cgs value is obtained by multiplying the modulus in psi by 6.895×10^4.

---

*According to MIL-STD-1621(NAVY) of 8 May 1973, the newton per square meter (N/m^2) is given the name pascal (Pa); however, this term has not yet received general usage in specifying moduli.

TABLE 1 – RELATIONSHIP BETWEEN ELASTIC CONSTANTS

Constant	K, G	K, E	K, σ	E, G	G, σ	E, σ
Lamé Constant μ Shear Modulus (G)	G	$\frac{3KE}{9K - E}$	$\frac{3K(1 - 2\sigma)}{2(1 + \sigma)}$	G	G	$\frac{E}{2(1 + \sigma)}$
Lamé Constant λ	$K - \frac{2G}{3}$	$\frac{3K(3K - E)}{9K - E}$	$\frac{3K\sigma}{1 + \sigma}$	$\frac{G(E - 2G)}{3G - E}$	$\frac{2\sigma G}{(1 - 2\sigma)}$	$\frac{E\sigma}{(1 + \sigma)(1 - 2\sigma)}$
Bulk Modulus (K)	K	K	K	$\frac{EG}{3(3G - E)}$	$\frac{2G(1 + \sigma)}{3(1 - 2\sigma)}$	$\frac{E}{3}\,\frac{1}{1 - 2\sigma}$
Young's Modulus (E)	$\frac{9KG}{3K + G}$	E	$3K(1 - 2\sigma)$	E	$2(1 + \sigma)G$	E
Poisson's Ratio (σ)	$\frac{3K - 2G}{6K + 2G}$	$\frac{3K - E}{6K}$	σ	$\frac{E}{2G} - 1$	σ	σ
Plane Wave Modulus (M)*	$K + \frac{4}{3}G$	$\frac{3K(3K - E)}{9K - E}$	–	$G\left(\frac{4G - E}{3G - E}\right)$	–	$\frac{E(1 - \sigma)}{(1 + \sigma)(1 - 2\sigma)}$

*This term will be defined later.

The mks values are lower than the cgs numbers by a factor of 10. The common use of all systems complicates the task of the worker in the field and requires him to acquire a "feel" for the different units. To further complicate the interpretation of data, many workers prefer to express their data as a compliance which is the inverse of the corresponding modulus. It requires extra effort to become accustomed to both methods of presentation, and this becomes even more difficult when the quantities are complex as will be discussed later.

To conclude the discussion about terminology and units, it should be pointed out that the bulk modulus is sometimes called the modulus of compressibility; the shear modulus, the rigidity modulus; the Young's modulus, the modulus of elasticity. The latter term is usually used in the German literature.

DYNAMIC ELASTIC MODULI

The moduli previously defined are called *static* moduli when the stress is constant. However, if the applied stress is a time-varying function such as that due to an acoustic wave, the resulting strain will vary in a similar manner. The ratio of stress to strain under these conditions, assuming small amplitude, is called the *dynamic* modulus. The static modulus is measured under isothermal conditions, and the dynamic modulus at greater than a critical frequency represents the adiabatic condition.

A difference between static and dynamic moduli occurs when there is failure to reach thermal equilibrium within the period of deformation. Another and more significant difference between static and dynamic moduli is due to structural relaxation processes. The strain for a given stress under static conditions is greater, and the modulus is smaller, because there is enough time for the internal reactions to achieve completion.

Disregarding relaxation effects, which will be discussed in a later section, it can be shown that the ratio of adiabatic to isothermal *bulk* moduli is

$$\frac{K_{ad}}{K_{is}} = \frac{C_p}{C_v} \tag{1}$$

where C_p and C_v are the heat capacities of the material at constant pressure and volume, respectively. A discussion of this relationship and some experimental data on solid polymers has been given by Warfield and associates[4] * at the Naval Ordnance Laboratory. They gave

[4]Warfield, R. W. et al., "Two Empirical Corresponding States Relationships for the Ratio of the Heat Capacities of Polymers," Naval Ordnance Laboratory TR 69-98 (21 Oct 1969).

*Warfield and his associates have been carrying on an extensive theoretical and experimental investigation at the Naval Ordnance Laboratory (now Naval Surface Weapons Center) for more than 10 years, concerning the static compressibility of solids.

data on a variety of solid polymers, ranging from K_{ad}/K_{is} = 1.01 for a polycarbonate to 1.18 for polyisobutylene. These data were taken at 25° C, where polycarbonate is a rigid plastic, and polyisobutylene is an elastomer. The critical frequency for the isothermal-adiabatic *transition* is somewhat dependent upon the dimensions of the specimen. However according to Ferry, this can occur at frequencies as low as 0.1 Hz so the majority of dynamic measurements of interest are adiabatic. For shear deformations at small strains, the moduli G_{ad} and G_{is} are identical in isotropic solids; see pages 140–141 of Reference 2.

The acoustic propagation properties of different types of stress waves are determined from the appropriate dynamic moduli. A general discussion of the various modes of propagation is available in texts such as Kolsky;[1] however, a summary will be given here of the modes of greatest interest in underwater acoustics.

The first case is a longitudinal wave propagated in a medium where the lateral dimensions are large, compared with a wavelength. The speed of the sound wave under this condition is

$$c_b = \sqrt{\frac{K + 4G/3}{\rho}} = \sqrt{\frac{M}{\rho}} \tag{2}$$

where M is defined as indicated, and ρ is the density. The modulus M is treated as one of the "basic" moduli due to the importance of this mode of propagation and is listed in Table 1. However, there does not seem to be general agreement about a name for it. Some authors refer to it only in terms of its components. Ferry[2] calls it the bulk longitudinal modulus, and Bobber[3] and others call it the plate modulus. None of these terms seems completely satisfactory, and, in recent informal correspondence with the author, Bobber has proposed the alternative of plane-wave modulus. His suggestion will be followed in this report. The G term is in the plane-wave modulus because deformation, in addition to compression, of the material occurs when this mode is propagated.

The propagation of a longitudinal wave in a rod or strip (Figure 2), when the lateral dimensions are small, compared with the wavelength, is also of interest and importance. In this case the sound speed is given by

$$c_e = \sqrt{\frac{E}{\rho}} \tag{3}$$

where E is Young's modulus.

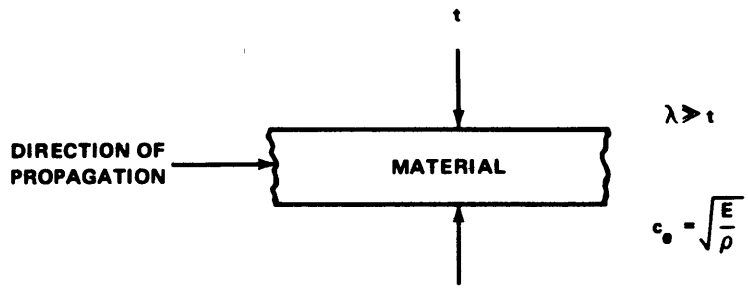


Figure 2 – Propagation of an Extensional Wave in a Rod

An intermediate case between the two types of waves discussed previously occurs when $t \approx \lambda$.* This case is rather complicated but has been studied both theoretically and experimentally.[1] The "rod" mode of propagation is approached in many situations in underwater acoustics in which microscopic or macroscopic voids are present in a matrix of elastomeric material.

When the direction of particle motion in a wave is perpendicular to the direction of propagation, contrasted with the parallel orientation of the longitudinal wave, a transverse or shear wave exists. In this case

$$c_s = \sqrt{\frac{G}{\rho}} \tag{4}$$

where G is the shear modulus.

ACOUSTIC DISSIPATION

If a significant portion of the acoustic energy put into compressing and accelerating the particles is dissipated as heat, the previous working formulas must be altered. The strain resulting from the stress in this case lags behind the stress by an angle δ, so, if the time dependence is given by $e^{i\omega t}$, any complex modulus m can be in the form

$$m^* = |m^*|\, e^{i\delta} = m' + i\, m'' \tag{5}$$

where m' is called the storage modulus and m'' the loss modulus; Equation (5) is usually written as

$$m^* = m'\,(1 + i\,\eta) \tag{6}$$

*The term λ refers here to the acoustic wavelength and should be distinguished from its previous usage as one of the Lamé constants. Both symbols are standard, and no confusion should arise in their respective usage.

The quantity η is called the *loss tangent* (tan δ) or *loss factor* and is commonly used to describe the acoustic dissipating properties of the material.

The loss factor in most cases indicates the dissipation which occurs in the material as it undergoes deformation. This is called the elastic loss factor. However, when a material in motion is subjected to viscous drag, there is a loss component in its inertial impedance called the density loss factor. Viscous loss occurs in a porous material where there is a displacement difference between the rigid matrix or framework and the liquid in the pores. Bobber[3] devoted pages 301–311 in his book to a discussion of the two types of loss and their relative effects.

Most materials have primarily elastic or viscous losses. In most cases the difference is not important because cyclic variation of both pressure and velocity occurs at a given point when a progressive wave passes. However, when a material is located in a standing wave pattern, the type of loss mechanism will be important. For example, if a thin material layer with a high viscous loss factor is mounted against a rigid backing, it is located in a pressure antinode and a velocity node. Therefore, very little motion of the particles of the medium takes place, and a viscous absorbing mechanism would not be effective. A material with a high elastic loss factor would, however, be quite effective because of the pressure maximum. If the viscous absorber is moved out a quarter wavelength to a pressure node and velocity antinode, the viscous loss mechanism would then predominate.

The relationship between the loss factor of the governing modulus and the spatial attenuation of the corresponding mode of propagation is as follows:*

$$\frac{m''}{m'} = \eta = \frac{2r}{1 - r^2} \tag{7}$$

where r is defined as

$$r = \frac{\alpha\lambda}{2\pi} \tag{8}$$

and α is the attenuation in nepers per unit length, and λ is the wavelength. It is usually more convenient to express α in decibels per unit length, in which case, $r \approx \alpha\lambda/54.6$. The term "r" is sometimes called the loss parameter.

In many cases it is preferable to indicate the dissipative properties of a material by using the term "Q" which is defined as

*This is derived in Section 6.4.4 of Reference 3.

$$Q = 2\pi \frac{\text{energy stored}}{\text{energy dissipated}} \tag{9}$$

during one cycle. This is related to η by $Q \approx 1/\eta$ for low absorption.

It should be noted that η is related to the attenuation *per wavelength* and α is the attenuation *per unit length*. The former is a more useful single number to describe the dissipating properties of a material under given environmental conditions.

The real part of the modulus is related to the propagation constants by

$$m' = \frac{\rho\, c^2\, (1 - r^2)}{\left(1 + r^2\right)^2} \tag{10}$$

where r is defined by Equation (8). With Equations (7), (8), and (10) one can obtain either m' and η from the propagation constants c and α (and ρ) or c and α from m' and η.* The set most suitable for the available sample size and the frequency range of interest is measured, and the other set can be computed if desired.

MEASUREMENTS OF BASIC ACOUSTIC PROPERTIES

INTRODUCTION

Collected data on acoustic properties of polymers are very extensive; however, the worker interested in a particular material is seldom able to find useful information. Data are scattered widely in the periodical and book literature of the world and in laboratory reports which are usually not readily available and frequently not even listed in bibliographies. These data are presented in many forms and units and each presentation usually covers a comparatively small range of frequency, temperature, and chemical composition. To obtain complete information concerning properties of a given material usually requires a literature search of mammoth scope as well as considerable work putting the data into a consistent form. Also, many of the measurements in the literature are made primarily to illustrate a new measurement technique, and composition and preparation of the material are not adequately described. Finally, one must cope with endless chemical variations in the polymers, such as different molecular structures for the same basic composition, amount and type of additives, method and degree of polymerization, amount of cross linking, etc. The result is that most workers find it more convenient to set up instrumentation and to make their own measurements. It is unfortunate

*Equations (7) and (10) result directly from the solution of the plane wave equation and are described various places including Reference 39.

that better use has not been made of the many excellent systematic studies of materials which have appeared in the report literature. More effort should be made to collect and particularly to collate this material.

BULK MODULUS

Measurement of dynamic bulk modulus K* is an especially challenging problem that has received little attention, compared with the effort expended on the study of shear and Young's moduli. However, it is an important quantity to consider when describing the acoustic properties of a material and should receive more attention. Probably the most comprehensive treatment of the theory and measurement of dynamic compressibility of polymers has been given by Marvin and McKinney.[5]

Techniques which have been used to measure the bulk modulus can be conveniently divided into four distinct categories, which will be described here. This discussion summarizes various attempts to measure this quantity.

Difference Method*

One of the more simple and obvious methods is to first obtain M*, which is equal to K* + 4G*/3, by measuring the speed and attenuation of the longitudinal plane wave in the material, calculating M* with Equations (7), (8), and (10), and then obtaining G* from corresponding data on the shear wave. A simple subtraction gives the real and imaginary parts of K*

$$\begin{aligned} K' &= M' - 4G'/3 \\ K'' &= M'' - 4G''/3 \end{aligned} \tag{11}$$

This method is not too accurate because it involves the difference between two measured quantities; however, it is the only one available to cover the frequency range from approximately 100 kHz to 30 megahertz.

Direct measurements of the propagation constants are made by introducing a train of pulses which is sufficiently long to approximate steady-state conditions. One type of experimental apparatus for doing this is shown in Figure 3.

---

[5]Marvin, R. S. and J. E. McKinney, "Volume Relaxation in Amorphous Polymers," Chapter 9 in Physical Acoustics, Volume 2, edited by W. P. Mason, Academic Press, New York (1965).

*The author has taken the liberty of applying short generic titles to the different methods.

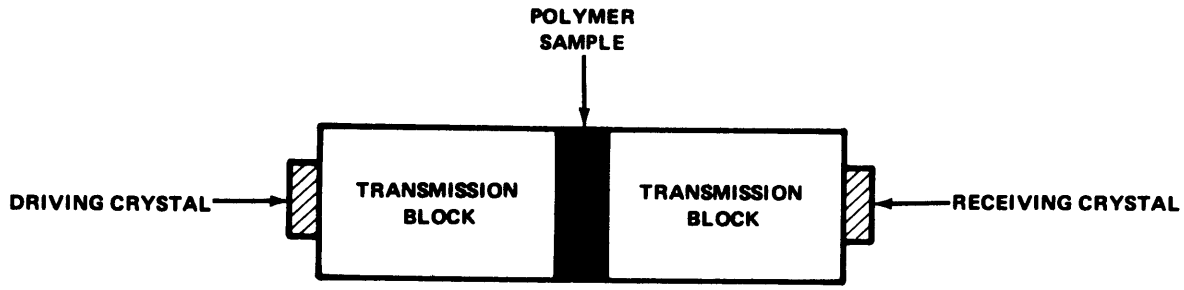


Figure 3 – Apparatus for Measuring Velocity and Attenuation of both Transverse and Longitudinal Modes in Polymer Sample in Megahertz Region

Essentially, the method involves measuring the transit time of an acoustic pulse in a transmission line both with and without the material to be studied. Experiments using versions of this basic apparatus have been described in the literature.[6–10] The transmitting medium shown in Figure 3 must be a solid to propagate the shear waves; however, a liquid medium can be used for the longitudinal waves. In some experiments a reflector was used instead of a receiver in which case the sender also acted as the receiver. Adhesive surfaces between sample and solid transmission line can becomes a problem, and proper precautions should be exercised.

Attenuation data are also obtained by using this method. A problem may arise, however, because of transmission loss or reflection during passage through the interfaces between sample and transmission blocks. If measurements are made of several thicknesses of the same material, this effect can be canceled.

A more direct method of obtaining both longitudinal and shear data is the so-called rotating-plate method, usually attributed to Schneider and Burton.[11] Summaries of this method are given in Chapter 8 of Reference 2 and in Reference 5. Figure 4 shows the basic concept.

---

[6] Ivey, D. G., "Propagation of Ultrasonic Bulk Waves in High Polymers," Journal of Applied Physics, Vol. 20, pp. 478–493 (1949).

[7] Nolle, A. W. and P. W. Sieck, "Longitudinal and Transverse Ultrasonic Waves in a Synthetic Rubber," Journal of Applied Physics, Vol. 23, pp. 888–894 (1952).

[8] Marvin, R. S. et al., "The Dynamic Bulk Viscosity of Polyisobutylene," Journal of Applied Physics, Vol. 25, pp. 1213–1218 (1954).

[9] Cunningham, J. R. and D. G. Ivey, "Dynamic Properties of Various Rubber at High Frequencies," Journal of Applied Physics, Vol. 27, pp. 967–974 (1956).

[10] Wada, Y. et al., "Comparison between Mechanical Relaxation Associated with Volume and Shear Deformation in Styrene-Butadiene Rubber," Journal of the Physical Society of Japan, Vol. 17, pp. 213–218 (1962).

[11] Schneider, W. C. and C. J. Burton, "Determination of the Elastic Constants by Ultrasonic Methods," Journal of Applied Physics, Vol. 20, pp. 48–58 (1949).

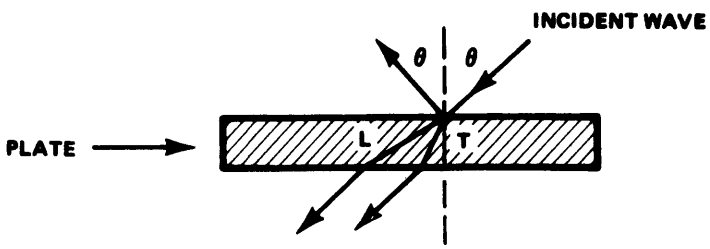


Figure 4a – Partial Transmission of Longitudinal Wave

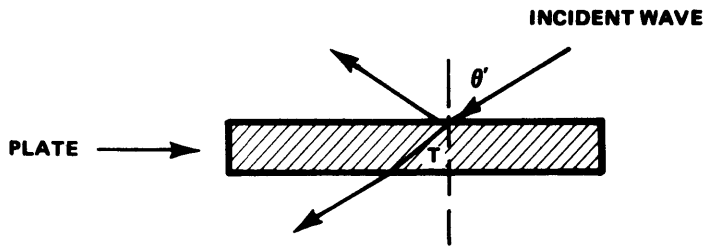


Figure 4b – Total Reflection of Longitudinal Wave

Figure 4 – Rotating-Plate Method for Separating Longitudinal and Transverse Waves in Viscoelastic Solids

(L refers to longitudinal wave in material; T, transverse wave)

A plate of the material to be studied is suspended in water or some other liquid with a plane sound wave incident upon one face at an angle θ. At normal incidence, no shear wave is generated in the material so that the longitudinal propagation parameters can be measured as described previously. However, both longitudinal and shear waves are excited when $\theta \neq 0$; these travel through the solid at different speeds and emerge into the liquid as separate longitudinal waves. If the angle of incidence is increased, a critical angle will be reached at which the longitudinal wave in the solid will undergo complete reflection and will reemerge into the water on the same side of the plate as it entered. The propagation constants of the transmitted shear wave can then be obtained from transit-time and attenuation measurements. This method is currently being used by Hartmann at the Naval Ordnance Laboratory[12] to study solid polymers. His apparatus is used over the range from 0.1 to 5.5 megahertz; Reference 12 gives a good description of this technique. Hartmann considers the speed data to be accurate to ±2 percent, while absorption measurements are accurate to ±10 percent.

[12]Hartmann, B. and J. Jarzynski, "Immersion Apparatus for Ultrasonic Measurements," Naval Ordnance Laboratory TR-72-73 (14 Apr 1972).

Direct Stress-Strain Measurements

A more direct way of measuring K* is to measure stress p and strain $\Delta V/V$ and to take the ratio; see Figure 1a. The best known use of this method was by McKinney, Edelman, and Marvin,[13] working at the National Bureau of Standards, who used the apparatus diagramed in Figure 5. The piezoelectric ceramic transducers were mounted in the center cavity, which was a cylinder 16 mm in diameter. One crystal drove the cavity at a frequency at which the wavelength was large, compared to the dimensions, and the other measured the phase and magnitude of the resulting strain. Measurements were made successively with the cavity filled with a transmitting liquid and with a sample in the liquid; the data obtained were used to calculate compressibility of the material. Measurements have been reported for the frequency ranges from 50 to 10,000 Hz, static pressures to 1000 atm and a range of temperatures. Using the same technique and with an improved detection that increased the frequency range, Heydemann,[14] working at Göttingen in Germany, measured the complex compressibility from 0.1 to 12 kHz at 20° C and at atmospheric pressure.

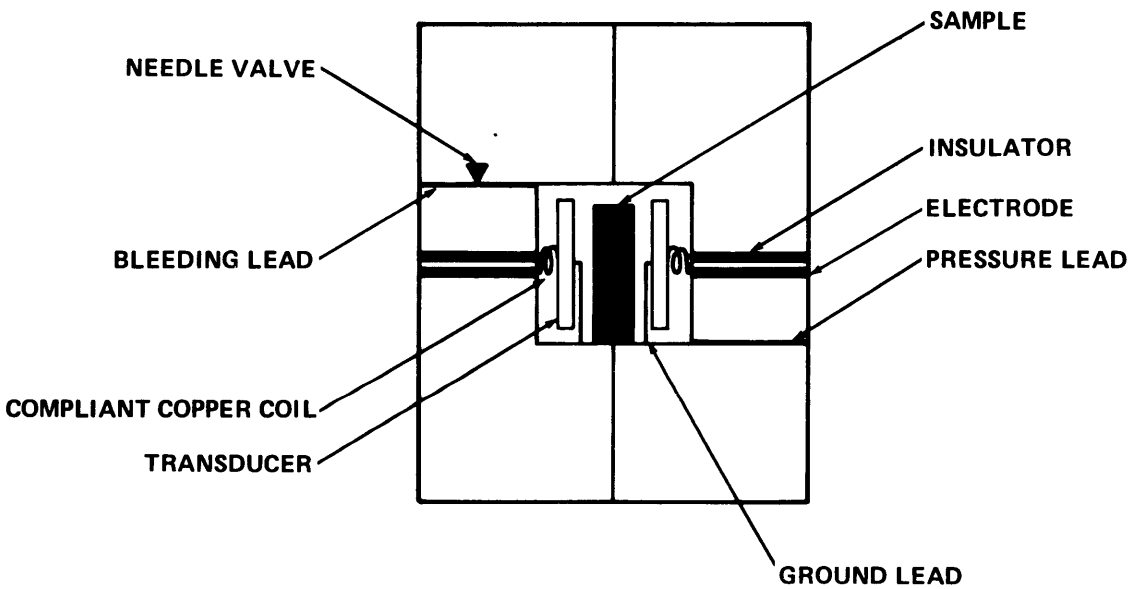


Figure 5 – National Bureau of Standards Dynamic Compressibility Chamber

---

[13] McKinney, J. E. et al., "Apparatus for the Direct Determination of the Dynamic Bulk Modulus," Journal of Applied Physics, Vol. 27, pp. 425–430 (1956).

[14] Heydemann, P., "The Dynamic Compressibility of Highpolymers in the Frequency Range from 0.1 c/s to 60 kc/s," Acustica, Vol. 9, pp. 446–452 (1959).

At very low frequencies a number of direct measurements have been made of dynamic bulk modulus. A frequently referenced example of this is the work of Philippoff and Brodnyan.[15] They used a mechanically driven piston to apply alternating stress to a specimen in a liquid-filled rigid cylinder, using strain gages to measure the volume strain. This equipment could be operated from 0.0003 to 6 hertz. This technique is particularly suitable for high-amplitude studies. A similar apparatus has been described by Sharma and McCarty[16] at The Pennsylvania State University, who worked in the range from 10^{-4} to 10^{-1} hertz.

Resonant Tube Method

Historically this method takes precedence; however, it has never had wide usage. As originally designed by Meyer and associates[17] working in Berlin during World War II, it consisted of a vertical steel tube filled with water (Figure 6), which was mechanically vibrated at the bottom and tuned to resonance by means of a monitoring hydrophone at the top. The resonant frequency (about 1200 Hz in their case) is first measured for the tube and water only; the sample is then placed a distance $\lambda/4$ down from the free surface as indicated in a pressure antinode to obtain a new resonance. The difference between these frequencies, along with parameters of the tube and sample, gives enough information to calculate the bulk modulus. The difference between the Q's of the tube with and without the sample is used to calculate the loss modulus K''. Hydrostatic pressure could be applied to the tube; however, this involves enclosing the whole unit in a large pressurized cylinder.* The resonant tube technique was revived in this country in the late 1940's by Sandler at the Naval Ordnance Laboratory (NOL).[18,19] A still newer version has been completed and put into use at the Naval Ship Research and Development Center (the Center) by Niemiec.[20] The method is comparatively easy to set up and is simple to use. The important advantage is that irregularly shaped samples may be used of materials ranging in compressibility from foam rubbers to hard plastics. Its chief drawbacks are a lack of frequency range and difficulty in applying static pressure. Also, it requires measuring frequency shifts to the order of one-tenth of a hertz.

---

[15] Philippoff, W. and J. Brodnyan, "Preliminary Results in Measuring Dynamic Compressibilities," Journal of Applied Physics, Vol. 26, pp 846–849 (1955).

[16] Sharma, M. G. and V. McCarty, "Experimental Investigations on the Dynamic Compressibilities of Polymeric Material," Applied Polymer Symposia, Vol. 1, pp. 201–215 (1965).

[17] Meyer, E. and K. Tamm, "Ein Akustisches Messverfahren Bestimmung der Dynamischen Kompressibilität und des Verlustfaktor Elastischer Stoffe," Akustische Zeitschrift, Vol. 7, pp. 45–50 (1942).

[18] Sandler, C. S., "Measurement of Bulk Modulus and Associated Loss Factor," Journal of the Acoustical Society of America, Vol. 22, Abstract, pp. 685–686 (1950).

[19] Sandler, C. S., "An Acoustic Technique for Measuring the Effective Dynamic Bulk Modulus of Elasticity and Associated Loss Factor of Rubber and Plastics," Naval Ordnance Laboratory Report 1534 (Sep 1950).

[20] Niemiec, J. M., "Determining the Dynamic Bulk Modulus and the Loss Factor of Polymers from Acoustic Measurements," NSRDC Report 3858 (Oct 1972).

*As will be discussed later, effect of static pressure on bulk modulus only becomes significant at several thousand pounds per square inch.

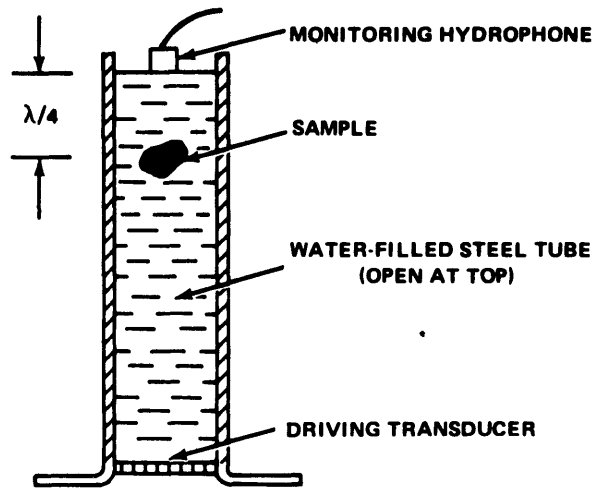


Figure 6 – Resonant Tube Method of Measuring Dynamic Bulk Modulus

Dispersed Material Method

A less practical method for general usage but of considerable value in basic studies involves measuring propagation constants in a liquid that contains a large dispersion of minute particles of the material to be studied. These small particles ($<1/20\ \lambda$) alter the propagation parameters of the liquid according to classic relationships advanced by Urick and Herzfeld, and the changes in propagation properties are used to calculate the complex bulk modulus. This method is said to be capable of giving high precision results with particle concentrations of the order of 10 percent by volume. The useful frequency range extends from approximately 0.3 to more than 10 megahertz. This method is usually credited to Wada and collaborators in Japan.[21] Other uses of this method were by Smithson at the U. S. Naval Academy[22] and Pullen, et al. in England.[23] This method is not used very often because of the difficulty encountered in preparing material in this form.

PLANE-WAVE MODULUS

This term has been defined previously in this report as $K^* + 4G^*/3$ and is the effective modulus for propagation of longitudinal waves in a medium in which the lateral dimensions

---

[21] Wada, Y. et al., "Volume Viscosity of Polymers and Other Highly Dissipative Materials," Journal of the Physical Society of Japan, Vol. 15, pp. 2324–2334 (1960).

[22] Smithson, J. R., "Measurement of Viscoelastic Properties of Polymers by Ultrasonic Methods," Journal of the Acoustical Society of America, Vol. 42, Abstract, p. 1162 (1967).

[23] Pullen, W. J. et al., "The Measurement of Dynamic Bulk Modulus Using an Ultrasonic Interferometer," Polymer, Vol. 5, pp. 471–478 (1964).

are large compared with a wavelength, and the waves are from the original source. One could, of course, determine M* by separate measurements of K* and G*; however, in most cases it is simpler to measure it directly.

Measurement of M* by direct transmission of high-frequency plane waves has already been described under Method 1 in the section about bulk modulus. In addition to the NOL work of Hartmann already mentioned, this technique is also being used by Folds[24] at the Naval Coastal Systems Laboratory in Panama City, Fla., to study materials for possible use in acoustic lenses. Folds used blocks of material 5 cm long to study propagation directly instead of using the transmitting medium and thin layer of material as previously described. He claimed an accuracy of ±8 meters per second. A similar technique was used by Morice at the Center to study propagation of longitudinal and transverse waves in syntactic foams. In a recent study at NOL, Madigosky and Von Bretzel[25] measured the longitudinal plane-wave properties of a polybutadiene elastomer over the range from 300 kHz to 5 megahertz. They used a water transmission path from 300 to 3000 kHz and a solid delay rod at 5 megahertz.

A recent development in measurement of the plane-wave modulus is the use of the acoustic impedance method. This technique has received comparatively little attention in the literature; however, it is of considerable interest because it can be used at low frequencies. It is of importance in underwater acoustics both because of its low-frequency capability and the relative ease of carrying out measurements under high hydrostatic pressures. The measurement technique will be discussed briefly here, and further details can be obtained from original sources or from the Bobber book.[3]

Referring to Figure 7 we consider a plane-wave incident on a layer of acoustically homogeneous material with properties as indicated. If wave equations are set up for the various waves, and the boundary conditions are applied, a relationship for the complex input impedance Z_i (ratio of acoustic pressure to particle velocity at the water-material interface) can be obtained. When the backing impedance Z_t is infinite this equation is

$$Z_i = Z_0 \coth (\alpha + i\, k)\, d \tag{12}$$

[24] Folds, D. L., "Experimental Determination of Ultrasonic Wave Velocities in Plastics, Elastomers, and Syntactic Foam as a Function of Temperature," Journal of the Acoustical Society of America, Letter to the Editor, Vol. 52, pp. 426–427 (1972).

[25] Madigosky, W. M. and J. von Bretzel, "Viscoelastic Properties of 1,2 Polybutadiene and Comparison with Other Rubbers," Naval Ordnance Laboratory TR 72-192 (23 Aug 1972).

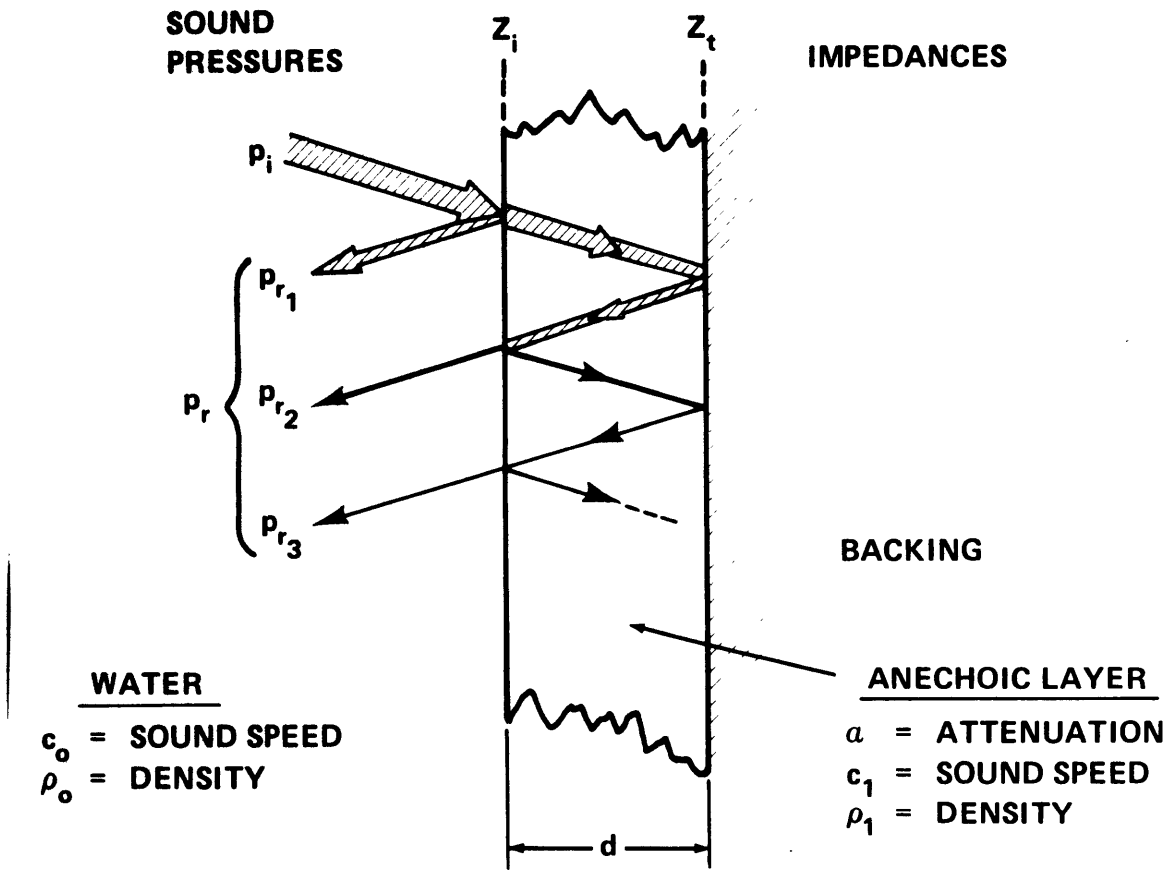


Figure 7 – Plane Wave Incident on Backed Absorbing Layer

(Radiation drawn at oblique incidence to show separate rays)

where Z_0 is the complex characteristic acoustic impedance of the material.* (This would represent the case where d is large enough that no echo will be returned from the backing, i.e., $p_{r_2} = 0$ in Figure 7.) In terms of the propagation properties

$$Z_0 = \rho_1 c_1 \left(\frac{1}{i - ir} \right) \tag{13}$$

or

$$Z_0 = \rho_1 c_1 \left(\frac{1}{1 + r^2} + i \frac{r}{1 + r^2} \right)$$

where, as previously, $r \approx \alpha\lambda/2\pi$ when α is in nepers per unit length, and approximately equal to $\alpha\lambda/54.6$ when α is in decibels per unit length. The derivation of and further discussion about Equations (12) and (13) can be found in Kosten and Zwikker,[26] Bobber,[3] and Cramer[27] as well as in the literature on electric transmission lines. For the present purpose, it is noted that if the acoustic impedance of the layer is measured with a rigid backing, the propagation terms c and α can be determined from Equations (12) and (13).

Unfortunately, Equation (12) is a transcendental equation for which unique solutions cannot be obtained. However, Sabin and associates at the Naval Research Laboratory, Underwater Sound Reference Division (USRD), Orlando, Fla., in 1966 developed charts from which α and k could be obtained from impedance data.[28] Subsequently, they developed a computer program for solving Equation (12) by successive approximations.[29] The acoustic impedance technique has been used extensively by Lastinger and associates at Orlando to study a variety of material.[30–32] The impedance method of determining propagation constants is in current

---

*When $Z_t = 0$, coth becomes tanh. For $Z_t \neq \infty$ or 0, Z becomes a more complicated expression as described in References 3, 26, and 27.

[26] Kosten, C. W. and C. Zwikker, "Sound Absorbing Materials," Elsevier Publishing Company, Amsterdam (1949).

[27] Cramer, W. S., "Theoretical Study of Underwater Sound Absorbing Layers," Naval Ordnance Laboratory Report 2803 (Jun 1953).

[28] Sabin, G. A., "Acoustic-Impedance Measurements at High Hydrostatic Pressures," Journal of the Acoustical Society of America, Vol. 40, pp. 1345–1353 (1966).

[29] Lastinger, J. L. and G. A. Sabin, "A PDP-8 Fortran Program for Reduction of Acoustic Impedance Data," Naval Research Laboratory Report 6906 (28 Mar 1969).

[30] Lastinger, J. L. and I. D. Groves, Jr., "Speed of Sound in Four Elastomers," Naval Research Laboratory Memorandum Report 2363 (3 Dec 1971).

[31] Lastinger, J. L., "The Acoustical Properties of Water-Saturated Fibrous Metal," Journal of the Acoustical Society of America, Vol. 52, pp. 1465–1470 (1972).

[32] Lastinger, J. L., "Acoustic Characteristics of Woods at High Hydrostatic Pressures," Journal of the Acoustical Society of America, Vol. 47, pp. 285–289 (1970).

use at the Center by Goodman and associates and has also been used extensively by Higgs and associates,[33] formerly at Honeywell, to study "pressure release" materials under hydrostatic pressure.

The method currently favored for measuring impedance in underwater sound work is the so-called pulse-tube method. This was developed in Germany by Meyer and associates[34,35] during World War II and was used in development of the anechoic coating concept. The first U. S. pulse tube was constructed in the early 1950's at NOL.[36] Since then many have been put into use, including those at the Center, the Mare Island Rubber Laboratory,* the Naval Underwater Systems Center, New London, several British naval laboratories, the University of Göttingen (Federal Republic of Germany), USRD, The Applied Research Laboratories (Texas), Honeywell, and probably others. Many of these are still operational.

The basic operation of the pulse tube is described with the aid of Figure 8. A pulsed acoustic sine wave is generated, which propagates up the tube to impinge on the interface between the water and the sample; the wave is then reflected to be picked up and measured either by the transmitter acting as a receiver or by separate pickups in the wall of the tube. The ratio $|p_r/p_i|$ (Figure 7) and the phase shift ϕ between p_r and p_i, which occur on reflection, are measured. This is a comparative method and values obtained must be related to corresponding values from a reflector of known characteristics. Either a rigid reflector ($|p_r/p_i| = 1$ and $\phi = 0°$) or an air interface ($|p_r/p_i| = 1$ and $\phi = 180°$) is used. The latter presents more experimental difficulties than does the rigid type. The ratio and phase shift due to the material give sufficient information to compute the acoustic impedance which is then substituted in Equation (12).

The pulse tube is most commonly used to measure reflection from and transmission through a structured layer of any thickness and with any backing desired that is designed for a specific acoustic application. The acoustic impedance method for obtaining the propagation constants should be considered a separate, but related, application of the tube requiring an acoustically homogeneous cylinder of the material of thickness approximately from 5 to 15 centimeters with a rigid or soft backing.

---

[33] Higgs, R. W. and L. J. Eriksson, "Acoustic Decoupling Properties of Onionskin Paper," Journal of the Acoustical Society of America, Vol. 46, pp. 211–215 (1969).

[34] Kuhl, W. et al., "Impulsverfahren zur Messung der Reflexion von Wasserschall–Absorben in Rohren," Acustica, Vol. 3, p. 421 (1953).

[35] Richardson, E. G., "Technical Aspects of Sound," Chapter 4, Section 2, Vol. II, Elsevier Publishing Company, Amsterdam (1957).

[36] Cramer, W. S., "Pulse Tube for Acoustic Measurements," Naval Ordnance Laboratory Report 2257 (Apr 1952).

*This laboratory was disestablished September 1973 and has become a part of the Annapolis Laboratory of the Center where work on coatings continues. The Rubber Laboratory under the able direction of Mr. Ross Morris made substantial contributions to the development of underwater acoustic structures.

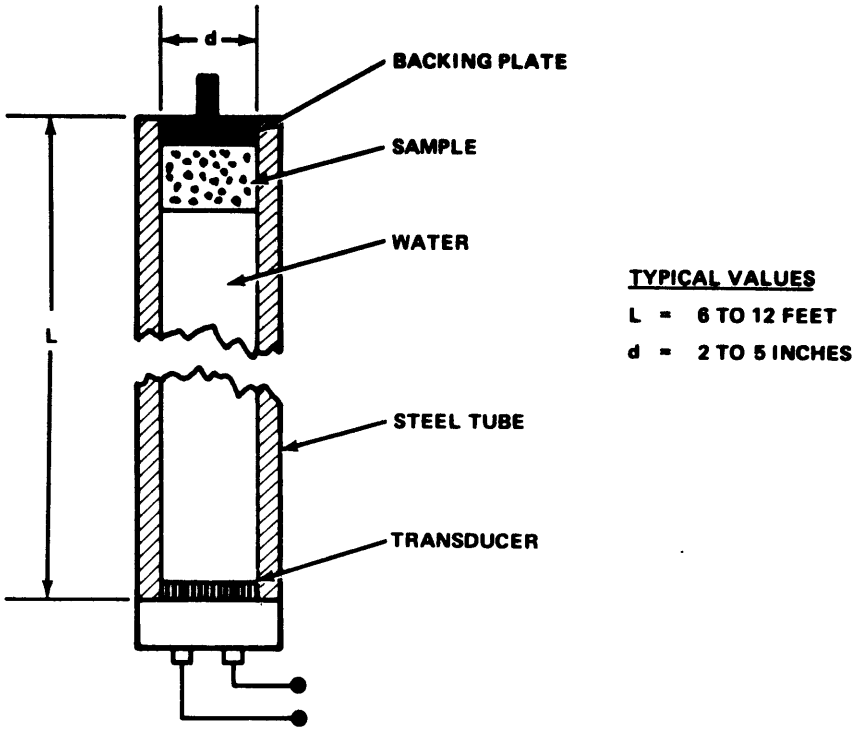


Figure 8 – Acoustic Pulse Tube

The pulse tube is by no means the only method of measuring acoustic impedance at an interface. Beranek[37] lists 12 different methods in his comprehensive chapter about *acoustic impedance* and, incidentally, discusses Equation (12) in some detail. A convenient and reliable method, which is favored in airborne sound work, involves the study of a standing wave pattern set up when a plane sound wave is normally incident on a sample. A traveling microphone is used to measure the locations and magnitude of the nodes and antinodes, and this information is used to calculate the input impedance. In the case of underwater applications where the effects of hydrostatic pressures on the material can be important, this method presents experimental difficulties in designing a traveling hydrophone for operation inside a pressurized tube. For this reason the pulse tube is favored; however, standing-wave studies in water-filled tubes have been reported. The standing-wave method has the substantial advantage that it does not require a reference reflector for comparison.

An interesting method of directly measuring and recording curves of acoustic impedance versus frequency for waterborne sound has been developed independently by Kreer[38] at

[37] Beranek, L. L., "Acoustic Measurements," Chapter 7, John Wiley and Sons, Inc., New York (1949).

[38] Kreer, U., "Aufbau einer Messapparatur zur direkten Aufzeichnung von Impedanzkurven für Wasserschall im Kundtschen Rohr," Acustica, Vol. 22, No. 1, pp. 23–34 (1969).

Göttingen and Ho, Heiner, and associates at the Center. Kreer used a water-filled waveguide with the sample mounted at one end and a continuous wave generator at the other. The sound pressure and particle velocity were determined simultaneously at a given location in the tube, and the complex ratio of these two quantities, formed by appropriate electronic apparatus, was recorded directly. The hydrophone system consisted of two identical pressure hydrophones separated by 10 millimeters. The sound pressure and the sound pressure gradient (from which the sound particle velocity can be calculated) were obtained from the sum and difference, respectively, of the two readings. His apparatus was used to measure the input impedance of absorbing samples over the frequency range from 2 to 14 kHz at variable temperatures and at static pressures up to 40 atmospheres. The Center apparatus is similar in principle.

YOUNG'S MODULUS

This modulus has already been defined as the governing modulus when sound is propagated as extensional waves in a rod or strip when the lateral dimensions are small compared with the wavelength. At more than $a/\lambda = 0.1$ a term, known as the Rayleigh correction,* is used to correct for the effects of radial inertia. This correction does reasonably well to approximately $a/\lambda = 0.7$. At more than that, the complicated relationships discussed in Chapter 3 of Kolsky must be used. Direct measurements of Young's modulus to 200 kHz have been made on fibers.

Measurements of Young's modulus have received attention from many investigators, and this short section will inevitably be highly selective. Techniques described will be those which have received wide usage and/or illustrate important physical principles.

Progressive Wave Techniques

Extensional Waves. Probably the best known and most convenient method of measuring Young's modulus for an elastomer is a progressive wave technique which is explained with the aid of Figure 9. A rod or strip of the material is suspended horizontally (or vertically) and is excited mechanically at one end so that an acoustic wave is propagated down the rod. Operating conditions are reached when attenuation is so great that negligible energy is returned from the far end, that is, no standing wave is formed. An acoustic pickup, usually a piezoelectric element which bears lightly on the material surface, is moved along the rod to measure

*In the Rayleigh correction the ratio of measured modulus to true modulus is $\left[1 - \sigma^2\pi^2 (a/\lambda)^2\right]^2$ where a is the radius of the rod and σ is the Poisson's ratio.

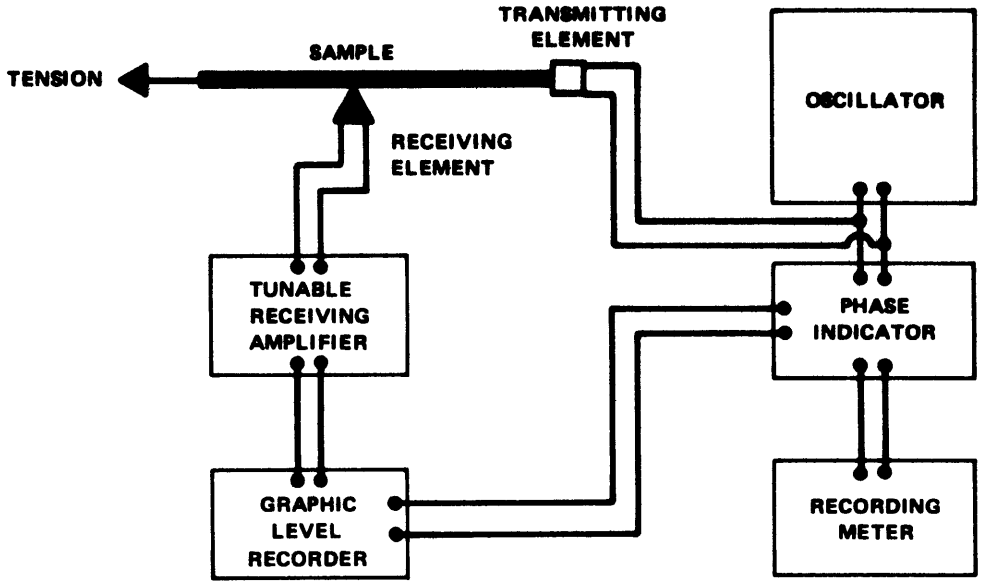


Figure 9 – Experimental Apparatus for Progressive Wave Technique

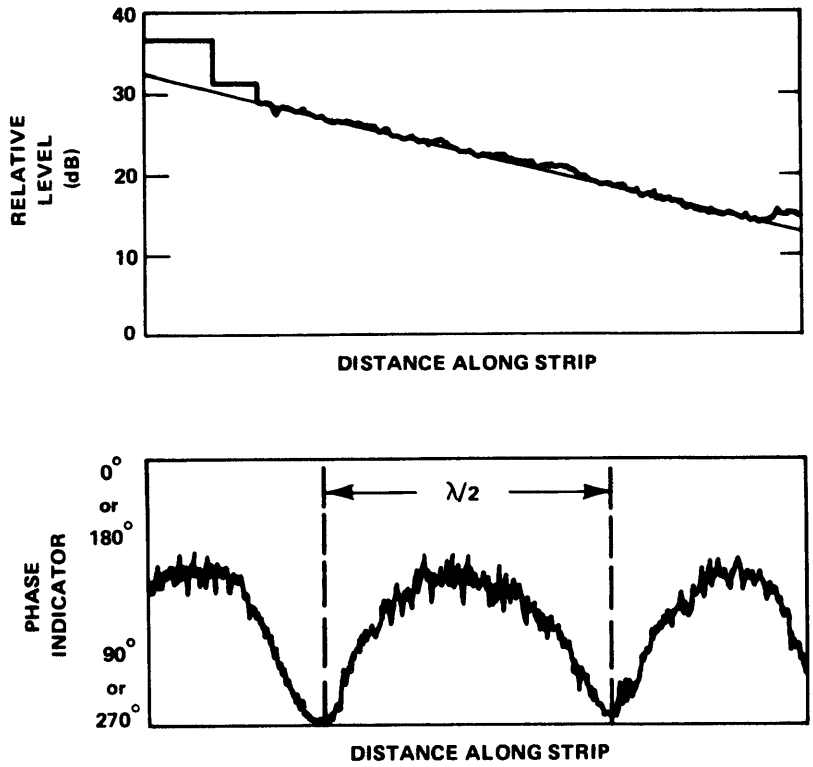


Figure 10 – Typical Attenuation Record for Strip Transmission Apparatus

(Figure 7 of Reference 62)

relative amplitude as a function of distance and also to measure the relative phase of the signal from which the wavelength can be obtained. With the preceding information and the density Equations (7) and (10) can be used to calculate the complex Young's modulus. Figure 10 shows the type of experimental data obtained.

The lower frequency limit of operation is reached when the attenuation, which increases with frequency, becomes too low to dissipate the energy in the length available. The upper limit is reached when the attenuation is too great to measure or when the acoustic wavelength approaches the cross sectional dimensions of the specimen as discussed previously. Satisfactory results in the range from 100 Hz to 40 kHz have been claimed.

Credit for this technique is usually given to Nolle,[39] working at the Acoustics Laboratory of the Massachusetts Institute of Technology during and shortly after World War II. However, this same principle was also employed by Meyer and associates in Germany during World War II. (His work is described in Section E4 of Chapter 3 of Reference 40.*) The progressive wave technique was also used at the Naval Ordnance Laboratory[41,42] (about 1950) and at the Mare Island Rubber Laboratory[43] (1967) to carry out extensive systematic investigations of the effects of rubber type and loading on the acoustic properties of elastomers. The literature contains many references to the use of this method. It is easy to set up and to use, and anamolous behavior can readily be spotted.

Flexural Waves. The previously described technique can also be applied to measurement of progressive flexural waves in rods or strips. Flexural waves, like extensional waves, are governed by Young's modulus; however, the relationships are different. Young's modulus applies because one side of the sample is stretched, and the other is compressed as the rod flexes. To measure flexural waves, the direction of the driver is changed, and the pickup is adjusted for lateral instead of longitudinal motion. The relationship between the sound speed of a flexural wave c_f and Young's modulus, no loss assumed, for a rod is derived from elementary consideration[1] to be

---

[39] Nolle, A. W., "Acoustic Determination of the Physical Constants of Rubberlike Materials," Journal of the Acoustical Society of America, Vol. 19, pp. 194–201 (1947).

[40] Meyer, E. and associates, "Sound Absorption and Sound Absorbers in Water," Naval Ship Systems Command Report 900, 164 (1947).

[41] Cramer, W. S., "Propagation of Stress Waves in Rubber Rods," Naval Ordnance Laboratory Report 4216 (Sep 1956); Journal of Polymer Science, Vol. 26, pp. 57–65 (1957).

[42] Cramer, W. S. and I. Silver, "Acoustical Properties of Rubber as a Function of Chemical Composition," Naval Ordnance Laboratory Report 1778 (Feb 1951).

[43] San Francisco Bay Naval Shipyard, "Determination of Dynamic Properties of Rubbers and Plastics," Rubber Laboratory, San Francisco Bay Naval Shipyard Report 104-2 (16 Feb 1967).

*Professor Meyer used an apparatus in which the sample (excited at its upper end) was lowered vertically into a water-filled tube. The maximum sound radiation occurs at the surface, which is closed with a brass plate. Attenuation and relative phase were measured by a hydrophone in the water column.

$$c_f = \sqrt[4]{\frac{E\,a^2\,\omega^2}{4\rho}} \quad \text{or} \quad E = \frac{4\rho\,c_f^4}{a^2\,\omega^2} \tag{14}$$

where a is the radius of the rod. If a strip of thickness h is used instead of the rod, the results are

$$c_f = \sqrt[4]{\frac{E\,h^2\,\omega^2}{12\rho}} \quad \text{or} \quad E = \frac{12\rho\,c_f^4}{h^2\,\omega^2} \tag{15}$$

when the width of a strip is less than a wavelength. When the complex modulus $E' + iE''$ is used instead of E in Equation (14), the previously shown results are modified as follows

$$E' = \frac{4\rho\,c^4}{a^2\,\omega^2}\left[\frac{(1 - 6r^2 + r^4)}{\left(1 + r^2\right)^4}\right]$$

and (16)

$$\frac{E''}{E'} = \eta = \left[\frac{4r\,(1 - r^2)}{(1 - 6r^2 + r^4)}\right]$$

where r is defined as previously. When a/λ is greater than 0.1 Equations (16) are in error because of neglect of rotational inertia and shearing effects. Corrections for these effects have been presented by Rayleigh (rotational energy) and Timoshenko (shearing correction) and must be added to Equations (16); see Equation (3.34), Chapter 3, Kolsky.[1]

An interesting experimental study of progressive flexural waves in strips of unsaturated polyurethane was made by Guicking[44] at Göttingen. Reference 44 discusses the correction terms to Equations (16) in detail and presents nomographs to simplify their calculation. To monitor the signals, Guicking used an optical technique in which the vibration of the strip modulated a light beam incident on a photoelectric cell. A similar experimental technique was used by Heydemann and Zosel,[45] also at Göttingen. Guicking worked in the range from 200 to 1000 Hz; Heydemann and Zosel, from 50 to 1000 hertz.

---

[44] Guicking, D., "Dynamisch-Mechanische Eigenshaften von Plastischen Massen und Vernetzten Polyurethanen," Acustica, Vol. 18, pp. 93–104 (1967).

[45] Heydemann, P. and A. Zosel, "Determination of the Complex Young's Modulus of Polymers at Audiofrequencies in a Wide Temperature Range," Acustica, Vol. 12, pp. 360–365 (1962).

The writer[41] investigated progressive flexural waves in rubber rods as a part of a study of propagation of extensional, flexural, and torsional waves in the same 1/8-inch-diameter rod. (Reference 41 also contained a discussion of the effects of the Rayleigh-Timoshenko corrections.) In this experiment a phonograph needle was used as the pickup. This study, based on very limited data, showed that $E'_{(flex)}$ and $E'_{(ext)}$ agreed to within about 10 percent under the same conditions. Flexural wave experiments in elastomers are currently being made at the Center by Holtz. He uses a miniature piezoelectric pickup* and a unique cross correlation method[46] for obtaining the wavelength. His operating range is from approximately 200 Hz to 20 kilohertz.

The alternative of using either extensional or flexural waves in the progressive wave technique to obtain Young's modulus invites comparisons between the two methods. The extensional wave calculations are much easier to handle analytically, and the Rayleigh correction, when needed, enables one to obtain reasonably accurate results for most conditions of interest in underwater acoustics. For flexural waves, the corrections are much more complicated and a computer program is recommended; however, according to Kolsky (Figure 16 of Reference 1), the corrected results agree well with those from exact theory. In general, the flexural waves are easier to excite and, in many cases, easier to pick up. The advantage of the optical pickup favored by the Germans is that all possibilities of mechanical interference with the wave have been eliminated. However, various investigations with light piezoelectric pickups show no evidence of significant mechanical interference, and this experimental method is much easier to set up. Another factor that must be considered is that the sound speed of a flexural wave is less than that of an extensional wave of the general frequency regions of interest, and the attenuation is greater. Consequently, choice of the mode of propagation to be used will be influenced by the length and type of material available and the frequencies of interest.

Another point of interest is that the flexural mode can be used to study materials which are too soft to be self supporting in the form of a rod or strip. This is done by applying the material to a strip of material of known properties. Measurements of flexural propagation in the composite are obtained and, using theories by Oberst[47] and van Oort,[48] Young's

---

*Designed and developed at the Center by C. A. Migliaccio and G. A. Smith.

[46] Holtz, L. C., "Traveling-Transducer Method of Measuring Cross-Correlation," U. S. Patent 3,597,962 (10 Aug 1971).

[47] Oberst, H. and K. Frankenfeld, "Über die Dämpfung der Biege-schwingungen dünner Bleche durch fest haftende Beläge," Part 1, Acustica, Vol. 2, Akustische Beihefte, Heft 4, AB 181–194 (1952); Part 2, Acustica, Vol. 4, 433–444 (1954).

[48] Van Oort, W. P., "A Method for the Measurement of Dynamic Mechanical Properties of Small Samples of Plastic Material," Microtechnic, Vol. 7, p. 246 (1952).

modulus of the unknown material is calculated. Also, this technique is sometimes useful to obtain the flexural wave propagation in strips of laminated materials as this represents conditions which exist in realistic situations such as the shell of an underwater body.

Standing Wave Techniques

Flexural Waves. The previous section dealt with acoustic waves that were attenuated to the extent that no appreciable standing wave pattern was set up. However, when a prominent standing wave pattern does exist, the resonance frequencies of the rod can be used to compute the dynamic modulus of the material. In one variation (Figure 11)[49] a rod or strip with unconstrained ends is excited in flexure in as many modes as possible. If the losses are small, $L = \lambda_n/2$, where $n = 1,2,3...$, and the values of λ_n obtained are used to compute the real part of Young's modulus at the various resonance frequencies. The dissipation of acoustic energy in the material is determined from the bandwidth* at the resonance frequency ($Q \approx 1/\eta$). A wide variety of methods has been used to drive such systems and to monitor the amplitude; however, such an instrument must not load the system, thereby affecting the resonance frequency. Also, the rod must be supported at a node of motion and Q of the system must be at least 10 ($\eta \leqslant 0.1$) to obtain reasonably accurate measurements. The general theory of rod and strips vibrating in flexure is available widely in the literature; see Kinsler and Frey.[50] However, additional correction terms are sometimes needed to achieve accurate results, and more detailed references must be consulted; see Chapter 3 of Reference 1.

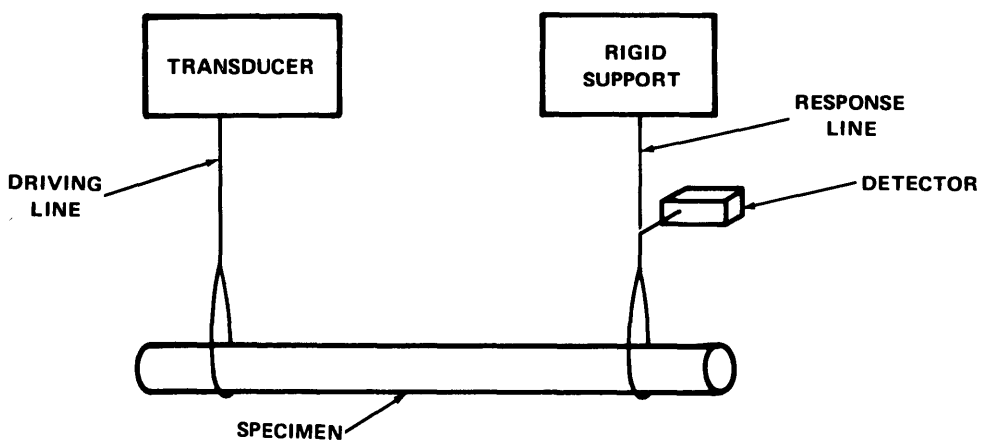


Figure 11 – Apparatus for Measuring Flexural Vibrations in a Freely Suspended Rod
(From Reference 50)

[49] Kline, D. E., "Effect of Branching on Dry Mechanical Properties of Polyethylene," Journal of Polymer Science, Vol. 22, p. 449 (1956).

[50] Kinsler, L. E. and A. R. Frey, "Fundamentals of Acoustics," Second Edition, Chapter 3, John Wiley and Sons, Inc., New York (1950).

*The frequency difference between the half-power points on the two sides of the resonance peak.

The flexural resonance technique is used frequently with rigid plastics; in fact, it is the most satisfactory method for studying these materials in the lower audiofrequencies. (Ferry says that it is effective from 10 to 5000 hertz.) The limitation to discrete frequencies is not a serious disadvantage for the rigid plastics because their properties do not change appreciably with frequency over the frequency and temperature ranges of general interest.

Figure 11 shows a free-free mode of oscillation. The clamped-free and clamped-clamped modes are also used. The large number of variations available of essentially the same principle makes it difficult and pointless to discuss this technique in detail in this survey. Almost every reference on dynamic properties of polymers treats the subject extensively; see Section C of Chapter 7 of Reference 2. Brief mention, however, will be made of a few cases of special interest. A variation of the technique of Figure 11 was used by Barnet and Cuevas[51] at NOL who studied the dynamic properties of epoxy resins filled with glass spheres or glass powder. Their article gave interesting results and presented a strong case for using dynamic evaluation of structural composite materials; it pointed out that this approach to the study of such materials had been largely neglected. A variation of the resonance technique that deserves special mention because of its frequent use is the "vibrating reed" or "vibrating cantilever." Figure 12 shows a reed driven at the clamped end; however, the free end is sometimes driven by a small coil mounted on it. It is a convenient method, frequently used for both rigid and rubberlike (if the loss is not too great) plastics. The useful frequency range is given as 10 to 500 hertz. Theoretical treatments are available to cover most situations.[52]

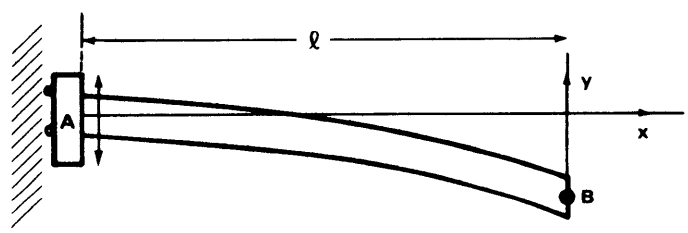


Figure 12 – Vibrating-Reed Method

The choice between the progressive wave and resonance techniques does not usually present a problem. However, when η is from 0.1 to 0.2, one usually contends with either a broad resonance and its inaccuracies or a progressive wave and a small standing wave pattern. For moderate losses the standing wave pattern near the reflecting end can be

[51] Barnet, F. R. and J. Cuevas, "Dynamic Response of Thermosetting Polymer System," Modern Plastics (Dec 1966).

[52] Bland, D. R. and E. H. Lee, "Calculation of the Complex Modulus of Linear Viscoelastic Materials from Vibrating Reed Measurements," Journal of Applied Physics, Vol. 26, pp. 1497–1503 (1955).

explored with a probe, and the location and magnitude of the nodes and antinodes can be measured. This information is then used to determine Young's modulus. This technique was used by Guicking[44] in one phase of his investigation of progressive flexural waves.

Finally, note will be made of a commercially available instrument by Brüel and Kjaer[53] (B & K) for measuring the complex Young's modulus by the technique described previously. The B & K apparatus is shown in Figure 13, which is reasonably self-explanatory. The apparatus can be used in the clamped-free or clamped-clamped mode of operation. Each transducer is a variable reluctance device; one is used as a velocity sensitive pickup, while the other is used as an electromagnetic vibration exciter. These transducers can be adjusted along the samples as desired. When the sample is nonmagnetic material, it can be made susceptible to magnetic fields by gluing a small ferromagnetic mass to the sample in front of the transducer. This apparatus is designed to operate in the range from 30 to 3000 Hz and at temperatures from −150 to +250 degrees centigrade. Its limitations and advantages have been discussed by Murtha[54] working at the Naval Applied Science Laboratory in Brooklyn, New York.* When the loss factor of the material under study exceeds 0.2, B & K recommends that the sample be glued to a thin steel strip, and the resonances of the combination be measured. The properties of the material can then be computed using theories previously referenced.[47,48] A decay-rate technique can be used with the B & K apparatus to measure the internal losses in the sample. The sample is brought to resonance, and the signal is abruptly stopped. The decay rate in decibels per second is then measured and converted to a loss factor. This method is rather severely limited at the upper frequency limit by instrument capability. In Figure 3.4 of Reference 53, it is shown that, for example, that at a moderate loss factor of 0.5, an upper frequency limit of approximately 70 Hz is set by the 1000-dB/sec limitation of the instrument. The decay method can be employed on most variations of the resonant bar technique.

Extensional Waves. The resonant rod technique discussed in the previous section can, in almost all cases, be adapted to use extensional waves. The analysis is easier; however, extensional waves are harder to excite and detect. The use of this mode is not referenced extensively in the literature; however, some cases may be cited. Sack, et al.[55] discuss a

---

[53] Brüel and Kjaer, "Instructions and Applications–Complex Modulus Apparatus, Type 3930," Naerum, Denmark (Jun 1964).

[54] Murtha, T. D., "Brüel and Kjaer Method for Determining Dynamic Mechanical Properties of Vibration Damping Materials," Laboratory Project 930-59, Progress Report 4, Naval Applied Science Laboratory (7 May 1970).

[55] Sack, H. S. et al., "Elastic Losses in Some High Polymers as a Function of Frequency and Temperature," Journal of Applied Physics, Vol. 18, pp. 450–456 (1947).

*Now disestablished. Much of the function and personnel of this Laboratory has been transferred to the Center.

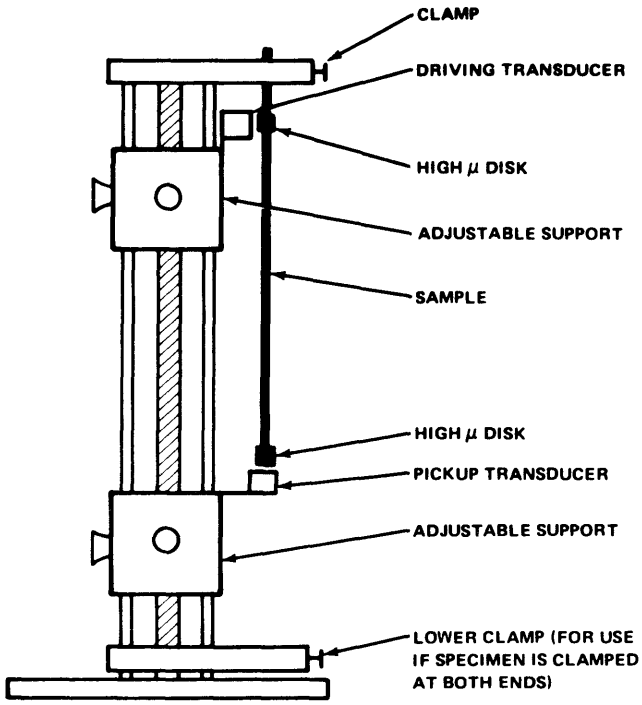


Figure 13 – Conceptual Drawing of Brüel and Kjaer Apparatus

(Shown here for free-end operation)

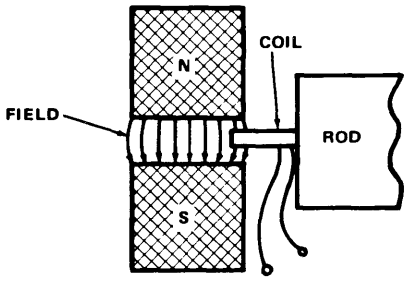


Figure 14a – Longitudinal Excitation

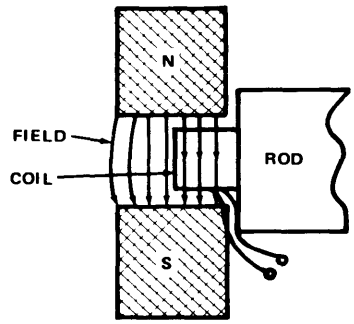


Figure 14b – Torsional Excitation

Figure 14 – Top View Showing Rod, Coil, and Magnetic Field for Rod Excitation and Resonance Detection

(Figure 5 of Reference 56)

vibrating reed, excited into longitudinal vibration with an electromagnetic driver attached to the free end. The writer contributed a short study[56] of TNT and polystyrene, using the free-free technique. In this case, the rod, which was 40 to 50 cm long and had a diameter of 2.5 cm, was suspended horizontally with both ends free and was excited in the first, second, and third modes. The corresponding resonance frequencies and Q's were measured, and the complex values of Young's modulus were calculated. The excitation and detection was accomplished with lightweight coils attached to both ends and placed in a magnetic field as shown in Figure 14a. Figure 14b, showing the excitation of torsional vibrations, will be discussed later. Corrections were included in the analysis for the effects of end loading, weight of coils; radial inertia, (Rayleigh correction); and acoustic dissipation. A later version of this technique was set up at the Center and has been reported by Castellucci.[57] In this case one face of the specimen and a vibrating piston were spaced close together, less than one-sixth of an inch, so that the airborne sound, projected normal to the end of the rod, produced the desired excitation. The vibrator face was servocontrolled for constant acceleration versus frequency, and the specimen response was monitored by the lightweight piezoelectric pickup, made at the Center, cemented to the other end. The report discussed the correction terms in some detail.

A study of rods of elastomers to determine the complex Young's modulus under static axial compression was carried out by Morris, James, and Guyton,[58] working at the Rubber Laboratory of the Mare Island Naval Shipyard. Figure 15 shows their apparatus. It has a clamped-clamped boundary condition, and the rods were tuned to multiples of $\lambda/2$. These investigators used rods 3/4 of an inch thick and 2 3/4 inches long, resulting in a primary resonance between 300 and 500 hertz. Measurements up to the fifth mode were made. This technique is particularly interesting because it is one of the few with a capability of making measurements of Young's modulus when the specimen is under static compressional stress. The usual Rayleigh corrections for radial inertia and acoustic dissipation must be made. Also, an additional correction may be needed for the lateral constraints on the end. No lateral displacement can take place at the steel-material interface if a rigid bond exists; this results in an increase of the apparent modulus as the ratio of diameter to height of the rod increases. Figure 16 shows, as an example, this effect on the static Young's modulus of a very soft

---

[56] Cramer, W. S., "Bulk Compressibility Data on Several High Explosives," Appendix, Naval Ordnance Laboratory Report 4380 (15 Sep 1956).

[57] Castellucci, G. R., "A Technique for the Measurement of the Dynamic Young's Modulus and Loss Factor," NSRDC Report 4149 (Jun 1973).

[58] Morris, R. E. et al., "A New Method for Determining the Dynamic Mechanical Properties of Rubber," Office of Naval Research Symposium Report ACR-4, Vol. 2 (1956); also, Proceedings of Joint Army-Navy-Air Force Conference on Elastomer Research and Development, pp. 204–228 (Jan 1956).

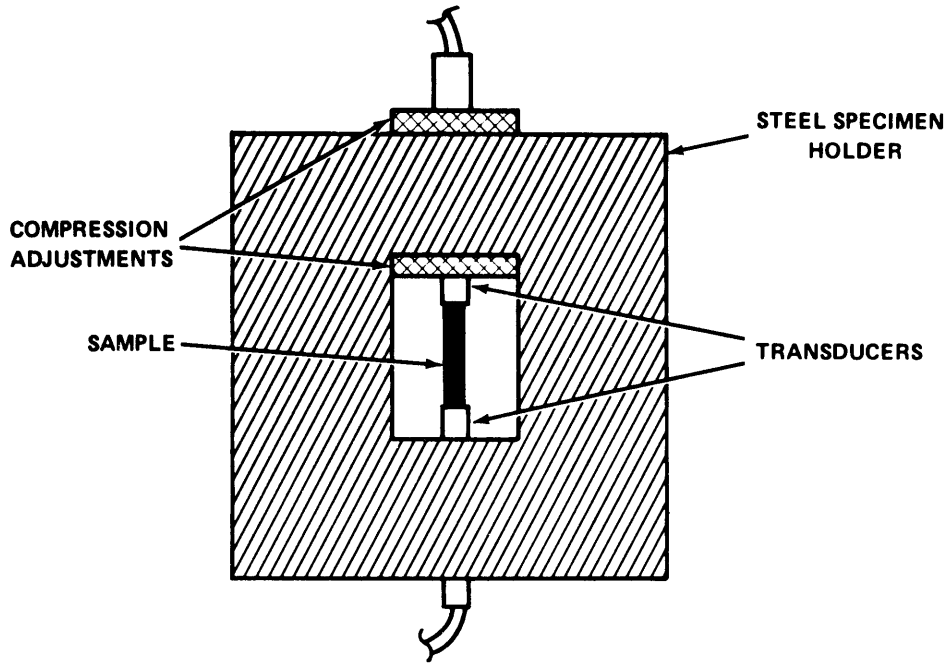


Figure 15 – Specimen Holder for Resonating Rubber Rods

(Adapted from Figure 3 of Reference 58)

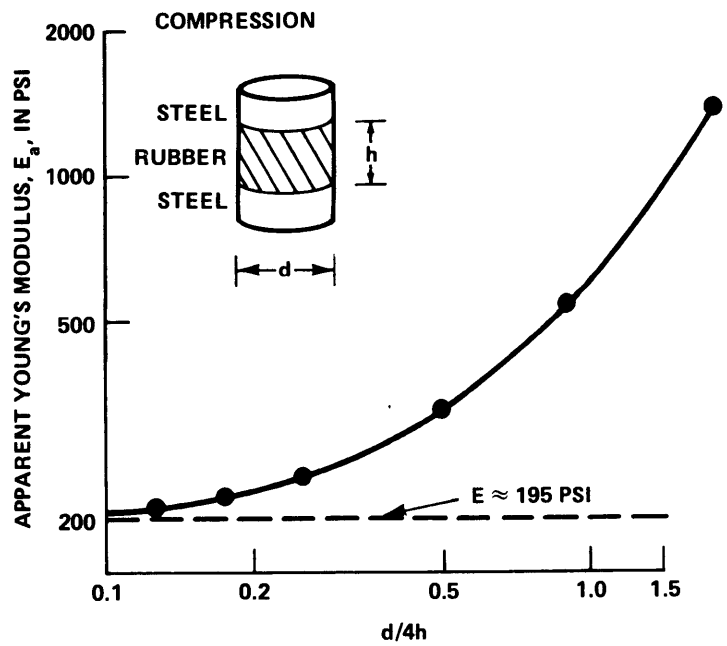


Figure 16 – Apparent Young's Modulus (E_a) in Compression for 1-Inch-Diameter, Gum Natural Rubber Cylinders over Shape Factor Range from 1/8 to 2

rubber rod 1 inch in diameter (received from an informant at B. F. Goodrich Company Research Center). In the Mare Island experimental work described, this effect is negligible; however, it should be considered in any such experiments. This end effect was also discussed by Harrison, Sykes, and Martin[59] working at the Center and by workers at the B. F. Goodrich Research Center.[60,61]

A similar technique but with the sample in tension instead of compression was described by Nolle in his classic paper[62] surveying methods for measuring the dynamic mechanical properties of rubberlike materials.* In this case a rubber strip, having a maximum length of 25 cm, was supported and driven in the extensional mode at one end and was attached to a detector at the other end. It was tuned to multiples of $\lambda/2$, and the usual measurements were made. Nolle claimed this method could be used between 60 and 300 hertz. It is only useful if $\eta < 0.2$.

In general, resonance techniques with the elastomers must consider that both the real and imaginary parts of the modulus change significantly in the usual frequency range of interest. However, a smooth curve through the experimental points is usually satisfactory.

Direct Stress-Strain Measurements

Other measurement techniques for Young's modulus involve direct measurements of stress and strain in contrast to the progressive wave and resonance techniques already discussed. One example was described by Parsons, Yater, and Schloss[63] working at the Center. A specimen (Figure 17) in the form of a thin rod was suspended vertically and was blocked at the upper end by a stiff, force gage attached to a large, isolated mass. A static tensile stress, which can be varied, insures proper vertical alignment for the axial excitation applied to the free end of the sample. The motion of the free end was measured with an accelerometer. The ratio of the two transducer signals and the phase angle between them were applied to a computer program to determine the propagation constants. The method was said to be effective from 20 to 10,000 Hz and could be used for materials from soft rubbers to hard

---

[59] Harrison, M. et al., "Wave Effects in Isolation Mounts," Journal of the Acoustical Society of America, Appendix, Vol. 24, pp. 62–71 (1952).

[60] Moghe, S. F. and H. F. Neff, "Elastic Deformation of Constrained Cylinders," Journal of Applied Mechanics, Vol. 38, pp. 393–400 (Jun 1971).

[61] Neff, H. F. and S. J. Caprette, Jr., "An Analysis of the Elastic Deformation of Cylinders with Constrained Ends under Axial Load," B. F. Goodrich Company Research Report for Office of Naval Research Contract Nonr-3304(00) (Aug 1966).

[62] Nolle, A. W., "Methods for Measuring Dynamic Mechanical Properties of Rubber-Like Materials," Journal of Applied Physics, Vol. 19, pp. 753–774 (1948).

[63] Parsons, J. S. et al., "The Measurement of Dynamic Properties of Materials Using a Transfer Impedance Technique," NSRDC Report 2981 (Apr 1969).

*This paper, published in 1948, was one of the earliest summaries of existing experimental methods of measuring Young's modulus of rubberlike material over a range of frequencies. Five methods–including vibrating reed, progressive wave, and resonant strip methods–were described.

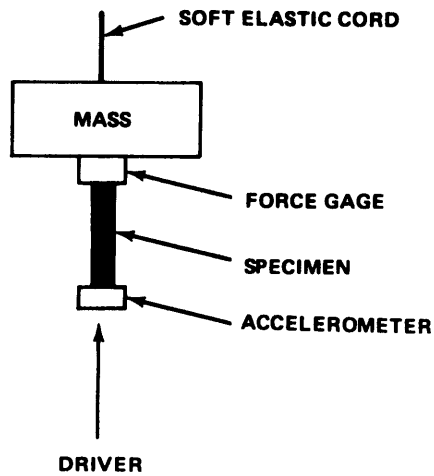


Figure 17 – Transfer Impedance Technique for Measuring Elastic Properties of Materials

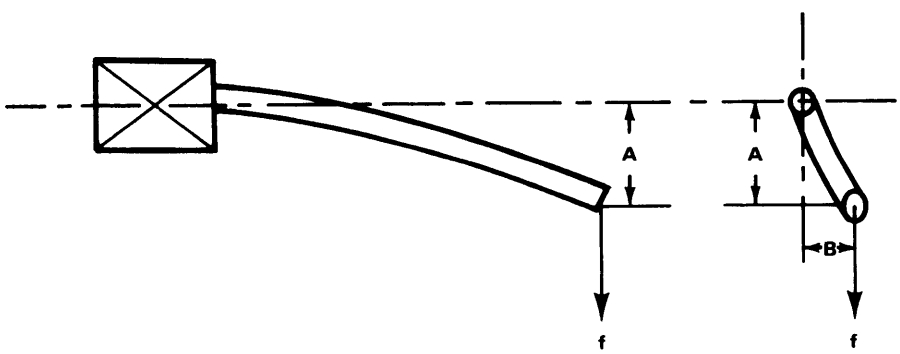


Figure 18 – Apparatus for Dynamic Measurements by a Rotating Rod Strained in Flexure

(Adapted from Figure 7-2 of Reference 2)

metals. Specimens to 6 inches in length were used in the study. The problems of end constraint and inertia of lateral motion previously mentioned must also be considered in this method.

The final item in this section is an ingenious direct measurement technique by Maxwell.[64] A rod of circular cross section is rotated at a given frequency and simultaneously is flexed by a yoke with negligible friction (Figure 18). Each element of the rod undergoes a sinusoidally varying strain in extension; if $\eta \neq 0$, the rod will have a steady-state deflection with components both in the direction of the flexing force (A) and perpendicular to it (B). The real and complex parts of Young's modulus are determined from the deflecting force f, the deflections, and the dimensions of the sample. This method can be used from 0.001 to more than 100 hertz. It is currently employed by the B. F. Goodrich Research Center.

SHEAR MODULUS

The last modulus to be considered is in some ways the most significant. It is very sensitive to parameters such as temperature, frequency, material variations, etc., and, in fact, most of the changes in acoustical characteristics studied so diligently by rheologists* and acousticians are due to changes in shear properties.

Measurements at more than 100 kHz have already been discussed under the bulk modulus section as Method 1—the difference method. This involves direct measurement of the transit time of a shear (transverse) wave over a measured-path length.

Progressive Wave Technique

A logical step in considering methods of measuring the shear modulus is to seek an analogy to the progressive and standing wave techniques already described for determining Young's modulus. The progressive wave technique as previously discussed considered only extensional or flexural waves; however, a similar procedure can be applied to the torsional wave which is governed by the shear modulus.** Extensive use does not seem to have been made of this propagation mode in studying properties of elastomeric materials. Ferry does not mention it at all in his book but a few cases have been described in the literature. One example was a study by Heydemann and Nägerl[65] made in Göttingen of torsional waves in a

---

[64] Maxwell, B., "An investigation of the Dynamic Mechanical Properties of Polymethyl Methacrylate," Journal of Polymer Science, Vol. 20, p. 551 (1956).

[65] Heydemann, P. and H. Nägerl, "Determination of the Complex Shear Modulus of Polymers at Audiofrequencies in a Wide Temperature Range," Acustica, Vol. 14, pp. 70–75 (1964).

*Rheology is the study of the deformation of matter.

**If a sinusoidal twisting moment about the axis is applied to the end of a rod, a torsional wave is propagated along the length of the rod with a speed of propagation (for a circular cross section) that equals that of a shear wave, viz., $c_t = \sqrt{G/\rho}$. For a square cross section, $c_t = 0.92\sqrt{G/\rho}$.

rubber rod strip. The signal was generated by an electromagnetic driver, and the amplitude was monitored by an optical technique. Figure 19 shows the principle of operation of the optical technique applied to a torsional wave. Alterations of the angle of the strip modulate the intensity of light passing the rod and cause a change in the output voltage of the photocell in proportion to the torsional amplitude. The modulation of the light intensity to monitor flexural waves, which has already been mentioned, works on the same principle. Theory relating the propagation measurements and shape factor to the shear modulus has been given. Spinner and Valore[66] discussed these relationships for rectangular strips and concluded that present theory was satisfactory.

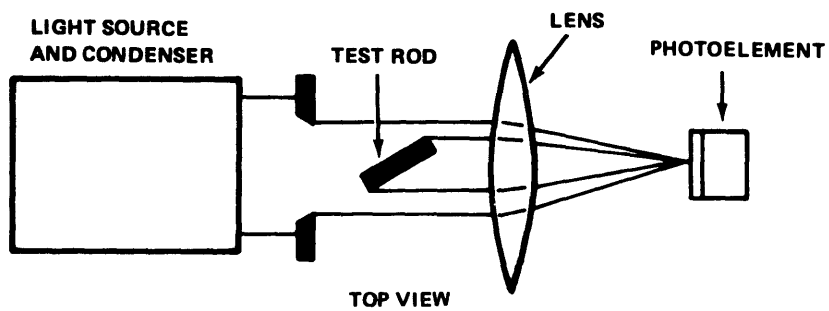


Figure 19 – Optical Vibration Detecting Device
(Adapted from Figure 4 of Reference 63)

In the case of torsional waves in rods, radial symmetry makes the correctional terms at low frequency negligible but the excitation and pickup (especially the latter) of the signal are more difficult than with strips.

The writer, working with 1/8-inch-diameter rods made of various kinds of rubber, obtained useful data about shear properties, using the progressive wave technique, over a range of frequencies from 1 to 10 kilohertz.[41] An electromagnetic drive similar to the movement of a D'Arsonval meter and a phonograph needle pickup were used. Reference 41 also presented data on longitudinal and flexural waves of the same samples and compared results.

Standing Wave Technique

Excitation of the resonant frequencies in torsion in hard plastic rods is also feasible but this technique is not used very much. Ferry cited several references on this subject, and the writer contributed the short study previously referenced[56] in which extensional and torsional waves were studied in rods of TNT and polystyrene. The excitation and pickup method of Reference 56 is shown in Figure 14b.

---

[66] Spinner, S. and R. C. Valore, Jr., "Comparison of Theoretical and Empirical Relations Between the Shear Modulus and Torsional Resonance Frequencies for Bars of Rectangular Cross Section," Journal of Research of the National Bureau of Standards Research Paper 2861, Vol. 60, No. 5 (May 1958).

Direct Stress-Strain Measurements

The most extensively used technique for measuring the shear modulus is the so-called Fitzgerald apparatus, introduced by Fitzgerald and Ferry at the University of Wisconsin in 1953.[67] Although the instrument is somewhat complicated, it is said to be capable of considerable accuracy (±2 percent) and versatility and has been widely used by Professor Fitzgerald (now at Johns Hopkins University) and many others. Its frequency of operation is from 10 to 5000 Hz for a wide temperature range. Samples from a soft gel to a hard glassy solid can be used as large as 1/4 inch in thickness and 1 inch in diameter. Strains from 10^{-3} to 10^{-6} are used.

> "The transducer of the Fitzgerald Apparatus is simple in principal: two coils are placed in separate radial magnetic fields, and are mechanically coupled to each other and to the sample by means of a rigid lightweight metal tube. [See Figure 20.] This tube is suspended so as to move in an axial direction, impeded only by the sample which is also attached to a heavy, suspended object in such a way that motion of the tube moves one face of the sample parallel to the opposite face. The two coils are isolated electrically and magnetically so that their only interaction can be through mechanical motion of the tube, but any such motion of the tube is opposed by the sample's resistance to shear.
>
> "An alternating current flowing through either coil will interact with the magnetic field to produce an alternating axial force on the coil and tube. This force will result in vibration of the tube, sample, and both coils in proportion to the shear compliance of the sample. When the second coil is vibrated in this way in its magnetic field, a voltage is produced which is proportional to its velocity and thus proportional to the shear compliance of the sample. Comparison of this voltage with the current applied to the first coil gives an indication of the shear compliance of the sample."*

---

[67] Fitzgerald, E. R., "Mechanical Resonance Dispersion in Metals at Audiofrequencies," Physical Review, Vol. 108, pp. 690–760 (1957).

*Figure 20 and the description quoted were taken from Technical Bulletin EM-1 of the Atlantic Research Corporation, Alexandria, Va. (Jul 1953).

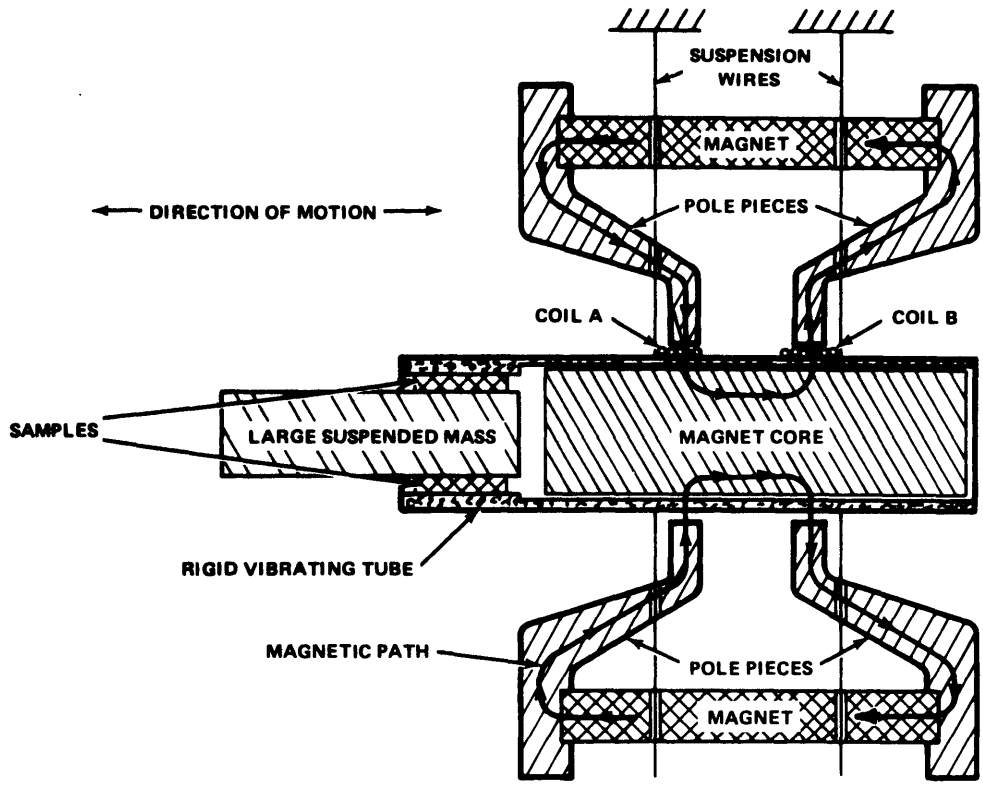


Figure 20 – Transducer for Fitzgerald Apparatus

In the last technique to be discussed in this section, the material under study is used as the stiffness element in a torsional pendulum. This is a popular method, and a number of versions have been described in the literature. Some have been discussed on pages 159–161 of Ferry's book, and a rather complete description of the theory and design of a torsional pendulum was given by Nielsen.[68] Figure 21 shows a recent design by Nederveen and van der Waal,[69] working at the Centraal Laboratorium TNO at Delft, the Netherlands. After a slight twist, the system is left free to perform damped oscillations, and the rotation of the upper clamp is converted into an electrical signal which is fed into a logarithmic recorder. The frequency of the free oscillations and the logarithmic decrement are used to calculate the complex shear modulus. Variations of this design have been favored by investigators

[68]Nielsen, L. E., "A Recording Torsion Pendulum for the Measurement of the Dynamic Mechanical Properties of Plastics and Rubbers," Review of Scientific Instruments, Vol. 22, pp. 690–693 (1951).

[69]Nederveen, C. J. and C. W. van der Waal, "A Torsion Pendulum for the Determination of Shear Modulus and Damping Around 1 Hertz," Rheologica Acta, Vol. 6, pp. 316–323 (1967).

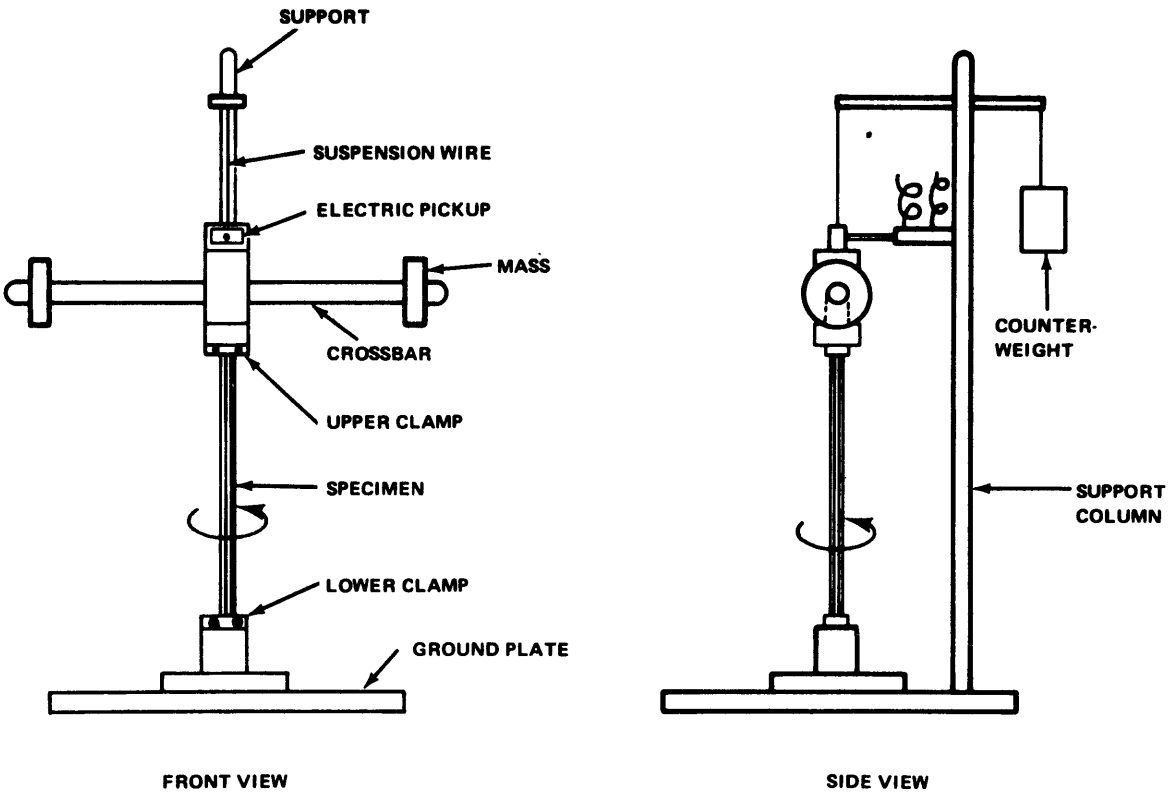


Figure 21 – Torsional Pendulum

(Adapted from Figure 1 of Reference 69)

in the Netherlands who have done extensive work on homogeneous and composite materials. This particular design was used between 1 and 20 hertz. The rotational inertia and, therefore, the resonance frequency, can be changed by using different masses on the cross bar. The design is used for materials from soft rubber to rigid plastic, and the accuracy is said to be about ±5 percent with a loss factor ranging from $0.005 < \eta < 0.3$.

ADDITIONAL REMARKS ON MEASUREMENTS

Relaxation Methods

This discussion of acoustical measurements would not be complete without mentioning relaxation techniques. These are given extensive treatment in rheology texts but are of little practical importance in most acoustical applications. In this type of measurement, a stress is applied to the material in one of the various modes previously described–bulk, extensional, or shear–however a transient signal is used instead of a steady state sinusoidal signal. If the strain is maintained constant, the stress will "relax" as shown in Figure 22 as internal

adjustments take place in the material. The modulus equals $\sigma(t)/\epsilon$, where σ is the time varying stress component, and ϵ is the constant strain component. This is called the *relaxation modulus*. Alternatively, we can apply the stress over a short period of time (Figure 23) and then maintain it constant while the strain as a function of time is measured. The ratio $\epsilon(t)/\sigma$ is called the *creep compliance.*

The complex dynamic modulus can be obtained from the relaxation modulus by a suitable transform (Chapter 3 of Reference 2), and an analogous relationship connects the creep compliance with the complex dynamic compliance. For a complete analysis of the acoustic properties of a material, the transformed creep or relaxation data are combined with the directly measured dynamic compliance or modulus data to cover a broad spectrum range—often 10 to 15 logarithmic decades. The relaxation data are used to cover the very low frequency region.

Calculation of Moduli

As indicated previously in discussing measurements of the plane-wave and bulk moduli, it is sometimes convenient to calculate the required modulus by using Table 1 measurements for two other moduli. In the case of the rubberlike (elastomeric) materials, certain characteristics exist which sometimes simplify the calculation and sometimes complicate it. The elastomers have relatively low compressibility (high K) and a high shear compliance (low G) which differ by two to three orders of magnitude. Taking the relationship for E in terms of K and G from Table 1, it is seen that under typical conditions $E = 3G$ to within less than 1 percent. This means that $E' \approx 3G'$, $E'' \approx 3G''$, and the respective loss factor $E''/E' = G''/G'$ is essentially equal. Unfortunately, however, the fact that elastomers have these properties severely restricts the ability to obtain K from E and G. From Table 1 we have $K = \frac{EG}{3(3G - E)}$, which involves a very small term in the denominator so that K cannot be determined with any precision.

SUMMARY OF MEASUREMENT TECHNIQUES DESCRIBED

The previous section undoubtedly presented too much detail for some and not enough for others. For the latter, the generous listing of references should be of value. Further discussions of measurement techniques can be found in almost all textbooks about the properties of viscoelastic materials.

To help consolidate this presentation of acoustic measurement techniques, Table 2 is presented, listing the various techniques mentioned in this report.

It will be emphasized again that any attempt to summarize the extensive literature about this subject inevitably reflects the interests and experiences of the author. Many interesting and useful techniques have been omitted. It is interesting to note that most

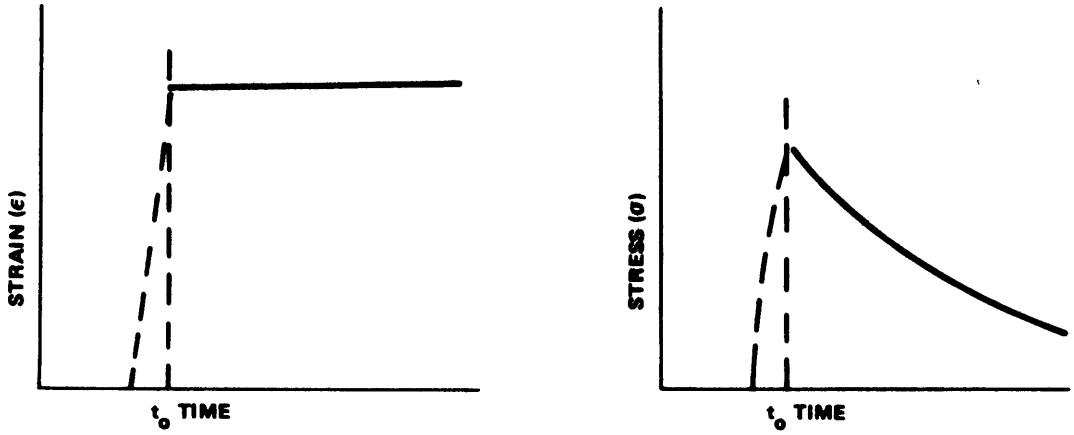


Figure 22 – Time Profile of a Simple Stress-Relaxation Experiment Following Sudden Strain

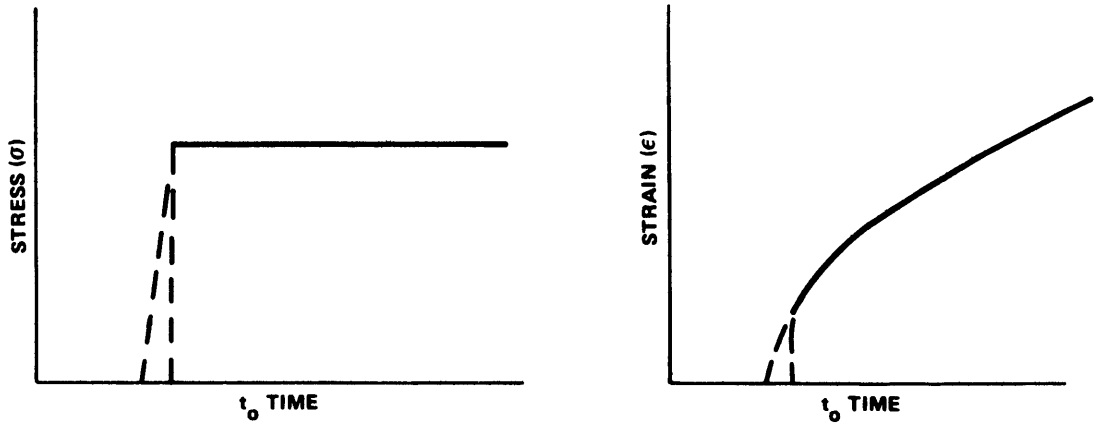


Figure 23 – Time Profile of a Creep Experiment

TABLE 2 – MEASUREMENT TECHNIQUES DESCRIBED

Modulus	Descriptive Name	Useful Frequency Range kHz	Material*	References Cited	Pages in Report Where Described
Bulk (K^*)	Resonant Water-Filled Tube	≈ 1.5	R, P	17–20	16–17
	Direct Stress-Strain Measurement	< 10	R, P	13–16	15–16
	Difference Method ($K^* = M^* - 4G^*/3$)	>100	R, P	6–12	12–14
	Propagation in Dispersions	>300	R, P	21–23	17
Plane-Wave (M^*)	Impedance	2–10	R, P	3 and 26–38	18–23
	Rotating Plate	>100	R, P	11 and 12	13–14
	Direct Transit Time Measurement	>100	R, P	6–12, 24 and 25	12–14, 18
Young's (E)	Vibrating Reed	0.01–0.5	R, P	52	29
	Resonant Rod (Flex)	0.01–3	P	50–54	28–29
	Transfer Impedance	0.02–10	R, P	63	34–36
	Resonant Rod (Ext) (Tension Load)	0.06–0.3	R	62	34
	Flexing Yoke	<0.1	R, P	64	36
	Resonant Rod (Ext)	0.1–10	P	55–57	30–32
	Progressive Wave (Flex)	0.1–10	R	41 and 44–48	25–28
	Progressive Wave (Ext)	0.1–40	R	39–43	23–25
	Resonant Rod (Ext) (Compression Load)	0.3–2.5	R	58 and 59	32–34
Shear (G)	Torsional Pendulum	0.001–0.02	R	68 and 69	39–40
	Fitzgerald Apparatus	0.01–5	R, P	67	38–39
	Progressive Wave (Tors)	0.5–10	R	41, 65, and 66	36–37
	Standing Wave (Tors)	0.1–10	P	54	37
	Direct Measurement	>100	R, P	6–12	12–14

*R is elastomers; P is rigid plastics.

methods cited date back a few years. Current developments usually involve more sophisticated versions of earlier methods, using computer and advanced instrumentation techniques. Also, much greater attention is now given to intensive studies of material rather than instrumentation development.

RESULTS OF ACOUSTICAL MEASUREMENTS

This report so far has been devoted exclusively to discussion of what acoustical quantities are measured and some of the techniques used for these measurements. Presentations of data on specific materials were purposely avoided. However, to give completeness to this discussion of measurement techniques, some insight should be provided concerning results to be expected as a function of type of stress applied, material, frequency, and temperature.

SHEAR PROPERTIES

The changes of a rubber to a rigid plastic at sufficiently low temperatures and of a rigid plastic to a rubbery solid at sufficiently high temperatures describe the obvious effects of temperatures on high polymers. The propagation constants and moduli reflect these physical changes. Qualitative curves showing the dynamic shear modulus and associated loss factor are presented in Figure 24 as a function of temperature. This transition region is usually from 30° to 60° C wide with a peak in the loss factor and a substantial change in the storage modulus. Below the transition temperature region the thermal energy of the molecule is not great enough to overcome the attractive force between portions of the polymer chain. The temperature below which the thermal motion essentially ceases is called the glass transition temperature T_g; for example, $T_g = -73°$ C for natural rubber and 100° C for polystyrene. The only deformation that can take place in the glassy region is Hookean elastic deformation from pulling the chains slightly away from their neighbors against the opposition of the van der Waal forces. Very little energy is dissipated during a cycle in this region. In the rubber stage, the thermal energy is enough for coiling and uncoiling of the chain molecule to take place when a sinusoidal stress is applied. This situation results in a high deformation (low modulus) and, also, little dissipation. In the relaxation region however the duration of the stress is comparable to the time it takes the molecule to respond. The result will be that there is a significant phase difference between stress and strain and a comparatively high dissipation per cycle.

The acoustical behavior of high polymers with frequency is very similar to that due to temperature if log ω is plotted as shown in Figure 24. The physical picture of why this occurs is quite straightforward. At low frequencies, the period of stress is sufficiently long, if the temperature is greater than T_g, for the chain molecules to have adequate time to

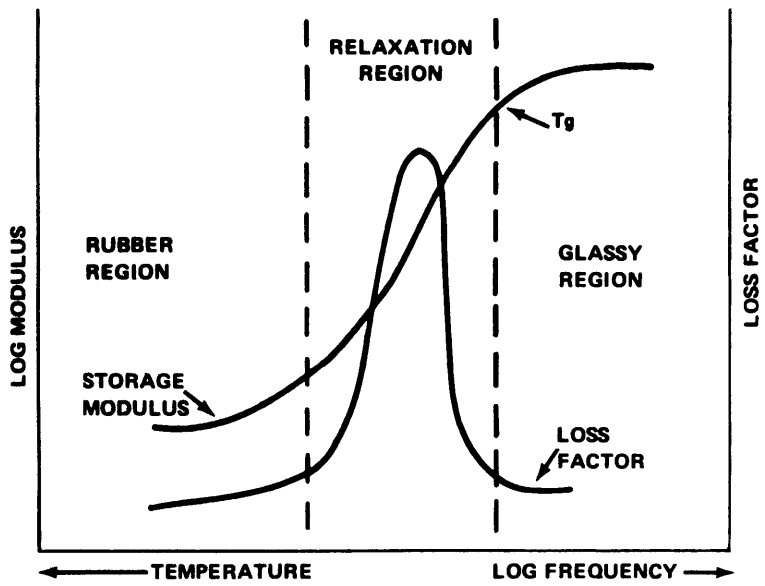


Figure 24 – General Shape of a Relaxation Curve for a Typical Polymer

extend and contract; therefore, the modulus is low. At the high frequencies, the action takes place so rapidly that an uncoiling and coiling cannot take place, and the material is acoustically stiff although it may feel like an ordinary piece of rubber. In the transition frequency region, the response time of the molecules and the stress duration are comparable, and there is high dissipation.

The similarity of behavior of acoustical properties with frequency and temperature suggests using a single parameter to simplify the presentation. Such a procedure was outlined by Williams, Landel, and Ferry in 1955[70] and is known as the WLF transformation. The abscissa is then log ωa_T, where a_T is given by

$$\log a_T = \frac{-c_1^0 (T - T_0)}{c_2^0 + (T - T_0)} \tag{17}$$

[70]Williams, M. L. et al., "The Temperature Dependence of Relaxation Mechanisms in Amorphous Polymers and Other Glassforming Liquids," Journal of the American Chemical Society, Vol. 77, p. 3701 (1955).

The temperature T_0 is arbitrarily selected, usually the temperature of greatest interest for the application in mind, and T is the temperature of the measurement; c_1^0 and c_2^0 are constants for the material. Detailed discussion of this transformation technique will not be attempted; however, further information can be obtained from the original paper as well as in most rheology texts, for example, Chapter 11 of the Ferry book.[2]

The technique of reduced variables enables the experimenter to cover a wide frequency band with a single apparatus instead of the two, three, or more that would ordinarily be used. One could, for example, work with a torsional pendulum at a few hertz and go through the entire relaxation region by lowering the temperature. It is desirable, however, to choose an instrument which operates in the frequency range of interest and not to force the temperature shift technique to extreme limits.

To provide some insight as to values of G' and η_G to be expected, Figures 25 and 26 are presented showing these terms as a function of frequency for various temperatures. The figures were taken from Snowdon's book[71] and show data on both low- and high-loss rubbers. Natural rubber, without reinforcing fillers, is a very compliant and low-loss material under most conditions of interest. It would be primarily located at the left side of the relaxation curve (Figure 24); although at lower temperatures and higher frequencies, it is starting to enter the relaxation region. The polysulfide rubber Thiokol RD, also unloaded, shows typical behavior when measurements are in the relaxation region.

For mechanical strength, most rubbers contain a reinforcing carbon black filler. An example of the effect of this loading on a typical rubber, taken from Reference 42, is shown in Figure 27. Note the interesting effect of particle size. The curves are drawn through measured values of loading of 10, 30, and 50 parts by weight to 100 parts rubber.

No single comprehensive collection of data on the shear properties of elastomers can be referenced. When one considers that about 15 different rubbers are in common use in approximately 240 available types, the magnitude of assembling such a collection is apparent. If one further introduces effects of additives, the compositions available are essentially limitless. In considering the literature, several extensive studies available only in report form should be considered. These are usually ignored in the open literature but contain useful systematic studies of important parameters. Among these should be mentioned the work at the Naval Ordnance Laboratory,[42] the Chesapeake Instrument Corporation,[72–74] sponsored

---

[71] Snowdon, J. C., "Vibration and Shock in Damped Mechanical Systems," Figures 1.6 and 1.11, John Wiley and Sons, Inc., New York (1968).

[72] Chesapeake Instrument Corporation, "Dynamic Mechanical Properties of Plastic Materials," Final Report, Chesapeake Instrument Corp., Shadyside, Md. (1 Jun 1957).

[73] Chesapeake Instrument Corporation, "Dynamic Mechanical Properties of Materials for Noise and Vibration Control," Technical Report 2, Vol. 1, Chesapeake Instrument Corp., Shadyside, Md. (1 Jul 1962).

[74] Chesapeake Instrument Corporation, "Dynamic Mechanical Properties of Materials for Noise and Vibration Control," Technical Report 2, Vol. 2, Chesapeake Instrument Corp., Shadyside, Md. (1 Jul 1962).

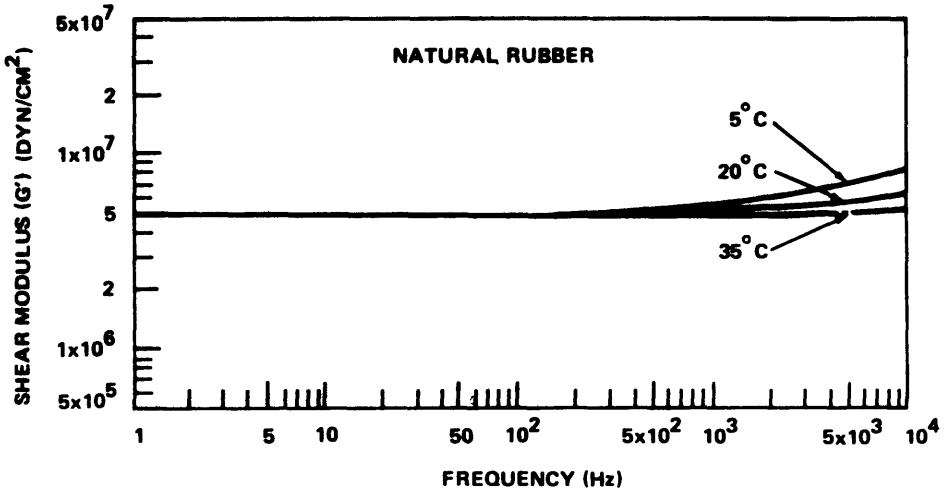


Figure 25a – Dynamic Shear Modulus

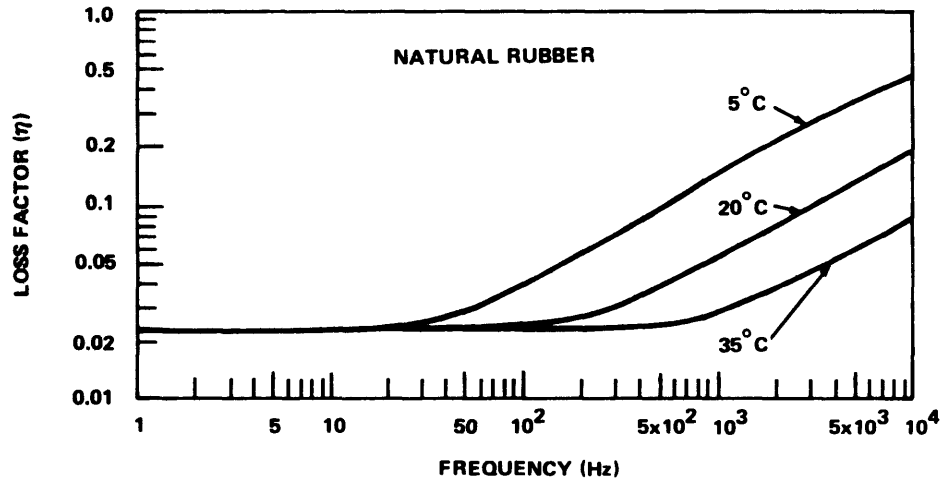


Figure 25b – Loss Factor

Figure 25 – Frequency Dependence of Dynamic Shear Modulus and Loss Factor Possessed by Unfilled Natural Rubber

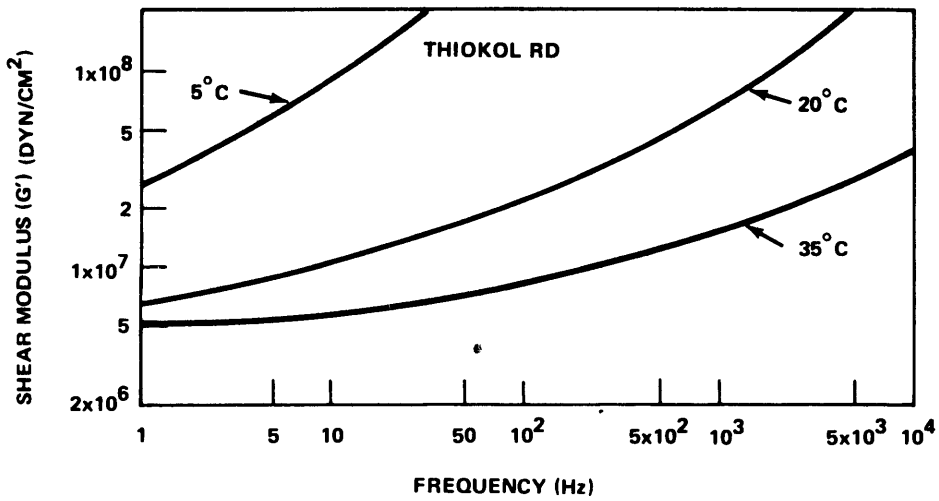


Figure 26a – Dynamic Shear Modulus

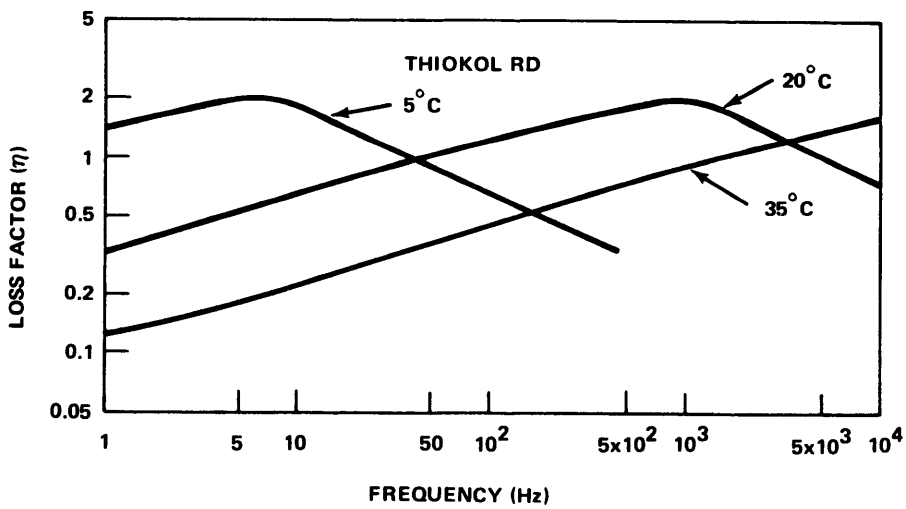


Figure 26b – Loss Factor

Figure 26 – Frequency Dependence of Dynamic Shear Modulus and Loss Factor Possessed by an Unfilled Rubber Thiokol RD

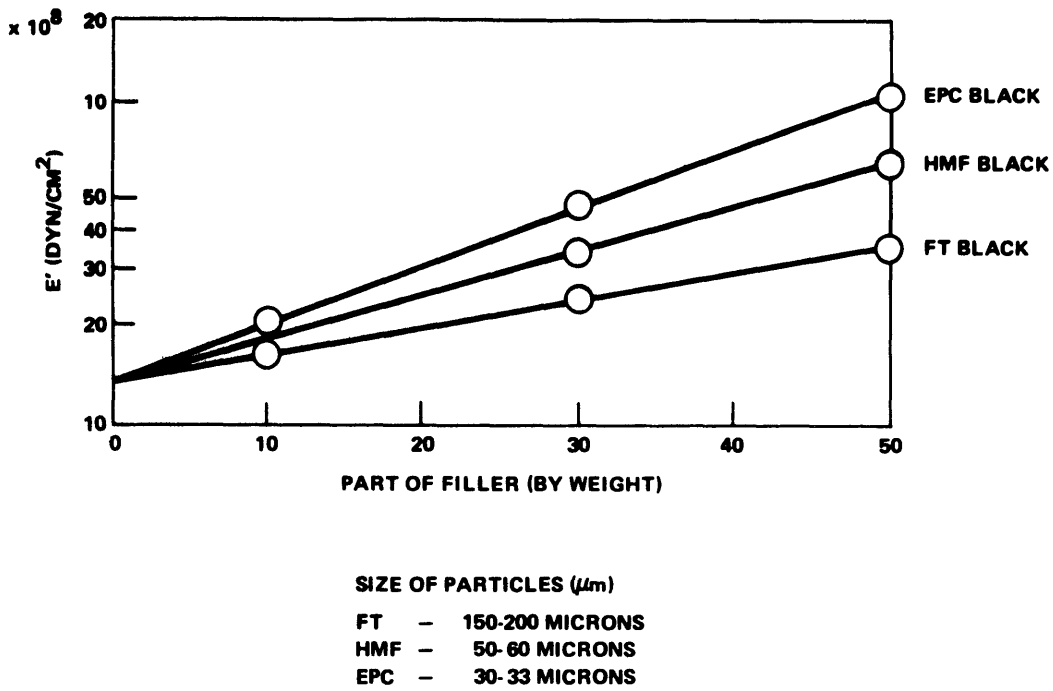


Figure 27 – Real Part of Young's Modulus of a Nitrile Rubber as a Function of Loading with Several Different Types of Carbon Black

(From Figure 16 of Reference 42)

by the Bureau of Ships and the Office of Naval Research, and the Mare Island Rubber Laboratory.[43] The NOL study emphasized the effects of various amounts and types of loading; see Figure 27. A progressive wave-type of measurement by Nolle was used on rubber rods in the range from 1 to 5 kHz at 30 degrees centigrade. The data give Young's modulus directly; however, the shear modulus can be taken for soft rubbers as one-third Young's modulus, and the loss factors can be taken as the same.

The Chesapeake Instrument study used the Fitzgerald apparatus[67] to measure shear modulus directly in the range from 25 to 5000 Hz at various temperatures. A vast amount of data have been presented in their report with emphasis on the properties of polyisobutylene and butadiene-acrylonitrile copolymers, showing effects of molecular weight, ratio of copolymers, amount, type and size of loading, temperature, and frequency.

The Mare Island study also used progressive waves in a rubber rod, Nolle-type measurements, and presented an interesting study of the effects of varying the ratio of butadiene to acrylonitrile in the nitrile copolymer. Measurements were made over the range from 25 to about 16,000 Hz at temperatures from 50 to 90 degrees Fahrenheit.

Considerable emphasis has been given to the variety of measurement techniques available for studying the shear properties of elastomers; see Table 2. A logical question then is how

do these various methods relate to each other and combine to explain the complete behavior of the material? The most ambitious attempt to answer this question was made by Marvin[75] and associates at the National Bureau of Standards during the early 1950's. They furnished samples of polyisobutylene made to precise standards to 20 investigators, who measured the shear properties from the very low frequency range covered by stress relaxation and creep techniques to acoustic measurements in the megahertz region. All data were reduced to a standard temperature of 25° C using the WLF transformation and curves of G′ and G″, plotted from log $\omega = -5$ to 9. A satisfactory consistency among measurements and agreement with predicted behavior of viscoelastic materials was found.

The values of G′ at frequencies and temperatures beyond the relaxation peak (right side of Figure 24) are of little interest for elastomers under most conditions. For the polymers having a glass transition temperature greater than the usual ambient temperature, this region is important. Typical values of G′ are 2.2×10^{10} dyn/cm^2 for Plexiglas, 1.9×10^{10} dyn/cm^2 for polystyrene, and 5.5×10^9 dyn/cm^2 for Teflon at 25° C; data were taken from Reference 12.

BULK PROPERTIES

Variations of the bulk modulus of a polymer with frequency and temperature are similar to those of the shear modulus shown in Figure 24. In the bulk case, however, the storage modulus changes by a factor of two or three instead of the several orders of magnitude associated with shear deformation. Also, the loss associated with bulk compression is lower than for shear. The work of Marvin and McKinney[5] discusses the subject in depth.

The bulk modulus of a polymer is of interest in many applications because it is an important constituent of the plane-wave modulus $M^* = K^* + 4/3(G^*)$ which governs the propagation of longitudinal waves in an extended medium. An interesting illustration of the relative contributions of K* and G* to M* in a typical rubber is provided in Figure 28, adapted from a 1954 study of polyisobutylene by Marvin et al.[6] Both the storage and the loss moduli were plotted against reduced frequency for 25 degrees centigrade. Note that the effect of 4/3(G*) on the storage modulus is not significant at less than 160 kHz, which corresponds to the frequency at 25° C for which log $\omega\, a_T = 6$, and that the relaxation frequency is about 15 megahertz. At lower frequencies, therefore, the sound speed of this wave

[75] Marvin, R. S., "The Dynamic Mechanical Properties of Polyisobutylene," Proceedings of the Second International Congress on Rheology, Oxford, England, Butterworths Scientific Publications, London, pp. 156–163, (26–31 Jul 1953).

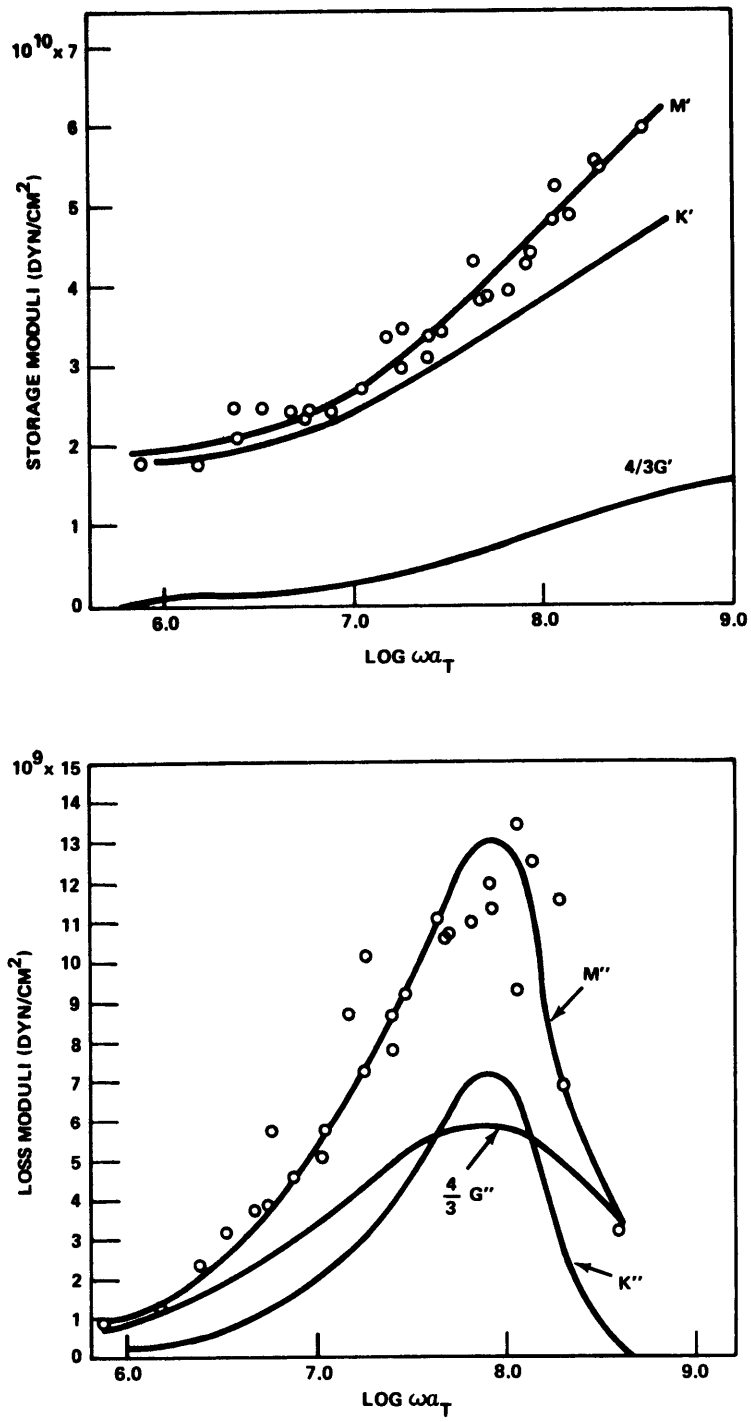


Figure 28 – Reduced Moduli versus Frequency at 25 Degrees Centigrade for Polyisobutylene

(K = Bulk, G = Shear, M = K + 4G/3; taken from Marvin, et al., Reference 8)

is governed effectively by the bulk modulus. The attenuation, however, is affected substantially by the loss associated with the shear component of the plane-wave modulus. The relative contributions of shear and bulk losses will vary with material.

Bulk properties will also be affected by the hydrostatic pressure. An interesting study of this effect was carried out by Lastinger and Groves[30] at NRL (USRD), using the acoustic impedance method, on four elastomers to 10,000 pounds per square inch gage. Results for natural rubber are shown in Figure 29 (Figure 2 of their report). An increase of approximately 15 percent in speed over this pressure range is noted.

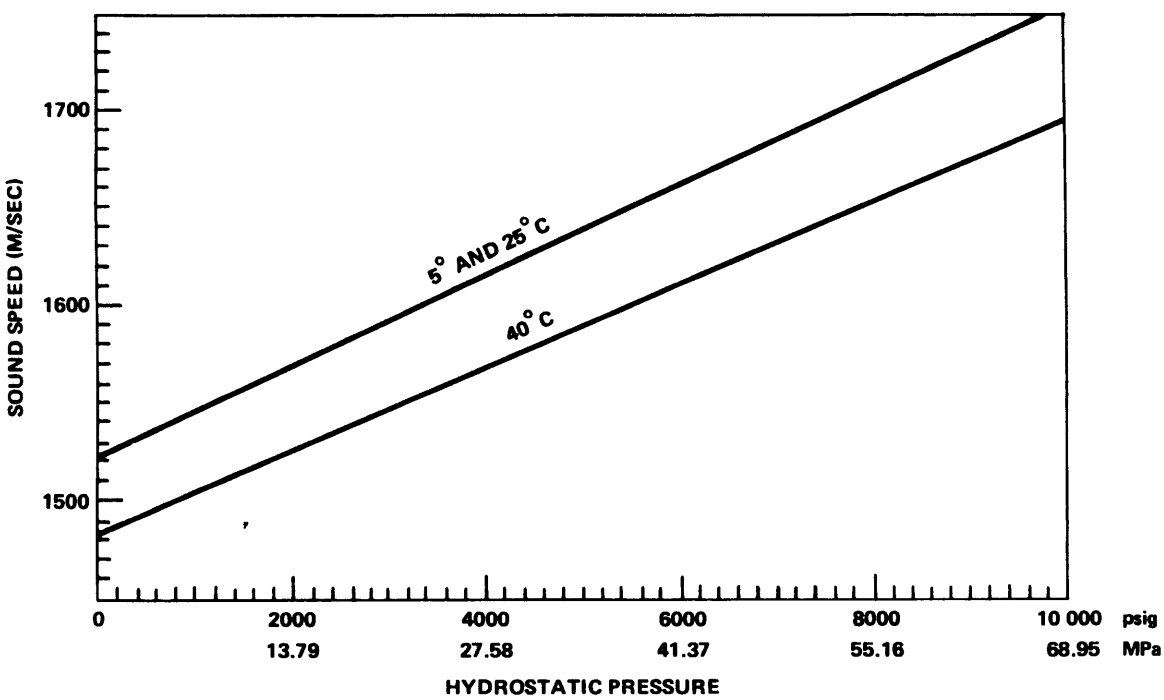


Figure 29 – Sound Speed in a Loaded Natural Rubber (Specific Gravity of 1.10) as a Function of Hydrostatic Pressure

(Taken from Reference 30)

One of the more extensive efforts to study variations in the bulk modulus as a function of rubber type was made by Cramer and Silver at NOL,[42] who measured the bulk moduli of 49 different variations of eight basic rubbers at 1.5 kHz and 30° C, using the resonant tube apparatus developed by Sandler.[18] The storage moduli ranged from 2.1×10^{10} dyn/cm^2 for an unloaded butyl to 3.30×10^{10} dyn/cm^2 for a neoprene loaded with 50 parts by weight of carbon black. The bulk loss factors of the same materials were also measured (Table 3 of Reference 42) and showed variations from a high of 0.112 for the unloaded butyl to zero, to three significant figures, for a heavily loaded nitrile rubber. Only 6 of the 49 samples had loss factors exceeding 0.020.

The loss factor for the plane-(bulk)-wave is given by

$$\eta_M = \frac{K'' + 4G''/3}{K' + 4G'/3} = \frac{K'\eta_B + 4G'\eta_G/3}{K' + 4G'/3} \tag{18}$$

In the denominator G' is negligible compared to K' for elastomers but in the numerator the two terms can be comparable in magnitude.

EXTENSIONAL PROPERTIES

Extensional properties involve the effects of both compression and shear; see Table 1. Using $E = 2(1 + \sigma)G$, for the elastomeric materials, where σ is approximately 0.5, $E' \approx 3G'$, and $\frac{E''}{E'} \approx \frac{G''}{G'}$. Using an average value for the bulk modulus, it can be shown that this approximation holds within 5 percent to a G' of 4×10^9 dynes per square centimeter (a rather stiff rubber). In the case of rigid plastics where σ is from 0.35 to 0.40, the approximation is no longer valid. Data for polystyrene[55] in which independent measurements of the three moduli were made on the same sample are $K = 3.85 \times 10^{10}$, $E = 3.65 \times 10^{10}$, $G = 1.36 \times 10^{10}$ all in dyn/cm^2, $E'/G' = 2.68$.

ACKNOWLEDGMENTS

The writer presented essentially the same material given in this report as an unpublished laboratory note, which was circulated to a number of his colleagues in the Navy laboratories known to have an interest in and experience with this subject. He was pleased and gratified to receive very many helpful comments and suggestions. In view of these suggestions, plus a few additional thoughts of his own, the report was substantially revised. Any errors that may have crept in during the rewriting are entirely his responsibility. Those who assisted critically reviewing the first draft are (in alphabetical order): J. Blue, USRD; R. Bobber, USRD; P. Granum, NSRDC; B. Hartmann, NOL; L. Holtz, NSRDC; J. Lastinger, USRD; G. Martin, NUC; R. Morris, MIRL; C. Nichols, NUC; W. Reader, NSRDC; and G. Sabin, USRD.

REFERENCES

1. Kolsky, H., "Stress Waves in Solids," Oxford University Press, London (1953).

2. Ferry, J. D., "Viscoelastic Properties of Polymers," Second Edition, John Wiley and Sons, Inc., New York (1970).

3. Bobber, R. J., "Underwater Electroacoustic Measurements," Naval Research Laboratory (Jul 1970).

4. Warfield, R. W. et al., "Two Empirical Corresponding States Relationships for the Ratio of the Heat Capacities of Polymers," Naval Ordnance Laboratory TR 69-98 (21 Oct 1969).

5. Marvin, R. S. and J. E. McKinney, "Volume Relaxation in Amorphous Polymers," in Chapter 9, "Physical Acoustics," Vol. 2, edited by W. P. Mason, Academic Press, New York (1965).

6. Ivey, D. G., "Propagation of Ultrasonic Bulk Waves in High Polymers," Journal of Applied Physics, Vol. 20, pp. 478–493. (1949).

7. Nolle, A. W. and P. W. Sieck, "Longitudinal and Transverse Ultrasonic Waves in a Synthetic Rubber," Journal of Applied Physics, Vol. 23, pp. 888–894 (1952).

8. Marvin, R. S. et al., "The Dynamic Bulk Viscosity of Polyisobutylene," Journal of Applied Physics, Vol. 25, pp. 1213–1218 (1954).

9. Cunningham, J. R. and D. G. Ivey, "Dynamic Properties of Various Rubber at High Frequencies," Journal of Applied Physics, Vol. 27, pp. 967–974 (1956).

10. Wada, Y. et al., "Comparison between Mechanical Relaxation Associated with Volume and Shear Deformation in Styrene-Butadiene Rubber," Journal of the Physical Society of Japan, Vol. 17, pp. 213–218 (1962).

11. Schneider, W. C. and C. J. Burton, "Determination of the Elastic Constants by Ultrasonic Methods," Journal of the Applied Physics, Vol. 20, pp. 48–58 (1949).

12. Hartmann, B. and J. Jarzynski, "Immersion Apparatus for Ultrasonic Measurements," Naval Ordnance Laboratory TR 72-73 (14 Apr 1972).

13. McKinney, J. E. et al., "Apparatus for the Direct Determination of the Dynamic Bulk Modulus," Journal of Applied Physics, Vol. 27, pp. 425–430 (1956).

14. Heydemann, P., "The Dynamic Compressibility of Highpolymers in the Frequency Range from 0.1 c/s to 60 kc/s," Acustica, Vol. 9, pp. 446–452 (1959).

15. Philippoff, W. and J. Brodnyan, "Preliminary Results in Measuring Dynamic Compressibilities," Journal of Applied Physics, Vol. 26, pp. 846–849 (1955).

16. Sharma, M. G. and V. McCarty, "Experimental Investigations on the Dynamic Compressibilities of Polymeric Material," Applied Polymer Symposia, Vol. 1, pp. 201–215 (1965).

17. Meyer, E. and K. Tamm, "Ein Akustisches Messverfahren Bestimmung der Dynamischen Kompressibilität und des Verlustfaktor Elastischer Stoffe," Akustische Zeitschrift, Vol. 7, pp. 45–50 (1942).

18. Sandler, C. S., "Measurement of Bulk Modulus and Associated Loss Factor," Journal of the Acoustical Society of America, Vol. 22, Abstract, pp. 685–686 (1950).

19. Sandler, C. S., "An Acoustic Technique for Measuring the Effective Dynamic Bulk Modulus of Elasticity and Associated Loss Factor of Rubber and Plastics," Naval Ordnance Laboratory Report 1534 (Sep 1950).

20. Niemiec, J. M., "Determining the Dynamic Bulk Modulus and the Loss Factor of Polymers from Acoustic Measurements," NSRDC Report 3858 (Oct 1972).

21. Wada, Y. et al., "Volume Viscosity of Polymers and Other Highly Dissipative Materials," Journal of the Physical Society of Japan, Vol. 15, pp. 2324–2334 (1960).

22. Smithson, J. R., "Measurement of Viscoelastic Properties of Polymers by Ultrasonic Methods," Journal of the Acoustical Society of America, Vol. 42, Abstract, p. 1162 (1967).

23. Pullen, W. J. et al., "The Measurement of Dynamic Bulk Modulus Using an Ultrasonic Interferometer," Polymer, Vol. 5, pp. 471–478 (1964).

24. Folds, D. L., "Experimental Determination of Ultrasonic Wave Velocities in Plastics, Elastomers and Syntactic Foam as a Function of Temperature," Journal of the Acoustical Society of America, Letter to the Editor, Vol. 52, pp. 426–427 (1972).

25. Madigosky, W. M. and J. von Bretzel, "Viscoelastic Properties of 1,2 Polybutadiene and Comparison with Other Rubbers," Naval Ordnance Laboratory TR 72-192 (23 Aug 1972).

26. Kosten, C. W. and C. Zwikker, "Sound Absorbing Materials," Elsevier Publishing Company, Amsterdam (1949).

27. Cramer, W. S., "Theoretical Study of Underwater Sound Absorbing Layers," Naval Ordnance Laboratory Report 2803 (Jun 1953).

28. Sabin, G. A., "Acoustic-Impedance Measurements at High Hydrostatic Pressures," Journal of the Acoustical Society of America, Vol. 40, pp. 1345–1353 (1966).

29. Lastinger, J. L. and G. A. Sabin, "A PDP-8 Fortran Program for Reduction of Acoustic Impedance Data," Naval Research Laboratory Report 6906 (28 Mar 1969).

30. Lastinger, J. L. and I. D. Groves, Jr., "Speed of Sound in Four Elastomers," Naval Research Laboratory Memorandum Report 2363 (3 Dec 1971).

31. Lastinger, J. L., "The Acoustical Properties of Water-Saturated Fibrous Metal," Journal of the Acoustical Society of America, Vol. 52, pp. 1465–1470 (1972).

32. Lastinger, J. L., "Acoustic Characteristics of Woods at High Hydrostatic Pressures," Journal of the Acoustical Society of America, Vol. 47, pp. 285–289 (1970).

33. Higgs, R. W. and L. J. Eriksson, "Acoustic Decoupling Properties of Onionskin Paper," Journal of the Acoustical Society of America, Vol. 46, pp. 211–215 (1969).

34. Kuhl, W. et al., "Impulsverfahren zur Messung der Reflexion von Wasserschall–Absorben in Rohren," Acustica, Vol. 3, p. 421 (1953).

35. Richardson, E. G., "Technical Aspects of Sound," Chapter 4, Section 2, Vol. II, Elsevier Publishing Company, Amsterdam (1957).

36. Cramer, W. S., "Pulse Tube for Acoustic Measurements," Naval Ordnance Laboratory Report 2257 (Apr 1952).

37. Beranek, L. L., "Acoustic Measurements," Chapter 7, John Wiley and Sons, Inc., New York (1949).

38. Kreer, U., "Aufbau einer Messapparatur zur direkten Aufzeichnung von Impedanzkurven für Wasserschall im Kundtschen Rohr," Acustica, Vol. 22, No. 1, pp. 23–34 (1969).

39. Nolle, A. W., "Acoustic Determination of the Physical Constants of Rubberlike Materials," Journal of the Acoustical Society of America, Vol. 19, pp. 194–201 (1947).

40. Meyer, E. and associates, "Sound Absorption and Sound Absorbers in Water," Naval Ship Systems Command Report 900, 164 (1947).

41. Cramer, W. S., "Propagation of Stress Waves in Rubber Rods," Naval Ordnance Laboratory Report 4216 (Sep 1956); also, Journal of Polymer Science, Vol. 26, pp. 57–65 (1957).

42. Cramer, W. S. and I. Silver, "Acoustical Properties of Rubber as a Function of Chemical Composition," Naval Ordnance Laboratory Report 1778 (Feb 1951).

43. San Francisco Bay Naval Shipyard, "Determination of Dynamic Properties of Rubbers and Plastics," Rubber Laboratory, San Francisco Bay Naval Shipyard Report 104-2 (16 Feb 1967).

44. Guicking, D., "Dynamisch-Mechanische Eigenshaften von Plastischen Massen und Vernetzten Polyurethanen," Acustica, Vol. 18, pp. 93–104 (1967).

45. Heydemann, P. and A. Zosel, "Determination of the Complex Young's Modulus of Polymers at Audiofrequencies in a Wide Temperature Range," Acustica, Vol. 12, pp. 360–365 (1962).

46. Holtz, L. C., "Traveling-Transducer Method of Measuring Cross-Correlation," U. S. Patent 3,597,962 (10 Aug 1971).

47. Oberst, H. and K. Frankenfeld, "Über die Dämpfung der Biege-schwingungen dünner Bleche durch festhallende Beläge," Acustica, Vol. 2, Leaflet 4, AB 181–194, Part 1 (1952); Vol. 4, p. 433, Part 2 (1954).

48. Van Oort, W. P., "A Method for the Measurement of Dynamic Mechanical Properties of Small Samples of Plastic Material," Microtechnic, Vol. 7, p. 246 (1952).

49. Kline, D. E., "Effect of Branching on Dry Mechanical Properties of Polyethylene," Journal of Polymer Science, Vol. 22, p. 449 (1956).

50. Kinsler, L. E. and A. R. Frey, "Fundamentals of Acoustics," Second Edition, Chapter 3, John Wiley and Sons, Inc., New York (1950).

51. Barnet, F. R. and J. Cuevas, "Dynamic Response of Thermosetting Polymer System," Modern Plastics (Dec 1966).

52. Bland, D. R. and E. H. Lee, "Calculation of the Complex Modulus of Linear Viscoelastic Materials from Vibrating Reed Measurements," Journal of Applied Physics, Vol. 26, pp. 1497–1503 (1955).

53. Brüel and Kjaer, "Instructions and Applications–Complex Modulus Apparatus, Type 3930," Naerum, Denmark (Jun 1964).

54. Murtha, T. D., "Brüel and Kjaer Method for Determining Dynamic Mechanical Properties of Vibration Damping Materials," Laboratory Project 930-59, Progress Report 4, Naval Applied Science Laboratory (7 May 1970).

55. Sack, H. S. et al., "Elastic Losses in Some High Polymers as a Function of Frequency and Temperature," Journal of Applied Physics, Vol. 18, pp. 450–456 (1947).

56. Cramer, W. S., "Bulk Compressibility Data on Several High Explosives," Appendix, Naval Ordnance Laboratory Report 4380 (15 Sep 1956).

57. Castellucci, G. R., "A Technique for the Measurement of the Dynamic Young's Modulus and Loss Factor," NSRDC Report 4149 (Jun 1973).

58. Morris, R. E. et al., "A New Method for Determining the Dynamic Mechanical Properties of Rubber," Office of Naval Research Symposium Report ACR-4, Vol. 2 (1956); also, Proceedings of Joint Army-Navy-Air Force Conference on Elastomer Research and Development, pp. 204–228 (Jan 1956).

59. Harrison, M. et al., "Wave Effects in Isolation Mounts," Journal of the Acoustical Society of America, Appendix, Vol. 24, pp. 62–71 (1952).

60. Moghe, S. F. and H. F. Neff, "Elastic Deformation of Constrained Cylinders," Journal of Applied Mechanics, Vol. 38, pp. 393–400 (Jun 1971).

61. Neff, H. F. and S. J. Caprette, Jr., "An Analysis of the Elastic Deformation of Cylinders with Constrained Ends under Axial Load," B. F. Goodrich Company Research Report for Office of Naval Research Contract Nonr-3304(00) (Aug 1966).

62. Nolle, A. W., "Methods for Measuring Dynamic Mechanical Properties of Rubber-Like Materials," Journal of Applied Physics, Vol. 19, pp. 753–774 (1948).

63. Parsons, J. S. et al., "The Measurement of Dynamic Properties of Materials Using a Transfer Impedance Technique," NSRDC Report 2981 (Apr 1969).

64. Maxwell, B., "An Investigation of the Dynamic Mechanical Properties of Polymethyl Methacrylate," Journal of Polymer Science, Vol. 20, p. 551 (1956).

65. Heydemann, P. and H. Nägerl, "Determination of the Complex Shear Modulus of Polymers at Audiofrequencies in a Wide Temperature Range," Acustica, Vol. 14, pp. 70–75 (1964).

66. Spinner, S. and R. C. Valore, Jr., "Comparison of Theoretical and Empirical Relations Between the Shear Modulus and Torsional Resonance Frequencies for Bars of Rectangular Cross Section," Journal of Research of the National Bureau of Standards Research Paper 2861, Vol. 60, No. 5 (May 1958).

67. Fitzgerald, E. R., "Mechanical Resonance Dispersion in Metals at Audiofrequencies," Physical Review, Vol. 108, pp. 690–760 (1957).

68. Nielsen, L. E., "A Recording Torsion Pendulum for the Measurement of the Dynamic Mechanical Properties of Plastics and Rubbers," Review of Scientific Instruments, Vol. 22, pp. 690–693 (1951).

69. Nederveen, C. J. and C. W. van der Waal, "A Torsion Pendulum for the Determination of Shear Modulus and Damping Around 1 Hertz," Rheologica Acta, Vol. 6, pp. 316–323 (1967).

70. Williams, M. L. et al., "The Temperature Dependence of Relaxation Mechanisms in Amorphous Polymers and Other Glassforming Liquids," Journal of the American Chemical Society, Vol. 77, p. 3701 (1955).

71. Snowdon, J. C., "Vibration and Shock in Damped Mechanical Systems," Figures 1.6 and 1.11, John Wiley and Sons, Inc., New York (1968).

72. Chesapeake Instrument Corporation, "Dynamic Mechanical Properties of Plastic Materials," Final Report, Chesapeake Instrument Corp., Shadyside, Md. (1 Jun 1957).

73. Chesapeake Instrument Corporation, "Dynamic Mechanical Properties of Materials for Noise and Vibration Control," Technical Report 2, Vol. 1, Chesapeake Instrument Corp., Shadyside, Md. (1 Jul 1962).

74. Chesapeake Instrument Corporation, "Dynamic Mechanical Properties of Materials for Noise and Vibration Control," Technical Report 2, Vol. 2, Chesapeake Instrument Corp., Shadyside, Md. (1 Jul 1962).

75. Marvin, R. S., "The Dynamic Mechanical Properties of Polyisobutylene," Proceedings of the Second International Congress on Rheology, Oxford, England, Butterworths Scientific Publications, London, pp. 156–163 (26–31 Jul 1953).

INITIAL DISTRIBUTION

Copies	
1	NRL Attn: C. Davis
2	NRL ORLANDO Attn: R. Bobber
1	USNA LIB
1	NAVPGSCOL LIB
1	NROTC & NAVADMINU, MIT
1	NAVWARCOL
2	NAVSEASYSCOM 1 SEA 035 1 SEA 037
1	NAVUSEACEN Attn: G. Martin
2	NAVCOASTSYSLAB 1 D. Folds 1 B. Nolte
2	NSWC 1 B. Hartmann 1 W. Madigosky
1	NPTLAB NUSC/LIB
1	NLONLAB NUSC
1	NISC Attn: B. Valenti
2	NAVSEC 1 SEC 6101E01/D. Pratt 1 SEC 6101E03/W. Graner
12	DDC
1	B. F. Goodrich Res Lab Attn: S. Caprette

CENTER DISTRIBUTION

Copies	Code	
1	0000	CAPT P. W. Nelson
1	1727	G. L. Becker
1	1900	M. M. Sevik
2	1940	G. J. Franz
3	1942	
	1	W. K. Blake
	1	P. J. Granum
	1	J. T. C. Shen
2	1945	
	1	J. Goodman
	1	L. C. Holtz
1	1945	W. T. Reader
25	1949	W. S. Cramer
1	2741	L. T. Ho
2	2843	
	1	J. J. Eynck
	1	W. F. Hern
30	5614	Reports Distribution
1	5641	Library (C)
1	5642	Library (A)