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Bethesda, Maryland 20034

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TECHNIQUE FOR MEASURING THE DYNAMIC YOUNG'S MODULUS AND LOSS FACTOR

by

Gerald R. Castellucci

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TECHNIQUE FOR MEASURING THE DYNAMIC YOUNG'S MODULUS AND LOSS FACTOR

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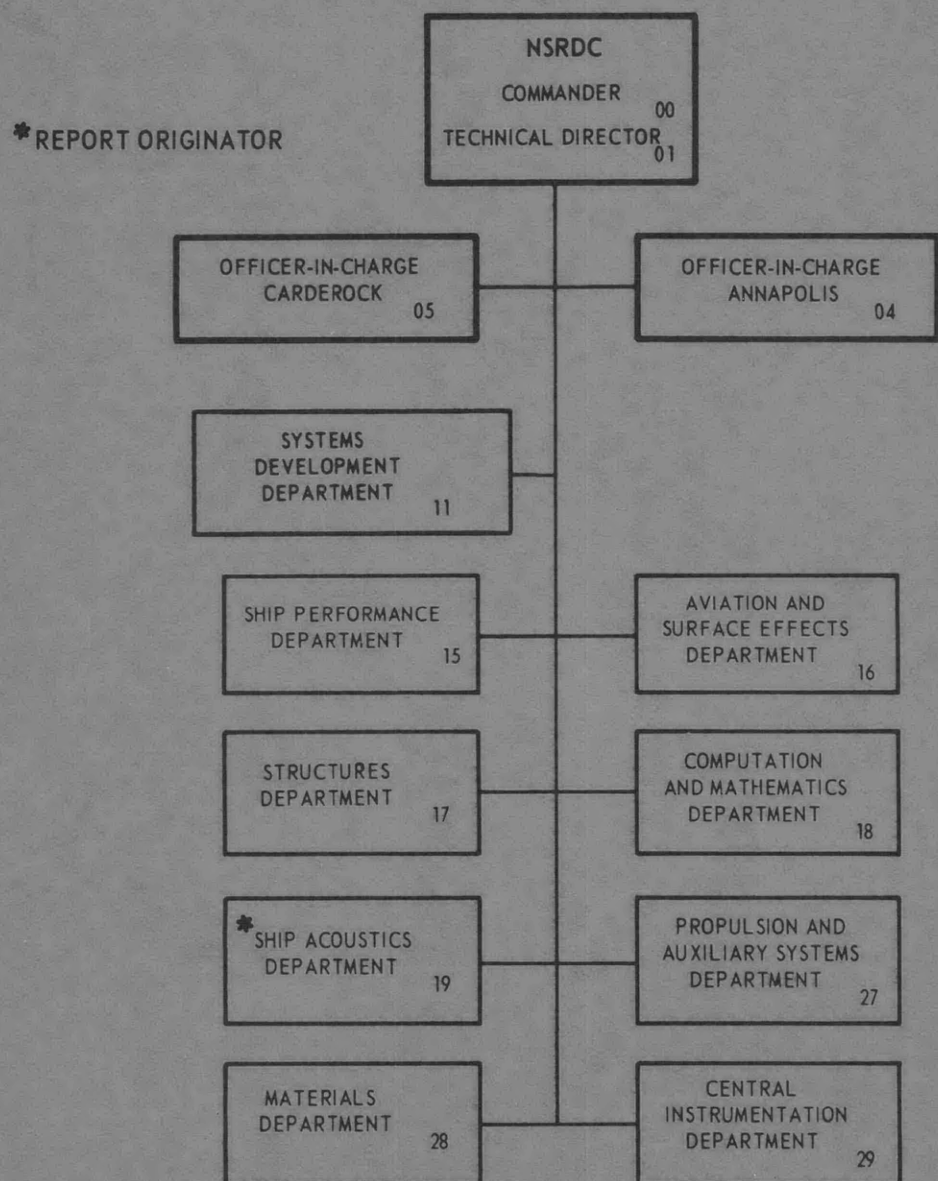

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NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

BETHESDA, MARYLAND 20034

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NOTATION

a	Radius of cylindrical sample
c	Speed of sound in a bar
c_p	Experimentally measured sound speed
f	Frequency
f_n	Resonance frequency of nth mode
h	Length-to-diameter ratio of sample
ℓ	Length of rod
m	Mass of objects attached to sample
m_b	Sample mass
n	Mode number
r	Loss parameter $\alpha c/\omega$
w	ω times radius/(shear modulus/density)$^{1/2}$
k	Wavenumber
Q	Quality factor
V	Root-mean-square volts
Y^*	Complex Young's modulus ($Y' + i\,Y''$)
ρ	Density
η	Loss factor
λ	Wavelength

ω	Angular frequency
σ	Poisson's ratio

ABSTRACT

As a part of an extensive program for measuring acoustical properties of polymeric materials, equipment has been designed and set up to measure the dynamic Young's modulus of stiff materials with low acoustic dissipation. Both the elastic modulus and the loss factor are obtained. The method described is based on the excitation of longitudinal resonances in a suspended uniform thin rod having free-free end conditions. For samples of actual material, the results must be corrected for the effect of the radial inertia of the bar and the effects of the measurement equipment on the results. The properties of very lossy or limp materials are measured more conveniently by other techniques.

ADMINISTRATIVE INFORMATION

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INTRODUCTION

As a part of an extensive program for measuring acoustical properties of polymeric materials, equipment has been designed and set up to measure the dynamic Young's modulus of rigid materials with low acoustic dissipation. Both the elastic modulus and the loss factor are obtained. The method described is based on the excitation of longitudinal resonances in a suspended uniform thin rod having free-free end conditions. Resonant frequency and 3-dB bandwidth are measured in the experiment along with such parameters as length, radius, mass, and density of the cylindrical samples. For the actual sample the results are corrected for the effects of radial inertia and, where necessary, the effects of the measuring equipment. Very good accuracy in the value of complex Young's modulus computed from this technique can be expected for low loss materials. Lossy materials for which the loss factor η is 0.1 or greater are measured more conveniently and accurately by other techniques.

THEORETICAL BASIS

VIBRATION OF CYLINDRICAL BARS

The longitudinal particle displacement ξ in a uniform thin strip or rod is described by the one-dimensional wave equation

$$Y^* \frac{\partial^2 \xi}{\partial x^2} = \rho \frac{\partial^2 \xi}{\partial t^2} \tag{1}$$

where Y^* is the complex Young's modulus. The complex form of the modulus, $Y^* = Y' + i\,Y''$, is used to include the effects of energy dissipation in the rod material and is often expressed in the form

$$Y^* = Y' \,(1 + i\,\eta) \tag{2}$$

where Y' is the real part of the modulus, and the term $\eta = Y''/Y'$ is defined as the "loss factor."

If a solution to Equation (1) of the form

$$\xi = A\, e^{i\omega t}\, e^{(\alpha + i\,\omega/c)x}$$

is assumed and substituted into the wave equation, the result is

$$Y^* \,(\alpha + i\,\omega/c)^2 = -\omega^2\rho$$

or

$$Y^* = -\frac{\rho\omega^2}{(\alpha + i\,\omega/c)^2}$$

$$Y^* = Y' + i\,Y'' = \frac{-\rho\omega^2(\alpha^2 - \omega^2/c^2)}{(\alpha^2 + \omega^2/c^2)^2} + i\,\frac{\rho\omega^2\; 2\alpha\omega/c}{(\alpha^2 + \omega^2/c^2)^2} \tag{3}$$

Simplifying the real part and substituting r (sometimes designated the "loss parameter") for $\alpha c/\omega$, we have

$$Y' = \rho c^2 \left[\frac{(1 - r^2)}{1 + r^2}{}^{2} \right] \tag{4}$$

In a completely nondissipative material, $r = 0$, and the bracketed term in Equation (4) vanishes. This term, therefore, is the correction applied to account for dissipation in the material.

The loss factor, $\eta = Y''/Y'$, can be expressed in terms of r as

$$\eta = \frac{2r}{1 - r^2} \tag{5}$$

If a bar is suspended in such a way that it is essentially free from restraint and is driven at various frequencies, it resonates at discrete frequencies f_n. At resonance, the acceleration is a maximum at the ends of the bar.

The wavelength λ at each resonance is approximately $2\ell/n$, where ℓ is the length of the rod, and n is the number of nodes present in the rod. The resonant frequency is then

$$f_n = \frac{c}{\lambda_n} \tag{6}$$

where c is the speed of sound in the rod.

Relative acceleration amplitudes as a function of longitudinal position on the bar are given in Figure 1 for several of the resonant modes.

Rearranging Equation (6) and substituting for λ_n

$$c = \frac{2\ \ell\ f_n}{n} \tag{7}$$

Therefore, if a resonant condition is established, the speed of propagation in a thin bar can be determined approximately if the bar length, the resonant frequency, and the mode number are known.

Three related quantities by which dissipation in a material can be described have thus far been mentioned. However, Y'', η, and r are difficult to measure directly. A fourth quantity, the quality factor, symbolized by the letter Q, is relatively simple to measure and for $Q \gg 0.5$ is

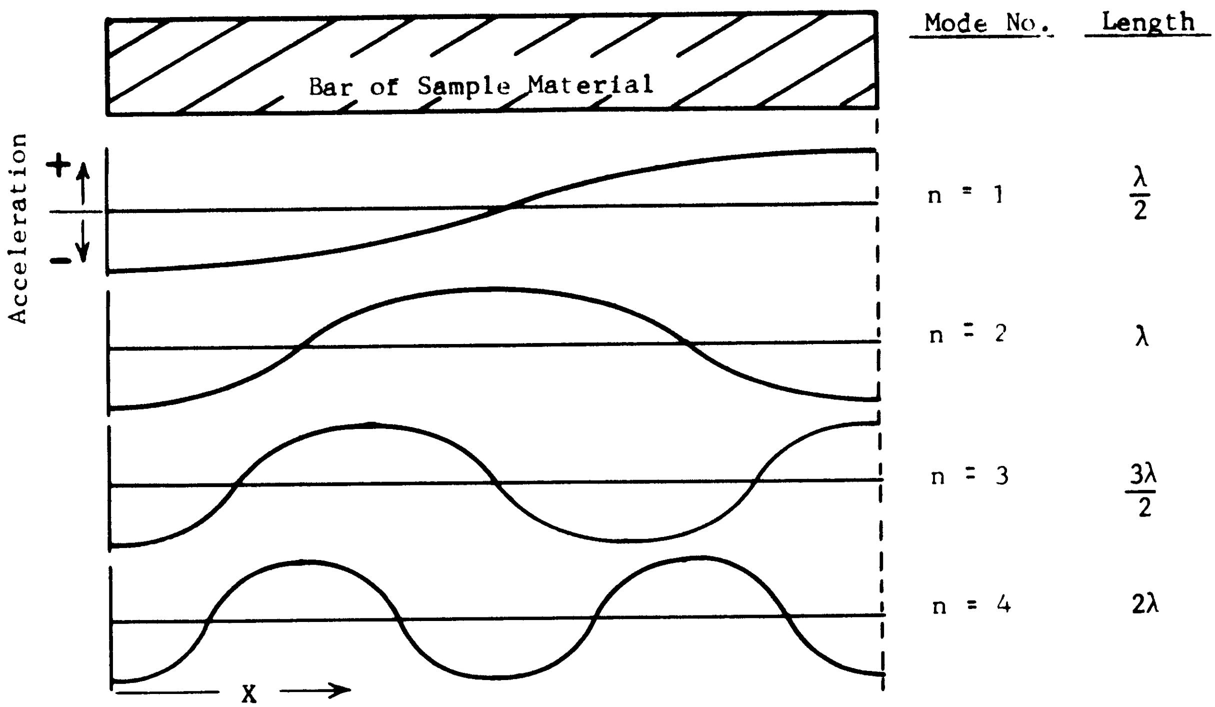

Figure 1 - Acceleration as a Function of Axial Position

approximately equal to $1/\eta$.[1] Q is defined as 2π times the ratio of the energy stored per cycle to the energy dissipated per cycle. It can be determined by a relationship involving the response of the system to excitation at or near its resonant frequencies

$$Q = \frac{f_n}{\Delta f_n} \tag{8}$$

where f_n is the resonant frequency of the nth mode, and Δf_n is the frequency bandwidth between the half power points, 3-dB down, of the system on each side of the resonance.[2]

The values of c and η obtained from f_n, λ, and Q are substituted in Equations (4) and (5) to obtain the complex Young's modulus.

CORRECTION TERM FOR MASS LOADING

When measuring the longitudinal resonant frequency of the rod, consideration must be given to the effects the measurement devices have on the results. The apparent resonant frequencies are actually those of a system, including the sample and everything mechanically attached to it. Over the frequency range of interest, we may consider that the attachments will behave as pure masses. Reference 3 contains a detailed discussion of this mass-loading problem, and the discussion results in the following equation for the resonant frequencies of the loaded bar

$$\frac{\tan k_n \ell}{k_n \ell} = - \frac{m}{m_b} \tag{9}$$

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[1] Kinsler, . and . Frey, "Fundamentals of Acoustics," John Wiley and Sons, Inc., New York (Apr 1967) Sec. 3.3. A complete listing of references is given on page 24 .

[2] Skilling, H. H., "Electrical Engineering Circuits," Second Edition, John Wiley and Sons, Inc., New York (1965).

[3] Kolsky, H., "Stress Waves in Solids," Dover Publications, Inc., New York (1963) p. 42.

where m is the mass of objects mechanically attached at one end of the rod,

m_b is the mass of the sample,

ℓ is the length of the rod, and

k_n is the wavenumber $(2\pi/\lambda)$.

The values of k_n which are solutions of this equation correspond to the modes of vibration. For the resonance frequencies of a free-free bar with no mass loading $k_n\ell = n\pi$.

Some idea of the effect of mass loading on the sample can be obtained from the following examples.

Example 1

In the experiments to be discussed, the only mass loading is due to a very small accelerometer weighing 0.51 g alone and 0.89 g including its cable. No mass loading is contributed by the vibration generator because the rods are driven through a small airgap which imposes no load on the system. For the smallest sample tested, m_b was 9.08, so

$$\frac{m}{m_b} = \frac{0.89}{9.08} < 0.1$$

If

$$\frac{\tan k_n \ell}{k_n \ell} = 0.1$$

the values of $k_n\ell$ that are solutions are

$$k_n\ell = 2.87,\ 5.76,\ 8.71,\ 11.40,\ \ldots.$$

for

$$n = 1,\ 2,\ 3,\ 4,\ \ldots.$$

For the first mode, $k_1\ell = 2.87$, resulting in an error of

$$\text{Error} = \frac{\pi - 2.87}{2.87} \times 100 \text{ percent}$$

or 9.5 percent, if mass loading effects were to be neglected.

Example 2

A sample having a mass of only 9.08 g is unusually small; a typical sample in the present experiment has a mass of 150 g. In this case

$$\frac{m}{m_b} = 0.005933$$

$$\frac{\tan k_n \ell}{k_n \ell} = -0.005933$$

$$k_n\ell = 3.12,\ 6.24,\ 9.37,\ 12.50,\ \ldots.$$

for

$$n = 1,\ 2,\ 3,\ 4,\ \ldots.$$

The error here for the first mode, if mass loading is neglected, is

$$\text{Error} = \frac{\pi - 3.12}{3.12} \times 100 \text{ percent}$$

or 0.69 percent.

For rods of steel in which the speed of sound is approximately 5000 m/sec, an error of 0.69 percent results in a decrease in the apparent sound speed of approximately 31.5 m/sec. Since the Young's modulus is proportional to c^2, the apparent Y is in error by -1.2 percent. This is an acceptable error, considering that Y for steel varies as much as 3 percent among reference books. Therefore, except for samples much smaller than 150 g, the mass loading effects in the present experiments are ignored.

CORRECTION FOR LATERAL INERTIA

As a longitudinal wave propagates through a rod, it generates areas of compression and rarefaction; associated with these are areas of lateral expansion and contraction. Longitudinal and lateral motion are related through Poisson's ratio σ

$$\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} \tag{10}$$

Consideration of the kinetic energy associated with the lateral motion leads to a modified relationship between resonance frequency and mode number and to a new expression for the sound speed. An approximate relationship[3] between c_p, the speed computed directly from measured f_n and λ_n, and c, the sound speed in absence of lateral effects, in terms of the Poisson's ratio, the physical dimensions of the sample, and the wavelength for a cylindrical rod are

$$\frac{c_p}{c} = 1 - \sigma^2 \pi^2 (a/\lambda)^2 \tag{11}$$

where a is the radius of the bar,

σ is the Poisson's ratio, and

λ is the wavelength

The correction for lateral inertia was originally derived by Lord Rayleigh[4] and is known as the Rayleigh correction. It is considered to be a useful approximation up to a/λ equal to approximately 0.7. The wavelength for various modes of vibration equals $2\ell/n$; therefore, substituting for λ

$$c = c_p \left(1 - \frac{\sigma^2 \pi^2 a^2 n}{4\ell^2}\right)^{-1} \tag{12}$$

[4] Lord Rayleigh, "The Theory of Sound," Dover Publications, New York (1945) p. 252.

Equation (4) requires the use of speed squared c^2; so, performing a binomial expansion on the square of Equation (12), we have the approximate value

$$c^2 = c_p^{\,2} \left[1 + 2 \left(\frac{\sigma \, \pi \, a \, n}{2\ell} \right)^2 \right] \tag{13}$$

provided $\left(\frac{\sigma \pi a n}{2\ell}\right) < 1$.

A simple experiment was conducted to investigate the importance of this correction. Three aluminum bars were made of different lengths and radii. The fundamental resonances and, where possible, the first harmonic frequencies were determined for these bars by the method discussed herein. The parameter a/λ was then calculated and used in Equation (13) along with a Poisson's ratio of 0.33 for aluminum to compute the ratio c/c_p. The results are presented in Table 1.

TABLE 1 - EXAMPLES OF THE RAYLEIGH CORRECTION

a(cms)	ℓ(cms)	n	a/λ	(c/c_p)
2.54	15.24	1	0.0833	1.0075
1.905	7.62	1	0.125	1.017
2.54	15.24	2	0.1666	1.030
5.08	10.16	1	0.250	1.065
5.08	10.16	2	0.506	1.240

The two values for the sound speed c and c_p differ in the worst case shown by approximately 20 percent. Consideration of this correction was, therefore, found to be necessary.

THE EXPERIMENT

Materials to be tested were fabricated in the form of cylindrical rods, and their density was measured. By means of appropriate transducers, the samples were excited and their response to excitation was monitored. From the response of a material to excitation at and near several resonances, the resonant frequency, the mode number, and the 3-dB bandwidth are determined. The various parameters, c, Q, n, and r, and, ultimately, the complex Young's modulus, are computed from these.

The equations previously developed are for free-free cylindrical rods having length-to-diameter ratios of two or greater, and these determine the general shape of the samples. Samples of other than circular cross section can be tested. It is only necessary to alter the form of the equation for the Rayleigh correction. The length of the samples is not critical; however, the length should be chosen so that the first few resonant frequencies fall within the frequency range of the system from 5 to 48 kHz and not near the resonance frequencies of the accelerometers, approximately 32 kHz in this experiment.

Figure 2 shows the sample; the exciting transducer, a crystal shaker effective at more than 1000 Hz; and the response-monitoring transducer, a piezoelectric accelerometer effective at more than 1000 Hz, attached to the sample opposite the shaker. The figure also shows a second accelerometer, identical to the first, attached to the shaker and a narrow airgap between the shaker and the sample that will be discussed later.

While performing the experiment there are three quantities to be determined--resonant frequency f_n, mode number n, and the quality factor Q. Resonant frequencies are determined by increasing the excitation frequency from approximately 1000 Hz, maintaining a constant driving acceleration at the shaker, and watching for a maximum response from the accelerometer attached directly to the end of the sample. Mode number n is determined by noting several of the lowest resonant frequencies in order and getting a coarse indication of their spacing in the frequency domain. By assuming that the harmonics are approximately integral multiples of the fundamental, the actual mode number can be determined. In most cases, identifying the first two modes is sufficient to determine the sound speed and to confirm its accuracy. In some cases, however, the frequency limitations of the shaker prevent excitation of the fundamental resonance. Since the sound speed is practically independent of frequency in the frequency range of interest for stiff materials, locating a particular resonant frequency is not important. It is only necessary to correctly identify the particular mode excited and to count its frequency with a suitable counter.

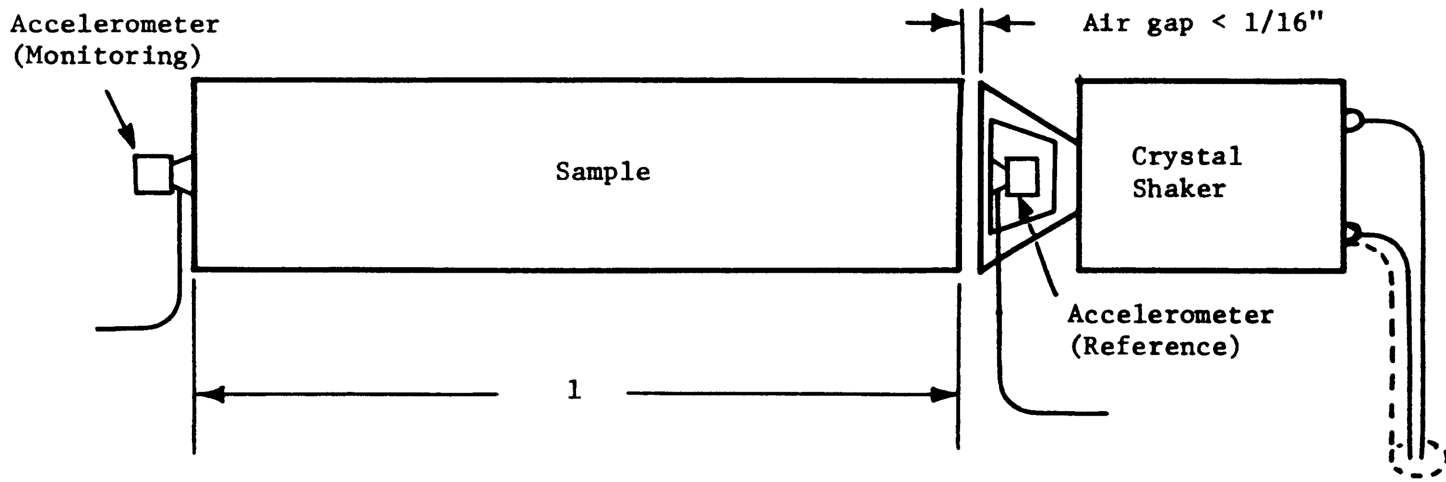

Figure 2 - Sample Instrumented for Testing

Having measured the resonant frequency and identified the particular mode, the next step is to determine the quality factor Q of the system from the 3-dB down bandwidth Δf_n. This bandwidth is measured by monitoring the response of the accelerometer with a General Radio 1900-A wave analyzer and noting the frequency of the 3-dB down points on either side of the resonant frequency. A typical response curve in the vicinity of the frequency f_n is shown in Figure 3.

Computations leading to the loss factor η and the real part of Young's modulus Y' proceed as follows. The sound speed c is derived from the resonant frequency f_n, the mode number n, and the length of the sample ℓ by Equation (14)

$$c_p = \frac{2\ell\ f_n}{n} \tag{14}$$

The loss factor η and loss parameter r are determined from Q by Equations (15) and (16)

$$\eta = \frac{1}{Q} \tag{15}$$

$$r = \frac{-2 + \sqrt{4 - 4\ \eta^2}}{2\eta} \tag{16}$$

From the radius and length of the sample a and ℓ, the mode number n, and Poisson's ratio σ, c--the sound speed in the absence of lateral effects--is computed using Equation (17)

$$c^2 = c_p^{\ 2} \left[1 + 2\left(\frac{\sigma\ \pi\ a\ n}{2\ell}\right)^2\right] \tag{17}$$

Having found c and r, and knowing the density ρ of the sample, the real part of Young's modulus may now be derived

$$Y' = \rho\ c^2 \left[\frac{1 - r^2}{\left(1 + r^2\right)^2}\right] \tag{18}$$

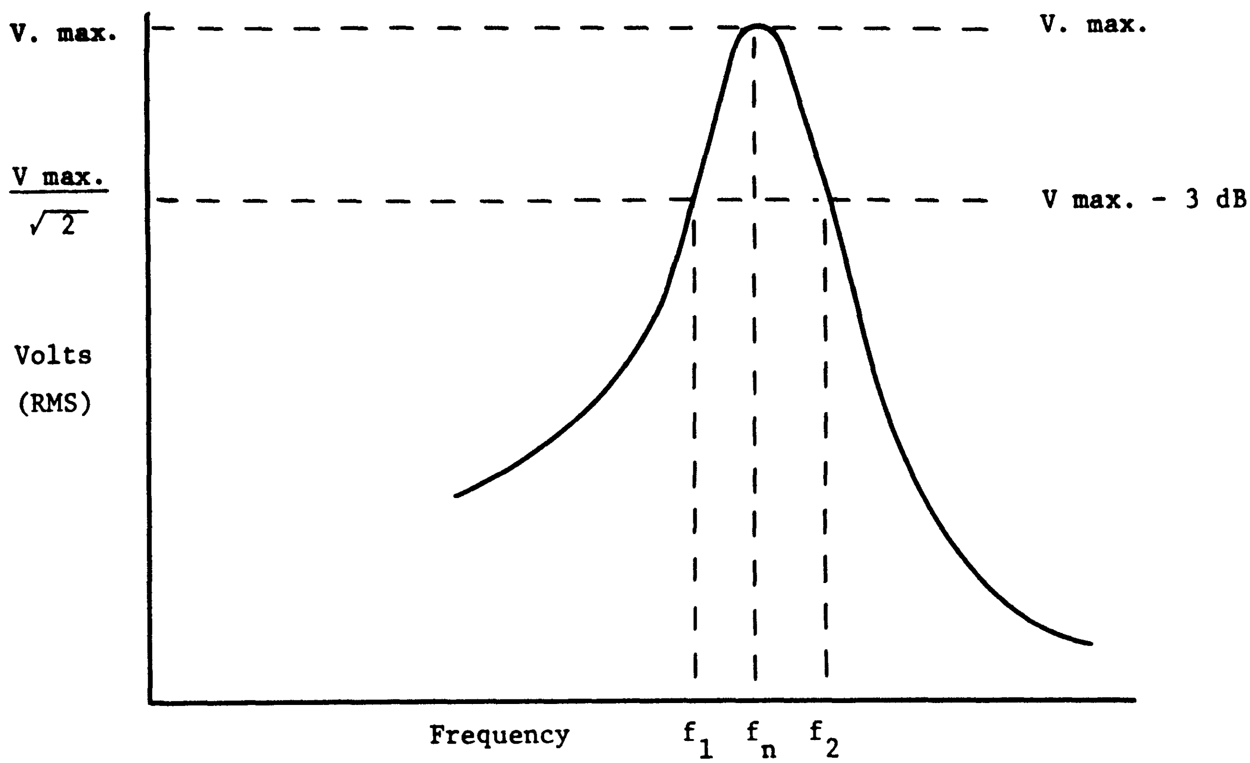

Figure 3 - Typical Response in Region of Resonant Frequency f_n

There is one major limitation to this technique. It does not work well for lossy materials. That is, material losses make it difficult to establish standing waves in lossy samples. In some instances, limits on the electrical system make it impossible to transmit enough energy into the material to overcome the difficulty. Furthermore, the ill-defined resonance curve makes it hard to decide just what frequency is the resonant frequency or where the 3-dB down points are.

The sample holder, drive system, and actual electronic equipment used in the experiment were selected to satisfy theoretical requirements and to minimize several problems. First, the previous theoretical development is based on a free-free rod. Figure 4 shows a cylindrical sample suspended horizontally by two fine threads in the test chamber. The direction of the excitation is along the axis of the cylinder in the horizontal direction, and it is assumed that this suspension provides negligible resistance to the small axial displacements we are dealing with. As a test of this assumption, several sets of measurements were made on an aluminum bar for which the results were available from published tables. The positions of the threads were changed, and the threads were replaced by rubber bands between successive measurements. No significant discrepancies appeared in the results. By adjusting the length of the threads and their position, the samples could be accurately positioned relative to the driving shaker.

Second, in a previous section, mass loading of the sample was discussed. To minimize mass loading, the crystal shaker is not physically attached to the samples. Samples are actually driven by the shaker through an airgap one-sixteenth of an inch wide. As shown in Figure 2, the driving surface of the shaker is fitted with a cone-shaped solid aluminum block to ensure that all regions of the surface adjoining the sample are moving in phase. L. C. Holtz, who devised this air-driven technique, checked the surface of the cone with very small accelerometers and found it to be moving uniformly in phase at a maximum frequency of approximately 20 kHz. Checks were made on the results of this technique for low-loss materials at frequencies as high as 45 kHz without notable discrepancies. The sample and shaker were carefully positioned so that their adjacent surfaces were parallel and no more than one-sixteenth of an inch apart. As long as the

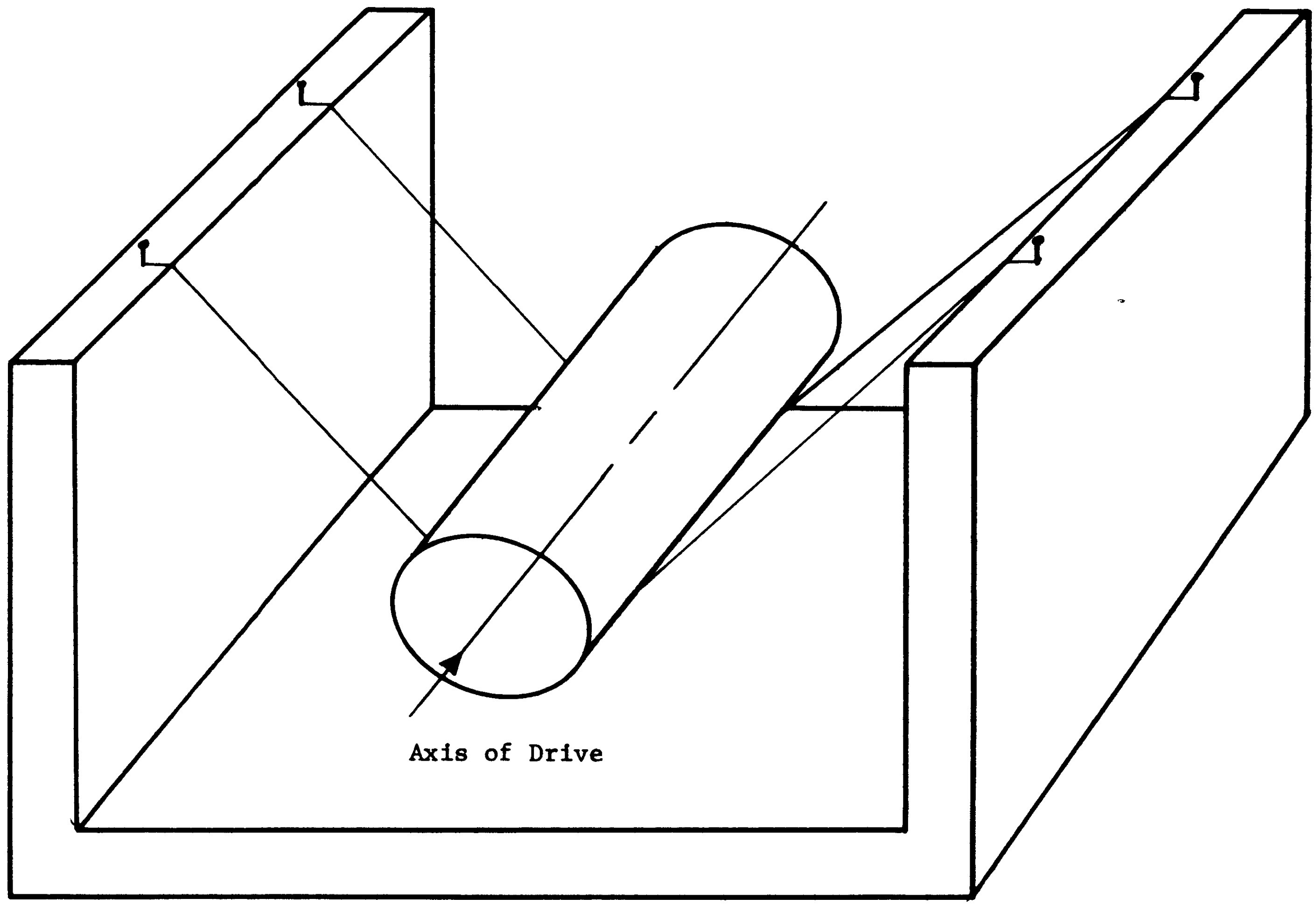

Figure 4 - Sample in Supporting Frame

frequency of the excitation is below the quarter wavelength of the airgap, both sides of the airgap are moving in phase, and, except for losses in efficiency, the effect of the coupling is negligible. The maximum frequency for this type of drive is approximately 48 kHz, set by the stiffness of the airgap. However, the inefficient coupling generates a new problem. In order to transfer sufficient energy to the samples, the shaker must be driven near its maximum level, and the acoustic signal in the sample picks up a great deal of noise caused by distortion in the shaker. Narrowband filters are used to remove this distortion from the signals of the reference and the monitoring accelerometers.

Third, the problem of narrowband filters able to track the same center frequency along with the need for a stable variable frequency, a-c source was a serious one. The pair of General Radio 1900-A wave analyzers shown in Figure 5 proved to be the solution. The analyzers are modified to operate in conjunction by disabling the main oscillator of one unit and operating both units in parallel from the remaining oscillator. The main oscillator is adjustable in the range from 100 to 154 kHz, and in each unit this signal is hetrodyned with a 100-kHz, crystal-controlled oscillator to generate a difference frequency. This difference or beat frequency is then isolated by a low-pass filter. Units for this type of tandem operation are especially selected for matched 100-kHz, crystal-controlled oscillators; therefore, the beat frequency, which is the center frequency of the narrowband analysis for each unit, is practically identical. These units provide two narrowband tracking filters which have as their center frequency the output of a common beat frequency oscillator (BFO). The BFO is very stable, allowing the experimenter to tune and hold sharp resonant peaks. In addition, the analyzer provided a convenient and accurate level meter for the sample-monitoring circuit.

Finally, the frequency dependence of the drive acceleration is compensated for in a servo circuit which employs a modified General Radio Graphic Level Recorder.[5] The reference accelerometer attached to the shaker

[5]General Radio Corporation, "General Radio Graphic Level Recorder Type 1521-A Operating Manual," p. 31 (Oct 1964).

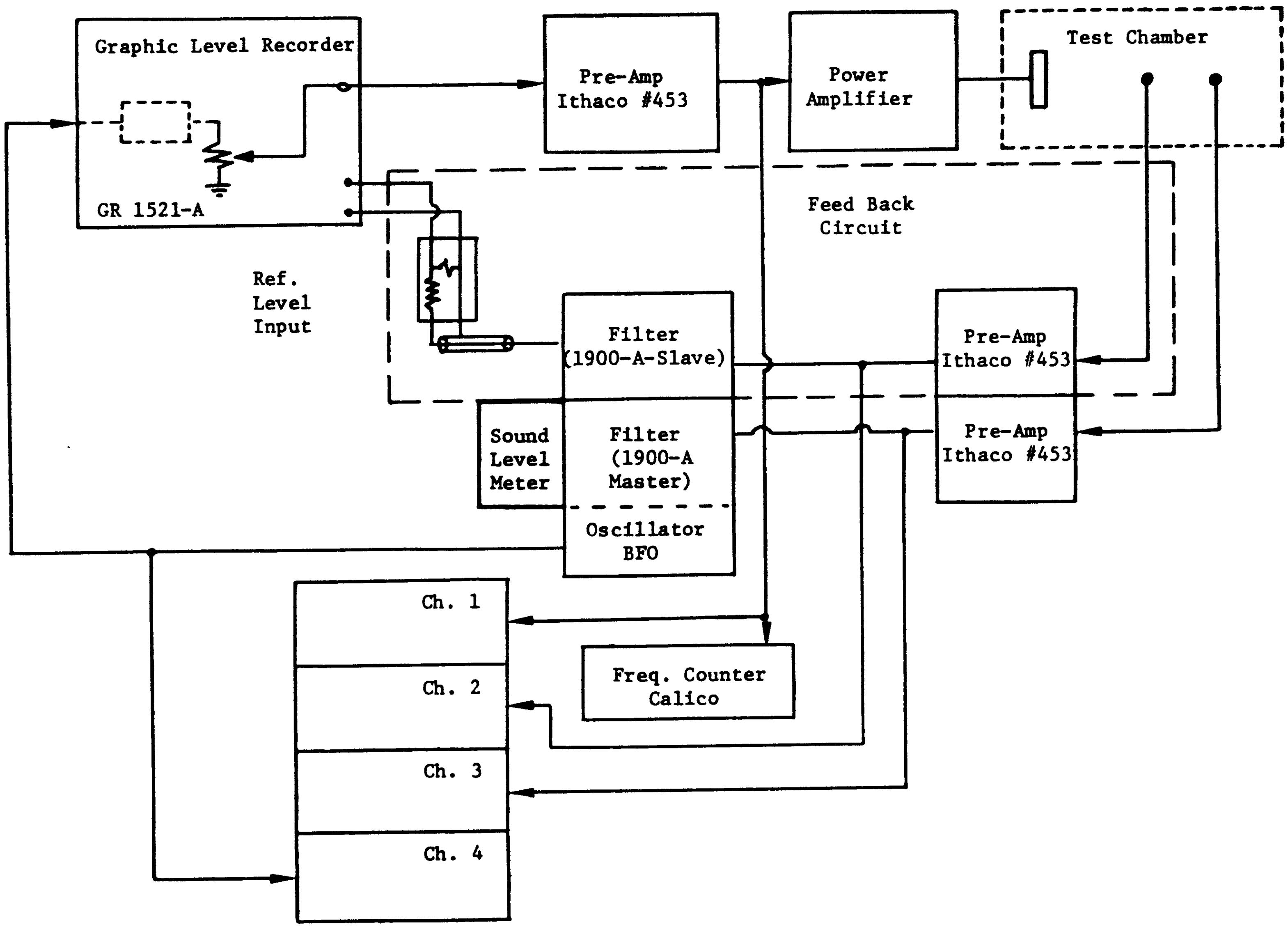

Figure 5 - Electronic Circuit

feeds back by way of a preamplifier to a level recorder which regulates the driving signal, thereby maintaining a constant driving acceleration in the region of each resonance. A decrease in the gain of the preamplifier just mentioned causes the driving level to increase to compensate. The opposite is also true. This provides for adjustment of the "constant" level. In this manner nonlinearities in the shaker and drive-circuit electronics are eliminated; the level, however, is only as independent of frequency as is the response of the reference transducer.

The reference transducer and also the monitoring transducer are very small piezoelectric accelerometers of the CGMS type, developed at the Naval Ship Research and Development Center (the Center) by G. A. Smith and C. A. Migliaccio.

VERIFICATION OF THE TECHNIQUE

COMPARISON WITH THE LITERATURE

Table 2 is a comparison with "handbook" values of the Young's modulus as measured by this technique for eight common materials.[6-10]

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[6] Cramer, W. S., "Bulk Compressibility Data on Several High Explosives," Naval Ordnance Systems Command Report 4380 (Sep 1956).

[7] Harris, C. M. and C. E. Crede, "Shock and Vibration Handbook," McGraw-Hill, New York (1961) p. 37-6.

[8] "Modern Plastics Encyclopedia," McGraw Hill, New York (1972).

[9] Chesapeake Instrument Corporation, "Dynamics of Mechanical Properties of Plastic Materials," Navy Department, Bureau of Ships, Contract Nobs-72106 (Jun 1957).

[10] Lazan, B. J., "Damping Mechanisms and Phenomenology in Materials," Proceedings of the 11th International Congress of Applied Mechanics, Edited by H. Gortler (1964).

TABLE 2 - COMPARISON OF HANDBOOK VALUES AND EXPERIMENTAL RESULTS

Material	Dimensions cm	Handbook		Experimental	
		$E (dyn/cm^2)$	η	$E (dyn/cm^2)$	η
Aluminum	ℓ-10.2 diam- 5.1	7.1×10^{11} (Ref 1)		7.0×10^{11}	0.0002
Brass	ℓ-20.3 diam- 5.1	10.4×10^{11} (Ref 1)		9.9×10^{11}	0.0003
Steel	ℓ-20.3 diam- 5.1	19.5×10^{11} (Ref 1)	0.0002 (Ref 10)	20×10^{11}	0.0002
PVC (Polyvinyl-chloride)	ℓ-10.0 diam- 5.0	4.13×10^{11} (Ref 8)		4.1×10^{11}	0.068
Polycar-bonate	ℓ-10.1 diam- 5.0	2.37×10^{11} (Ref 8)		2.3×10^{11}	0.022
Nylon-6	ℓ-10.1 diam- 4.98	3.1×10^{11} (Ref 8)	0.237 (Ref 9)	3.6×10^{11}	0.021
Polystyrene	ℓ-10.1 diam- 4.97	3.45×10^{11} (Ref 8) 3.65×10^{11} (Ref 4)	0.013 (Ref 8)	3.5×10^{11}	0.013

All the experimental data presented are for samples larger than 150 g. Corrections were applied to the basic data for loss and radial inertia; however, mass-loading effects were ignored. The quantity η refers to the loss factor. There is wide variation in the properties of plastics of the same nature among manufacturers and even among different batches produced by the same manufacturer. The occasional large variations in data regarding plastics may be attributed to these variations since the comparisons were not made on the same samples.

COMPARISON WITH EXACT SOLUTION FOR FINITE ROD

The results of this investigation were also compared with those of J. R. Hutchinson[11] for aluminum. In his paper Hutchinson solved the case of a finite rod with all stress-free boundaries, using exact solutions for the general equations of linear elasticity. His method yielded an infinite eigenvalue matrix which for the case of a bar of aluminum was found to converge to sufficient accuracy with a 20-by-20 term approximation. Tracings of the Hutchinson computer-generated curves for the first three axial modes are presented in Figure 6. Hutchinson defines a nondimensional frequency w

$$w = \frac{\text{Angular Frequency} \times \text{Radius}}{(\text{Shear Modulus/Density})^{1/2}}$$

and also a quantity h, which is the ratio of length to diameter for a right cylindrical sample. The frequency w is plotted on the ordinate in Figure 6; the ratio h is on the abscissa.

Tests were performed on five aluminum bars having values of h of 2, 4, 5, and 6. The resonant frequencies were put in the form of the nondimensional frequency, and the information has been presented in Table 3. As a means of comparison these points have been superimposed on the Hutchinson curves; see Figure 6.

Values of shear modulus and density for aluminum were obtained from Reference 1. The theoretical curves plotted by Hutchinson and the points experimentally measured by the method of this report are seen to agree very acceptably.

From the agreement of these few points and the rigorously computed curves, it seems reasonable to say that simple rod theory with an applied Rayleigh correction for radial inertia is accurate for the first two modes

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[11]Hutchinson, J. R., "Axisymmetric Vibrations of a Free Finite-Length Rod," The Journal of the Acoustical Society of America, Vol. 51, No. 1, Part 2 (Jan 1972).

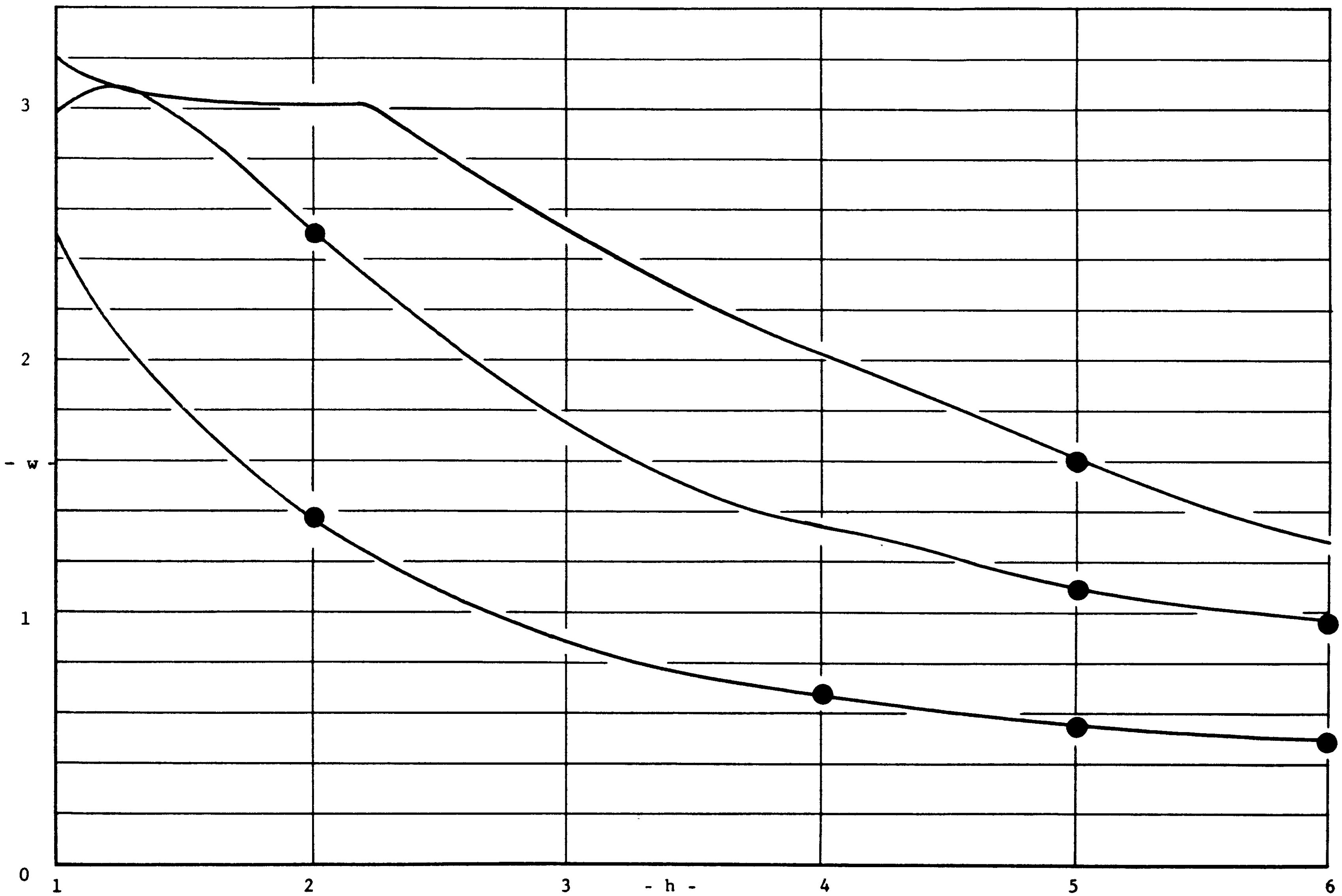

Figure 6 - Frequency w as a Function of the Length-to-Diameter Ratio h

(w = ω • radius/(shear modulus/density)$^{1/2}$)

TABLE 3 - FREQUENCY w COMPUTED FROM THE EXPERIMENTALLY DETERMINED RESONANT FREQUENCY

h	Length m	Radius m	Modes	w	Resonant Frequency Hz
2	0.1016	0.0254	1 2	1.26 2.44	24,659 45,615
4	0.0762	0.00953	1	0.673	33,468
5	0.254	0.0254	1 2 3	0.547 1.084 1.600	10,214 20,248 29,883
6	0.1524	0.0127	1 2	0.4463 0.8926	16,666 33,336

of vibration for rods having length-to-diameter ratios of 2 or more, and for a larger length-to-diameter ratio, the higher modes may be used accurately.

CONCLUSIONS

A convenient technique for measuring the dynamic Young's modulus and loss factor has been presented. The technique is effective and accurate for solid materials having relatively small loss, i.e., $Q \gg 0.5$. Materials are best tested in the form of rods with a length-to-diameter ratio of 2 or higher so that simple rod theory and a Rayleigh correction is accurate. Samples should weigh in excess of 150 g to avoid the need for considering mass loading; however, if not convenient, the loading can be accounted for. Length should be selected so that the fundamental resonance frequency is well above the lower limit of the electronic equipment, in this case, 1000 Hz. In this experiment there is less than 1 percent of experimental error in the Young's modulus, barring judgment errors on the part of the operator. This error is from mass loading and a 1-digit uncertainty in the frequency counter. For very low loss materials, $Q > 5000$, accuracies of 1 or 2 percent can be expected. As the technique is applied to lossier materials, the accuracy becomes more dependent upon the judgment of the operator. Finally, for very lossy materials with a Q of less than 10, this technique is of questionable value.

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13 ABSTRACT

As a part of an extensive program for measuring acoustical properties of polymeric materials, equipment has been designed and set up to measure the dynamic Young's modulus of stiff materials with low acoustic dissipation. Both the elastic modulus and the loss factor are obtained. The method described is based on the excitation of longitudinal resonances in a suspended uniform thin rod having free-free end conditions. For samples of actual material, the results must be corrected for the effect of the radial inertia of the bar and the effects of the measurement equipment on the results. The properties of very lossy or limp materials are measured more conveniently by other techniques.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Acoustic Measurement of Low Loss Materials						
(1) Dynamic Young's Modulus						
(2) Complex Young's Modulus						
Longitudinal Waves in Bars						
Acoustic Measurement of Low Loss Polymers						