Report 4038

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Bethesda, Maryland 20034

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BOUNDARY-LAYER SUCTION WITH SLOTS ON AXISYMMETRIC BODIES

Hans J. Lugt and Sin K. Oh

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COMPUTATION AND MATHEMATICS DEPARTMENT

November 1972 Report 4038

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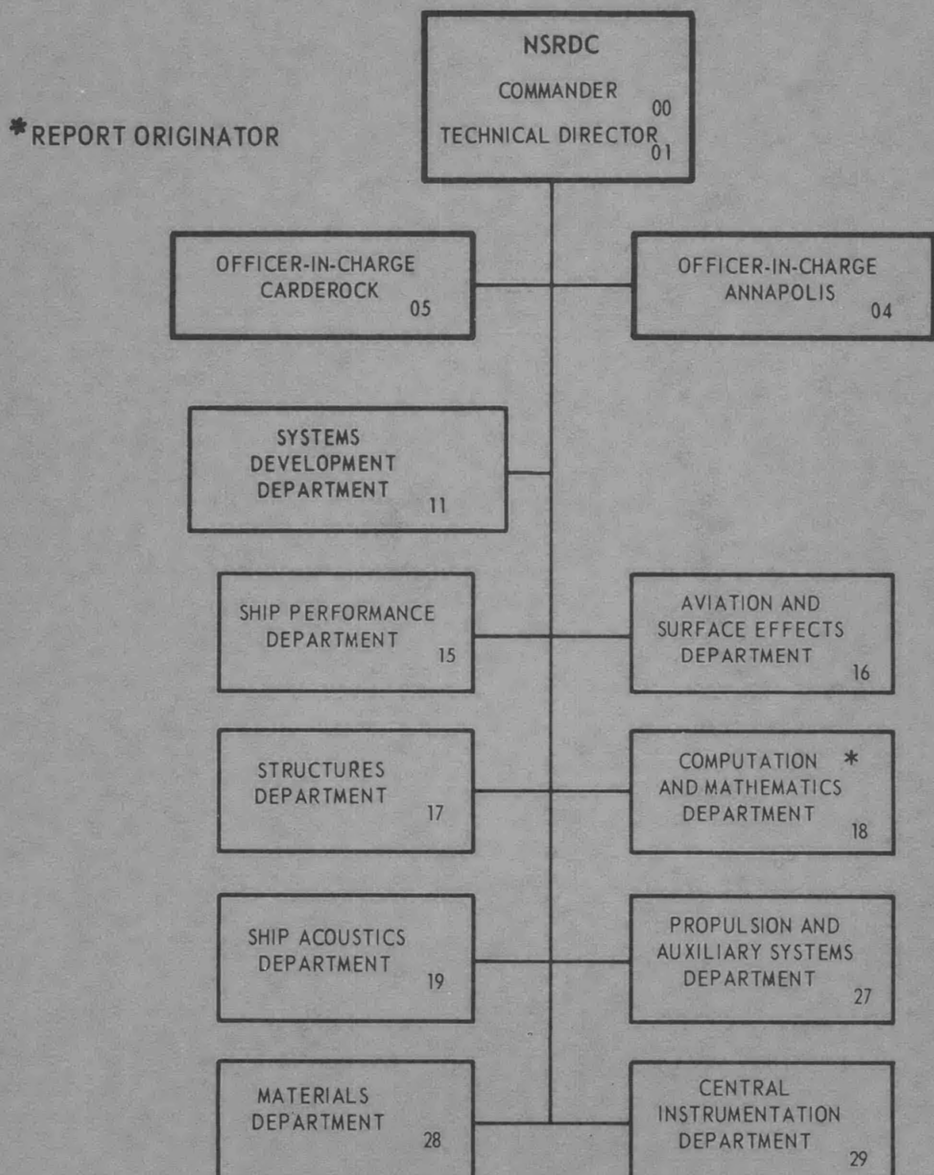

NDW-NSRDC 3960/44 (REV. 8/71)
GPO 917-872

DEPARTMENT OF THE NAVY

NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

BETHESDA, MD. 20034

BOUNDARY-LAYER SUCTION WITH SLOTS ON AXISYMMETRIC BODIES

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Hans J. Lugt and Sin K. Oh

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NOTATION

A, B	Upper limits for Re_{s1} and $Re_{\delta_{2J}}$
C_D	Drag coefficient
C_T	Total drag coefficient $= C_D + C_Q$
C_f	Friction coefficient
C_F	Dissipation coefficient
C_Q	Suction-drag coefficient
D	Drag
$F_1, \ldots, F_4$	Functions defined in Equation (6)
H_{12}	Form parameter $= \delta_1/\delta_2$
H_{32}	Form parameter $= \delta_3/\delta_2$
$\overline{H}_{32}, \tilde{\overline{H}}_{32}$	Mean values of H_{32} defined in Equation (13)
L	Length of body
l	Length of surface contour from bow to stern
M	Mangler-transformation function
n	Exponent defined in $Z = \delta_2 Re_{\delta_2}^n$
N	Exponent defined in Equation (6)
N_i	Theoretical pumping power for the i^{th} slot
p	Pressure
Q	Rate of fluid sucked into a slot
r, x	Cylindrical polar coordinates in the meridional plane
Re_{δ_2}	Reynolds number based on δ_2

$\widetilde{Re}_{\delta_2}$	Reynolds number based on δ_2 for plane motion
Re_L	Length Reynolds number
Re_{sl}	Slot Reynolds number
S	Surface area of body
s	Width of slot
u	Velocity component parallel to the body surface
u_δ	Velocity of potential flow at the body surface
u_∞	Constant velocity at infinity
v	Mean velocity in the slot
x'	Coordinate along the body surface in the meridional plane
$\bar{x}, \bar{x}'$	Dimensionless coordinates = x/L, x'/L
y	Coordinate normal to x'
Z	Boundary-layer variable = $\delta_2 \, Re_{\delta_2}^n$
α, β	Functions defined in Equations (7) and (8)
$\hat{\beta}$	Cone angle defined in Equation (27)
δ	Boundary-layer thickness
δ_1	Displacement thickness
δ_2	Momentum-loss thickness
δ_3	Energy-loss thickness
δ_Q	Quantity defined in Equation (18)
η	Dimensionless coordinate = y/δ
η_Q	Quantity defined in Equation (14)
ν	Kinematic viscosity of the fluid
ρ	Density of the fluid
τ	Shear Stress

Subscripts

E	Endpoint of calculation
i	i^{th} slot
J	Instability
p	Plane motion
Q	Suction
r	Last point of instability before turbulence
s	Surface of body
sl	Slot
T	Transition to turbulence
I	Immediately before slot
II	Immediately after slot

ABSTRACT

The engineering exploitation of boundary-layer suction with slots for drag reduction is controversial. Two major obstacles appear to exist: (a) Clogging and fouling of the slots and the body surface, (b) limitation in the order of magnitude of the length Reynolds number. Extensive studies will be needed to resolve the first problem. The present paper shows that the second problem can be circumvented by partial laminarization. For high Reynolds numbers, when full laminarization cannot be achieved, partial laminarization also yields substantial drag reduction and reduces acoustic noise. The computation is based on an integral method for solving the boundary-layer equations of an incompressible fluid around an axisymmetric body. Laminar as well as turbulent flow is considered. Suction is limited by an upper slot Reynolds number which prevents disturbances from the slots.

ADMINISTRATIVE INFORMATION

This study was funded by the Naval Material Command under the Independent Research Program, Task Area ZR 0140201. The date of completion was 14 November 1972. The Work Unit was 1-1804-002.

BACKGROUND AND MOTIVATION

The idea of controlling boundary-layer flow by means of suction is as old as the concept of boundary layer itself. In fact, Prandtl already described boundary-layer suction (BLS) in his famous paper of 1904 on boundary layers. About 1920 practical interest in using BLS to obtain high lift coefficients initiated systematic research. Since then the flood of papers on BLS has not ceased, and reference to the vast amount of literature is restricted here to some survey articles[1,2,3,4] and to papers pertinent to this study.

The notion of BLS is simple. Fluid elements near the solid wall, which are decelerated through their adherence to the wall and which may cause the flow to become unstable, are sucked away so that fluid elements with larger momentum are moved closer to the wall. This stabilizing process can prevent or delay flow separation or the transition from laminar to turbulent motion. The main engineering applications are in the increase in lift, the reduction of drag, and the control of acoustic noise. The latter two are of interest in this paper.

There are two principal ways to remove fluid from the boundary layer: through continuous suction (in practice by means of porous or densely perforated walls) and through distributed suction (by means of discrete

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1 "Boundary Layer and Flow Control," edited by G. V. Lachmann, Pergamon Press, 1961.

2 Schlichting, H., "Boundary-Layer Theory," McGraw-Hill Book Co., Sixth Edition, 1968.

3 Thwaites, B., "Incompressible Aerodynamics," Oxford Clarendon Press, 1960.

4 Smith, A. M. O., "A Decade of Boundary-Layer Research," Applied Mechanics Reviews, 23 (1970) No. 1, 1.

slots or holes). The first technique is ideal since continuous suction gives optimum results. The second is more practical with regard to clogging and fouling (although this argument has been questioned). This study considers BLS for vehicles submersed in the sea, and computations are restricted to axisymmetric bodies with suction distributed by means of slots.

Over the years the idea of BLS has been discredited for various technical reasons, and experimental work on this method has almost ceased, at least in the U.S. and in England. An appraisal of the situation, made recently by the senior author in an unpublished study, indicated that the most serious restrictions for engineering applications are:

(1) Clogging and fouling of the slots and the surface.

(2) Limitation in the size of the upper length Reynolds number to achieve full laminarization.

(3) Influence of ocean turbulence.

Besides these fluid dynamic restrictions, engineering design studies are necessary to determine such factors as optimum hull structure, propeller efficiency, and sucked-flow ejection.

The motivation for reconsidering the method of BLS despite these shortcomings is this:

There is reason to believe that fluid-dynamic difficulties can be overcome or diminished. Indications are that the frequency range of turbulence in the oceans does not endanger BLS except in strong ocean boundary layers. The limitation in the upper length Reynolds number is no obstacle to applying BLS to any vehicle if partial laminarization is considered. It is the main objective of this paper to demonstrate that partial laminarization can result in high drag reduction. The most serious difficulty for engineering application of BLS appears to be clogging and fouling. Experimental data which verify or negate this conjecture are still missing, but new research[5] in solid-surface treatment may lead to a solution of this problem.

BASIC EQUATIONS AND ASSUMPTIONS

1. THE BOUNDARY-LAYER EQUATIONS FOR MOMENTUM AND ENERGY

The steady boundary-layer flow of an incompressible fluid around an axisymmetric body moving parallel to the line of symmetry is considered. The contour of the body is divided into three parts and is described by a polynomial function for each section. (Figure 1.)

The flow field consists of the potential flow outside the boundary-layer and the boundary-layer flow adjacent to the body surface. The potential-flow field is calculated by an existing CMD-computer code.[6] For the

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5 Symposium on Gas-Surface Interactions May 3-5, 1972, Meersburg, Germany, organized and proceedings published by Dornier System, Friedrichshafen, Germany.

6 Dawson, C.W. and Dean, J.S., "The XYZ Potential Flow Program," Naval Ship Research and Development Center Rept. 3892, June 1972.

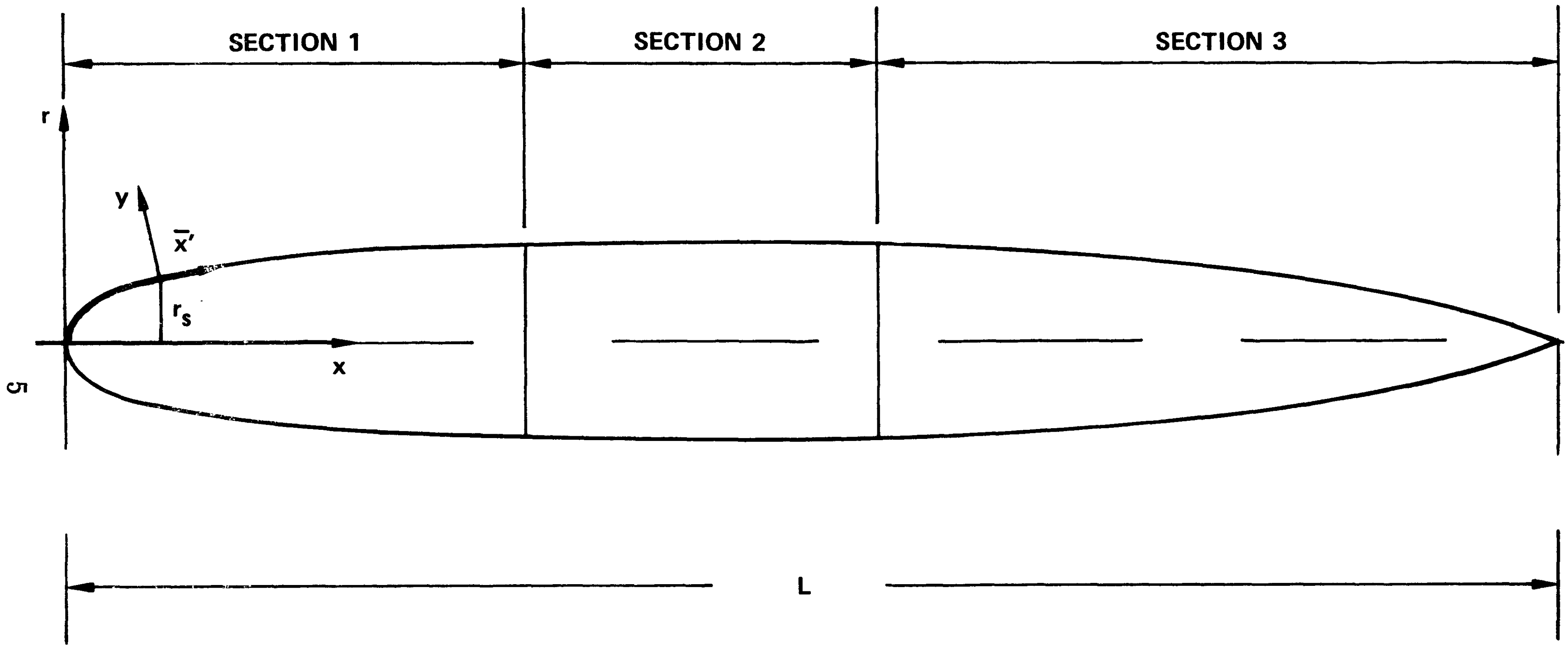

Figure 1 - Sketch of Axisymmetric Body with Coordinate System

boundary-layer flow an integral method[7] is chosen. This technique gives quite reliable data with a minimum amount of computer time so that a multitude of cases can be considered. For a one-parameter family of velocity profiles (which is sufficient for the present study) incorporation of suction slots is simple. The procedure is based on a recent study by Thiede[8], but essential improvements and extensions have been included.

The following standard boundary-layer parameters for integral methods are introduced. (The notation is adopted from Walz[7] and Thiede[8] with a few exceptions.)

$$\delta_1 = \int_0^\delta (1 - \frac{u}{u_\delta})dy \quad , \quad \text{displacement thickness}$$

$$\delta_2 = \int_0^\delta \frac{u}{u_\delta}(1 - \frac{u}{u_\delta})dy, \quad \text{momentum-loss thickness}$$

$$\delta_3 = \int_0^\delta \frac{u}{u_\delta}[1 - (\frac{u}{u_\delta})^2]\, dy \; , \quad \text{energy-loss thickness}$$

$$H_{12} = \delta_1/\delta_2 \; , \quad \text{form parameter}$$

$$C_f = \tau_s/\rho u_\delta^2 \; , \quad \text{friction coefficient}$$

$$C_F = \int_0^{u_\delta} \tau\, du/\rho u_\delta^3 \; , \quad \text{dissipation coefficient} \; ,$$

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7 Walz, A., "Boundary Layers of Flow and Temperature," MIT-Press, 1969.

8 Thiede, P., "Theoretische Untersuchungen zur Laminarhaltung der Grenzschicht an Rotationskörpern durch Absaugeschlitze bei inkompressibler Strömung," Fortschritt-Berichte der VDI Zeitschriften, Reihe 7, Nr. 27, November 1970.

where δ is the boundary-layer thickness,

u the velocity component parallel to the wall,

u_δ is the u-value of the potential flow at the border of the boundary layer,

ρ is the fluid density, and

τ and τ_s are the shear stress and the wall-shear stress, respectively.

The coordinate y is normal to the wall in the meridional (x, r) plane. (See Figure 1.) Under the assumption that $\delta_1/r_s << 1$, the boundary-layer equations representing the conservation of momentum and energy yield the form[7]:

$$\frac{d\delta_2}{dx'} + \delta_2(2 + H_{12}) \frac{1}{u_\delta} \frac{du_\delta}{dx'} + \delta_2 \frac{1}{r} \frac{dr}{dx'} - C_f = 0 \quad , \tag{1}$$

$$\frac{d\delta_3}{dx'} + 3\,\delta_3 \frac{1}{u_\delta} \frac{du_\delta}{dx'} + \delta_3 \frac{1}{r} \frac{dr}{dx'} - 2\,C_F = 0 \quad . \tag{2}$$

Here, x' is the coordinate along the wall. With the new parameters

$$Z = \delta_2\, Re^n_{\delta_2} \quad , \qquad Re_{\delta_2} = u_\delta\, \delta_2/\nu \quad , \qquad H_{32} = \delta_3/\delta_2 \quad ,$$

where ν is the kinematic viscosity of the fluid, and

Re_{δ_2} is the Reynolds number based on the local momentum-loss thickness,

Equations (1) and (2) are rewritten as

$$\frac{dZ}{dx'} + [F_1 \frac{1}{u_\delta} \frac{du_\delta}{dx'} + (n+1) \frac{1}{r} \frac{dr}{dx'}]\, Z - F_2 = 0 \quad , \tag{3}$$

$$\frac{dH_{32}}{dx'} + F_3 \frac{1}{u_\delta} \frac{du_\delta}{dx'} \cdot H_{32} - \frac{F_4}{Z} = 0 \quad . \tag{4}$$

Here, the following abbreviations are used[7]

$$\left.\begin{aligned} F_1 &= 2 + n + (n+1)\, H_{12} \\ F_2 &= (1+n)\, C_f \cdot Re^n_{\delta_2} \\ F_3 &= 1 - H_{12} \\ F_4 &= (2\, C_F - C_f\, H_{32}) Re^n_{\delta_2} \end{aligned}\right] \tag{5}$$

For a one-parameter family of velocity profiles C_f and C_F depend only on H_{32} [7], and Equation (5) can be simplified to

$$\left.\begin{aligned} F_1 &= 2 + n + (n+1)\, H_{12} \\ F_2 &= (1+n)\, \alpha \\ F_3 &= 1 - H_{12} \\ F_4 &= 2\beta\, Re^{n-N}_{\delta_2} - \alpha\, H_{32} \end{aligned}\right] , \tag{6}$$

where, for laminar flows, $n = 1$ and $N = 1$; and for turbulent motions, $n = 0.268$ and $N = N(H_{32})$. The quantities α, β, and H_{12}, which are functions of H_{32}, depend on the velocity profile. Approximations which describe laminar flow situations for both pressure increase and pressure drop are constructed by Eppler[9]:

9 Eppler, R., "Praktische Berechnung laminarer und turbulenter Absauge-Grenzschichten," Ing. Archiv 32 (1963), 221.

$$\left.\begin{aligned}
\alpha &= 2.512589 - 1.686095 \cdot H_{12} + 0.391541 \cdot H_{12}^2 \\
&\quad -0.031729 \cdot H_{12}^3 \quad \text{for } 1.51509 \le H_{32} \le 1.57258 \\
\alpha &= 1.372391 - 4.226253 \cdot H_{32} + 2.221687 \cdot H_{32}^2 \\
&\quad \text{for } 1.57258 < H_{32} \le 1.66667 \\
\beta &= 7.853976 - 10.260551 \cdot H_{32} + 3.418898 \cdot H_{32}^2 \\
&\quad \text{for } 1.51509 \le H_{32} \le 1.66667 \\
H_{12} &= 4.02922 - (583.60182 - 724.55916 \cdot H_{32} \\
&\quad + 227.18220 \cdot H_{32}^2) \cdot \sqrt{H_{32} - 1.51509} \\
&\quad \text{for } 1.51509 \le H_{32} \le 1.57258 \\
H_{12} &= 79.870845 - 89.582142 \cdot H_{32} + 25.715786 \cdot H_{32}^2 \\
&\quad \text{for } 1.57258 < H_{32} \le 1.66667 \\
H_{12} &= 3.738 - \sqrt{13.43 \cdot (H_{32} - 1.4418)} \\
&\quad \text{for } 1.66667 < H_{32} \le 2 \quad .
\end{aligned}\right\} \quad (7)$$

For turbulent flows the semi-empirical relations[7] are

$$
\left.\begin{aligned}
\alpha &= 0.0566\,H_{32} - 0.0842 \\
\beta &\doteq 0.0056 \\
N &= 0.168 \\
H_{12} &= 1 + 1.48(2 - H_{32}) + 104(2 - H_{32})^{6.7} \\
&\qquad \text{for } 1.50 \le H_{32} \le 2 \,.
\end{aligned}\right] \tag{8}
$$

If the function $u_\delta(x')$ for the potential flow is given, Equations (3) and (4) are two ordinary differential equations of first order for Z and H_{32}. The method of their solution is described later.

2. CRITERIA FOR INSTABILITY AND TRANSITION TO TURBULENCE

When solving Equations (3) and (4) stepwise from the initial value at $x' = 0$, two criteria are incorporated. The first indicates the onset of instability at $x' = x'_J$, the second permits the computation of the distance from x'_J to x'_T, where the flow becomes turbulent. The theoretical instability criteria for various velocity profiles can be approximated[8] by the single formula (see Figure 2)

$$
\begin{aligned}
\log_{10} Re_{\delta_{2J}} &= 4.556 - 76.87(1.670 - H_{32})^{1.542} \\
\text{for } 1.5150 \le H_{32} &\le 1.6667 \,.
\end{aligned} \tag{9}
$$

This criterion is claimed to be valid for both plane and axisymmetric motions. The transition point x'_T at which the boundary-layer flow becomes turbulent must be determined by a criterion valid for plane motions only. Thiede[8] applies the following technique: Starting from the point of instability x'_J one proceeds to the next point x'_{J+1}. With

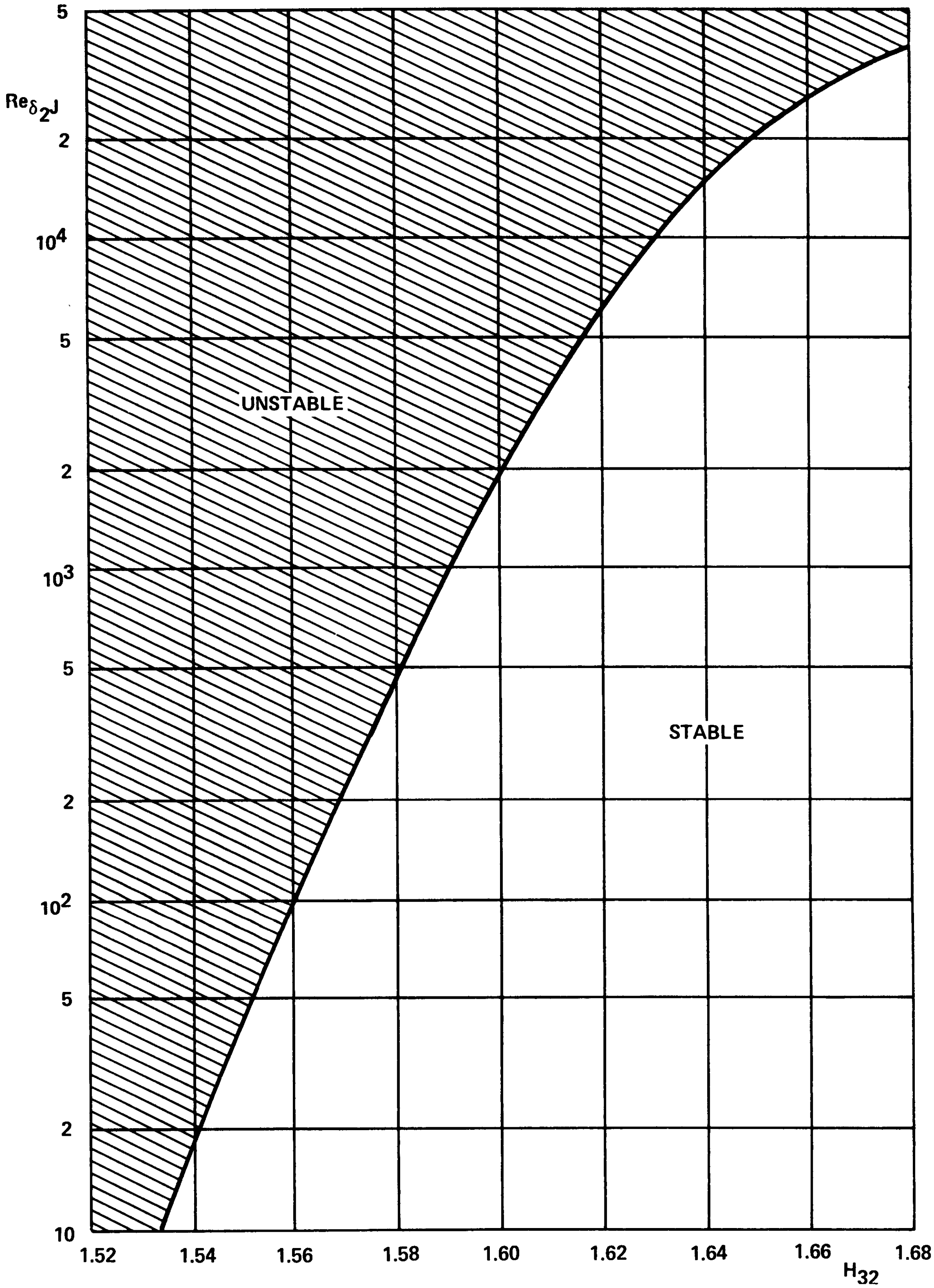

Figure 2 - Reynolds Number for Unstable Boundary Layer $Re_{\delta_{2J}}$ Plotted Against Form Parameter H_{32}

the Mangler transformation

$$M(x') = \left[\frac{1}{x' - x'_J} \int_{x'_J}^{x'} r^2 \, dx' \right]^{\frac{1}{2}} \quad , \tag{10}$$

the value $\widetilde{Re}_{\delta_2}$ for the auxiliary plane boundary layer is computed by

$$\widetilde{Re}_{\delta_2} = \frac{r}{M} Re_{\delta_2} \quad \text{at } x'_{J+1} \quad . \tag{11}$$

This value is compared with the turbulence criterion for plane motions

$$\left.\begin{aligned} \log_{10}(\widetilde{Re}_{\delta_{2T}} - \widetilde{Re}_{\delta_{2J}}) &= 1.6435 - 24.20 \cdot (1.5150 - \tilde{\bar{H}}_{32}) \\ &\text{for } 1.5150 \leq \tilde{\bar{H}}_{32} \leq 1.5600 \\ \log_{10}(\widetilde{Re}_{\delta_{2T}} - \widetilde{Re}_{\delta_{2J}}) &= 3.312 - 967.5 \cdot (1.6250 \\ &\quad - \tilde{\bar{H}}_{32})^{2.715} \\ &\text{for } 1.5600 < \tilde{\bar{H}}_{32} \leq 1.6250 \end{aligned}\right\} \tag{12}$$

where $\widetilde{Re}_{\delta_{2J}} = Re_{\delta_{2J}}$ and H_{32} is averaged according to

$$\tilde{\bar{H}}_{32} = \bar{H}_{32} = \frac{1}{x' - x'_J} \int_{x'_J}^{x'} H_{32} \, dx' \quad . \tag{13}$$

If $\widetilde{Re}_{\delta_2} < \widetilde{Re}_{\delta_{2T}}$ at x'_{J+1}, the value for the next step at x'_{J+2} is computed until the criterion of Equation (12) is met at x'_T.

3. SUCTION SLOTS

Suction slots are considered whenever an instability point x'_J is reached. Thiede[8] claims that the first suction slot on the front of the body can be placed at the transition point x'_T instead of x'_J, since disturbances could be eliminated by the following slots. His argument is based on experiments by Pfenninger and Groth[1] for two-dimensional bodies. His view is not shared here. The nose region of axisymmetric bodies is very sensitive to disturbances which must be suppressed immediately. Hence, the first suction slot is introduced at the first x'_J. The effects of both assumptions are compared in Figure 3. The density of slots in the front of the body indicates the importance of avoiding any onset of instability.

The boundary-layer parameters behind the slots, Z_{II} and $H_{32_{II}}$, are computed from the corresponding quantities in front of the slots, that is, Z_I and H_{32_I}. Since boundary-layer theory is not valid at the slots, a heuristic approach is applied in the form of "Walz's amputation principle[7, 10]. It states that Z_{II} and $H_{32_{II}}$ can be determined from the velocity profile which is left after part of the boundary layer is sucked into the slot. (Figure 4.) The consideration of slots requires a new input parameter which specifies the amount of fluid taken away:

$$\eta_Q = y_Q/\delta_I \ . \tag{14}$$

Here, y_Q is the part of the boundary layer, of thickness δ_I before the slot, which is sucked in. The rate of fluid inside the slot of

10 Walz, A., "Theorie zur Absaugung der Reibungsschicht," Zentrale für wissenschaftliches Berichtswesen-Bericht 1775, AVA Göttingen, 1943.

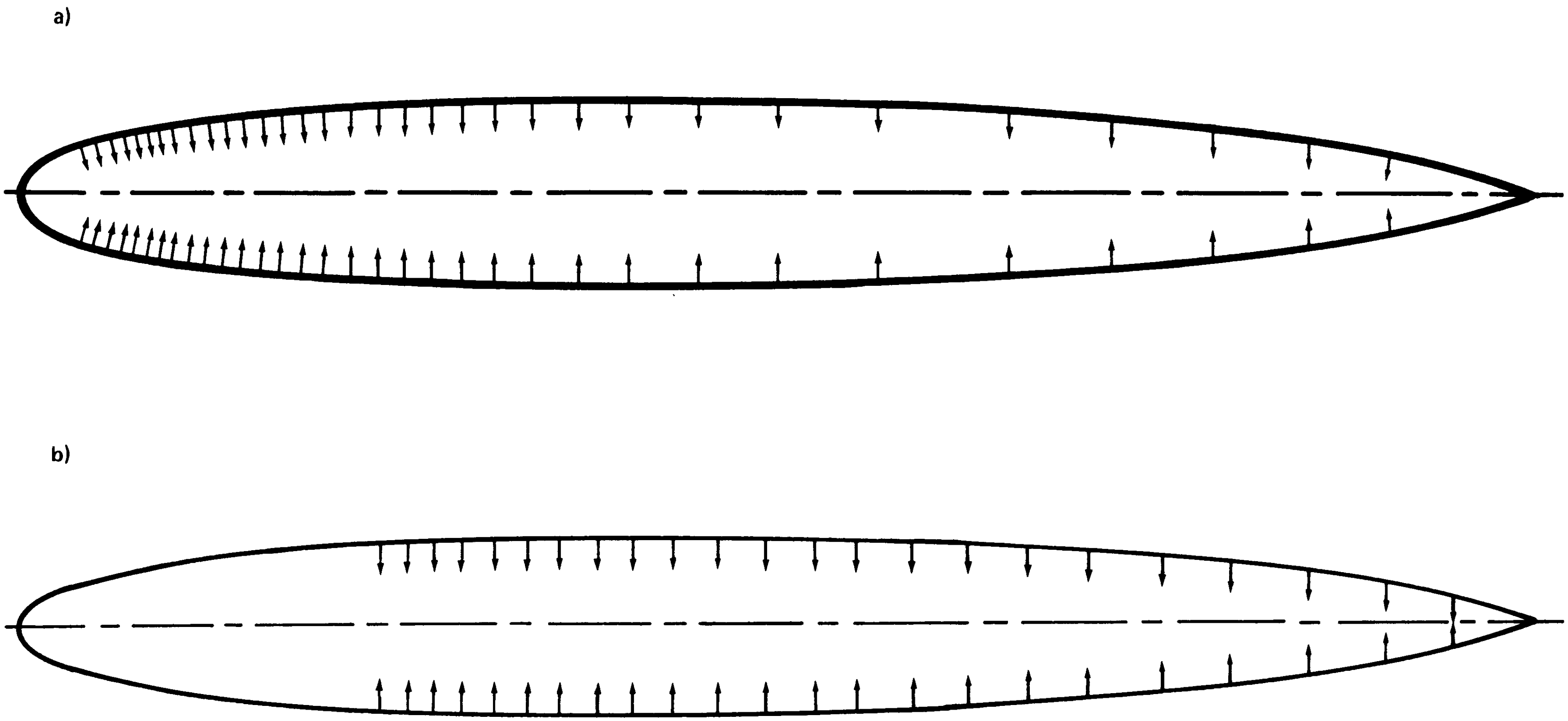

Figure 3 - Location of Suction Slots for $Re_L = 5 \cdot 10^7$, $Re_{sl} < \infty$, $\eta_Q = 0.1$ if the First Slot is Placed at a) x'_J, b) x'_T. Shape I of Figure 8.

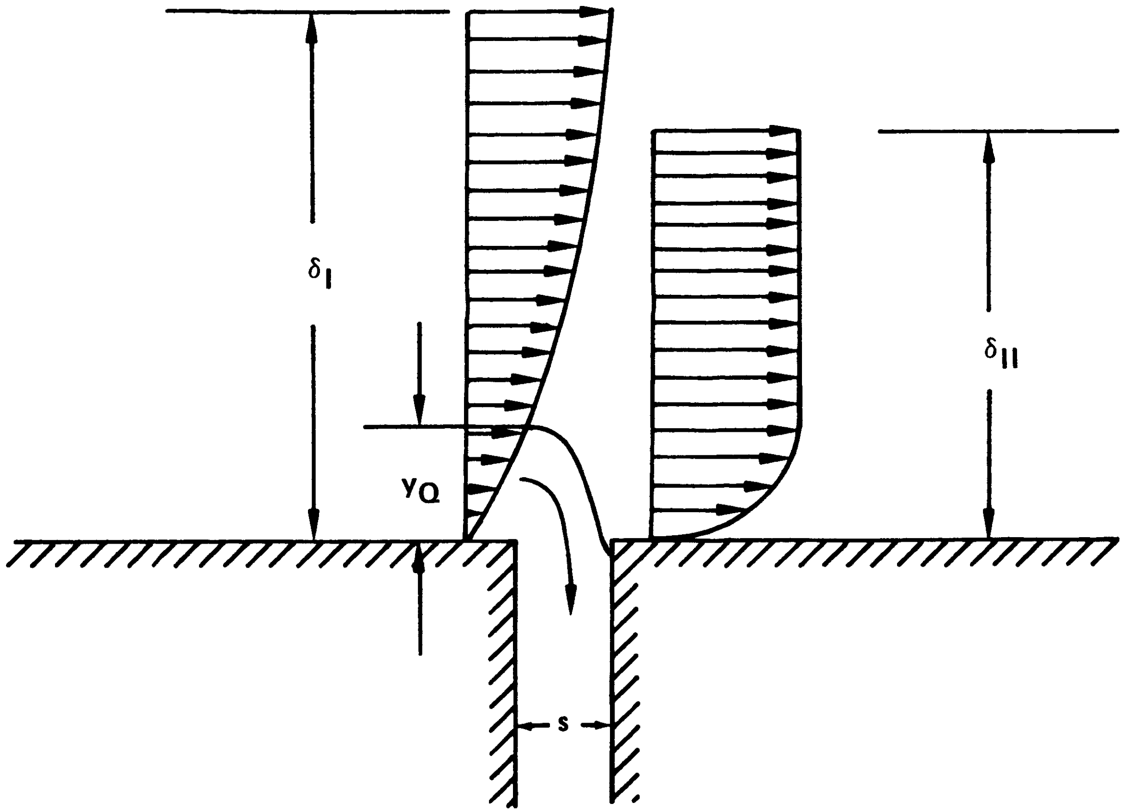

Figure 4 - Walz's Amputation Hypothesis

width s is

$$Q = 2\pi\, r_s\, s\, v \quad , \tag{15}$$

where v is the mean velocity in the slot. The slot Reynolds number Re_{sl} is defined by

$$Re_{sl} = s\, v/\nu \quad . \tag{16}$$

The rate of fluid Q is related to η_Q by

$$Q = 2\pi\, r_s\, u_\delta\, \delta_Q \quad , \tag{17}$$

where

$$\delta_Q = \delta_I \int_0^{\eta_Q} \left(\frac{u}{u_\delta}\right)_I d\eta \ . \tag{18}$$

For a one-parameter family of velocity profiles the flow quantities behind the slot are functions of H_{32_I} and η_Q only. The lengthy computation is omitted, and reference is made to Thiede's paper[8].

Walz's amputation principle is hypothetical and must be verified either by experiments or by analytical means. Thiede[8] has made comparisons with known analytical solutions within the framework of boundary-layer theory. Computations are under way using the full Navier-Stokes equations; they will be reported elsewhere (see Figure 5).

In this paper two new limitations are introduced which are based on Pfenninger's experiments*. Slots and plenum chambers underneath can generate disturbances which affect the boundary-layer flow. To avoid these instabilities the slot-Reynolds number must not exceed a certain value A:

$$Re_{sl} \le A < \infty \ . \tag{19}$$

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* Private communication.

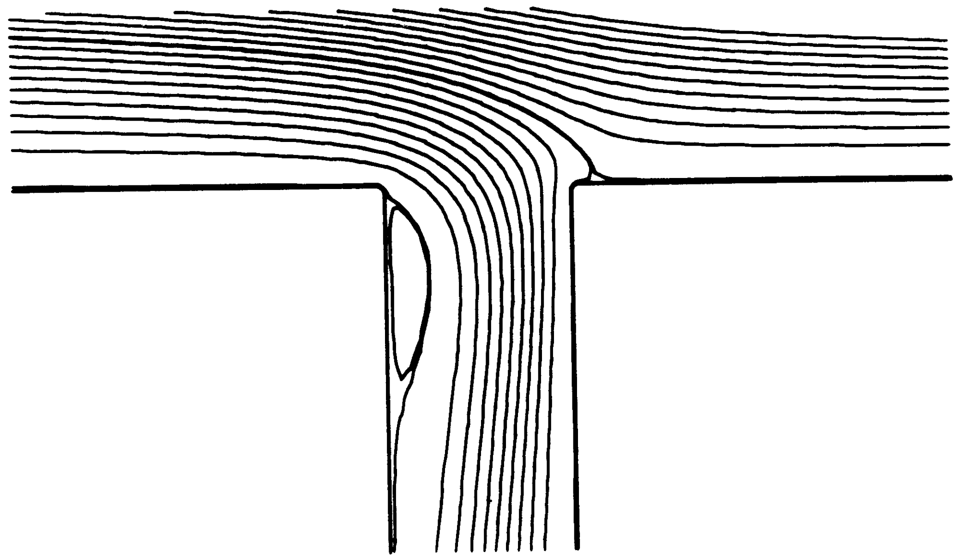

Figure 5 - Computer Generated Streamline Pattern for $Re_{sl} = 100$
(Courtesy Dawson and Marcus)

From his experiments Pfenninger finds A to be about 100. He concludes that, with better design of the slots and plenum chambers, the value of A can safely be assumed to be 250. Thiede[8] obtained uniqueness by assuming $\eta_Q = 0.1$ for the whole body. Calculations show that the value $A = 250$ is quite a severe restriction and that, except for the nose region, the upper limit $Re_{sl} = 250$ must be enforced everywhere by reducing η_Q. (It is assumed now that $Re_{sl} = A$ everywhere by varying η_Q. Thus, the physically meaningful condition $Re_{sl} = A$ replaces the rather arbitrary assumption $\eta_Q = 0.1$ of Thiede.)

The second restriction is concerned with an upper limit for $Re_{\delta_{2J}}$. Although theory does not predict the existence of a finite upper limit (the value $Re_{\delta_{2J}}$ in Figure 2 approaches infinity for $H_{32} \rightarrow 2$), experiments indicate that, due to inherent irregularities of the surface and other influences, there might be a finite upper limit from the practical point of view. In fact, Pfenninger* concludes on the basis of his experiments that the value $2 \cdot 10^8$ for the length Reynolds number Re_L is hard to exceed. Of course, Re_L cannot be a characteristic number for the boundary layer but Re_{δ_2} can. This leads to the heuristic postulate

$$Re_{\delta_{2J}} \leq B < \infty . \tag{20}$$

It may be emphasized that this restriction is not based on physical reasoning but is a safety measure for engineering applications.

The replacement of Pfenninger's upper-limit Re_L by Equation (20) has an important consequence. It makes it possible to consider partial laminarization. The technique of BLS thus becomes applicable for

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* Private communication.

Naval vehicles with length-Reynolds numbers higher than $2 \cdot 10^8$. For the body contour shown in Figure 1 and under the assumption of (approximately) optimal slot arrangement, the value for B can be determined from $Re_L = 2 \cdot 10^8$. The upper limit is then $B = 17800$. The corresponding value for H_{32} is 1.645.

4. DRAG COEFFICIENTS

The drag coefficient C_D, which is the sum of the pressure and friction-drag coefficients, is computed from Young's formula[7]

$$C_D = \frac{D}{S\,\rho/2\,u_\infty^2} = 4\pi\,\frac{r_E}{S}\,\delta_{2E}\left(1+\tfrac{1}{2}\,\frac{\delta_{2E}}{r_E}\right)(u_{\delta E})^{\frac{5+H_{12E}}{2}} , \tag{21}$$

where D is the drag,
S is the surface of the body, and
E is the subscript indicating the end point of the calculation.

The pumping power must be considered in the calculation of the drag. If one assumes that the sucked fluid does not contribute to the thrust, and if one does not consider pressure loss in the suction-pipe system, the theoretical pumping power required for the i^{th} slot is

$$N_i = Q_i\left(\frac{\rho}{2}\,u_\infty^2 + p_\infty - p_i\right) , \tag{22}$$

where p_i is the pressure at the i^{th} slot. The equivalent drag coefficient C_Q for all slots is then

$$C_Q = \frac{1}{\rho/2\,u_\infty^3\,S}\,\sum_i N_i = \frac{1}{S u_\infty}\,\sum_i Q_i\left(1 - \frac{p_i - p_\infty}{\rho/2\,u_\infty^2}\right) . \tag{23}$$

The total drag coefficient C_T is

$$C_T = C_D + C_Q . \tag{24}$$

NUMERICAL METHOD OF SOLUTION

Solutions of Equations (3) and (4) are constructed by means of the Runge-Kutta-Gill method[11]. Since the solution is singular at the initial point $x' = 0$, the values Z and H_{32} at the first point $x' = x'_1$ must be approximated by a known solution. For a conical front with the cone angle $\hat{\beta}$ the flow parameters at the first two points are (by using Hartree-profiles)

$$H_{32}(0) = H_{32}(1) = \text{function of } \hat{\beta} \ , \tag{25}$$

$$Z(0) = 0, \quad Z(1) = \frac{F_2}{3} x'_1 \quad \frac{1}{1 + F_1 \dfrac{\hat{\beta}}{6 - 3\hat{\beta}}} \ , \tag{26}$$

where the cone angle is

$$\hat{\beta} = \frac{2}{\pi} \tan^{-1} \frac{r_{s1}}{x_1} \quad . \tag{27}$$

In order to use Hartree-profiles the plane auxiliary angle $\hat{\beta}_p = \hat{\beta}/(3-\hat{\beta})$ is introduced. Then, function (25) can be approximated[8] by

$$\left.\begin{aligned} H_{32} &= 1.5720 + 0.1561 \cdot \hat{\beta}_p - 0.2470 \cdot \hat{\beta}_p^2 + 0.2244 \cdot \hat{\beta}_p^3 - 0.0804 \cdot \hat{\beta}_p^4 \\ &\qquad \text{for } \ 0 \le \hat{\beta}_p \le 1 \ , \\ H_{32} &= 1.5720 + 0.2258 \cdot \hat{\beta}_p + 0.7663 \cdot \hat{\beta}_p^2 + 5.3850 \cdot \hat{\beta}_p^3 \\ &\qquad \text{for } \ -0.1988 < \hat{\beta}_p < 0 \ . \end{aligned}\right\} \tag{28}$$

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11 Ralston, A. and Wilf, H. S., "Mathematical Methods for Digital Computers," Vol. 1, Part III, Sec. 9, Wiley and Sons, 1960.

For a blunt nose, $\beta = 1$, $\beta_p = 0.5$; the value of H_{32} at x'_1 is $H_{32}(1) = 1.6113$.

The coordinate x' is made dimensionless by L: $x'/L = \bar{x}'$. Then, the body contour starts at $\bar{x}' = 0$ and ends at $\bar{x}' = l/L$. The Runge-Kutta-Gill method operates from $\bar{x}'_2 = 0.0116$ to $\bar{x}' = \bar{x}'_E$, where $\bar{x}_E$ is determined by the upper limit $\delta_2/r_s = 1/15$*. The size of the interval $\Delta\bar{x}' = 0.00127$ means that the contour is represented by about 800 points (the exact number depends on the particular contour). This number is necessary to obtain sufficient resolution for placing densely located slots. The potential flow is computed at 400 points. Values in-between are obtained by linear interpolation.

The accuracy of H_{32} and Z is controlled by Collatz's estimation[11] for the Runge-Kutta method. When the Collatz accuracy factor ϵ exceeds 0.03, the interval $\Delta\bar{x}'$ is divided into 5 equal subintervals. This procedure continues until the derived accuracy is met. (See Figure 6.)

The width s of the slot is determined by Thiede[8] under the assumption that $s = y_Q$, based on Gregory's studies[1]. The width s does not enter the computer program except for determining the position $(x'_J + s)$ of $H_{32_{II}}$ and Z_{II}. (See Figure 7.) The assumption $x = y_Q$ is used in this paper, too, although it is recognized that this assumption leads to unrealistically small s for practical applications. However, the influence of this assumption on the overall results is negligible. Of larger importance is the question of whether s can be made wider in practice. The answer is affirmative for the restriction $Re_{sl} \leq 250$. The purpose of this limitation is just to keep the slot flow sufficiently viscous. The calculation of s is given by Thiede[8].

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* Beyond this value the boundary-layer assumption is not valid.

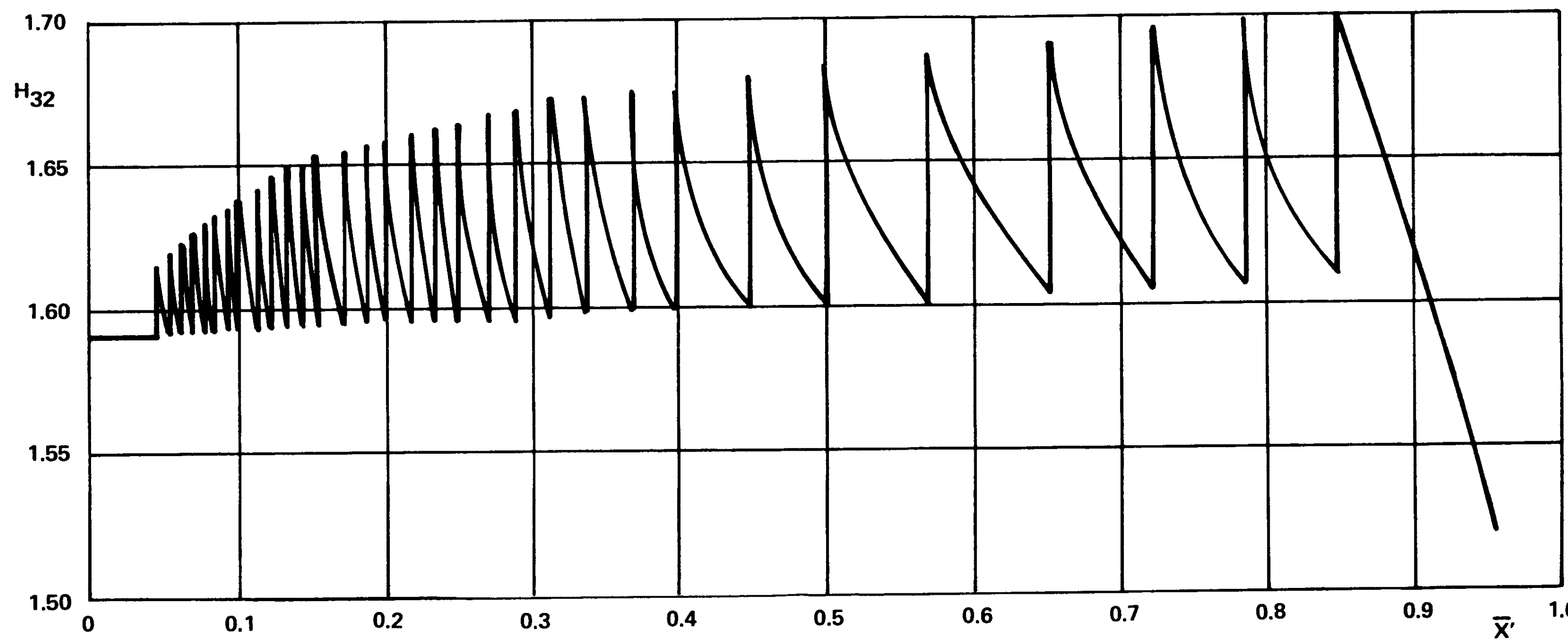


Figure 6 - Form Parameter H_{32} Plotted Against $\overline{x}'$ for $Re_L = 5 \cdot 10^7$, $Re_{s1} < \infty$, $\eta_Q = 0.1$. Shape I of Figure 8.

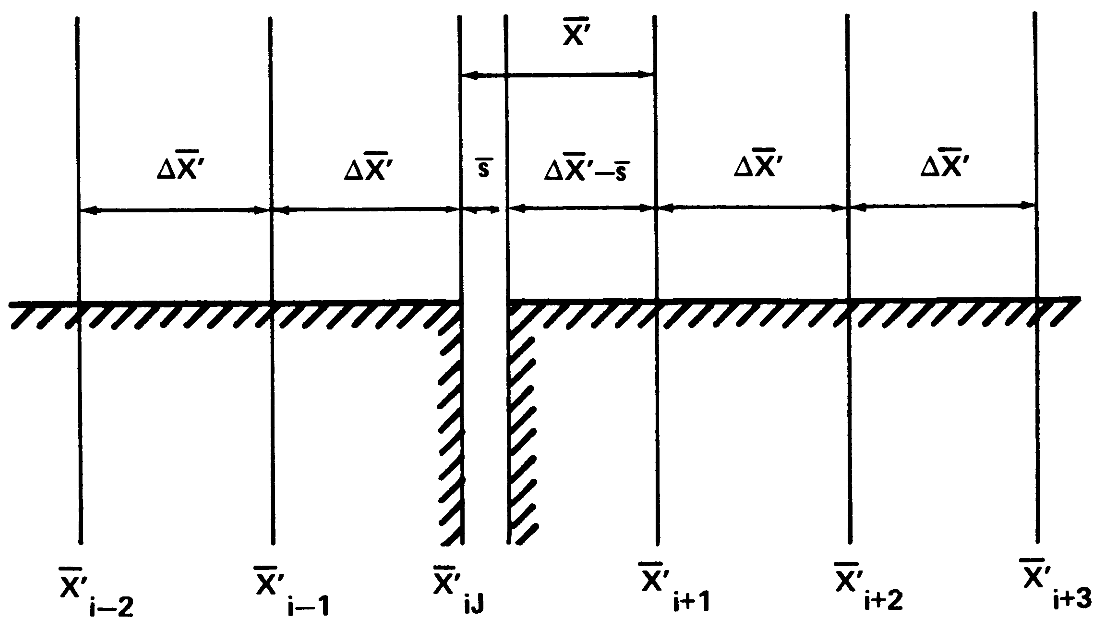


Figure 7 - Spacing Near a Suction Slot

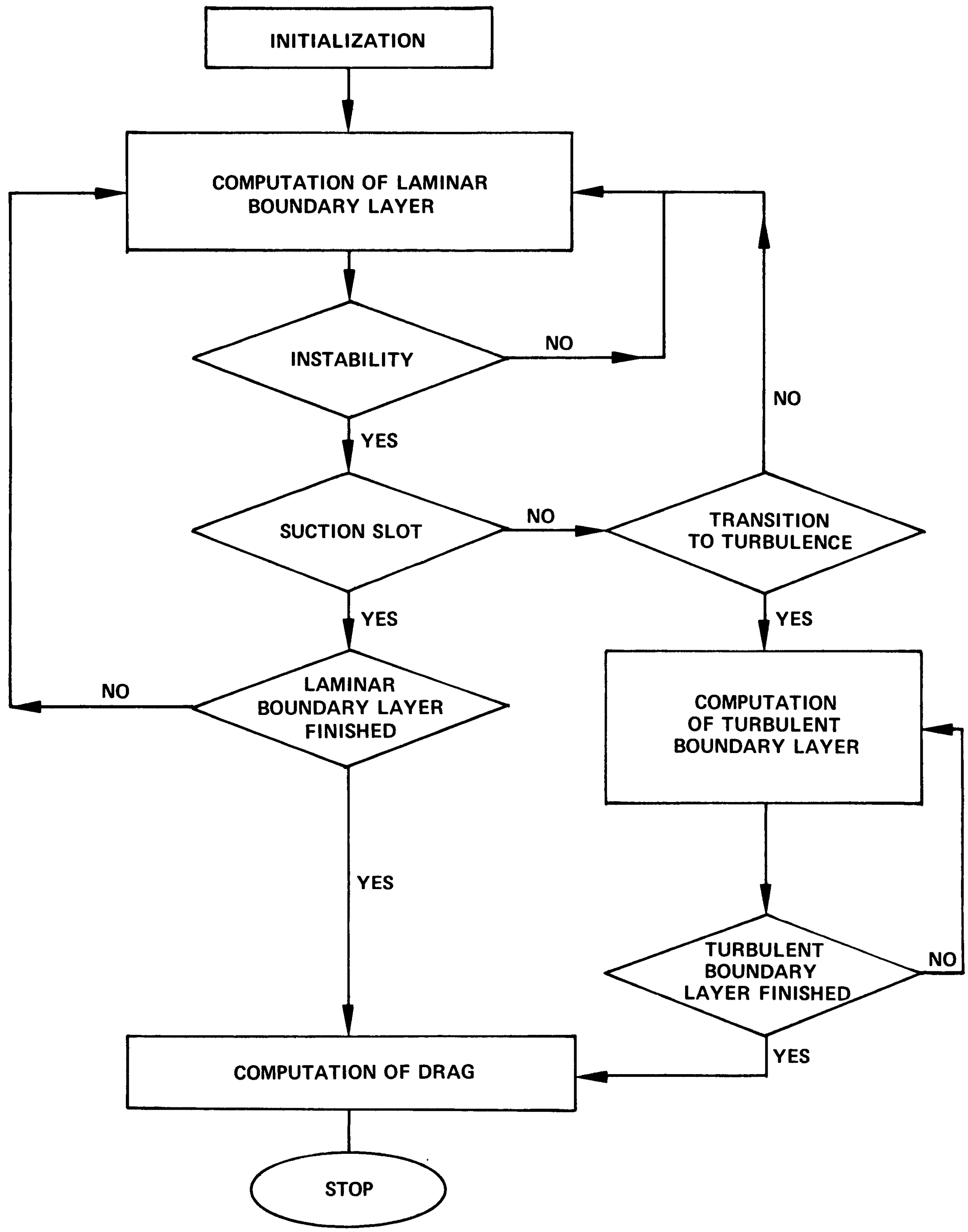
The possible number of slots is restricted in the computer program to 300. Whenever Re_{sl} exceeds the maximum A, η_Q is reduced by $\Delta \eta_Q$.

When $\bar{x}' = \bar{x}'_T$ is reached, the computation of the turbulent boundary layer starts. At this point the value Z (which is computed for laminar motion) is recalculated for turbulent flow by using $n = 0.268$.

The sequence of the computer calculation is shown in the flow chart. Input values are the shape of the body, Re_L, A, B, $\Delta \bar{x}'$, and η_Q for the first slot. Output values are the drag coefficients, H_{32}, Z, η_Q, position and number of slots.

The computer time on the CDC 6700 is of the order of 30 seconds for the boundary-layer program. This time is negligibly small in comparison with the 5 minutes computer time required for the potential flow program.

FLOW CHART



INITIALIZATION
COMPUTATION OF LAMINAR BOUNDARY LAYER
INSTABILITY
NO
YES
NO
SUCTION SLOT
NO
TRANSITION TO TURBULENCE
YES
YES
NO
LAMINAR BOUNDARY LAYER FINISHED
COMPUTATION OF TURBULENT BOUNDARY LAYER
YES
TURBULENT BOUNDARY LAYER FINISHED
NO
YES
COMPUTATION OF DRAG
STOP

RESULTS

1. FULL LAMINARIZATION

In this section it is assumed that flow past the total body surface can be kept laminar by means of BLS. According to Pfenninger this assumption has been verified experimentally only up to $Re_L = 6 \cdot 10^7$ in the low-turbulence wind tunnel of Ames. Pfenninger* concludes from other experiments that $Re_L = 2 \cdot 10^8$ may be reached. Such an upper limit for Re_L is disregarded here in the study of full laminarization but will be of major concern in the investigation of partial laminarization.

Thiede[8] has presented extensive data on the influence of shape, suction, and Re_L on the drag coefficients. His Re_L values are limited to $2 \cdot 10^8$ with a few exceptions for $Re_L = 5 \cdot 10^8$. In this context it must be remembered that Thiede's first suction slot is placed at the first transition point $\bar{x}_T^*$, whereas the present method uses the first instability point $\bar{x}_J'$.

The contour of shape I in Figure 8 was chosen as the basic body form for the calculations. Two additional shapes with the same volume and length but with higher curvature of the nose were checked for comparison (Figure 8). In Figure 9 the dimensionless surface velocities and surface pressures for the potential flow are plotted against $\bar{x}$.

In Figure 10 the total drag coefficient C_T is displayed for laminar and turbulent flows as well as for full laminarization due to BLS. Two cases are considered: (a) $Re_{s1} < \infty$, $\eta_Q = 0.1$, (b) $Re_{s1} \leq 250$, $\eta_Q = 0.1$ at the first slot. At $Re_L = 2 \cdot 10^8$ the reduction in drag is 1:12 for (a) and 1:12.5 for (b). The greater drag reduction in the latter case must be paid for by a larger number of slots. In Figure 11 the number of slots is

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* Private communication.

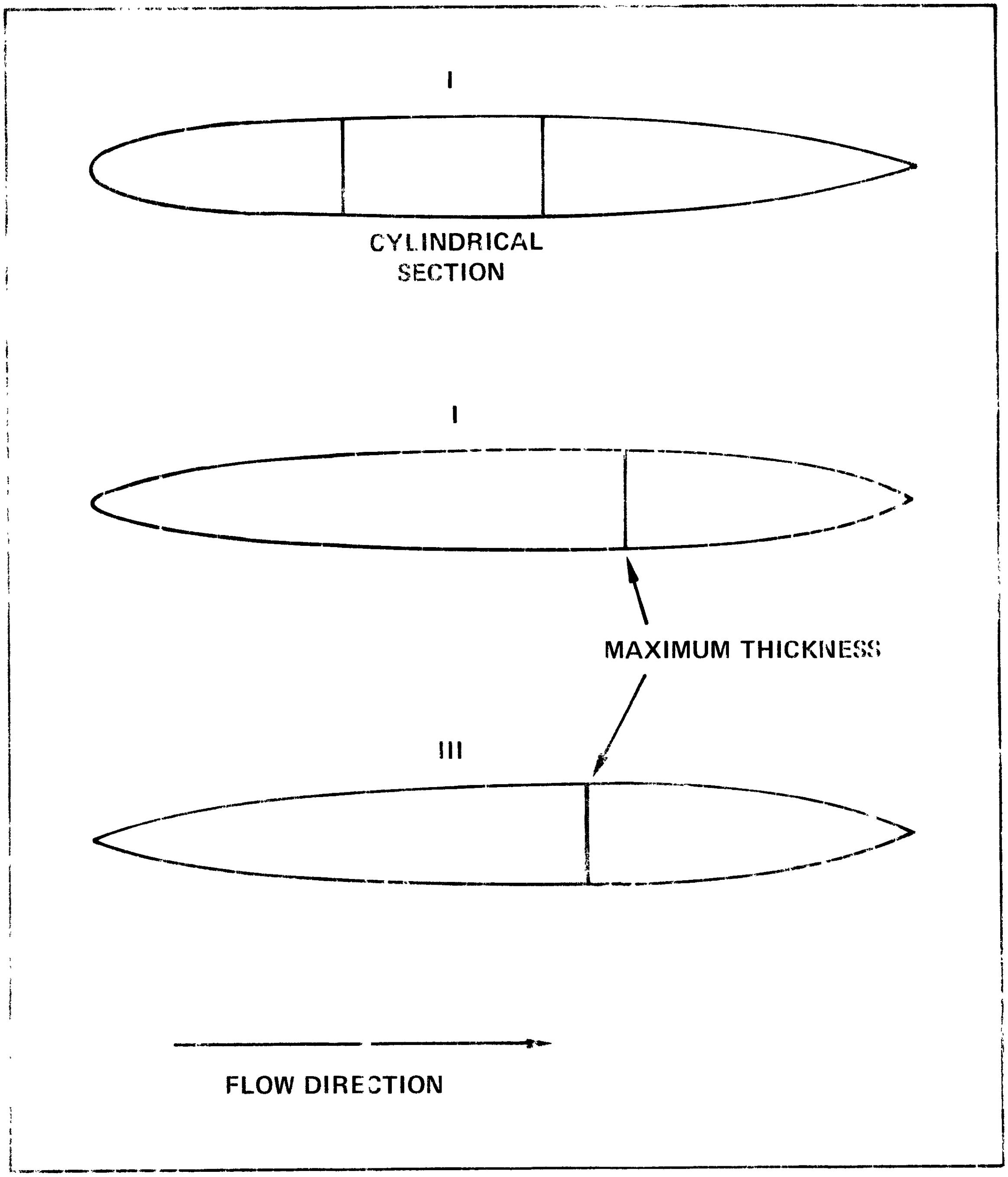

Figure 8 - Various Shapes Considered in this Study. Volume and Length are the Same for all Three Bodies. Nose Radius Scaled by the Body Length for I: 0.011, II: 0.005, III: 0.

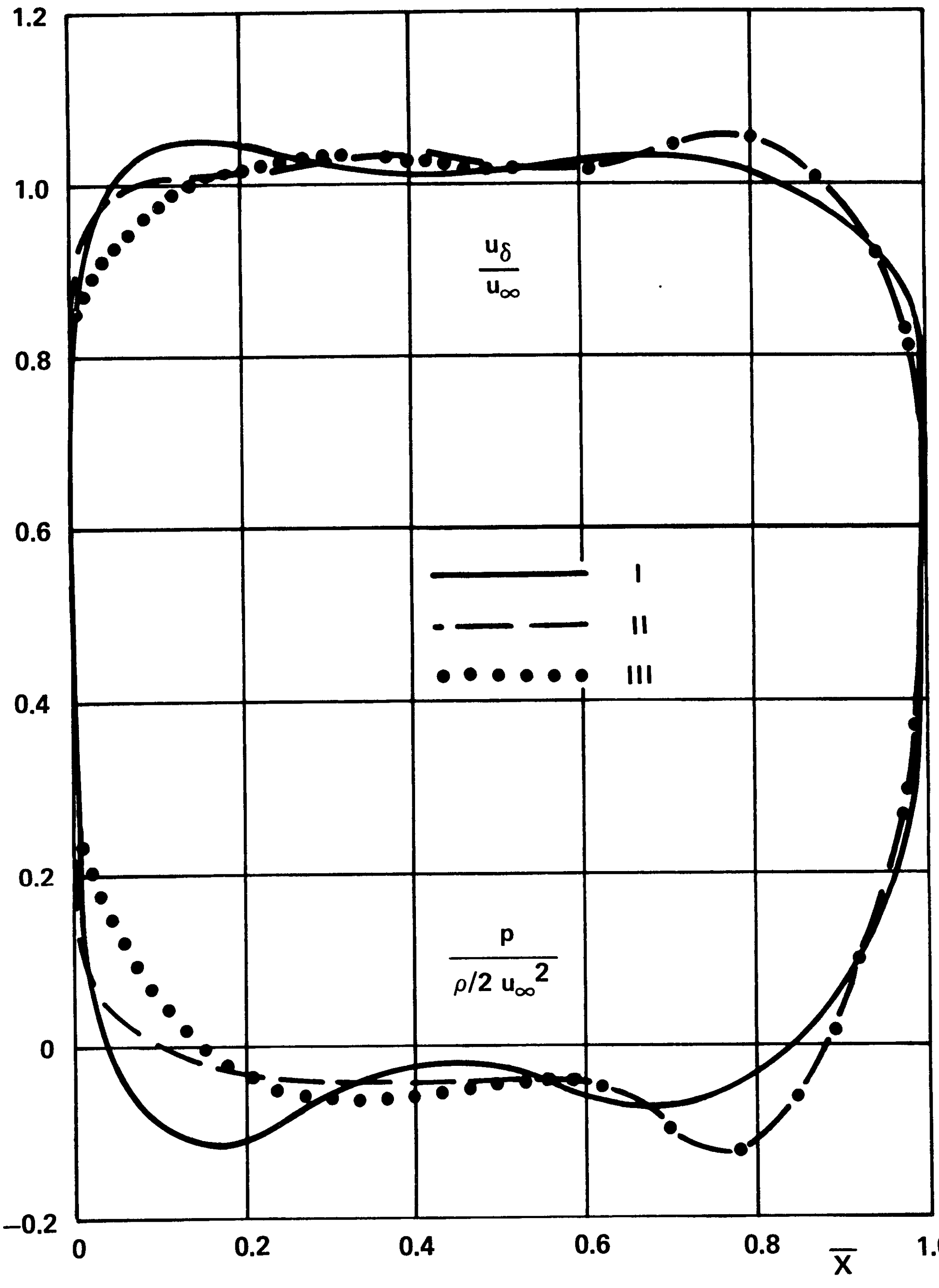

Figure 9 - Surface Velocities and Surface Pressures of the Potential Flow Around the Three Bodies I, II, and III of Figure 8.

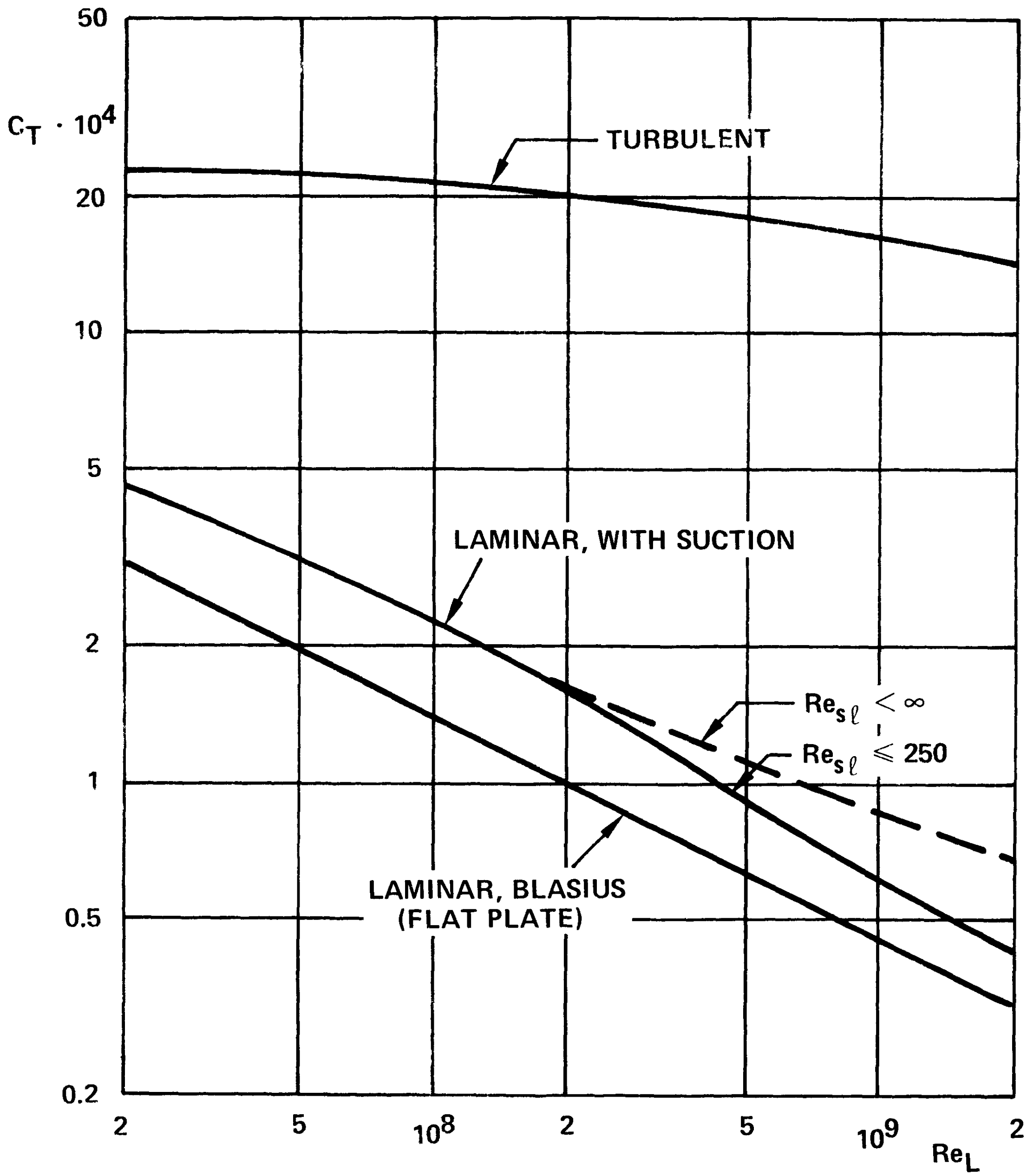

Figure 10 - Total Drag Coefficient C_T Plotted Against Re_L with $Re_{sl} \leq 250$ and $Re_{sl} < \infty$ for Shape I.

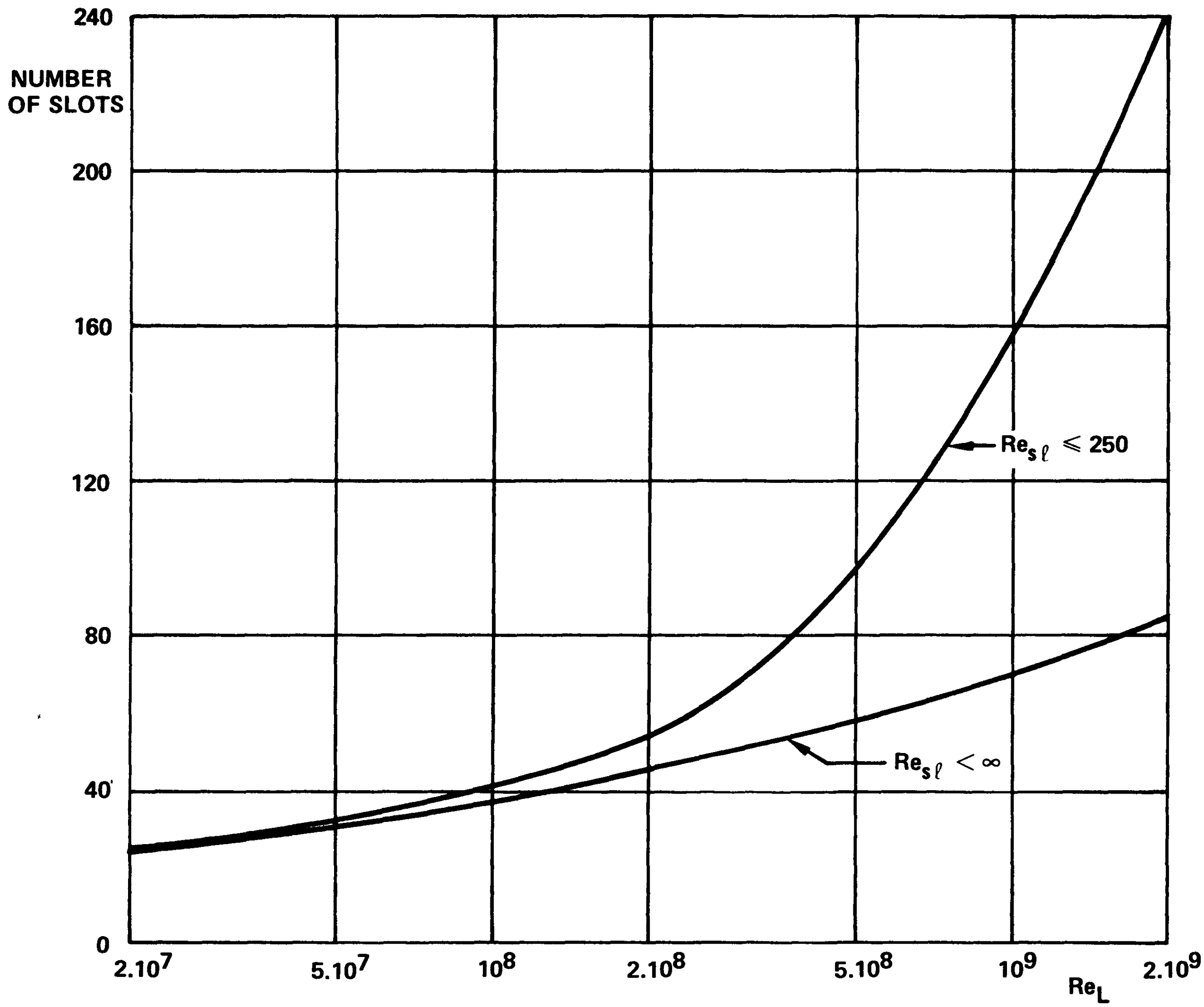

Figure 11 - Number of Slots Plotted Against Re_L for $Re_{sl} \leq 250$ and $Re_{sl} < \infty$ for Shape I.

plotted against Re_L for the cases (a) and (b). At $Re_L = 2 \cdot 10^8$ the numbers are 47 and 53, respectively. However, the suction-drag coefficient C_Q, which is a measure of the rate of suction, diminishes with increasing number of slots. Figure 12 shows C_Q as a function of Re_L for (a) and (b). The positions of the slots are displayed in Figure 3 for the special case $Re_L = 5 \cdot 10^7$, $Re_{sl} < \infty$, $\eta_Q = 0.1$. In case (a), for which $\eta_Q = 0.1$ everywhere, the slot-Reynolds number increases rapidly with $\bar{x}'$ (Figures 13 and 14). If one enforces $Re_{sl} \leq 250$ as in case (b), the local rate of suction η_Q must be reduced with $\bar{x}'$.

In practical applications of BLS the number and position of slots cannot be changed and must be kept fixed under various operating conditions. To study the influence of fixed slots for different Re_L, the position of the slots for $Re_L = 2 \cdot 10^8$ is chosen, and then Re_L is lowered. The resulting C_T values do not show deviations visible in the curves of Figure 10. However, it is believed that optimization of the η_Q-distribution would decrease the drag below the curves in Figure 10 for $Re_L < 2 \cdot 10^8$.

The influence of various η_Q for $Re_{sl} < \infty$ and of various η_Q at the first slot for $Re_{sl} \leq 250$ shows the same tendency as that found by Thiede[8]. With decreasing η_Q the drag coefficient diminishes and approaches the optimal situation of continuous suction. However, the number of slots increases rapidly.

The three shapes of Figure 8 were selected to investigate the influence of body contours on drag and suction. In the region $2 \cdot 10^8 \leq Re_L \leq 2 \cdot 10^9$ the values for C_T and C_Q are almost the same. For instance, at $Re_L = 2 \cdot 10^8$, $Re_{sl} < \infty$, $\eta_Q = 0.1$ the total drag coefficient is 0.000168 for I, 0.000169 for II, and 0.000172 for III. However, the number of slots at $Re_L = 2 \cdot 10^8$, $Re_{sl} < \infty$, $\eta_Q = 0.1$ is 47 for I, 54 for II, and 66 for III. At $Re_L = 2 \cdot 10^9$ the corresponding values are 86, 89, and 101. Hence, the bluntest of the three bodies requires the least number of slots.

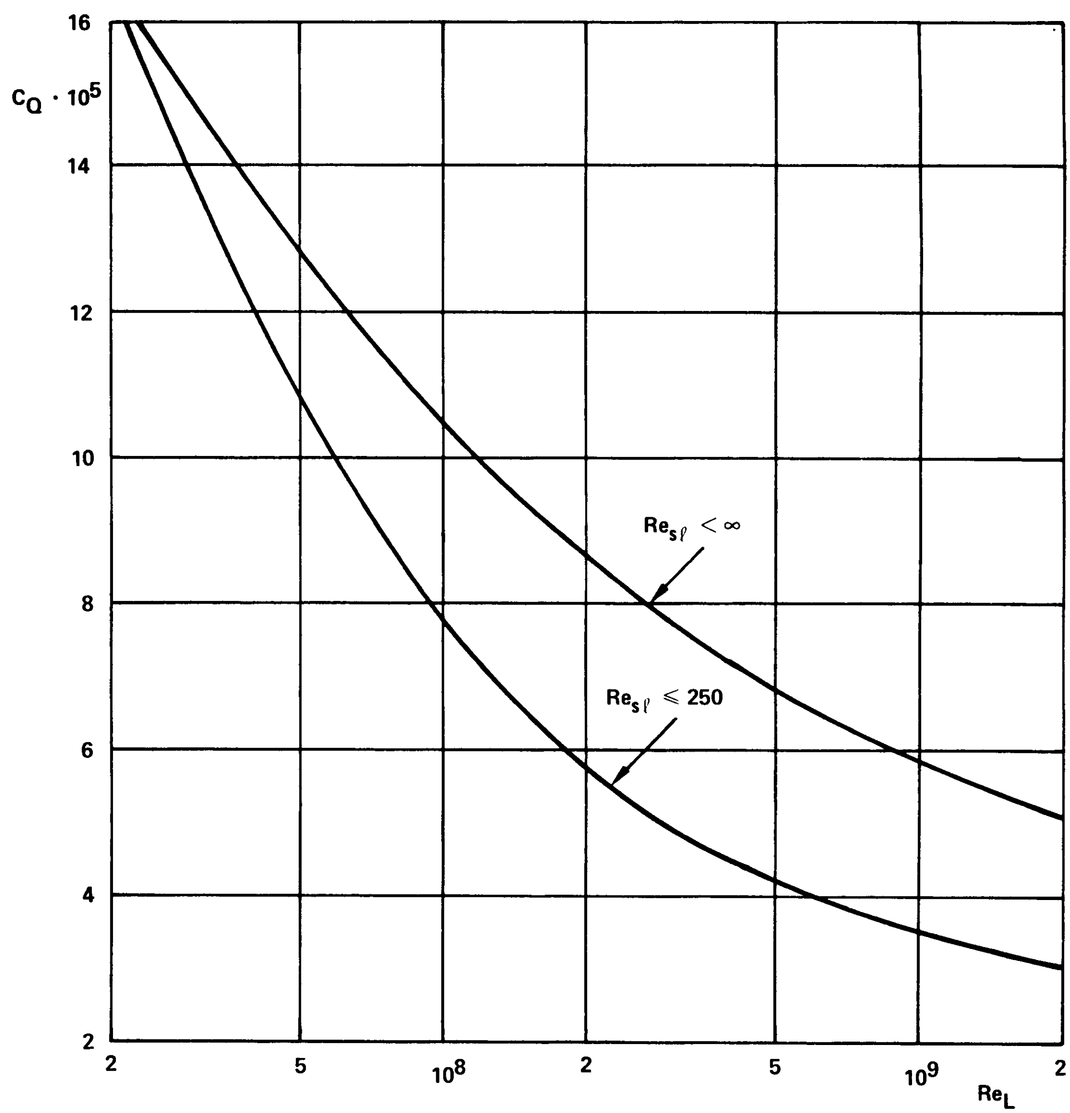

Figure 12 - Suction-Drag Coefficient C_Q Plotted Against Re_L for $Re_{sl} \leq 250$ and $Re_{sl} < \infty$ for Shape I.

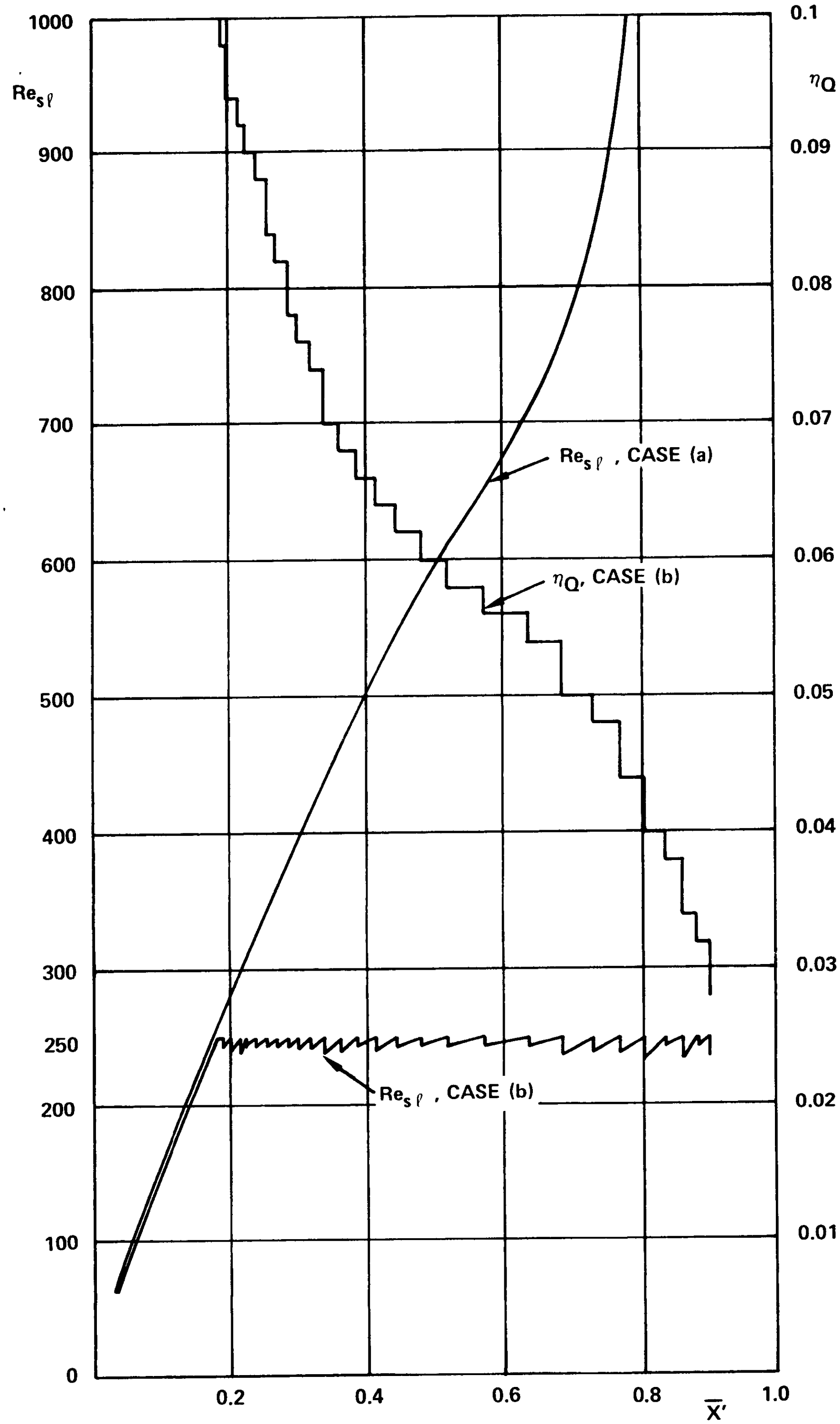

Figure 13 - Slot-Reynolds Number Re_{sl} and η_Q Plotted Against $\overline{x}'$ for $Re_L = 2 \cdot 10^8$ with (a) $Re_{sl} < \infty$, $\eta_Q = 0.1$ and (b) $Re_{sl} \leq 250$, $\eta_Q = 0.1$ at First Slot, $\Delta\eta_Q = 0.002$ for Shape I.

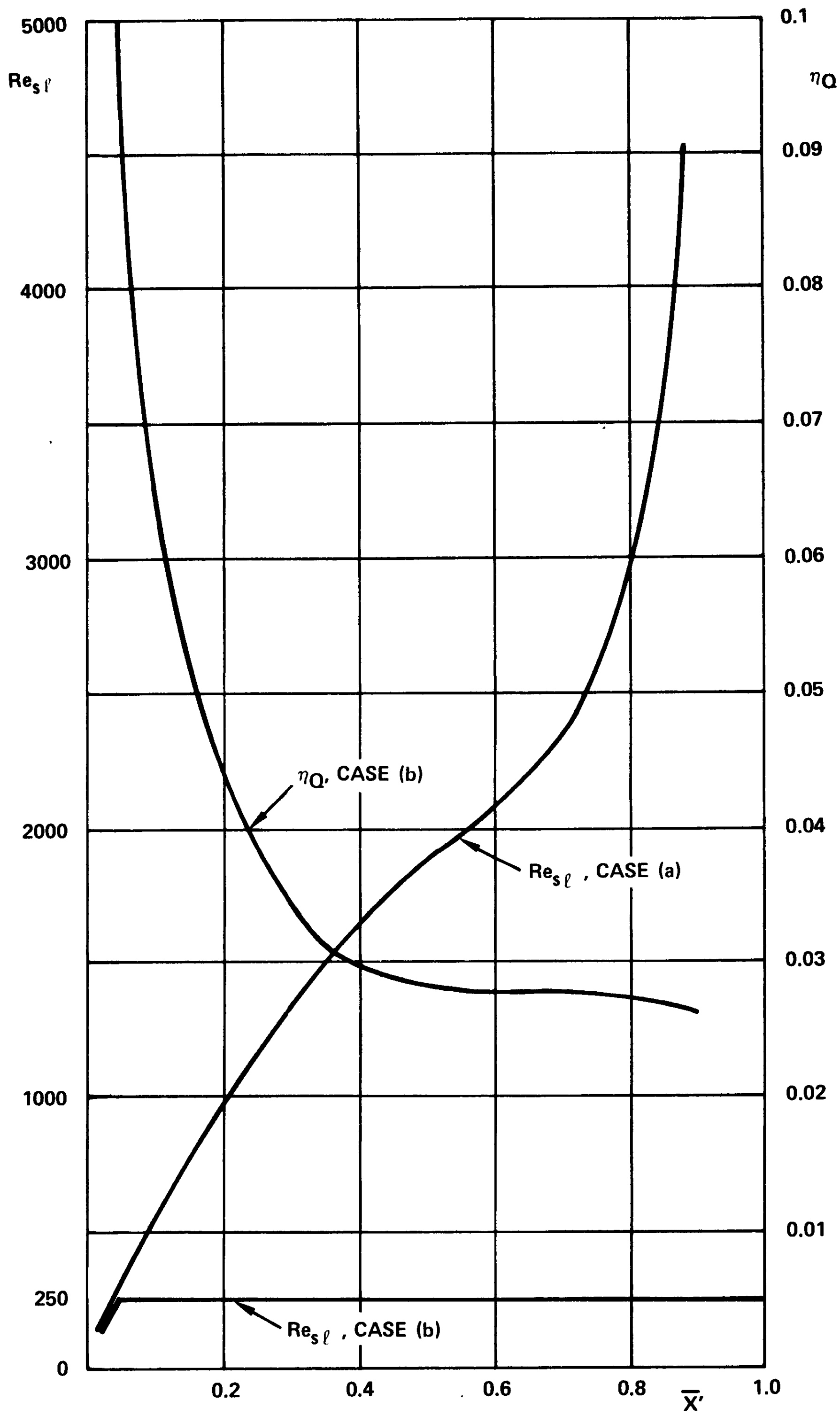

Figure 14 - Slot-Reynolds Number Re_{sl} and η_Q Plotted Against $\overline{x}'$ for $Re_L = 1.65 \cdot 10^9$ with (a) $Re_{sl} < \infty$, $\eta_Q = 0.1$ and (b) $Re_{sl} \leq 250$, $\eta_Q = 0.1$ at First Slot, $\Delta\eta_Q = 0.0002$. Shape I.

The fact that the various body shapes have little effect in changing drag and rate of suction is not surprising. Parsons and Goodson[12] found by means of an optimization process that the extent of drag reduction depends on how far the transition from laminar to turbulent motion can be delayed. The optimum shape is the "Dolphin-Griffith" type, and the body becomes fatter with increasing Re_L. Since BLS delays the transition to the stern of the body, the drag is insensitive to changes in the contour.

2. PARTIAL LAMINARIZATION

If one accepts Pfenninger's argument that the upper length Reynolds number for which BLS works is $2 \cdot 10^8$ (according to the present state of knowledge), then full laminarization cannot be achieved for Naval vehicles with Re_L beyond $2 \cdot 10^8$. This raises the question of how effective partial laminarization by means of BLS would be.

The last sentence implies that the upper limit for Re_L can be used to determine B in the postulate (20) which enables one to compute partial laminarization. The value of B = 17800 obtained from $Re_L = 2 \cdot 10^8$ can only be approximate. It suffices here since the limitation of B is only a safety measure and does not affect the validity of the calculation.

If one increases Re_L beyond $2 \cdot 10^8$, full laminarization cannot be achieved since $Re_{\delta_{2J}} = 17800$ is reached at a point $\bar{x}'_r < \bar{x}'_E$. The location $\bar{x}' = \bar{x}'_r$ marks the onset of turbulence which is obtained at the point $\bar{x}'_T$ following $\bar{x}'_r$. This point divides the region for which laminarization due to BLS is obtained from the region of turbulence. With increasing Re_L

12 Parsons, J.S. and Goodson, R.E., "The Optimum Shaping of Axisymmetric Bodies for Minimum Drag in Incompressible Flow," Purdue University, Lafayette, Indiana, Automatic Control Center Report ACC-72-5, June 1972.

the point $\bar{x}_r^{\prime}$ (and thus the next $\bar{x}_T^{\prime}$ behind $\bar{x}_r^{\prime}$) migrates toward the bow.

In Figure 15 the drag coefficient C_T is plotted against Re_L by considering partial laminarization from $Re_L = 2 \cdot 10^8$ on for $Re_{sl} \leq 250$. The drag reduction is smaller, of course, for partial laminarization than for full laminarization. However, it is important to note that the gain in drag reduction is still substantial. For $Re_L = 1.2 \cdot 10^9$ the reduction would be 1:5.5!

The length of the body $\bar{x}_{r-1}^{\prime}$ for which slots must be considered ($\bar{x}_{r-1}^{\prime}$ is the position of the last slot) is plotted versus Re_L in Figure 16. The number of slots is 66 for $Re_L = 5 \cdot 10^8$, 103 for $Re_L = 1.1 \cdot 10^9$, and 179 for $Re_L = 2 \cdot 10^9$. If one places the last slot at $\bar{x}_{r-1}^{\prime}$ without regard to the limitation of B, one obtains curves for C_T as displayed in Figures 17 and 18.

CONCLUSIONS

One of the main obstacles for the engineering application of BLS is the existence of a restricting upper limit for the length Reynolds number due to imperfections of the body surface. This difficulty can be circumvented by partial laminarization. The drag reduction in the range $2 \cdot 10^8 \leq Re_L \leq 2 \cdot 10^9$ is still high enough to compete with present-day drag-reduction techniques. Another problem for the engineering application of BLS is the influence of slot disturbances on the stability of the surface-boundary layer. This problem, too, can be solved by restricting the slot Reynolds number to values below a certain level. With this limitation the dominance of viscous effects can be assured. However, this procedure must be paid for with an increased number of slots which, on the other hand, will have an additional drag-reducing effect.

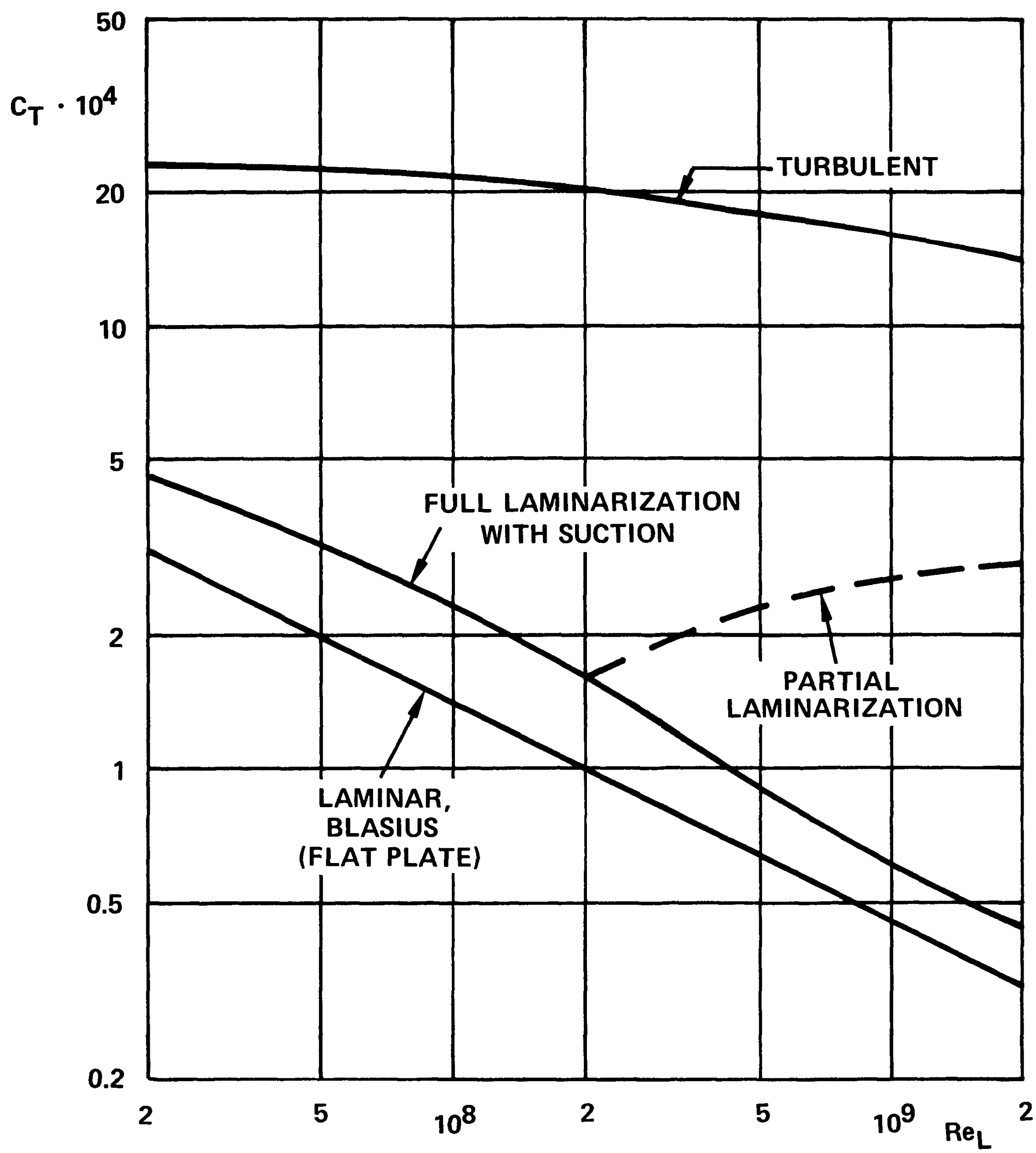

Figure 15 - Partial and Full Laminarization with BLS. C_T is Plotted Against Re_L for $Re_{s1} \leq 250$, $Re_{\delta_{2J}} \leq 17800$. Shape I.

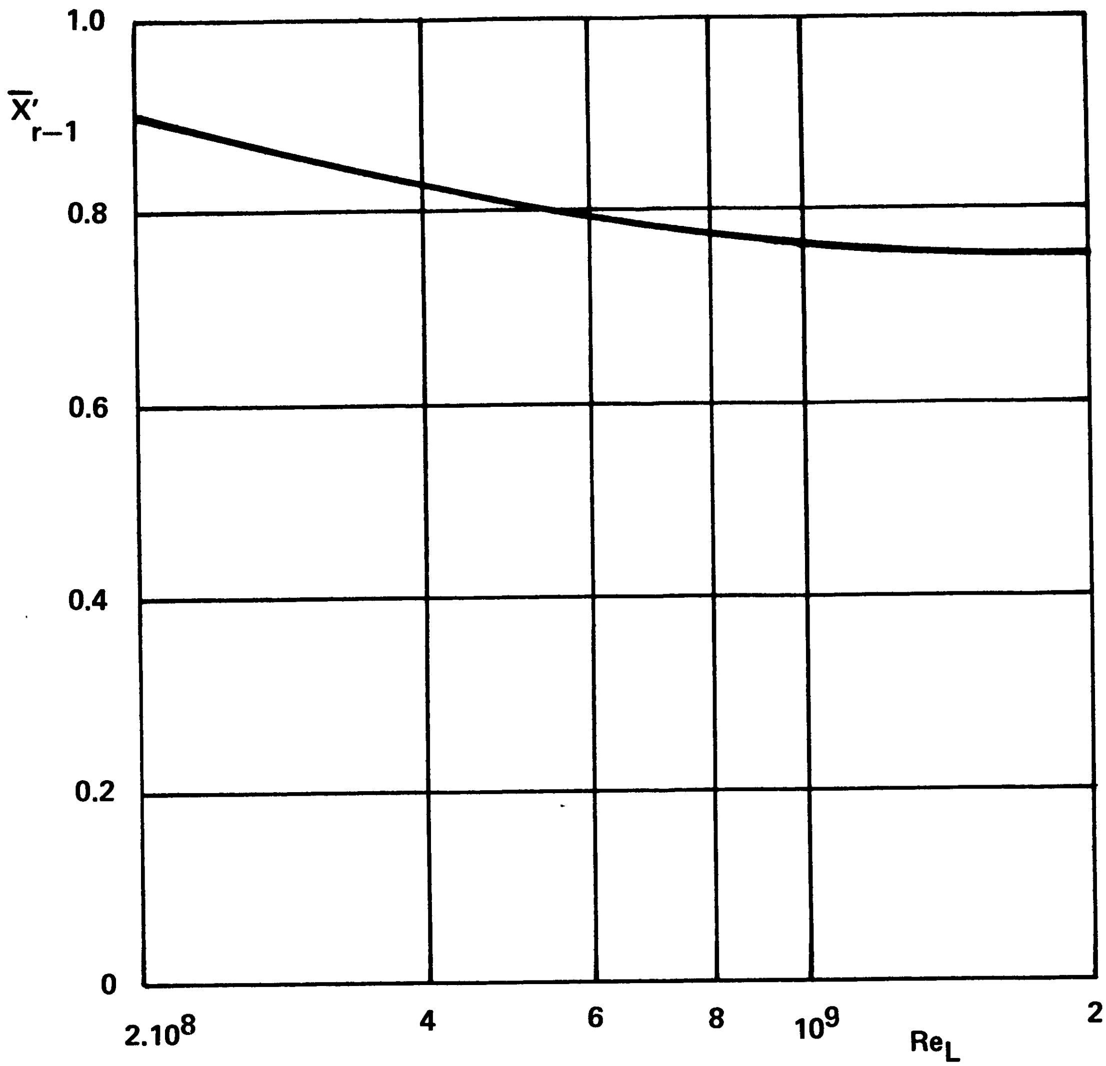

Figure 16 - Partial Laminarization. Position of Last Slot $\bar{x}'_{r-1}$ Plotted Against Re_L for $Re_{sl} \leq 250$, $Re_{\delta_{2J}} \leq 17800$. Shape I.

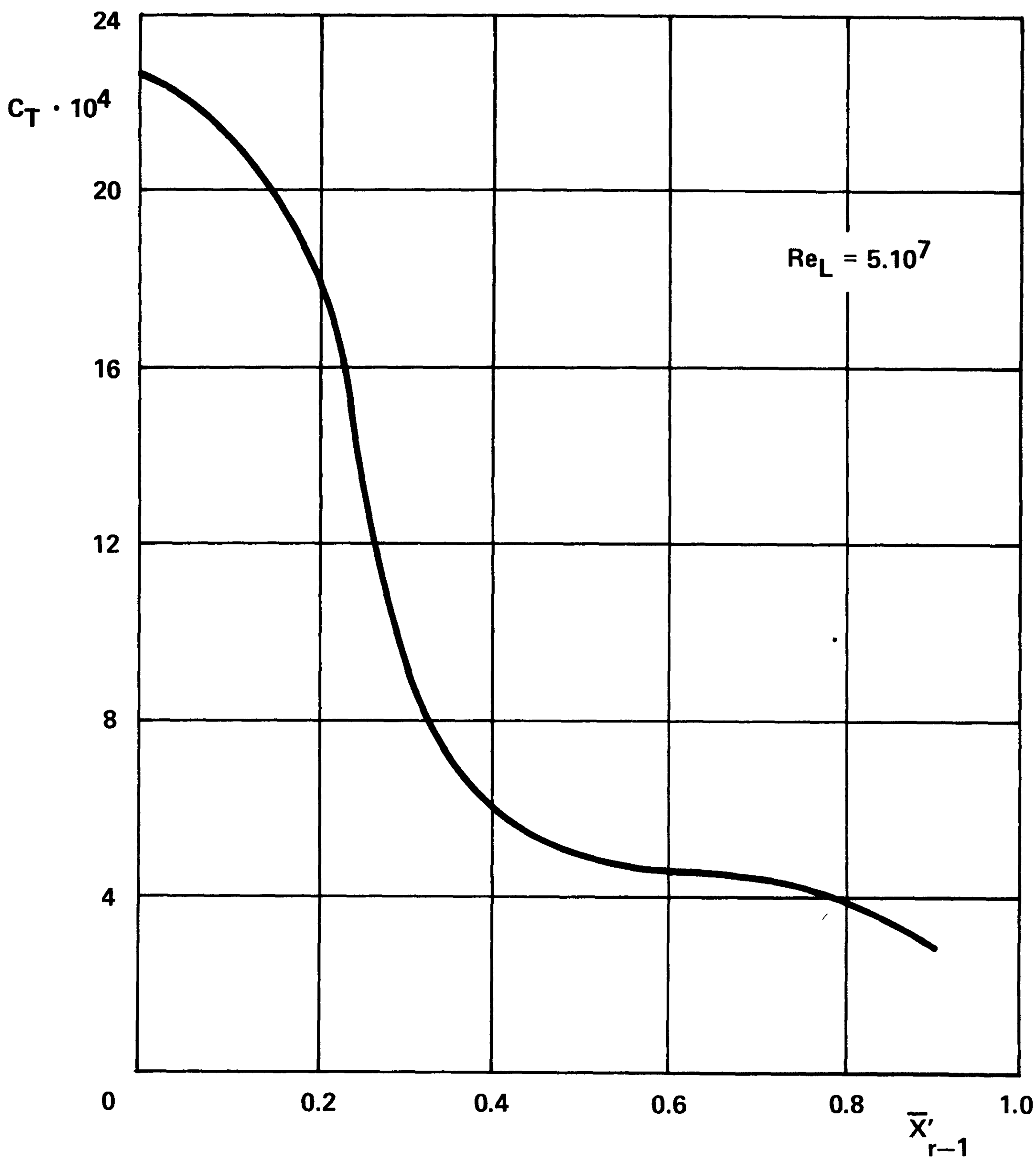

Figure 17 - Partial Laminarization. C_T is Plotted Against Position of Last Slot $\bar{x}'_{r-1}$ for $Re_L = 5 \cdot 10^7$, $Re_{s1} < \infty$, $\eta_Q = 0.1$ Shape I.

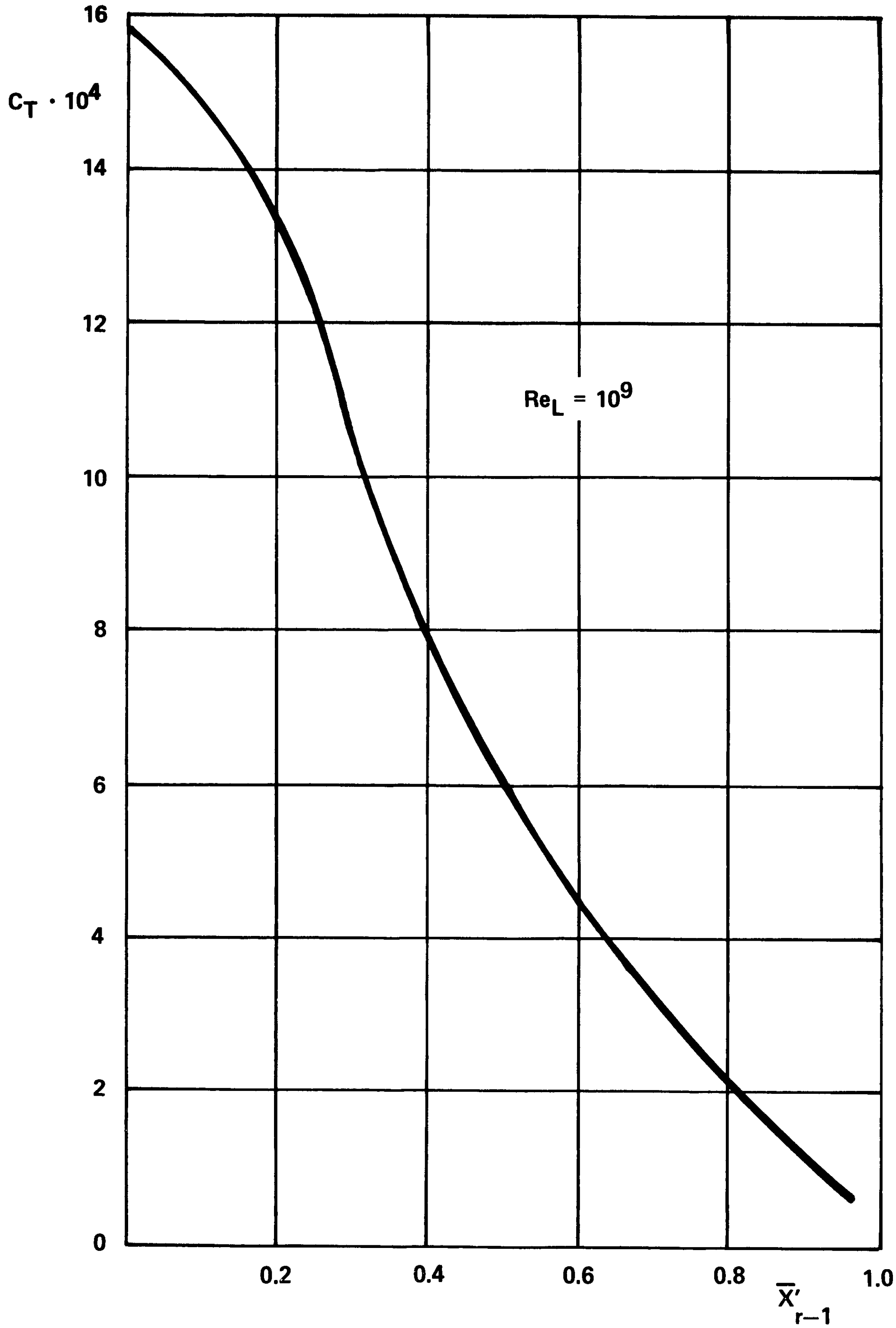

Figure 18 - Partial Laminarization. C_T is Plotted Against Position of Last Slot $\overline{x}'_{r-1}$ for $Re_L = 10^9$, $Re_{s1} \leq 250$, $\eta_Q = 0.1$ at First Slot, $\Delta\eta_Q = 0.002$. Shape I.

ACKNOWLEDGMENTS

The authors would like to thank Professor A. Walz, Technical University Berlin, for making available Thiede's PhD-thesis before publication and for many stimulating discussions. The authors are indebted to Dr. W. Pfenninger for valuable information and comments. Mr. S. Ohring's help in setting up the computer code is also acknowledged.

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1 ORIGINATING ACTIVITY (Corporate author) Naval Ship R&D Center Bethesda, Maryland 20034		2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP	
3 REPORT TITLE Boundary-Layer Suction with Slots on Axisymmetric Bodies			
4 DESCRIPTIVE NOTES (Type of report and inclusive dates)			
5. AUTHOR(S) (First name, middle initial, last name) Hans J. Lugt Sin K. Oh			
6. REPORT DATE November 1972		7a. TOTAL NO. OF PAGES 51	7b. NO. OF REFS 12
8a. CONTRACT OR GRANT NO b. PROJECT NO. c. ZR 014-02-01 d.		9a. ORIGINATOR'S REPORT NUMBER(S) 4038 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
10. DISTRIBUTION STATEMENT Distribution limited to U.S. Gov't. agencies only; test and evaluation; November 1972. Other requests for this document must be referred to Head, Computation and Mathematics Department.			
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13 ABSTRACT

The engineering exploitation of boundary-layer suction with slots for drag reduction is controversial. Two major obstacles appear to exist: (a) clogging and fouling of the slots and the body surface, (b) limitation in the order of magnitude of the length Reynolds number. Extensive studies will be needed to resolve the first problem. The present paper shows that the second problem can be circumvented by partial laminarization. For high Reynolds numbers, when full laminarization cannot be achieved, partial laminarization also yields substantial drag reduction and reduces acoustic noise. The computation is based on an integral method for solving the boundary-layer equations of an incompressible fluid around an axisymmetric body. Laminar as well as turbulent flow is considered. Suction is limited by an upper slot Reynolds number which prevents disturbances from the slots.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Drag reduction						
Boundary-layer suction						
Slots						
Partial laminarization						