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## DRAG REDUCTION OF FLAT PLATES WITH SLOT EJECTION OF POLYMER SOLUTION

by

Paul S. Granville

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Naval Ship Research and Development Center  
Washington, D.C. 20007

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## NOTATION

A	Slope of logarithmic velocity law in natural logarithms
$B_1$	Law-of-the-wall factor
$B_2$	Law-of-the-wake factor
$B_{1,0}$	Value of $B_1$ for smooth surfaces in ordinary fluids
$\Delta B$	Polymer characterization as drag-reducing agent
C	Concentration of polymer
$C_{ej}$	Ejected concentration of polymer
$C_F$	Drag coefficient
$C_u$	Concentration of polymer for uniform conditions
$C_w$	Wall concentration of polymer
D	Drag
$D_1, D_2$	Velocity profile factors, Equation [28]
E	Velocity profile-concentration profile factor, Equation [10]
e	Base of natural logarithms
F	Velocity-defect law, Equation [3]
g	Concentration profile, Equation [7]
i	Subscript for initial conditions
$\ell$	Characteristic length of polymer molecule
$\ell^*$	Nondimensional characteristic length, Equation [14]
$\dot{m}$	Mass flux of polymer from slot
$Q_{ej}$	Volume flux of solution from slot
$R_x$	Reynolds number, $R_x = Ux/\nu_0$
$R_\theta$	Reynolds number, $R_\theta = U\theta/\nu_0$
$\Delta R_x$	Change in $R_x$
U	Velocity outside boundary layer
u	Tangential velocity of flow in boundary layer
$u_\tau$	Shear velocity, $u_\tau = \sqrt{\tau_w/\rho}$
x	Streamwise distance
y	Normal distance from wall
$\gamma$	Average concentration across boundary layer, Equation [9]
$\delta$	Boundary-layer thickness
$\eta$	Boundary-layer Reynolds number, $\eta = u_\tau \delta/\nu_0$

$\theta$	Momentum thickness
$\nu_0$	Kinematic viscosity of solvent
$\rho$	Density of solution
$\sigma$	Local drag parameter, $\sigma = U/u_\tau$
$\tau_w$	Wall shear stress

## ABSTRACT

The drag reduction due to emission of polymer from a slot is analytically treated for the fourth stage for which the concentration boundary layer coincides with the momentum boundary layer.

## ADMINISTRATIVE INFORMATION

This work was funded by the Naval Ordnance Systems Command under Subproject UR109 01 03.

## INTRODUCTION

The problem of analytically predicting the drag reduction of a flat plate moving in a uniform concentration of polymer solution has been previously treated<sup>1</sup> on the basis of the similarity laws of turbulent boundary layers. A more likely practical application is to have the polymer solution ejected in concentrated form from a slot for diffusion into the turbulent boundary layer downstream. A concentration boundary layer with a nonuniform profile develops which grows in thickness downstream until it coincides with the regular or momentum boundary layer of the flat plate.

An experimental study by Poreh and Cermak<sup>2</sup> of the diffusion of ammonia gas from a slot into the turbulent boundary layer of an air flow indicates four stages of development of the concentration boundary layer in the downstream direction:

1. A very slow growth by molecular diffusion through the laminar sublayer on the wall.
2. A very rapid growth due to the high turbulence intensity near the wall.
3. A somewhat slower growth due to the lesser turbulence intensity in the outer region of the boundary layer.
4. The concentration boundary layer coincides with the flat-plate boundary layer.

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<sup>1</sup>References are listed on page 13.

The situation with respect to the ejection of polymer solution is somewhat different since the presence of the polymer modifies the velocity profile and turbulence structure of the original boundary layer. It is to be expected that the adjustment of the boundary layer to the presence of the polymer will be completed by the time the fourth stage is reached. Then the similarity laws are assumed to hold. The effective concentration is also assumed to be that at the wall since the drag-reduction effects operate directly only on the similarity law of the wall.

An analytical study is performed on the drag reduction occurring in the fourth stage on the basis of the similarity laws incorporating the polymer effects obtained from tests with uniform concentration. A similar concentration profile is assumed to hold throughout the fourth stage. The dilution arising from the thickening of the boundary layer downstream alters the average concentration and hence the drag reduction. Analytical expressions are developed to calculate the resulting momentum changes and hence drag reduction in the fourth stage.

#### DILUTION OF POLYMER CONCENTRATION

If  $\dot{m}$  is the mass flux of polymer from a slot of unit breadth, then

$$\dot{m} = \rho C_{ej} Q_{ej} \quad [1]$$

where  $C_{ej}$  is the mass concentration of polymer in the ejected solution  
 $\rho$  is the density of the solution, and  $Q_{ej}$  is the volume flux of solution from a slot of unit breadth.

It is evident that from considerations of conservation of mass

$$\dot{m} = \rho \int_0^{\delta} u C dy \quad [2]$$

at every station downstream of the slot.

Here  $u$  is the streamwise component of velocity in the boundary layer,

$y$  is the normal distance from the wall,

$\delta$  is the boundary layer thickness, and

$C$  is the concentration of polymer at any point in the boundary layer.



Fabula and Burns<sup>3</sup> show that Equation [2] holds to a sufficiently close degree for turbulent flow where  $u$  is now the temporal average of the fluctuating velocity and  $C$  is the temporal average of the fluctuating concentration.

The velocity profile  $u[y]$  is obtained from the outer similarity law or velocity-defect law which is considered in terms of momentum changes to hold also to the wall

$$\frac{U-u}{u_\tau} = F\left[\frac{y}{\delta}\right] \quad [3]$$

where  $U$  is the velocity outside the boundary layer, a constant for flat plates (with zero pressure gradient) and  $u_\tau$  is the shear velocity defined as

$$u_\tau \equiv \sqrt{\frac{\tau_w}{\rho}} \quad [4]$$

where  $\tau_w$  is the wall shear stress.

Then

$$u = U\left(1 - \frac{F}{\sigma}\right) \quad [5]$$

where  $\sigma$  is a local skin friction factor defined as

$$\sigma \equiv \frac{U}{u_\tau} \quad [6]$$

In the subsequent analysis,  $\sigma$  is to be used as the principal streamwise parameter instead of  $x$ , the streamwise coordinate.

For the fourth stage where the concentration boundary layer coincides with the momentum boundary layer, tests<sup>2,4</sup> indicate that the concentration profile has similarity in the streamwise direction.

$$\frac{C}{C_w} = g\left[\frac{y}{\delta}\right] \quad [7]$$

where  $C$  is the mass concentration of polymer at any position within the boundary layer and  $C_w$  is the value of  $C$  at the wall,  $y = 0$ .

Substitution of Equations [5] and [7] into [2] produces

$$C_w = \frac{\dot{m}}{\rho U \delta \gamma (1-E/\sigma)} \quad [8]$$

where

$$\gamma \equiv \int_0^1 g d\left(\frac{y}{\delta}\right) \quad [9]$$

the average concentration (normalized) across the boundary layer, a constant, and

$$E \equiv \frac{\int_0^1 F g d\left(\frac{y}{\delta}\right)}{\gamma} \quad [10]$$

a constant.

Equation [8] is equivalent to Equation [8] of Reference 3;  $\sigma$  is given by Equation [9] of Reference 1 as

$$\sigma = A \ln \eta + B_1 + B_2 \quad [11]$$

where

$$\eta \equiv u_\tau \delta / \nu_0 \quad [12]$$

or

$$\delta = \frac{\sigma \nu_0}{U} e^{\frac{1}{A} (\sigma - B_1 - B_2)} \quad [13]$$

where

$\nu_0$  is the kinematic viscosity of the solvent;

$A$  is a constant, the reciprocal of von Kármán's constant;

$B_2$  is the law-of-the-wake factor which is constant for flat plates with zero pressure gradient; and

$B_1$  is the law-of-the-wall factor which for uniform concentration  $C_u$  of a particular polymer in a particular solvent  $B_1 = f[\ell^*, C_u]$ .

$$\ell^* \equiv u_\tau \ell / \nu_0 \quad [14]$$

where  $\ell$  is the length parameter of the polymer. It is assumed that the  $B_1$  characterization obtained empirically for uniform concentration  $C_u$  holds for  $C_w$ .

From Equations [8] and [13]

$$C_w = \left( \frac{\dot{m}}{\gamma \rho v_o} \right) \frac{1}{(\sigma-E) e^{\frac{1}{A} (\sigma-B_1-B_2)}} \quad [15]$$

#### DRAG-REDUCTION CHARACTERIZATION

The  $B_1$  characterization for a polymer solution may be restated as

$$B_1 = B_{1,0} + \Delta B[\ell^*, C_u] \quad [16]$$

where

$B_{1,0} = B_1$  for ordinary fluids, a constant for smooth surfaces. For Stage 4 of the injected polymer, it is assumed  $C_w = C_u$  or

$$B_1 = B_{1,0} + \Delta B[\ell^*, C_w] \quad [17]$$

Now Equation [15] may be rewritten

$$C_w e^{-\frac{\Delta B}{A}} = \left( \frac{\dot{m} e^{\frac{B_{1,0}+B_2}{A}}}{\gamma \rho v_o} \right) \frac{1}{(\sigma-E) e^{\sigma/A}} \quad [18]$$

where the only variable on the right-hand side is  $\sigma$  for a particular flow situation. Also since

$$\ell^* \equiv \frac{u_\tau \ell}{v_o} = \left( \frac{U\ell}{v_o} \right) \frac{1}{\sigma} \quad [19]$$

and  $\frac{U\ell}{v_o}$  is a constant for a particular flow situation,  $\ell^*$  is also only a function of  $\sigma$ .

Hence it is more convenient to restate  $\Delta B$  of Equation [17] as

$$\Delta B = f \left[ \ell^*, C_w e^{-\frac{\Delta B}{A}} \right] \quad [20]$$

Then for a particular flow situation,  $\Delta B$  is a function only of  $\sigma$ .

Also from Equations [8] and [12]

$$\eta = \left( \frac{\dot{m}}{\gamma \rho v_o} \right) \frac{1}{(\sigma-E) C_w} \quad [21]$$

where  $C_w$  is determined from  $C_w e^{-\frac{\Delta B}{A}}$  and hence is also only a function of  $\sigma$  for a particular flow situation.

The drag-reduction characterization chart  $\Delta B[l, C_w^*]$  for a particular polymer solution is converted to  $C_w e^{-\frac{\Delta B}{A}} [l, C_w^*]$  for convenience of calculation. See Figure 1 for Guar Gum.

#### DRAG COEFFICIENT

The drag coefficient  $C_F$  is defined for a unit breadth of flat plate as

$$C_F \equiv \frac{D}{\frac{1}{2} \rho U^2 x} \quad [22]$$

where  $D$  is the frictional resistance or drag of one side and  $x$  is the length of the plate.

The Reynolds number  $R_x$  is defined as

$$R_x \equiv Ux/v_o \quad [23]$$

From momentum considerations

$$C_F = 2 R_\theta / R_x \quad [24]$$

where

$$R_\theta \equiv U \theta / v_o \quad [25]$$

and  $\theta$  is the momentum thickness,  $\theta \equiv \int_0^\delta \left( 1 - \frac{u}{U} \right) \frac{u}{U} dy$ .

From Equation [59] of Reference 1

$$R_x - R_{x,i} = \int_{R_{\theta,i}}^{R_\theta} \sigma^2 d R_\theta \quad [26]$$

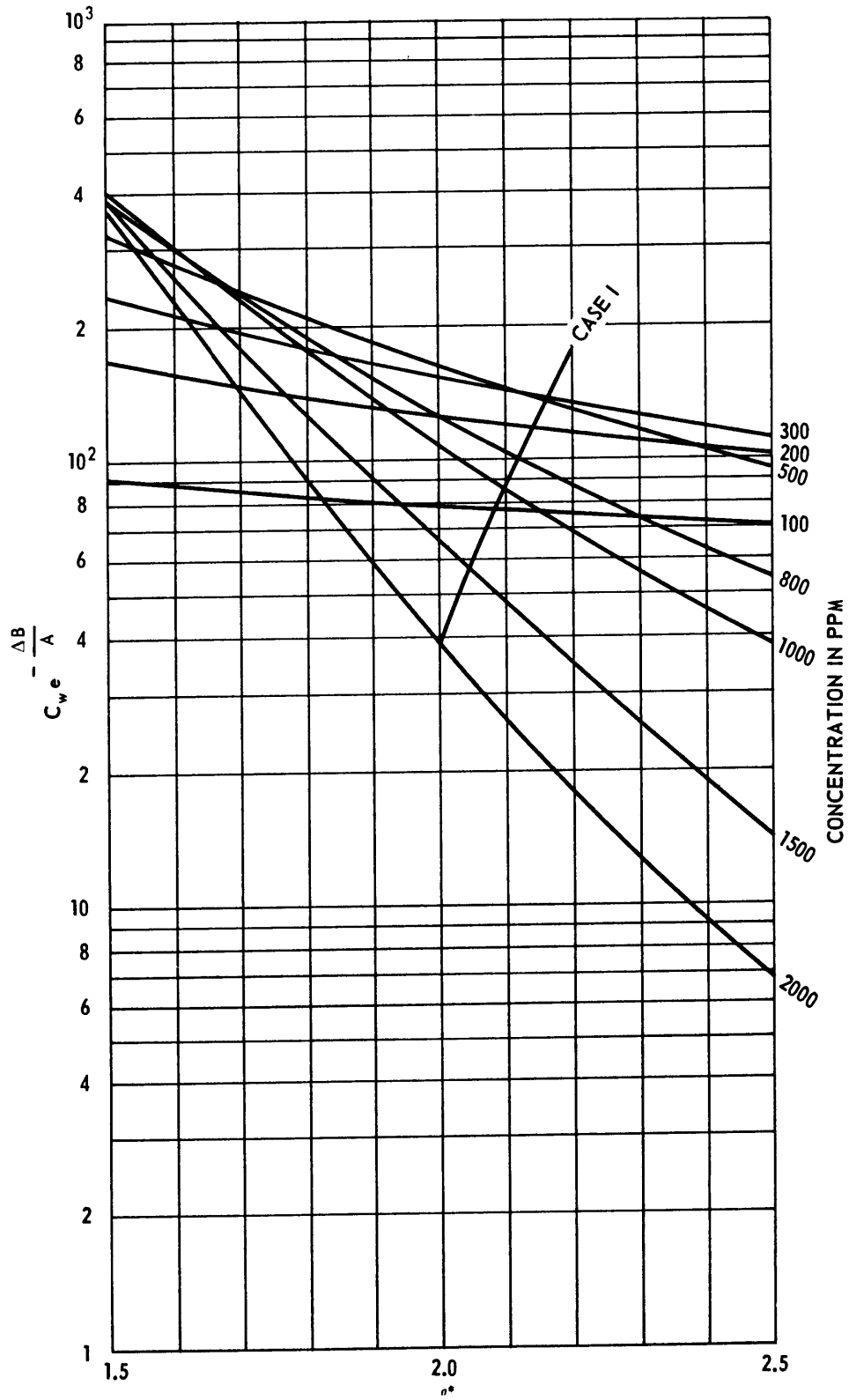


Figure 1 - Drag-Reduction Characterization of Guar Gum

or

$$R_x - R_{x,i} = \sigma^2 R_\theta - \sigma_i^2 R_{\theta,i} - 2 \int_{\sigma_i}^{\sigma} R_\theta \sigma \, d\sigma \quad [27]$$

where the subscript  $i$  refers to the initial point of integration which in the present analysis is the start of the fourth stage.

From Equation [52] of Reference 1, at higher Reynolds numbers

$$R_\theta = \eta(D_1 - D_2/\sigma) \quad [28]$$

where  $D_1$  and  $D_2$  are constants.

Then

$$\begin{aligned} \Delta R_x = R_x - R_{x,i} &= \eta(D_1 \sigma^2 - D_2 \sigma) - \eta_i(D_1 \sigma_i^2 - D_2 \sigma_i) \\ &- 2 \int_{\sigma_i}^{\sigma} \eta(D_1 \sigma - D_2) \, d\sigma \end{aligned} \quad [29]$$

#### EFFECT OF DILUTION ON LOCAL SKIN FRICTION

For ordinary fluids, the local skin function  $\tau_w$  decreases downstream with  $x$  as the boundary layer thickens,  $\frac{d\tau_w}{dx} < 0$  or  $\frac{d\sigma}{dx} > 0$ . However for the injected boundary layer with polymer in Stage 4, the increasing dilution with boundary layer growth reduces the concentration which may reduce the drag reduction and thus increase the local skin friction,  $\frac{d\tau_w}{dx} > 0$  or  $\frac{d\sigma}{dx} < 0$ . To investigate this,  $\frac{d\sigma}{dx}$  is determined as follows.

The von Kármán momentum equation<sup>1</sup> is stated as

$$\frac{d\theta}{dx} = \frac{\tau_w}{\rho U^2} = \frac{1}{\sigma^2} \quad [30]$$

Now at higher Reynolds numbers<sup>1</sup>

$$\theta = \delta \left( \frac{D_1}{\sigma} - \frac{D_2}{\sigma^2} \right) \quad [31]$$

Differentiating Equation [31] and utilizing Equation [30] produces

$$1 = \delta \left( -D_1 + \frac{2 D_2}{\sigma} \right) \frac{d\sigma}{dx} + (D_1 \sigma - D_2) \frac{d\delta}{dx} \quad [32]$$

Differentiating  $\sigma$  in Equation [13] produces

$$\frac{d\delta}{dx} = \frac{v_o \eta}{U} \left[ 1 + \frac{\sigma}{A} \left( 1 - \frac{d \Delta B}{d\sigma} \right) \right] \frac{d\sigma}{dx} \quad [33]$$

Eliminating  $\sigma$  by Equation [13] and  $\frac{d\delta}{dx}$  by Equation [33] converts Equation [32] to

$$\frac{d\sigma}{dx} = \left\{ \frac{v_o}{U} \eta \left[ D_2 + \left( \frac{D_1 \sigma}{A} - \frac{D_2}{A} \right) \sigma \left( 1 - \frac{d \Delta B}{d\sigma} \right) \right] \right\}^{-1} \quad [34]$$

For the condition  $\frac{d\sigma}{dx} = 0$ , examination of the terms of Equation [34] indicates that  $\frac{d \Delta B}{d\sigma} \rightarrow \infty$ .

Differentiating Equation [20] yields

$$\frac{d \Delta B}{d\sigma} = \left( \frac{\partial \Delta B}{\partial \ell^*} \right)_{C_w e^{-\frac{\Delta B}{A}}} \frac{d\ell^*}{d\sigma} + \left( \frac{\partial \Delta B}{\partial C_w e^{-\frac{\Delta B}{A}}} \right)_{\ell^*} \frac{d C_w e^{-\frac{\Delta B}{A}}}{d\sigma} \quad [35]$$

Since  $\ell^* = \left( \frac{U\ell}{v_o} \right) \frac{1}{\sigma}$

$$\frac{d\ell^*}{d\sigma} = -\frac{\ell^*}{\sigma} \quad [36]$$

Differentiating Equation [15] results in

$$\frac{d C_w e^{-\frac{\Delta B}{A}}}{d\sigma} = -C_w e^{-\frac{\Delta B}{A}} \left( \frac{1}{\sigma-E} + \frac{1}{A} \right) \quad [37]$$

Finally

$$\frac{d\Delta B}{d\sigma} = - \left( \frac{\partial \Delta B}{\partial \ell^*} \right)_{C_w} e^{-\frac{\Delta B}{A}} \frac{\ell^*}{\sigma} - \left( \frac{\partial \Delta B}{\partial C_w} \right)_{\ell^*} e^{-\frac{\Delta B}{A}} \left( \frac{1}{\sigma-E} + \frac{1}{A} \right) \quad [38]$$

It is sometimes more convenient to determine  $\frac{d\Delta B}{d\sigma}$  from  $\Delta B = f[\ell^*, C_w]$ .

Then

$$\frac{d\Delta B}{d\sigma} = \left( \frac{\partial \Delta B}{\partial \ell^*} \right)_{C_w} \frac{d\ell^*}{d\sigma} + \left( \frac{\partial \Delta B}{\partial C_w} \right)_{\ell^*} \frac{dC_w}{d\sigma} \quad [39]$$

From differentiating Equation [15]

$$\frac{dC_w}{d\sigma} = C_w \left( \frac{1}{A} \frac{d\Delta B}{d\sigma} - \frac{1}{\sigma-E} - \frac{1}{A} \right) \quad [40]$$

With  $\frac{d\ell^*}{d\sigma}$  from Equation [36]

$$\frac{d\Delta B}{d\sigma} = \frac{- \left( \frac{\partial \Delta B}{\partial \ell^*} \right)_{C_w} \frac{\ell^*}{\sigma} - \left( \frac{\partial \Delta B}{\partial C_w} \right)_{\ell^*} \left( \frac{1}{\sigma-E} + \frac{1}{A} \right)}{1 - \frac{C_w}{A} \left( \frac{\partial \Delta B}{\partial C_w} \right)_{\ell^*}} \quad [41]$$

Hence for the condition  $\frac{d\sigma}{dx} = 0$  for which  $\frac{d\Delta B}{d\sigma} \rightarrow \infty$ ,  $\frac{C_w}{A} \left( \frac{\partial \Delta B}{\partial C_w} \right)_{\ell^*} = 1$ .

#### ILLUSTRATIVE EXAMPLE

Case I is a flat plate moving at 20 knots with a solution of Guar Gum being injected from a slot. It is assumed that at the beginning of Stage 4, the wall concentration is 2000 ppm (parts per million) and  $\ell^* = 2$ . A kinematic viscosity of  $\nu_0 = 1.5 \times 10^{-5}$  ft<sup>2</sup>/sec is assumed. From the data for Guar Gum from References 1 and 5, a chart of  $C_w e^{-\frac{\Delta B}{A}}$  as a function of  $\ell^*$  and  $C_w$  is prepared; see Figure 1. For Guar Gum  $\ell = 4 \times 10^{-4}$  in. and then  $\frac{U\ell}{\nu_0} = 75$ . Since  $\sigma = \left( \frac{U\ell}{\nu_0} \right) / \ell^*$ , the initial value of  $\sigma$  is 37.5.



From Equation [15]  $\frac{\dot{m}}{\gamma \rho v_o} = 8.380 \times 10^9$ , and  $A = 2.39$  and  $E = 5.17$  (Reference 3). For the initial values,  $\frac{d\sigma}{dR_X} = -0.844 \times 10^{-7}$  from Equation [34].

Initially then,  $\sigma$  decreases with increasing  $R_X$ . For Case I,  $C_w e^{-\frac{\Delta B}{A}}$  as a function of  $\ell^*$  is determined from Equations [18] and [19] and is plotted in Figure 1.

$B_{1,0} = 5.45$  and  $B_2 = 2.6$  (Reference 1). The intersections in Figure 1 provide the values of  $C_w$  for determining  $\eta$  from Equation [21].  $\Delta R_X$  is determined from Equation [29], the integral being numerically determined.  $D_1 = 3.499$  and  $D_2 = 23.23$  from Reference 6.  $\tau_w / \rho U^2 = 1/\sigma^2$  is plotted against  $\Delta R_X$  in Figure 2. For comparison,  $\tau_w / \rho U^2$  is also plotted for the cases of no drag reduction and uniform concentration of 2000 ppm.

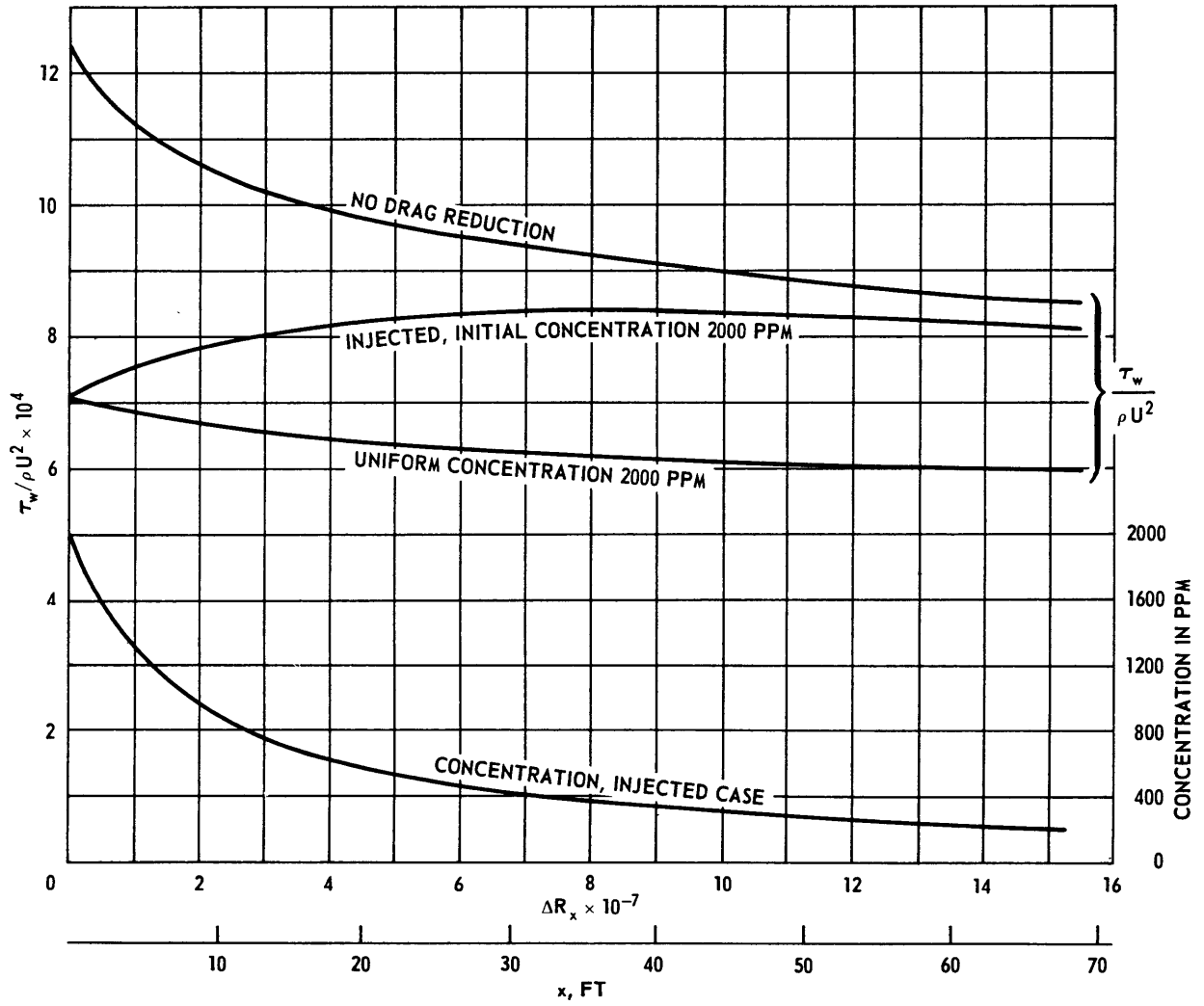


Figure 2 – Illustrative Example for Case I

$U = 20$  knots, initial concentration of 2000 ppm Guar Gum,

initial  $l^* = 2$ ,  $\nu_0 = 1.5 \times 10^{-5}$  ft<sup>2</sup>/sec.

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