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THE FRICTIONAL RESISTANCE AND VELOCITY SIMILARITY
LAWS OF DRAG-REDUCING DILUTE POLYMER SOLUTIONS

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HYDROMECHANICS LABORATORY

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Naval Ship Research and Development Center
Washington, D. C. 20007

ABSTRACT

The velocity similarity laws of shear flows are developed for drag-reducing dilute polymer solutions. Relations for boundary-layer parameters and frictional resistance formulas of the logarithmic type are derived for pipe flow and for flat plates in parallel flow.

ADMINISTRATIVE INFORMATION

The work described in this report was sponsored by the Bureau of Ordnance (now Naval Ordnance Systems Command). Funding was under Sub-project WR 109-0103, Task 402. The material was previously published informally as Hydromechanics Laboratory Technical Note 61, Nov 1966.

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NOMENCLATURE

A	Slope of logarithmic velocity law in natural logarithms
A_1	Slope of logarithmic velocity law in common logarithms
a	Factor in pipe friction formula, Equation [40]
B_1, B_2	Intercepts of logarithmic velocity law, Equations [7] and [8]
B_1'	Derivative of B_1 with respect to $\ln \ell^*$
B_1''	Derivative of B_1' with respect to $\ln \ell^*$
$B_{1,0}$	B_1 for solvent (zero concentration)
β	Factor in pipe friction formula, Equation [41]
C	Concentration of polymer solution
C_f	Coefficient of frictional resistance of flat plates, Equation [56]
c_1, c_2, c_3	Linearization factors in Equations [37] and [38]
D	Diameter of pipe
D_1, D_2, D_3	Velocity profile factors, Equations [18], [21], and [24]
\mathcal{D}	Drag
e	Base of natural logarithms
F	Outer law function, Equation [4]
f	Fanning friction factor, Equation [29]
f_1	Inner law function, Equation [2a]
G	Subscript for quantities at junction of inner and outer turbulent sublayers
H	Shape parameter, $H = \delta^* / \theta$
h	Deviation of outer law from logarithmic law
J	Transitional sublayer factor, Equation [13]
k, k_1, k_2, \dots	Roughness length parameters
L	Subscript for quantities at junction of laminar and transitional sublayers
$\ell, \ell_1, \ell_2, \dots$	Characteristic lengths of polymer molecules
ℓ^*	Non-dimensional characteristic length, $\ell^* = u_\tau \ell / \nu_0$
ℓ_o^*	Threshold value of ℓ^*

M	Subscript for quantities corresponding to minimum diameter for logarithmic similarity law
m, m_1, m_2, \dots	Characteristic masses of polymer molecules
P	Polymer characteristics for particular chemical species and concentration
q	Drag-reducing factor in Equation [81]
r	Radial distance from center of pipe
R_D	Reynolds number of pipe, $R_D = VD/\nu_o$
R_x	Reynolds number, $R_x = Ux/\nu_o$
R_θ	Reynolds number, $R_\theta = U\theta/\nu_o$
T	Subscript for quantities at junction of transitional and inner turbulent sublayers
t, t_1, t_2, \dots	Characteristic times of polymer molecules
U	Velocity in center of pipe or outside of boundary layer of flat plates
u	Tangential velocity of shear flow
u_τ	Shear velocity
V	Average velocity in pipe
x	Streamwise distance along flat plate
y	Normal distance from wall
y^*	Non-dimensional y, $y^* = u_\tau y/\nu_o$
α	Velocity profile factor, Equation [19]
β	Velocity profile factor, Equation [22]
γ	Velocity profile factor, Equation [25]
δ	Pipe radius or boundary layer thickness
δ^*	Displacement thickness
η	Boundary layer Reynolds number, $\eta = u_\tau \delta/\nu_o$
θ	Momentum thickness
κ	von Kármán's constant
λ	Darcy-Weisbach friction factor, Equation [31]
μ	Coefficient of viscosity of solution

μ_o	Coefficient of viscosity of solvent
ν	Kinematic viscosity of solution
ν_o	Kinematic viscosity of solvent
ρ	Density of solution
ρ_o	Density of solvent
ρ_p	Density of dispersion of polymer molecules
σ	Local resistance parameter, $\sigma = U/u_\tau$
τ	Shearing stress in fluid
τ_w	Shearing stress at wall

THE FRICTIONAL RESISTANCE AND VELOCITY SIMILARITY
LAWS OF DRAG-REDUCING DILUTE POLYMER SOLUTIONS

by

Paul S. Granville

INTRODUCTION

The remarkable reduction in frictional resistance of the turbulent flow of liquids in pipes and in the boundary layers of moving bodies by the addition of small amounts of certain organic polymers has engendered many scientific and engineering questions and problems. It was originally found that the more concentrated solutions exhibited non-Newtonian behavior in the variation of the coefficient of viscosity with rate of shear. Accordingly, analyses^{1,2} of the turbulent flow in pipes and on flat plates were developed for power-law analytical models of the non-Newtonian viscous behavior. Then it was discovered^{3,4}, or rather rediscovered, that dilute solutions still gave drag reductions for turbulent flow even though the coefficient of viscosity was Newtonian with shear rate. This effect was termed the Toms phenomenon in honor of Toms who originally reported the anomalous behavior in 1948 to the first Congress of Rheology; after which it seems to have not become generally known.

However, an ordinary Newtonian analysis of a viscous fluid proves unsatisfactory for drag-reducing solutions, since the pipe friction factors vary with both Reynolds number and pipe diameter as well as with the size and configuration of the polymer molecules and their concentrations.

Work by Ernst⁵ and Meyer⁶ has shown that the friction factors for different pipe diameters of a polymer solution of the same material and concentration can be correlated on the basis of the inner similarity law. For the logarithmic portion of the inner law which has two factors, it was found that von Kármán's constant which controls the plotted slope did remain constant but that the factor (which unfortunately does not have a

¹References are listed on page 30.

convenient name) which controls the plotted intercept varied with shear velocity. It was also shown that a threshold value of shear velocity exists below which no drag-reducing effect occurs. Previous investigators⁷ had also noticed a threshold shearing stress. This procedure was continued by Elata et al,⁸ who introduced a macromolecular relaxation time in the analysis.

The present work develops the similarity laws for homogeneous solutions from a dimensional analysis of the inner similarity law in terms of characteristic lengths, masses, and/or times introduced into the fluid by the polymer molecules. The outer similarity law remains the same. The overlapping of the inner and outer laws results in a logarithmic relation with von Kármán's constant unchanged and the intercept factors a function of a nondimensional ratio containing the shear velocity.

Various integral relations are developed to be applied to the friction factors of pipe flow and to the boundary layer parameters of displacement thickness, momentum thickness, and shape parameter of flat-plate flow. It is shown that a minimum diameter exists for pipe flow below which the logarithmic similarity law vanishes. Formulas are developed for the total resistance coefficients of flat plates in parallel flow as functions of Reynolds number and a dimensionless ratio containing the length of the plate. A numerical example is worked out for a sample concentration of Guar Gum.

VELOCITY SIMILARITY LAWS

GENERAL

Since research into the fundamentals of the mechanics of turbulence has yet to achieve methods for predicting frictional resistance and the behavior of turbulent flows, recourse is made to the phenomenological properties of turbulent flows in terms of the similarity properties of the mean-velocity profiles.

For turbulent shear flows, such as fully developed flow in pipes or boundary-layer flow, the two laws which provide similarity in the mean-velocity profile by linking it to the wall shearing stress are:

1. The inner law or law of the wall which applies to the flow immediately adjacent to the solid boundary.

2. The outer law or velocity-defect law which applies to the remaining outer region of the shear flow.

The overlapping across the shear flow of the two laws results in a logarithmic functional form for both similarity laws within the common region of overlap.

The similarity laws of the velocity profile of shear flows were originally developed by Prandtl and von Kármán among others^{9,10,11} for ordinary Newtonian fluids.

INNER LAW OR LAW OF THE WALL

A similarity law may be developed for the mean turbulent flow of dilute polymer solutions close to the wall or solid boundary by adding additional parameters to account for the physical presence of the polymer molecules in the solutions such as characteristic lengths, masses, and/or times of the molecules. Specifically, then, the mean velocity component u of the flow in a direction parallel to the wall may be considered to depend on the usual attributes of the shear flow such as the normal distance y away from the wall, the shearing stress at the wall τ_w , the density ρ_0 and the coefficient of viscosity* μ_0 of the solvent. In addition the presence of the polymer molecules is indicated by an unknown number of characteristic lengths $\ell, \ell_1, \ell_2, \dots$, by an unknown number of characteristic masses m, m_1, m_2, \dots , by an unknown number of characteristic times t, t_1, t_2, \dots , and the density of dispersion of the polymer molecules in the fluid ρ_p . Future investigation will have to ascertain which characteristic quantities have significance to the drag-reducing qualities of the polymer molecules. A discussion of current molecular theories appears in Reference 7.

*The coefficient of viscosity μ of the solution is not used in the analysis since 1) it is not an independent variable but is dependent on the presence of the polymer molecules and hence implicitly specified; and 2) its use in Reynolds numbers in drag relations would mask the increase in drag due any increased viscosity of the solution.

Examples of such characteristic lengths may be the molecular dimensions of factors like radii of gyration. A characteristic time may be a relaxation time. A characteristic mass may be the molecular weight. In fact, Hoyt¹⁷ has proposed using drag reduction as a means of measuring the molecular weights of homologous polymers.

The analysis does not involve viscoelasticity per se. However, characteristic time may be considered as a measure of the viscoelastic property of the solution. Gadd^{18,19} has experimentally shown that there is no apparent correlation between drag reduction ability and normal stress difference which is the usual measure of viscoelastic behavior.

The analytical statement of the inner law then becomes

$$u = f \left[\tau_w, \rho_o, \mu_o, \gamma, \rho_p, \ell, \ell_1, \ell_2, \dots, m, m_1, m_2, \dots, t, t_1, t_2, \dots \right] \quad [1]$$

The boundary condition at the wall as usual is $u = 0$ at $y = 0$.

By dimensional analysis the variables may be grouped into the following nondimensional ratios:

$$\frac{u}{u_\tau} = f_1 \left[\frac{u_\tau y}{v_o}, \frac{\rho_p}{\rho_o}, \frac{u_\tau \ell}{v_o}, \left(\frac{\ell}{\ell_1}, \frac{\ell_1}{\ell_2}, \dots, \frac{m}{\rho_o \ell^3}, \frac{m}{m_1}, \frac{m_1}{m_2}, \dots, \frac{t v_o}{\ell^2}, \frac{t}{t_1}, \frac{t_1}{t_2}, \dots \right) \right] \quad [2a]$$

or

$$\frac{u}{u_\tau} = f_2 \left[\frac{u_\tau y}{v_o}, \frac{\rho_p}{\rho_o}, \frac{u_\tau}{v_o} \left(\frac{m}{\rho_o} \right)^{1/3}, \left(\frac{m}{m_1}, \frac{m_1}{m_2}, \dots, \frac{m}{\rho_o \ell^3}, \frac{\ell}{\ell_1}, \frac{\ell_1}{\ell_2}, \dots, \frac{t v_o}{\ell^2}, \frac{t}{t_1}, \frac{t_1}{t_2}, \dots \right) \right] \quad [2b]$$

or

$$\frac{u}{u_\tau} = f_3 \left[\frac{u_\tau y}{v_o}, \frac{\rho_p}{\rho_o}, u_\tau \sqrt{\frac{t}{v_o}}, \left(\frac{t}{t_1}, \frac{t_1}{t_2}, \dots, \frac{t v_o}{\ell^2}, \frac{\ell}{\ell_1}, \frac{\ell_1}{\ell_2}, \dots, \frac{m}{\rho_o \ell^3}, \frac{m}{m_1}, \frac{m_1}{m_2}, \dots \right) \right] \quad [2c]$$

where $u_\tau = \sqrt{\frac{\tau_w}{\rho_o}}$ is the friction or shear velocity

and $\nu_o = \frac{\mu_o}{\rho_o}$ is the kinematic viscosity of the solvent.

It is thus possible to use either a characteristic length, mass, or time to indicate the hydrodynamic properties of the solution. The ratio $\frac{\rho_p}{\rho_o}$ may be converted to the usual definition for concentration C,

$C = \frac{\rho_p/\rho_o}{1 + \rho_p/\rho_o}$. The ratios within the dashed parentheses are constant for a particular chemical species in a particular solvent.

Equations [2] are statements of the inner law or law of the wall and are applicable to shear flows like fully developed turbulent flow in pipes and boundary-layer flows.

For convenience, Equation [2a] based on the characteristic length is used in the analyses of this paper. The following notation is also adopted

$$\ell^* = \frac{u_\tau \ell}{\nu_o} \quad y^* = \frac{u_\tau y}{\nu_o}$$

and

$$P = \frac{\ell}{\ell_1}, \frac{\ell_1}{\ell_2}, \dots, \frac{m}{\rho_o \ell^3}, \frac{m}{m_1}, \frac{m_1}{m_2}, \dots, \frac{t \nu_o}{\ell^2}, \frac{t}{t_1}, \frac{t_1}{t_2}, \dots$$

OUTER LAW OR VELOCITY-DEFECT LAW

For the turbulent flow of ordinary fluids in pipes and for flat plates* at some distance away from the wall, where U is the velocity in the center of the pipe or at the outer edge of the boundary layer, the velocity defect U-u has been found experimentally to be directly independent of

* In the terminology of this report, flat-plate flow implies zero pressure gradient.

viscosity and only a function of τ_w , ρ , and distance $\delta - y$ or

$$U - u = f \left[\tau_w, \rho, y, \delta \right] \quad [3]$$

where δ is the thickness of the boundary layer or the pipe radius for fully-developed pipe flow. For dilute solutions, $\rho \cong \rho_0$. Then by dimensional analysis

$$\frac{U - u}{u_\tau} = F \left[\frac{y}{\delta} \right] \quad [4]$$

which is the statement of the outer law or velocity-defect law.

Since the outer law is not directly dependent on viscous behavior, it is to be expected that it should also be the same for the flow of dilute polymer solutions as for the flow of the pure solvents. This has been confirmed by the pipe tests of Ernst.⁵ Additional confirmation has come from studies by Jackley¹² on the flow in free jets wherein polymer additives showed no effects on jet behavior. The flow in jets represents outer-law flow in effect.

LOGARITHMIC VELOCITY LAW

Within the shear layer there is a region where both the inner and outer laws are considered to hold. This overlapping leads to a logarithmic relation as follows: Equating the derivative of velocity u with respect to distance y for the inner and outer laws, Equations [2] and [4], gives

$$\frac{\partial u}{\partial y} = \frac{u_\tau^2}{v_o} \frac{\partial f_1}{\partial y^*} = - \frac{u_\tau}{\delta} \frac{dF}{d(y/\delta)} \quad [5]$$

or

$$y^* \frac{\partial f_1}{\partial y^*} = - \left(\frac{y}{\delta} \right) \frac{dF}{d(y/\delta)} = A \quad [6]$$

Since the left-hand side of Equation [6] is only a function of y^* and polymer parameters C , ℓ^* , and P , and the right-hand side is only a function of y/δ , the only quantity satisfying these conditions is a constant, say A , independent of all these variables.

From the left-hand side of Equation [6] there results after integration

$$f_1 = \frac{u}{u_\tau} = A \ln y^* + B_1 [\ell^*, C, P] \quad [7]^\dagger$$

where the constant of integration B_1 is necessarily a function of ℓ^* , C , and P from the integration of a partial derivative. Equation [7] is the statement of the inner law in the overlapping region.

The constancy of A and the variation of B_1 with ℓ^* has been experimentally confirmed in References 5, 6, and 8. In Reference 6 B_1 is correlated with u_τ for CMC solutions flowing in different sized pipes and in Reference 8 B_1 is correlated with $\frac{u_\tau^2 t}{\nu}$ for Guar Gum Solutions. For a particular concentration C and polymer species P , the factors ℓ , t , and ν are constant and hence B_1 correlates with u_τ or any function of u_τ such as ℓ^* .

From the right-hand side of Equation [6] the outer law in the overlapping region becomes

$$F = \frac{U - u}{u_\tau} = -A \ln y/\delta + B_2 \quad [8]$$

An important consequence of the preceding analysis is that the hydrodynamic effects of the polymer additives are characterized by the behavior of B_1 as a function of ℓ^* , C , P which is to be experimentally determined. B_1 will be termed the drag-reducing characterization for a given concentration of a solution of a given polymer.

Equating the velocities of the inner and outer laws in the overlapping region, Equations [7] and [8] provides a relation for local skin friction:

$$\sigma \equiv \frac{U}{u_\tau} = A \ln \eta + B_1 + B_2 \quad [9]$$

where σ is a local resistance parameter and $\eta = u_\tau \delta/\nu_0$.

[†]ln = natural logarithm.

It is to be noted that A may be also written as $1/\kappa$ where κ is known as von Kármán's constant.

SUBLAYERS

Various sublayers may be distinguished in the boundary layer according to the behavior of the velocity profiles. These are:

1. The laminar sublayer next to the wall wherein the turbulent fluctuations are effectively damped out.
2. The transitional sublayer wherein the shearing stresses are affected by both laminar and turbulent contributions.
3. The inner turbulent sublayer wherein the inner and outer laws overlap.
4. The outer turbulent sublayer where only the outer law prevails.

VELOCITY LAW FOR LAMINAR SUBLAYER

The inner law, Equation [2] holds here without any specification as to the form of the functional relationship. For essentially parallel laminar flow within the laminar sublayer, the shearing stress τ is given as

$$\tau = \mu \frac{du}{dy} \quad [10]$$

where μ = coefficient of viscosity of the solution.

If the variation of τ with y is neglected within the thin sublayer,

$$\tau = \tau_w.$$

Then $\frac{du}{dy} = \frac{\tau_w}{\mu}$, $u = \frac{\tau_w}{\mu} y$ with $u = 0$ at $y = 0$ and consequently

$$\frac{u}{u_\tau} = \frac{\nu_0}{\nu} y^* \quad \text{for } 0 \leq y^* \leq y_L^* \quad [11]$$

where ν = kinematic viscosity of the solution.

VELOCITY LAW FOR TRANSITIONAL SUBLAYER

The velocity law for the transitional sublayer derived by Squire¹³ for ordinary flows may be considered also valid for dilute polymer solutions. Then

$$\frac{u}{u_\tau} = A \ln (y^* - J) + B_1 \quad \text{for } y_L^* \leq y^* \leq y_T^* \quad [12]$$

where

$$J = \frac{v}{v_o} (B_1 + A \ln \frac{v}{v_o} \frac{A}{e}) \quad [13]$$

e = base of natural logarithms

This relation starts at the outer edge of the laminar sublayer y_L and merges asymptotically with the logarithmic velocity law, Equation [7]. A measure of the thickness of the laminar sublayer is given by

$$y_L^* = \frac{v}{v_o} (B_1 + A \ln \frac{v}{v_o} \frac{A}{e}) \quad [14]$$

It is seen that the laminar sublayer increases with increasing values of B_1 . Figure 1 shows a sample plot of the inner law.

VELOCITY LAW FOR INNER TURBULENT SUBLAYER

The inner turbulent sublayer represents the region of the overlapping of the inner and outer similarity law. Here Equations [7] and [8] apply.

VELOCITY LAW FOR OUTER TURBULENT SUBLAYER

Instead of a completely undefined statement of the outer law as given by Equation [4] for the region outside of the validity of the logarithmic velocity law, Equation [8], it is sometimes of value to consider the velocity law in terms of a deviation of the outer law from the logarithmic law: $h[y/\delta]$. Then the outer law becomes

$$\frac{U - u}{u_\tau} = F[y/\delta] = -A \ln y/\delta + B_2 - h[y/\delta] \quad [15]$$

The limiting values of h are

$$h = 0 \quad \text{at} \quad y/\delta = (y/\delta)_G$$

$$h = B_2 \quad y/\delta = 1$$

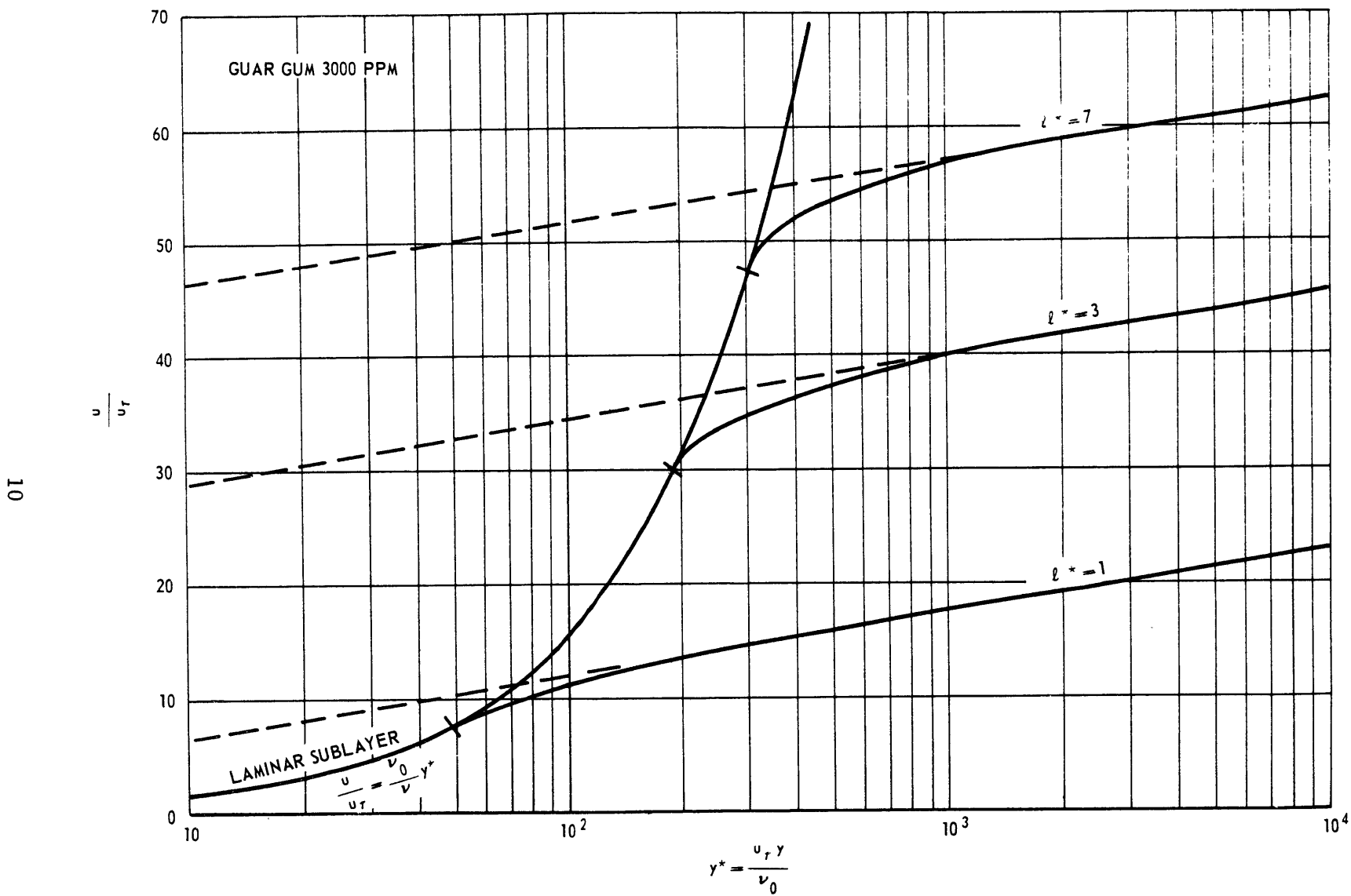


Figure 1 - Inner Similarity Law

For the inner and outer turbulent sublayer the velocity law becomes

$$\frac{u}{u_\tau} = A \ln y^* + B_1 + h[y/\delta] \quad \text{for } y_T^* \leq y^* \leq \eta \quad [16]$$

Figure 2 shows plots of the outer law for pipes and flat plates.

INTEGRAL RELATIONS OF VELOCITY SIMILARITY LAWS

For subsequent analyses of frictional resistance, integration of the velocity similarity laws are required over the whole shear layer such as

$$\int_0^\eta \frac{u}{u_\tau} dy^*, \quad \int_0^\eta \left(\frac{u}{u_\tau} \right)^2 y^* dy^* \quad \text{and} \quad \int_0^\eta \frac{u}{u_\tau} y^* dy^*$$

These are performed in a piecewise manner using the appropriate velocity similarity law for each sublayer, Equations [11], [12], and [16]. By definition $y^* = \eta(y/\delta)$ which is needed for transforming integrations from y^* to y/δ . Consequently

$$\int_0^\eta \frac{u}{u_\tau} dy^* = \eta (\sigma - D_1) + \alpha \quad [17]$$

where

$$D_1 = A + B_2 - \int_{(y/\delta)_G}^1 h d(y/\delta) \quad [18]$$

$$\alpha = \frac{v}{v_o} A^2 \left(1 - \ln \frac{v}{v_o} A + \frac{1}{2} \ln^2 \frac{v}{v_o} A \right) - \frac{v}{v_o} \frac{B_1^2}{2} \quad [19]$$

$$\int_0^\eta \left(\frac{u}{u_\tau} \right)^2 dy^* = \eta (\sigma^2 - 2D_1 \sigma + D_2) + \beta \quad [20]$$

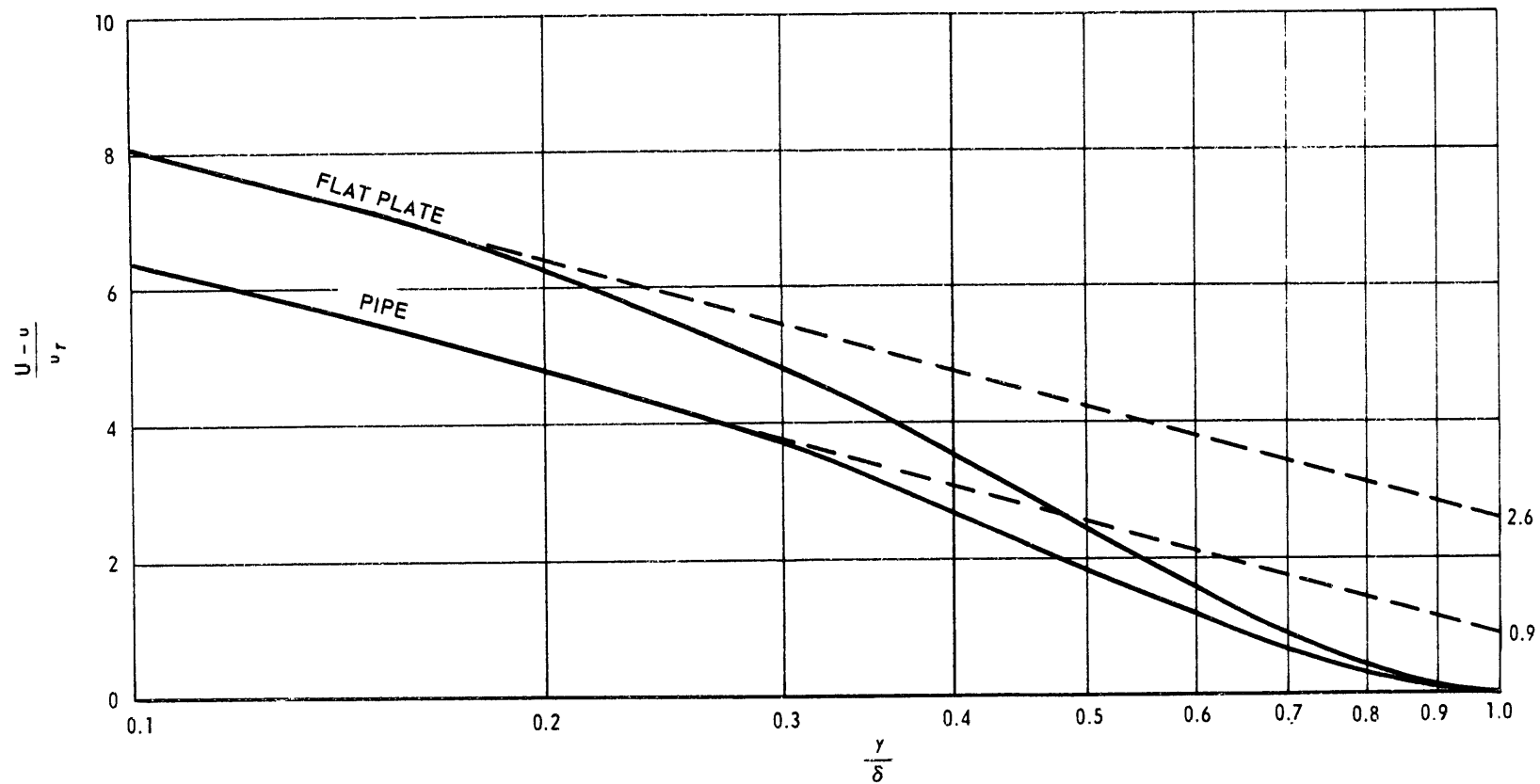


Figure 2 - Outer Similarity Law

where

$$D_2 = B_2^2 + 2A B_2 + 2A^2 - 2B_2 \int_{(y/\delta)_G}^1 h \, d(y/\delta) + 2A \int_{(y/\delta)_G}^1 h \ln y/\delta \, d(y/\delta) + \int_{(y/\delta)_G}^1 h^2 \, d(y/\delta) \quad [21]$$

$$\beta = \frac{v}{v_o} \left[A^3 \left(\frac{1}{3} \ln^3 \frac{v}{v_o} A - \ln^2 \frac{v}{v_o} A + 2 \ln \frac{v}{v_o} A - 2 \right) + B_1 A^2 \left(\ln^2 \frac{v}{v_o} A - 2 \ln \frac{v}{v_o} A + 2 \right) - \frac{2 B_1^3}{3} \right] \quad [22]$$

and

$$\int_0^\eta \left(\frac{u}{u_\tau} \right) y^* \, dy^* = \frac{\eta^2}{2} (\sigma - D_3) + \gamma \quad [23]$$

where

$$D_3 = \frac{A}{2} + B_2 - 2 \int_{(y/\delta)_G}^1 h \, y/\delta \, d(y/\delta) \quad [24]$$

$$\begin{aligned} \gamma = & \left(\frac{v}{v_o} \right)^2 A^2 \left(\frac{A}{3} \ln^3 \frac{v}{v_o} A - \frac{A}{2} \ln^2 \frac{v}{v_o} A + 2 \ln \frac{v}{v_o} A - \ln^2 \frac{v}{v_o} A - \frac{A}{4} - 1 \right) \\ & + \left(\frac{v}{v_o} \right)^2 B_1 \left(A - A \ln \frac{v}{v_o} A + \frac{B_1 A^2}{2} \ln^2 \frac{v}{v_o} A - \frac{B_1^2}{6} \right) \\ & + \frac{v}{v_o} \left(B_1 - A + A \ln \frac{v}{v_o} A \right) y_T^* \left(\ln y_T^* - 1 \right) \end{aligned} \quad [25]$$

PIPE FLOW

AVERAGE VELOCITY

For pipe flow the most meaningful velocity for engineering application is the average velocity of the flow V which for a circular cross section is:

$$V = \frac{2\pi \int_0^{D/2} u r dr}{\frac{\pi}{4} D^2} \quad [26]$$

where r is the radial distance from the center of the pipe and D is the diameter of the pipe.

In terms of similarity parameters

$$V = \frac{2 u_\tau}{\eta} \left(\int_0^\eta \frac{u}{u_\tau} dy^* - \frac{1}{\eta} \int_0^\eta \frac{u}{u_\tau} y^* dy^* \right) \quad [27]$$

where y is the radial distance from the pipe wall towards the center.

Substituting the values of the integrals, Equations [17] and [23], produces

$$\frac{V}{u_\tau} = \sigma - 2D_1 + D_3 + \frac{2\alpha}{\eta} - \frac{2\gamma}{\eta^2} \quad [28]$$

FRICTION FACTOR

The Fanning friction factor f for pipe flow is defined as

$$f \equiv \frac{\tau_w}{\frac{1}{2}\rho V^2} \quad [29]$$

or

$$\sqrt{\frac{2}{f}} = \frac{V}{u_\tau} \quad [30]$$

The Darcy-Weisbach friction factor λ is defined as

$$\lambda \equiv \frac{8\tau_w}{\rho V^2} \quad [31]$$

or

$$\lambda = 4f \quad [32]$$

Substituting average velocity, Equation [28], for maximum velocity in the pipe flow in the relation for skin friction, Equation [9], produces

$$\frac{V}{u_\tau} = A \ln \eta + B_1 + B_2 - 2D_1 + D_3 + \frac{2\alpha}{\eta} - \frac{2\gamma}{\eta^2} \quad [33]$$

Using the expression for friction factor, Equation [30], and the fact that by definition

$$\eta \equiv \frac{\sqrt{2}}{4} \sqrt{f} R_D \quad [34]$$

and

$$R_D \equiv \frac{VD}{v_o} \quad [35]$$

results in

$$\sqrt{\frac{1}{f}} = \frac{A}{\sqrt{2}} \ln f R_D - \frac{A}{\sqrt{2}} \ln 2 \sqrt{2} + \frac{1}{\sqrt{2}} (B_1 + B_2 - 2D_1 + D_3) + \frac{4\alpha}{\sqrt{f} R_D} - \frac{8\sqrt{2}\gamma}{f R_D^2} \quad [36]$$

Within the range of R_D of interest the last two terms of Equation [36] can be absorbed by linearization:

$$\frac{1}{\sqrt{f} R_D} = c_1 \ln \sqrt{f} R_D + c_2 \quad [37]$$

and

$$\frac{1}{f R_D^2} = c_3 \ln \sqrt{f} R_D + c_4 \quad [38]$$

Then

$$\frac{1}{\sqrt{f}} = a \log \sqrt{f} R_D + b + \frac{1}{\sqrt{2}} (B_1 - B_{1,0}) \quad [39]^*$$

* log refers to common logarithms to the base 10.

where

$$B_1 = f [\ell^*]$$

$$B_{1,0} = B_1 \text{ for solvent}$$

$$a = \frac{2.3026}{\sqrt{2}} (A + 4 c_1 \alpha - 16 c_3 \gamma) \quad [40]$$

and

$$\mathcal{B} = \frac{1}{\sqrt{2}} (B_{1,0} + B_2 - 2D_1 + D_3 + 4 c_2 \alpha - 16 c_4 \alpha - 3.4539 A \log 2) \quad [41]$$

MINIMUM DIAMETER FOR LOGARITHMIC LAW

The extent of overlapping of the inner and outer similarity laws wherein the logarithmic law holds varies with η . The outer limit of the logarithmic law is given by $(y/\delta)_G$ and the inner limit by y_T^* . Hence for the inner law the extent is $\eta(y/\delta)_G - y_T^*$ and for the outer law the extent is $(y/\delta)_G - y_T^*/\eta$. The extent diminishes with decreasing η so that the value of η for the disappearance of the logarithm law or η_M is

$$\eta_M = \frac{y_T^*}{(y/\delta)_G} \quad [42]$$

For ordinary fluids, then from Equation [34]

$$R_{D,M} = \frac{4\eta_M}{\sqrt{2}} \frac{1}{\sqrt{f_M}}$$

and from Equation [39]

$$R_{D,M} = \frac{4}{\sqrt{2}} \eta_M (a \log \frac{4}{\sqrt{2}} \eta_M + \mathcal{B}) \quad [43]$$

For dilute drag-reducing fluids $B_1 = f [\ell^*]$ and $y_T = f [\ell^*]$.

Hence

$$\frac{D_M}{\ell} = \frac{2\eta_M}{\ell^*} = \frac{2 y_T^*}{(y/\delta)_G \ell^*} \quad [44]$$

The corresponding

$$R_{D,M} = \frac{4\eta_M}{\sqrt{2}} \left(a \log \frac{4}{\sqrt{2}} \eta_M + B + \frac{B_1 - B_{1,0}}{\sqrt{2}} \right) \quad [45]$$

Since $B_1 = f [\ell^*]$

$$R_{D,M} = f [\ell^*] = f [D_M/\ell]$$

or

$$\frac{D_M}{\ell} = f \left[\frac{V\ell}{v_o} \right]$$

For a particular polymer solution the minimum diameter is then specified by $\frac{V}{v_o}$. At each value of ℓ^* , a value of $\frac{D_M}{\ell}$ and $R_{D,M}$ is calculated. $\frac{V\ell}{v_o}$ is

then determined from $\frac{R_{D,M}}{D_M/\ell}$.

An example is shown in Figure 3 for a particular concentration of Guar Gum. It is obvious that too small a pipe diameter will provide data not susceptible to generalization.

BOUNDARY-LAYER FLOW

BOUNDARY-LAYER PARAMETERS

The boundary-layer parameters of displacement thickness δ^* , momentum thickness θ , and shape parameter H are obtained from the integrated values of the similarity laws over the boundary layer.

Since the displacement thickness δ^* is defined as

$$\delta^* \equiv \int_0^{\delta} \left(1 - \frac{u}{U} \right) dy \quad [46]$$

or

$$\frac{\delta^*}{\delta} = 1 - \frac{1}{\sigma\eta} \int_0^{\eta} \frac{u}{u_\tau} dy^* \quad [47]$$

and from Equation [17]

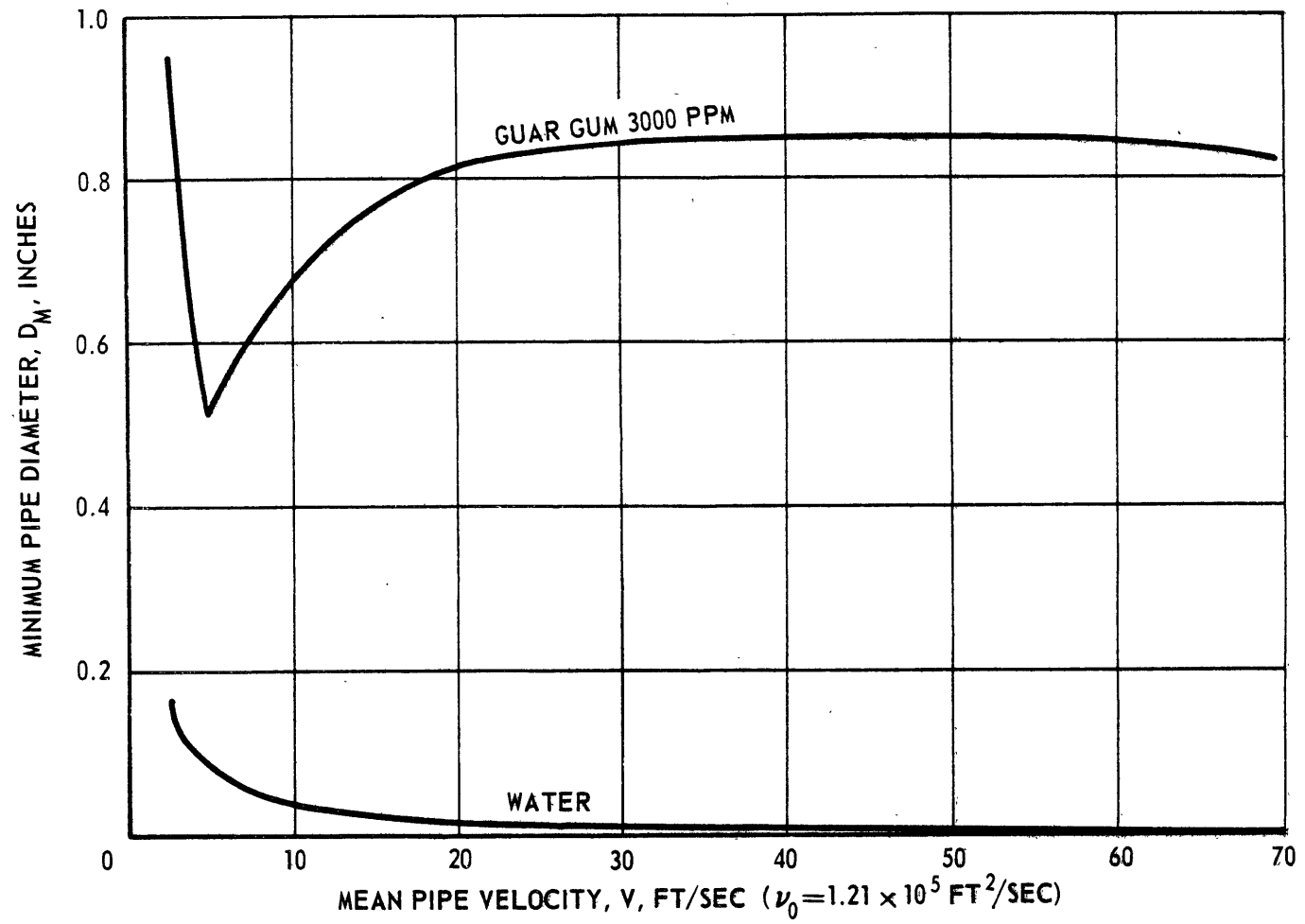


Figure 3 - Minimum Pipe Diameter for Logarithmic Similarity Law

$$\frac{\delta^*}{\delta} = \frac{1}{\sigma} (D_1 - \frac{\alpha}{\eta}) \quad [48]$$

Momentum thickness θ is defined as

$$\theta \equiv \int_0^{\delta} (1 - \frac{u}{U}) \frac{u}{U} dy \quad [49]$$

or

$$\frac{\theta}{\delta} = \frac{1}{\sigma\eta} \int_0^{\eta} \frac{u}{u_{\tau}} dy^* - \frac{1}{\sigma^2\eta} \int_0^{\eta} \left(\frac{u}{u_{\tau}}\right)^2 dy^* \quad [50]$$

and from Equations [17] and [20]

$$\frac{\theta}{\delta} = \frac{1}{\sigma} (D_1 + \frac{\alpha}{\eta}) - \frac{1}{\sigma^2} (D_2 + \frac{\beta}{\eta}) \quad [51]$$

and

$$R_{\theta} \equiv \frac{U\theta}{\nu} = \eta (D_1 - \frac{D_2}{\sigma}) + \alpha - \frac{\beta}{\sigma} \quad [52]$$

For shape parameter H defined as

$$H \equiv \delta^* / \theta$$

$$\frac{1}{H} = 1 - \frac{1}{\sigma} \left[(D_2 + \beta/\eta) / (D_1 - \alpha/\eta) \right] \quad [53]$$

FRICTIONAL RESISTANCE OF FLAT PLATES

The frictional resistance or drag of a flat plate in two-dimensional flow without pressure gradients which corresponds to a flat plate moving lengthwise in an infinite fluid is determined by the momentum thickness of the boundary-layer flow leaving the trailing edge as a wake. The von Kármán momentum equation after neglecting the small effect of the normal Reynolds stress term holds also for dilute solutions of polymers since the boundary-layer equations of motion are the same as for the pure solvent:

$$\frac{d\theta}{dx} = \frac{\tau_w}{\rho U^2} \quad [54]$$

where x is the streamwise distance from the leading edge.

Since the frictional resistance or drag \mathfrak{D} for a flat plate of unit breadth is

$$\mathfrak{D} = \int_0^x \tau_w dx \quad [55]$$

and the drag coefficient

$$C_f \equiv \frac{\mathfrak{D}}{\frac{1}{2} \rho U^2 x} \quad [56]$$

then

$$C_f = 2 \frac{\theta}{x} = \frac{2 R_\theta}{R_x} \quad [57]$$

where

$$R_x = \frac{Ux}{v_o} \quad [58]$$

Since $\frac{1}{\sigma^2} \equiv \frac{\tau_w}{\rho U^2}$, the momentum equation, [54], becomes

$$R_x = \int \sigma^2 dR_\theta \quad [59]$$

or by integrating by parts

$$R_x = \sigma^2 R_\theta - 2 \int R_\theta \sigma d\sigma + \text{const} \quad [60]$$

Inserting the relation for η from [9] into the relation for R_θ , Equation [52], and integrating by parts repeatedly results in a series:

$$\int R_\theta \sigma d\sigma = A D_1 \eta \sigma \left\{ 1 - \left(A + \frac{D_2}{D_1} + B_1' \right) \frac{1}{\sigma} + \left[\left(A + \frac{D_2}{D_1} \right) B_1' + B_1'^2 - A B_1'' \right] \frac{1}{\sigma^2} + \dots \right\} + \int \alpha \sigma d\sigma - \int \beta d\sigma + \text{const} \quad [61]$$

where

$$B_1' = \frac{dB_1}{d(\ln \ell^*)} \quad [62]$$

and

$$B_1'' = \frac{d^2 B_1}{d(\ln \ell^*)^2} \quad [63]$$

Then from Equations [60], [52], and [61]

$$R_x = D_1 \sigma^2 \eta \left\{ 1 - \frac{(2A + D_2/D_1)}{\sigma} + \frac{2A(A + D_2/D_1 + B_1')}{\sigma^2} - \frac{2A[(A + D_2/D_1)B_1' + B_1'^2 - AB_1'']}{\sigma^3} + \dots \right\} + \alpha\sigma^2 - \beta\sigma - 2 \int \alpha\sigma d\sigma + 2 \int \beta d\sigma + \text{constant} \quad [64]$$

The frictional-resistance coefficient of flat plates is then given implicitly in terms of σ by Equations [57], [52], and [64].

LOGARITHMIC RESISTANCE FORMULA

It is seen from Equation [64] that C_f is a function of R_x in terms of parameter σ . When σ is eliminated, logarithmic resistance formulas arise. The procedure is as follows: Equations [57] and [60] are combined to give, after neglecting the constant of integration,

$$\frac{1}{\sigma^2} = \frac{C_f}{2} \left(1 - \frac{2 \int R_\theta \sigma d\sigma}{\sigma^2 R_\theta} \right) \quad [65]$$

Substituting the expressions for R_θ and $\int R_\theta \sigma d\sigma$ from Equations [52] and [61] gives, with α and β neglected:

$$\frac{1}{\sigma^2} = \frac{C_f}{2} \left\{ 1 - \frac{2A}{\sigma} + \frac{2A(A + B_1')}{\sigma^2} - \frac{2A[A(B_1' - D_2/D_1) + B_1'^2 - AB_1'']}{\sigma^3} + \dots \right\} \quad [66]$$

Through reiteration σ is replaced by C_f within the brackets so that

$$\frac{1}{\sigma^2} = \frac{C_f}{2} \left[1 - 2A \left(\frac{C_f}{2} \right)^{1/2} + 2A \left(2A + B_1' \right) \left(\frac{C_f}{2} \right) + \dots \right] \quad [67]$$

and by the binomial theorem

$$\frac{1}{\sigma} = \sqrt{\frac{C_f}{2}} \left[1 - A \left(\frac{C_f}{2} \right)^{1/2} + A \left(\frac{3}{2} A + B_1' \right) \left(\frac{C_f}{2} \right) + \dots \right] \quad [68]$$

and by inversion

$$\sigma = \sqrt{\frac{2}{C_f}} \left[1 + A \left(\frac{C_f}{2} \right)^{1/2} - A \left(\frac{A}{2} + B_1' \right) \left(\frac{C_f}{2} \right) + \dots \right] \quad [69]$$

Now, after substituting for η from Equation [9] and ignoring α , β , and the constant of integration, R_x in Equation [64] is written in logarithmic form as:

$$\ln R_x = \frac{\sigma}{A} - \frac{B_1}{A} - \frac{B_2}{A} + \ln D_1 - \ln \frac{1}{\sigma^2} + \ln \left[1 - (2A + D_2/D_1) \frac{1}{\sigma} + \dots \right] \quad [70]$$

Substituting the appropriate expressions for $\frac{1}{\sigma^2}$, $\frac{1}{\sigma}$, and σ from Equations [67], [68], and [69] and expanding the logarithm as series results in

$$\ln R_x C_f = \frac{\sqrt{2}}{A} \sqrt{\frac{1}{C_f}} + \left(1 - \frac{B_1}{A} - \frac{B_2}{A} + \ln 2D_1 \right) - \frac{1}{\sqrt{2}} \left(\frac{A}{2} + \frac{D_2}{D_1} + B_1' \right) \sqrt{C_f} \quad [71]$$

wherein terms of higher order than $\sqrt{C_f}$ have been neglected. For the case of the pure solvent $B_1' = 0$ and $B_1 = B_{1,0}$, Equation [71] becomes

$$\ln R_x C_f = \frac{\sqrt{2}}{A} \sqrt{\frac{1}{C_f}} + \left(1 - \frac{B_{1,0}}{A} - \frac{B_2}{A} + \ln 2D_1 \right) - \frac{1}{\sqrt{2}} \left(\frac{A}{2} + \frac{D_2}{D_1} \right) \sqrt{C_f} \quad [72]$$

Equation [72] is reducible to the form of the well-known Kármán-Schoenherr formula for flat plates by linearizing C_f with respect to $\frac{1}{C_f}$ to absorb the last term.

LOCAL SKIN-FRICTION COEFFICIENTS AND SHAPE PARAMETERS

For the approximate calculation of turbulent boundary layers in pressure gradients such as on bodies there is still use for relations which derive from the flat-plate conditions such as the local skin friction and shape parameter as functions of momentum thickness. For drag-reducing solutions the local skin friction is expressed as

$$\frac{1}{\sigma^2} \equiv \frac{\tau_w}{\rho U^2} = f \left[R_\theta, \theta/\ell \right] \quad [73]$$

and the shape parameter H as

$$H = f \left[R_\theta, \theta/\ell \right] \quad [74]$$

From Equations [9] and [52] the local skin friction is expressed as

$$R_\theta = D_1 \left(1 - \frac{D_2}{D_1 \sigma} \right) \exp \left[\frac{1}{A} (\sigma - B_2 - B_1) \right] \quad [75]$$

where α and β have been neglected. Logarithmically, Equation [75] becomes

$$\ln R_\theta = \frac{1}{A} \sqrt{\frac{\rho U^2}{\tau_w}} - \frac{B_2}{A} - \frac{B_1}{A} + \ln D_1 - \frac{D_2}{D_1} \sqrt{\frac{\tau_w}{\rho U^2}} \quad [76]$$

wherein one term has been retained in the series expansion of $\ln \left(1 - \frac{D_2}{D_1 \sigma} \right)$.

This equation represents the general case with B_1 as a function of ℓ^* .

In the case of the shape parameter H, Equation [53] is simplified to

$$\frac{H}{H-1} = \frac{D_1}{D_2 \sigma} = \frac{D_1}{D_2} \sqrt{\frac{\rho U^2}{\tau_w}} \quad [77]$$

by dropping α and β .

COMBINED ROUGHNESS AND DRAG-REDUCING EFFECTS

The flow of a dilute polymer solution over a rough surface produces mutually antagonistic effects: the presence of the polymer molecules tends to reduce the wall shearing stress while the roughness of the wall tends to increase the wall shearing stress. The similarity laws may be applied to the combined situation by confining the effects to the inner law and leaving the outer law unaffected.

The inner law then includes all the pertinent parameters: (1) for roughness: k, k_1, k_2, \dots length parameters defining the roughness geometry and (2) for drag-reducing elements: $\rho_p, \ell, \ell_1, \ell_2, \dots, m, m_1, m_2, \dots, t, t_1, t_2, \dots$ as previously defined. Then

$$u = f \left[y, \tau_w, \mu_o, \rho_o, k, k_1, k_2, \dots, \rho_p, \ell, \ell_1, \ell_2, \dots, m, m_1, m_2, \dots, t, t_1, t_2, \dots \right] \quad [78]$$

and nondimensionally

$$\frac{u}{u_\tau} = f \left[y^*, \ell^*, C, P, \ell/k, k/k_1, k_1/k_2, \dots \right] \quad [79]$$

The outer law is considered unaffected so that overlapping produces the usual logarithmic relations with

$$B_1 = f \left[\ell^*, C, P, \ell/k, k/k_1, k_1/k_2, \dots \right] \quad [80]$$

For a specific uniform roughness and a specific concentration of a given polymer $B_1 = f [\ell^*]$. All the other relations then follow as before.

LINEAR LOGARITHMIC DRAG-REDUCING CHARACTERIZATION

PIPE FLOW

For the range of Reynolds numbers tested in pipe flow, Meyer⁶ found B_1 to be a linear logarithmic function of u_τ . By introducing the polymer parameter ℓ this can be generalized to a function of $\frac{u_\tau \ell}{\nu}$ or ℓ^* as required by the similarity laws as previously shown or

$$B_1 = B_{1,0} - A \ln \frac{v}{v_0} + q (\log \ell^* - \log \ell_0^*) , \ell^* > \ell_0^* \quad [81]$$

$$B_1 = B_{1,0} - A \ln \frac{v}{v_0} , \ell^* \leq \ell_0^*$$

where

$B_{1,0} = B_1$ for zero concentration or pure solvent

$\ell_0^* = \ell^*$ - value for $B_1 = B_{1,0} - A \ln \frac{v}{v_0}$, threshold value of ℓ^*

q = slope of logarithmic relation

and

$$q = f [C, P]$$

Therefore, q varies with concentration and type of polymer. The friction relation for pipe flow, Equation [39], can be further deduced by considering that by definition

$$\ell^* = \frac{u_\tau \ell}{v_0} = \sqrt{\frac{f}{2}} R_D \left(\frac{\ell}{D} \right) \quad [82]$$

Then Equation [39] becomes

$$\frac{1}{\sqrt{f}} = \left(a + \frac{q}{2} \right) \log \sqrt{f} R_D + B - \frac{q}{\sqrt{2}} \log \sqrt{2} \ell_0^* - \frac{q}{\sqrt{2}} \log \left(\frac{D}{\ell} \right) \quad [83]$$

$$- \frac{A_1}{\sqrt{2}} \log \frac{v}{v_0}$$

where $A_1 = 2.3026 A$. This result was substantially found by Meyer⁶ who pointed out that this explains the effect of diameter on frictional factor as given by the next to the last term on the right-hand side. Furthermore, it was also pointed out that the slope of plots of $\frac{1}{\sqrt{f}}$ against $\log \sqrt{f} R_D$ is altered by the factor q and not by changes in a or its equivalent von Kármán's constant κ .¹⁴

FLAT-PLATE FLOW

By definition for flat-plate flow

$$\ell^* = \frac{1}{\sigma} R_x \left(\frac{\ell}{x} \right) \quad [84]$$

Then the resistance formula for flat plates, Equation [71], becomes

$$\begin{aligned} \left(1 + \frac{q}{A_1}\right) \log R_x = & - \left(1 + \frac{q}{2A_1}\right) \log C_f + \frac{\sqrt{2}}{A_1} \sqrt{\frac{1}{C_f}} + \frac{1}{A_1} \left(A - B_{1,0} - B_2\right) \\ & + \log 2D_1 + \log \frac{v}{v_o} + \frac{q}{A_1} \log \sqrt{2} \ell_o^* - \frac{1}{2.3026 \sqrt{2}} \left(\frac{A}{2} + \frac{D_2}{D_1}\right) \sqrt{C_f} + \frac{q}{A_1} \log x/\ell \end{aligned} \quad [85]$$

For local skin friction by definition

$$\ell^* = \frac{1}{\sigma} R_\theta \left(\frac{\ell}{\theta} \right) \quad [86]$$

Then the local skin friction formula, Equation [76], becomes

$$\begin{aligned} \left(1 + \frac{q}{A_1}\right) \log R_\theta = & \frac{1}{A_1} \sqrt{\frac{\rho U^2}{\tau_w}} - \frac{(B_2 + B_{1,0})}{A_1} + \log \frac{v}{v_o} + \log D_1 \\ & + \frac{q}{A_1} \log \ell_o^* - \frac{D_2}{2.3026 D_1} \sqrt{\frac{\tau_w}{\rho U^2}} \\ & - \frac{q}{A_1} \log \sqrt{\frac{\tau_w}{\rho U^2}} + \frac{q}{A_1} \log \theta/\ell \end{aligned} \quad [87]$$

EXPERIMENTAL DATA

SIMILARITY LAWS OF ORDINARY FLUIDS

The measured values of A and B_1 for the inner law velocity profiles have had the usual discrepancies due to experimental errors. Pitot tubes are subject to wall and finite-size effects in shear flows. More serious have been the discrepancies between pipe flow and flat plate boundary-layer flows.¹¹ This has resulted in controversy concerning the calibration of Preston tubes for pipes and flat plates which implicitly depends on the values of A and B_1 . In an effort to resolve this difficulty, Patel¹⁵ made measurements of the velocity profiles of the inner law of pipe and flat-plate flows under very carefully controlled conditions and did find agreement on this score. The results are

$$A_1 = 5.5$$

$$A = \frac{5.5}{2.3026} = 2.39$$

and

$$B_1 = 5.45$$

This means that the data from proper pipe measurements can be applied to flat plates without reservation.

The velocity profiles for the outer law of pipe and flat-plate flow are, however, quite different as shown in Figure 2. An adjustment of the values of B_2 ¹¹ to conform to $A = 2.39$ gives

$$B_2 = 2.6 \text{ for flat plates}$$

and

$$B_2 = 0.9 \text{ for pipes}$$

The nonlogarithmic portion of the outer law may be tentatively fitted by

$$\frac{U - u}{u_\tau} = 9.87 (1 - y/\delta)^2 \text{ for flat plates} \quad [88]$$

and

$$\frac{U - u}{u_\tau} = 7.52 (1 - y/\delta)^2 \text{ for pipes} \quad [89]$$

$$\text{for } y/\delta \geq 0.15$$

FRICTIONAL RESISTANCE

In the case of pipes the generally accepted values are $a = 4.0$ and $b = -0.4$ for determining the Fanning friction factors. No doubt some adjustment would result from more careful measurements.

In the case of flat plates the Schoenherr formula gives acceptable values of frictional resistance for ordinary fluids

$$\log\left(\frac{U_x}{v} C_f\right) = \frac{0.242}{\sqrt{C_f}} \quad [90]$$

Equation [83] for drag reducing fluids then becomes

$$\begin{aligned}
\left(1 + \frac{q}{A_1}\right) \log R_x = & - \left(1 + \frac{q}{2A_1}\right) \log C_f \\
& + \frac{0.242}{\sqrt{C_f}} + \log \frac{v}{v_o} + \frac{q}{A_1} \log \sqrt{2} \ell_o^* \\
& + \frac{q}{A_1} \log \frac{x}{\ell}
\end{aligned} \tag{91}$$

GUAR GUM

The most extensive data available in friction-reducing properties of aqueous polymer solutions in pipes are those of Guar Gum, a natural polymer used in the food industry. Guar Gum requires much higher concentrations than Polyox for the same drag-reducing effects but has the advantage of being resistant to mechanical degradation. Elata et al.⁸ obtained values of q (α in their notation, $q = 2\alpha$) as a function of concentration, a maximum of $q = 47$ occurring at $C = 3000$ ppm. Up to 800 ppm, $q = 0.026C$. (ppm = parts per million by weight)

An examination of Elata's data gives a threshold value of drag reducing property of $\frac{u_\tau}{v_o} = 2.5 \times 10^3 \text{ in}^{-1}$ independent of concentration. The reciprocal of this value gives a threshold length, $\frac{v_o}{u_\tau} = 4.0 \times 10^{-4} \text{ in}$. If the characteristic length ℓ is taken as this threshold length,

$$\ell = 4.0 \times 10^{-4} \text{ in}$$

then

$$\ell_o^* = 1$$

The viscosity dependence of Guar Gum solutions on concentration is given by Rubin and Elata.¹⁶ Their data may be fitted to give

$$\frac{v}{v_o} - 1 = 5.25 \times 10^{-4} C^{1.157} \tag{92}$$

C in ppm.

The drag reduction due to Guar Gum on a flat plate is illustrated in Figure 4 for the 3000 ppm concentration. The effects of flat-plate length and threshold value of shearing stress are very pronounced.

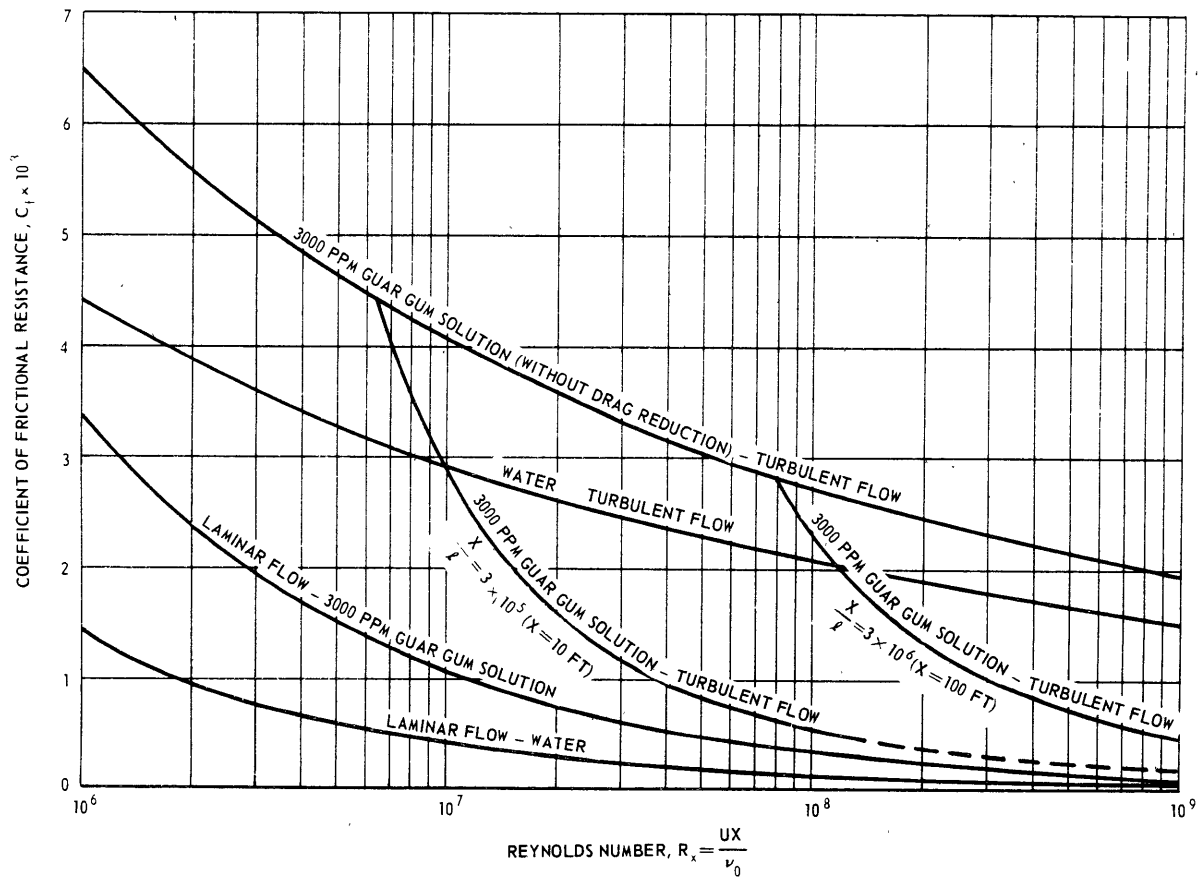


Figure 4 - Frictional Resistance of Flat Plate

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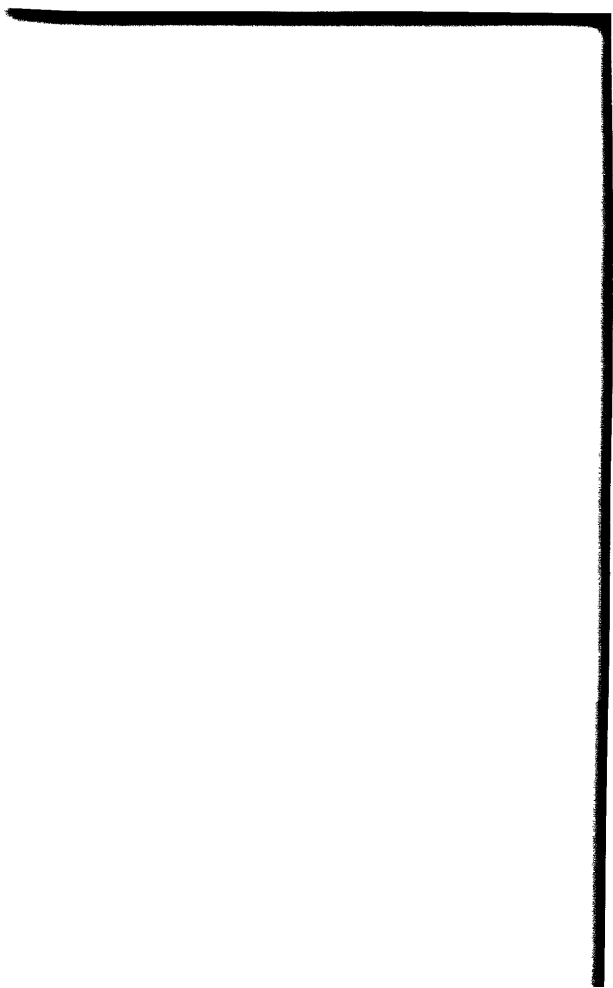
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