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SYSTEM OF SMALL-SIZE TRANSDUCERS AS ELEMENTAL UNIT IN SONAR SYSTEM

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Naval Ship Research and Development Center
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NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER
WASHINGTON, D. C. 20007**

**SYSTEM OF SMALL-SIZE TRANSDUCERS AS ELEMENTAL
UNIT IN SONAR SYSTEM**

by

G. Maidanik

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System of Small-Size Transducers as Elemental Unit in Sonar System

G. MAIDANIK

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Can a transducer system consisting of a few small-size transducers replace an extended transducer as an elemental unit in a sonar system? This question is analyzed in some detail, assuming that the external pressure field to which the transducer system of a sonar system is subjected is generated by an incident plane acoustic pressure field and a subsonic turbulent boundary layer. The analysis is conducted, by and large, in spectral space, $\{\mathbf{k}, \omega\}$ space. The conceptual advantages of utilizing spectral space for the analysis of the response of transducer systems are exemplified and discussed.

INTRODUCTION

THE utilization of spatially extended flush-mounted pressure transducers to increase the "signal-to-noise (S/N) ratio" of a receiving array system is well known.¹ The signal is the response associated with the boundary pressure field that is induced by an incident plane acoustic pressure field. The noise is the response associated with the pressure field in a subsonic turbulent boundary layer. Since practical situations may arise where spatially extended transducers cannot be conveniently used, it was proposed that a system of small-size transducers might provide an alternative, the parameters of the transducer system to be adjusted to produce the same effect as the replaced spatially extended transducer.² The criterion was that the small-size transducers must be placed within the spatial boundaries of the replaced transducer. Jorgensen and Maidanik recently explored this proposition and found it impractical for sonar systems.² The analysis showed that this alternative system is effective only when rather small spatial extents are involved. However, the analysis may prove useful in considering the response of extended transducers with nonuniform spatial sensitivities and in providing an elementary exercise pertaining to the analysis of sonar systems.

In Ref. 2, the analysis was conducted in $\{\mathbf{x}, \omega\}$ space. In this paper, by and large, the same problem is analyzed; however, the analysis here is primarily con-

ducted and illustrated in $\{\mathbf{k}, \omega\}$ space, the Fourier conjugate of the real physical $\{\mathbf{x}, t\}$ space. It is argued that defining the filtering properties of the transducer systems in $\{\mathbf{k}, \omega\}$ space offers certain conceptual advantages. This is particularly relevant when one considers the response of the transducer system to pressure fields that are stationary, both spatially and temporally. The variations in the filtering properties of these systems as functions of the sizes and the sensitivities of the individual transducers and the separations between the centers of the transducers are discussed. The conditions that are required to maximize the S/N ratio of the transducer system are of particular concern.

I. DESCRIPTION OF THE TRANSDUCER SYSTEM

The transducer system considered in this paper consists of an infinite plane boundary in which a number of electrically interconnected transducers are flush mounted. The surface formed by the boundary and the surfaces of the transducers is termed the baffle. The baffle faces a semi-infinite space occupied by a fluid medium. The fluid medium is characterized by a density ρ and a speed of sound c . The fluid medium moves relative to the baffle with a free stream velocity U_∞ ; this flow generates a turbulent boundary layer that, in turn, induces a pressure field on the baffle. In addition, the baffle is subjected to a pressure field that is induced by an incident plane acoustic pressure field.

A typical transducer system and the relevant features of the external pressure fields are sketched in Fig. 1. The coordinate system employed in this paper is also shown on this Figure.

¹ G. M. Corcos, "Resolution of Pressure in Turbulence," J. Acoust. Soc. Am. 35, 192-199 (1963).

² D. W. Jorgensen and G. Maidanik, "Response of a System of Point Transducers to Turbulent Boundary Layer Pressure Field," J. Acoust. Soc. Am. 43, 1390-1394 (1968).

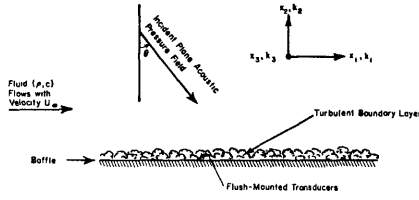


FIG. 1. Typical transducer system, sources of external pressure field and coordinate system.

II. FILTERING ACTION OF THE TRANSDUCER SYSTEM

The impulse response function (the Green function) $g(\mathbf{y}-\mathbf{x}, \tau+t)$ for a flush-mounted transducer system is defined^{1,3,4}

$$g(\mathbf{y}-\mathbf{x}, \tau+t) = \sum_i g_i(\mathbf{y}+\mathbf{y}_i-\mathbf{x}, \tau+\tau_i+t), \quad (1)$$

where g_i is the impulse response function of the i th transducer; \mathbf{y} and τ define the spatial and temporal centers of the transducer system; \mathbf{y}_i is the spatial position vector of the center of the i th transducer relative to \mathbf{y} , and τ_i is the time delay of the i th transducer relative to τ ; \mathbf{x} is the spatial position vector variable in the plane of the baffle; and t is the temporal variable. In Eq. 1, it is assumed that the impulse response function of the i th transducer is stationary, both spatially and temporally. A class of transducer systems for which this assumption is valid are those possessing baffles that exhibit surface admittances that are spatially uniform.

In terms of the impulse response function g , the output $p_m(\mathbf{y}, \tau)$ of the transducer system to an arbitrary external pressure field $p_e(\mathbf{x}, t)$ is⁵

$$p_m(\mathbf{y}, \tau) = \int \int d\mathbf{x} \int_{-\infty}^{\infty} dt g(\mathbf{y}-\mathbf{x}, \tau+t) p_e(\mathbf{x}, t). \quad (2)$$

The external pressure field is defined as that pressure field that would be generated by external sources when the baffle is made rigid.

One is usually interested in the response S of the transducer system; the response is defined as the mean-square value of the output of the transducer system, $S = \langle p_m^2(\mathbf{y}, \tau) \rangle$, where the angular brackets $\langle \rangle$ indicate the appropriate statistical averaging process. From Eq. 2, one obtains

$$S = \int \int d\mathbf{x} \int_{-\infty}^{\infty} dt \int \int d\mathbf{x}' \int_{-\infty}^{\infty} dt' g(\mathbf{y}-\mathbf{x}, \tau+t) \times g(\mathbf{y}-\mathbf{x}', \tau+t') \langle p_e(\mathbf{x}, t) p_e(\mathbf{x}', t') \rangle. \quad (3)$$

³ G. Maidanik and D. W. Jorgensen, "Boundary Wave-Vector Filters for the Study of the Pressure Field in a Turbulent Boundary Layer," *J. Acoust. Soc. Am.* **42**, 494-501 (1967).

⁴ G. Maidanik, "Flush-Mounted Pressure Transducer Systems as Spatial and Spectral Filters," *J. Acoust. Soc. Am.* **42**, 1017-1024 (1967).

⁵ P. M. Morse and H. Feshbach, *Method of Theoretical Physics* (McGraw-Hill Book Co., New York, 1953), Chaps. 4 and 8.

The angular bracket in the integrand spans the external pressure field only, since the transducer system is considered to possess no statistical properties; the impulse response function is fixed.

If the external pressure field is stationary, both spatially and temporally, Eq. 3 can be readily reduced^{1,3,4}

$$S = \int \int d\mathbf{x} \int_{-\infty}^{\infty} dt w(\mathbf{x}, t) r(\mathbf{x}, t), \quad (4)$$

where

$$r(\mathbf{x}, t) = \langle p_e(\mathbf{x}'+\mathbf{x}, t'+t) p_e(\mathbf{x}', t') \rangle, \quad (5)$$

$$w(\mathbf{x}, t) = \int \int d\mathbf{x}' \int_{-\infty}^{\infty} dt' g(\mathbf{y}-\mathbf{x}-\mathbf{x}', \tau+t+t') \times g(\mathbf{y}-\mathbf{x}', \tau+t'). \quad (6)$$

(The variables \mathbf{x} and t , when appropriate, define a spatial and a temporal separation between two positions on the baffles and two instants of time, respectively—e.g., in Eqs. 5 and 6.)

The function $w(\mathbf{x}, t)$ is termed the spatial-temporal filtering operator of the transducer system. This operator is defined only if the response is sought to external pressure fields that are stationary, both spatially and temporally. Except for this requirement on the form of the pressure field, the operator is solely a function of the parameters of the transducer system. Indeed, a knowledge of the spatial-temporal filtering operator is all one needs to know about the transducer system. To determine the response of the transducer system, one also requires a knowledge of the external pressure field. However, the spatial-temporal filtering operator and the external pressure field are mutually exclusive quantities and can be studied separately. In this Section, the properties of the transducer system are analyzed; the properties of the external pressure field are defined in a subsequent Section. The arbitrary external pressure field defined in Eq. 2 is employed merely as a test pressure field to assist in the analytical definition of the transducer system.

Equation 4 can be readily transformed from the temporal to the frequency domain⁵

$$\tilde{S}(\omega) = \int \int d\mathbf{x} \tilde{w}(\mathbf{x}, \omega) \tilde{r}(\mathbf{x}, \omega), \quad (7)$$

$$S = (2\pi)^{-1} \int_{-\infty}^{\infty} d\omega \tilde{S}(\omega), \quad (8)$$

where

$$\tilde{w}(\mathbf{x}, \omega) = (2\pi)^{-1} \int_{-\infty}^{\infty} dt w(\mathbf{x}, t) \exp(i\omega t), \quad (9)$$

$$\tilde{r}(\mathbf{x}, \omega) = (2\pi)^{-1} \int_{-\infty}^{\infty} dt r(\mathbf{x}, t) \exp(i\omega t), \quad (10)$$

the frequency variable ω is the Fourier conjugate pair of the temporal variable t , $\tilde{S}(\omega)$ is the frequency spectral density of the response, $\tilde{r}(\mathbf{x}, \omega)$ is the cross-frequency spectral density of the external pressure field, and $\tilde{w}(\mathbf{x}, \omega)$ is termed the spatial-frequency filtering operator of the transducer system. The spatial-frequency filtering operator in component form is

$$\begin{aligned} \tilde{w}(\mathbf{x}, \omega) = \sum_i \sum_j \int \int d\mathbf{x}' \tilde{g}_i(\mathbf{y} + \mathbf{y}_i - \mathbf{x} + \mathbf{x}', \omega) \\ \times \tilde{g}_j^*(\mathbf{y} + \mathbf{y}_j - \mathbf{x}', \omega) \exp[-i\omega(\tau_i - \tau_j)], \end{aligned} \quad (11)$$

where

$$\begin{aligned} \tilde{g}_i(\mathbf{y} + \mathbf{y}_i - \mathbf{x}, \omega) \\ = (2\pi)^{-1} \int_{-\infty}^{\infty} g_i(\mathbf{y} + \mathbf{y}_i - \mathbf{x}, t) \exp(i\omega t). \end{aligned} \quad (12)$$

Utilizing the frequency domain to analyze the response of the transducer system to external pressure fields is advantageous in that it affords a means of examining the nature of the response in greater detail. Also, the temporal dependence of the spatial-temporal filtering operator, Eq. 6, which is given in terms of a convolution integral, becomes algebraic in the frequency domain, Eq. 11, and the time delays between transducers appear as simple phase factors, Eq. 11. The behavior of the frequency spectral component $\tilde{S}(\omega)$ of the response S is inherently less complex to decipher than the response itself, and, therefore, more complex situations can be analyzed without loss of physical insight. In fact, each component $\tilde{S}(\omega)$ of the response can be examined separately; the use of frequency-band filters to augment the frequency filtering action of a responding system is a common procedure in present-day analysis. To account for the frequency filter, one need only multiply each term on the right-hand side of Eq. 11 by the filtering action $D(\omega, \omega_0, \Delta)$ of the frequency-band filter, where ω_0 is the center frequency of the band and Δ is its half-bandwidth.³

Examination of the response equation, Eq. 4, reveals similarities between the components of the spatial variables and the temporal variables. Since a Fourier transformation, pertaining to the temporal variables, proved advantageous in dealing with the temporal operations, it is quite natural to assume that a transformation into the Fourier conjugate domain, pertaining to the spatial variables, would prove advantageous in dealing with the spatial operations. Following a procedure similar to that involved with respect to the

transformation from the temporal to the frequency domain, one derives

$$\Phi_m(\mathbf{k}, \omega) = W(\mathbf{k}, \omega) \Phi_e(\mathbf{k}, \omega), \quad (13)$$

$$\tilde{S}(\omega) = (2\pi)^{-1} \int \int d\mathbf{k} \Phi_m(\mathbf{k}, \omega), \quad (14)$$

$$S = (2\pi)^{-1} \int \int d\mathbf{k} \int_{-\infty}^{\infty} d\omega \Phi_m(\mathbf{k}, \omega), \quad (15)$$

where

$$W(\mathbf{k}, \omega) = (2\pi)^{-1} \int \int d\mathbf{x} \tilde{w}(\mathbf{x}, \omega) \exp(-i\mathbf{k} \cdot \mathbf{x}), \quad (16)$$

$$\Phi_e(\mathbf{k}, \omega) = (2\pi)^{-1} \int \int d\mathbf{x} \tilde{r}(\mathbf{x}, \omega) \exp(-i\mathbf{k} \cdot \mathbf{x}), \quad (17)$$

the wave-vector variable \mathbf{k} is the Fourier conjugate pair of the spatial variable \mathbf{x} , $\Phi_m(\mathbf{k}, \omega)$ is the spectral density of the response, $\Phi_e(\mathbf{k}, \omega)$ is the spectral density of the external pressure field, and $W(\mathbf{k}, \omega)$ is termed the filtering action of the transducer system. The filtering action in component form is

$$\begin{aligned} W(\mathbf{k}, \omega) = \sum_i \sum_j G_i(\mathbf{k}, \omega) G_j^*(\mathbf{k}, \omega) \\ \times \exp[-i\mathbf{k} \cdot (\mathbf{y}_i - \mathbf{y}_j) - i\omega(\tau_i - \tau_j)], \end{aligned} \quad (18)$$

where

$$G_i(\mathbf{k}, \omega) = (2\pi)^{-1} \int \int d\mathbf{x} \tilde{g}_i(\mathbf{x}, \omega) \exp(-i\mathbf{k} \cdot \mathbf{x}). \quad (19)$$

Employing the wave-vector domain in the analysis of the transducer system has advantages similar to those enumerated with respect to the frequency domain. The exception is the auxiliary frequency filter; auxiliary wave-vector filters are not commonly available. If one desires a strong filtering action in the wave-vector domain—rejection of most wave-vector components and acceptance of chosen few—one must construct the transducer system itself to possess this wave-vector filtering action.^{3,4}

A point of interest is the relationship of the spatial separations between transducers and the corresponding time delays. It is observed in Eq. 18 that a spatial separation can be interpreted as a time delay and vice versa.⁴

Before proceeding to analyze specific situations, it is convenient to pause at this stage and to relate the elements of the present formulism with those preceding it.

III. RELATIONSHIP TO PREVIOUS ANALYSIS

Previous analyses by Maidanik and Jorgensen^{2-4,6} assumed that the spatial and temporal dependence of the impulse response function are separable

$$g(\mathbf{y}-\mathbf{x}, \tau+t)=\mathbf{h}(\mathbf{y}-\mathbf{x}) \cdot \mathbf{f}(\tau+t), \quad (20)$$

where

$$\mathbf{h}(\mathbf{y}-\mathbf{x})=\{h_i(\mathbf{y}+\mathbf{y}_i-\mathbf{x})\}, \quad (21)$$

$$\mathbf{f}(\tau+t)=\{f_i(\tau+\tau_i+t)\}. \quad (22)$$

In Eq. 20, $\mathbf{h}(\mathbf{y}-\mathbf{x})$ is the spatial sensitivity vector of the transducer system and $\mathbf{f}(\tau+t)$ is the temporal sensitivity vector of the transducer system. The spatial sensitivity function h_i , Eq. 21, of the i th transducer is assumed to be spatially stationary. The temporal sensitivity function f_i , Eq. 22, of the i th transducer is assumed to be temporally stationary. Strictly, the analysis presented in this Section is applicable to a class of baffles that are characterized by point-reacting admittances. A specific baffle that falls within this class is a rigid baffle. Indeed, previous analyses by Jorgensen and Maidanik assumed rigid baffles.²⁻⁴ Those transducer systems that do not fall within this class require special treatment. Nevertheless, the analysis, using Eq. 20, may serve as a first step to gain some physical insight into the working of those transducer systems that show a more complex behavior than is implied by Eq. 20.

Under the assumption of a rigid baffle, the spatial-temporal filtering operator of the transducer system, employing Eq. 22, is given by

$$w(\mathbf{x}, t)=\sum_i \sum_j s_i s_j m_{ij}(\mathbf{x}) n_{ij}(t), \quad (23)$$

where

$$n_{ij}(t)=\int_{-\infty}^{\infty} dt' f_i(\tau+\tau_i+t+t') f_j(\tau+\tau_j+t'), \quad (24)$$

$$s_i s_j m_{ij}(\mathbf{x})=\int_{-\infty}^{\infty} d\mathbf{x}' h_i(\mathbf{y}+\mathbf{y}_i-\mathbf{x}-\mathbf{x}') h_j(\mathbf{y}+\mathbf{y}_j-\mathbf{x}'), \quad (25)$$

s_i is the sensitivity of the i th transducer. This parameter is independent of the spatial and the temporal variables. The parameter s_i may assume a complex value.⁷

The spatial-frequency filtering operator of the transducer system, employing Eq. 20, is given by

$$\tilde{w}(\mathbf{x}, \omega)=\sum_i \sum_j s_i s_j m_{ij}(\mathbf{x}) N_{ij}(\omega) \exp[-i\omega(\tau_i-\tau_j)], \quad (26)$$

where

$$N_{ij}(\omega)=F_i(\omega) F_j^*(\omega), \quad (27)$$

⁶ G. Maidanik, "A Domed Sonar System," J. Acoust. Soc. Am. 44, 113-124 (1968).

⁷ N. Brown suggested, in a private conversation, that complex sensitivities could be utilized to generate phasings similar to those generated by time delays. The phasings generated by complex sensitivities are not frequency dependent and may, therefore, be more convenient to implement than time delays in a computer program that is designed to produce given filtering actions at various frequencies.

$$F_i(\omega)=(2\pi)^{-1} \int_{-\infty}^{\infty} dt f_i(t) \exp(i\omega t) \quad (28)$$

(cf. Eqs. 11 and 12).

Jorgensen and Maidanik employed Eq. 26 to analyze the S/N ratio of a singlet, a single transducer, a doublet, and triplet transducer system.² In this study, the authors imposed $N_{ij}(\omega) \equiv 1$ and $\tau_i = \tau_j \equiv 0$. The form of $\tilde{w}(\mathbf{x}, \omega)$ under these prescribed conditions was presented in Ref. 2 for the various transducer systems just mentioned.

The filtering action of the transducer system, employing Eq. 20, is given by

$$W(\mathbf{k}, \omega)=\sum_i \sum_j s_i s_j M_{ij}(\mathbf{k}) N_{ij}(\omega) \times \exp[-i\mathbf{k} \cdot (\mathbf{y}_i-\mathbf{y}_j)-i\omega(\tau_i-\tau_j)], \quad (29)$$

where

$$s_i s_j M_{ij}(\mathbf{k})=H_i(\mathbf{k}) H_j^*(\mathbf{k}), \quad (30)$$

$$H_i(\mathbf{k})=(2\pi)^{-1} \int_{-\infty}^{\infty} d\mathbf{x} h_i(\mathbf{x}) \exp(-i\mathbf{k} \cdot \mathbf{x}). \quad (31)$$

To relax the condition of rigid baffles, the function $M_{ij}(\mathbf{k})$ must be multiplied by $|(1+R_b)/2|^2$, where R_b is the spectral reflection coefficient of the baffle.⁶ (For a rigid baffle, $R_b \equiv 1$.) The factor $|(1+R_b)/2|^2$ merely accounts for the change in the boundary pressure on the baffle as its rigidity is relaxed.⁶

Some workers^{1,8,9} employed analyses that differ in detail from those presented by the author in the present paper and in Refs. 2-4. These analyses do not specify the sensitivity functions of the transducers in response to an external pressure field as defined in Eq. 2, but rather, they specify the sensitivity functions of the transducers in response to the actual pressure field on the baffle. These analyses essentially define an impulse response function in the form

$$g(\mathbf{y}-\mathbf{x}, \tau+t)=\mathbf{K}(\mathbf{y}-\mathbf{x}) \cdot \mathbf{e}(\tau+t), \quad (32)$$

where

$$\mathbf{K}(\mathbf{y}-\mathbf{x})=\{K_i(\mathbf{y}+\mathbf{y}_i-\mathbf{x})\}; \quad \mathbf{e}(\tau+t)=\{\delta_i(\tau+\tau_i+t)\}, \quad (33)$$

K_i is the spatial sensitivity function of the i th transducer, and δ_i is the Dirac delta function representing the temporal sensitivity function of the i th transducer. The output $p_m(\mathbf{y}, \tau)$ of the set of flush-mounted transducers to the actual pressure field $p(\mathbf{x}, t)$ on the baffle is given by an equation like Eq. 2 except that $p(\mathbf{x}, t)$ replaces $p_e(\mathbf{x}, t)$. Analyses utilizing this approach need not impose the assumption that the baffle possesses uniform and

⁸ K. L. Chandiramani, "Interpretation of Wall Pressure Measurements Under a Turbulent Boundary Layer," Bolt Beranek and Newman Rept. No. 1310, Contract No. Nonr 2321(00) (Aug. 1965).

⁹ P. H. White, "Effect of Transducer Size, Shape, and Surface Sensitivity on the Measurement of Boundary-Layer Pressures," J. Acoust. Soc. Am. 41, 1358-1363, 1967.

point reacting surface admittance to validate the stationarity and separability expressed in Eqs. 32 and 33. The complexity that is associated with a baffle that exhibits surface admittance that varies spatially is transferred to the definition of the pressure field rather than the impulse-response function. The difficulty is to ascertain that the actual pressure field is indeed stationary, both spatially and temporally, because now this pressure field may depend on the characteristics of the baffle, in particular its surface admittance variabilities. If a specific external pressure field is not modified by the baffle, except in the manner specified in the paragraph following Eq. 31, and this external pressure field is stationary, both spatially and temporally, the difficulty is suppressed. In this sense, the baffle may be considered to exhibit uniform surface admittance. Then the two formulisms are identical; however, in the author's formulism the condition that the nature of the spatial-temporal filtering operator (or the filtering action) is not contingent on the external pressure field must be relaxed.

A practical situation relevant to these discussions arises in the case of a "pinhole" transducer that is designed to respond to the pressure field in a turbulent boundary layer. The pinhole is assumed bored in an otherwise flat rigid boundary. Obviously, there exists a surface discontinuity in the admittance of the baffle. However, experimentalists assume that this surface discontinuity in the admittance does not modify the pressure field of a turbulent boundary layer that is generated on the baffle.¹⁰ If they are correct in their assumption, the surface admittance discontinuity may be neglected and the baffle may be considered uniform in this experiment.

IV. PROPERTIES OF THE EXTERNAL PRESSURE FIELD

To estimate the S/N ratio of a transducer system, the external pressure field must be specified in terms of the part that contributes to the signal and the part that contributes to the noise in the response. The signal is induced by an external pressure field that arises as a consequence of an incident plane acoustic pressure field, and the baffle is blocked to present a rigid surface. The spectral density of this external pressure field is denoted by $\Phi_a(\mathbf{k}, \omega)$. The incident plane acoustic pressure field is assumed to be pure tone of frequency ω_a . The plane of incidence makes an angle θ with a plane normal to the baffle, and the induced pressure field on the baffle is assumed convected in the positive x_1 direction. It can be readily shown that the spectral density $\Phi_a(\mathbf{k}, \omega)$ has the form⁶

$$\Phi_a(\mathbf{k}, \omega) = (2\pi)^{\frac{1}{2}} \phi_a(\omega_a) \delta(\omega_a c_a^{-1} - k_1) \delta(k_3) \delta(\omega - \omega_a), \quad (34)$$

¹⁰ F. E. Gieb, Jr., "Measurements on the Effect of Transducer Size on the Resolution of Boundary-Layer Pressure Fluctuations" (to be published).

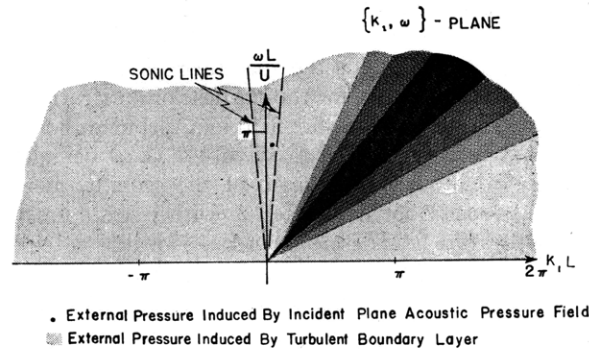


FIG. 2. Spectral density of external pressure field on normalized $\{k_1, \omega\}$ plane (normalizing parameters are length L and convection velocity U).

where $c_a = c/\sin\theta$, c_a being the convection velocity of this part of the external pressure field. It is clear that the convection velocity c_a associated with the spectral density $\Phi_a(\mathbf{k}, \omega)$ is invariably supersonic. The quantity $\phi_a(\omega_a)$ is the amplitude of the acoustic pressure field on the baffle, taking account of the pressure doubling at the rigidized baffle.

The pressure field described in Eq. 34 is illustrated on the normalized $\{k_1, \omega\}$ plane in Fig. 2. The illustration is confined to the first and second quadrants because, for purposes of the analysis presented in this paper, the first quadrant is identical to the third and the second to the fourth (the sensitivities s_i are kept real in this paper).⁴

The noise is assumed to be induced by the pressure field in a subsonic turbulent boundary layer, $c \gg U_\infty$. This turbulent boundary layer is developed on the baffle; the baffle in this case is assumed rigid. It is further assumed that the structure of the turbulent boundary layer does not change when the rigidity of the baffle is relaxed. The spectral density of the external pressure field in the turbulent boundary layer is denoted by $\Phi_t(\mathbf{k}, \omega)$ (Refs. 3 and 4)

$$\Phi_t(\mathbf{k}, \omega) = \Phi_{t1}(k_1, \omega) \Phi_{t3}(k_3, \omega) \Phi_{t0}(\omega - k_1 U). \quad (35)$$

It is assumed in Eq. 35 that the spectral density $\Phi_t(\mathbf{k}, \omega)$ is separable and that Φ_{t1} and Φ_{t3} are symmetric and reasonably smooth functions of ω , k_1 and k_3 . It is further assumed that the convection velocity U of the pressure field in the turbulent boundary layer has the same direction chosen for the convection velocity c_a , namely, the positive x_1 direction. The function $\Phi_{t0}(\omega - k_1 U)$ peaks substantially in the range where $\omega \simeq k_1 U$.^{3,4,6} Since the convection velocity U ($U < U_\infty$) is subsonic, this peak in Φ_{t0} implies that the spectral components of the pressure field in a subsonic turbulent boundary layer are highly concentrated in the subsonic region of spectral space, this region is defined by $\omega \simeq k_1 U$. Equivalently stated, the predominant part of the spectral components of the pressure field in a subsonic turbulent boundary layer have phase velocities that approach or equal the convection velocity U .⁴ At the present time, however, the detail distribution of the spectral components of the

pressure field in a turbulent boundary layer is not known. This is particularly true of those spectral components that lie outside the spectral region defined by $\omega \simeq k_1 U$. Nevertheless, for the purpose of illustration, it is assumed that the pressure field in a turbulent boundary layer is as given in Eq. 35 with $\Phi_i(\mathbf{k}, \omega)$ being the appropriate Fourier transform of the cross-frequency spectral density defined in Ref. 2 (with γ set to unity).

The pressure field described in Eq. 35 is illustrated on the normalized $\{k_1, \omega\}$ plane in Fig. 2. Again, only the first and second quadrants need be shown because of the symmetry.

The assumption is imposed that the external pressure field generated by the incident plane acoustic pressure field and the turbulent boundary layer are uncorrelated, so that the spectral density of the external pressure field is

$$\Phi_e(\mathbf{k}, \omega) = \Phi_a(\mathbf{k}, \omega) + \Phi_t(\mathbf{k}, \omega). \quad (36)$$

As stated previously, the filtering action of the transducer system is not contingent on the external pressure field except for the requirement that this pressure field be stationary, both spatially and temporally. The chosen external pressure field is utilized here to demonstrate the method of analysis and is not intended to represent an exact and an actual practical situation. In a practical situation, the sources that generate the external pressure field that contribute to the noise may not always be exclusively associated with the pressure field in a turbulent boundary layer.

V. CONDITIONS FOR MAXIMIZING THE S/N RATIO

From Eqs. 13, 15, 34, and 35, the signal S_a is given by

$$S_a = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\mathbf{k} \int_{-\infty}^{\infty} d\omega W(\mathbf{k}, \omega) \Phi_a(\mathbf{k}, \omega) \quad (37)$$

and the noise S_t is given by

$$S_t = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\mathbf{k} \int_{-\infty}^{\infty} d\omega W(\mathbf{k}, \omega) \Phi_t(\mathbf{k}, \omega). \quad (38)$$

The S/N ratio is then simply S_a/S_t .

One way to maximize the S/N ratio is to adjust the parameters of the filtering action of the transducer system so that the ratio S_a/S_t attains its maximum value. Another method of maximizing this ratio is to reduce the external pressure that contributes to the noise, especially in that part of spectral space where, owing to the signal requirements, the filtering action has unavoidably high values. This latter method is, however, outside the scope of this paper; here the external pressure field that induces the noise is assumed specified, and one, therefore, must resort to the former method.

In practice, it is desirable to keep the value of the signal S_a high to avoid having it masked by other noises—e.g., electrical. Thus, one seeks to maximize the ratio S_a/S_t with the auxiliary condition that S_a attains close to its highest possible value. In view of the simplicity assumed for the spectral density $\Phi_a(\mathbf{k}, \omega)$, the signal S_a can be readily evaluated:

$$S_a = W(\omega_a c_a^{-1}, 0, \omega_a) \phi_a(\omega_a). \quad (39)$$

Thus, to ensure that S_a attains high value, one must choose the parameters of the transducer system so that the filtering action has a high value for $W(\omega_a c_a^{-1}, 0, \omega_a)$. This choice represents the auxiliary condition just discussed.

It is usual to incorporate a frequency-band filter in the output of the transducer system. The way to account for the filtering action of the frequency-band filter has already been explained in Sec. II. It is assumed in the subsequent discussions that a narrow-frequency-band filter is incorporated in the transducer system.

The evaluation of the noise S_t is more complex than the evaluations just performed for the signal, Eq. 39. It is not intended in this paper to present extensive computations, the results of such computations would be dependent on the assumed external pressure field and, therefore, would be rather restrictive. Instead, the analysis is illustrated for a few simple situations so as to provide a generalized physical insight as to the rôle played by the transducer system as a device for enhancing the S/N ratio. As in Ref. 2, the analysis is illustrated for the cases of a "line transducer," a doublet, and a triplet transducer system. All the transducers are assumed to have rectangular shapes, equal widths, and uniform spatial sensitivities.¹¹ The transducer systems are assumed aligned in the x_1 direction. The total spatial extents of the transducer systems in this direction are assumed equal, this total spatial extent is denoted by L (see Figs. 3-5).

Assuming that Eq. 29 is valid for the transducer systems under consideration, one readily obtains the filtering actions for these transducer systems. To simplify the illustrations, it is assumed that the transducer systems are incorporated with nominally identical frequency band filters. It is further assumed that all the transducers have equal frequency filtering actions; this filtering action is denoted by $N(\omega)$. The combined frequency filtering action of each transducer system is thus designated by $N(\omega)D(\omega, \omega_0, \Delta)$. The wave-vector filtering action of a transducer system is designated by $A_a(k_1, \omega\tau)B(k_3)$. The separability is possible because of

¹¹ A line transducer is defined here as a rectangular transducer having a finite length. A doublet transducer system consists of two identical and similarly oriented rectangular transducers separated by a distance that substantially exceeds their individual lengths. (In the strict sense, the transducers are of vanishing lengths.) The transducers are aligned so that a line joining their centers is perpendicular to their widths. A triplet transducer system consists of three collinear transducers, two successive transducers form a doublet.

the rectangular shapes of the transducers.³ The subscript α is written for l , d , and t denoting, respectively, the line transducer, the doublet, and the triplet transducer system. The absence of a subscript in $B(k_3)$ is possible because the transducers are of equal widths. The filtering actions of the transducer systems can thus be collectively written

$$W_\alpha(k, \omega) = [N(\omega)D(\omega, \omega_0, \Delta)][A_\alpha(k_1, \omega\tau)][B(k_3)]. \quad (40)$$

For the line transducer,

$$A_l(k_1, \omega\tau) = s_l^2 \left[\frac{\sin(k_1 L/2)}{(k_1 L/2)} \right]^2, \quad (41)$$

where s_l is the sensitivity of the transducer.

For the doublet transducer system,

$$A_d(k_1, \omega\tau) = 4s_d^2 \cos^2[(k_1 L + \omega\tau)/2]; \quad s_d = s_1 = s_2, \quad (42a)$$

$$\sin^2[k_1 L + \omega\tau/2]; \quad s_d = s_1 = -s_2, \quad (42b)$$

where s_1 and s_2 are the sensitivities of the two transducers, and τ is the time delay between them.

For the triplet transducer system

$$\{s_2 + 2s_0 \cos[(k_1 L + \omega\tau)/2]\}^2; \quad s_0 = s_1 = s_3, \quad (43a)$$

$$A_t(k_1, \omega\tau) = s_2^2 + 2s_0^2 \sin^2[(k_1 L + \omega\tau)/2]; \quad s_0 = s_1 = -s_3, \quad (43b)$$

where s_1 , s_2 , and s_3 are the sensitivities of the transducers in order (see Fig. 5). The time delay τ is the time delay between the first and third transducers. The time delays between successive transducers are assumed equal.

The filtering action $W_l(k, \omega)$ of the line transducer, utilizing Eq. 41, is illustrated on the normalized $\{k_1, \omega\}$ plane in Fig. 3. The frequency filtering action $[N(\omega)D(\omega, \omega_0, \Delta)]$ is plotted on a displaced frequency axis, and the wavenumber filtering action $A_l(k_1, \omega\tau)$ is

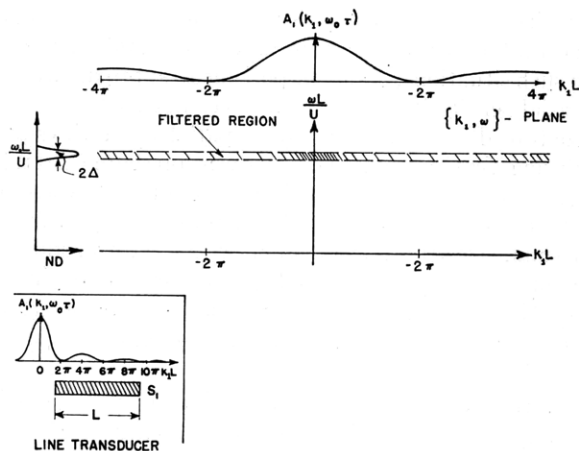


FIG. 3. Filtering action of a line transducer on normalized $\{k_1, \omega\}$ plane (normalizing parameters are length L and convection velocity U .)

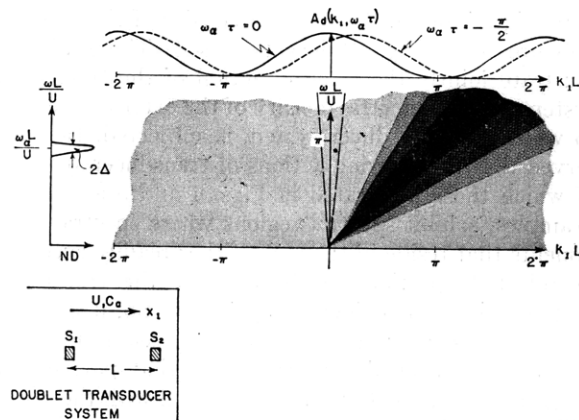


FIG. 4. Filtering action of a doublet transducer system and external pressure field on normalized $\{k_1, \omega\}$ plane (normalizing parameters are length L and convection velocity U .)

plotted on a displaced wavenumber k_1 axis. The nature of the filtering action on the $\{k_1, \omega\}$ plane can then be deduced from Eq. 40. Only two quadrants of this plane are shown, because the filtering action in the first quadrant is identical to that in the third, and the filtering action in the second quadrant is identical to that in the fourth. Similar illustrations can be given for the filtering action in any plane of the $\{k, \omega\}$ space. Three-dimensional illustrations can also be constructed for the filtering action. The filtering action $W_d(k, \omega)$ of the doublet transducer system, utilizing Eq. 42a, is analogously illustrated on the normalized $\{k_1, \omega\}$ plane in Fig. 4. The effect of time delay between the transducers [phasing] on the filtering action of the doublet transducer system is also illustrated in Fig. 4. The filtering action $W_t(k, \omega)$ of the triplet transducer system, utilizing Eq. 43a, is illustrated similarly in Fig. 5. The effect of adjusting the sensitivities of the transducers (shading) on the filtering action of the triplet transducer system is also illustrated in Fig. 5.

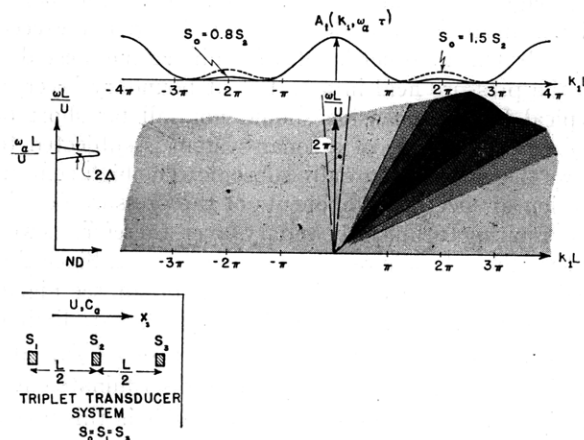


FIG. 5. Filtering action of a triplet transducer system and external pressure field on $\{k_1, \omega\}$ plane (normalizing parameters are length L and convection velocity U .)

In Figs. 4 and 5, the spectral density of the external pressure field (Fig. 2) is superposed. These Figures illustrate jointly the filtering action of the transducer system and the spectral density of the external pressure to which the transducer system is subjected. It is observed that the filtering actions of transducer systems, of which those illustrated in Figs. 4 and 5 are simple examples, exhibit spectral regions where spectral components that reside in them are either readily accepted or rejected (partially or totally) by the transducer system. Thus, to achieve a high S/N ratio, it is necessary to adjust the parameters of the transducer system so that the spectral components of that part of the external pressure field that is associated with the signal will be readily accepted and that the spectral components of that part of the external pressure field that may induce noise will be substantially rejected by the transducer system. The degree to which this can be achieved with a given transducer system determines the magnitude of the S/N ratio that can be achieved with this transducer system. The illustrations in Figs. 4 and 5 pertain to situations where the parameters of the transducer systems are adjusted so as to result in approximately maximum values for the signal to noise ratios with the proper auxiliary condition on the signal S_a .

Figures 2-5 are not intended to represent exact forms of the filtering action and the spectral density of the external pressure field. Rather, they just emphasize those salient features of the filtering action and the external pressure field that are necessary to demonstrate the interaction between the two quantities. Such rough Figures are useful when one seeks physical insight into this problem and when one desires to examine methods of approximations in computing the S/N ratio.

VI. ADDITIONAL COMMENTS

1. From Figs. 2 and 3, it is deduced that, on the one hand, to ensure the full supersonic range without loss in the magnitude of the signal, a typical linear dimension L of a transducer in a sonar system should not exceed $2c/\omega_a$.^{1,6} On the other hand, to minimize the noise due to the pressure field in a turbulent boundary layer, a typical linear dimension should not fall far short of $2c/\omega_a$.^{1,6} The baffle of a sonar system is subjected to flows that are substantially subsonic. The high concentration of spectral components of the pressure field in the resulting turbulent boundary layer, therefore, is well within the subsonic region of spectral space. Since this region of high concentration is rather broad, see Fig. 2, the doublet and triplet transducer systems are limited to small total extents, of the order of $\pi U/\omega_a$ and $2\pi U/\omega_a$, respectively, if they are to discriminate effectively against the components that reside in this region (see Figs. 4 and 5). These extents are generally impractical for use in sonar systems. Moreover, these extents are dependent on the convection velocity U and

would, therefore, require adjustment as the speed of the sonar system through the fluid is varied. The adjustment can be made by using time delays (see Fig. 4). However, because of the auxiliary condition that S_a be maintained high, the range of time delays that can be employed is rather limited. The conclusion is then that a doublet and a triplet transducer system cannot be made to replace effectively an appropriately chosen transducer in a sonar system.

2. If the transducers in the doublet and the triplet transducer systems are changed so as to provide them with length b , small as compared with L ($b \ll L$) but nevertheless finite, the effect would be to reduce the filtering actions of these transducer systems by the factor $\sin^2(k_1 b/2)/(k_1 b/2)^2$ (see Eq. 41). This factor is effective only at the higher wavenumber range, $k_1 b \geq 1$, and, therefore, would not substantially change the conclusion stated in 1.

3. In the preceding Section, the line transducer was assumed to have uniform spatial sensitivity. The spatial sensitivity function for this transducer is

$$h(y-x) = \frac{2\pi s_l}{LL_3} U\left(\frac{L}{2} - |x_1|\right) U\left(\frac{L_3}{2} - |x_3|\right), \quad (44)$$

where

$$U(a) = 1, \quad a > 0 \\ = 0, \quad a < 0, \quad (45)$$

the center of the transducer is assumed coincident with its geometrical center; L_3 is the width of the transducer, and s_l is its sensitivity. From Eq. 30, one readily derives

$$H(\mathbf{k}) = s_l \frac{\sin(k_1 L/2)}{(k_1 L/2)} \cdot \frac{\sin(k_3 L_3/2)}{(k_3 L_3/2)}. \quad (46)$$

It is apparent that the analysis of a transducer having a uniform spatial sensitivity is quite simple to handle. However, situations may arise where the spatial nonuniformity of the sensitivity of the transducer is complex; and its Fourier transform is difficult to obtain directly. In such situations, it may be possible to approximate the spatial sensitivity of the transducer by a set of small-size transducers having uniform spatial sensitivities but varying sensitivities.² The analysis presented in this paper can be employed to obtain the filtering action of this transducer. Equation 29 indicates that a proper set of small-size transducers is one where the transducers are of nominally equal sizes because then the $M_{ij}(\mathbf{k})$ in Eq. 29 are all equal. White proposed a similar method for analyzing a transducer of nonuniform spatial sensitivity.⁹

4. It was stated, in connection with Eq. 3, that one is usually interested in the response S . There are situations, however, where one may be interested in the autocorrelation $R(0, \tau_0)$ of the output. This quantity can be readily obtained from the spectral density $\tilde{S}(\omega)$, Eq.

7, of the response

$$R(0, \tau_0) = (2\pi)^{-1} \int_{-\infty}^{\infty} d\omega \tilde{S}(\omega) \exp(i\omega\tau_0), \quad (47)$$

where τ_0 is the time delay between the two outputs.

One may, in certain situations, be interested in the correlation of the outputs of two nominally identical transducer systems whose centers are spatially separated by \mathbf{y}_0 . This type of correlation is termed the cross correlation and can be expressed as

$$R(\mathbf{y}_0, \tau_0) = (2\pi)^{-1} \int_{-\infty}^{\infty} d\mathbf{k} \int_{-\infty}^{\infty} d\omega \Phi_m(\mathbf{k}, \omega) \times \exp(i\mathbf{k} \cdot \mathbf{y}_0 + i\omega\tau_0), \quad (48)$$

where $\Phi_m(\mathbf{k}, \omega)$ is given in Eq. 13.

5. Although the analysis has been exemplified for transducer systems embodying a small number of transducers, it is clear that the analysis is also applicable to transducer systems consisting of a large number of flush-mounted transducers. Indeed, the analysis is applicable to the entire transducer system in a planar array sonar system, and not just to the elemental unit of such a system.⁶ It is also applicable to the analysis of a device that employs a large number of small flush-mounted transducers to explore the pressure field in a turbulent boundary layer.^{3,4} The larger number of transducers is employed to construct a more refined wave-vector filtering action than can be achieved with the few considered in this paper.

6. Frequency filters have been employed extensively for the purpose of enhancing S/N ratios in many physical situations. Frequency filters have also been employed extensively to decipher the nature of the temporal behavior of physical systems. Analysts have, therefore, no conceptual difficulty in visualizing the

filtering action of these frequency filters. In this paper, the spatial behavior of the system was transformed into its Fourier conjugate domain. The analogy of this transformation with the transformation from the temporal domain to the frequency domain was demonstrated, and the wave-vector filtering action was analogously defined. The result of the analysis in this paper demands that one think in terms of a filter having three orthogonal dimensions (e.g., see Eq. 40). Provided that one could extend one's visualization to absorb this increase in dimensionality of the filtering actions, no further amplification of the conceptual advantages of performing the analysis in the $\{\mathbf{k}, \omega\}$ space would be required.

7. Finally, in practice, one can rarely achieve the idealization imposed on the baffle in order to validate the analysis presented in this paper. The modifications in the analysis that are required to relax some of this idealization must await further research; different types of spatial nonuniformity of the surface admittance of the baffle may admit different modifications. As stated in the text, the modifications required may be dependent on the external pressure field to which the baffle is subjected. Until these modifications are studied in some detail, it is difficult to estimate the deviations from the idealization that can be tolerated without affecting the response of the system as expressed in this paper.

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13 ABSTRACT Can a transducer system consisting of a few small-size transducers replace an extended transducer as an elemental unit in a sonar system? This question is analyzed in some detail, assuming that the external pressure field to which the transducer system of a sonar system is subjected is generated by an incident plane acoustic pressure field and a subsonic turbulent boundary layer. The analysis is conducted, by and large, in spectral space (k, ω) space. The conceptual advantages of utilizing spectral space for the analysis of the response of transducer systems are exemplified and discussed.		

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